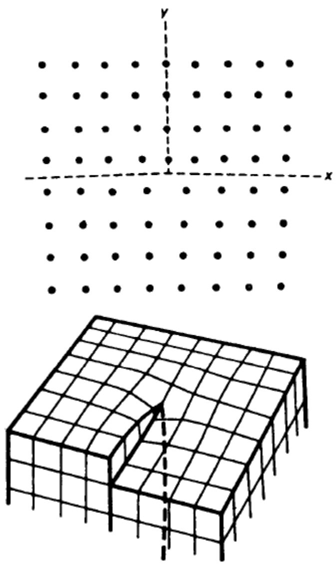
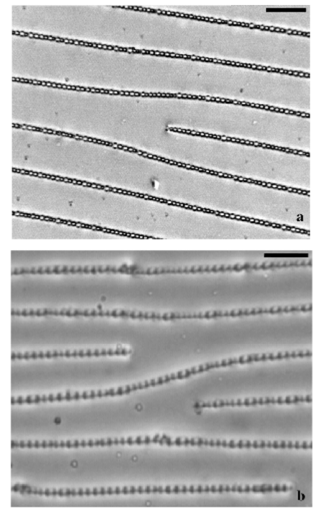
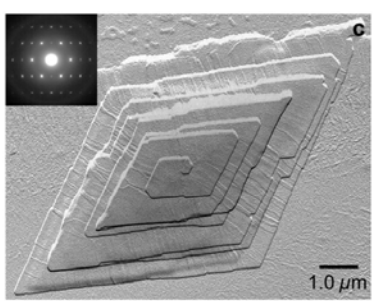
Dislocations





History

- Taylor, Orowan, and Polanyi (1934)
  - How crystals flow
- Crystal growth theory by F. C. Frank



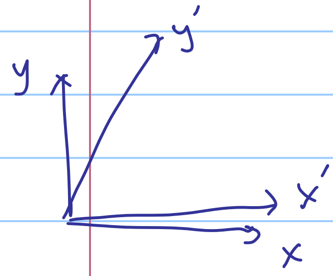
TEM image of a precipitate C-LA_8 showing a multilayered lozenge-shaped single crystal formed by spiral dislocations. (*Inset*) Electron diffraction pattern revealing an orthorhombic unit cell.

Hwang J J et al. PNAS 2002;99:9662-9667

Elasticity (Landau + Lifshitz)

Note Title

Strain Tensor



$\vec{r}' = (x_1', x_2', x_3')$ $\quad \vec{r} = (x_1, x_2, x_3)$.

$\vec{u} = \vec{r}' - \vec{r}$

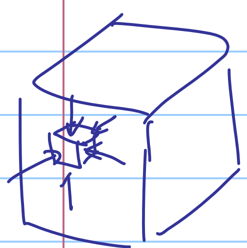
$(dl')^2 = (dl)^2 + 2u_{ik}\, dx_i dx_k$

$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right)$ for small deformation

Stress Tensor

When deformation occurs, forces arise which tend to return the body to equilibrium.

$\rightarrow$ internal stress (depends on molecular forces)



Total force on the volume element dV is

$\int F_i\, dV \Rightarrow$ want to make surface integral (similar to divergence theorem).

$$\int F_i\, dV = \int \frac{\partial \sigma_{ik}}{\partial x_k} dV = \oint \sigma_{ik}\, df_k$$

$\uparrow$ stress tensor

$\int \rho\, dV = \int \nabla \cdot \vec{E}\, dV = \oint \vec{E} \cdot d\vec{A}$

$$F_i = \frac{\partial \sigma_{ik}}{\partial x_k}$$

force

Energy of deformation

$$dE = \underbrace{T\,dS}_{\text{Heat}} + \underbrace{\sigma_{ik}\, du_{ik}}_{\text{Work done by internal stress}}$$

free energy

$$dF = dE - T\,dS - S\,dT = -S\,dT + \sigma_{ik}\, du_{ik}$$

$$\frac{\partial F}{\partial u_{ik}} = \sigma_{ik}$$

$$F = A u_{ik} + B u_{ik}^2 + C u_{ik}^3 + \cdots \qquad E = Ax + Bx^2$$

Free energy can be expanded as powers of u_{ik}

No linear term in u_{ik}. ($u_{ik} = 0$ is the equil. position)

$$F = F_0 + \frac{1}{2}\lambda u_{ii}^2 + \mu u_{ik}^2$$ (general expression for isotropic body)

λ, μ: Lamé coefficents

For isotropic body $\quad u_{ik} = \underbrace{\left(u_{ik} - \frac{1}{3}\delta_{ik} u_{ll}\right)}_{\text{shear}} + \underbrace{\frac{1}{3} u_{ik} u_{ll}}_{\text{hydrostatic deformation}}$

$$F = \mu\left(u_{ik} - \frac{1}{3}\delta_{ik} u_{ll}\right)^2 + \frac{1}{2} K u_{ll}^2 \qquad K = \lambda + \frac{2}{3}\mu$$

μ: Shear Modulus $\quad K$: Bulk modulus

<u>For solids (crystals)</u>

In general $\quad F = \frac{1}{2} K_{iklm} u_{ik} u_{lm}$

K_{iklm}: elastic modulus tensor (4th rank tensor)

$\rightarrow$ ($3^4 = 81$ components)

u_{ik} : symmetric $\Rightarrow$ $K_{iklm} = K_{kilm} \cdots$

$\Rightarrow$ 21 independent components

For cubic crystals $\rightarrow$ <u>only 3 independent components</u>