## MA835 - Non Linear Dynamics and Methods

# Assignment - Analysis of Rössler Attractor

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## 1 Rössler System

Rössler System is a system consisting of 3 Ordinary Differential equations,

$$\dot{x} = -y - z$$

$$\dot{y} = x + ay$$

$$\dot{z} = b + z(x - c)$$

Here , (a,b,c) are the parameters of the equations.

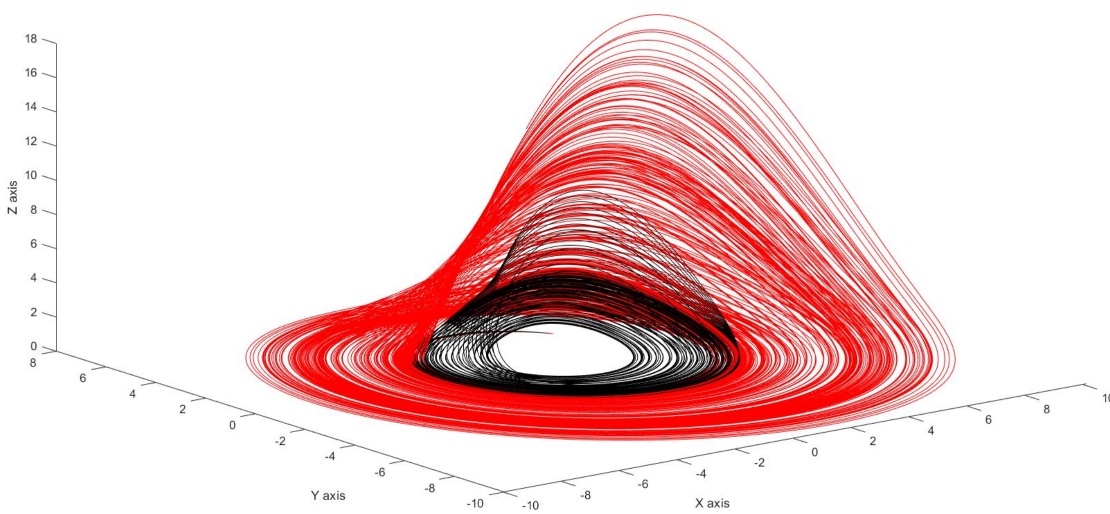


Figure 1: Rössler System

## 2 Equilibrium Points

To Obtain the Equilibrium points, $\dot{x}$ , $\dot{y}$ & $\dot{z}$ respectively were equated to 0 and the following equation was obtained,

$$x = \frac{c \pm \sqrt{c^2 - 4ab}}{2}$$
$$y = -\left(\frac{c \pm \sqrt{c^2 - 4ab}}{2a}\right)$$
$$z = \frac{c \pm \sqrt{c^2 - 4ab}}{2}$$

From the above equations , a condition for the existence of the roots arises, that is

$$c^2 > 4ab$$

This puts a condition on the roots to be either Real or Imaginary.

Using the Above equations, the roots for the following cases of (a,b,c) were obtained:

- (0.25 , 0.5 , 1) roots are (0.146 , -0.586 , 0.586) and (0.853 , -3.414 , 3.414)
- (0.5 , 1 , 1) roots are (0.5 $\pm 0.5i$ , -1 ∓ i , 1 ± i)
- (1 , 1.5 , 1) roots are (0.5 ± 1.12i , -0.5 ∓ 1.12i , 0.5 ± 1.12i)
- (0.5 , 0.5 , 1) roots are (0.5 , -1 , 1)

# 3 Bifurcations

Bifurcation plots for each Parameter was obtained and is shown below. It can be observed that only **b** converges to a single solution whereas **a** and **c** begin to diverge.

- The areas where we cannot see a distinct line are the ranges where chaos is observed due to the corresponding Parameter
- **a** is stable initially but shows chaos after crossing a values of 0.16 , after which it exhibits stable limit cycles for some short intervals in between the Chaos.
- **b** starts stabilising approximately after 0.6, before which is instils chaotic behaviours excluding a really small interval between 0.25 and 0.4
- **c** , similar to **a** destabilizes the system and exhibits chaotic behaviour after crossing the value of 4 .

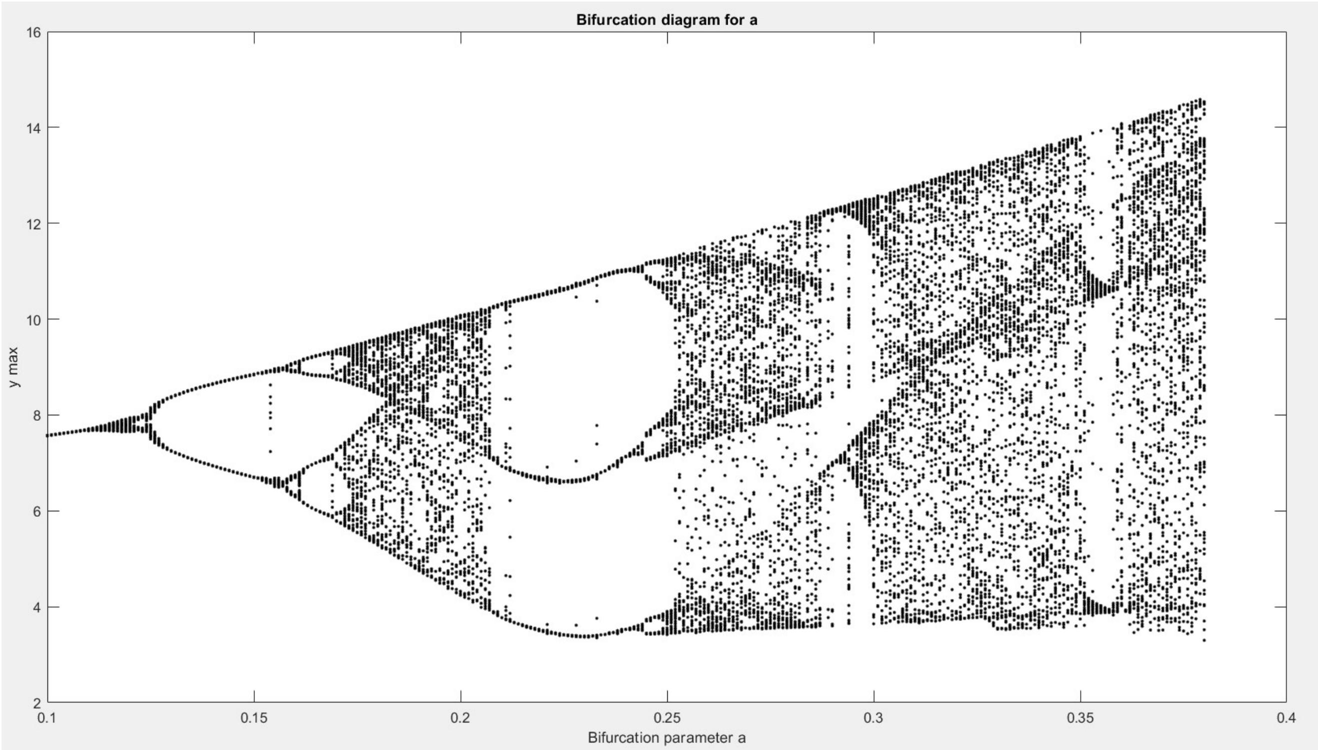


Figure 2: Bifurcation of a

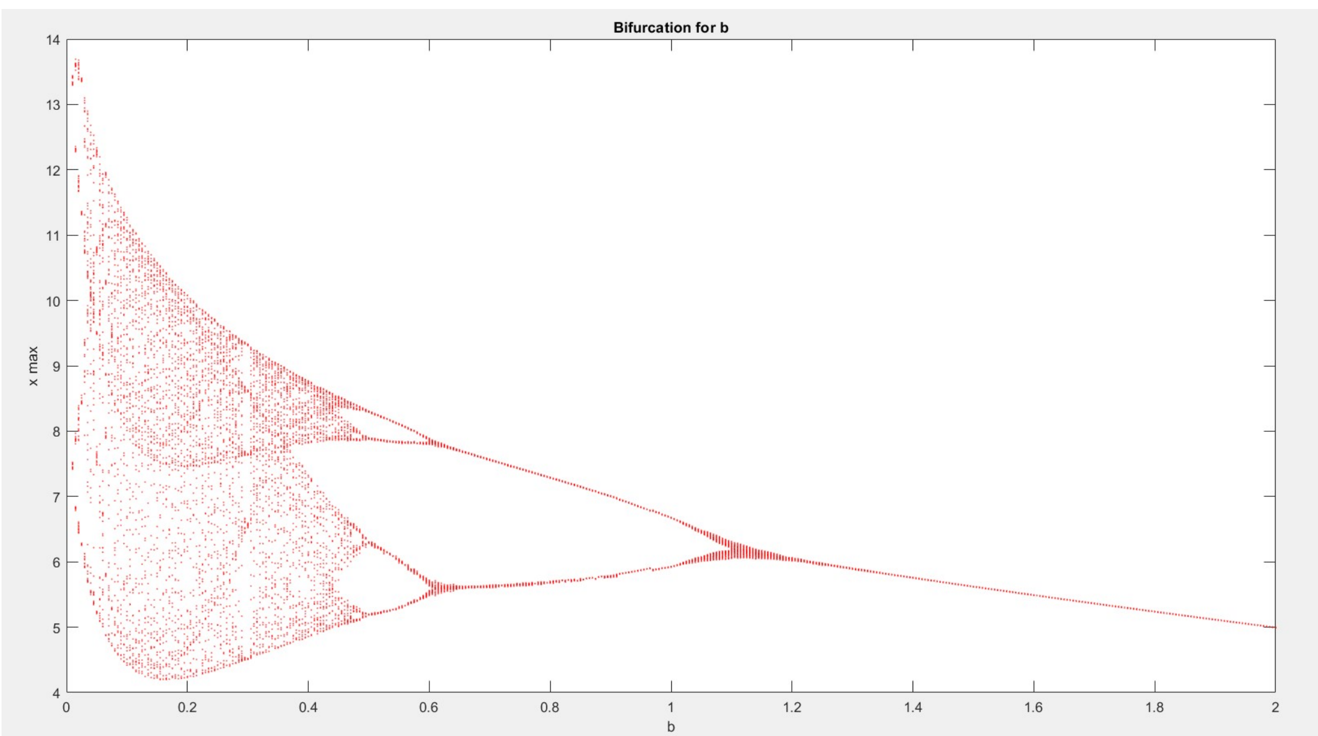


Figure 3: Bifurcation of b

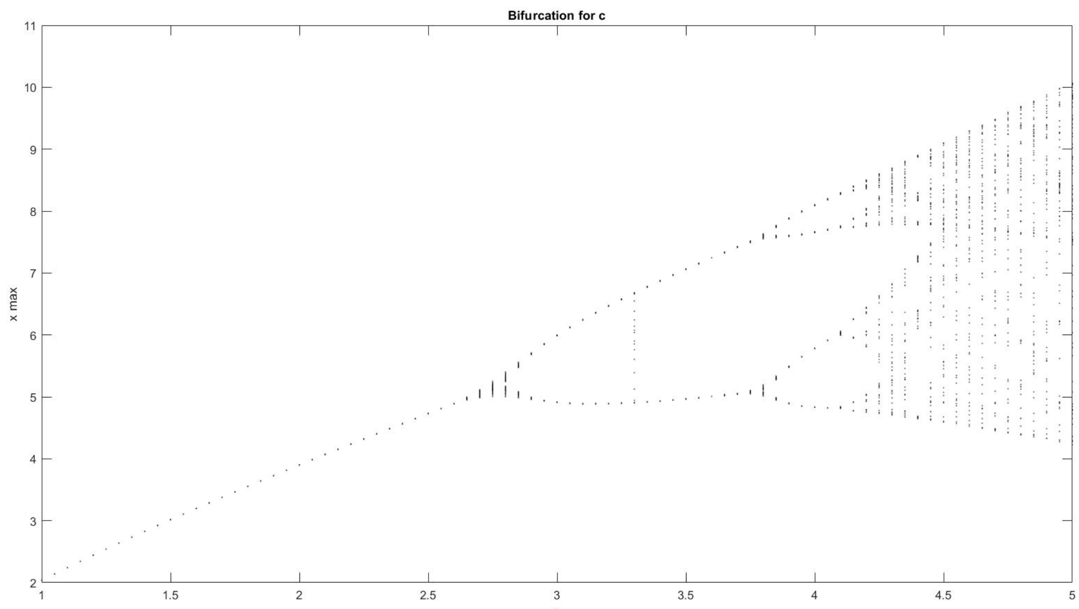


Figure 4: Bifurcation of c

# 4 Behaviour of System as ’c’ is increased

- Values of Parameters **a** & **b** were fixed for 4 cases while 3 values of c were used and the Simulation was run for 100sec.(Roots are represented in the plot by Astrix(*) symbol)
- It can be observed that for the case in top left, as **c** increases, the loops seem to be stabilising to a single loop implying the reduction in Chaotic behaviour for this particular case.
- For the rest of the cases, the plots show a straight line with a single root. Single root means that the roots are actually Complex and only the real part is represented ($c^2 < 4ab$).
- If we reduce the run time and re-run the cases where we obtained straight lines, we see what actually is happening. For these cases, the system spiral is destabilising rapidly 8. Due to comparatively large initial run time, the spiral radius(along X) diminishes compared to the System value reached with T=100s along Z and Y axes.

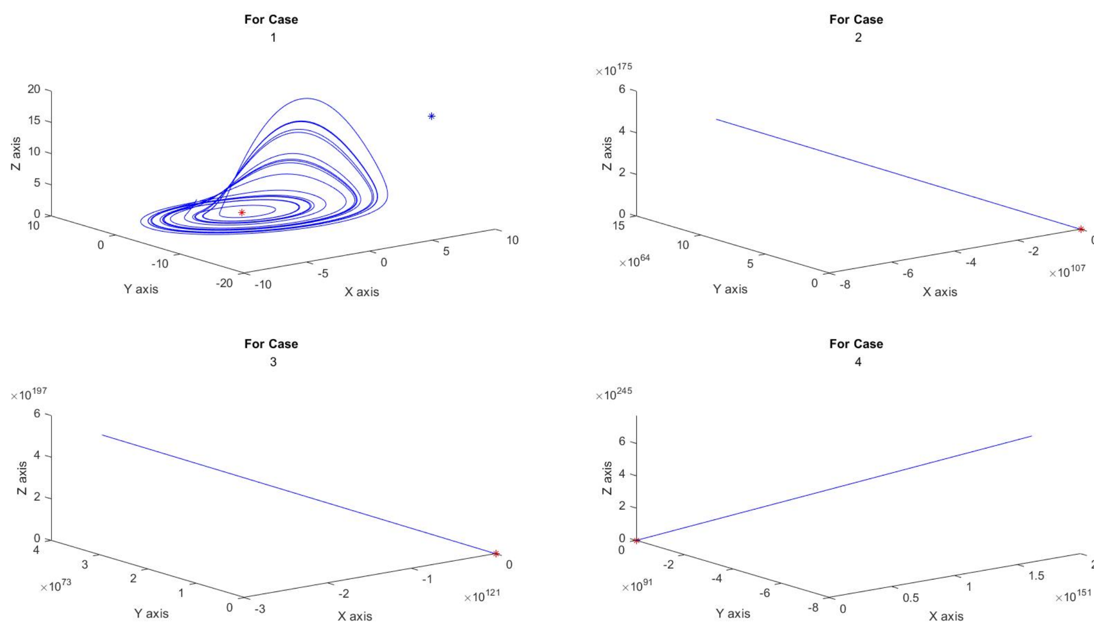


Figure 5: c = 5.0

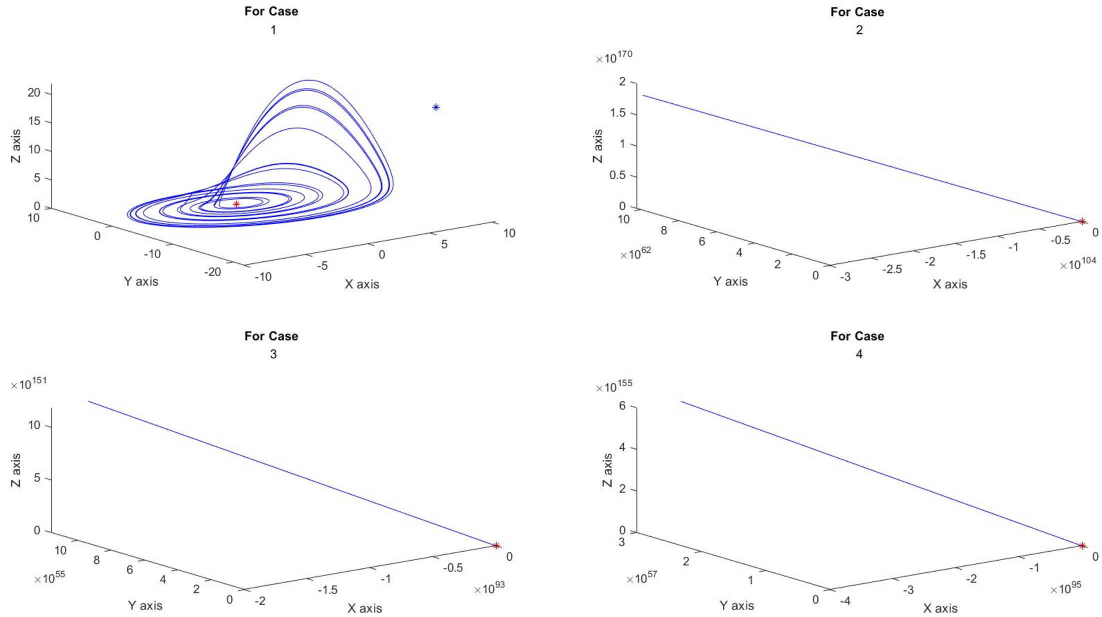


Figure 6: c = 5.5

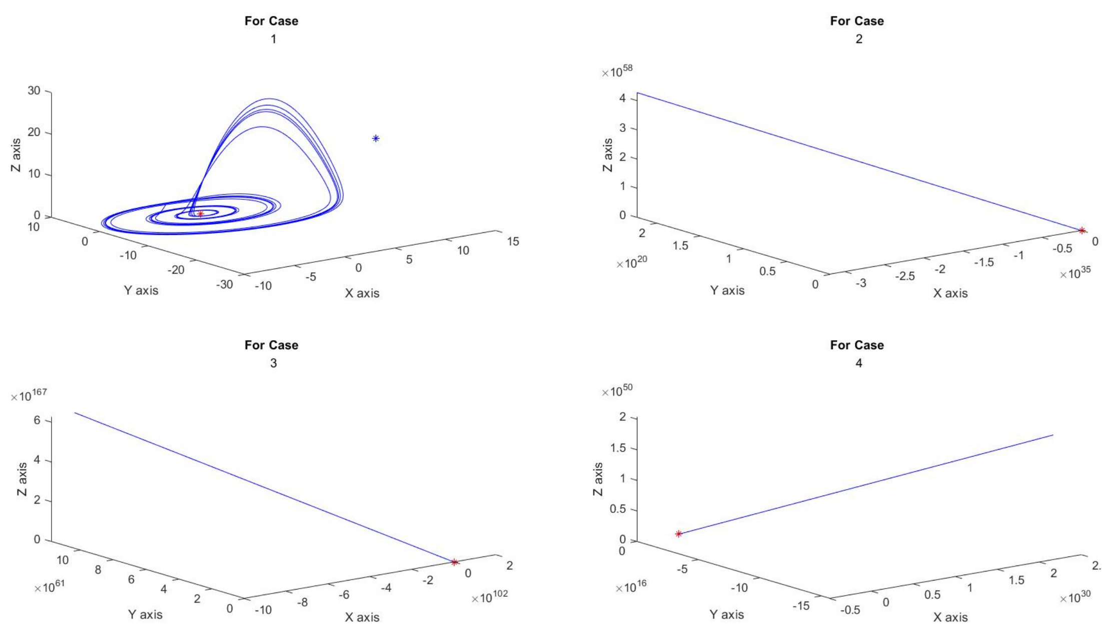


Figure 7: c = 6.0

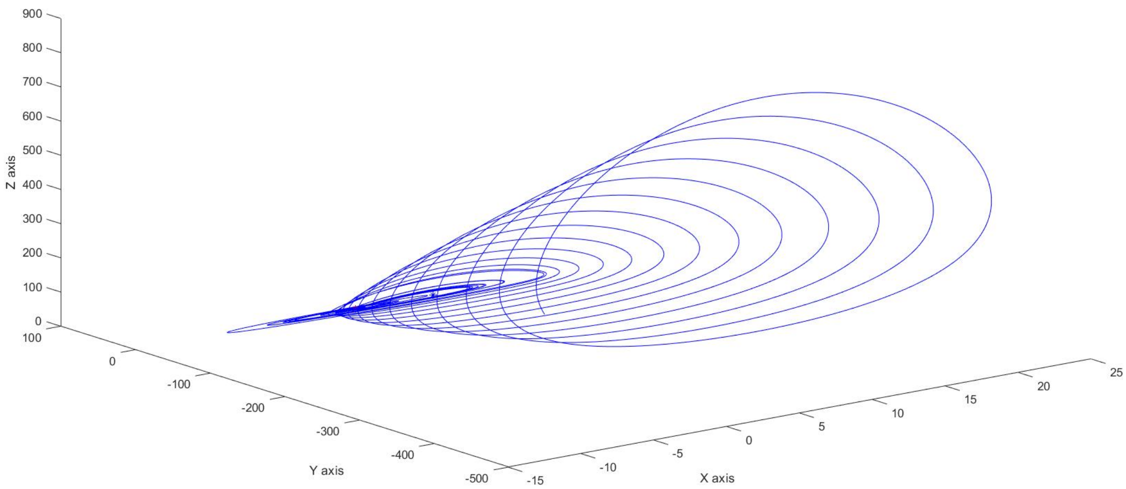


Figure 8: For a short run time

# 5 Conclusions

- When real roots exist, the root with lower Norm is *Stable Spiral Fixed point* whereas the other is a *Saddle Node*
- System can be said to show Attractor behaviour for some regions of $R^3$ for the Parameters.
- The 3D System can be reduced to 2D using Poincare Mapping.
- Rössler System is a Periodic System.

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