3D Particle in a Box

Using separation of variables [$\Psi(x,y,z) = X(x)Y(y)Z(z)$] to solve the 3D Schrodinger Equation for a particle in a (cubic) box, we find:

$$-\frac{\hbar^2}{2m}\frac{X''}{X} = W_1$$
$$-\frac{\hbar^2}{2m}\frac{Y''}{Y} = W_2$$
$$-\frac{\hbar^2}{2m}\frac{Z''}{Z} = W_3$$

Where each equation is equivalent to that describing a 1D particle in the box with the same boundary conditions ($[X(0), Y(0), Z(0)] = 0$ and $[X(a), Y(a), Z(a)] = 0$) Therefore, we can write:

$$X(x)_n = \sqrt{\frac{1}{2a}}\sin(\frac{n_x\pi}{a}x)$$
$$Y(y)_n = \sqrt{\frac{1}{2a}}\sin(\frac{n_y\pi}{a}y)$$
$$Z(z)_n = \sqrt{\frac{1}{2a}}\sin(\frac{n_z\pi}{a}z)$$

The overall wavefunction is then:

$$\Psi(x,y,z)_n = \sqrt{\frac{8}{a^3}}\sin(\frac{n_x\pi}{a}x)\sin(\frac{n_y\pi}{a}y)\sin(\frac{n_z\pi}{a}z)$$

The energies are:

$$W_{1n} = \frac{n_x^2\pi^2\hbar^2}{2ma^2}$$
$$W_{2n} = \frac{n_y^2\pi^2\hbar^2}{2ma^2}$$
$$W_{3n} = \frac{n_z^2\pi^2\hbar^2}{2ma^2}$$

We observe what is formally known as degeneracy, where different combinations of of n_x, n_y, n_z, bring about the same total energy of the system.

$$W_n = W_{1n} + W_{2n} + W_{3n}$$

$\frac{W}{W_{1n}}$	(n_x, n_y, n_z)	Degeneracy
1	(1,1,1)	1
3	(2,1,1),(1,2,1),(1,1,2)	3
9	(2,2,1),(2,1,2),(1,2,2)	3
11	(3,1,1),(1,3,1),(1,1,3)	3
12	(2,2,2)	1