Question 6

From the fact the fundamental relation for a simple-material is first order homogenous and that

$$S^A = S^B = C(U^2VN)^{1/4}$$

the addition of A and B to an enclosed container will yield another simple-material system with a fundamental relation given by

$$S = C(U^2VN)^{1/4}$$

Question 8

Given

$$S_i = C(N_1+N_2) + (N_1+N_2)R\ln\left(\frac{U^{3/2}V}{(N_1+N_2)^{5/2}}\right) - N_1R\ln\left(\frac{N_1}{N_1+N_2}\right) - N_2R\ln\left(\frac{N_2}{N_1+N_2}\right)$$

We can write the overall entropy of the container

$$\begin{aligned}
S =& S_A + S_B \\
=& C(N_1^A+N_2^A) + (N_1^A+N_2^A)R\ln\left(\frac{U_A^{3/2}V^A}{(N_1^A+N_2^A)^{5/2}}\right) - N_1^AR\ln\left(\frac{N_1^A}{N_1^A+N_2^A}\right) - N_2^AR\ln\left(\frac{N_2^A}{N_1^A+N_2^A}\right) \\
&+ C(N_1^B+N_2^B) + (N_1^B+N_2^B)R\ln\left(\frac{U_B^{3/2}V^B}{(N_1^B+N_2^B)^{5/2}}\right) - N_1^BR\ln\left(\frac{N_1^B}{N_1^B+N_2^B}\right) - N_2^BR\ln\left(\frac{N_2^B}{N_1^B+N_2^B}\right)
\end{aligned}$$

The system is subject to the following constraints

$$\begin{aligned}
V^A =& V^B = 5[L] \\
V =& V^A + V^B = 10[L] \\
N_2^A =& 1[mole] \\
N_2^B =& 0.75[mole] \\
N =& N_1^A + N_2^A + N_1^B + N_2^B = 3.5[mole] \\
U =& U_A + U_B
\end{aligned}$$

Taking derivatives (i.e.: virtual displacements)

$$\begin{aligned}
\frac{\partial S}{\partial N_1^A} &= C + R\ln\frac{V^AU_A^{\frac{3}{2}}}{(N_1^A+N_2^A)^{\frac{5}{2}}} - R\ln\frac{N_1^A}{N_1^A+N_2^A} - \frac{5}{2}R \\
\frac{\partial S}{\partial N_1^B} &= C + R\ln\frac{V^BU_B^{\frac{3}{2}}}{(N_1^B+N_2^B)^{\frac{5}{2}}} - R\ln\frac{N_1^B}{N_1^B+N_2^B} - \frac{5}{2}R \\
\frac{\partial S}{\partial U^A} &= \frac{1}{T^A} = \frac{3R(N_1^A+N_2^A)}{2U^A} \\
\frac{\partial S}{\partial V^A} &= \frac{P^A}{T^A} = \frac{R(N_1^A+N_2^A)}{V^A}
\end{aligned}$$

We can write

$$\frac{3P_AV_A}{2} = U_A$$

Substituting the previous expression and setting $\frac{\partial S}{\partial N_1^i} = 0$

$$\begin{aligned}0 =& C + R\ln\frac{V^A(\frac{3P^AV^A}{2})^{\frac{3}{2}}}{(N_1^A+N_2^A)^{\frac{5}{2}}} - R\ln\frac{N_1^A}{N_1^A+N_2^A} - \frac{5}{2}R \\ 0 =& C + R\ln\frac{V^B(\frac{3P^BV^B}{2})^{\frac{3}{2}}}{(N_1^B+N_2^B)^{\frac{5}{2}}} - R\ln\frac{N_1^B}{N_1^B+N_2^B} - \frac{5}{2}R\end{aligned}$$

Using $N = N_1^A + N_2^A + N_1^B + N_2^B$ and $P^A = P^B$ and $V^A = V^B$

$$R\ln\frac{V^A(\frac{3P^AV^A}{2})^{\frac{3}{2}}}{(N_1^A+N_2^A)^{\frac{5}{2}}} - R\ln\frac{N_1^A}{N_1^A+N_2^A} = R\ln\frac{V^A(\frac{3P^AV^A}{2})^{\frac{3}{2}}}{(N_1^B+N_2^B)^{\frac{5}{2}}} - R\ln\frac{N_1^B}{N_1^B+N_2^B}$$