

Design a narrow-sense binary BCH code of length  $n = 127$  with designed distance  $\delta = 15$ . Find the generator polynomial and the parameters of the code. Assume that the field  $\mathbb{F}_{128}$  is constructed using  $x^7 + x^3 + 1$ .

- (a) (30%) Implement a systematic encoder for the code.
- Encoder inputs:
    - The input will be a binary vector  $m \in \mathbb{F}_2^k$ . The input will be in a file named “msg.txt”, with a binary vector of length  $k$  in each line. All the vectors should be encoded.
  - Encoder outputs:
    - The encoder should return a vector in  $\mathbb{F}_2^n$ . The program must print the output to a file named “codeword.txt”.
- (b) (60%) Implement a decoder for this code. The simplified Berlekamp-Massey algorithm for binary codes (as given in Lin and Costello or Ryan and Lin) should be used.
- Decoder inputs:
    - The input to the decoder will be a vector of length  $n$  containing errors and erasures. An erasure is denoted as 2. The input will be in a file named “rx.txt” with a received vector in each line. All the vectors should be decoded.
  - Decoder outputs:
    - The program should print out all the intermediate steps and intermediate variables as shown in see Table 6.8 of Lin and Costello or Table 3.5 of Ryan and Lin. Additionally, the second column of the output should contain the syndrome.
    - The elements of  $\mathbb{F}_{128}$  must be displayed in exponential form.
    - For the polynomials it is sufficient to display the coefficients. For instance the polynomial  $1 + x + \alpha^5 x^3$  can be represented as 0 0 -1 5 . Only the exponent of  $\alpha$  needs to be displayed and the exponent -1 stands for 0 not  $\alpha^{-1}$ .
    - The program should print the decoded codeword and the message on separate lines.
    - When the decoder fails to correct the received codeword, it should report failure and print an error message why it failed.
    - The output should be in a file “decoderOut.txt”.
- (c) (10%) Return a typed report of 3 pages, including sample output.

#### Further guidelines

- Only partial credit is given if the decoder cannot correct erasures.
- For arithmetic over  $\mathbb{F}_{128}$  you can use the matlab functions such as gfadd, gfsb, gfmul etc.
- Document the program and submit it along with the report.