

ARNOLDI'S Method.

Q $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ $v = [1 \ 0 \ 0]^t$ $m=2$

Find an orthonormal vector $\{v_1, \dots, v_m\}$ and a 3×2 Hessenberg matrix \bar{H}_m s.t.

$$AV_m = V_{m+1} \bar{H}_m \text{ where } V = (v_1, v_2, \dots, v_m)$$

Solⁿ :- Step 0: $v_1 = \frac{v}{\|v\|} = [1 \ 0 \ 0]^t$

Step 1: for $k=1$

$$\hat{v} = Av_1 = [1 \ 3 \ 6]^t$$

$$h_{11} = v_1^T \hat{v} = 1$$

$$\hat{v} = \hat{v} - h_{11}v_1 = [0 \ 3 \ 6]^t$$

~~$$h_{21} = v_2^T \hat{v} =$$~~

then,

$$h_{21} = \|\hat{v}\|_2 = 6.7082$$

and,

$$v_2 = \frac{\hat{v}}{h_{21}} = [0 \ 0.4472 \ 0.8944]^t$$

for $k=2$

$$\hat{v} = Av_2 = [\quad]^t$$

$$h_{12} = v_1^T \hat{v} = 3.5777$$

$$\hat{v} = \hat{v} - h_{12}v_1$$

$$h_{22} = v_2^T \hat{v} = 12$$

$$\hat{v} = \hat{v} - h_{22}v_2 = [\quad]^t$$

then

$$h_{32} = \|\hat{v}\|_2 = 1$$

$$v_3 = \frac{\hat{v}}{h_{32}} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

 \times

$$\text{Ans} = \{v_1, v_2, v_3\}$$

$$H = \begin{bmatrix} 1 & 3.5779 \\ 6.708 & 12 \\ 0 & 1 \end{bmatrix}$$