Note Title 27/02/2012

General O.P. for L.C.: traceless, symmetric, 2^{nd} rank tensor

$$Q_{\alpha\beta} = n_\alpha n_\beta - \frac{1}{3}\delta_{\alpha\beta}$$

$\rightarrow$ diagonalize (for uniaxial nematics) $\hat{z} : \hat{n}$ (director of nematic)

$$\bar{\bar{Q}} = \begin{pmatrix} -S/3 & 0 & 0 \\ 0 & -S/3 & 0 \\ 0 & 0 & 2S/3 \end{pmatrix}$$

$$S \equiv \frac{1}{2}\langle(3\cos^2\theta - 1)\rangle = \begin{cases} 1 & \text{for } \theta = 0, \pi \\ 0 & \text{for random orientation} \end{cases}$$

Landau free energy

$$f = \frac{3A}{4}\sum_{\alpha\beta} Q_{\alpha\beta}Q_{\beta\alpha} - \frac{3B}{2}\sum_{\alpha\beta\gamma} Q_{\alpha\beta}Q_{\beta\gamma}Q_{\gamma\alpha}$$

$$+ \frac{9D}{16}\left|\sum_{\alpha\beta} Q_{\alpha\beta}Q_{\beta\alpha}\right|^2 + \frac{D'}{4}\sum_{\alpha\beta\gamma\delta} Q_{\alpha\beta}Q_{\beta\gamma}Q_{\gamma\delta}Q_{\delta\alpha} + \cdots$$

$$= \frac{A}{2}S^2 - \frac{B}{3}S^3 + \frac{D}{4}S^4 + \cdots \qquad B, D > 0$$

$$= \left(\frac{A}{2} - \frac{B^2}{9D}\right)S^2 + \frac{D}{4}\left(S - \frac{2B}{3D}\right)^2 S^2$$

Set $A \equiv a(T - T^*)$ $\quad a > 0$

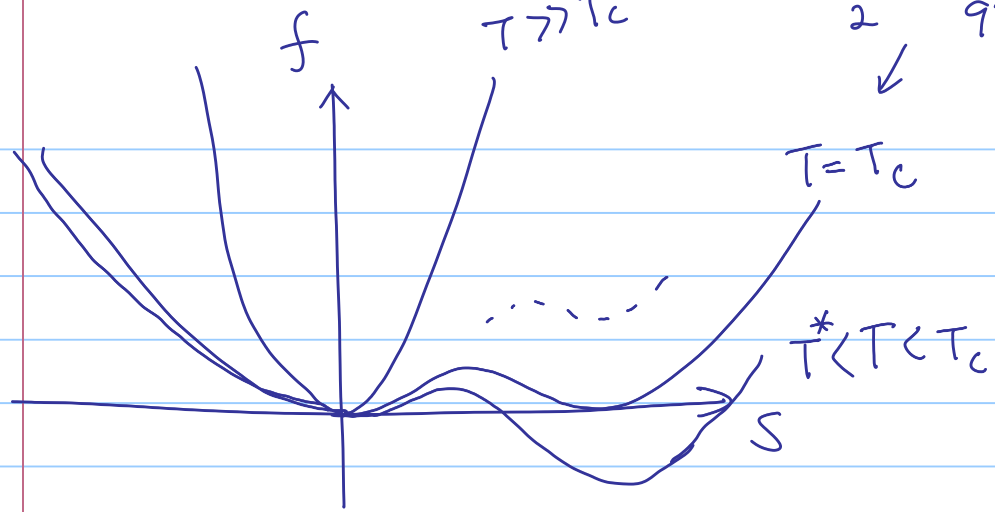
For $T > T^*$

$\frac{A}{2} > \frac{B^2}{9D}$: minimum for $f = 0$, $S = 0$

$\frac{A}{2} = \frac{B^2}{9D}$: minimum $f = 0$ for $\left[\begin{array}{l} S = 0 \\ S = \frac{2B}{3D} \end{array}\right.$

$$\frac{A}{2}=\frac{B^2}{9D} \rightarrow \frac{1}{2}a(T-T^*)=\frac{B^2}{9D}$$

$$T_c = T^* + \frac{1}{a}\frac{2B^2}{9D}$$



$T>T_c$: $s=0$

$T<T_c$: $s=s_0$

$T=T_c$: Two-phase coexistence.

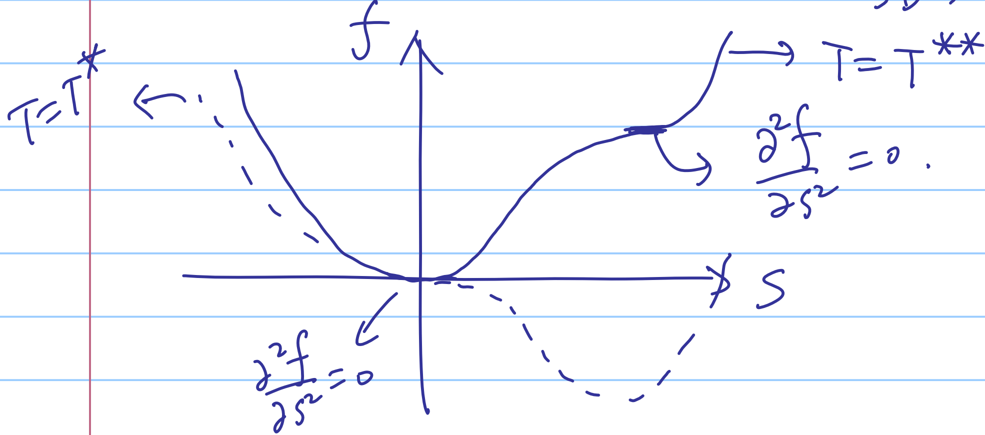
$T=T^* \Rightarrow \quad \frac{\partial^2 f}{\partial s^2} = A + 2Bs + \cdots = 0$ for $s=0$

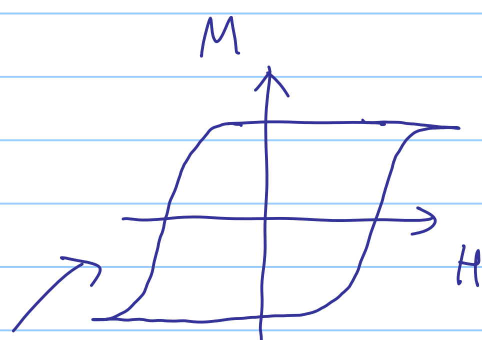
$T>T^{**}$: $s=0$ local and global minimum.

$T^{**}>T>T_c$: $s=0$ (2 minimum), $s=0$ global

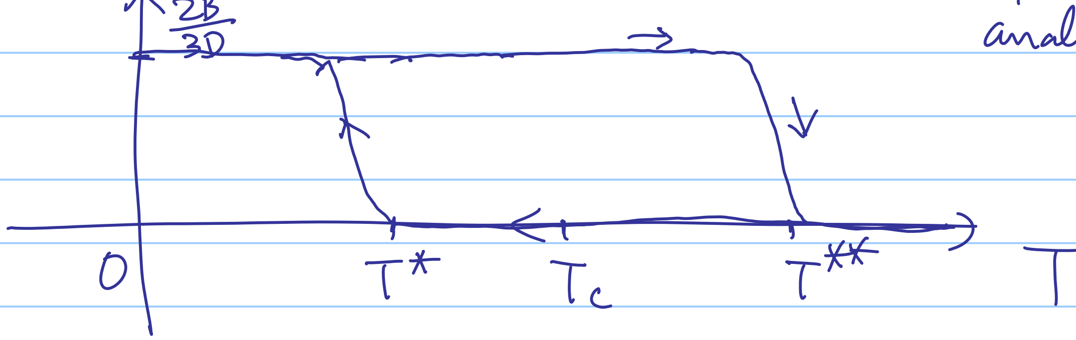
$T^*<T<T_c$: 2 local minimum, $s=\frac{2B}{3D}$: global minimum

$T<T^*$: minimum @ $s=\frac{2B}{3D}$.





analogy

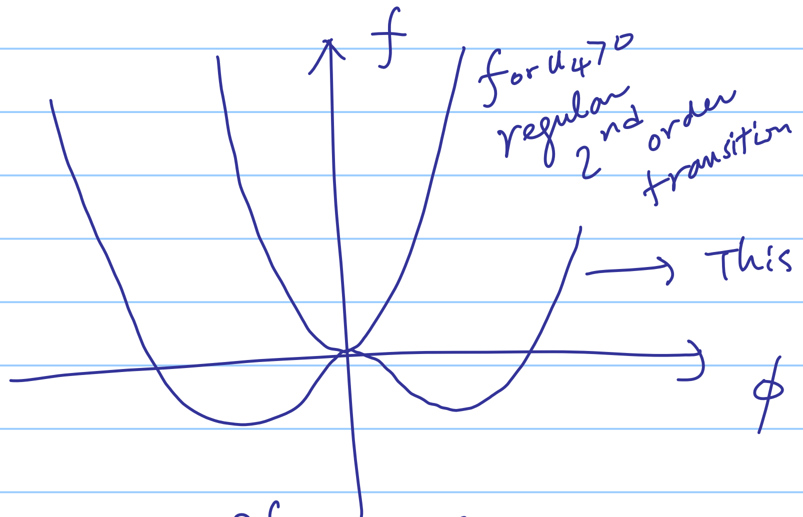


→ 1st phase transition (cubic term drives this)

Q: What if there is no cubic term?

A: You can still have 1st order transition

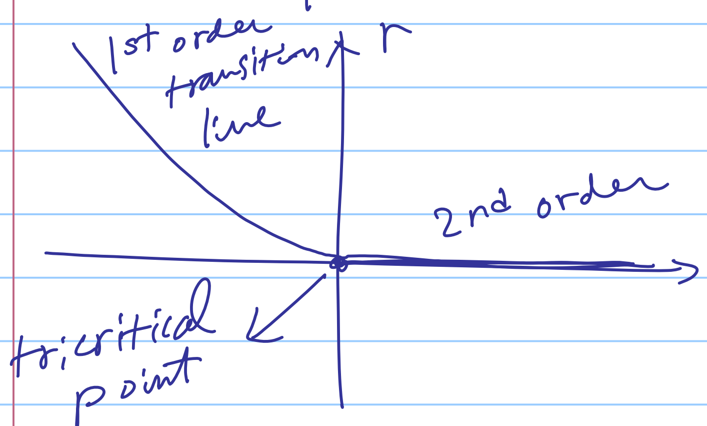
Consider $f = \frac{1}{2} r \phi^2 + u_4 \phi^4 + u_6 \phi^6 \qquad (u_6 > 0)$



ϕ: order parameter

$r = a(T - T^*)$

At T_c: $\frac{\partial f}{\partial \phi} = 0$, $f = 0 \Rightarrow r_c = a(T_c - T^*) = \frac{u_4^2}{2u_6}$



At T.P.

$\frac{1}{2} r \phi^2 + a_6 \phi^6 = 0$

$\downarrow$

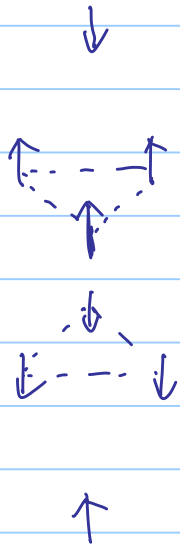
$\phi = \pm \left(\frac{-r}{2a_6} \right)^{1/4}$

$\beta = 1/4$ @ T.P.

Examples: ① metamagnetic transition (§ 4.6.2)

② $^3He - {}^4He$ mixture (C+L § 4.6.3.)

FeCl$_2$ metamagnet (triangular lattice)



$$\mathcal{H} = -J_1 \sum_{\langle \parallel \rangle} \vec{S}_1 \cdot \vec{S}_j + J_2 \sum_{\langle \perp \rangle} \vec{S}_i \cdot \vec{S}_j$$

$\Rightarrow$ Ising anisotropy $\rightarrow \propto S_i^z S_j^z$

$$\vec{S}_i \cdot \vec{S}_j = S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z$$

$\rightarrow$ approximate with Ising model.

$$\mathcal{H} = -J_1 \sum_{\langle \parallel \rangle} S_i S_j + J_2 \sum_{\langle \perp \rangle} S_i S_j$$