ASSIGNMENT – II

(Q1)a) To prove λ is the root of equation :

$$1 + \rho u^T (D - \lambda I)^{-1} u = 0$$

where,

(λ, q) is eigenpair of rank-one perturbed diagonal matrix $\hat{D} = D + \rho u u^T$.

Proof

Since, (λ, q) is an eigen pair of $\hat{D}$, we must have,

$$(D + \rho u u^T) q = \lambda q \quad (\text{for some } q \neq 0)$$

i.e.,

$$(D - \lambda I) q = -\rho (u^T q) u.$$

Now assumption that $\rho \neq 0$ that $d_1 < d_2 < \cdots d_n$ and that none of the components of u is zero imply that

a) $(D - \lambda I)$ is non sigular

b) $u^T q$ is nonzero

Then, multiplying by $(D - \lambda I)^{-1}$, we have,

$$q = -\rho (u^T q) (D - \lambda I)^{-1} u$$

$\Rightarrow$ multiplying both sides by u^T and divide by nonzero scalar $u^T q$ we have,

$$1 + \rho u^T (D - \lambda I)^{-1} u = 0$$

Hence proved

Q b) $q = (D - \lambda I)^{-1} u$ when q is an eigenvector of $(D + \rho u u^T)$ corresponding to λ.

proof $(D + \rho u u^T)(D - \lambda I)^{-1} u = (D - \lambda I + \lambda I + \rho u u^T)(D - \lambda I)^{-1} u$

$= u + \lambda (D - \lambda I)^{-1} u + u \rho u^T (D - \lambda I)^{-1} u$

$= u + \lambda (D - \lambda I)^{-1} u + u(-1)$

$= u + \lambda (D - \lambda I)^{-1} u - u$

$= \lambda (D - \lambda I)^{-1} u$

Hence proved

Q2) $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ $v = [1 \ 0 \ 0]^t$

$w = [2 \ 2 \ 0]^t$

construct orthogonal vectors $\{v_1, v_2, v_3\}$ and $\{w_1, w_2\}$ and T_m = non-symmetric tridiagonal matrix using Lanczos Method.

Solutn Step 0 : set $\beta_1 = 0$, $\delta_1 = 0$, $w_0 = v_0 = 0$

and $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = v$

and $w_1 = w/2$ $(\because \langle v_1, w_1 \rangle = 1)$

Step 1 : a) for $j = 1$

$\alpha_1 = w_1^T A v_1 = 4$

$\hat{v}_2 = A v_1 - \alpha_1 v_1 - \beta_1 v_0 = [-3 \ 3 \ 6]^t$

$\hat{w}_2 = A^T w_1 - \alpha_1 w_1 - \beta_1 w_0 = [0 \ 2 \ 8]^t$

$$\delta_2 = \sqrt{|\hat{w}_2^T \cdot \hat{v}_2|} = 7.3485$$

$$\beta_2 = (\hat{w}_2^T \cdot \hat{v}_2)/\delta_2 = 7.3485$$

$$w_2 = \hat{w}_2/\beta_2 = [0 \quad 0.2722 \quad 1.0887]^t$$

$$v_2 = \hat{v}_2/\delta_2 = [-0.4082 \quad 0.4082 \quad 0.8165]^t$$

b) for $j = 2$

$$\alpha_2 = 8.7778$$

$$\hat{v}_3 = [-0.9072 \quad 0.9072 \quad -0.2268]^t$$

$$\hat{w}_3 = [0 \quad -1.0282 \quad 0.5141]^t$$

$$\delta_3 = 1.0244$$

$$\beta_3 = -1.0244$$

$$w_3 = [0 \quad 1.0037 \quad -0.5018]^t$$

$$v_3 = [-0.8856 \quad 0.8856 \quad -0.2214]^t$$

Ans $\{v_1, v_2, v_3\} = \{[1, 0, 0], [-0.4082, 0.4082, 0.8165], [-0.8856, 0.8856, -0.2214]\}$

$\{w_2, w_3\}$; $T_m = \begin{bmatrix} \alpha_1 & \beta_2 \\ \delta_2 & \alpha_2 \end{bmatrix} = \begin{bmatrix} 4 & 7.348 \\ 7.3484 & 8.778 \end{bmatrix}$