Let anchors $r_1, r_2, r_3, \ldots r_N$ are placed

P is target unknown.

t_0 is unknown start time

C is signal propagation speed

$$\text{speed } c = \frac{d}{t} \qquad d = ct \rightarrow c(t_N - t_0)$$

So distance from r_1 & P

$$d^2(r_1, P) = d^2 = c^2(t_1 - t_0)^2$$

$$\hookrightarrow (r_{1x} - P_x)^2 + (r_{1y} - P_y)^2 + (r_{1z} - P_z)^2$$

$$(r_{1x} - P_x)^2 + (r_{1y} - P_y^2) + \underbrace{(r_{1z} - P_z)^2}_{\text{omit}} = c^2(t_1 - t_0)^2 \quad \text{— (1)}$$

$$(r_{2x} - P_x)^2 + (r_{2y} - P_y)^2 + \underbrace{(r_{2z} - P_z)^2}_{\text{omit}} = c^2(t_2 - t_0)^2 \quad \text{— (2)}$$

$$(r_{3x} - P_x)^2 + (r_{3y} - P_y)^2 + \underbrace{(r_{3z} - P_z)^2}_{\text{omit}} = c^2(t_3 - t_0)^2 \quad \text{— (3)}$$

$$(r_{4x} - P_x)^2 + (r_{4y} - P_y)^2 + \ldots = c^2(t_4 - t_0)^2 \quad \text{— (4)}$$

$$\vdots$$

$$(r_{Nx} - P_x)^2 + (r_{Ny} - P_y)^2 + \ldots = c^2(t_N - t_0)^2 \quad \text{— (N)}$$

consider only 2D case

(2) − (1)
(3) − (1)
(4) − (1)
⋮
(N) − (1)

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$$r_{1x}^2 + P_x^2 - 2r_{1x}P_x + r_{1y}^2 + P_y^2 - 2r_{1y}P_y = c^2(t_1^2 + t_0^2 - 2t_1t_0) \quad -(1)$$

$$r_{2x}^2 + P_x^2 - 2r_{2x}P_x + r_{2y}^2 + P_y^2 - 2r_{2y}P_y = c^2(t_2^2 + t_0^2 - 2t_2t_0) \quad -(2)$$

$$r_{3x}^2 + P_x^2 - 2r_{3x}P_x + r_{3y}^2 + P_y^2 - 2r_{3y}P_y = c^2(t_3^2 + t_0^2 - 2t_3t_0) \quad -(3)$$

$$r_{4x}^2 + P_x^2 - 2r_{4x}P_x + r_{4y}^2 + P_y^2 - 2r_{4y}P_y = c^2(t_4^2 + t_0^2 - 2t_4t_0) \quad -(4)$$

$$r_{Nx}^2 + P_x^2 - 2r_{Nx}P_x + r_{Ny}^2 + P_y^2 - 2r_{Ny}P_y = c^2(t_N^2 + t_0^2 - 2t_Nt_0) \quad -(N)$$

(2) - (1)

$$(r_{2x}^2 - r_{1x}^2) - 2(r_{2x} - r_{1x})P_x + (r_{2y}^2 - r_{1y}^2) - 2(r_{2y} - r_{1y})P_y = c^2(t_2^2 - t_1^2 - 2(t_2 - t_1)t_0)$$

$$(r_{2x}^2 - r_{1x}^2) + (r_{2y}^2 - r_{1y}^2) - 2(r_{2x} - r_{1x})P_x - 2(r_{2y} - r_{1y})P_y = c^2(t_2^2 - t_1^2) - 2c^2(t_2 - t_1)t_0$$

$$2(r_{2x} - r_{1x})P_x + 2(r_{2y} - r_{1y})P_y - 2c^2(t_2 - t_1)t_0 = (r_{2x}^2 - r_{1x}^2) + (r_{2y}^2 - r_{1y}^2) - c^2(t_2^2 - t_1^2) \quad -(a)$$

(3) - (1)

$$2(r_{3x} - r_{1x})P_x + 2(r_{3y} - r_{1y})P_y - 2c^2(t_3 - t_1)t_0 = (r_{3x}^2 - r_{1x}^2) + (r_{3y}^2 - r_{1y}^2) - c^2(t_3^2 - t_1^2) \quad -(b)$$

(4) - (1)

$$2(r_{4x} - r_{1x})P_x + 2(r_{4y} - r_{1y})P_y - 2c^2(t_4 - t_1)t_0 = (r_{4x}^2 - r_{1x}^2) + (r_{3y}^2 - r_{1y}^2) - c^2(t_4^2 - t_1^2) \quad -(c)$$

$\vdots$

(N) - (1)

$$2(r_{Nx} - r_{1x})P_x + 2(r_{Ny} - r_{1y})P_y - 2c^2(t_N - t_1) = (r_{Nx}^2 - r_{1x}^2) + (r_{Ny}^2 - r_{1y}^2) - c^2(t_N^2 - t_1^2) \quad -(n)$$

$$
\begin{bmatrix}
2(\gamma_{2x}-\gamma_{1x}) & 2(\gamma_{2y}-\gamma_{1y}) & -2c^2(t_2-t_1) \\
2(\gamma_{3x}-\gamma_{1x}) & 2(\gamma_{3y}-\gamma_{1y}) & -2c^2(t_3-t_1) \\
2(\gamma_{4x}-\gamma_{1x}) & 2(\gamma_{4y}-\gamma_{1y}) & -2c^2(t_4-t_1) \\
\vdots & & \\
2(\gamma_{Nx}-\gamma_{1x}) & 2(\gamma_{Ny}-\gamma_{1y}) & -2c^2(t_N-t_1)
\end{bmatrix} = A
$$

$$
x = \begin{bmatrix} P_x \\ P_y \\ t_0 \end{bmatrix}
$$

$$
b = \begin{bmatrix}
(\gamma_{2x}^2-\gamma_{1x}^2)+(\gamma_{2y}^2-\gamma_{1y}^2)-c^2(t_2^2-t_1^2) \\
(\gamma_{3x}^2-\gamma_{1x}^2)+(\gamma_{3y}^2-\gamma_{1y}^2)-c^2(t_3^2-t_1^2) \\
(\gamma_{4x}^2-\gamma_{1x}^2)+(\gamma_{4y}^2-\gamma_{1y}^2)-c^2(t_4^2-t_1^2) \\
\vdots \\
(\gamma_{Nx}^2-\gamma_{1x}^2)+(\gamma_{Ny}^2-\gamma_{1y}^2)-c^2(t_N^2-t_1^2)
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\overbrace{(\gamma_{2x}^2+\gamma_{2y}^2)}^{\|\mathrm{norm}(\gamma_2)\|^2}-\overbrace{(\gamma_{1x}^2+\gamma_{2y}^2)}^{\|\mathrm{norm}(\gamma_1)\|^2}-c^2(t_2^2-t_1^2) \\
(\gamma_{3x}^2+\gamma_{3y}^2)-(\gamma_{1x}^2+\gamma_{1y}^2)-c^2(t_3^2-t_1^2) \\
(\gamma_{4x}^2+\gamma_{4y}^2)-(\gamma_{1x}^2+\gamma_{1y}^2)-c^2(t_4^2-t_1^2) \\
\vdots \\
(\gamma_{Nx}^2-\gamma_{1x}^2)-(\gamma_{1x}^2+\gamma_{1y}^2)-c^2(t_N^2-t_1^2)
\end{bmatrix}
$$

$$b = \begin{bmatrix} \|\gamma_2\|^2 - \|\gamma_1\|^2 - c^2(t_2^2 - t_1^2) \\ \|\gamma_3\|^2 - \|\gamma_1\|^2 - c^2(t_3^2 - t_1^2) \\ \|\gamma_4\|^2 - \|\gamma_1\|^2 - c^2(t_4^2 - t_1^2) \\ \vdots \\ \|\gamma_N\|^2 - \|\gamma_1\|^2 - c^2(t_N^2 - t_1^2) \end{bmatrix}$$

$Ax = b \qquad x = A^{-1}b$

taking 2 out of A,

$$b = \frac{1}{2} \begin{bmatrix} \|\gamma_2\|^2 - \|\gamma_1\|^2 - c^2(t_2^2 - t_1^2) \\ \|\gamma_3\|^2 - \|\gamma_1\|^2 - c^2(t_3^2 - t_1^2) \\ \|\gamma_4\|^2 - \|\gamma_1\|^2 - c^2(t_4^2 - t_1^2) \\ \vdots \\ \|\gamma_N\|^2 - \|\gamma_1\|^2 - c^2(t_N^2 - t_1^2) \end{bmatrix}$$

$$\text{So } A = \begin{bmatrix} \gamma_{2x} - \gamma_{1x} & \gamma_{2y} - \gamma_{1y} & -c^2(t_2 - t_1) \\ \gamma_{3x} - \gamma_{1x} & \gamma_{3y} - \gamma_{1y} & -c^2(t_3 - t_1) \\ \gamma_{4x} - \gamma_{1x} & \gamma_{4y} - \gamma_{1y} & -c^2(t_4 - t_1) \\ \vdots & \vdots & \\ \gamma_{Nx} - \gamma_{1x} & \gamma_{Ny} - \gamma_{1y} & -c^2(t_N - t_1) \end{bmatrix}$$

$$\boxed{x = A^{-1}b}$$