Question 6

From the fact the fundamental relation for a simple-material is first order homogenous and that

$$S^A = S^B = C(U^2VN)^{1/4}$$

the addition of A and B to an enclosed container is analogous to the scenario of A and B separated by a non-fixed, diathermal permeable piston. Thus the new fundamental relation is

$$S = S_A + S_B = C(U_A^2V_AN_A)^{1/4} + C(U_B^2V_BN_B)^{1/4}$$

with the following constraints

$$\begin{aligned} U_T &= U_A + U_B \\ V_T &= V_A + V_B \\ N_T &= N_A + N_B \end{aligned}$$

Question 8

Given

$$S_i = C(N_1 + N_2) + (N_1 + N_2)R\ln\left(\frac{U^{3/2}V}{(N_1+N_2)^{5/2}}\right) - N_1R\ln\left(\frac{N_1}{N_1+N_2}\right) - N_2R\ln\left(\frac{N_2}{N_1+N_2}\right)$$

We can write the overall entropy of the container

$$\begin{aligned} S =& S_A + S_B \\ =& C(N_1^A + N_2^A) + (N_1^A + N_2^A)R\ln\left(\frac{U_A^{3/2}V^A}{(N_1^A+N_2^A)^{5/2}}\right) - N_1^AR\ln\left(\frac{N_1^A}{N_1^A+N_2^A}\right) - N_2^AR\ln\left(\frac{N_2^A}{N_1^A+N_2^A}\right) \\ &+ C(N_1^B + N_2^B) + (N_1^B + N_2^B)R\ln\left(\frac{U_B^{3/2}V^B}{(N_1^B+N_2^B)^{5/2}}\right) - N_1^BR\ln\left(\frac{N_1^B}{N_1^B+N_2^B}\right) - N_2^BR\ln\left(\frac{N_2^B}{N_1^B+N_2^B}\right) \end{aligned}$$

The system is subject to the following constraints

$$\begin{aligned} V^A =& V^B = 5[L] \\ V =& V^A + V^B = 10[L] \\ N_2^A =& 1[mole] \\ N_2^B =& 0.75[mole] \\ N =& N_1^A + N_2^A + N_1^B + N_2^B = 3.5[mole] \\ U =& U_A + U_B \end{aligned}$$

Taking derivatives (i.e.: virtual displacements)

$$\frac{\partial S}{\partial N_1^A} = \frac{\mu_1^A}{T} = C + R\ln\frac{V^A U_A^{\frac{3}{2}}}{(N_1^A+N_2^A)^{\frac{5}{2}}} - R\ln\frac{N_1^A}{N_1^A+N_2^A} - \frac{5}{2}R$$
$$\frac{\partial S}{\partial N_1^B} = \frac{\mu_1^B}{T} = C + R\ln\frac{V^B U_B^{\frac{3}{2}}}{(N_1^B+N_2^B)^{\frac{5}{2}}} - R\ln\frac{N_1^B}{N_1^B+N_2^B} - \frac{5}{2}R$$
$$\frac{\partial S}{\partial U^A} = \frac{1}{T^A} = \frac{3R(N_1^A+N_2^A)}{2U^A}$$
$$\frac{\partial S}{\partial V^A} = \frac{P^A}{T^A} = \frac{R(N_1^A+N_2^A)}{V^A}$$

At equilibrium, we must have $\mu_1^A = \mu_1^B$, thus we can write

$$\ln\frac{V^A(\frac{3RT}{2})^{3/2}}{(N_1^A)} = \ln\frac{V^B(\frac{3RT}{2})^{3/2}}{(N_1^B)}$$
$$\ln(N_1^A) = \ln(N_1^B)$$

From the conservation of moles, we find

$$N_1^A = N_1^B = \frac{N - N_2^A - N_2^B}{2}$$
$$=0.875[mole]$$

From the conservation of energy (no heat lost to the surroundings), the equilibrium temperature is the same on both sides of the piston

$$U = \frac{3RT^A(N_1^A+N_2^A)}{2} + \frac{3RT^A(N_1^A+N_2^A)}{2} = \frac{3RT(N_1^A+N_2^A+N_1^B+N_2^B)}{2}$$
$$T = \frac{T^A(N_1^A+N_2^A)+T^A(N_1^A+N_2^A)}{N_1^A+N_2^A+N_1^B+N_2^B}$$
$$= \frac{320(1.75)+250(1.75)}{3.5}$$
$$=285K$$

The pressures are obtained from the equations of state

$$P^A = \frac{RT(N_1^A+N_2^A)}{V^A}$$
$$= \frac{285R(1.875)}{5*10^{-3}}$$
$$= 888.5[kPa]$$
$$P^B = \frac{RT(N_1^B+N_2^B)}{V^B}$$
$$= \frac{285R(1.625)}{5*10^{-3}}$$
$$= 770.0[kPa]$$