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In [9]: import math
import pandas as pd
import matplotlib.pyplot as plt
from matplotlib.patches import Ellipse
from itertools import combinations
from collections import OrderedDict
import os

def load_train_test_data(base_path):
    def load_data(folder_path):
        df_list = []
        for entry in os.listdir(folder_path):
            class_label = entry.split('_')[0]
            file_path = os.path.join(folder_path, entry)
            temp_df = pd.read_csv(file_path, sep=' ', names=['Feature1', 'Feature2'])
            temp_df['Class'] = class_label
            df_list.append(temp_df)
        return pd.concat(df_list, ignore_index=True)

    train_path = os.path.join(base_path, "train")
    test_path = os.path.join(base_path, "test")

    train_df = load_data(train_path)
    test_df = load_data(test_path)

    return train_df, test_df

# -----
# Matrix & small-linalg helpers (2x2)
# -----
def det2x2(M):
    return M[0][0]*M[1][1] - M[0][1]*M[1][0]

def inv2x2(M):
    d = det2x2(M)
    if abs(d) < 1e-12:
        eps = 1e-6
        M = [[M[0][0] + eps, M[0][1]], [M[1][0], M[1][1] + eps]]
        d = det2x2(M)
        if abs(d) < 1e-12:
            raise ValueError("Matrix not invertible")
    return [[ M[1][1]/d, -M[0][1]/d],
            [-M[1][0]/d, M[0][0]/d]]

def matvec(M, v):
    return [M[0][0]*v[0] + M[0][1]*v[1],
            M[1][0]*v[0] + M[1][1]*v[1]]

def dot(u, v):
    return u[0]*v[0] + u[1]*v[1]

def eig_2x2_symmetric(M):
    # M = [[a,b],[b,c]]
    a = M[0][0]; b = M[0][1]; c = M[1][1]
    tr = a + c
    disc = tr*tr - 4*(a*c - b*b)
    disc = max(disc, 0.0)
    sqrt_disc = math.sqrt(disc)
    l1 = (tr + sqrt_disc) / 2.0
    l2 = (tr - sqrt_disc) / 2.0
    def vec_for(l):
        if abs(b) > 1e-12:
            v = [-b, a - l]
            norm = math.sqrt(v[0]*v[0] + v[1]*v[1])
            return [v[0]/norm, v[1]/norm] if norm>0 else [1.0,0.0]
        else:
            # diagonal matrix
            if abs(a - l) < abs(c - l):
                return [1.0, 0.0]
            else:
                return [0.0, 1.0]
    v1 = vec_for(l1)
    v2 = vec_for(l2)
    return (l1, v1), (l2, v2)

# -----
# Statistics from training set
# -----
def compute_class_means_and_priors(train_df):
    class_means = {}
    priors = {}
    total = len(train_df)
    for cls in train_df['Class'].unique():
        subset = train_df[train_df['Class'] == cls]
        f1 = subset['Feature1'].tolist()
        f2 = subset['Feature2'].tolist()
        mean1 = sum(f1)/len(f1)
        mean2 = sum(f2)/len(f2)
        class_means[cls] = [mean1, mean2]

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        priors[cls] = len(subset)/total
    return class_means, priors

def covariance_matrix(df):
    x = df['Feature1'].tolist()
    y = df['Feature2'].tolist()
    n = len(x)
    mx = sum(x)/n
    my = sum(y)/n
    varx = sum((xi-mx)**2 for xi in x)/(n)
    vary = sum((yi-my)**2 for yi in y)/(n)
    covxy = sum((xi-mx)*(yi-my) for xi,yi in zip(x,y))/(n)
    return [[varx, covxy],[covxy, vary]]

def compute_class_covariances(train_df):
    covs = OrderedDict()
    for cls in train_df['Class'].unique():
        subset = train_df[train_df['Class'] == cls]
        covs[cls] = covariance_matrix(subset)
    return covs

# -----
# Generic Discriminant & Predict (Gaussian Bayes form)
# -----
def discriminant_score(x, mu, Sigma, prior):
    Sigma_inv = inv2x2(Sigma)
    detS = det2x2(Sigma)
    if detS <= 0:
        # tiny ridge
        eps = 1e-6
        Sigma = [[Sigma[0][0]+eps, Sigma[0][1]], [Sigma[1][0], Sigma[1][1]+eps]]
        Sigma_inv = inv2x2(Sigma)
        detS = det2x2(Sigma)
    diff = [x[0]-mu[0], x[1]-mu[1]]
    qf = dot(diff, matvec(Sigma_inv, diff))
    return -0.5 * qf - 0.5 * math.log(detS) + math.log(prior)

def predict_with_params(test_df, means, priors, covs, shared=False, diagonal=False):
    preds = []
    for _, row in test_df.iterrows():
        x = [row['Feature1'], row['Feature2']]
        scores = {}
        for cls in means:
            mu = means[cls]
            Sigma = covs["shared"] if shared else covs[cls]
            if diagonal:
                Sigma = [[Sigma[0][0], 0.0], [0.0, Sigma[1][1]]]
            scores[cls] = discriminant_score(x, mu, Sigma, priors[cls])
        preds.append(max(scores, key=scores.get))
    return preds

# -----
# Classifier wrappers
# -----
def classifier_sigma2I(train_df, test_df):
    means, priors = compute_class_means_and_priors(train_df)
    covs = compute_class_covariances(train_df)
    # average of variances
    allvars = []
    for C in covs.values():
        allvars.append(C[0][0]); allvars.append(C[1][1])
    sigma2 = sum(allvars)/len(allvars)
    cov_shared = [[sigma2, 0.0], [0.0, sigma2]]
    covs_shared = {"shared": cov_shared}
    return predict_with_params(test_df, means, priors, covs_shared, shared=True)

def classifier_shared_full(train_df, test_df):
    means, priors = compute_class_means_and_priors(train_df)
    covs = compute_class_covariances(train_df)
    n = len(covs)
    s = [[0.0, 0.0], [0.0, 0.0]]
    for C in covs.values():
        s[0][0] += C[0][0]; s[0][1] += C[0][1]
        s[1][0] += C[1][0]; s[1][1] += C[1][1]
    shared = [[s[i][j]/n for j in range(2)] for i in range(2)]
    return predict_with_params(test_df, *compute_class_means_and_priors(train_df), {"shared": shared}, shared=True)

def classifier_diag_per_class(train_df, test_df):
    means, priors = compute_class_means_and_priors(train_df)
    covs = compute_class_covariances(train_df)
    return predict_with_params(test_df, means, priors, covs, diagonal=True)

def classifier_full_per_class(train_df, test_df):
    means, priors = compute_class_means_and_priors(train_df)
    covs = compute_class_covariances(train_df)
    return predict_with_params(test_df, means, priors, covs)

# -----
# Evaluation utilities
# -----
def confusion_matrix(true_labels, pred_labels):
    labels = sorted(set(true_labels) | set(pred_labels))
    cm = {l: {k:0 for k in labels} for l in labels}

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for t,p in zip(true_labels, pred_labels):
    cm[t][p] += 1
return labels, cm

def classification_report_print(true_labels, pred_labels):
    labels, cm = confusion_matrix(true_labels, pred_labels)
    total = len(true_labels)
    print("\n=== Confusion Matrix ===")
    hdr = "\t" + "\t".join(labels)
    print(hdr)
    for l in labels:
        row = [str(cm[l][p]) for p in labels]
        print(l + "\t" + "\t".join(row))
    # metrics
    print("\n=== Classification Report ===")
    print("Class\tPrecision\tRecall\tF1-score\tSupport")
    precisions = []
    recalls = []
    f1s = []
    supports = []
    for l in labels:
        tp = cm[l][l]
        fp = sum(cm[other][l] for other in labels if other!=l)
        fn = sum(cm[l][other] for other in labels if other!=l)
        support = tp + fn
        prec = tp/(tp+fp) if (tp+fp)>0 else 0.0
        rec = tp/(tp+fn) if (tp+fn)>0 else 0.0
        f1 = (2*prec*rec/(prec+rec)) if (prec+rec)>0 else 0.0
        print(f"{l}\t{prec:.4f}\t{rec:.4f}\t{f1:.4f}\t{support}")
        precisions.append(prec); recalls.append(rec); f1s.append(f1); supports.append(support)
    mean_prec = sum(precisions)/len(precisions) if precisions else 0
    mean_rec = sum(recalls)/len(recalls) if recalls else 0
    mean_f1 = sum(f1s)/len(f1s) if f1s else 0
    acc = sum(cm[l][l] for l in labels)/total if total>0 else 0
    print(f"\nAccuracy: {acc:.4f}")
    print(f"Mean Precision: {mean_prec:.4f}")
    print(f"Mean Recall : {mean_rec:.4f}")
    print(f"Mean F1 Score : {mean_f1:.4f}")
    return labels, cm

# pretty heatmap of confusion matrix
def plot_confusion_matrix_heatmap(labels, cm, title="Confusion Matrix"):
    # create 2D numeric grid
    grid = [[cm[row][col] for col in labels] for row in labels]
    fig, ax = plt.subplots(figsize=(6,5))
    im = ax.imshow(grid, interpolation='nearest', cmap=plt.cm.Blues)
    ax.set_xticks(range(len(labels))); ax.set_yticks(range(len(labels)))
    ax.set_xticklabels(labels); ax.set_yticklabels(labels)
    plt.colorbar(im, ax=ax)
    ax.set_xlabel("Predicted"); ax.set_ylabel("True")
    ax.set_title(title)
    # annotate
    for i in range(len(labels)):
        for j in range(len(labels)):
            ax.text(j, i, str(grid[i][j]), ha="center", va="center", color="black")
    plt.tight_layout()
    plt.show()

# -----
# Plot: training scatter (start)
# -----
def plot_training_scatter(train_df, title="Training Data Scatter"):
    plt.figure(figsize=(8,6))
    markers = ['o','s','^','x','D','P','*']
    labels = train_df['Class'].unique()
    for i, cls in enumerate(labels):
        sub = train_df[train_df['Class']==cls]
        plt.scatter(sub['Feature1'], sub['Feature2'], marker=markers[i%len(markers)], label=cls, edgecolor='k', alpha=0.8)
    plt.xlabel("Feature1"); plt.ylabel("Feature2"); plt.title(title); plt.legend()
    plt.grid(True)
    plt.show()

# -----
# Plot constant-density Gaussian contours (1σ,2σ,3σ)
# -----
def plot_density_contours(train_df, class_means, class_covs, title="Constant Density Contours (1σ,2σ,3σ)":
    plt.figure(figsize=(8,6))
    colors = ["red","blue","green","purple","orange","brown","cyan"]
    labels = list(class_means.keys())
    for idx, cls in enumerate(labels):
        sub = train_df[train_df['Class']==cls]
        plt.scatter(sub['Feature1'], sub['Feature2'], label=cls, alpha=0.6, marker='o', edgecolor='k')
        mu = class_means[cls]
        cov = class_covs[cls]
        (l1,v1),(l2,v2) = eig_2x2_symmetric(cov)
        l1 = max(l1, 1e-6); l2 = max(l2, 1e-6)
        angle = math.degrees(math.atan2(v1[1], v1[0]))
        for scale in [1,2,3]:
            width = 2*scale*math.sqrt(l1)
            height = 2*scale*math.sqrt(l2)
            e = Ellipse((mu[0], mu[1]), width, height, angle=angle, edgecolor=colors[idx%len(colors)], facecolor='none', linewidth=1)
            plt.gca().add_patch(e)
    plt.xlabel("Feature1"); plt.ylabel("Feature2"); plt.title(title); plt.legend()

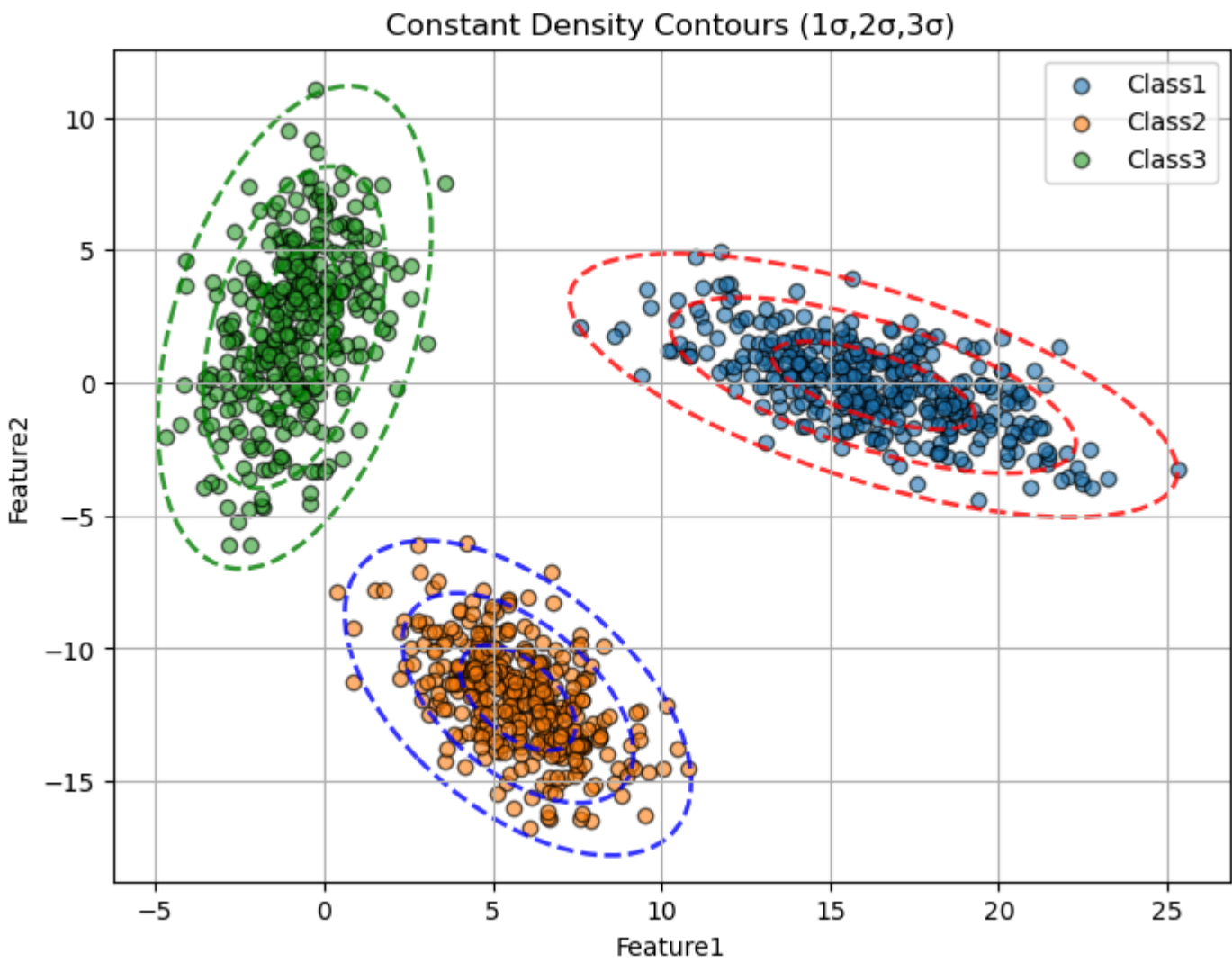
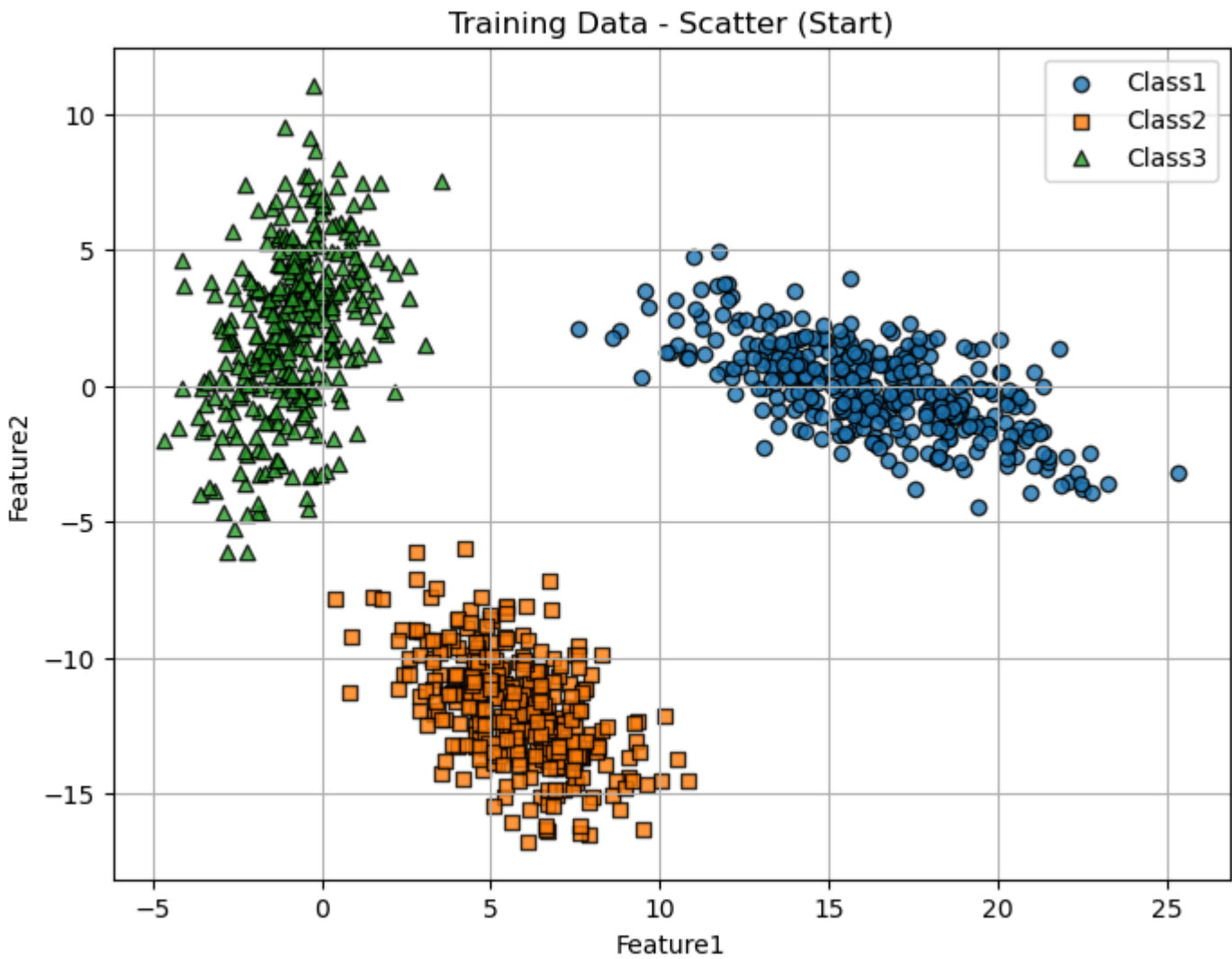
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localhost:8889/nbconvert/html/Untitled3 - Copy.ipynb?download=false

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for C in class_covs.values():
    s[0][0]+=C[0][0]; s[0][1]+=C[0][1]; s[1][0]+=C[1][0]; s[1][1]+=C[1][1]
n = len(class_covs)
covs_for_plot = {"shared": [[s[0][0]/n, s[0][1]/n],[s[1][0]/n, s[1][1]/n]]}
plot_decision_regions(train_df, means, priors, covs_for_plot, shared=flags["shared"], diagonal=flags["diagonal"], tit
```

```
In [10]: base_path = "../../Dataset/Group04/LS_Group04/"
# base_path = "../../Dataset/Group04/NLS_Group04/"
# base_path = "../../Dataset/Group04/rd_group4/"

train_df, test_df = load_train_test_data(base_path)
run_all(train_df, test_df, res=200)
```



=====
Classifier: sigma2I

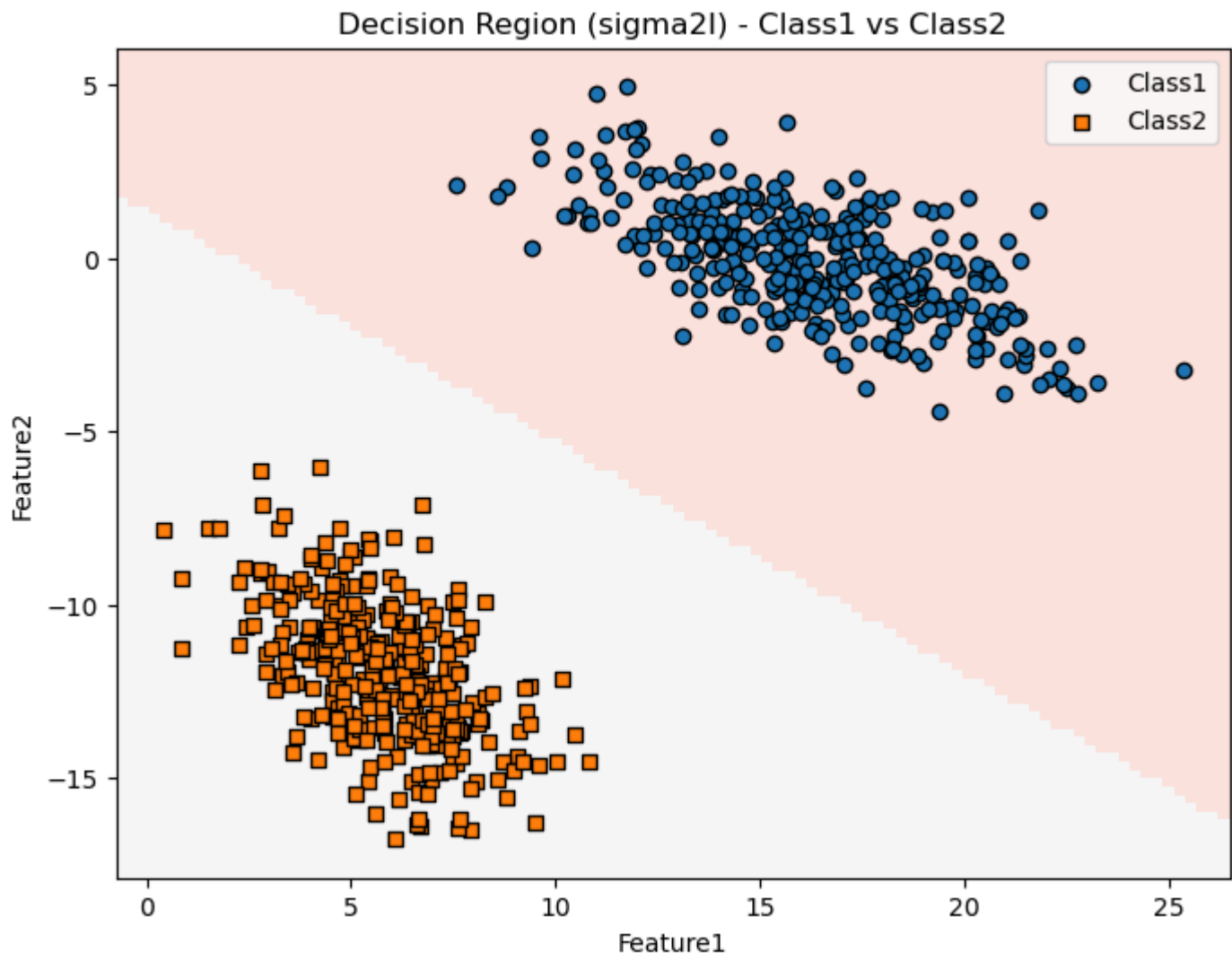
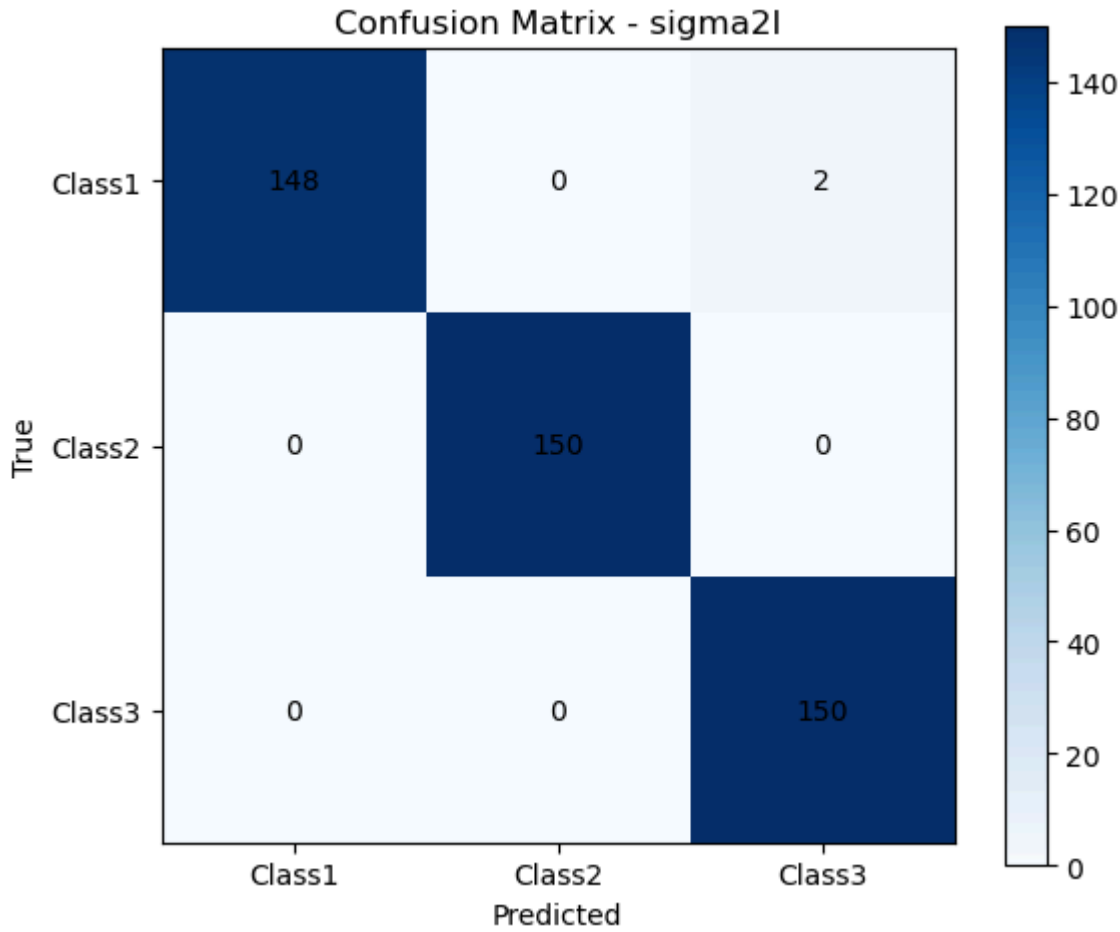
=== Confusion Matrix ===

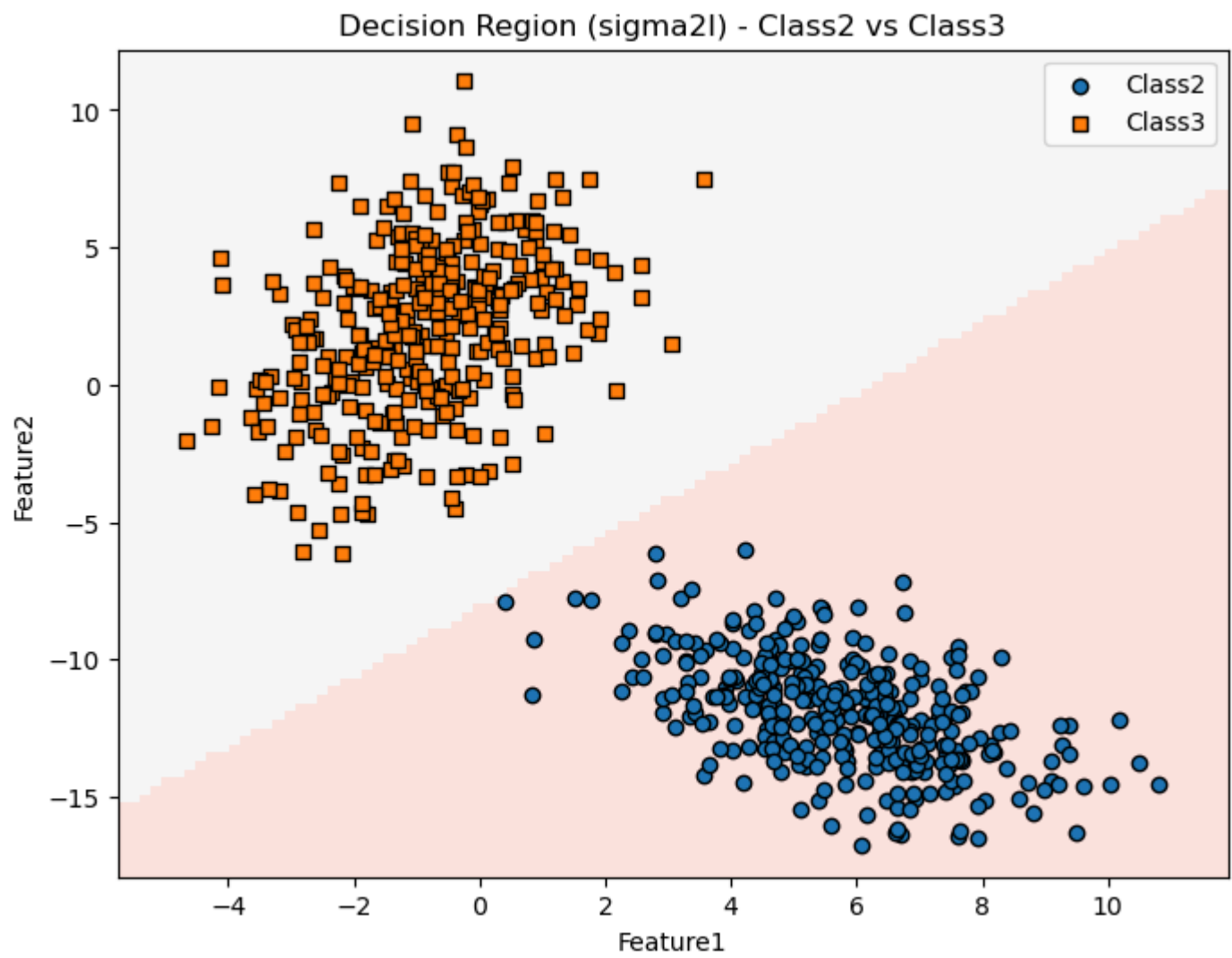
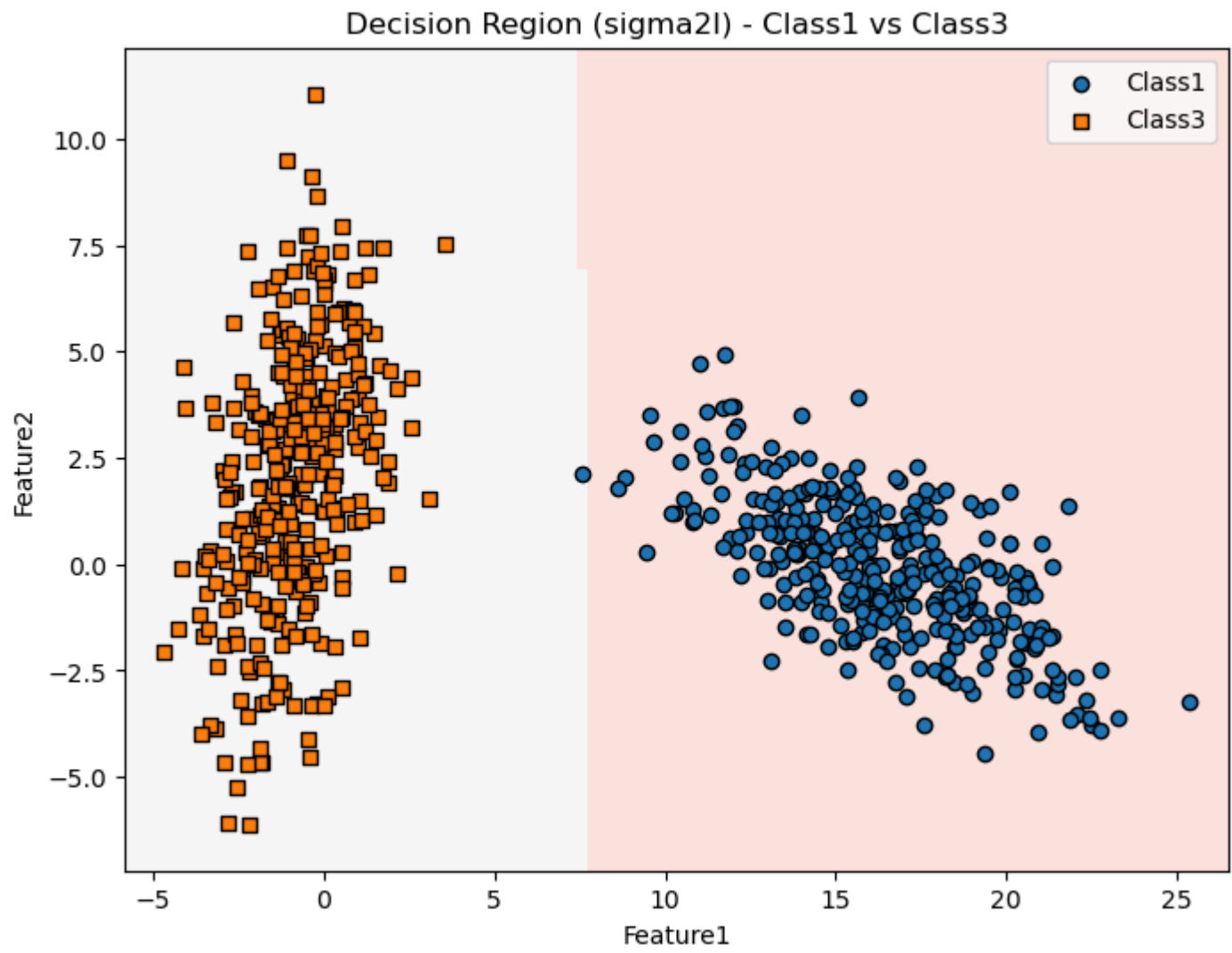
| | Class1 | Class2 | Class3 |
|--------|--------|--------|--------|
| Class1 | 148 | 0 | 2 |
| Class2 | 0 | 150 | 0 |
| Class3 | 0 | 0 | 150 |

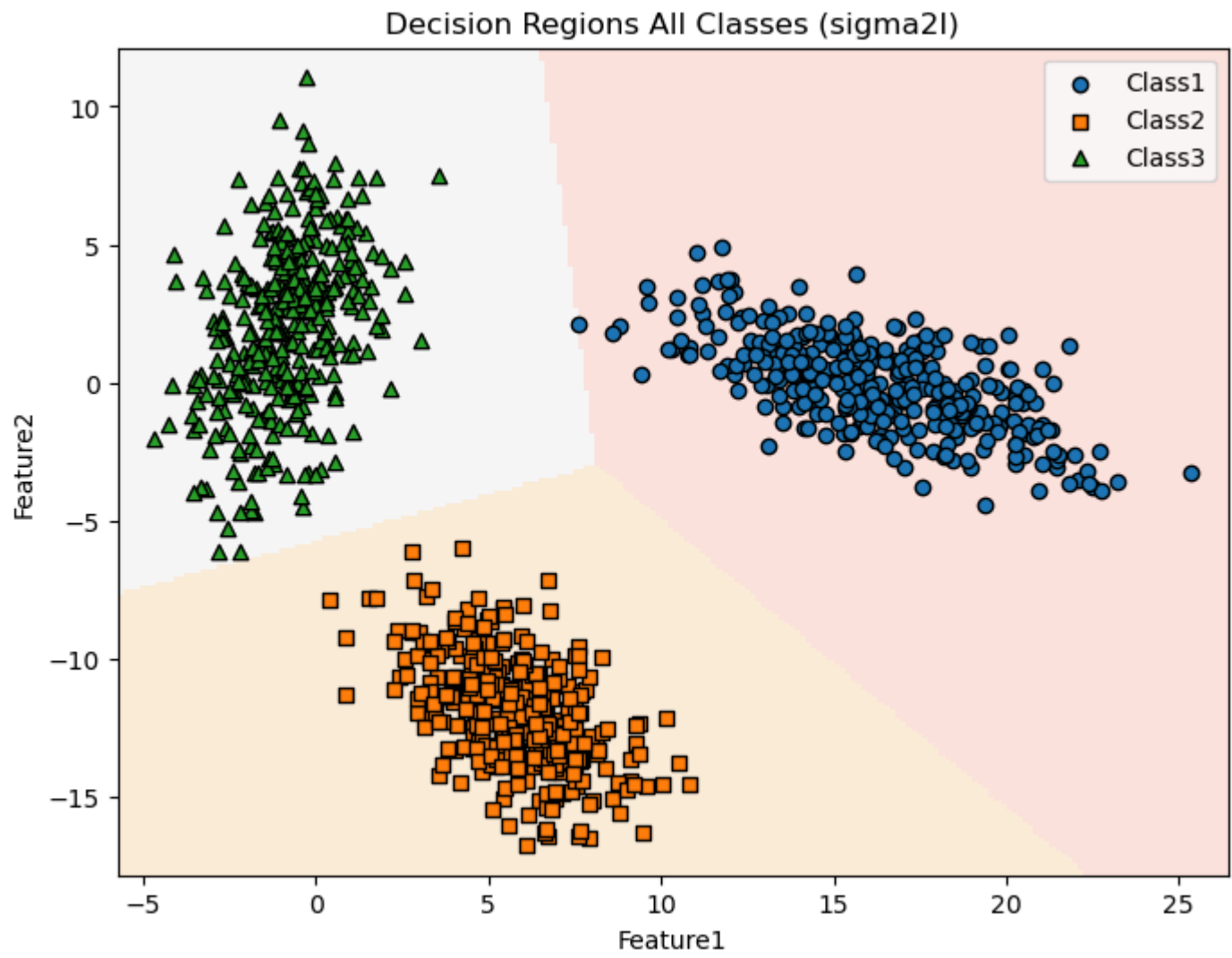
=== Classification Report ===

| Class | Precision | Recall | F1-score | Support |
|--------|-----------|--------|----------|---------|
| Class1 | 1.0000 | 0.9867 | 0.9933 | 150 |
| Class2 | 1.0000 | 1.0000 | 1.0000 | 150 |
| Class3 | 0.9868 | 1.0000 | 0.9934 | 150 |

Accuracy: 0.9956
Mean Precision: 0.9956
Mean Recall : 0.9956
Mean F1 Score : 0.9956







=====
Classifier: shared_full

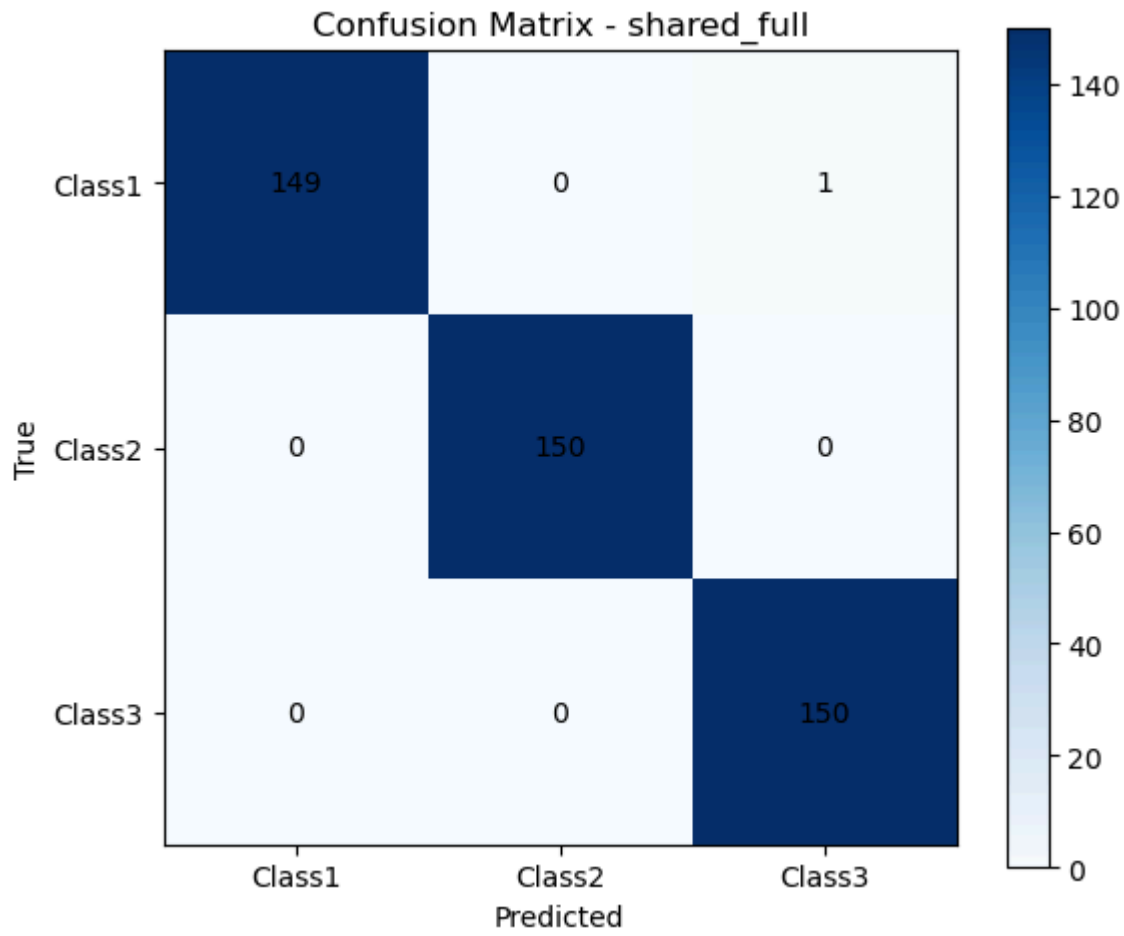
=== Confusion Matrix ===

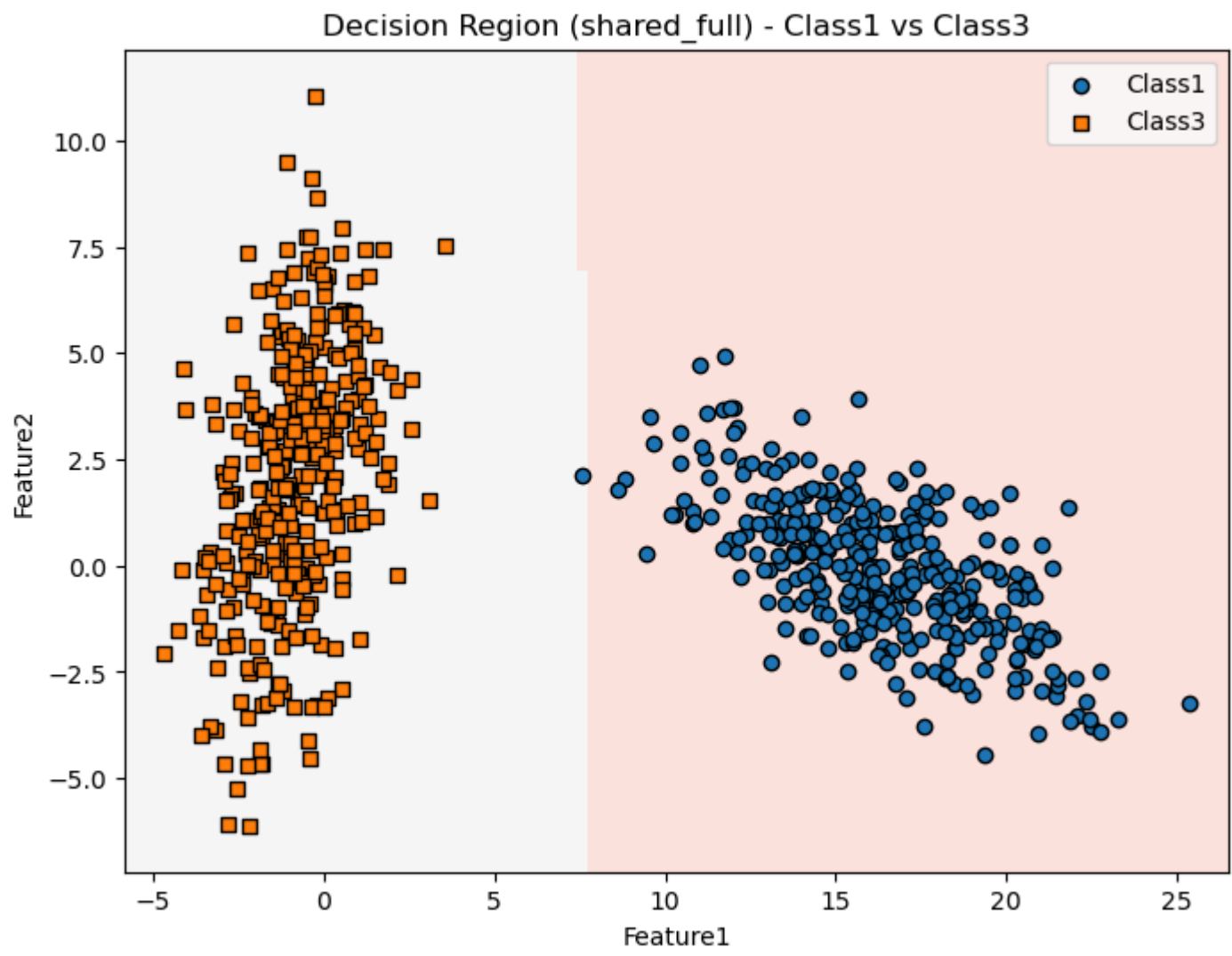
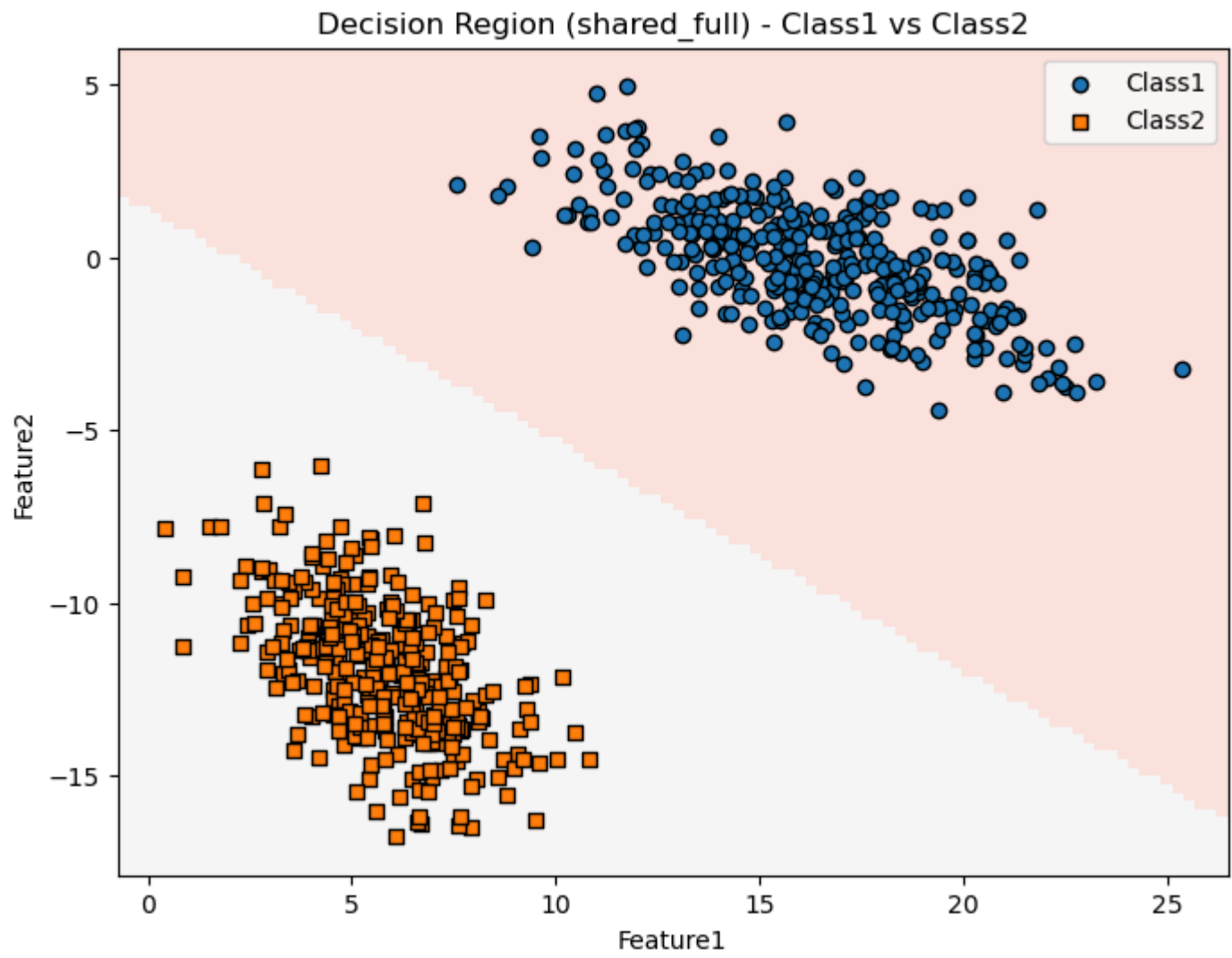
| | Class1 | Class2 | Class3 |
|--------|--------|--------|--------|
| Class1 | 149 | 0 | 1 |
| Class2 | 0 | 150 | 0 |
| Class3 | 0 | 0 | 150 |

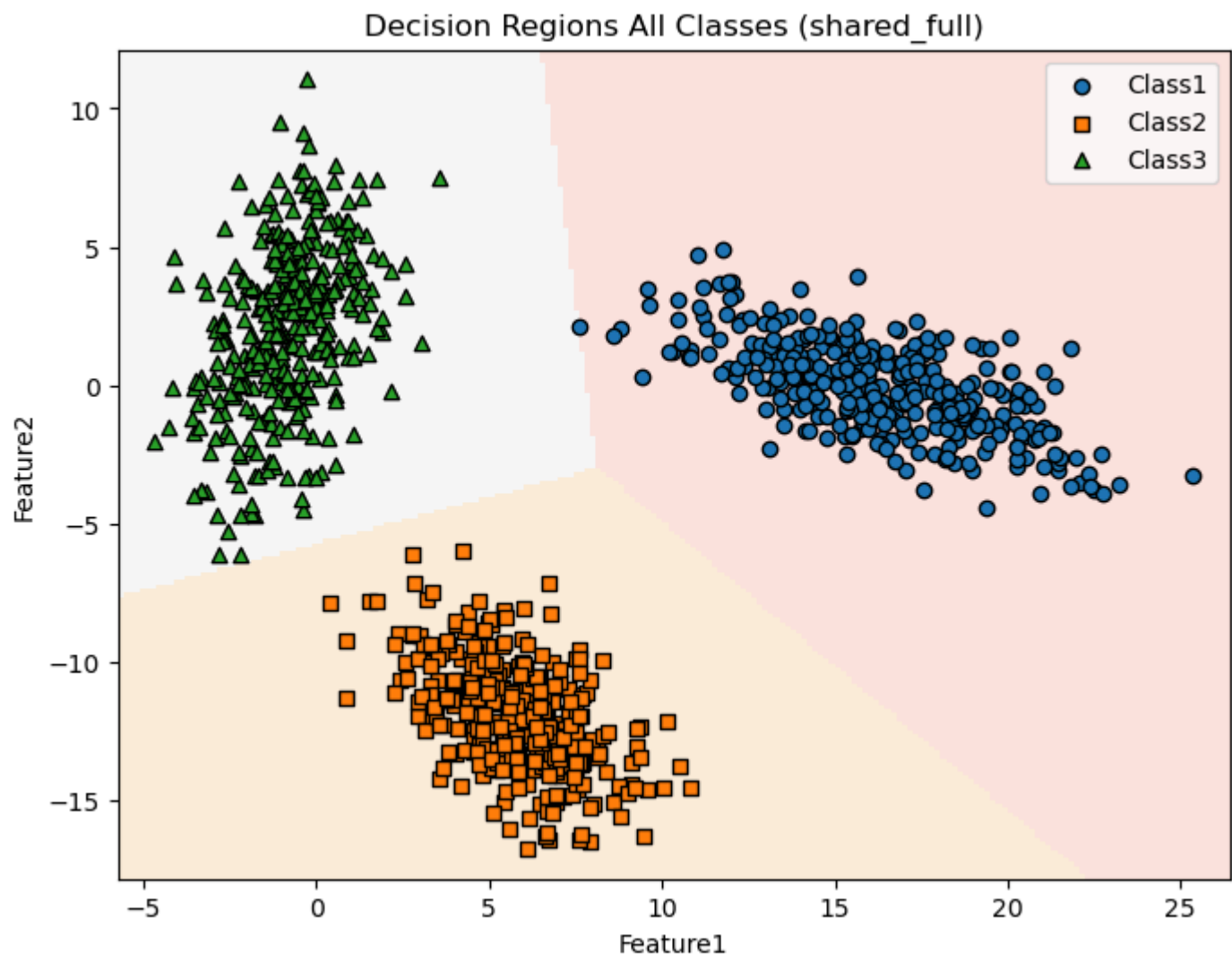
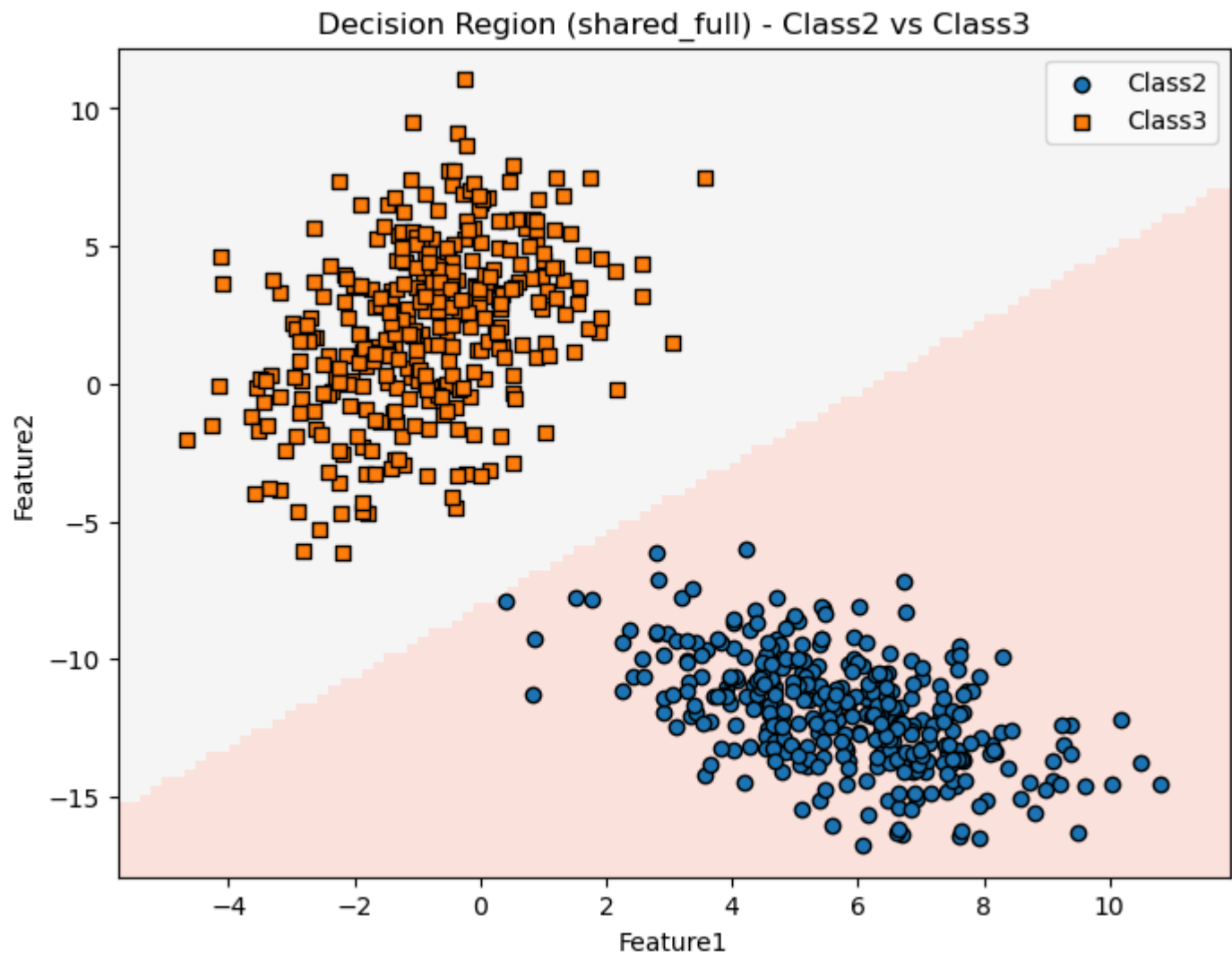
=== Classification Report ===

| Class | Precision | Recall | F1-score | Support |
|--------|-----------|--------|----------|---------|
| Class1 | 1.0000 | 0.9933 | 0.9967 | 150 |
| Class2 | 1.0000 | 1.0000 | 1.0000 | 150 |
| Class3 | 0.9934 | 1.0000 | 0.9967 | 150 |

Accuracy: 0.9978
Mean Precision: 0.9978
Mean Recall : 0.9978
Mean F1 Score : 0.9978





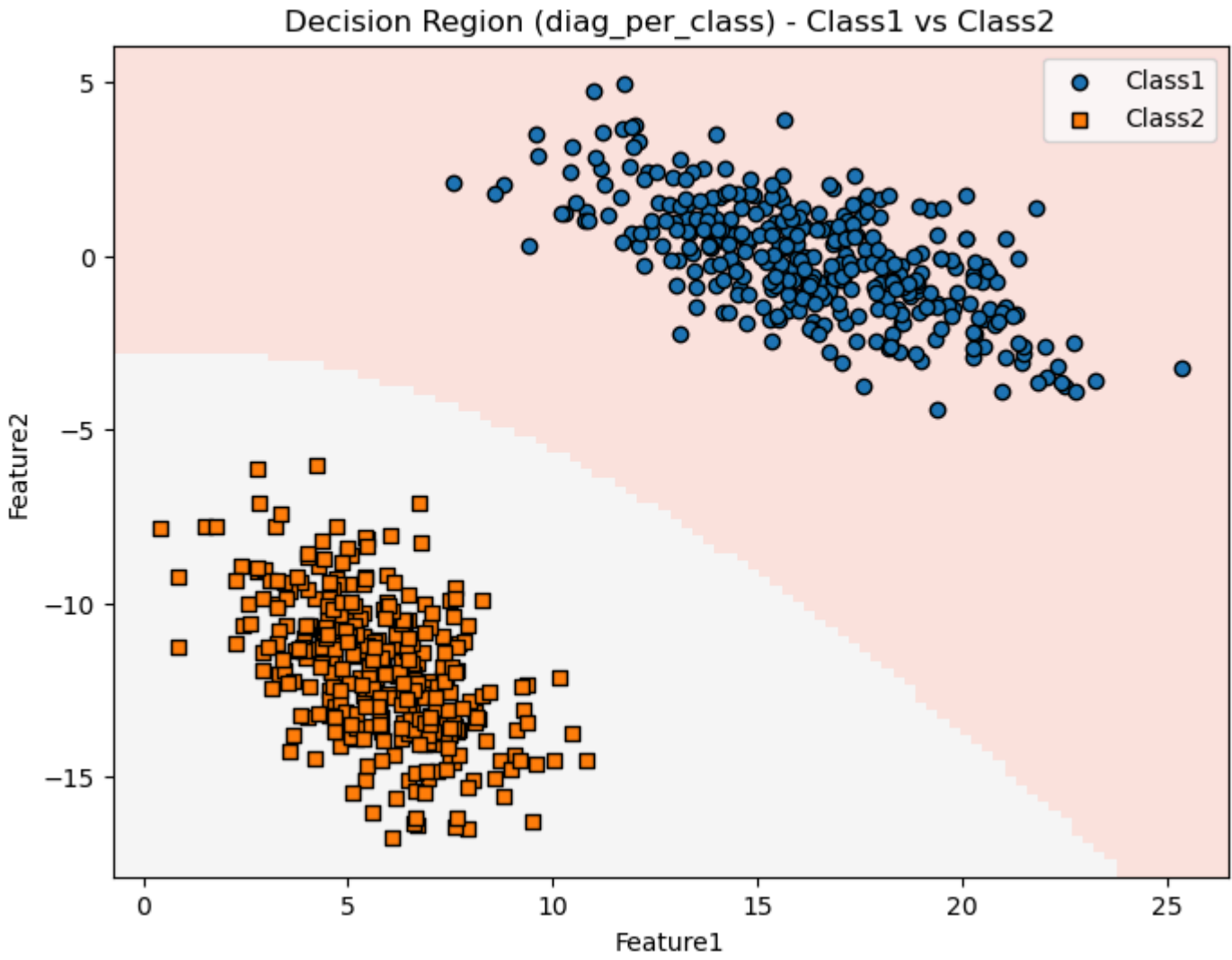
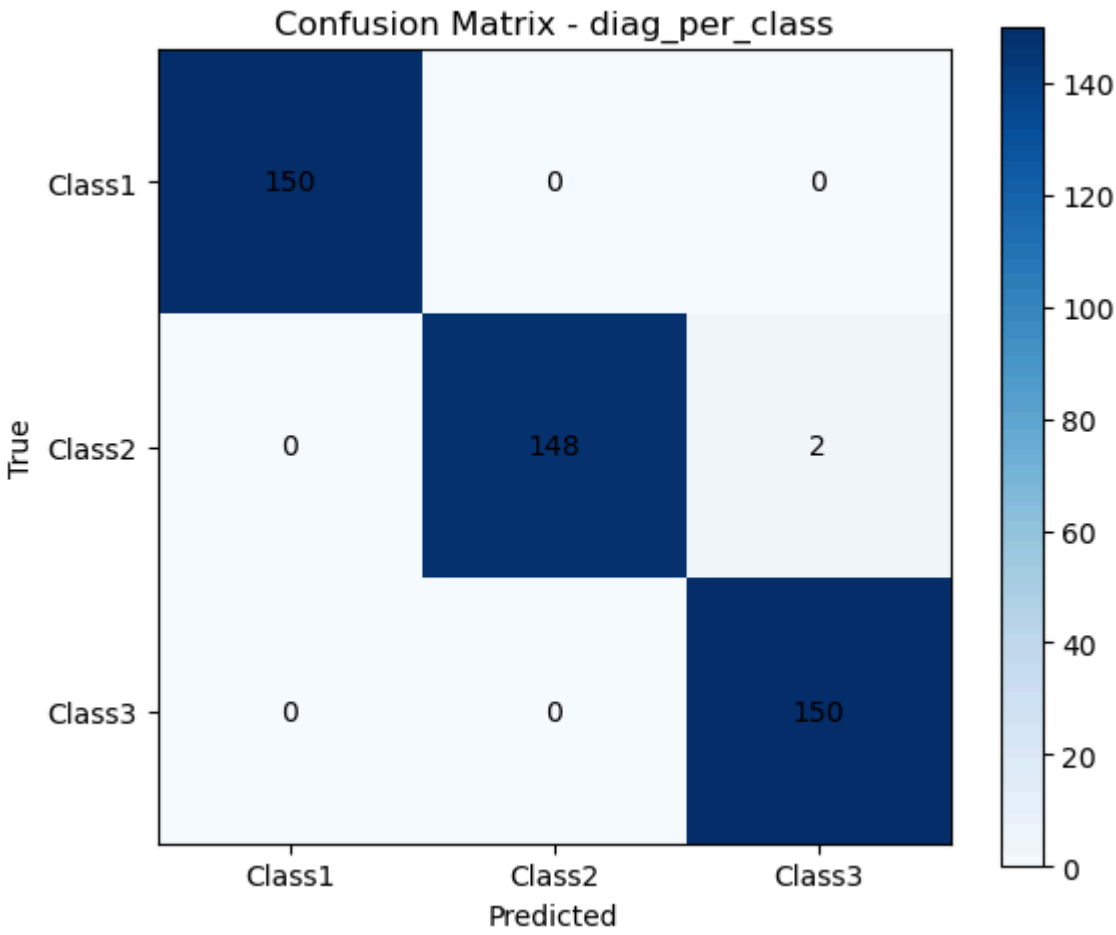


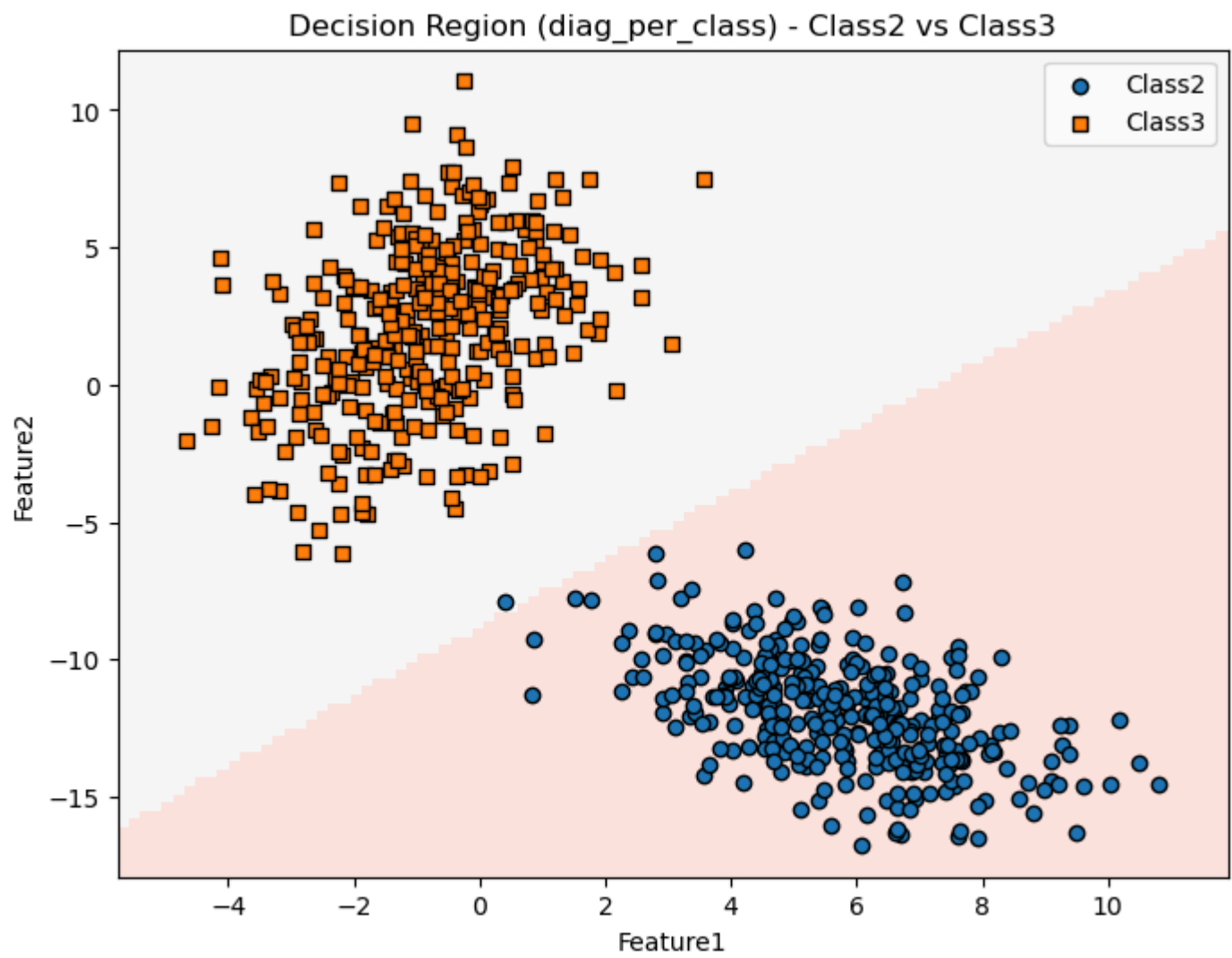
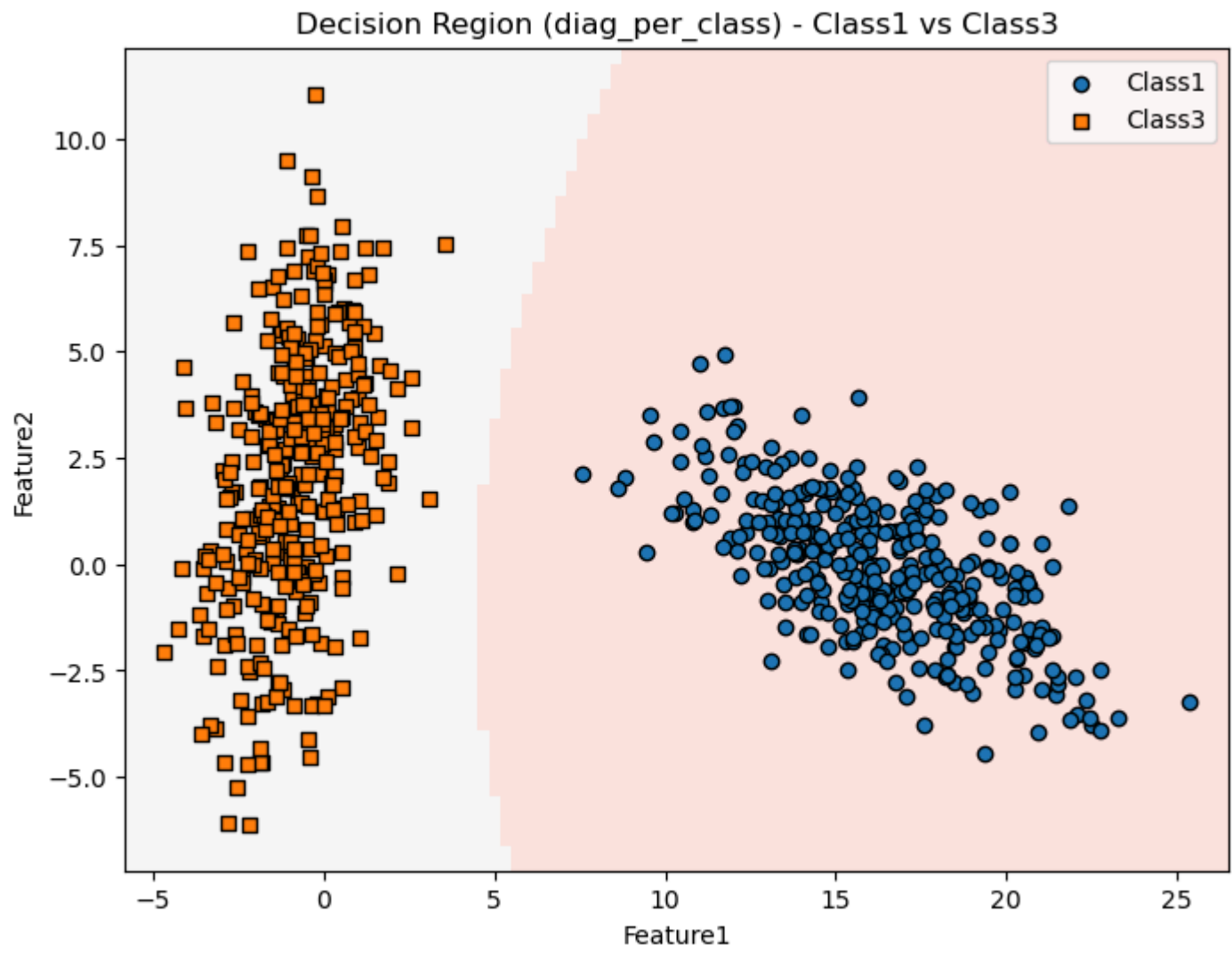
=====
Classifier: diag_per_class

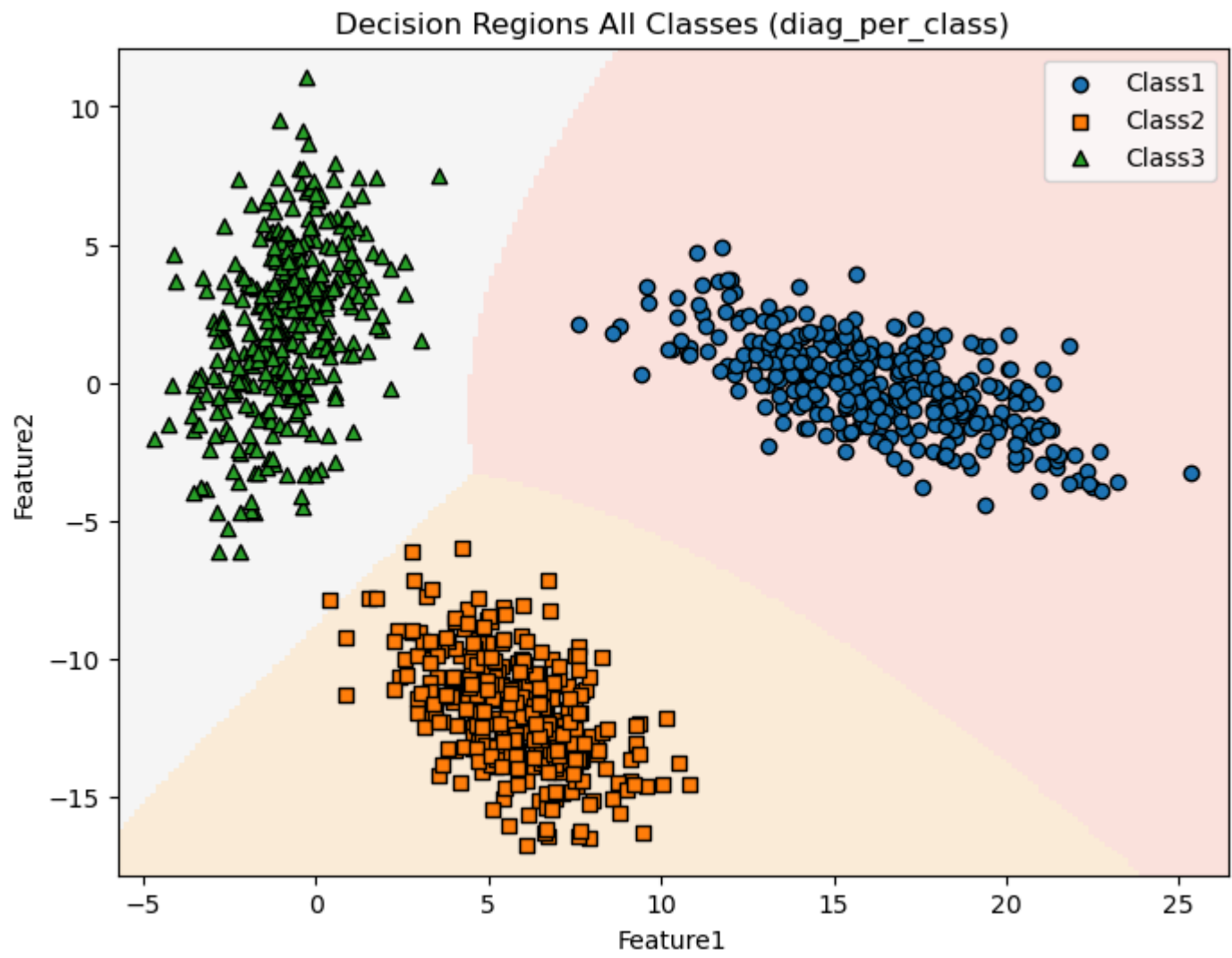
=== Confusion Matrix ===
Class1 Class2 Class3
Class1 150 0 0
Class2 0 148 2
Class3 0 0 150

=== Classification Report ===
Class Precision Recall F1-score Support
Class1 1.0000 1.0000 1.0000 150
Class2 1.0000 0.9867 0.9933 150
Class3 0.9868 1.0000 0.9934 150

Accuracy: 0.9956
Mean Precision: 0.9956
Mean Recall : 0.9956
Mean F1 Score : 0.9956





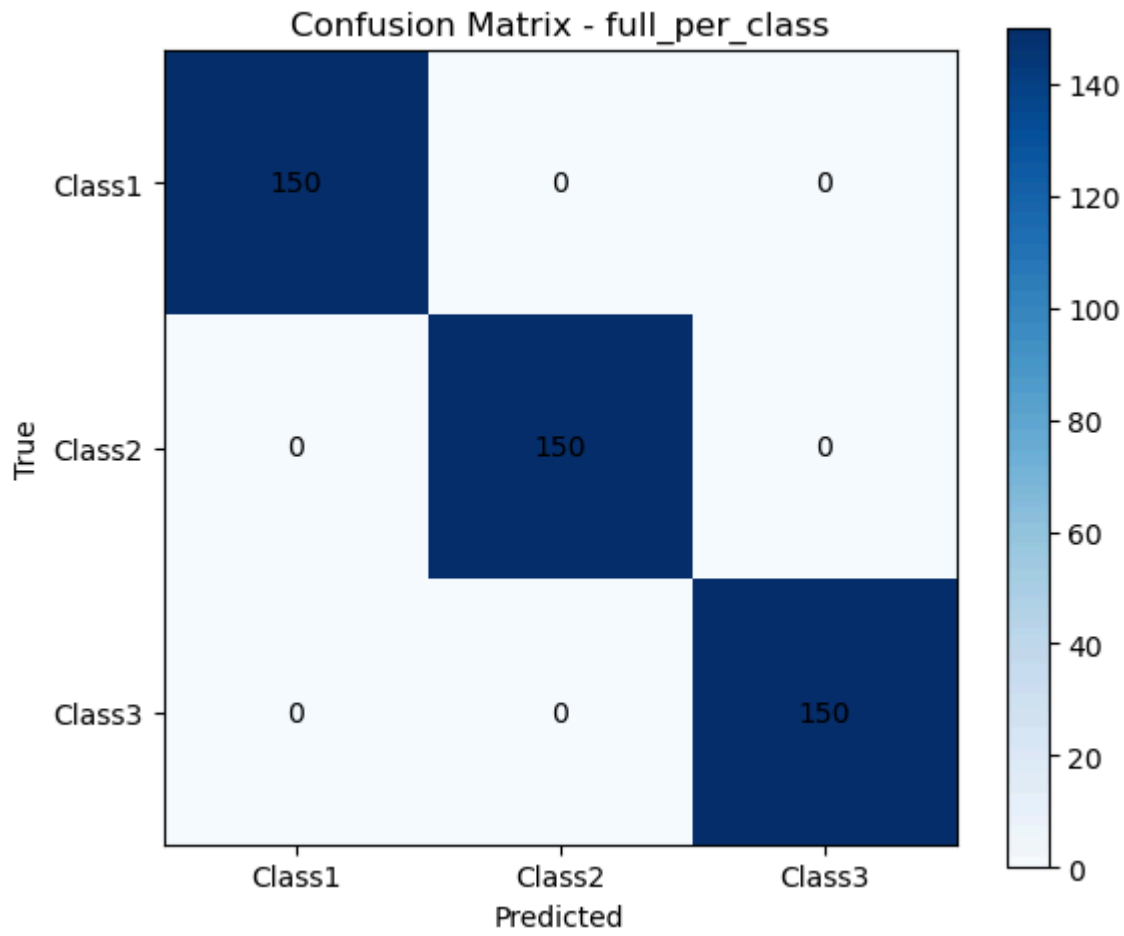


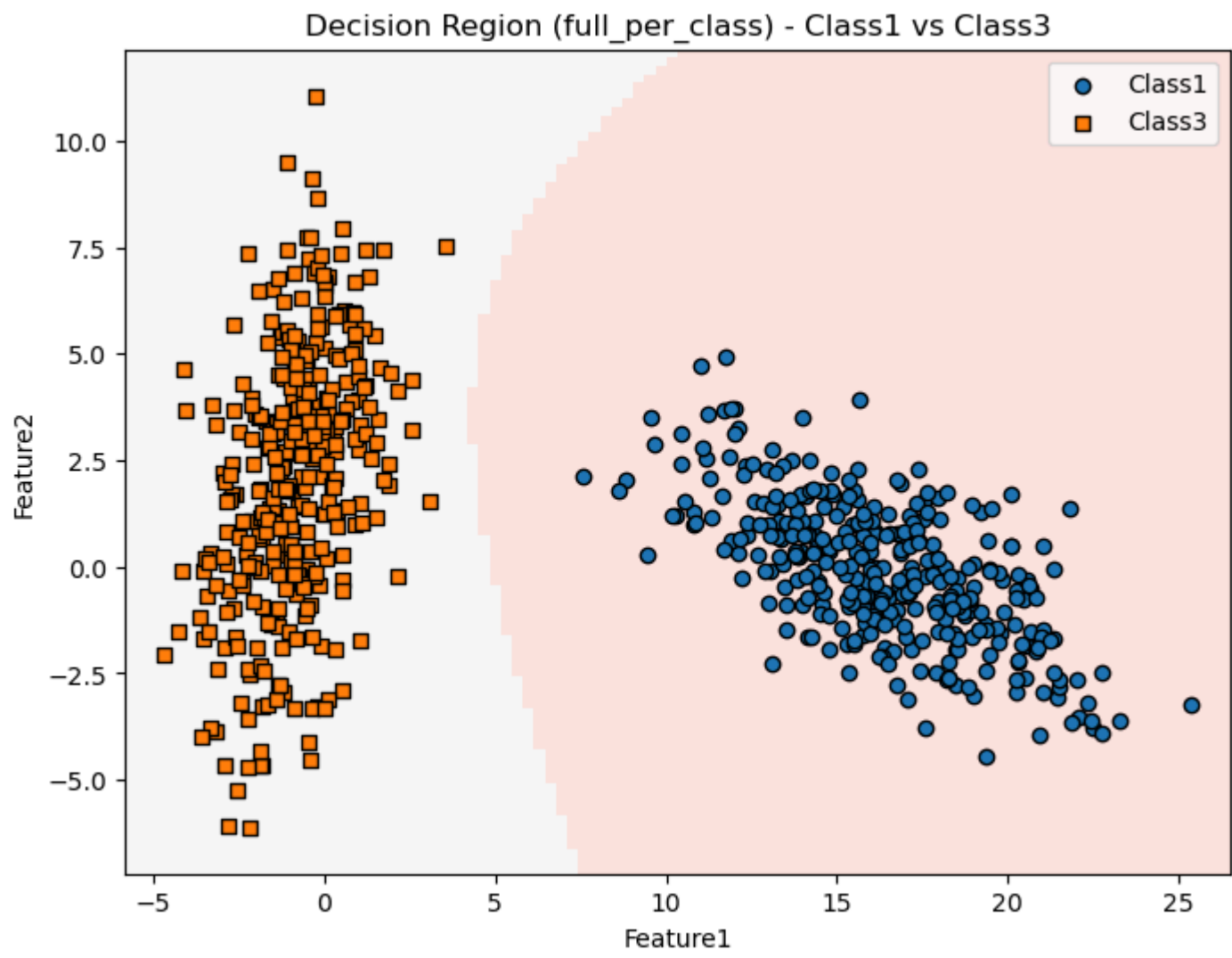
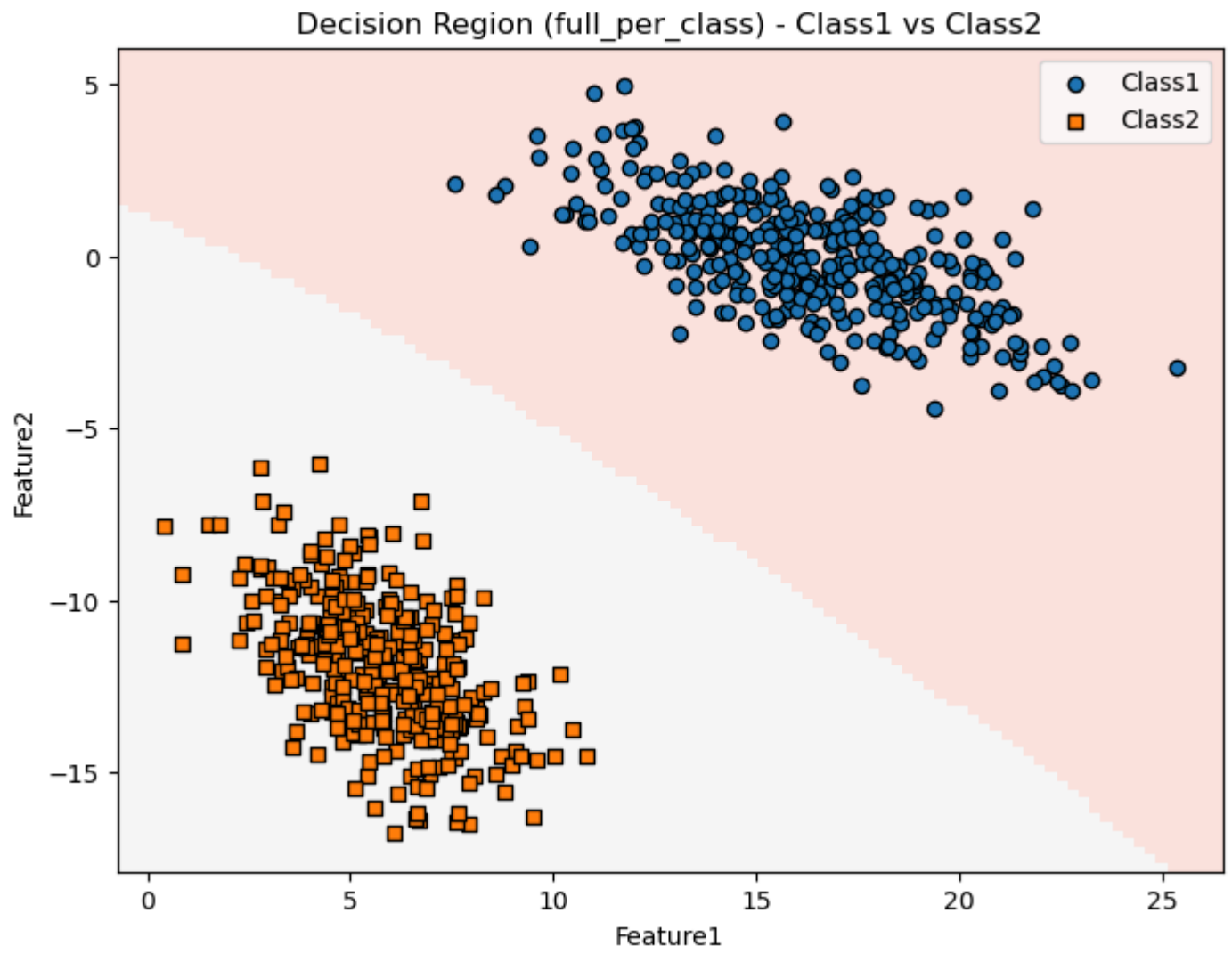
=====
Classifier: full_per_class

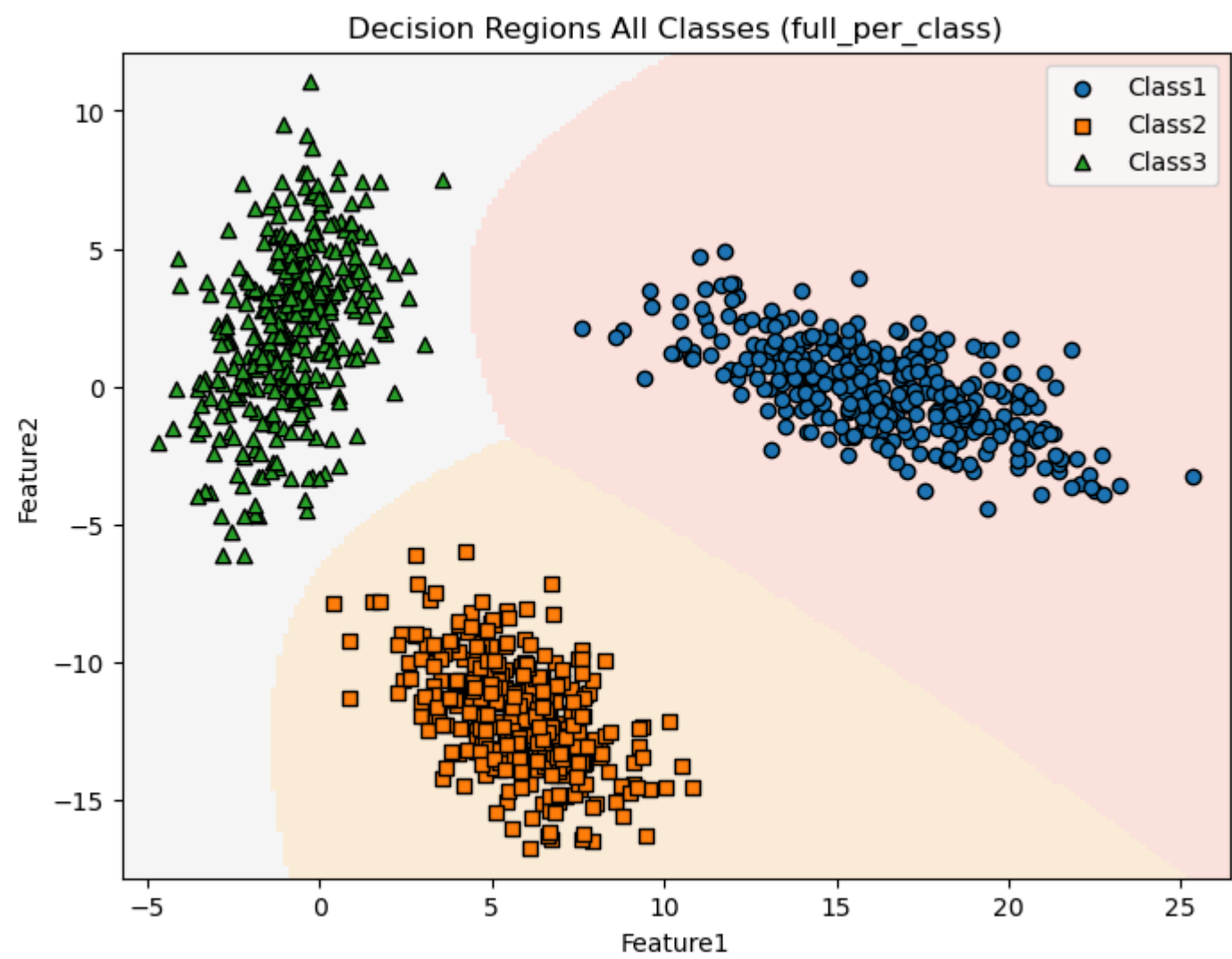
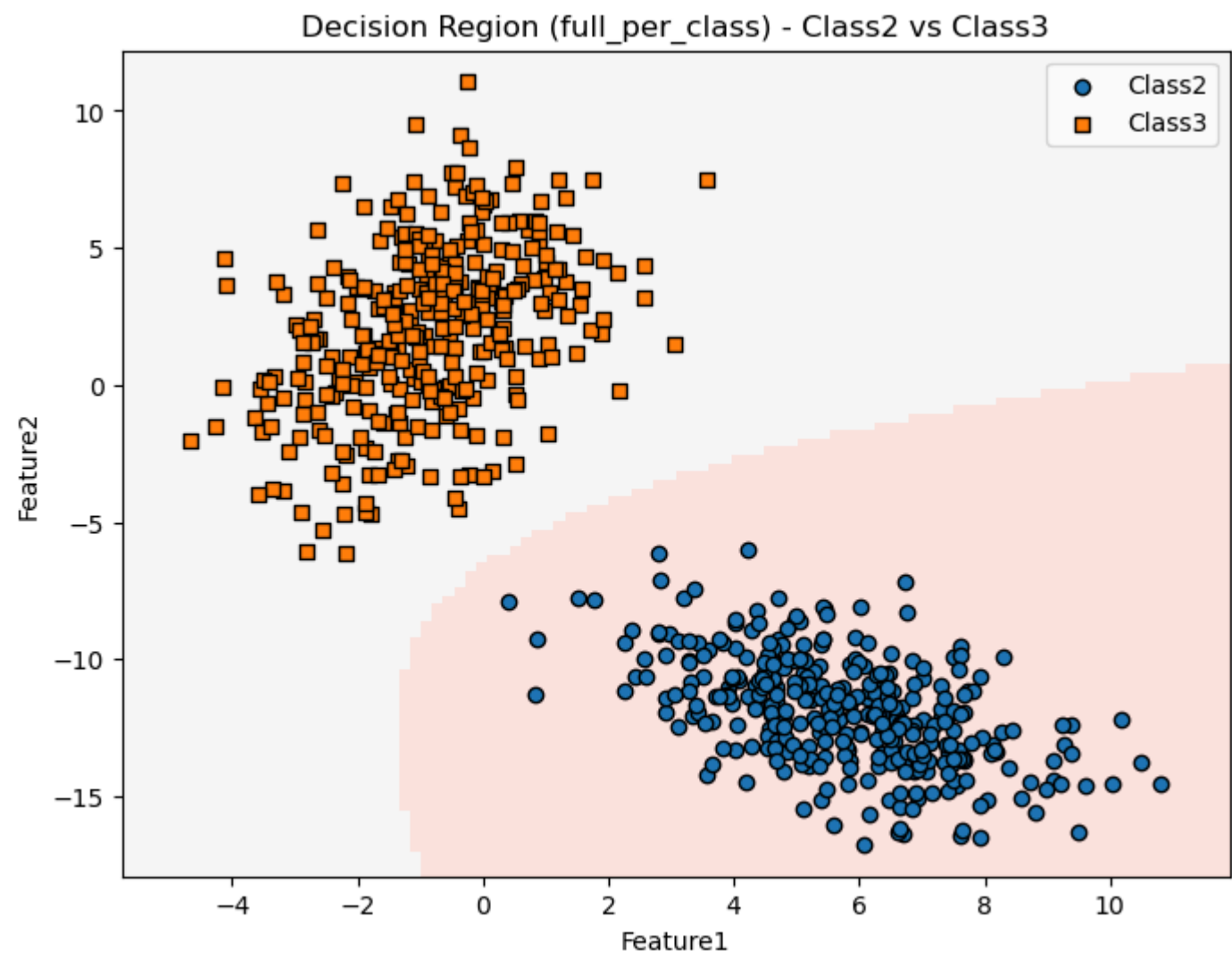
=== Confusion Matrix ===
Class1 Class2 Class3
Class1 150 0 0
Class2 0 150 0
Class3 0 0 150

=== Classification Report ===
Class Precision Recall F1-score Support
Class1 1.0000 1.0000 1.0000 150
Class2 1.0000 1.0000 1.0000 150
Class3 1.0000 1.0000 1.0000 150

Accuracy: 1.0000
Mean Precision: 1.0000
Mean Recall : 1.0000
Mean F1 Score : 1.0000

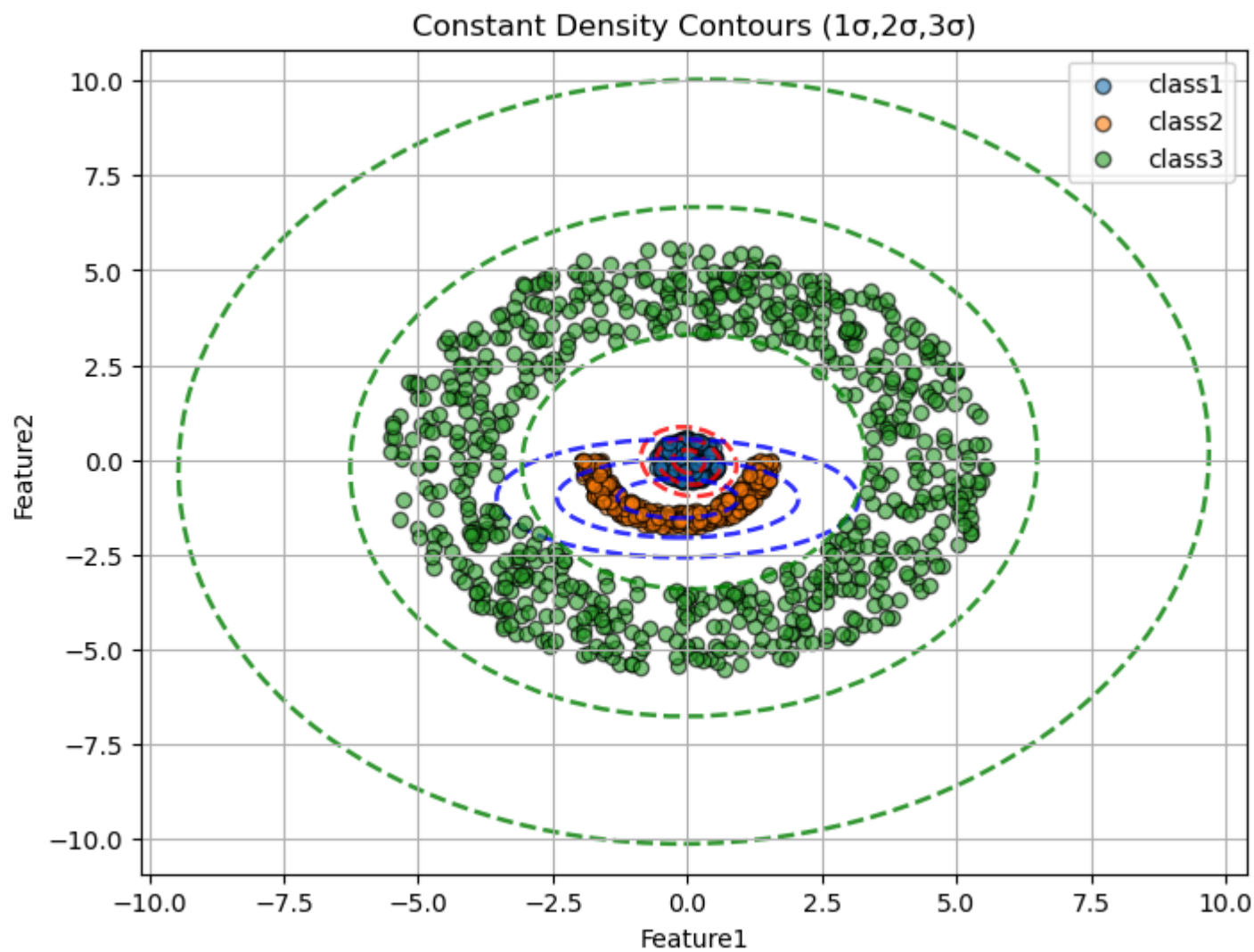
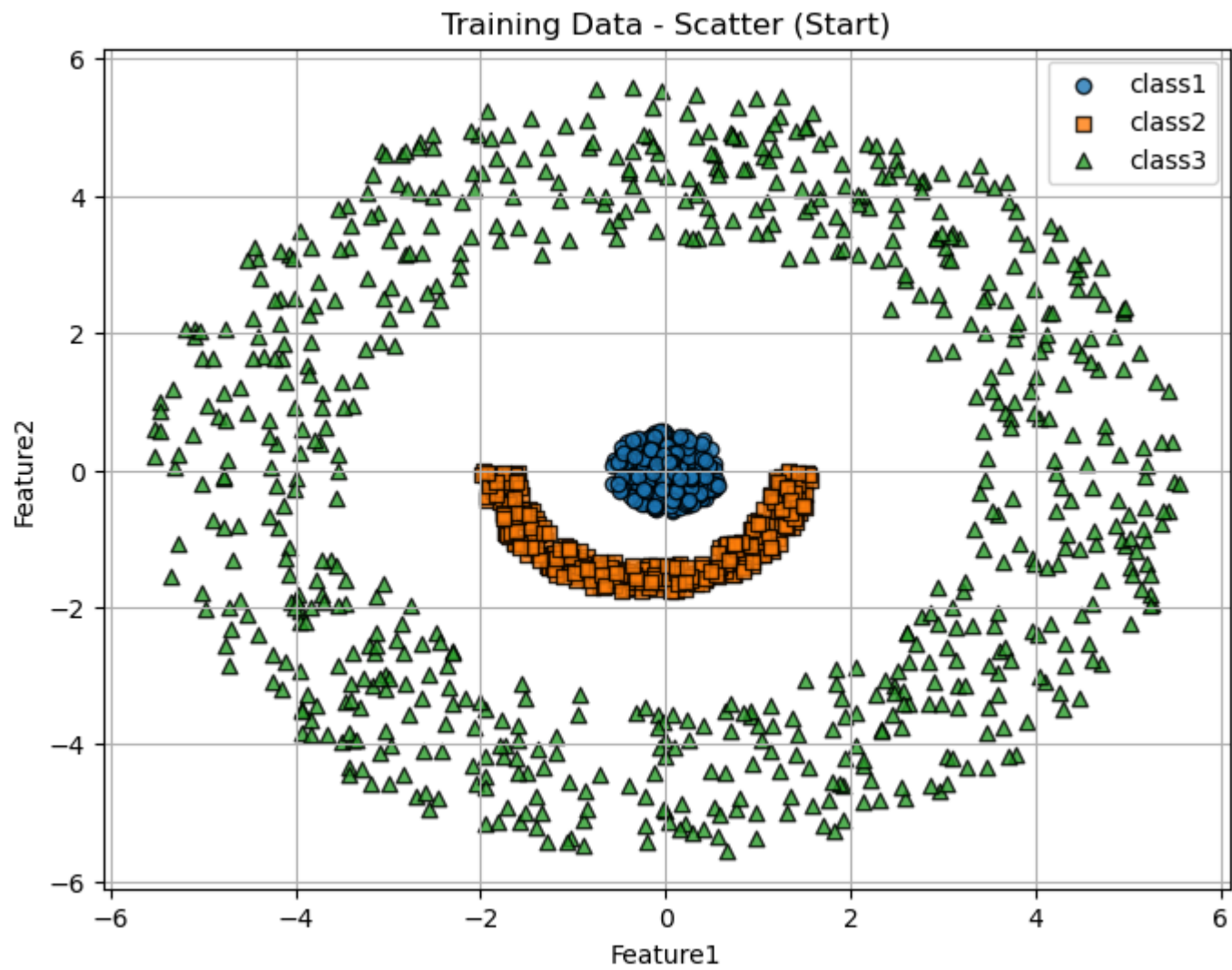






```
In [11]: # base_path = "../../Dataset/Group04/LS_Group04/"
base_path = "../../Dataset/Group04/NLS_Group04/"
# base_path = "../../Dataset/Group04/rd_group4/"

train_df, test_df = load_train_test_data(base_path)
run_all(train_df, test_df, res=200)
```



=====
Classifier: sigma2I

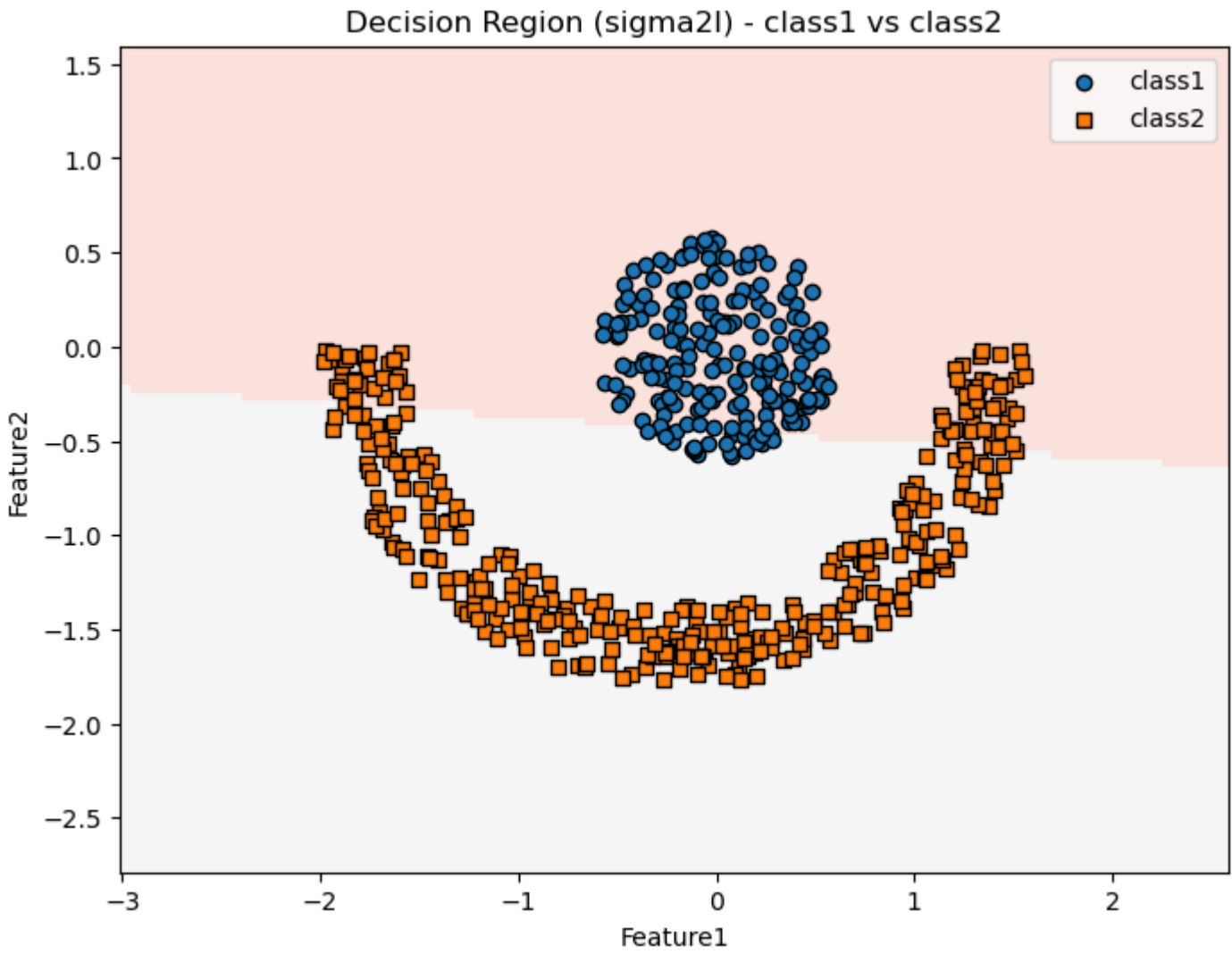
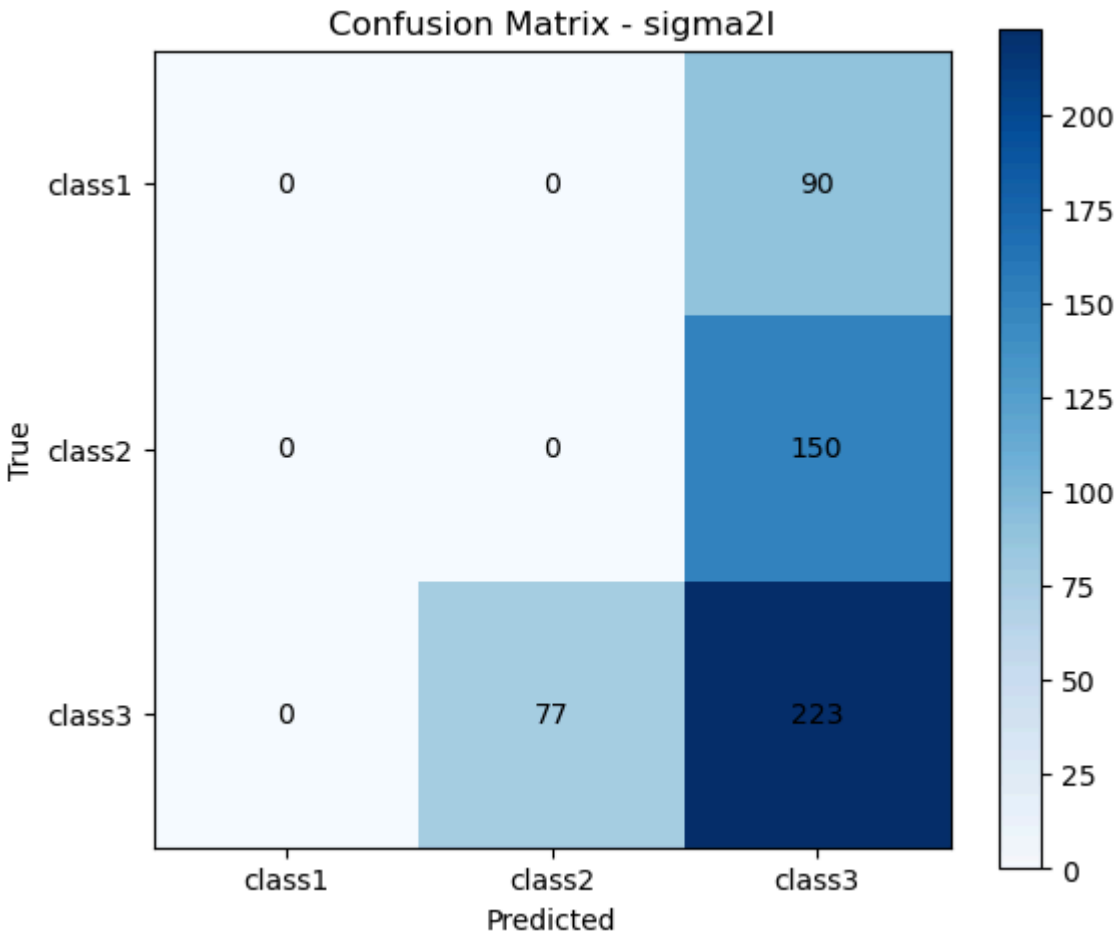
=== Confusion Matrix ===

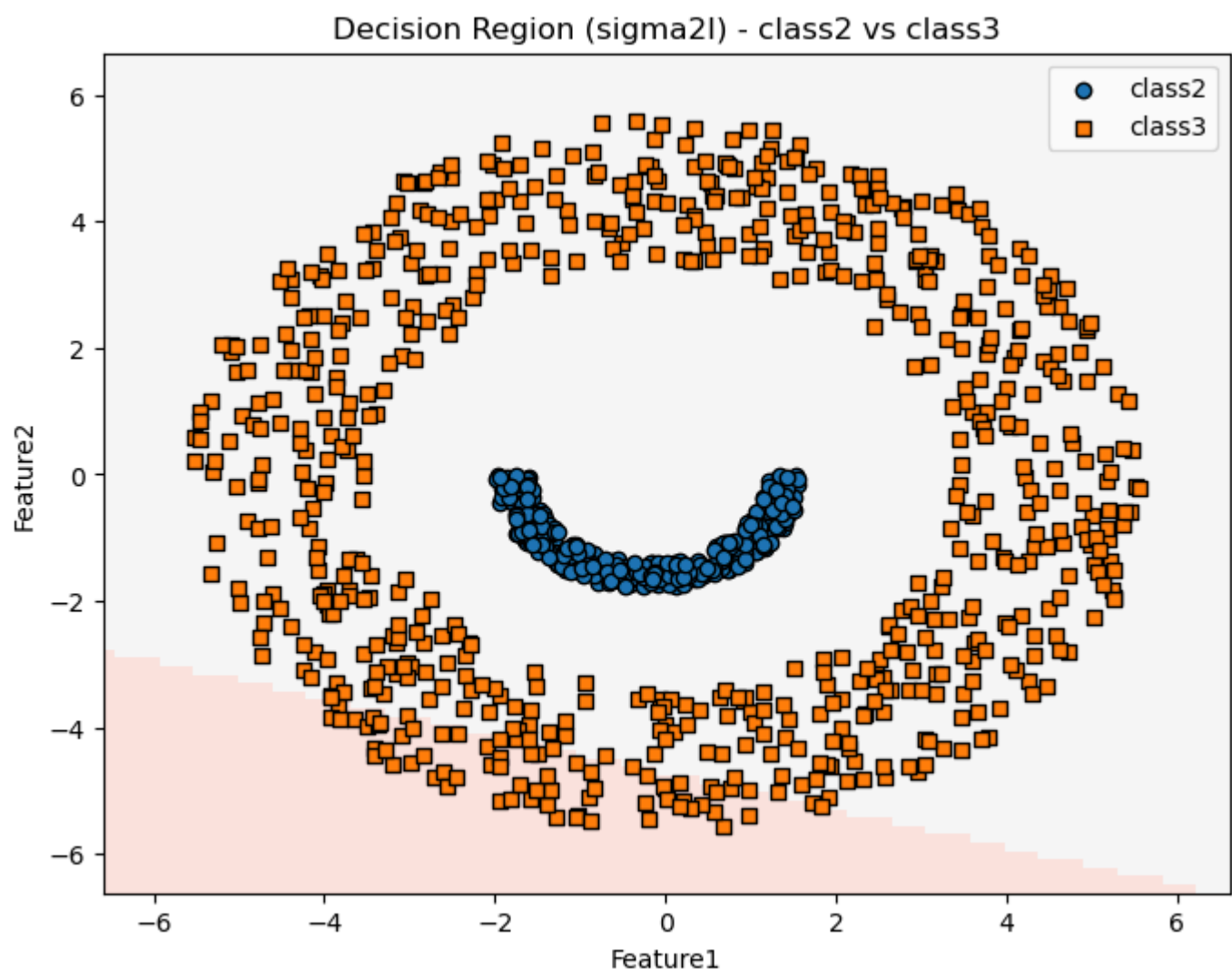
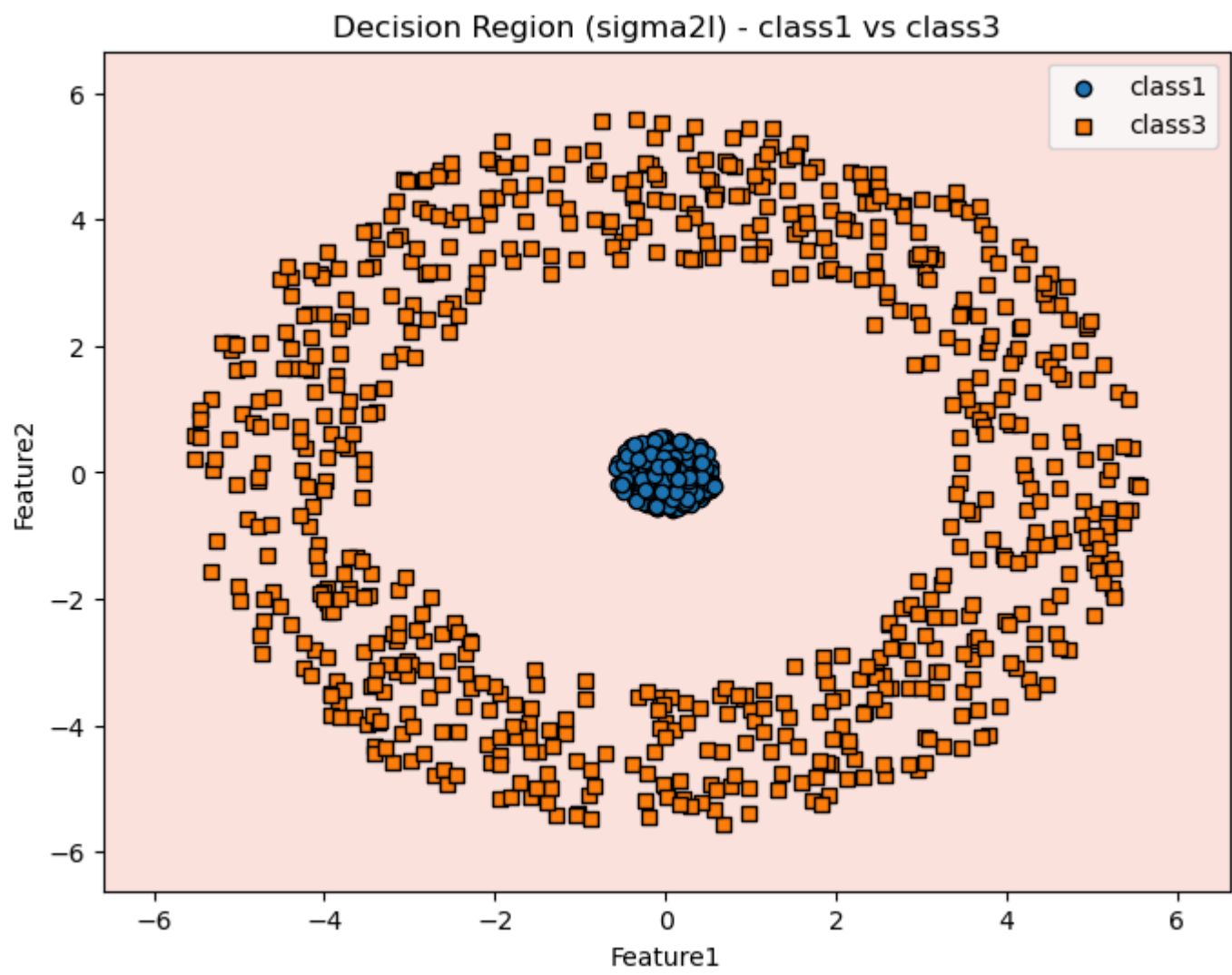
| | class1 | class2 | class3 |
|--------|--------|--------|--------|
| class1 | 0 | 0 | 90 |
| class2 | 0 | 0 | 150 |
| class3 | 0 | 77 | 223 |

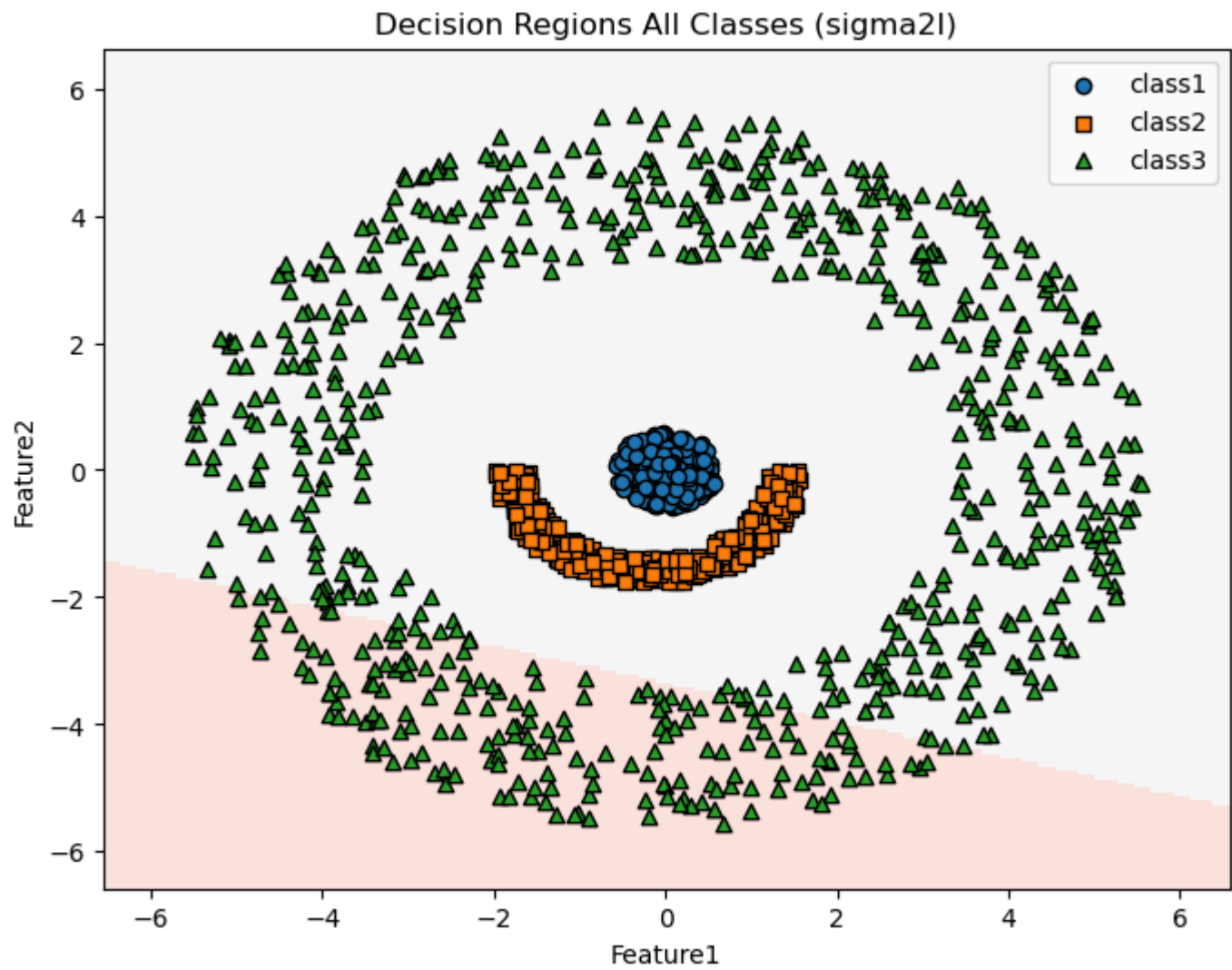
=== Classification Report ===

| Class | Precision | Recall | F1-score | Support |
|--------|-----------|--------|----------|---------|
| class1 | 0.0000 | 0.0000 | 0.0000 | 90 |
| class2 | 0.0000 | 0.0000 | 0.0000 | 150 |
| class3 | 0.4816 | 0.7433 | 0.5845 | 300 |

Accuracy: 0.4130
Mean Precision: 0.1605
Mean Recall : 0.2478
Mean F1 Score : 0.1948







=====
Classifier: shared_full

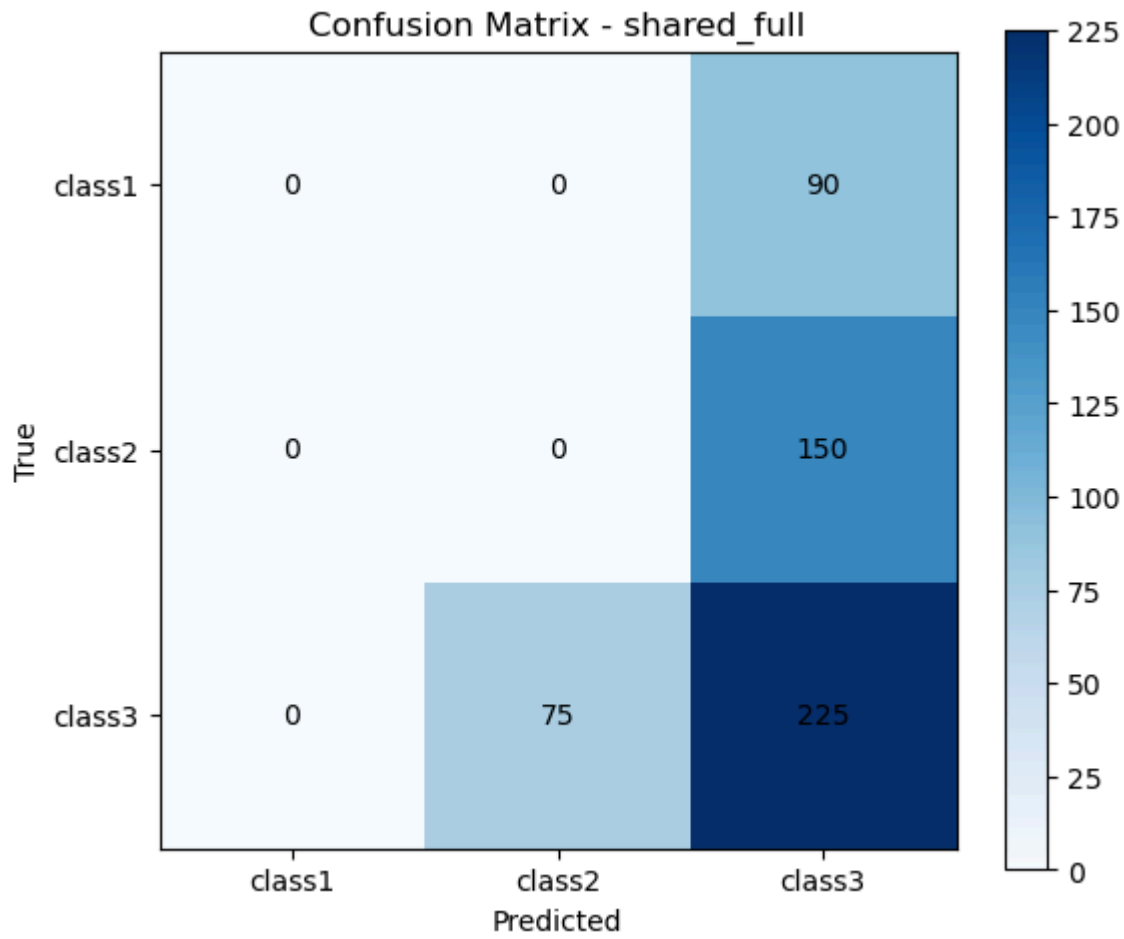
=== Confusion Matrix ===

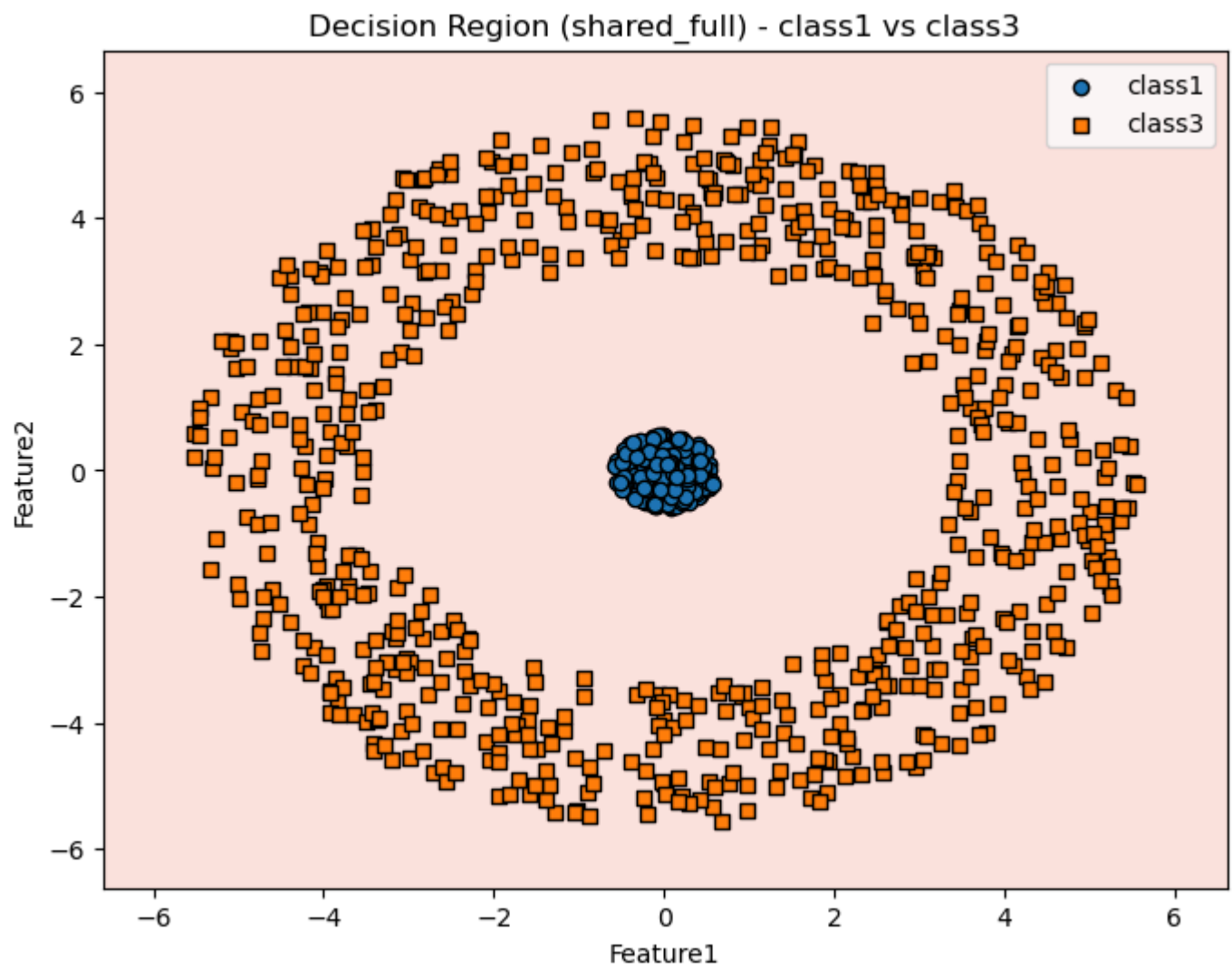
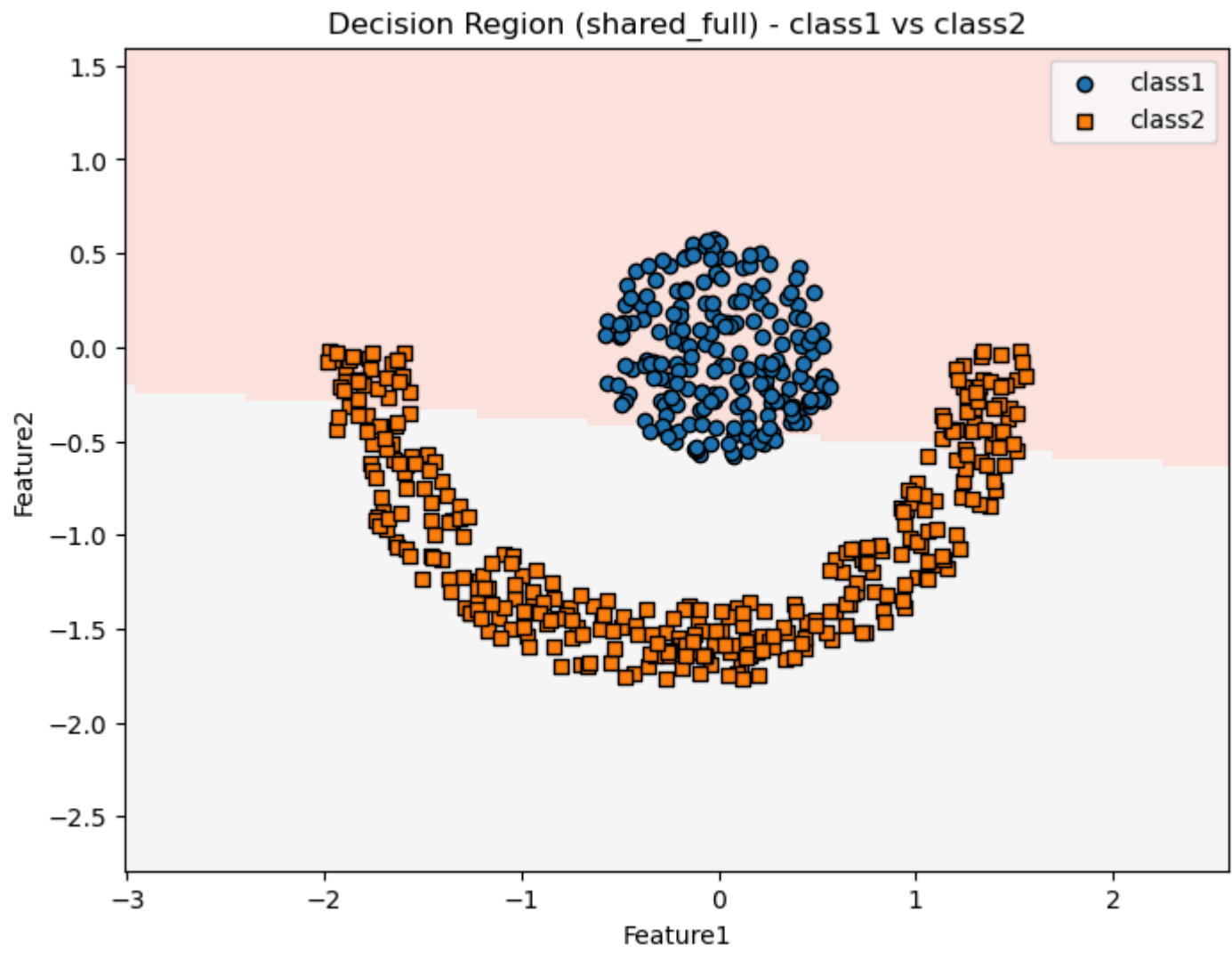
| | class1 | class2 | class3 |
|--------|--------|--------|--------|
| class1 | 0 | 0 | 90 |
| class2 | 0 | 0 | 150 |
| class3 | 0 | 75 | 225 |

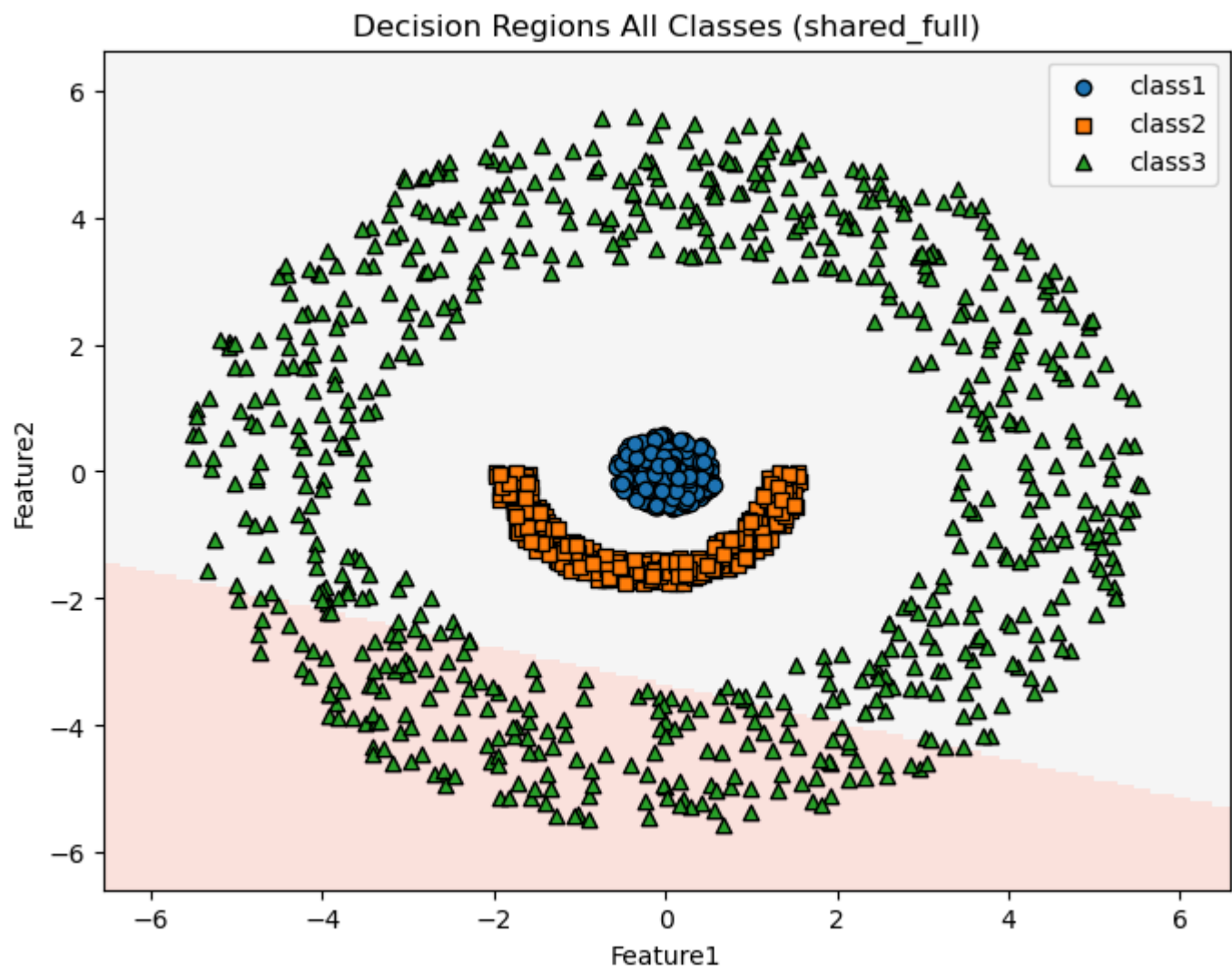
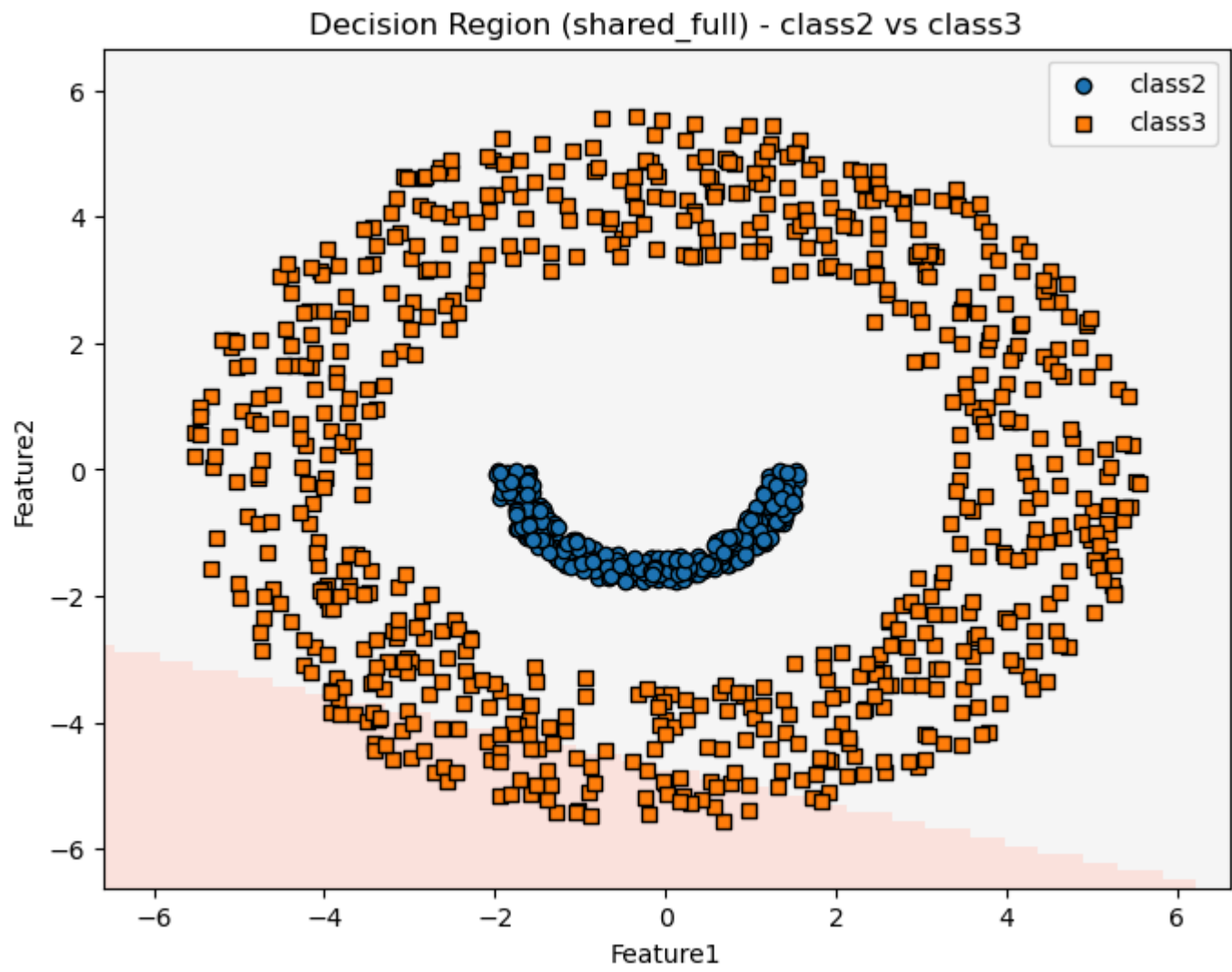
=== Classification Report ===

| Class | Precision | Recall | F1-score | Support |
|--------|-----------|--------|----------|---------|
| class1 | 0.0000 | 0.0000 | 0.0000 | 90 |
| class2 | 0.0000 | 0.0000 | 0.0000 | 150 |
| class3 | 0.4839 | 0.7500 | 0.5882 | 300 |

Accuracy: 0.4167
Mean Precision: 0.1613
Mean Recall : 0.2500
Mean F1 Score : 0.1961







=====
Classifier: diag_per_class

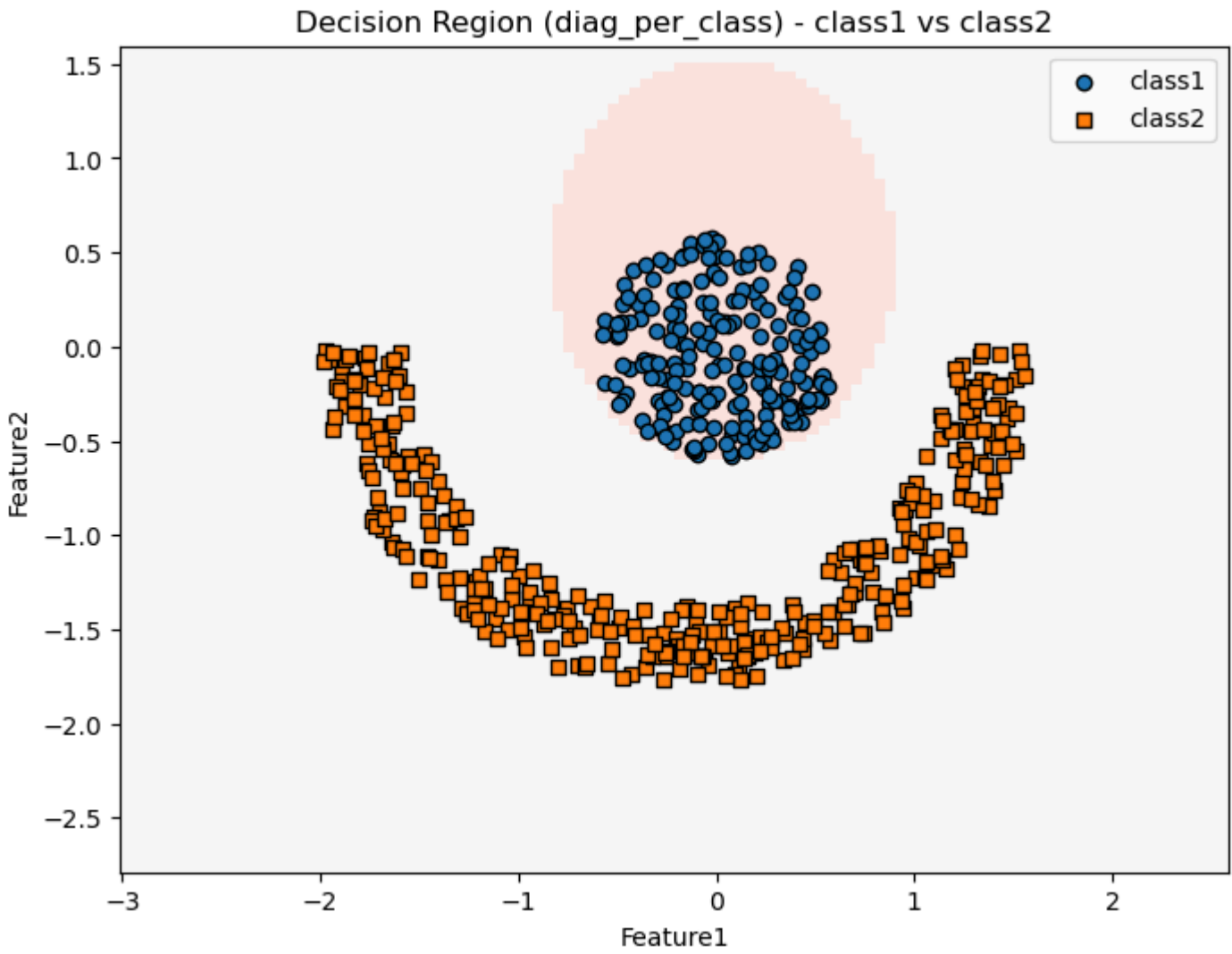
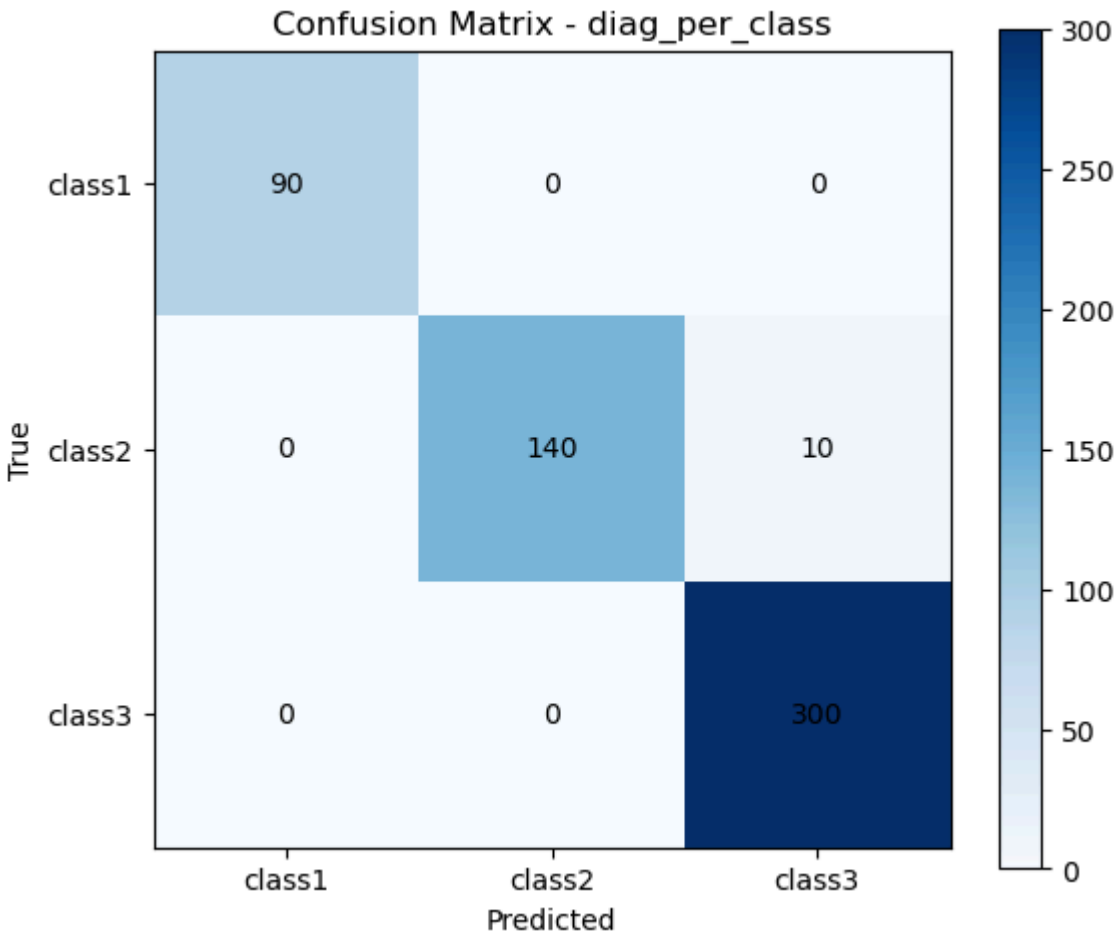
=== Confusion Matrix ===

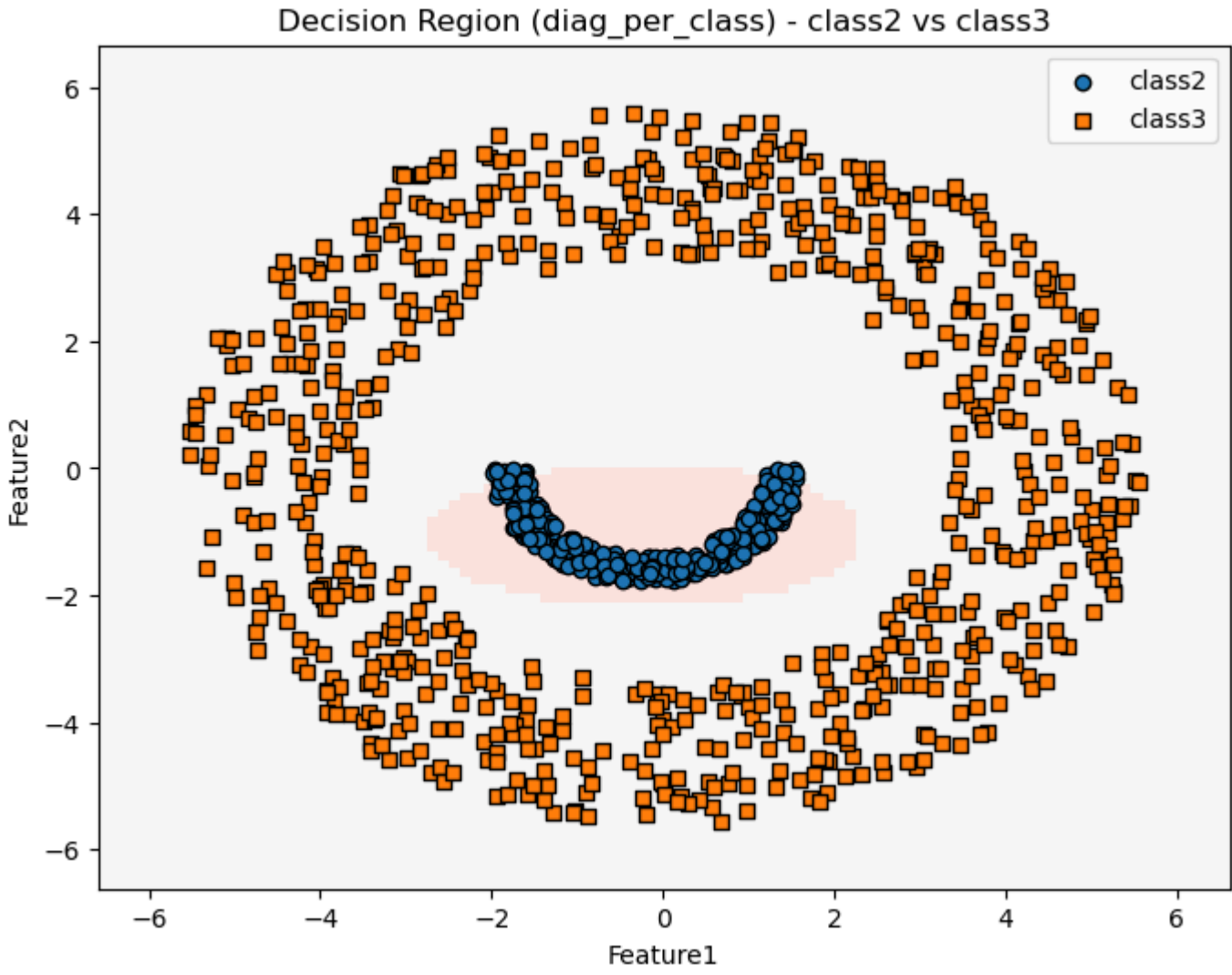
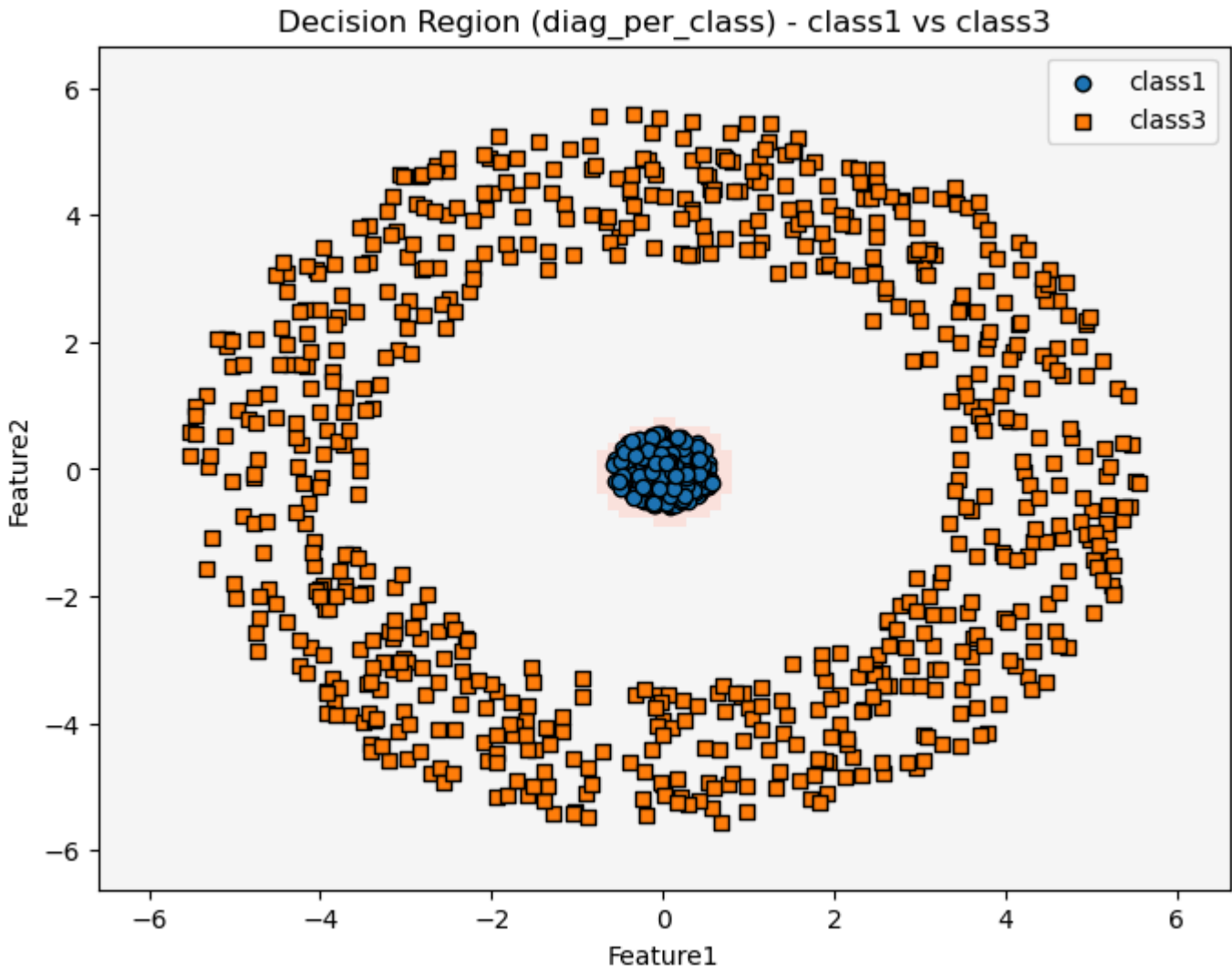
| | class1 | class2 | class3 |
|--------|--------|--------|--------|
| class1 | 90 | 0 | 0 |
| class2 | 0 | 140 | 10 |
| class3 | 0 | 0 | 300 |

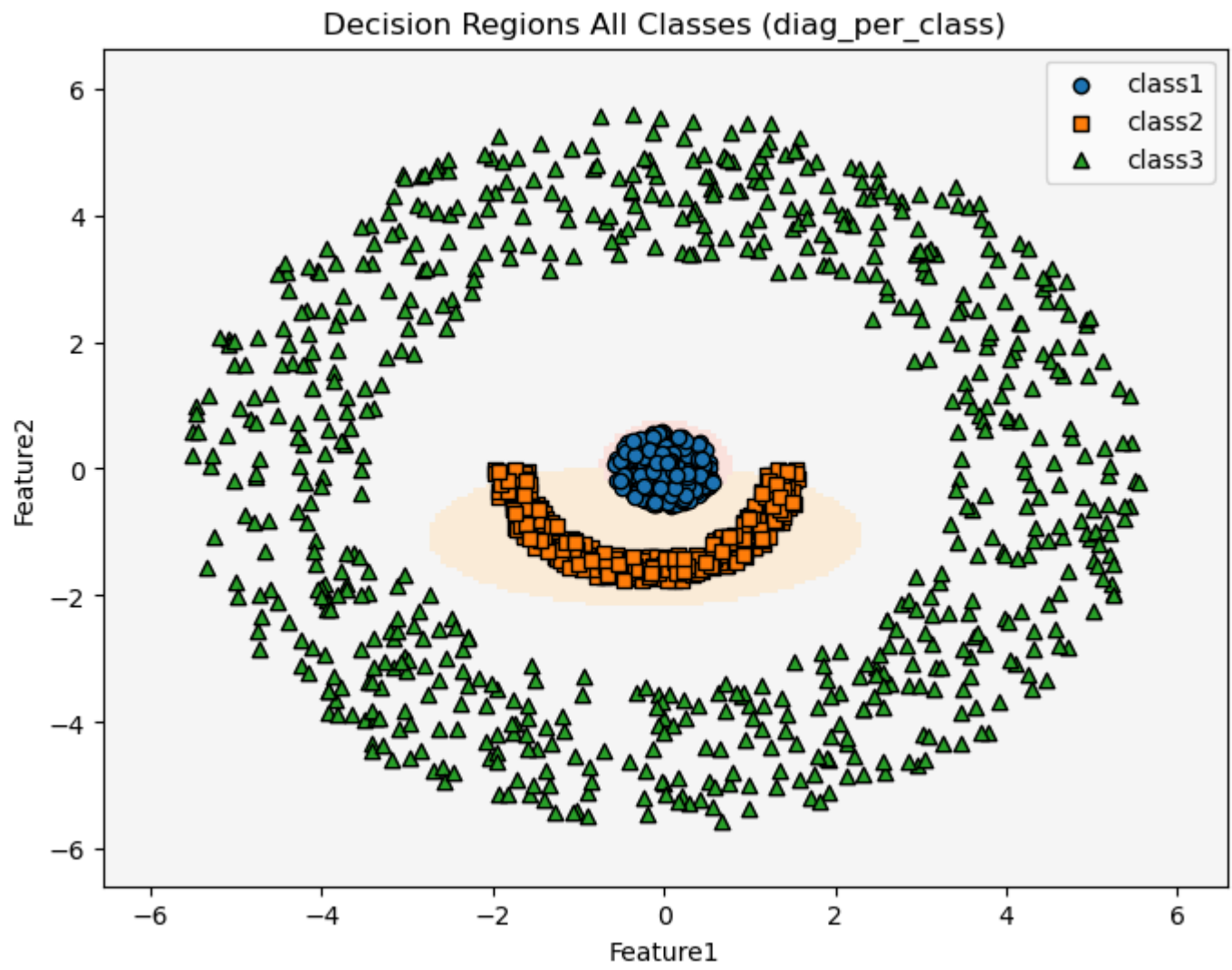
=== Classification Report ===

| Class | Precision | Recall | F1-score | Support |
|--------|-----------|--------|----------|---------|
| class1 | 1.0000 | 1.0000 | 1.0000 | 90 |
| class2 | 1.0000 | 0.9333 | 0.9655 | 150 |
| class3 | 0.9677 | 1.0000 | 0.9836 | 300 |

Accuracy: 0.9815
Mean Precision: 0.9892
Mean Recall : 0.9778
Mean F1 Score : 0.9830





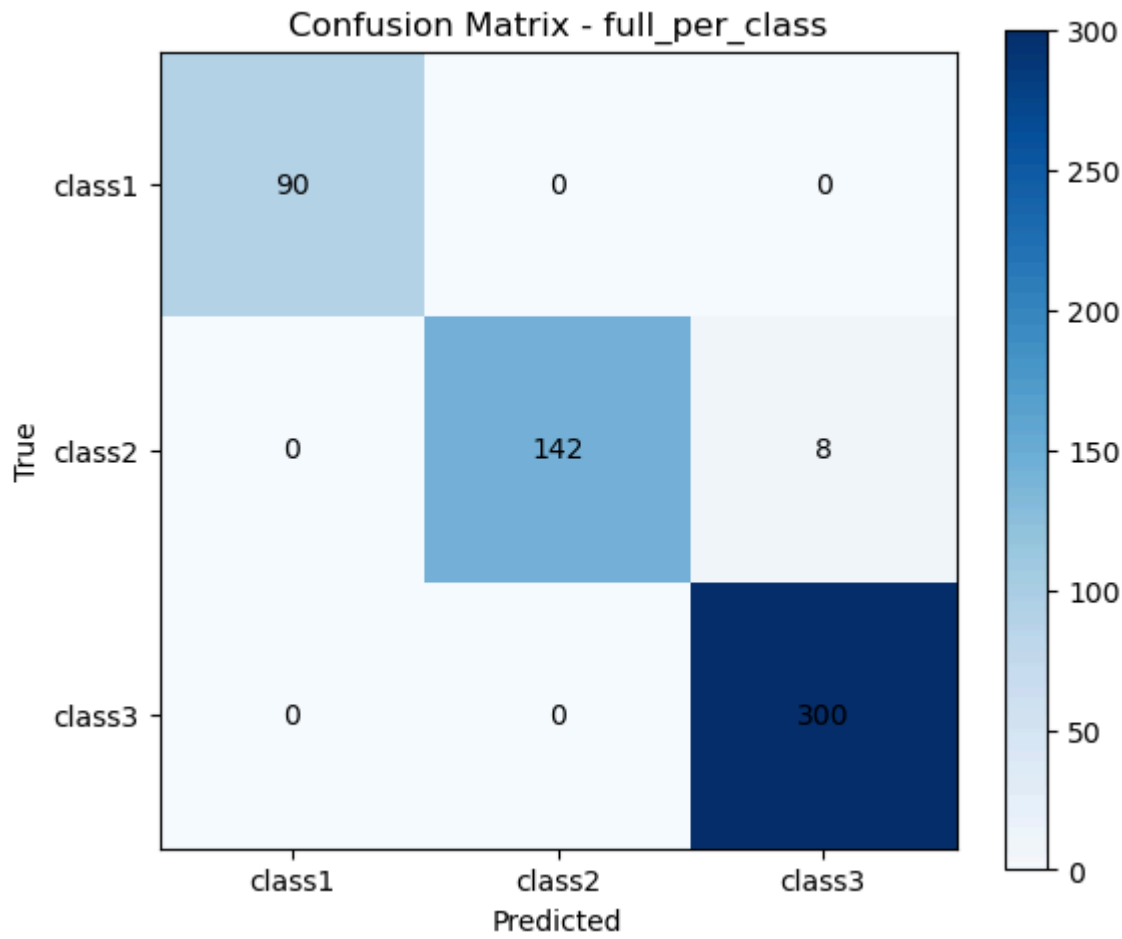


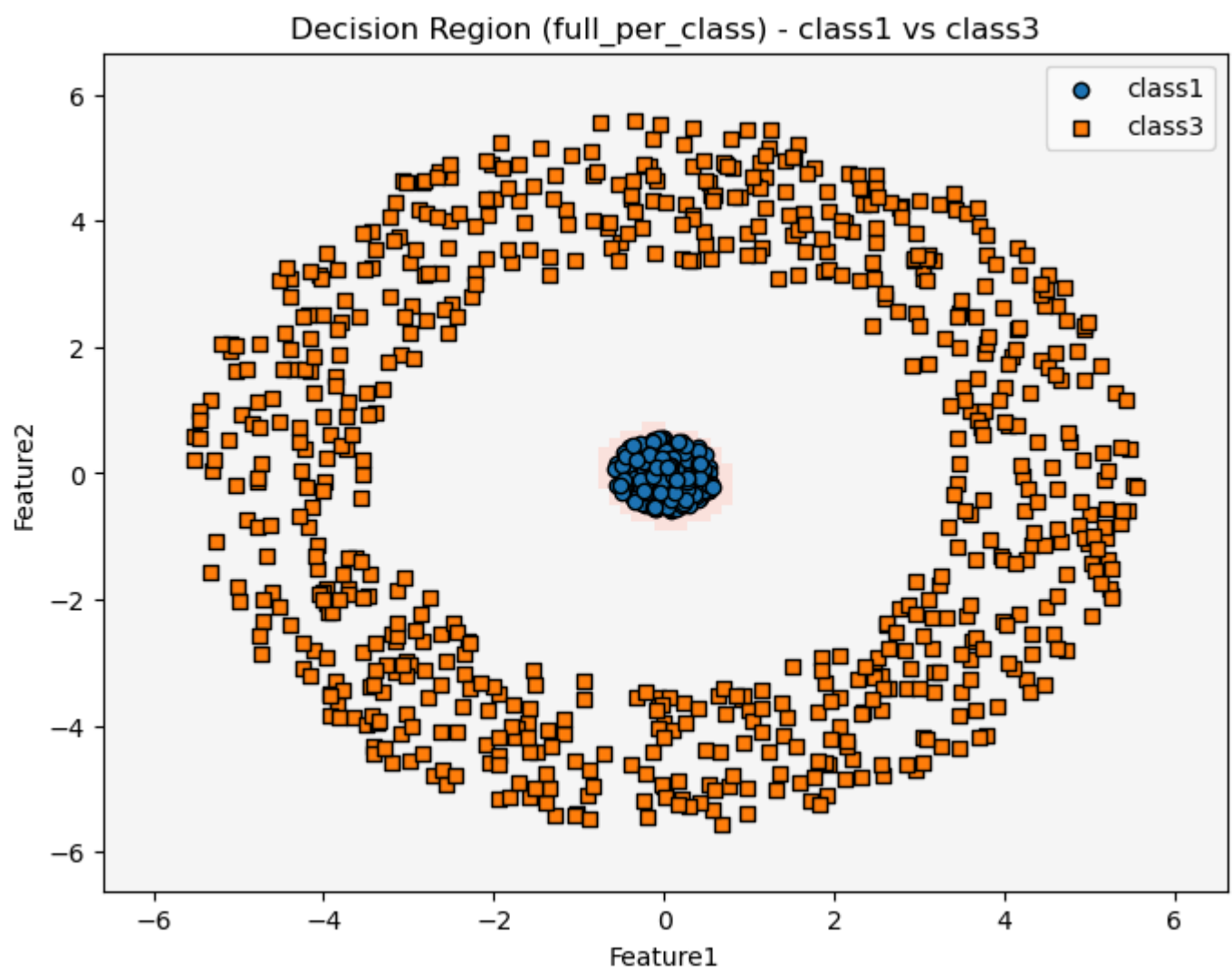
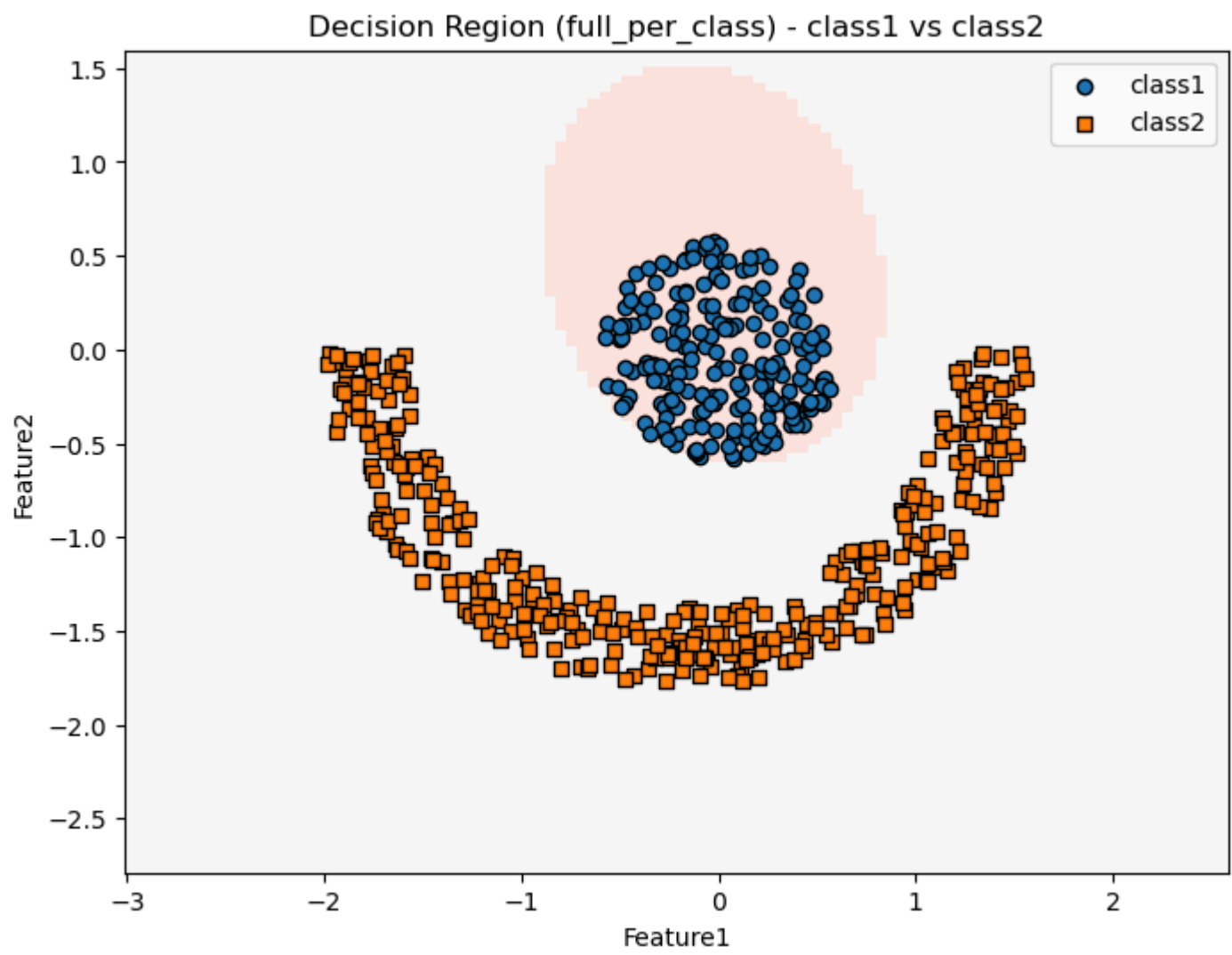
=====
Classifier: full_per_class

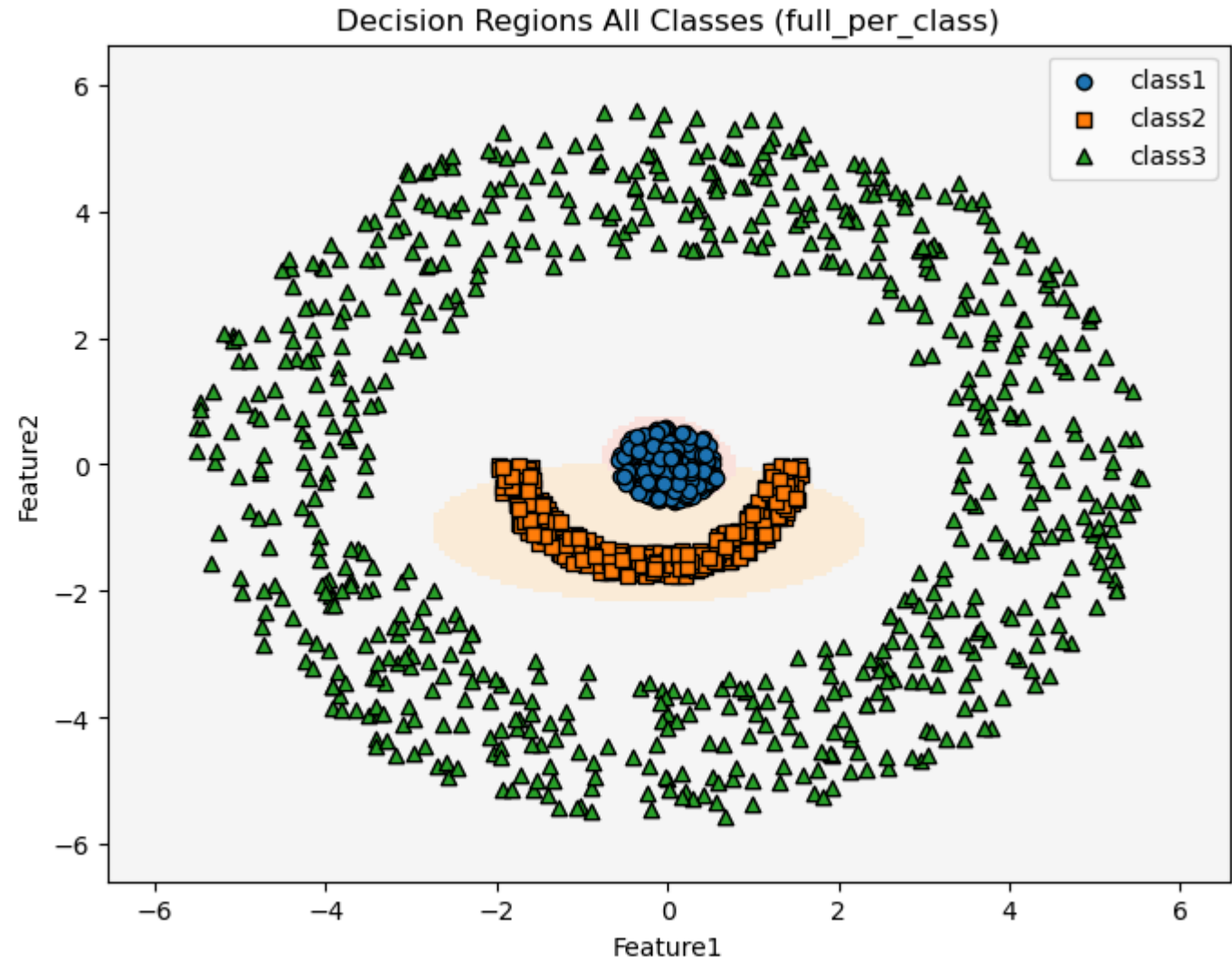
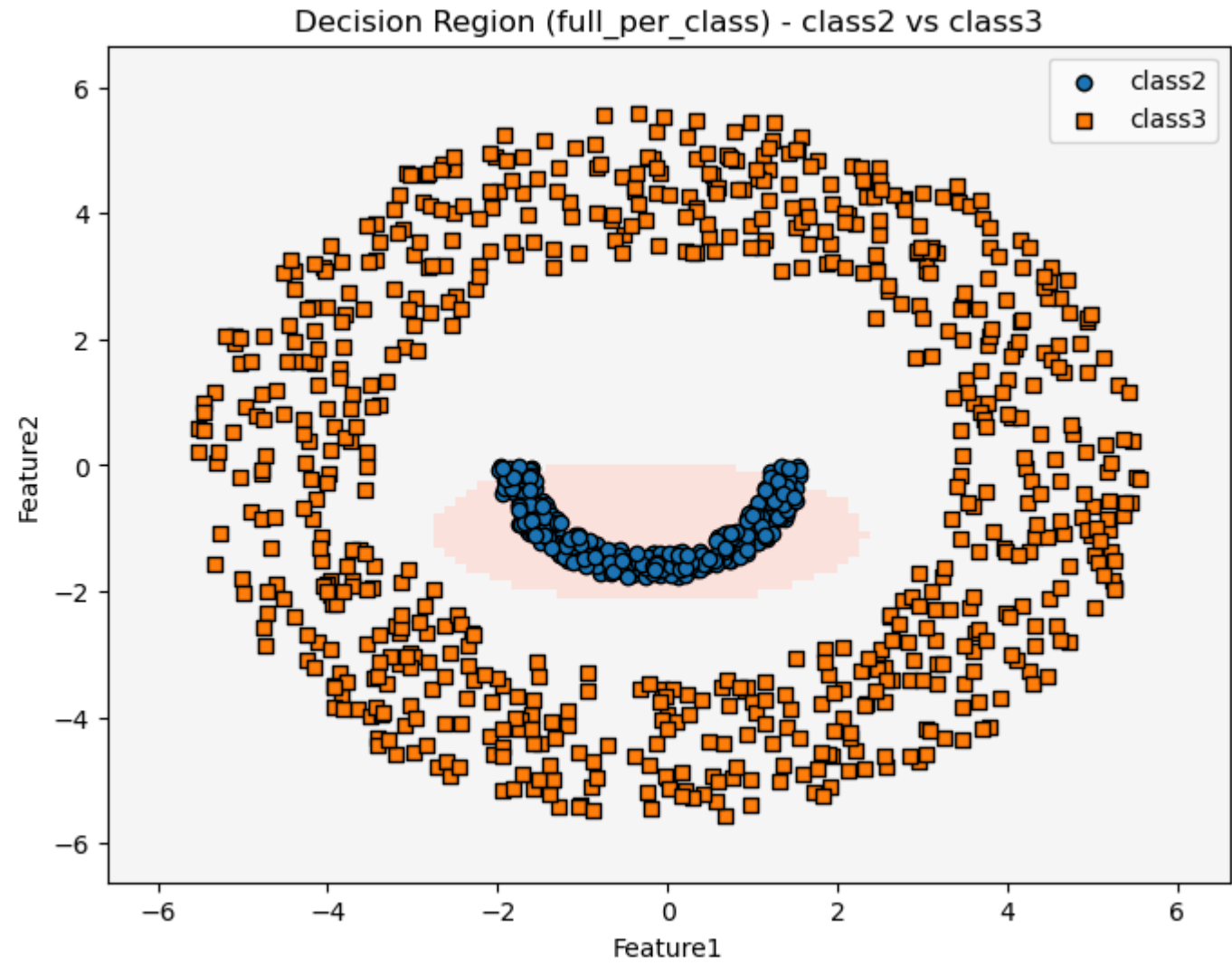
=== Confusion Matrix ===
class1 class2 class3
class1 90 0 0
class2 0 142 8
class3 0 0 300

=== Classification Report ===
Class Precision Recall F1-score Support
class1 1.0000 1.0000 1.0000 90
class2 1.0000 0.9467 0.9726 150
class3 0.9740 1.0000 0.9868 300

Accuracy: 0.9852
Mean Precision: 0.9913
Mean Recall : 0.9822
Mean F1 Score : 0.9865

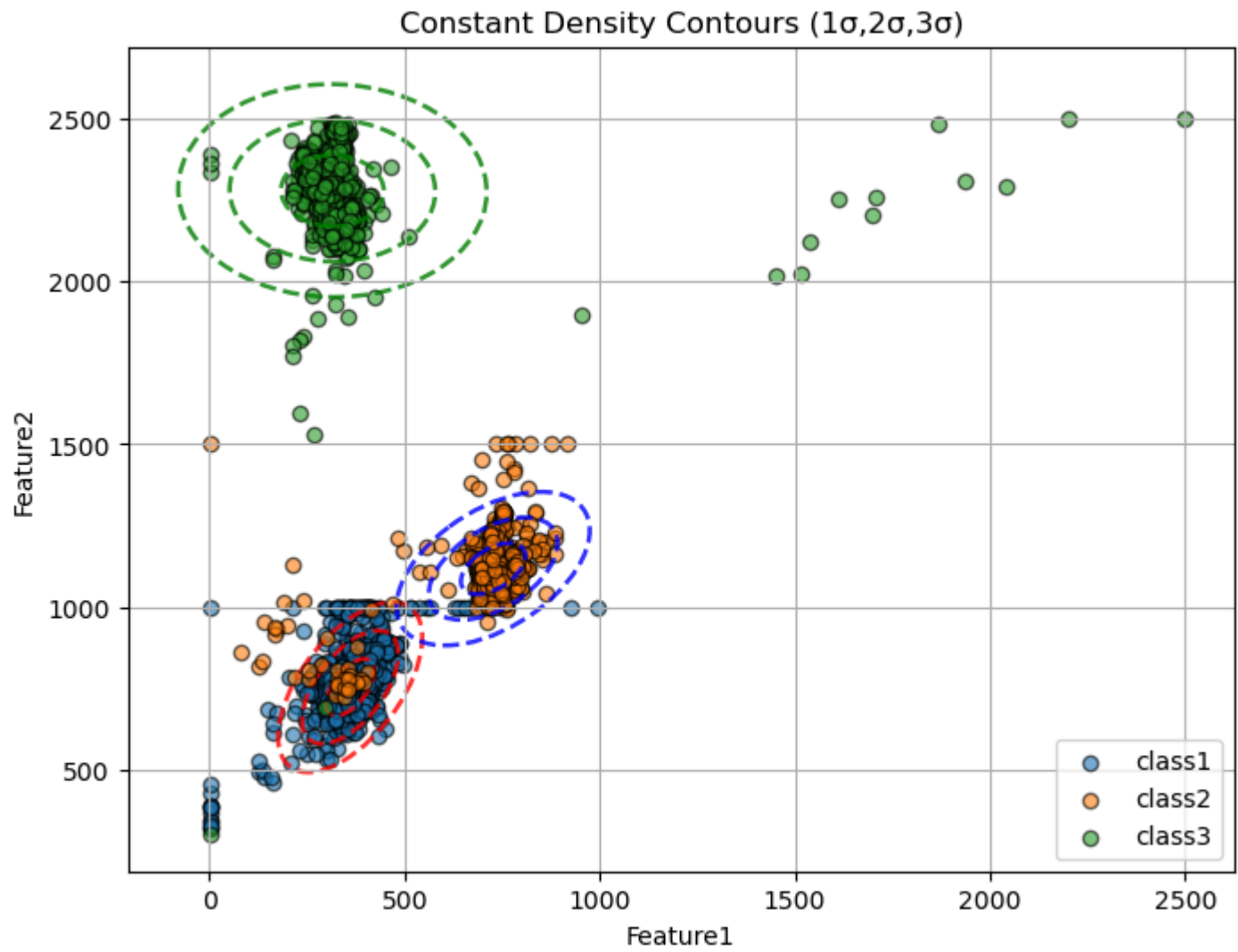




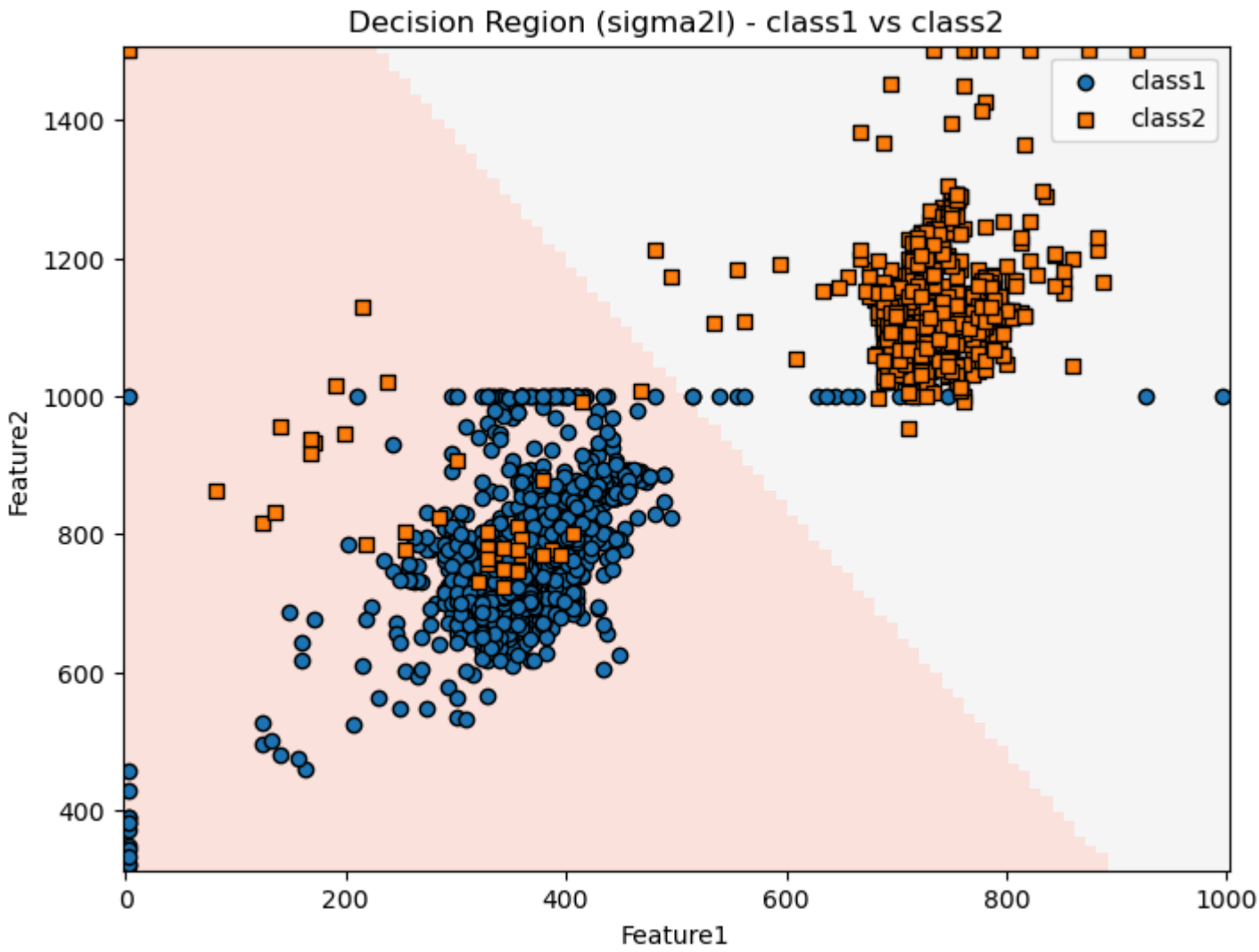
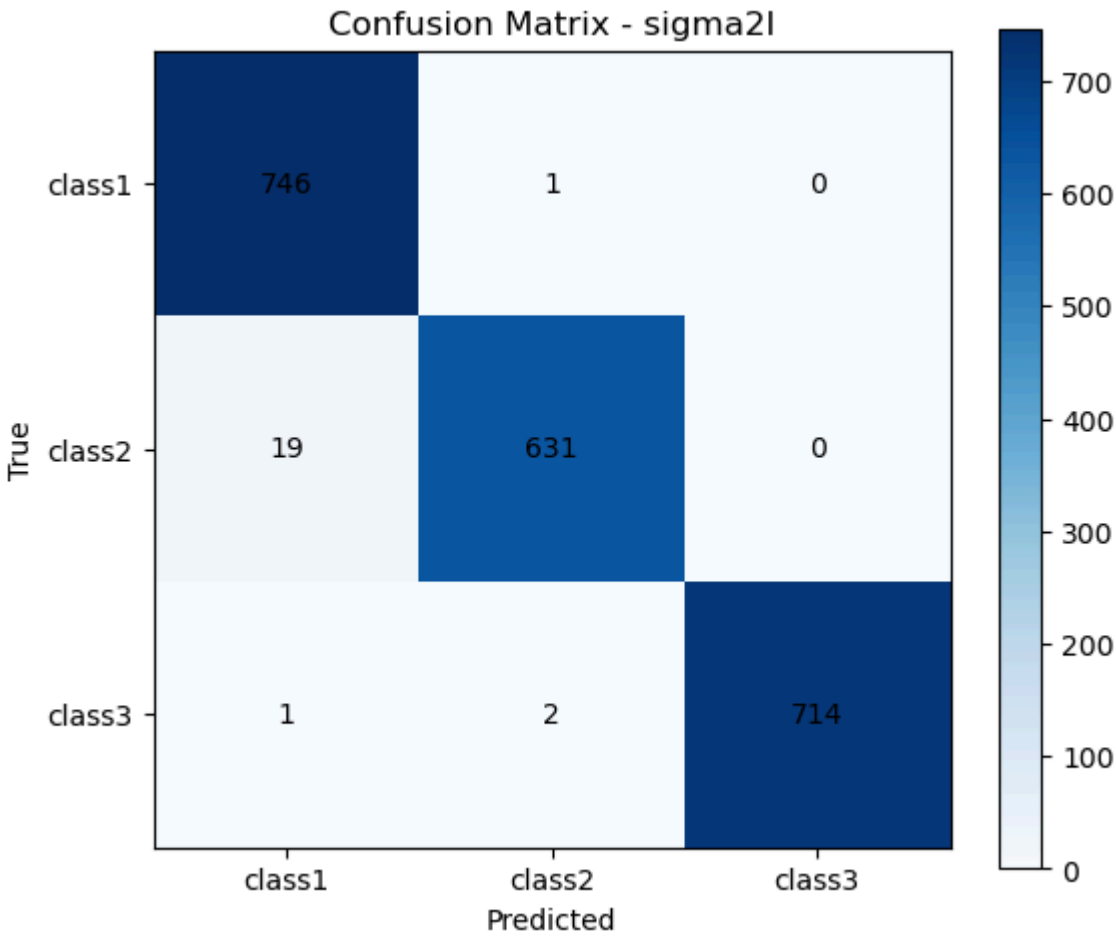


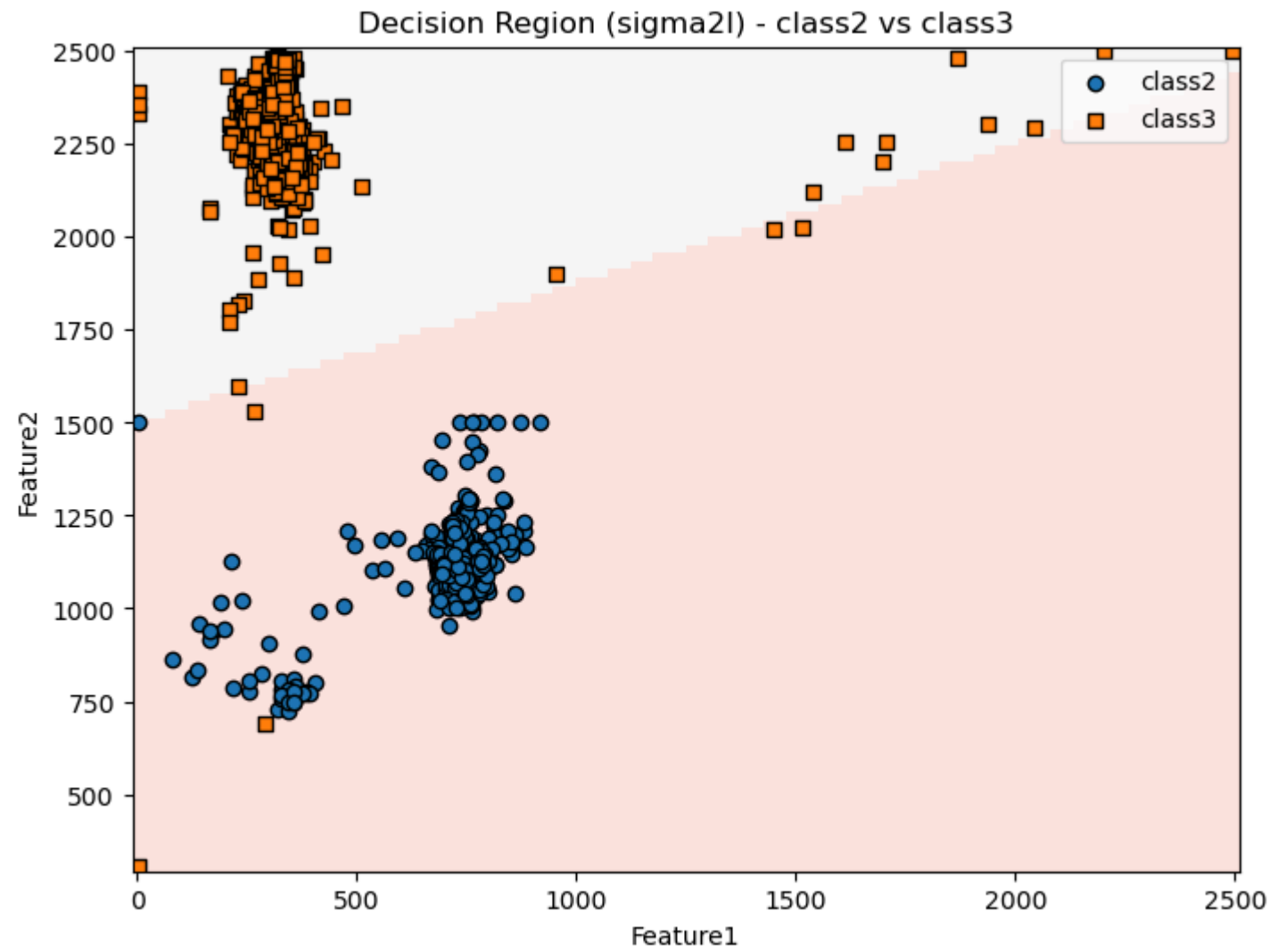
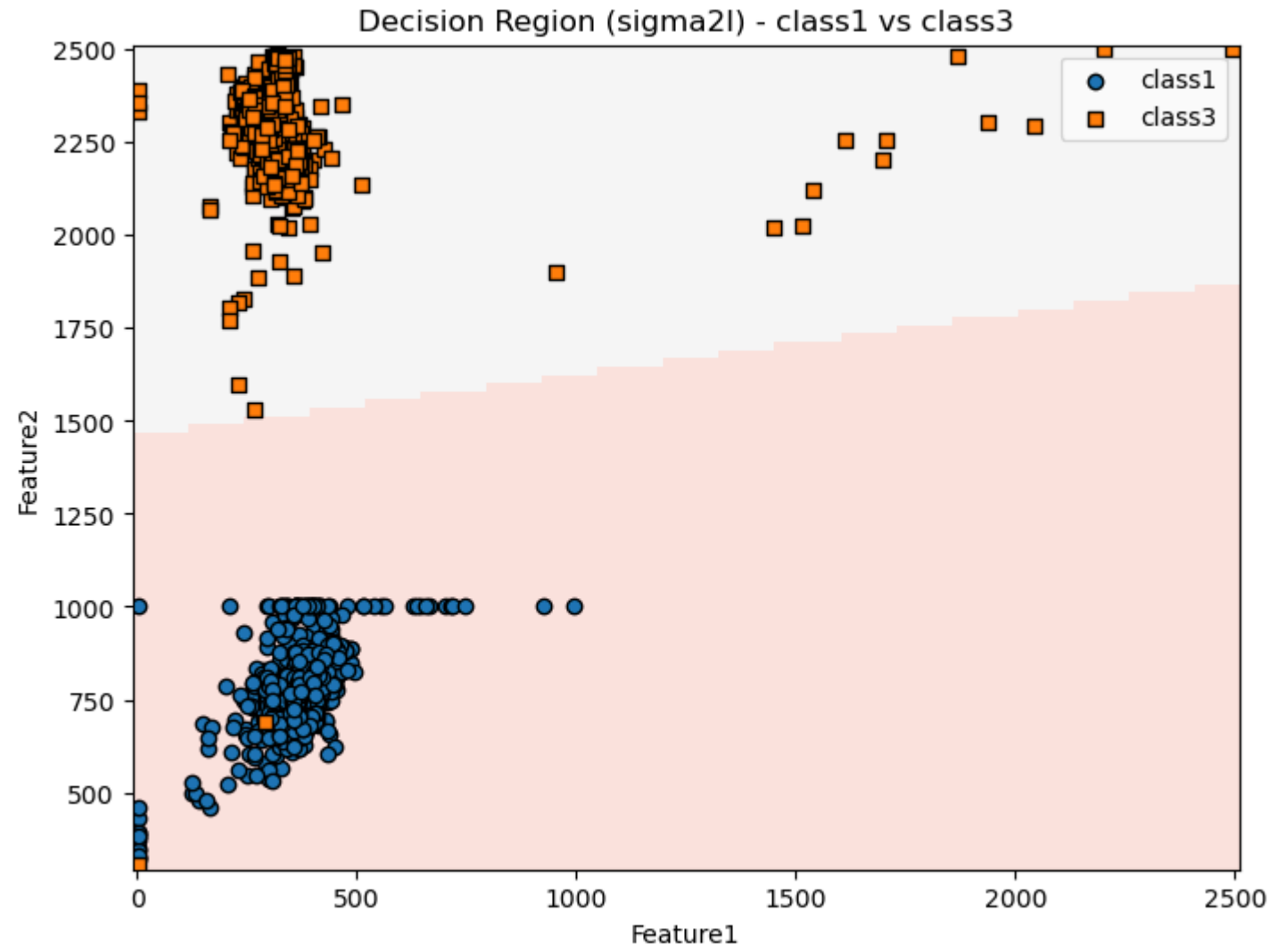
```
In [12]: # base_path = "../../../Dataset/Group04/LS_Group04/"
# base_path = "../../../Dataset/Group04/NLS_Group04/"
base_path = "../../../Dataset/Group04/rd_group4/"

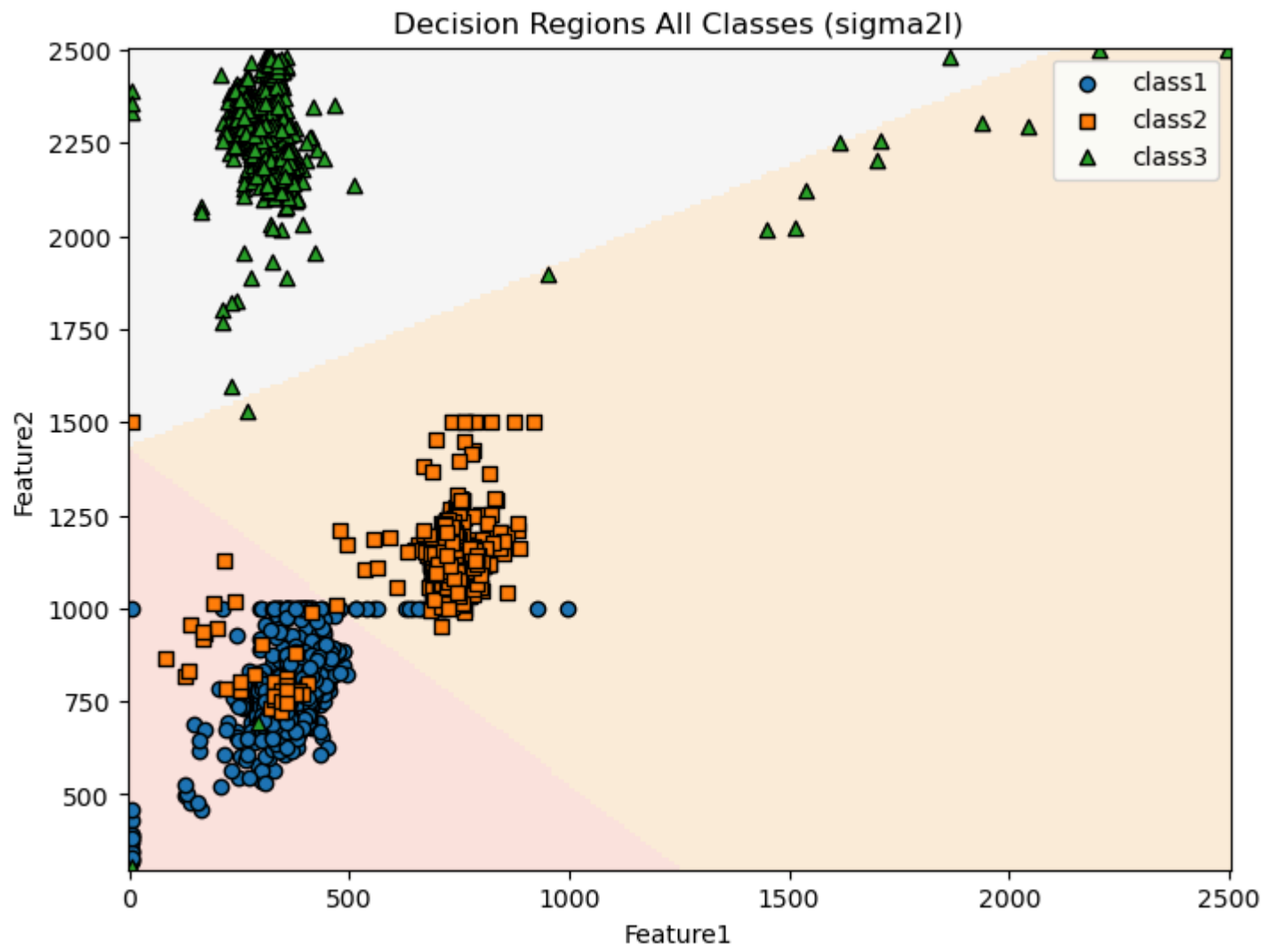
train_df, test_df = load_train_test_data(base_path)
run_all(train_df, test_df, res=200)
```



```
=====  
Classifier: sigma2I  
  
=== Confusion Matrix ===  
      class1 class2 class3  
class1  746     1      0  
class2   19    631     0  
class3    1      2    714  
  
=== Classification Report ===  
Class  Precision      Recall  F1-score      Support  
class1  0.9739          0.9987  0.9861         747  
class2  0.9953          0.9708  0.9829         650  
class3  1.0000          0.9958  0.9979         717  
  
Accuracy: 0.9891  
Mean Precision: 0.9897  
Mean Recall : 0.9884  
Mean F1 Score : 0.9890
```







=====
Classifier: shared_full

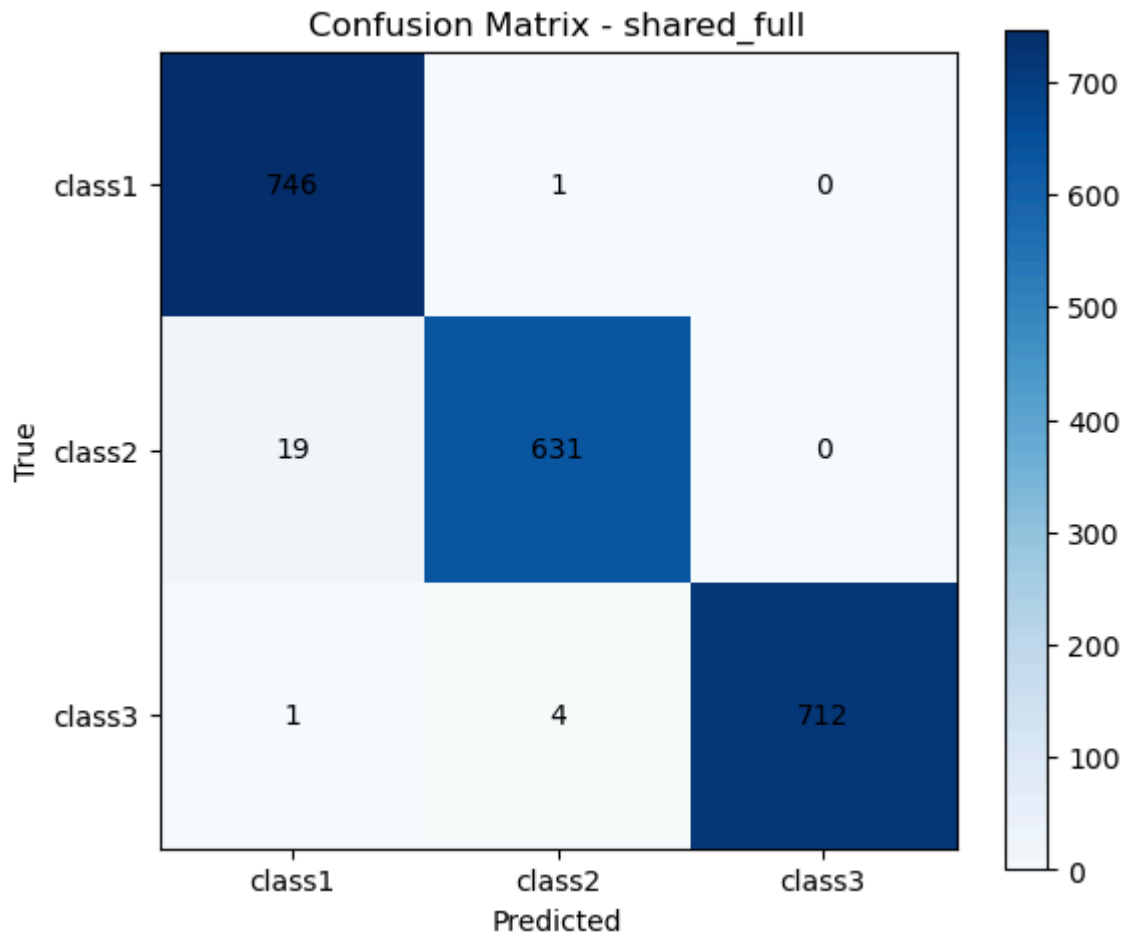
=== Confusion Matrix ===

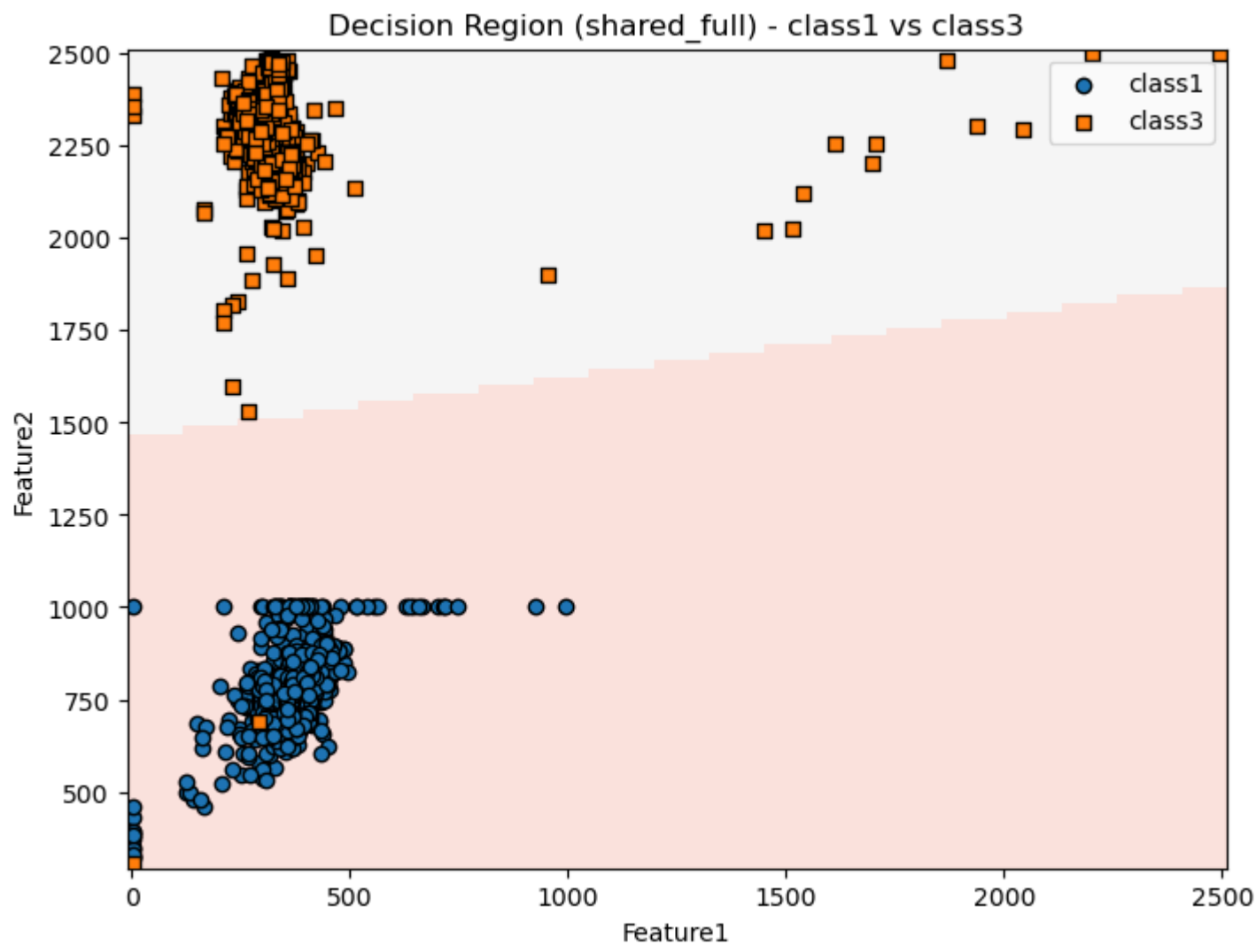
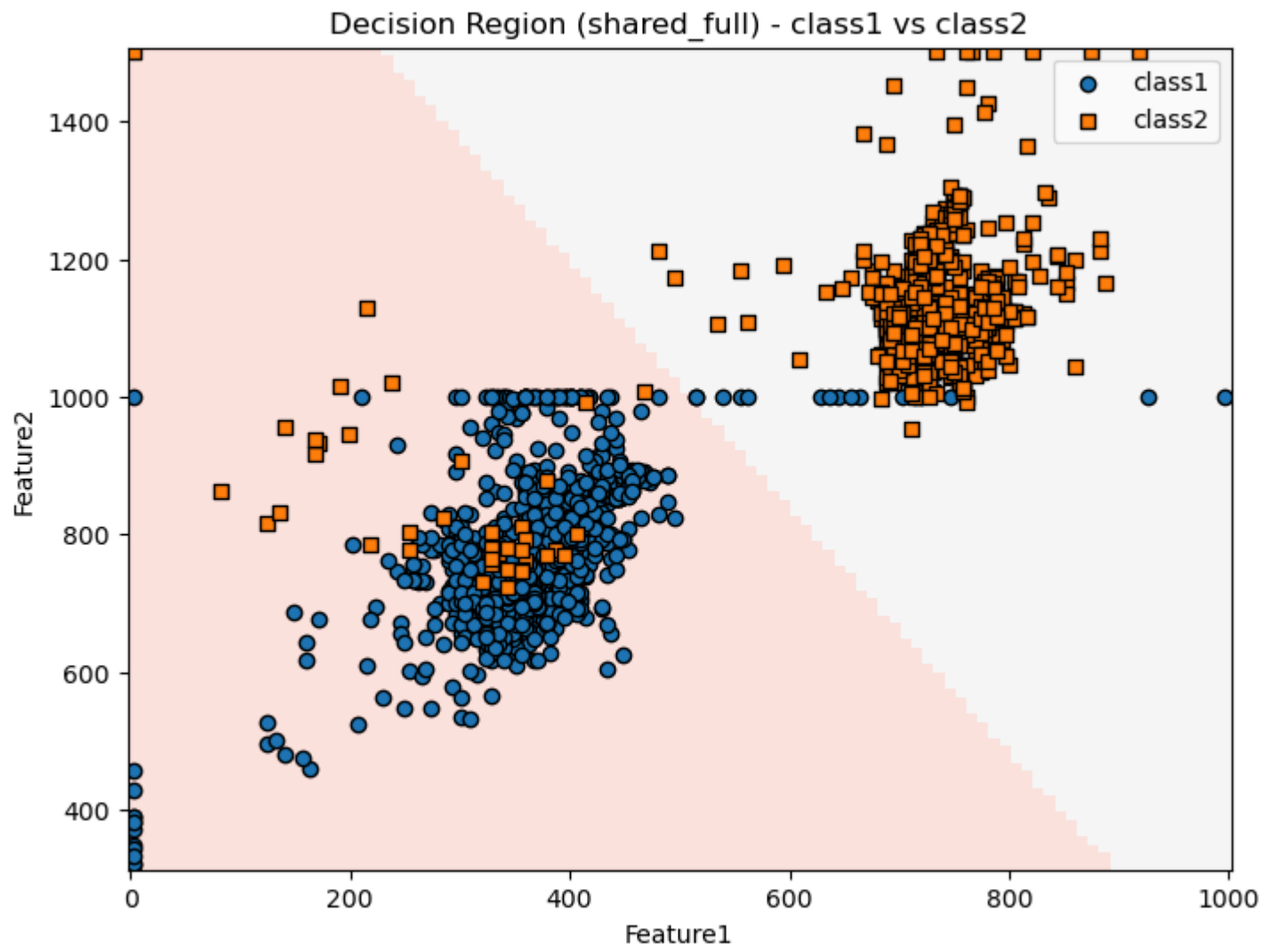
| | class1 | class2 | class3 |
|--------|--------|--------|--------|
| class1 | 746 | 1 | 0 |
| class2 | 19 | 631 | 0 |
| class3 | 1 | 4 | 712 |

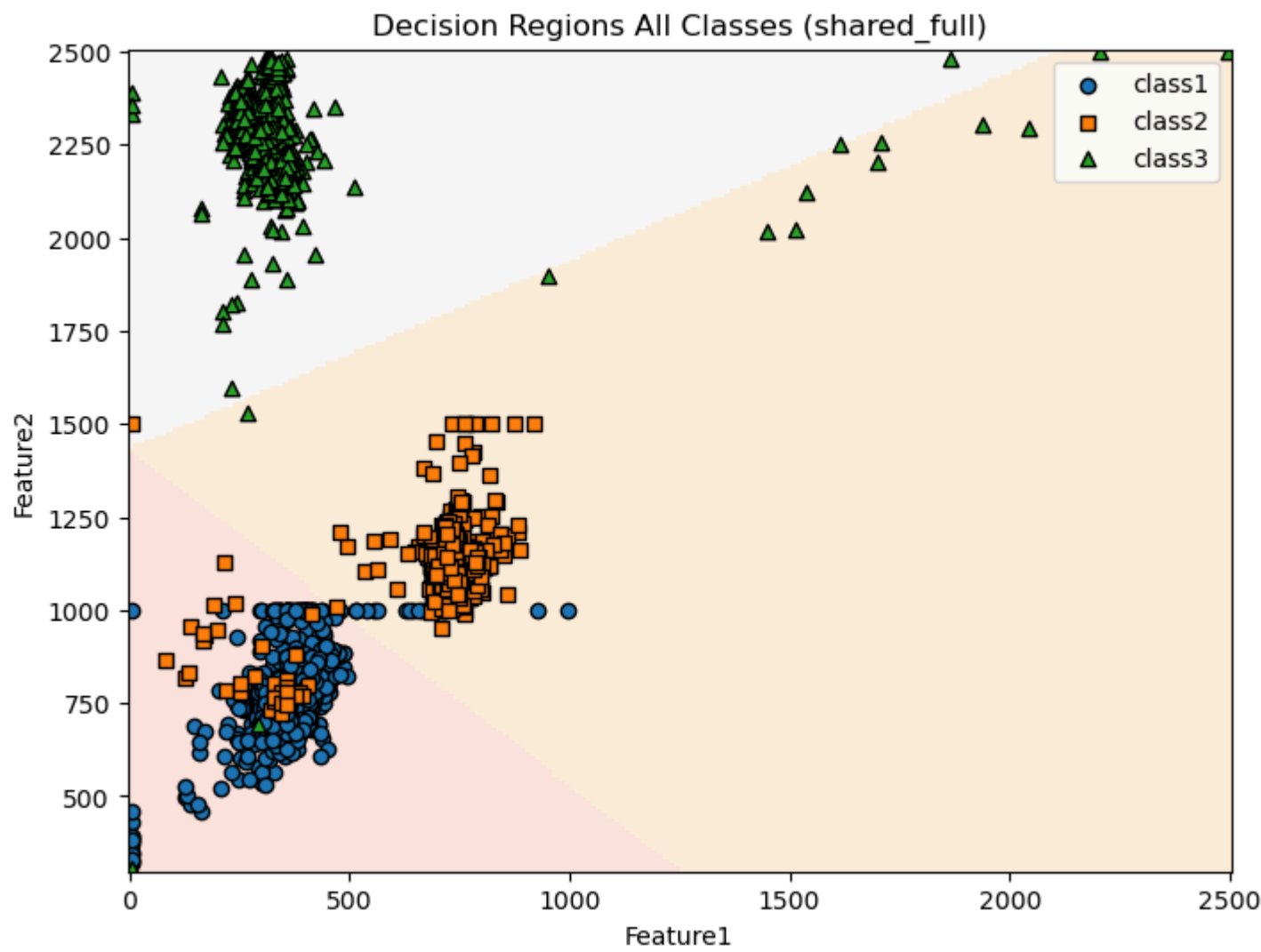
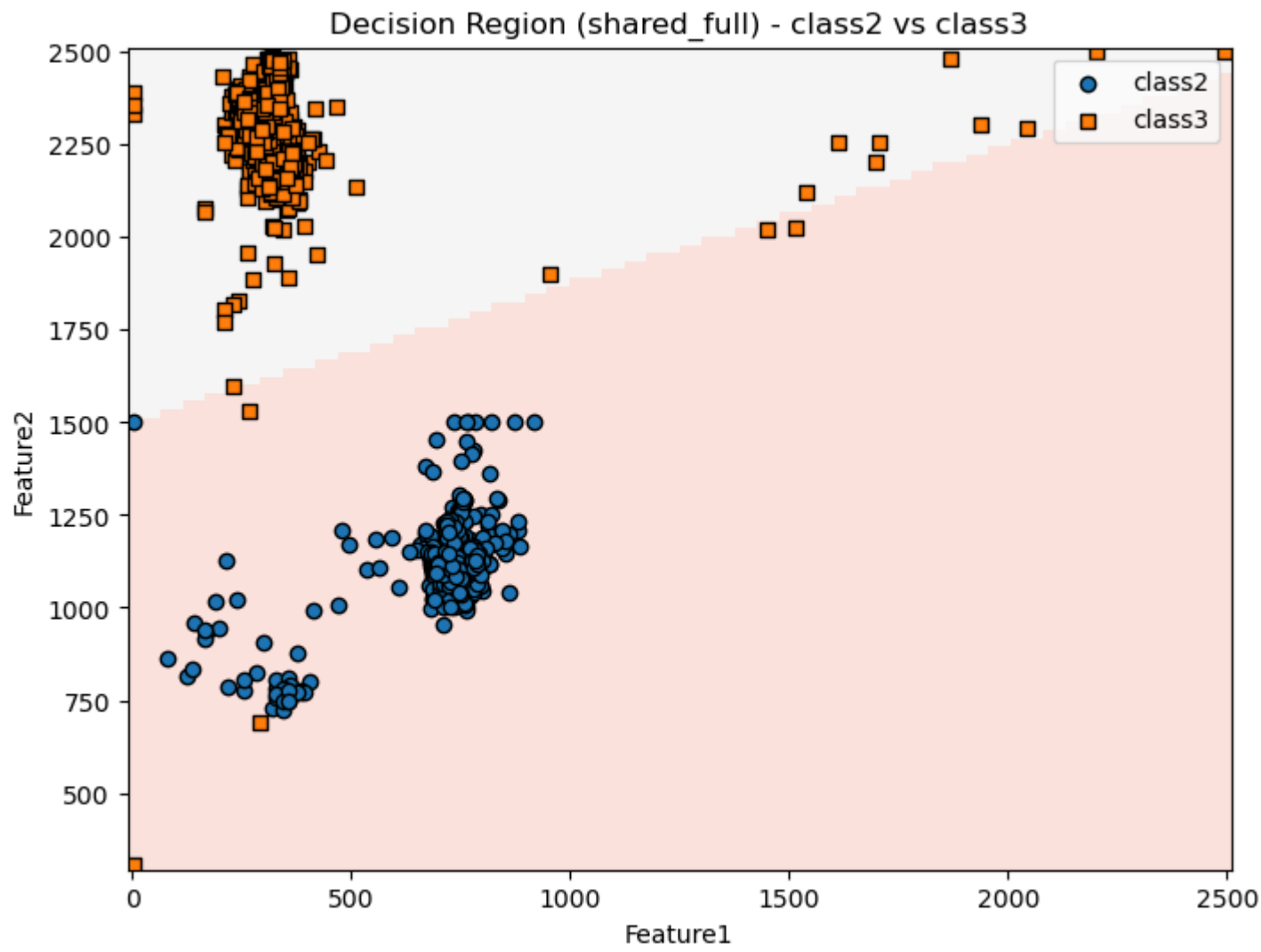
=== Classification Report ===

| Class | Precision | Recall | F1-score | Support |
|--------|-----------|--------|----------|---------|
| class1 | 0.9739 | 0.9987 | 0.9861 | 747 |
| class2 | 0.9921 | 0.9708 | 0.9813 | 650 |
| class3 | 1.0000 | 0.9930 | 0.9965 | 717 |

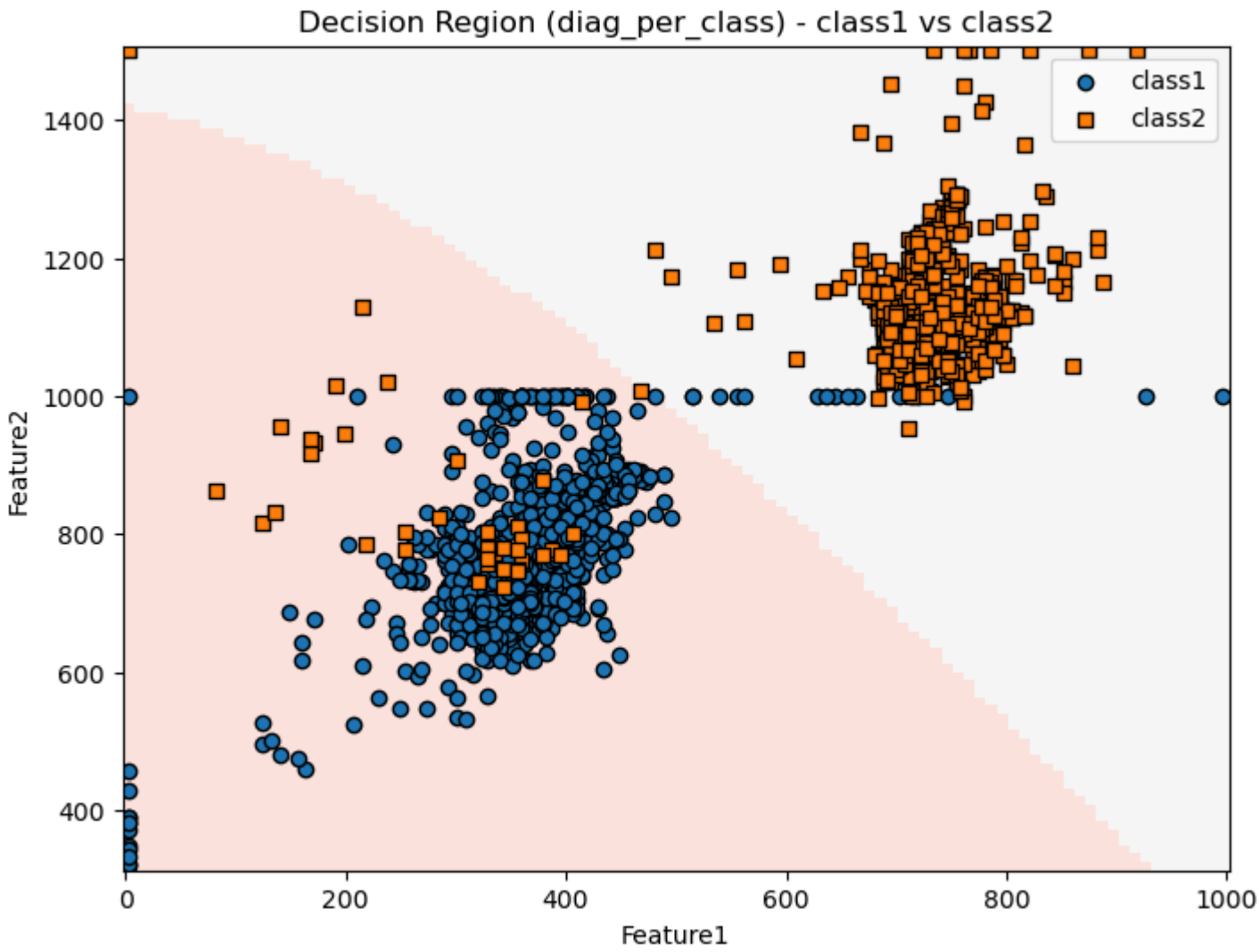
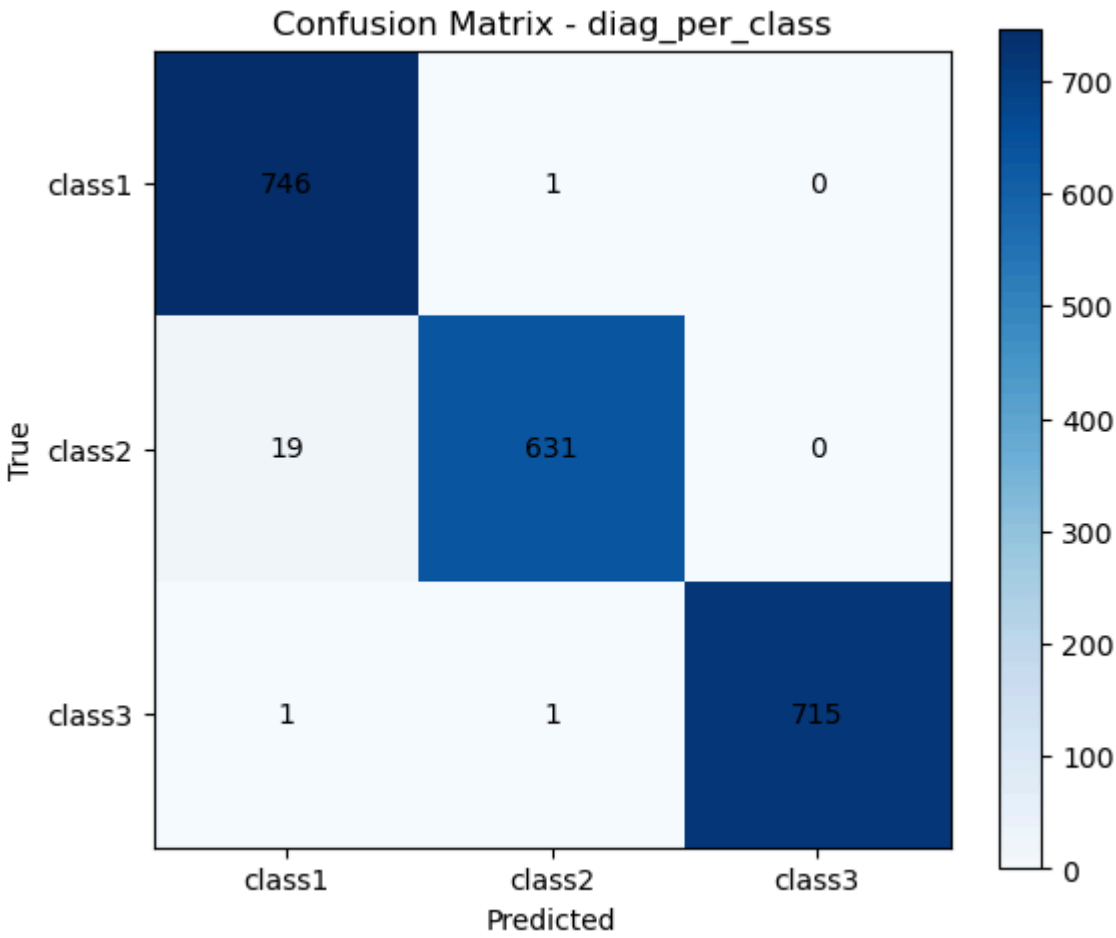
Accuracy: 0.9882
Mean Precision: 0.9887
Mean Recall : 0.9875
Mean F1 Score : 0.9880

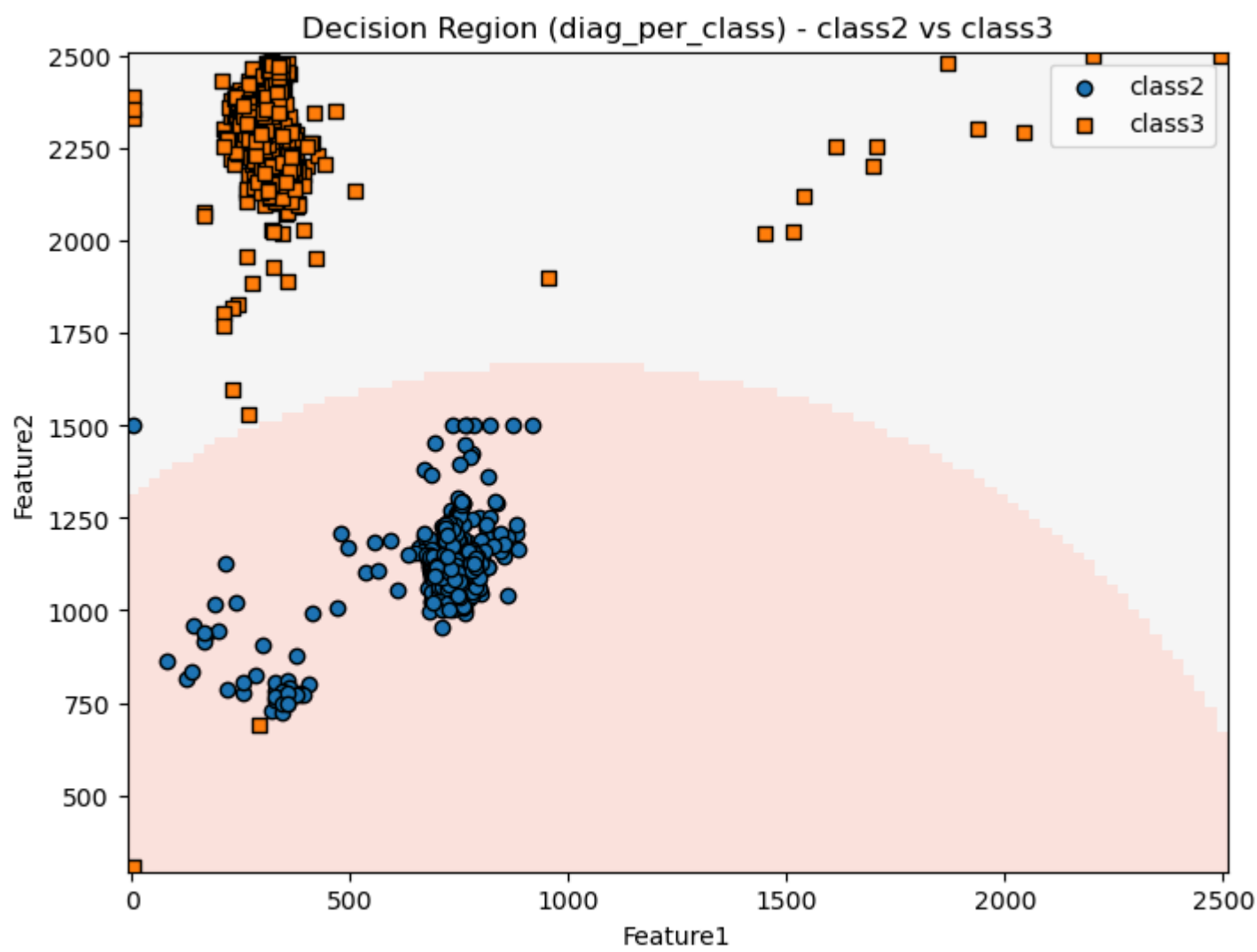
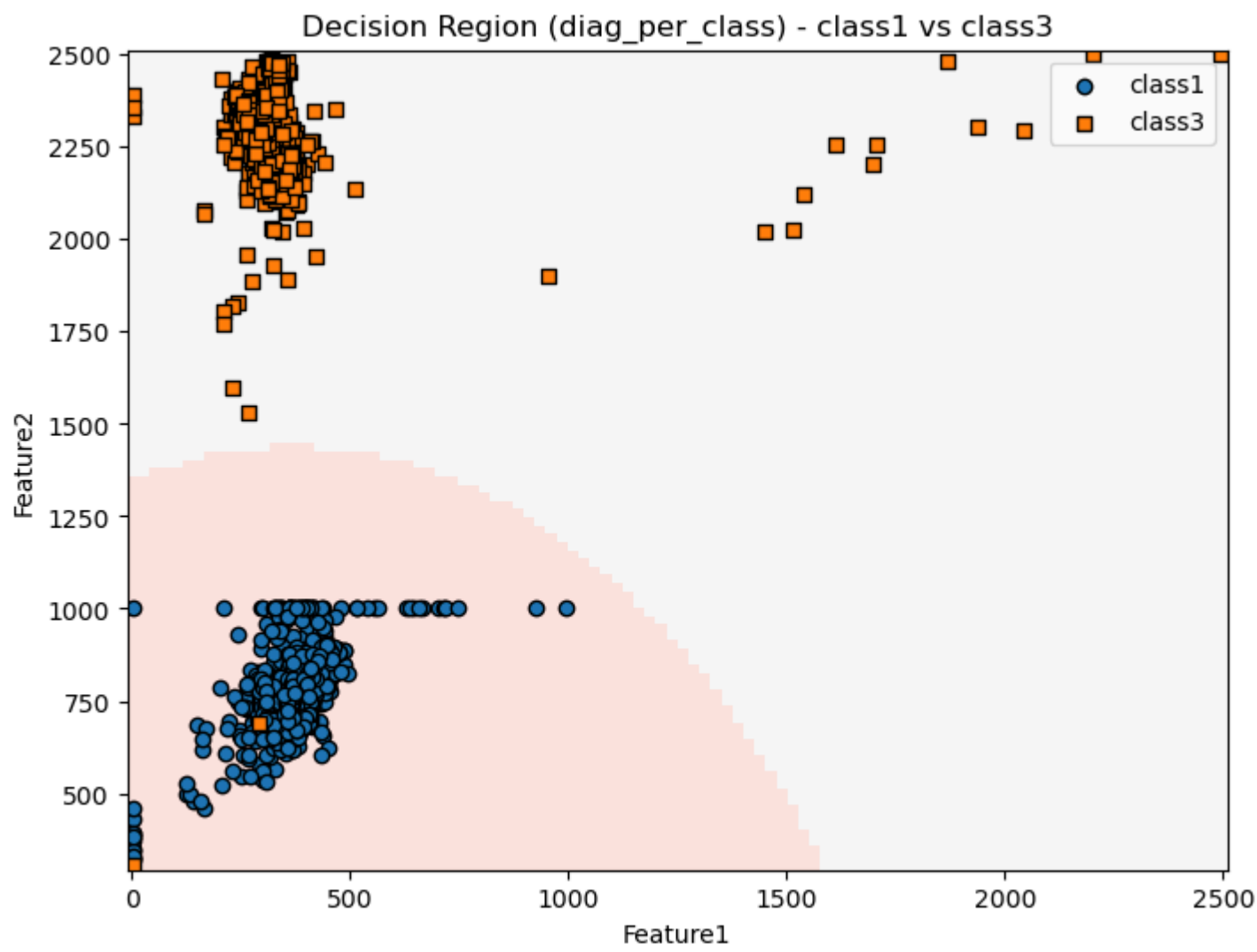


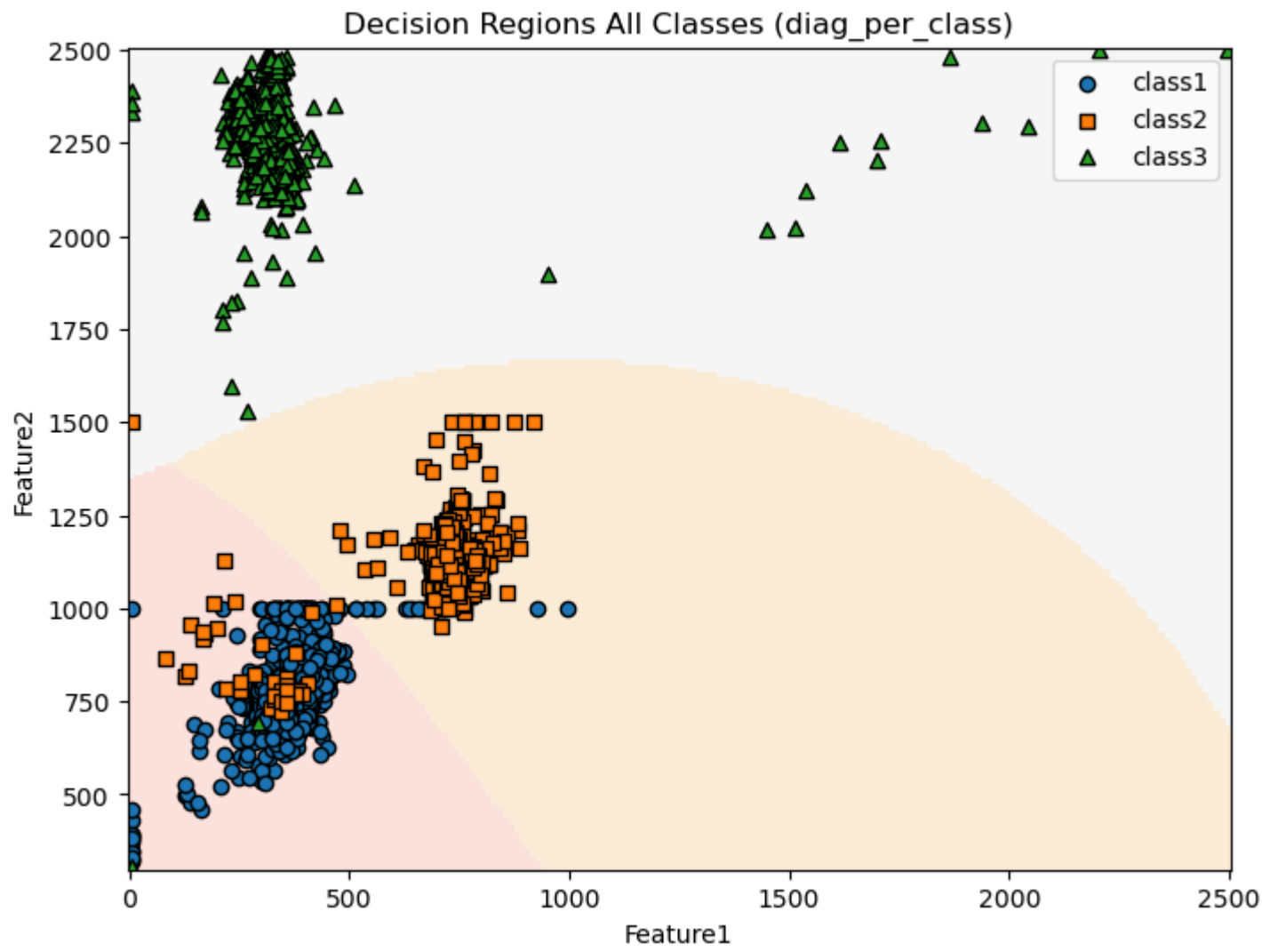




```
=====  
Classifier: diag_per_class  
  
=== Confusion Matrix ===  
      class1  class2  class3  
class1   746      1      0  
class2    19     631      0  
class3     1      1     715  
  
=== Classification Report ===  
Class  Precision      Recall  F1-score      Support  
class1  0.9739         0.9987  0.9861         747  
class2  0.9968         0.9708  0.9836         650  
class3  1.0000         0.9972  0.9986         717  
  
Accuracy: 0.9896  
Mean Precision: 0.9902  
Mean Recall   : 0.9889  
Mean F1 Score : 0.9895
```







=====
Classifier: full_per_class

=== Confusion Matrix ===

| | class1 | class2 | class3 |
|--------|--------|--------|--------|
| class1 | 745 | 2 | 0 |
| class2 | 19 | 631 | 0 |
| class3 | 1 | 1 | 715 |

=== Classification Report ===

| Class | Precision | Recall | F1-score | Support |
|--------|-----------|--------|----------|---------|
| class1 | 0.9739 | 0.9973 | 0.9854 | 747 |
| class2 | 0.9953 | 0.9708 | 0.9829 | 650 |
| class3 | 1.0000 | 0.9972 | 0.9986 | 717 |

Accuracy: 0.9891
Mean Precision: 0.9897
Mean Recall : 0.9884
Mean F1 Score : 0.9890

