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EMS714 Modern Robotics

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CONTENTS

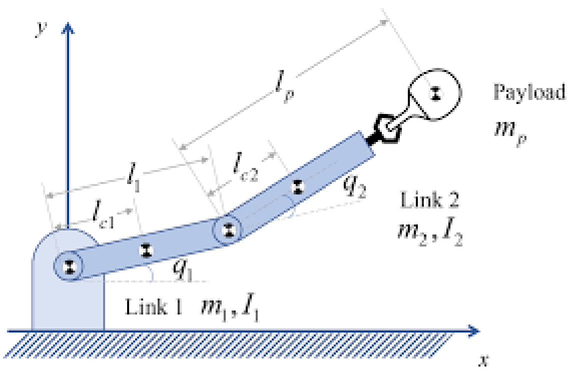
Introduction

Aim: To investigate the factors governing the performance of a simple servo-controlled two link robot manipulator, and relate these to basic analytical techniques.

Introduction:

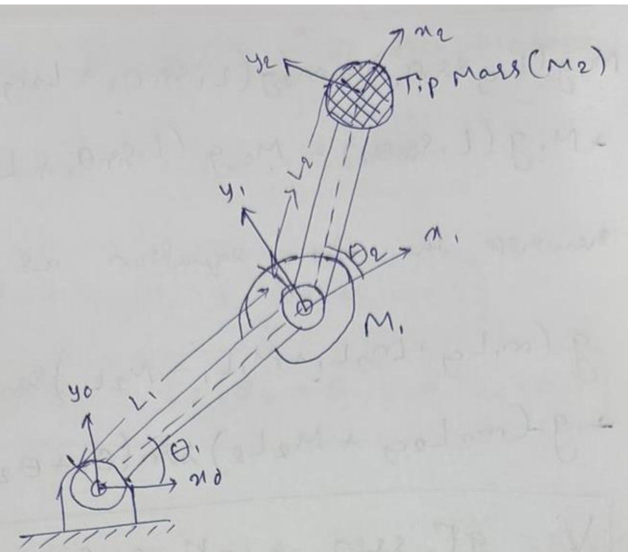
In robotic applications, the efficiency of a two-link servo-controlled robot manipulator is essential, especially for jobs requiring dexterity and accuracy. This simulation looks at the variables that affect its performance and assesses them using basic analytical methods. The study emphasizes the interaction between system parameters, control techniques, and overall functioning by concentrating on the manipulator's dynamics.

The calculated torque method, a popular robotics control strategy, is used in the analysis. This technique makes it possible to precisely regulate the manipulator's movements by simplifying its dynamics. The end-effector and payload are roughly represented as a lumped mass at the tip of the outer link in this study's ideal model of the two-link manipulator. Clear insights into the behavior of the system can be derived with the help of this modeling assumption.



Two link manipulator[1]

Background Theory



All the z-axis are Normally aligned to the plane

~~...~~ We know that M_2 is the tip mass m_i, L_i, L_{ig} & K_{ig} are respectively the i^{th} link mass, length, link position of Center of mass with reference to the i^{th} joint & i^{th} link radius of gyration about its Centre of mass

Let $\boxed{q_1 = \theta_1}$ be the angle of rotation with respect to
$\boxed{q_2 = \theta_2}$ local horizontal with respect to link 1 & 2

Assuming Counter Clockwise as positive.

$\therefore$ The height of the C.G of the 1^{st} link from the axis of 1^{st} joint is $\boxed{Y_1 = L_{1g} \sin\theta}$, & for 2^{nd} link $\boxed{Y_2 = L_1 \sin\theta_1 + L_2 \sin(\theta_2+\theta_1)}$

& for tip mass $\boxed{Y_{tip} = L_1 \sin\theta_1 + L_2 \sin(\theta_2+\theta_1)}$

$\therefore$ Potential energy of the body is

$V = m_1 g h_1 + m_2 g h_2 + M_1 g h_1 + M_2 g (h_{tip})$.

$\therefore \quad V = m_1 g (L_{1cg} \sin\theta_1) + m_2 g (L_1 \sin\theta_1 + L_{2cg} \sin(\theta_2 + \theta_1))$
$+ M_1 g (L_1 \sin\theta_1) + M_2 g (L_1 \sin\theta_1 + L_2 \sin(\theta_2 + \theta_1))$

We can rewrite the above equation as

$$V = g(m_1 L_{cg} + m_2 L_1 + M_1 L_1 + M_2 L_1)\sin\theta_1 + g(m_2 L_{2cg} + M_2 L_2)\sin(\theta_1 + \theta_2)$$

Let $\therefore \boxed{V = g r_{11} \sin\theta_1 + g r_{22} \sin(\theta_1 + \theta_2)}$

where $r_{11} = m_1 L_{1cg} + m_2 L_1 + M_1 L_1 + M_2 L_1$
$r_{22} = m_2 L_{2cg} + M_2 L_2$

Similarly the Horizontal position of the C.g of the 1st & 2nd link and the tip mass are.

$\boxed{x_1 = L_{1cg} \cos\theta}$, $\boxed{x_2 = L_1 \cos\theta_1 + L_{2cg} \cos(\theta_2 + \theta_1)}$

$\boxed{x_{tip} = L_1 \cos\theta_1 + L_2 \cos(\theta_2 + \theta_1)}$

$\boxed{x_{j1} = L_1 \cos\theta_1}$

Horizontal velocities of the c.g's of the masses.

$\boxed{\dot{x}_1 = -L_{1cg}\dot{\theta}_1 \sin\theta_1}$ $\boxed{\dot{x}_{j1} = -\dot{\theta}_1 L_1 \sin\theta_1}$

$\boxed{\dot{x}_2 = -L_1\ddot{\theta}_1 \sin\theta_1 - L_{2cg}(\dot{\theta}_2 + \dot{\theta}_1)\sin(\theta_2 + \theta_1)}$

$\boxed{\dot{x}_{tip} = -L_1\dot{\theta}_1 \sin\theta_1 - L_2(\dot{\theta}_2 + \dot{\theta}_1)\sin(\theta_2 + \theta_1)}$

Vertical velocity of the c.g of the masses

$\boxed{\dot{y}_1 = L_{1cg}\dot{\theta}_1 \cos\theta_1}$ $\boxed{\dot{y}_{j1} = L_1 \dot{\theta}_1 \cos\theta_1}$

$\boxed{\dot{y}_2 = L_1\dot{\theta}_1 \cos\theta_1 + L_{2cg}(\dot{\theta}_2 + \dot{\theta}_1)\cos(\theta_2 + \theta_1)}$

$\boxed{\dot{y}_{tip} = L_1\dot{\theta}_1 \cos\theta_1 + L_2(\dot{\theta}_2 + \dot{\theta}_1)\cos(\theta_2 + \theta_1)}$

Let T_1 & T_2 be Translational and Rotational kinetic Energy of the Manipulator such that $T = T_1 + T_2$ be the total kinetic Energy

$$\therefore T_1 = \frac{1}{2} m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} M_1(\dot{x}_{j1}^2 + \dot{y}_{j1}^2) + \frac{1}{2} m_2(\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2} M_2(\dot{x}_{tip}^2 + \dot{y}_{tip}^2)$$

Substituting ~~and~~ simplifying we get

$$T_1 = \frac{1}{2} m_1 L_{1cg}^2 \dot{\theta}_1^2 + \frac{1}{2} M_1 L_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2(L_1^2\dot{\theta}_1^2 + L_{2cg}^2(\dot{\theta}_1 + \dot{\theta}_2)^2) + \frac{1}{2} M_2(L_1^2\dot{\theta}_1^2 + L_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2) + (m_2 L_{2cg} + M_2 L_2) L_1 \cos\theta_2 (\dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2))$$

$$T_2 = \frac{1}{2} m_1 k_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 k_2^2 (\dot{\theta}_1 + \dot{\theta}_2)^2$$

$$\therefore T = T_1 + T_2 = \frac{1}{2}(m_1(L_{1cg}^2 + k_1^2) + (m_2 + M_1 + M_2)L_1^2)\dot{\theta}_1^2 + \frac{1}{2}(m_2(L_{2cg}^2 + k_2^2) + M_2 L_2^2)(\dot{\theta}_1 + \dot{\theta}_2)^2 + (m_2 L_{2cg} + M_2 L_2) L_1 \cos\theta_2 (\dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2))$$

$\therefore$ Total kinetic Energy can be expressed as.

$$T = \frac{1}{2} I_{11} \dot{\theta}_1^2 + \frac{1}{2} I_{22} (\dot{\theta}_1 + \dot{\theta}_2)^2 + I_{21} \cos\theta_2 \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2)$$

Here $\boxed{I_{11} = m_1 (L_{1cg}^2 + K_{1cg}^2) + M_1 L_1^2 + (m_2 + M_2) L_1^2}$

$\boxed{I_{21} = (m_2 L_{2cg} + M_2 L_2) L_1}$ $\boxed{I_{22} = m_2 (L_{2cg}^2 + K_{2cg}^2) + M_2 L_2^2}$

$I_{21} = r_{22} L_1$

According to Lagrangian Equation $L = T - V$.

and Euler Lagrange equations are $\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i$

where $q_i = \theta_i$

$$\frac{\partial T}{\partial \dot{\theta}_1} = I_{11} \dot{\theta}_1 + I_{22} (\dot{\theta}_1 + \dot{\theta}_2) + I_{21} (2\dot{\theta}_1 + \dot{\theta}_2) \cos\theta_2 .$$

$$\frac{\partial T}{\partial \dot{\theta}_2} = I_{21} \dot{\theta}_1 \cos\theta_2 + I_{22} (\dot{\theta}_1 + \dot{\theta}_2)$$

$$\frac{\partial T}{\partial \theta_1} = 0 \qquad \frac{\partial T}{\partial \theta_2} = - I_{21} \sin\theta_2 (\dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2))$$

$$\frac{\partial V}{\partial q_1} = g r_{11} \cos\theta_1 + g r_{22} \cos(\theta_1 + \theta_2)$$

$$\frac{\partial V}{\partial q_2} = g r_{22} \cos(\theta_1 + \theta_2)$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = I_{11} \theta_1 + I_{22} (\dot{\theta}_1 + \dot{\theta}_2) + I_{21} (2\dot{\theta}_1 + \dot{\theta}_2) \cos\theta_2 .$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = I_{21} \dot{\theta}_1 \cos\theta_2 + I_{22} (\dot{\theta}_1 + \dot{\theta}_2)$$

$$\frac{\partial L}{\partial q_1} = - g r_1 \quad , \quad \frac{\partial L}{\partial q_2} = - g r_2 - I_{21} \sin\theta_2 (\dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2))$$

with $r_1 = r_{11}\cos\theta_1 + r_{22}\cos(\theta_1+\theta_2)$

$r_2 = r_{22}\cos(\theta_1+\theta_2)$

$\therefore$ the Euler-Lagrange Equation can be put in Matrix form as

$$\begin{bmatrix} I_{11}+I_{21}\cos\theta_2 & I_{22}+I_{21}\cos\theta_2 \\ I_{21}\cos\theta_2 & I_{22} \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_1+\ddot{\theta}_2 \end{bmatrix} + I_{21}\sin\theta_2\begin{bmatrix} -\dot{\theta}_2\ (2\dot{\theta}_1+\dot{\theta}_2) \\ \dot{\theta}_1^2 \end{bmatrix}$$

$$+ g\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

also $\omega_1 = \dot{\theta}_1$

$w_2 = \dot{\theta}_1 + \dot{\theta}_2$

We can also write Euler-Lagrange Equation can be written in state-space Equation as

$$\begin{bmatrix} I_{11}+I_{21}\cos\theta_2 & I_{22}+I_{21}\cos\theta_2 \\ I_{21}\cos\theta_2 & I_{22} \end{bmatrix}\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} + I_{21}\sin\theta_2\left\{\begin{pmatrix} \omega_1^2 - w_2^2 \\ \omega_1^2 \end{pmatrix}\right\}$$

$$+ g\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

where $r_1 = r_{11}\cos(\theta_1) + r_{22}\cos(\theta_1+\theta_2)$

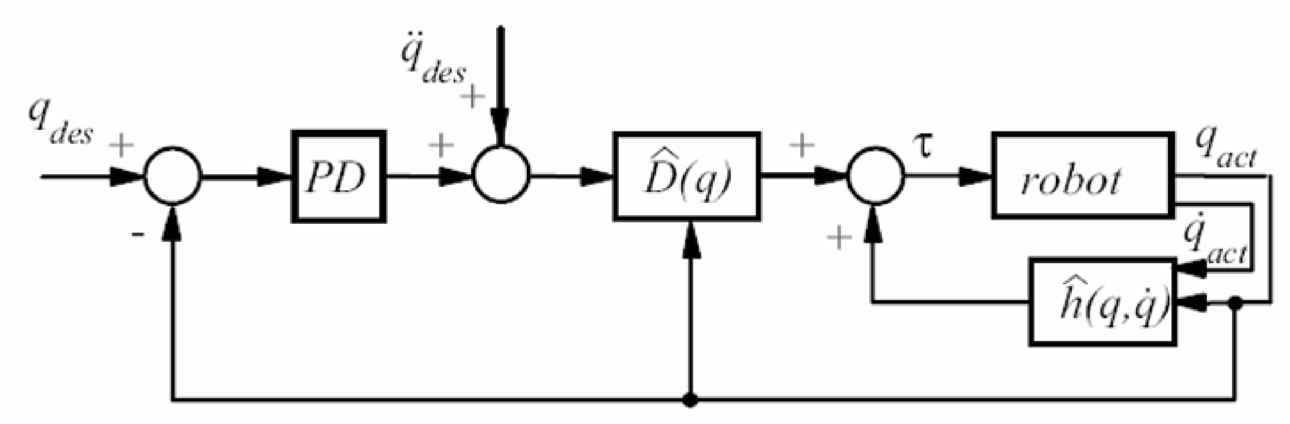
$r_2 = r_{22}\cos(\theta_1+\theta_2)$

Computed Torque Control model:

Computed torque control is a model-based control strategy designed to ensure precise motion of robot manipulators by compensating for the nonlinear dynamics affecting the system. This approach feeds back signals to each joint actuator to cancel disturbances arising from gravity, friction, manipulator inertia, as well as Coriolis and centrifugal forces. These disturbances are calculated using the robot's dynamic model, typically derived from the Euler-Lagrange equations. Once these effects are negated, additional feedback mechanisms introduce desired dynamics such as inertia, viscous damping, and stiffness, ensuring that the error dynamics achieve the desired performance.

Strategy for Computed torque control model:

The strategy of Computed torque control involves feeding back, to each of the joint servos, a signal that cancels all of the effects of gravity, friction, the manipulator inertia torques as well as the Coriolis and centrifugal torques. All these forces are computed on the basis of Euler-Lagrange dynamic model. All these effects are treated as disturbances that must be cancelled at each of the joints. Additional feedbacks are then provided to put in place the desired inertia torques, viscous friction torques and stiffness torques which are selected so the error dynamics behaves in a desirable and prescribed manner



Strategy for computed torque control [2]

PROCEDURE

Step 1: Derive the Dynamic Equations of the System

1. Using the **Euler-Lagrange formulation** we derive the equations of motion for the two-link manipulator:
2. We define the joint coordinates for q1 and q2 and also find velocity and acceleration terms for the same
3. Using the above data we can find the Kinetic energy (T) anad potential energy(V) of the system
4. Using the kinetic and the potential energy we can find the lagrangian, L=T-V, and further use the lagrange equation to obtain the manipulator dynamics

Step 2: Defining the control Law

$$\tau=M(q)v+C(q,\dot{q})\dot{q}+G(q),$$

where:

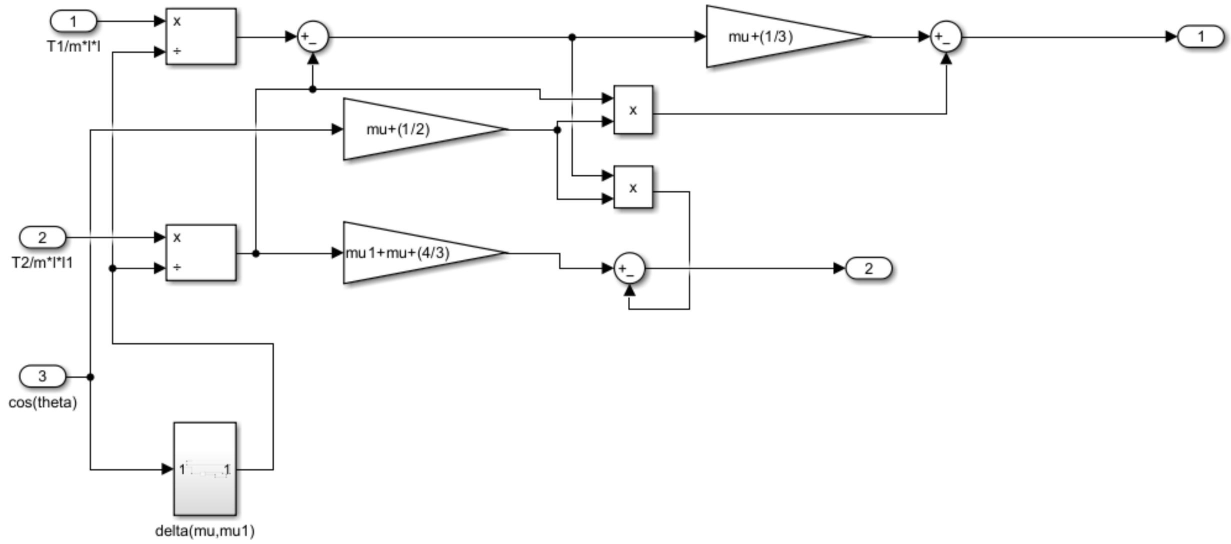
$$v = \ddot{q}_d + K_v(\dot{q}_d - \dot{q}) + K_p(q_d - q).$$

choosing the appropriate Kp and Kv gains for the desired closed loop dynamics

For the first instance of the Simulink model we employ the below equations :

$$\theta_1 = \omega_1,\ \theta_2 = \omega_2 - \omega_1,$$

$$\begin{bmatrix}\dot{\omega}_1\\ \dot{\omega}_2\end{bmatrix} = \mathbf{I}_{robot}^{-1}\left\{\frac{1}{mL^2}\begin{bmatrix}T_1\\ T_2\end{bmatrix} - \left(\frac{1}{2}+\mu_2\right)\sin(\theta_2)\begin{bmatrix}\omega_1^2-\omega_2^2\\ \omega_1^2\end{bmatrix} - \frac{g}{L}\begin{bmatrix}\left(\frac{3}{2}+\mu_1+\mu_2\right)\\ \left(\frac{1}{2}+\mu_2\right)\end{bmatrix}\right\},$$

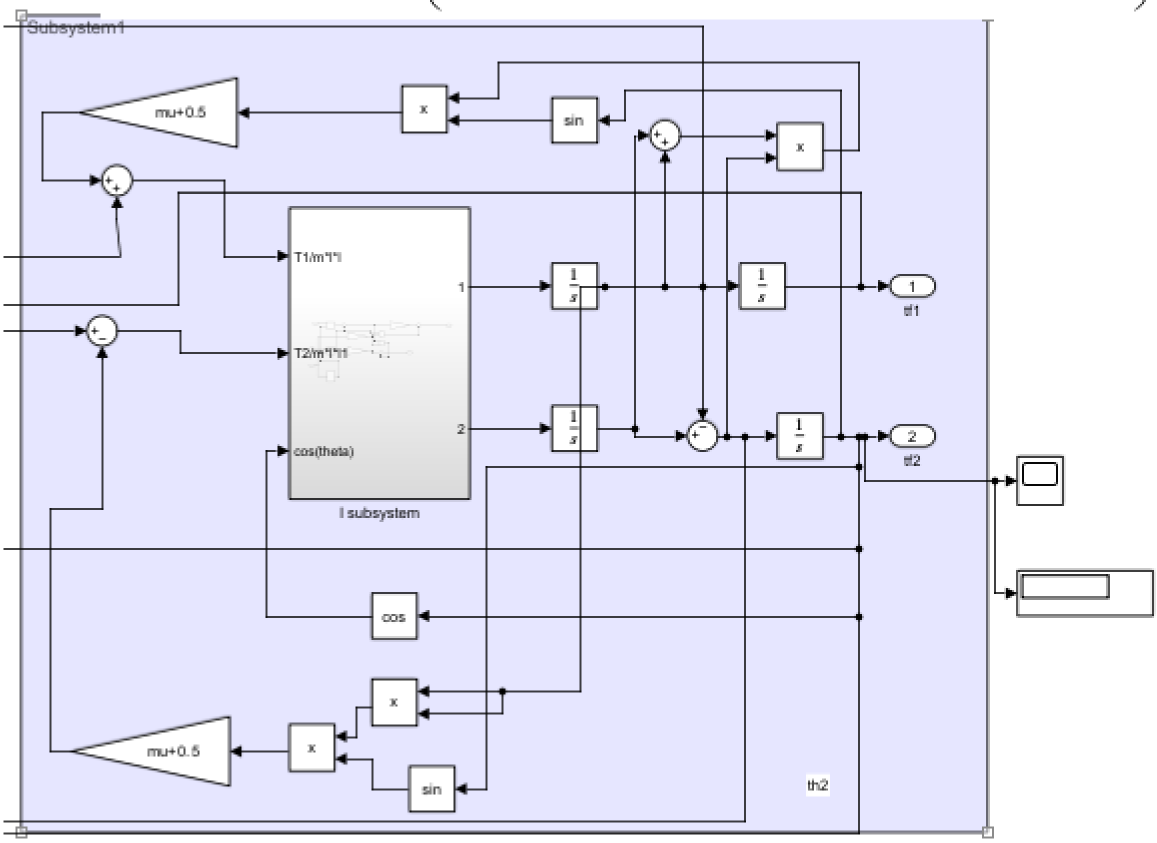


[3] The $\mathbf{I}_{robot}^{-1}$ subsystem

Where ,

$$\mathbf{I}_{robot}^{-1}=\frac{1}{\Delta(\mu_1,\mu_2)}\begin{bmatrix}\left(\frac{1}{3}+\mu_2\right) & -\left(\frac{1}{2}+\mu_2\right)\cos(\theta_2)-\left(\frac{1}{3}+\mu_2\right)\\ -\left(\frac{1}{2}+\mu_2\right)\cos(\theta_2) & \left(\frac{4}{3}+\mu_1+\mu_2\right)+\left(\frac{1}{2}+\mu_2\right)\cos(\theta_2)\end{bmatrix},$$

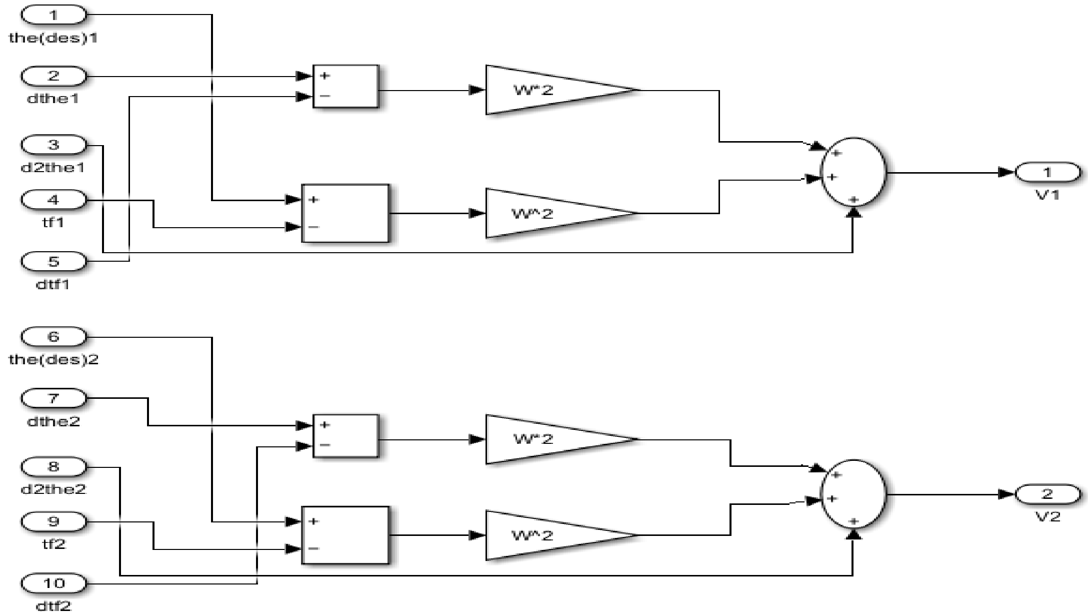
$$\Delta(\mu_1,\mu_2)=\left(\frac{7}{36}+\frac{2}{3}\mu_2+\mu_1\left(\mu_2+\frac{1}{3}\right)+\left(\mu_2+\frac{1}{2}\right)^2\sin^2\theta_2\right).$$



[3]

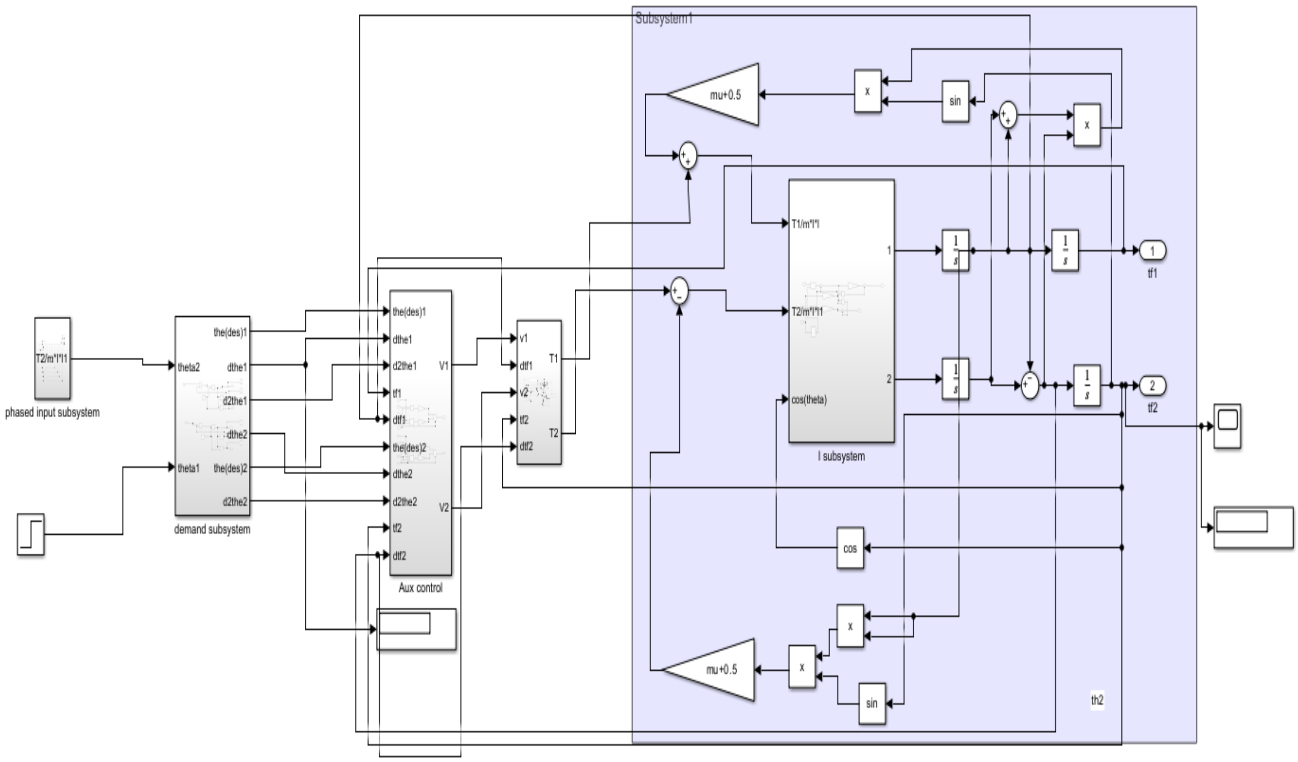
The auxiliary control inputs for the system are given by the equation:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_{1d} \\ \ddot{\theta}_{2d} \end{bmatrix} + 2\omega_n \begin{bmatrix} \dot{\theta}_{1d} \\ \dot{\theta}_{2d} \end{bmatrix} + \omega_n^2 \begin{bmatrix} \theta_{1d} \\ \theta_{2d} \end{bmatrix} - 2\omega_n \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} - \omega_n^2 \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}.$$



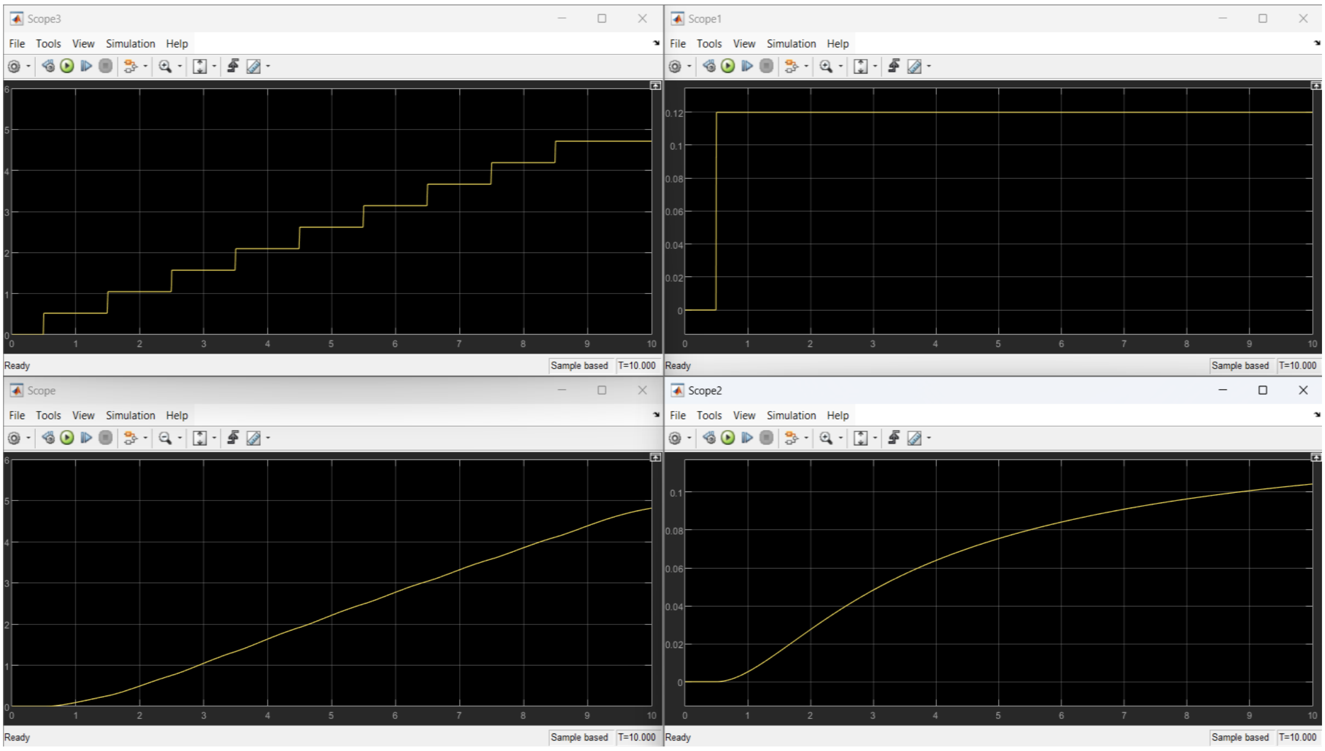
Aux control input in Simulink model[3]

The entire Simulink model can be represented as:



[3]

OBSERVATIONS

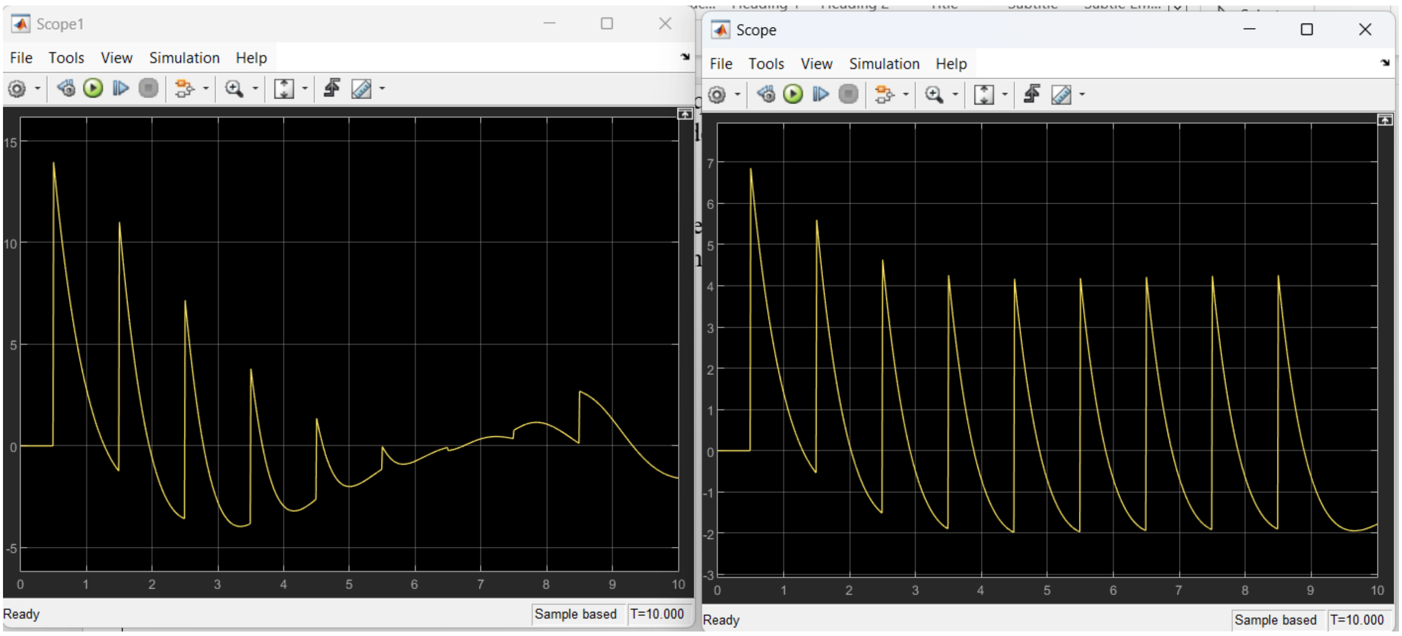


[3]

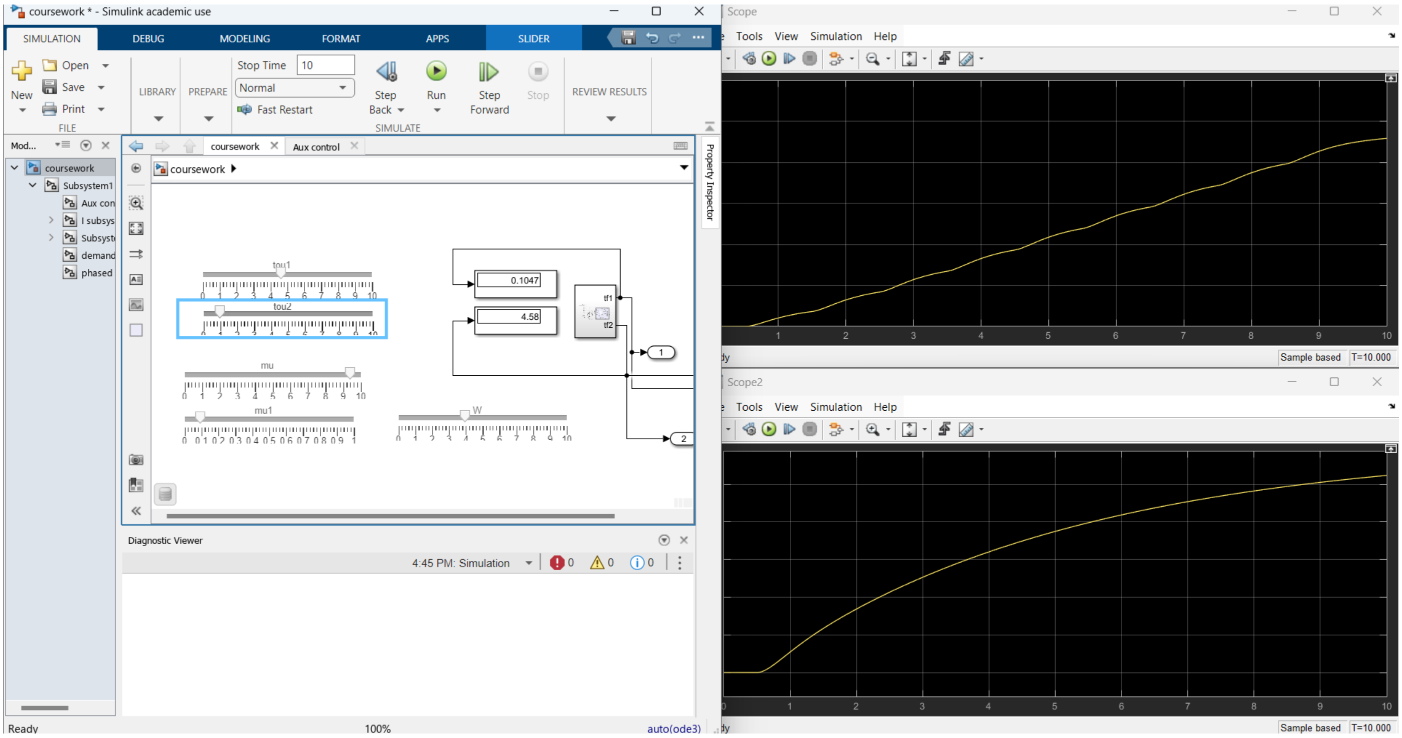
The above scope represents the step and the phase input provided to q1 and q2 (q1des and q2des) and the corresponding output (q1act and q2 act)

From the results it is observed that the computed torque control gives a smooth curve such that the robot can smoothly reach to the desired end point.

Accordingly it applies more torque to arm 1 compared to arm2 as the intial toruq must be high for the first arm which is observed in the below graphs

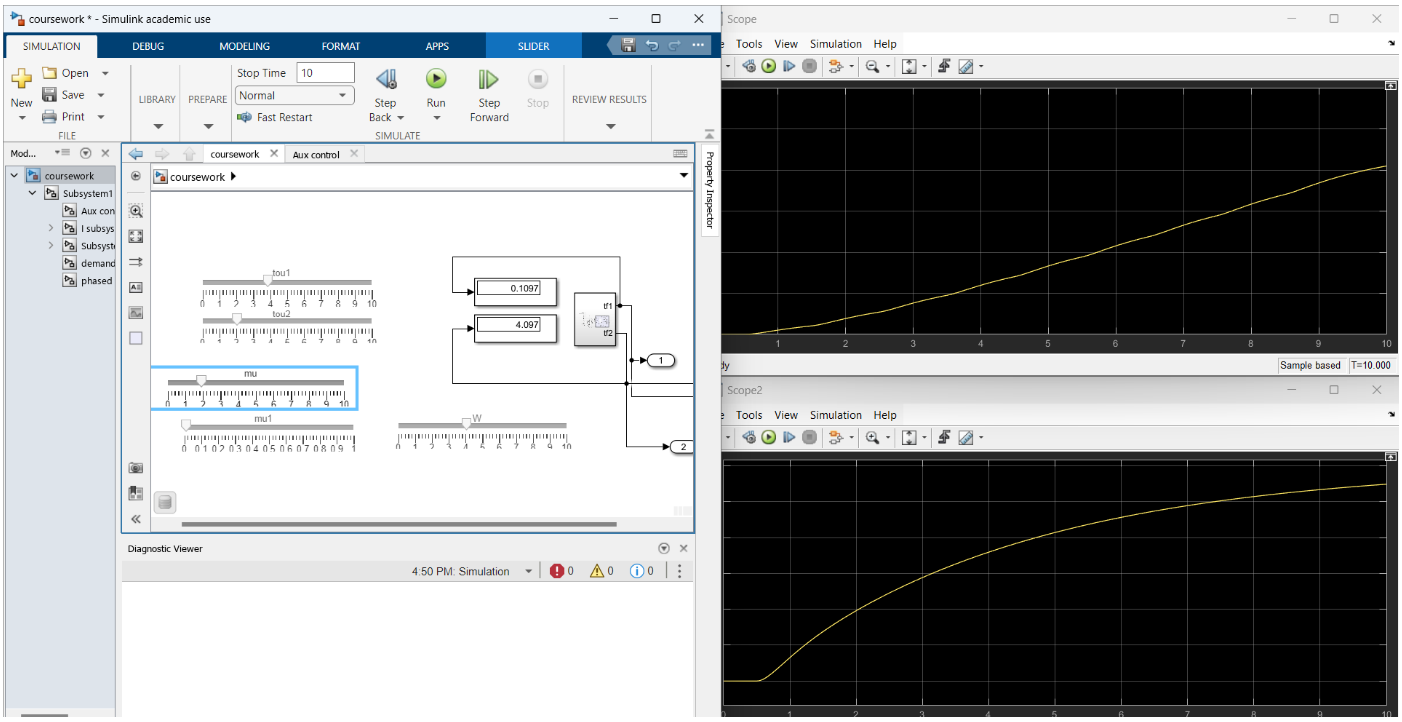


T1 and T2 scope [3]



We can observe the changes in the graph when we change the constant parameter values, by altering these values I was able to look for changes and then finalize the w,tou1,tou2,mu and mu1

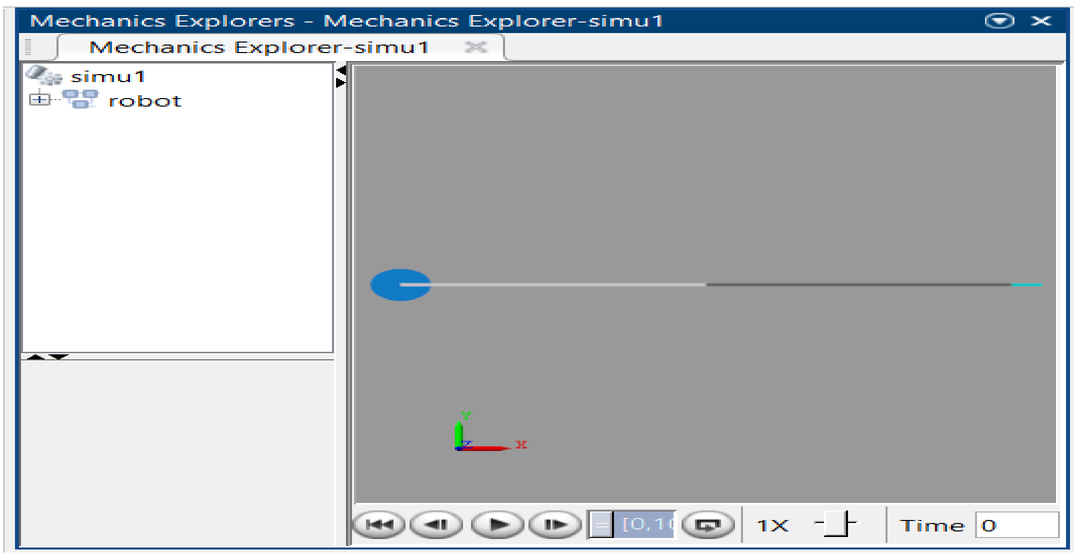
When it was altered we observe a smooth graph as shown below:



Results:

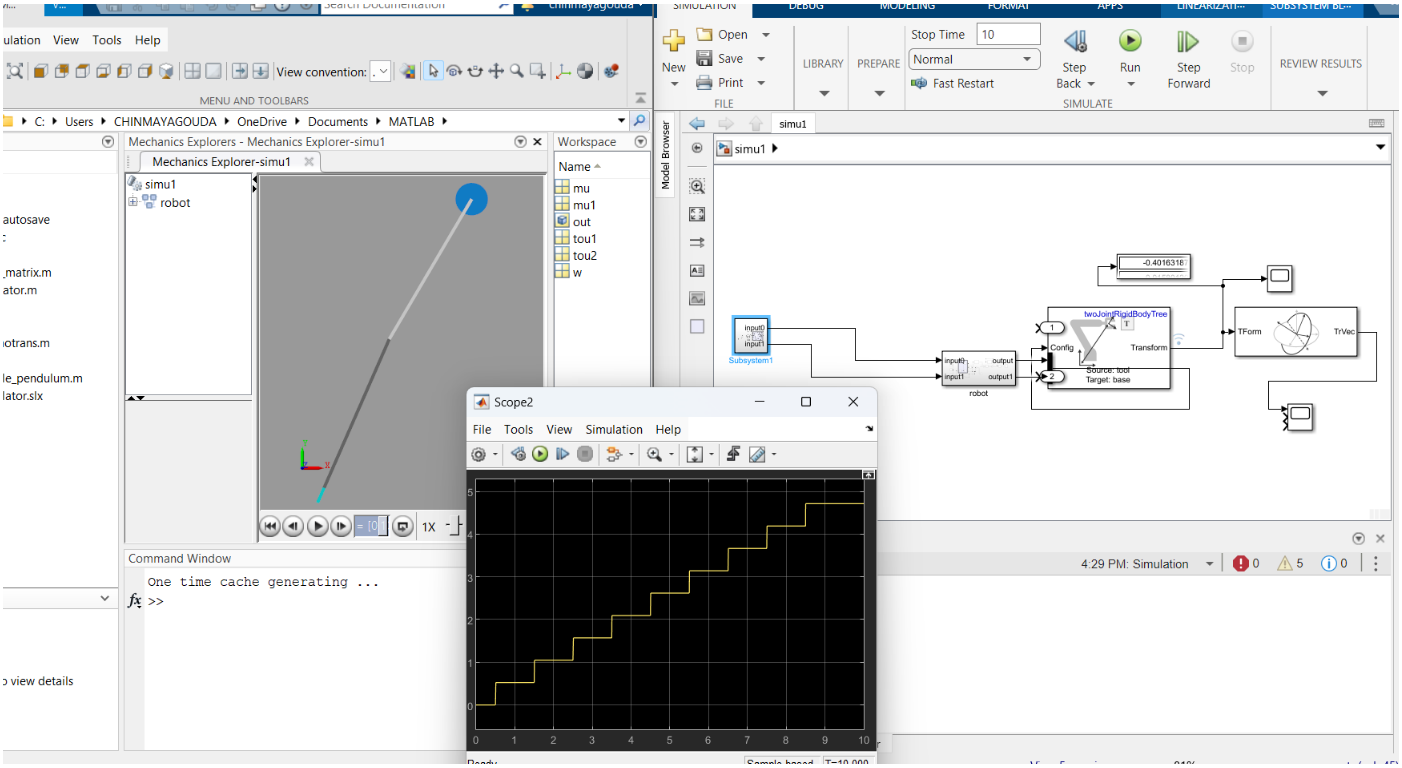
Mu1	Mu2	T1	T2
0	0	-0.093	-0.11
	0.1	-0.124	-0.1445
	0.2	-0.1535	-0.1775
	0.5	-0.241	-0.2753
	1	-0.3887	-0.4403
	2	-0.6845	-0.771
	10	-3.038	-3.401
0.1	0	-0.093	-0.11
	0.1	-0.124	-0.1445
	0.2	-0.1535	-0.1775
	0.5	-0.241	-0.2753
	1	-0.3887	-0.4403
	2	-0.6845	-0.771
	10	-3.038	-3.401
Tou1	q1	Tou2	q2
1	0.1199	1	4.614
2	0.1191	2	4.142
3	0.1153	3	3.63
4	0.1091	4	3.29
5	0.1022	5	2.946

Before running the simulation the robot is placed (0,0)



[3]

Once the simulation runs the robot arm 1 rotates by 6.87 degree and arm 2 rotates by 240 degree at 30 degree steps



Simulink model[3]

DISCUSSION

A two-link manipulator's nonlinear dynamics can be effectively managed by the calculated torque control technique, which compensates for disturbances including joint friction, Coriolis and centrifugal effects, and gravity forces. The Lagrangian method is used to validate the generated equations of motion, which offer a strong basis for this control strategy. Based on simulation results, the control system accomplishes precise trajectory tracking by precisely modeling the dynamics of the manipulator.

The modelling errors such as inaccuracies in mass, inertia or friction estimates can affect the performance leading to change in the path of the trajectory of the robot or instability in maintaining its position. The most common errors are originated form unmodeled dynamics and parameters mismatched. Other strategies or control techniques employ sensor feedback to estimate and compensate for unmodeled forces dynamically.

Key features of the simulation include the decoupling of joint dynamics and the elimination of nonlinearities, enabling simplified linear system behavior. However, practical implementation introduces challenges. Setting the acceleration due to gravity to zero (g=0g = 0g=0) in simulations simplifies calculations but may limit the applicability of results to real-world scenarios, where gravitational forces significantly impact system dynamics. The analysis confirms that neglecting gravity is valid only for specific configurations or environments, such as space applications.

Despite the good efficiency of the calculated torque control approach is not always the best one in terms of computational ease or energy efficiency. Alternative approaches, however more sophisticated, including robust or adaptive controllers, could be able to better handle modeling uncertainty. However, under optimal circumstances, the simulation demonstrates the high-precision control that can be achieved with the computed torque method.

CONCLUSION

The Simulink model is working efficiently and delivering smooth motion to the desired input angle. Since we have a step input it is necessary that the robot follows a curved path so that it achieves a smooth motion rather than a sudden movement to the desired angle. The error tracking takes feedback from the input and output to know the current value of the theta angle and what changes it must make to adjust its path back on track.

The Simulink modelling gave me a good idea as to how to convert the equation of motions into block diagram and also its working in a more efficient way. The application of slider for some constant parameters enabled me to achieve a smoother curve by varying the different parameters and how they affect the robot trajectories.

Despite achieving accurate control, the computed torque control strategy is not always the best course of action. It calls for precise system dynamics modeling, which is difficult to do in real-world situations because of parameter uncertainty and outside perturbations. Performance can be greatly impacted by environmental conditions, such as friction and unforeseen external forces. Although techniques like adaptive or robust control may help address these issues, they frequently come at the expense of a more complex system.

Future application:

In order to improve flexibility, accuracy, and durability, artificial intelligence (AI) will be incorporated into computed torque control in robotic manipulators in the future. AI can guarantee precise control in dynamic situations by enabling real-time parameter estimation and machine learning algorithm correction for modeling flaws. By learning from system behavior and outside disruptions, neural networks and reinforcement learning can refine control tactics and gradually increase performance. While adaptive algorithms can react dynamically to changes in the manipulator's environment or payload, AI-powered predictive models can improve trajectory planning by striking a balance between speed, energy efficiency, and safety. AI can also make defect detection and disturbance rejection easier, which increases the system's resistance to outside influences and hardware failures. Smarter, more adaptable manipulators that can thrive in challenging and uncertain situations are promised by this integration.

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