Recap

- XY-model in 2D
- For continuous symmetry (e.g. XY model)

$$H = -J \sum_{<i,j>} \cos(\theta_i - \theta_i)$$

- In continuum limit

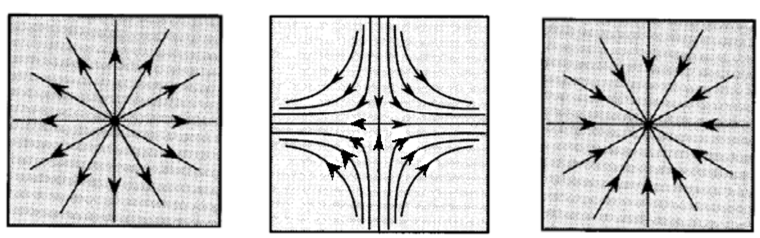
$$H = E_0 + \frac{J}{2} \int d^2 r (\nabla \theta)^2$$

- Order parameter correlation function:

$$C(r) = \exp[-\frac{k_B T}{2\pi J} \log({}^{r}\!/_{a})] \propto r^{-\eta} \qquad \eta = \frac{k_B T}{2\pi J}$$

- B-K-T Transition: vortex unbinding transition

What is a vortex?

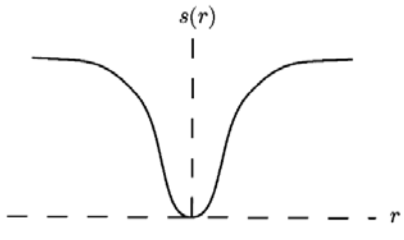


(a) (b) (c)

- For example, XY-model $\psi = S[\cos\theta(\mathbf{x}), \sin\theta(\mathbf{x})]$
- <ψ> is continuous everywhere except for origin $\nabla\theta = \frac{1}{r}$

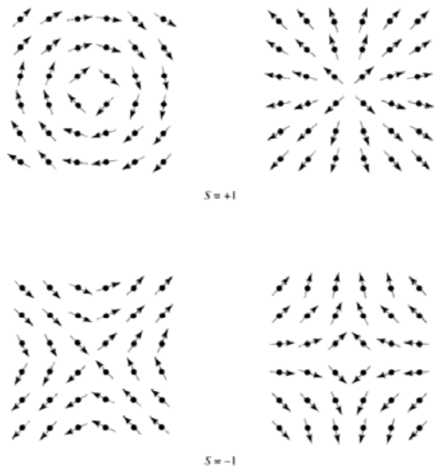
$$\oint dr \cdot (\nabla\theta) = 2\pi k \quad (k : \text{winding number})$$

Topological defect



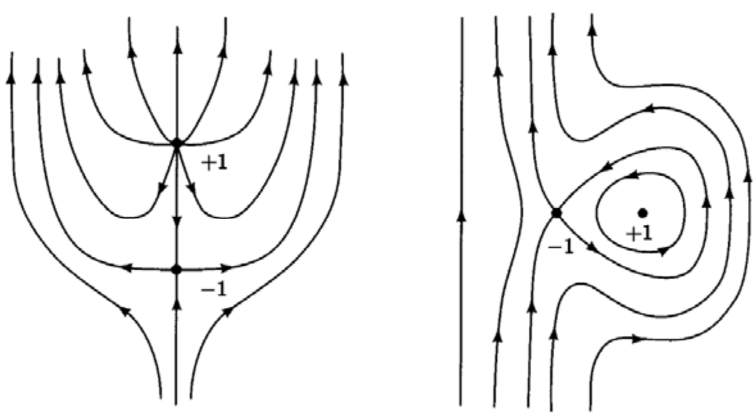
(b)

thematical singularity by cutting a hole in the
e order parameter $s(r)$ near the core of the



Vortex pairs

- Far from the pair, $<\psi>$ are nearly parallel.
- Topologically equivalent to the uniform state.



(a) (b)

Bishop+Reppy (1978)

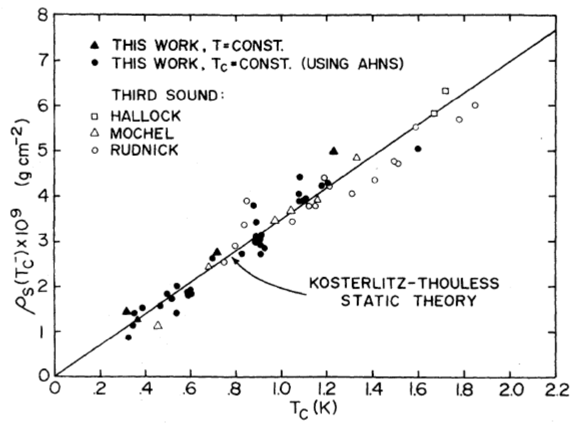
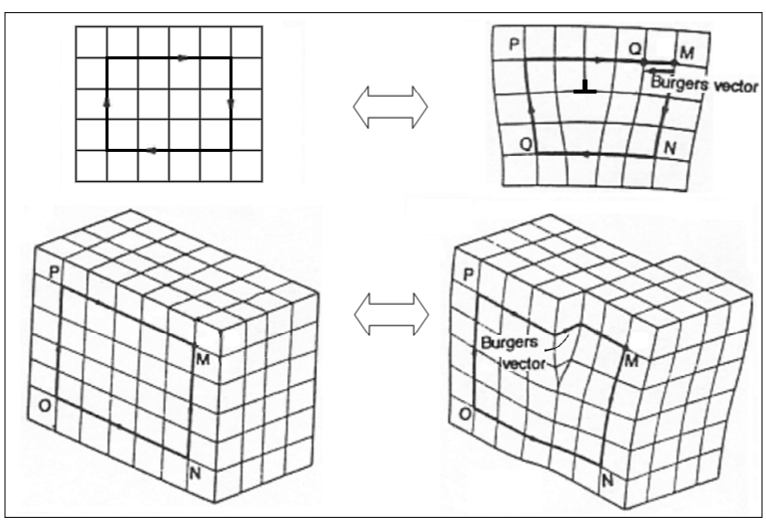


FIG. 3. Results of all of our data, in addition to previous third-sound results for the discontinuous superfluid density jump $\rho_s(T_c^-)$ as a function of temperature. The solid line is the Kosterlitz-Thouless (Refs. 3 and 4) static theory.

Dislocations



Note Title

Vortex unbinding transition.

$$E_1 = \frac{1}{2} J \oint \underbrace{(\nabla\theta)^2}_{\frac{1}{r}} \underbrace{d^2r}_{r\,dr\,d\theta} = \frac{1}{\cancel{2}} J \cdot \cancel{2}\pi \cdot \log\left(\frac{L}{a}\right)$$

L : size of vortex

a : cutoff scale

(c.f. $E_{pair} = \pi J \log\left(\frac{r}{a}\right)$ $\quad r$: distance between the pair)

(entropy of a single vortex)

$$S_1 = k_B \log\left(\frac{L}{a}\right)^2$$

$\left(\frac{L}{a}\right)^2$ ← # of possible sites for vortex core.

$$F = E_1 - TS_1 = (\pi J - 2k_B T)\log\left(\frac{L}{a}\right)$$

$$T_{KT} = \frac{\pi J}{2k_B}$$

Recall $C(r) = r^{-\eta}$ $\qquad \eta = \frac{k_B T}{2\pi J} = \frac{\cancel{k_B}}{2\pi \cancel{J}} \cdot \frac{\pi \cancel{J}}{2\cancel{k_B}} = \frac{1}{4}$

$$\eta = 1/4 \text{ @ } T = T_{KT}$$

Using RG. $\qquad \eta = \frac{k_B T_{KT}}{2\pi R} = \frac{1}{4}$

R ← rigidity

$T_{KT} \propto R$ rigidity (e.g. superfluid density)

Elasticity

Consider deformation $(x_1, x_2, x_3) \to (x_1', x_2', x_3')$

$$u_i = x_i' - x_i$$

$\vec{u}(u_1, u_2, u_3)$: deformation vector.

Distance transformation

$$dl = \sqrt{dx_1^2 + dx_2^2 + dx_3^2}$$

$$dl' = \sqrt{dx_1'^2 + dx_2'^2 + dx_3'^2}$$

$$du_i = \frac{\partial u_i}{\partial x_k} dx_k$$

$$dl'^2 = dx_i'^2 = (dx_i + du_i)^2$$

$$= dx_i^2 + \underbrace{2\frac{\partial u_i}{\partial x_k} dx_i dx_k}_{2\sum_i dx_i \left(\sum_k \frac{\partial u_i}{\partial x_k} dx_k\right) \cdots = \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) dx_i dx_k} + \frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_l} dx_k dx_l$$

$$= dl^2 + \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} + \frac{\partial u_l}{\partial x_k}\frac{\partial u_l}{\partial x_i}\right) dx_i dx_k$$

$$= dl^2 + 2u_{ik} dx_i dx_k$$

$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} + \cancel{\frac{\partial u_l}{\partial x_k}\frac{\partial u_l}{\partial x_i}}\right)$$

small strain approximation.

: Strain tensor.

(change in length as a result of deformation)