Question 9

Given that an ideal gas satisfies

$$\mu(T,P) = \mu(T,P_r) + \bar{R}T \ln \frac{P}{P_r}$$

The Gibbs function is

$$G = N(\mu(T,P_r) + \bar{R}T \ln \frac{P}{P_r})$$

The specific internal energy is

$$u = Ts - Pv + \mu$$

and specific enthalpy is

$$\begin{aligned} h &= Ts - Pv + \mu + Pv \\ &= Ts + \mu \end{aligned}$$

Entropy is

$$\begin{aligned} s &= -\frac{\partial \mu}{\partial T} \\ &= -\frac{\partial \mu(T,P_r)}{\partial T} - \bar{R} \ln \frac{P}{P_r} \end{aligned}$$

Substituting back into the equation for enthalpy gives

$$\begin{aligned} h &= -T\frac{\partial \mu(T,P_r)}{\partial T} - \bar{R}T \ln \frac{P}{P_r} + \mu(T,P_r) + \bar{R}T \ln \frac{P}{P_r} \\ h &= -T\frac{\partial \mu(T,P_r)}{\partial T} + \mu(T,P_r) \end{aligned}$$

Thus h is only a function of T (but also P_r).

Question 12

For an isotherm, T_0, the pressure as a function of specific volume is

$$P(v) = \begin{cases} P_1 + a(v_1 - v) & : v < v_1 (I) \\ v\frac{P_2 - P_1}{v_2 - v_1} + \frac{v_2 P_1 - v_1 P_2}{v_2 - v_1} & : v_1 < v < v_2 (II) \\ P_2 + a(v_2 - v) & : v_2 < v (III) \end{cases}$$

Restricting a to positive values, the criterion for the areas to be equal is only satisfied if the saturation pressure is the midpoint between P_1 and P_2 since the areas are defined by similar triangles.

$$P_s = \frac{P_1 + P_2}{2}$$

Following Table 1.3 in the notes, the stable regions are

$$\begin{aligned} P_s < P < P_2 \quad & (I) \\ P_1 < P < P_s \quad & (III) \end{aligned}$$