Leading E.value of $A = 6$ and E.vect : $[1, 1, 1]^T$.

Least Square Method :-

$Ax = b$, when b is not in column space of A.

Ex :- $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{4\times 3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Soln This has no soln, thus we find x^* which is close to x.

$$A^T A x^* = A^T b$$

$\Rightarrow r(A) = 3$, thus no soln.

$$A^T A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$A^T b = [1 \quad 1 \quad 1]^T$$

$$\Rightarrow \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} 1/7 & 1/7 & 1/7 \end{bmatrix}$$