

Richardson Method :- $[Q = I]$

$$x^k = x^{k-1} + r^{k-1}$$

1) find $r^{k-1} = b - Ax^{k-1}$

2) $x^k = x^{k-1} + r^{k-1}$

Ex:-

$$\begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/3 & 1 & 1/2 \\ 1/2 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 11/18 \\ 11/18 \\ 11/18 \end{bmatrix}$$

given $x^{(0)} = (0, 0, 0)^T$

[I] a) $r^{(0)} = b - Ax^{(0)}$

$$= \begin{bmatrix} 11/18 \\ 11/18 \\ 11/18 \end{bmatrix} - \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/3 & 1 & 1/2 \\ 1/2 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 11/18 & 11/18 & 11/18 \end{bmatrix}^T - \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

$$r^{(0)} = \begin{bmatrix} 11/18 & 11/18 & 11/18 \end{bmatrix}^T$$

b) $x^{(1)} = x^{(0)} + r^{(0)} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T + \begin{bmatrix} 11/18 & 11/18 & 11/18 \end{bmatrix}^T$

$$= \begin{bmatrix} 0.611 & 0.611 & 0.611 \end{bmatrix}^T$$

[II] a) $r^{(1)} = b - Ax^{(1)}$

$$= \begin{bmatrix} 11/18 & 11/18 & 11/18 \end{bmatrix}^T - \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/3 & 1 & 1/2 \\ 1/2 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 0.611 & 0.611 & 0.611 \end{bmatrix}^T$$

$$= \begin{bmatrix} 11/18 & 11/18 & 11/18 \end{bmatrix}^T - \begin{bmatrix} 1.120 & 1.120 & 1.120 \end{bmatrix}^T$$
$$= \begin{bmatrix} -0.509 & -0.509 & -0.509 \end{bmatrix}^T$$

$$b) x^{(2)} = x^{(1)} + x^{(1)}$$

$$= [0.611 \quad 0.611 \quad 0.611]^T + [-0.509 \quad -0.509 \quad -0.509]^T$$

$$= [0.102 \quad 0.102 \quad 0.102]^T$$

And so on...

1: Method : (2) diagonal matrix with diagonal element same as