Magnetization as a function of temperature

From degenerate pertubation theory, the energy correction from the applied magnetic field B is governed by the following equation:

$$\frac{e\hbar B}{2m_e}[H'][A] = W'[A]$$

Where the elements of $[H']$:

$$H'_{mn} = \int \psi *_{0m} L_z \psi_{0n} dV$$

This yields the following eigenvalue problem:

$$\frac{e\hbar B}{2m_e}\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}A_1\\ A_0\\ A_{-1}\end{bmatrix} = W'\begin{bmatrix}A_1\\ A_0\\ A_{-1}\end{bmatrix}$$

The eigenvalues are found from solving the cubic equation:

$$\frac{e\hbar B}{2m_e}(1-W')(0-W')(-1-W') = 0$$

The splitting of the degeneracy results in three distinct energy states (The lower energy being in direction of B):

$$W'_1 = -1.7365E - 05[eV]$$
$$W'_0 = 0.0000[eV]$$
$$W'_{-1} = 1.7365E - 05[eV]$$

The Maxwell-Boltzmann distribution has the form:

$$N_i = \frac{N exp(-W_i/kT)}{\sum_i exp(-W_i/kT)}$$
$$W_i = W_0 + W'_i$$

The $exp(W_0/kT)$ will cancel in the numerator and denominator, giving:

$$N_i = \frac{N exp(-W'_i/kT)}{\sum_i exp(-W'_i/kT)}$$

Substituting in $W_i' = -1.7365E-05, 0, 1.7365E-05K$ and repeating the calcution for $T = 300, 77, 4.2K$:

N_i	$T = 300$	$T = 77$	$T = 4.2K$
N_{-1}	3.3356	3.3421	3.4945
N_0	3.3333	3.3333	3.3308
N_1	3.3311	3.3246	3.1747

Table 1: Distribution particles into the three energies (all values are to be multiplied by 1^{23}

Recalling the eigenvalues of L_z, the average z-component of angular momentum for a given temperature is:

$$\begin{aligned}\langle \bar{L_z} \rangle &= \frac{\sum_i N_i \langle L_{zi} \rangle}{N} \\ &= \frac{N_{-1} \times (1) + N_0 \times (0) + N_1 \times (-1)}{N}\end{aligned}$$

Therefore, the macroscopic z-component of magnetization for a given temperature is:

$$\vec{\mu_z} = \frac{\langle \bar{L_z} \rangle QN}{2m_e}$$

$Temperature$	$u_z[J/T]$
300	0.0004153
77	0.0016180
4.2	0.029653

Table 2: Z-component of the magnetization