UNIVERSITÄT ZU LÜBECK
INSTITUT FÜR TECHNISCHE INFORMATIK

Exercise Sheet 5

Control engineering

Lecture *Real-Time Systems*, Summer semester 2021

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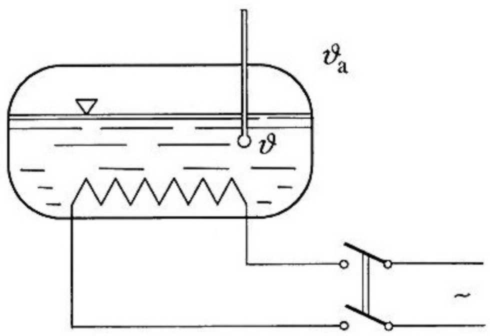
A discussion forum for the exercise can be found at: `moodle.uni-luebeck.de`.

Exercise 1 Modelling of a controlled system

In this task, you will model, analyse and simulate a given system. Bring your calculations as well as your prepared simulations to the practical exercise.

In an electrically heated water tank (fig. 1.1) are 1000 L of water. The temperature of the water should be controlled. The process variable $\mathbf{x(t)}$ is the water temperature $\vartheta(t)$, the control value $\mathbf{y_r(t)}$ is the electrical heating power $p(t)$.



$m_w = 1000kg$	Water mass
$c_w = 1.163\frac{Wh}{kgK}$	Specific heat capacity of water
$m_b = 75kg$	Container mass
$c_b = 1.3\frac{Wh}{kgK}$	Specific heat capacity of the container
$d = 8mm$	Thickness of insulation layer
$A = 7.8m^2$	Surface area of the container
$\lambda = 0.128\frac{W}{m\cdot K}$	Thermal conductivity of the insulation layer
$\vartheta_a = (273+15)K$	Outside temperature
$\vartheta(0) = \vartheta_a$	Initial water temperature

Figure 1.1: On the left: The electrically heated water tank. On the right: Relevant physical parameters.

The following differential equation describes the system of the hot water tank:

$$T_1 \cdot \frac{d\vartheta(t)}{dt} + \vartheta(t) = K_S \cdot p(t) + \vartheta_a \tag{1}$$

T_1 and K_S are parameters whose values are derived from the parameters listed in fig. 1.1. These values will be calculated in a later task. $p(t)$ is the heat output that heats the water. $\vartheta(t)$ is the temperature of the water in the tank, which is initially equal to the outside temperature. ϑ_a is the outside temperature and assumed to be constant.

(a) Laplace-Transformation

Compute the Laplace transform of the differential equation given in Eq. 1 with the help of the calculation rules and tables for the Laplace transformation known from the lecture. When applying the differential theorem, note that $\vartheta(0) = \vartheta_a$. Solve your result for $\vartheta(s)$.

(b) Step response in image domain

The step response for an input step $p(t) = P_0 \cdot \Theta(t)$ is to be sought. First transform the input signal into the image domain and insert it into the equation.

Note: If you cannot solve the previous task, continue with $\vartheta(s) = \frac{K_S}{(T_1 s+1)} \cdot P(s) + \frac{\vartheta_a}{s}$.

(c) Step response in time domain

Determine the corresponding step response $\vartheta(t)$ in the time domain by transforming back the equation of the previous subtask with the tables for Laplace transformation.

Note: If you cannot solve the previous task, continue with $\vartheta(s) = \frac{K_S \cdot P_0}{s(T_1 s+1)} + \frac{\vartheta_a}{s}$.

Sketch the signals waveform. What kind of control process are you dealing with? What influence does the outside temperature have?

(d) Transfer function

From now on we do not look at the absolute temperature $\vartheta(t)$, but at the temperature increase $\vartheta_\Delta(t)$ compared to the outside temperature: $\vartheta_\Delta(t) = \vartheta(t) - \vartheta_a$.

$$p(t) \longrightarrow \boxed{G_S(s)} \longrightarrow \vartheta_\Delta(t)$$

$\vartheta_\Delta(t)$ thus describes the offset of the system from the resting point $\vartheta = 288\,\mathrm{K}$. In control engineering, linear models typically describe deviations from an operating point, e.g. $\Delta v = v - v_0$ or $\Delta s = s - s_0$, which however are often only referred to as v or s. In this sense you can still use the variable ϑ instead of ϑ_Δ or $\Delta\vartheta$.

Calculate the transfer function of the system $G_S(s) = \frac{\vartheta_\Delta(s)}{P(s)}$.

(e) Closed-loop transfer function

What is the Closed-loop transfer function $G_F(s)$ if you control the system with a P-controller with controller gain K_P?

Note: If you were unable to solve the previous task, continue with $G_S(s) = \frac{K_S}{(T_1 s+1)}$.

Calculate the asymptotic behaviour of the controlled system for $t \to \infty$ for a step-shaped input $w(t) = \Theta(t)$ with the limit theorem. Do you expect a permanent system deviation? If so, how can you minimize it?

(f) Simulation of the system

Simulate the system with MATLAB/Simulink. Create a new, empty Simulink model and add the hot water tank system by using the *Transfer Fcn* block. Use the equations

$$T_1 = \frac{d}{\lambda A} \cdot (m_w c_w + m_b c_b) \quad \text{und} \quad K_S = \frac{d}{\lambda A} \tag{2}$$

and the values shown in Figure 1.1 to calculate the system parameters. Use a jump (*Step*) to $P_0 = 5000\,\mathrm{W}$ as the input signal. You can display the waveform of signals with the *Scope* block.

(g) Extension of the simulation by a controller

Extend the simulation with a controller. Use the block *Sum* for the calculation of the control deviation and *PID Controller* for the controller. You can find the controller form used by Simulink in the block parameters of the PID controller under the heading *Compensator formula*.

The input should be a jump from 0 K to 15 K. First use a pure P controller, whereby the gain should be selected so that the maximum heating power $P_{max} = 5000\,\mathrm{W}$ is not exceeded. Is the set point reached?

Change the controller to a PI controller. The target temperature should be reached as quickly as possible, whereby a maximum overshoot of 0.5 K is permitted. Also experiment with a D component. What problems does this pose? Why do you not want to reach the maximum heating power?

Exercise 2 Stability

Given is the system $G_S(s) = \frac{1}{8s^2+4s+5}$ and the PI controller with the parameters $K_P = 3$ and $T_n = 2$.

Note: If you want to check your calculations with Simulink, note that the PID block in Simulink calculates with $I = \frac{1}{T_n}$.

(a) Transfer function of the PI controller

Determine the transfer function $G_R(s)$ of the PI controller. Use the formula for PI controllers known from the script. Give your result in the form of $\frac{B(s)}{A(s)}$.

(b) Closed-loop transfer function

Calculate the closed-loop transfer function of the controlled system. Construct your result in the form $\frac{B(s)}{A(s)}$. What is the characteristic polynomial?

Note: If you cannot solve the previous task, continue with $G_R(s) = \frac{sT_nK_P+K_P}{sT_n}$.

(c) Hurwitz Criteria

Conduct a stability analysis using the Hurwitz criterion. Is the system controlled by this PI regulator stable?

Note: If you cannot solve the previous task, continue with the characteristic polynomial $s^3 8T_n + s^2 4T_n + sT_n(5+Kp) + K_P$.

(d) Stability limit

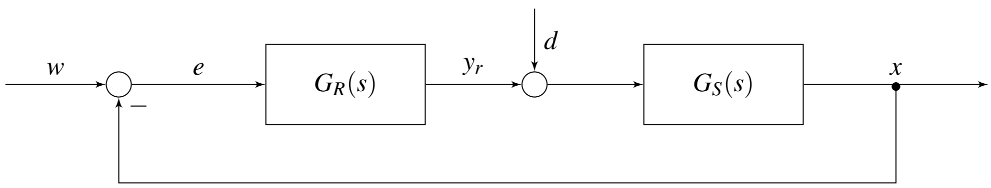
Which value range for T_n is allowed for stable operation under the assumption $K_P = 3$? (T_n be positive.)

Does it make sense to use this entire range for the realization of a controller? If not, why?

Note: If you were not able to solve the previous task, continue with the inequality $4T_n^2(5+K_P) - 8T_nK_P > 0$.

(e) Disturbance transfer function

What is the disturbance transfer function of the system for an disturbance $d(t)$ at the system input of the system $G_S(s)$?



Which part of your stability analysis would you have to redo if not an input signal is acting on the system, but an input disturbance $d(t)$? Is the controlled system stable in this case?

Exercise 3 Questions on control processes

In this task short comprehension questions about systems and controllers are asked. Answer the questions briefly and explain your answer if necessary.

1) Name the properties of a linear time invariant system.

2) What does the closed-loop transfer function describe.

3) What does disturbance transfer function describe.

4) What can be seen from the step response of a system.

5) Which type of controllers exist and in which systems can they be used?
   Give an example and explain why the controller is suitable for the system.

6) Under which circumstances can a PID-controler be used to control a non linear system?

7) Name and explain a heuristic approach for tuning controllers.

8) What are the requierments to apply the Hurwitz criteria?
   What does it say, when a system is stable according to Hurwitz?

9) What kind of differences and problems do you expect when characterizing a real system and testing control parameters?