Lecture 13

Note Title 27/02/2012

$FeCl_2$ (C+L § 4.6.2)

triangular layers

A
B
A
B

Assume two sublattices: A + B

$FeCl_2$ $\quad \mathcal{H} = -J_1 \sum_{\langle // \rangle} S_i S_j + J_2 \sum_{\langle \perp \rangle} S_i S_j$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$-J_1 \cdot z_1 \cdot \frac{1}{2}\left(m_A^2 + m_B^2\right) \cdot \frac{1}{2} \qquad\qquad J_2 \cdot z_2 \cdot \frac{1}{2} \cdot m_A m_B$$

$$f = \frac{1}{2} z_2 J_2 m_A m_B - \frac{1}{4} z_1 J_1 \left(m_A^2 + m_B^2\right)$$
$$- \frac{1}{2} T \left[\ln 2 - \frac{1}{2}(1+m_A)\ln(1+m_A) - \frac{1}{2}(1-m_A)\ln(1-m_A) + \cdots \right]$$

$$m = \frac{m_A + m_B}{2} \qquad m_s = \frac{m_A - m_B}{2}$$

$$f = f_0(m) + \frac{1}{2} r_s(m) m_s^2 + u_4(m) m_s^4 + u_6(m) m_s^6$$

$$f_0(m) \equiv \frac{1}{2}\left(z_2 J_2 - z_1 J_1\right) m^2$$

$$r_s(m) = \frac{T}{1-m^2} - \left(z_1 J_1 + z_2 J_2\right)$$

$$u_4(m) = \frac{T}{12} \frac{1+3m^2}{\left(1-m^2\right)^3}$$

But $f(m, m_s)$, $\Rightarrow$ make $g(m_s)$

Use $\frac{\partial f}{\partial m} = H$ to calculate $m \equiv m_0$ for $m_s = 0$

Then expand f around $m = m_0 + \delta m$

$$f(m, m_s) = f(m_0 + \delta m, m_s) = f(m_0, m_s) + \left.\frac{\partial f}{\partial m}\right|_{m=m_0} \delta m + \cdots$$

$$g(m_s, \delta m) = \frac{1}{2}\chi_0^{-1}(\delta m)^2 + \frac{1}{2} r_s(m_0) m_s^2 + u_4(m_0) m_s^4 + u_6(m_0) m_s^6$$
$$+ \lambda_0 m_s^2 \delta m$$

$$\chi_0^{-1} \equiv z_2 J_2 - z_1 J_1 + \frac{T}{(1-m_0^2)^2} \qquad \lambda_0 = \frac{m_0 T}{1-m_0^2}$$

$$g(m_s, \delta m) = \frac{1}{2}\chi_0^{-1}\left(\delta m + \lambda_0 \chi_0 m_s^2\right)^2 +$$
$$\frac{1}{2} r_s(m_0) m_s^2 + \left(u_4 - \frac{1}{2}\chi_0^{-1}\lambda_0^2 \chi_0^2 m_s^4\right) m_s^4 + u_6 m_s^6$$

At equilibrium if $m_s \neq 0$ $\underline{\delta m = -\lambda_0 \chi_0 m_s^2}$

$u_4 - \frac{1}{2}\chi_0 \lambda_0^2 = 0$ is tricritical point.

Ginzburg criterion (V.L. Ginzburg, 1960)

Fluctuation averaged over coherence volume $V_\xi \sim \xi^d$ should be much less than $\langle\phi\rangle^2$ itself.

$$\langle(\delta\phi)^2\rangle \ll \langle\phi\rangle^2$$

$$\delta\phi = \phi(\vec{x}) - \langle\phi\rangle$$

$$\langle(\delta\phi)^2\rangle = \frac{1}{V_\xi^2}\int d^d\vec{x}\, d^d\vec{x}'\, \langle \underbrace{\delta\phi(\vec{x})\,\delta\phi(\vec{x}')}_{\text{Ursell function}} \rangle$$

$$C(\vec{x}-\vec{x}') = C(\vec{x},0)$$

$$= \frac{1}{V_\xi}\underbrace{\int d^d\vec{x}\, C(\vec{x},0)}$$

Static Susceptibility sum rule

$$k_B T \chi = S(q=0)$$

$$= \int C(\vec{x},0)\, d^d\vec{x}$$

$$\sim \frac{1}{\xi^d}\cdot T\chi$$

$$\xi^{-d}\cdot T\cdot\chi \ll \langle\phi\rangle^2$$

$$\begin{cases} \langle\phi\rangle^2 \sim t^{2\beta} \\ \chi \sim t^{-\gamma} \\ \xi \sim t^{-\nu} \end{cases} \qquad t = \frac{|T-T_c|}{T_c}$$

$$t^{+\nu d}\cdot t^{-\gamma}\cdot t^{-2\beta} \ll 1$$

$$t^{\nu\left(d - \frac{\gamma+2\beta}{\nu}\right)} \ll 1$$

$$\beta = \frac{1}{2} \quad \gamma = 1, \quad \nu = \frac{1}{2}$$

$$t^{\frac{1}{2}(d-4)} \ll 1$$

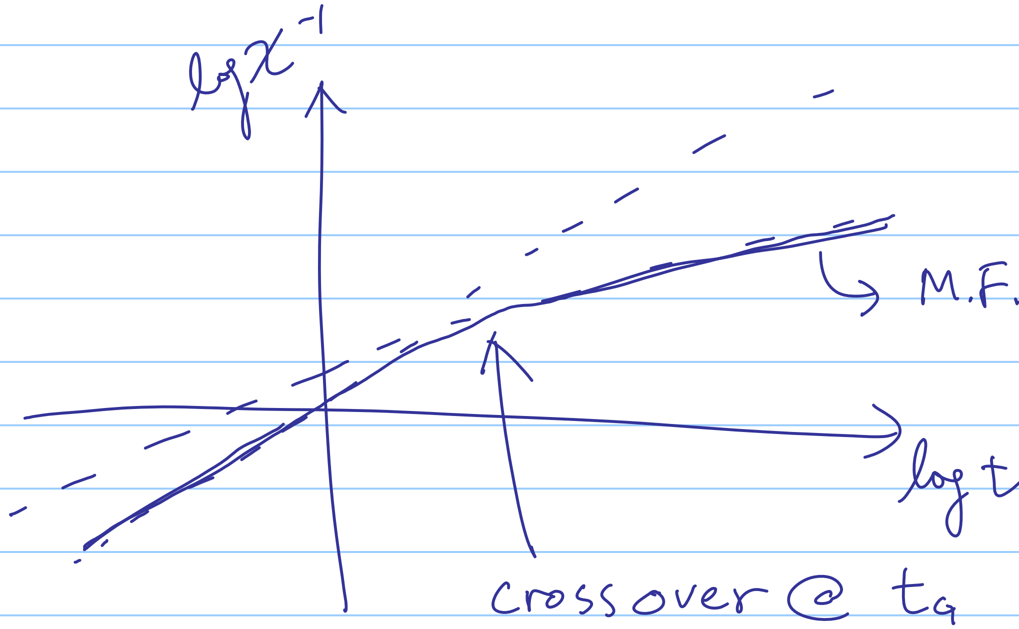
$$d_c \equiv \frac{\gamma+2\beta}{\nu}$$

upper critical dimension

$d > 4$: OK

$d < 4$: not OK.

Tricritical $\beta = 1/4 \Rightarrow d_c = 3$



$$t_G = \frac{k_B^2}{32\pi^2 (\Delta C_v)^2 \xi_0^6}$$

C_v jump.

bare coherence length $\sim 10^4$ Å for BCS s.c. (e.g. Al)

S.C.: $t_G \approx 10^{-16}$

L.C.: $t_G \approx 10^{-5}$

Magnet: $t_G \approx 1$

But cuprates ξ_0 is short $\Rightarrow$ $t_G \approx 0.1$