Rayleigh Quotient :-

Ex $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ $x = [1 \;\; -0.5]^T$

Then,

$$Rq = \frac{x^t A x}{||x||_2} = 0.894 [\quad] = -0.25$$

Ex $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ $x_0 = [0.5256 \;\; 0.7622 \;\; 1.0000]^T$

[I]

a) $$\sigma_0 = \frac{x_0^t A x_0}{x_0^t x_0} = \frac{17.8728}{1.8572} = 9.6235$$

b) $$[A - \cancel{9.6235}\, \sigma_0 I]\, \hat{x}_1 = x_0$$

$$\hat{x}_1 = [0.5256 \;\; 0.7622 \;\; 1.00]^t \begin{bmatrix} -8.6235 & 2 & 3 \\ 2 & -6.6235 & 4 \\ 3 & 4 & -4.6235 \end{bmatrix}^{-1}$$

$$\hat{x}_1 = [-21318.48 \;\; -30974.35 \;\; -40630.22]$$

c) $$x_1 = \frac{\hat{x}_1}{\cancel{\min(x_1)}\ \max(x_1)} = [1 \;\; 1.4529 \;\; 1.9059]$$

[62]

[II] a) $\sigma_1 = \dfrac{x_1^t A x_1}{x_1^t x_1} = \dfrac{64.8947}{6.7434} = 9.6235$

b) $[A - \sigma_1 I]\hat{x}_2 = x_1$

$$\hat{x}_2 = [A - \sigma_1 I]^{-1} x_1$$

$$= \begin{bmatrix} -8.6235 & 2 & 3 \\ 2 & -6.6235 & 4 \\ 3 & 4 & -4.6235 \end{bmatrix}^{-1}_{3\times 3} \begin{bmatrix} 1 \\ 1.4529 \\ 1.9059 \end{bmatrix}_{3\times 1}$$

$$\hat{x}_2 = [-40622.3573 \quad -59021.6213 \quad -77420.885]^t$$

c) $x_2 = \dfrac{\hat{x}_2}{\cancel{\min(\hat{x}_2)}\ \max(\hat{x}_2)} = [1 \quad 1.4529 \quad 1.9059]^t$

So on ...