Assignment #2

Due : 10am March 20th 2012 (before the lecture)

1. Choose a phase diagram of your choice and background. Give a one paragraph explanation.
2. Find the ground state (stable configuration at $T = 0$) of the following spin models:

(a) The one-dimensional Ising model with first and second neighbor interactions

$$\mathcal{H} = -J_1 \sum_i \sigma_i \sigma_{i+1} - J_2 \sum_i \sigma_i \sigma_{i+2}, \quad \sigma_i = \pm 1.$$

Consider both positive and negative values of the exchange couplings. Present your answer as a phase diagram with J_1 and J_2 as horizontal and vertical axes, respectively.

(b) The one-dimensional, 3-state chiral clock model

$$\mathcal{H} = -J \sum_i \cos 2\pi (n_i - n_j + \Delta)/3, \quad n_i = 1, 2, 3.$$

for $J > 0$ and all values of Δ. Your phase diagram in this case is one dimensional with Δ as a variable.

3. (Chaikin and Lubensky problem 4.4 on p. 210) Show that the tricritical point T_t and H_t (for $z_2 J_2 / z_1 J_1 > 3/5$) is given by

$$\begin{aligned} \frac{T_t}{T_N} &= 1 - \frac{z_2 J_2}{3 z_1 J_1}, \\ \frac{H_t}{T_N} &= \frac{1}{2}\zeta \ln\left(\frac{1+\sqrt{1-\zeta}}{1-\sqrt{1-\zeta}}\right) + \frac{T_m}{T_N}\sqrt{1-\zeta}, \end{aligned}$$

where $\zeta \equiv T_t / T_N$. Note that there is a typo in the textbook; the correct formula is $\chi^{-1}(m_0) = T_m + T(1 - m_0^2)^{-1}$ and $\lambda(m_0) = m_0 T (1 - m_0^2)^{-2}$.

4. Peer-reviewing is an important part of scientific research. Critical and unbiased referee reports often help improving the paper. Read the following article and write a referee report of about 1 page long. You can find helpful information at the publisher's website, for example, Physical Review has information available at http://publish.aps.org/refinfo.html.

- C.-H. Du *et al.* Phys. Rev. Lett. **84**, 3911 (2000)