Given

$$\psi = ce^{-\frac{\alpha r}{2a}}$$

Normalization, see `http://www.wolframalpha.com/input/?i=Integrate%5Bx%5E2*e%5E%28-%28b*x%29%2Fa%29%29%2C+%7Bx%2C+0%2C+%5C%5BInfinity%5D%7D%5D++`

$$\begin{aligned}
<\psi^*|\psi> &= 1 \\
&= \int_0^\infty \int_0^\pi \int_0^{2\pi} c^2 e^{-\frac{\alpha r}{a}} r^2 sin\theta d\phi d\theta dr \\
&= 4\pi c^2 \int_0^\infty r^2 e^{-\frac{\alpha r}{a}} dr \\
c^2 &= \frac{\alpha^3}{8\pi a^3}
\end{aligned}$$

Energy expectation

$$\begin{aligned}
E &=<\psi^*|H|\psi> \\
&= \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi[\frac{-\hbar^2}{2m}\nabla^2 + V]\psi r^2 sin\theta d\phi d\theta dr
\end{aligned}$$

Kinetic portion of H

$$\begin{aligned}
T &= \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi \left[\frac{-\hbar^2}{2m}\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}(sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2}{\partial^2\phi}\right)\right]\psi r^2 sin\theta d\phi d\theta dr \\
&= 4\pi\frac{-\hbar^2}{2m}\int_0^\infty \psi\left[\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi}{\partial r})\right)\right] r^2 dr
\end{aligned}$$

Working out the derivative

$$\begin{aligned}
\left[\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi}{\partial r})\right)\right] &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{-\alpha c}{2a}e^{-\frac{\alpha r}{2a}}\right) \\
&= \frac{1}{r}\frac{-\alpha c}{a}e^{-\frac{\alpha r}{2a}} + \frac{\alpha^2 c}{4a^2}e^{-\frac{\alpha r}{2a}}
\end{aligned}$$

Multiplying with $\psi*$ and r^2

$$T = 4\pi c^2\frac{-\hbar^2}{2m}\int_0^\infty \frac{-\alpha r}{a}e^{-\frac{\alpha r}{a}} + \frac{\alpha^2 r^2}{4a^2}e^{-\frac{\alpha r}{a}} dr$$

Integrating, see `http://www.wolframalpha.com/input/?i=Integrate%5Bb%5E2%2F%284*a%5E2%29*x%5E2*e%5E%28-%28b*x%29%2Fa%29%29%2C+%7Bx%2C+0%2C+%5C%5BInfinity%5D%7D%5D++` and `http://www.wolframalpha.com/input/?i=Integrate%5Bb%2F%28a%29*x*e%5E%28-%28b*x%29%2Fa%29%29%2C+%7Bx%2C+0%2C+%5C%5BInfinity%5D%7D%5D++`

$$\begin{aligned}
T &= 4\pi c^2\frac{-\hbar^2}{2m}[\frac{a}{2\alpha} - \frac{a}{\alpha}] \\
&= \frac{\hbar^2\alpha^2}{8ma^2}
\end{aligned}$$

Potential portion of H

$$\begin{aligned} V &= 4\pi \int_0^\infty \psi V \psi r^2 dr \\ &= -4\pi c^2 A \int_0^\infty r^2 e^{-\frac{\alpha+1}{a}} dr \\ &= -8\pi c^2 A \frac{a^3}{(1+\alpha)^3} \\ &= -A \frac{\alpha^3}{(1+\alpha)^3} \end{aligned}$$

Energy functional

$$\begin{aligned} E[\alpha] &= T + V \\ &= \frac{\hbar^2 \alpha^2}{8ma^2} - A\frac{\alpha^3}{(1+\alpha)^3} \end{aligned}$$

Minimizing see `http://www.wolframalpha.com/input/?i=d%2Fdx+%28h%5E2*x%5E2%2F%288*m*a%5E2%29+-A*x%5E3%2F%281%2Bx%29%5E3%29`

$$\frac{dE[\alpha]}{d\alpha} = \frac{\hbar^2 \alpha}{4ma^2} - 3A\frac{\alpha^2}{(1+\alpha)^4}$$

Solving for $\frac{dE[\alpha]}{d\alpha} = 0$, see `http://www.wolframalpha.com/input/?i=solve+%281.054571726*10%5E%E2%88%9234%29%5E2*x%2F%288*1.672621777*10%5E%E2%88%9227*%282.2*10%5E-15%29%5E2%29-3*5.12696468*10%5E-12*x%5E2%2F%281%2Bx%29%5E4%3D0+for+x`

$$\begin{aligned} \alpha &= 0 \\ &= 0.011696 \\ &= 3.07324 \end{aligned}$$

Choosing $\alpha = 3.07324$

$$E[\alpha] = -3.621[MeV]$$

Expectation of r

$$\begin{aligned} < r > &= < \psi^* | r | \psi > \\ &= \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi r \psi r^2 sin\theta d\phi d\theta dr \\ &= 4\pi c^2 \int_0^\infty r^3 e^{-\frac{\alpha r}{a}} dr \\ &= 4\pi c^2 \frac{6a^4}{\alpha^4} \\ &= \frac{3a}{\alpha} \end{aligned}$$

Expectation of r^2

$$\begin{aligned}
< r^2 > &=< \psi^* | r^2 | \psi > \\
&= \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi r^2 \psi r^2 sin\theta d\phi d\theta dr \\
&= 4\pi c^2 \int_0^\infty r^4 e^{-\frac{\alpha r}{a}} dr \\
&= 4\pi c^2 \frac{24a^5}{\alpha^5} \\
&= \frac{12a^2}{\alpha^2}
\end{aligned}$$