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Date
Page

Q7) Given output = [H, T, H, H, T, H, H, H, H, H].

We can use the Bayes rule to find the posterior probability distribution of the coin parameter x given the observed data.

$$P(x|data) = \frac{P(data|x) \times P(x)}{P(data)}$$

likelihood

Prior probability.

normalizing constant.

$$P(X=x|Y=8) = \frac{P(Y=8|X=x) \cdot P(X=x)}{P(Y=8)}$$

likelihood of observing 8 heads.

The prior distribution of X is given by the Kumaraswamy distribution with $a=2$ and $b=3$:

~~$P(X=x)=$~~

$$P(X=x) = ab\, x^{(a-1)} \times (1-x^{a})^{(b-1)}$$
$$= 2 \cdot 3 x \times (1-x^2)^2.$$

$$P(Y=8|X=x) = {}^{10}C_8\, x^8 (1-x)^2.$$

likelihood of observing 8 heads given the value of X can be calculated using binomial distribution.

The marginal likelihood of observing 8 heads regardless of the value of X can be obtained: ~~by~~

$$P(Y=8) = \int_0^1 P(Y=8|X=x)P(X=x)\,dx.$$

Date
Page

This integral is not available in closed form but it can be approximated.

Substituting these expressions into Bayes' rule, we get:

$$P(X=x|Y=8) = \frac{{}^{10}C_8\, x^8 (1-x)^2 * 23x(1-x^2)^2}{P(Y=8)}.$$

To find $P(Y=8)$, we can integrate the joint distribution of X & Y over all possible values of X:

$$P(Y=8) = \int_0^1 P(Y=8|X=x)\, P(X=x)\, dx$$

$$= \int_0^1 {}^{10}C_8\, x^8 (1-x)^2 \times 23x(1-x^2)^2 dx.$$

$$= {}^{10}C_8\, 23 \int_0^1 x^9 (1-x)^2 (1-x^2)^2 dx.$$

~~This integral is not available in closed~~

$$= \frac{315}{64}.$$

$$\therefore P(X=x|Y=8) = 768x(1-x^2)^2.$$

This is the probability distribution of a heads result from the coin which gave us 8 heads & 2 tails in ten coin flips