Problem 1

The crystal momentum is defined as $\hbar k$, where k is a real-valued wavevector determined by the solution of the one-electron Schrodinger equation in a periodic potential under appropriate boundary conditions (as was first done by Kronig and Penney). If energy is a parabolic function of k, the crystal momentum corresponds to the free particle momentum with a given effective mass.

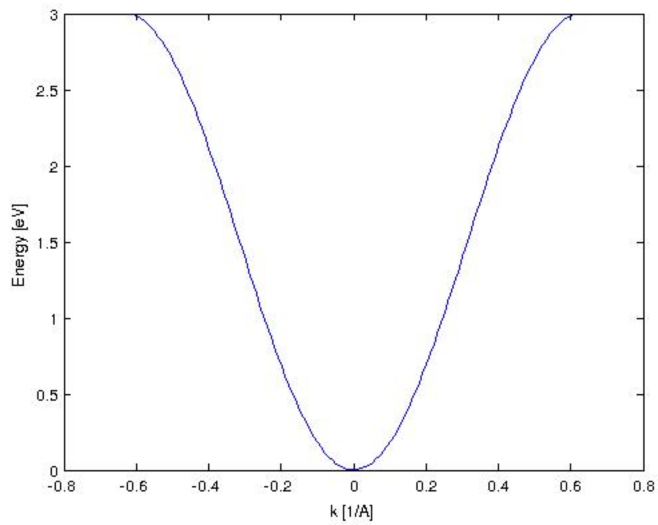
Problem 2

- a. Effective mass attempts to include the contribution of a periodic potential experienced by an electron and thereby provide a comparable picture with of free-electron. An electron in a periodic potential can be interpreted to be "carrying around" the extra energy of this potential.

$$m* = \frac{\hbar^2}{d^2\epsilon/dk^2}$$

- b. The dashed line has a smaller effective mass (larger curvature) than that of the solid line (smaller curvature).
- c. The effective mass as defined, affects the curvature of the E-k curve. The density of states describes the number of available states around an infinitesimal portion of energy. If we split the E-k curve into parcels of δE, we observe that a band with a larger curvature, corresponding to a smaller effective mass, has fewer states within this region δE. Qualitatively speaking, we can say the a larger effective mass corresponds to a greater density of states in a band with the vice versa being equally true.

Problem 3



- a. See plot.
- b.

$$m* = \frac{\hbar^2}{d^2E/dk^2}$$
$$\frac{d^2E}{dk^2} = a^2E_0\cos(ka)$$

For $k = 0$:

$$\begin{aligned} m* &= \frac{\hbar^2}{a^2E_0} \\ &= 1.85E-31[kg] \end{aligned}$$

For $k = \pi/a$:

$$\begin{aligned} m* &= \frac{\hbar^2}{-a^2E_0} \\ &= -1.85E-31[kg] \end{aligned}$$

- c. Negative effective mass corresponds to the presence of vacant state, known as a hole.