$a_{k+1} - b_k$

$a_k - b_k$

Divide & Conquer.

Q Compute Eignal and vector of $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & -1 \\ 4 & -1 & 1 \end{bmatrix}$

Solⁿ: a) $S_1 = \sqrt{a_{12}^2 + a_{13}^2} = \sqrt{9+16} = 5$

b) $x_2 = \frac{1}{2}\left[1 + \frac{a_{12}}{S_1}\right] = \frac{1}{2}\left[1 + \frac{3}{5}\right] = 0.8944$

$x_3 = \frac{a_{13}}{2x_2 S_1} = 0.447$

★ $T = \begin{bmatrix} a_1 & b_1 & & 0 \\ b_1 & \ddots & \ddots & \\ & \ddots & & b_{n-1} \\ 0 & & b_{n-1} & a_n \end{bmatrix}$

$V_1 = \begin{bmatrix} 0 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8944 \\ 0.447 \end{bmatrix}$

c) $T_1 = I - 2V_1V_1^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.6 & -0.8 \\ 0 & -0.8 & 0.6 \end{bmatrix}$

d) Triadigonal form $= T = T_1 A T_1 =$

$$\begin{array}{c|c|l} & \begin{matrix} a_1 & & a_2 \end{matrix} & \\ b_1 \rightarrow & \begin{bmatrix} 1 & -5 & 0 \\ -5 & 0.4 & 0.2 \\ 0 & 0.2 & 2.6 \end{bmatrix} & \begin{matrix} \\ \leftarrow b_2 \\ \leftarrow a_3 \end{matrix} \end{array}$$

Then, Defining T_1 and T_2 from T for $k=2$, ~~$b_2 = 0.2$~~

$T_1 = \begin{bmatrix} a_1 & b_1 & & 0 \\ b_1 & \ddots & & \\ & & a_{k-1} & b_{k-1} \\ 0 & & b_{k-1} & a_k - b_k \end{bmatrix}$

where,

$a_1 = 1$ $\quad b_1 = 5$

$a_2 = 0.4$ $\quad b_2 = 0.2$

$a_3 = 2.6$

1) $T_1 = \begin{bmatrix} a_1 & b_1 \\ b_1 & a_k - b_k \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -5 & 0.4-0.2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -5 & 0.2 \end{bmatrix}$

& $T_2 = \begin{bmatrix} a_{k+1} - b_k & b_{k+1} & & 0 \\ b_{k+1} & \ddots & & \\ & & & b_{n-1} \\ 0 & & b_{n-1} & a_n \end{bmatrix} = \begin{bmatrix} a_3 - a_2 & \cancel{b_3} \\ \cancel{b_3} & \cancel{a_3} \end{bmatrix}$

$= [a_3 - a_2] = [2.4]$

★ Since, T_1 and T_2 are symmetric tridiagonal, we can find orthogonal matrices Q_1 and Q_2 such that,

$$T_1 = Q_1 D_1 Q_1^t \text{ and } T_2 = Q_2 D_2 Q_2^t$$

Then, using spectral decomposition,

$$D_1 = \begin{bmatrix} 5.61 & 0 \\ 0 & -4.41 \end{bmatrix} \quad \& \quad Q_1 = \begin{bmatrix} 0.73 & 0.67 \\ -0.67 & 0.73 \end{bmatrix}$$

and

$$D_2 = [2.4] \quad \& \quad Q_2 = [1]$$

★ Also,

$$u = \begin{pmatrix} Q_1^T & 0 \\ 0 & Q_2^T \end{pmatrix} v = [-0.67 \quad 0.73 \quad 1]^t$$

Then,

$$\hat{D} = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix} + b_K u u^T$$

$$= \begin{bmatrix} 5.61 & 0 & 0 \\ 0 & -4.41 & 0 \\ 0 & 0 & 2.4 \end{bmatrix} + (0.2) \begin{bmatrix} -0.67 \\ 0.73 \\ 1 \end{bmatrix} [0.67 \ 0.73 \ 1]$$

$$= \begin{bmatrix} \quad \end{bmatrix}_{3 \times 3}$$

[38]

Ans Eigen values (λ) and Eig vect (q) of $\hat{D}$ are.

$(\lambda_1 = 5.7, q_1) \ldots (\lambda_3, q_3)$ etc

which are same as those of T