For a plane curve expressed as $y = f(x)$, the curvature is given as:

$$\kappa = \frac{|y''|}{(1 + y'^2)^{3/2}}$$

For any point on a curve, we can define a unique osculating circle, which passes through the point P and a pair of points infinitesimally close to P. The curvature of this circle is the same has the curvature at point P. The radius of this circle becomes the radius of curvature:

$$\kappa = \frac{1}{R}$$

In some applications, like the bending of a beam, the slope y' is assumed to be much smaller than unity, giving:

$$\kappa = |y''|$$

The moment relationship for the bending of a beam is, M is the moment, E is the elastic modulus of the material, I is the second moment of inertia about the bending axis, R is the radius of curvature:

$$M = \frac{EI}{R}$$

Examining the geometry of the beam:

$$\frac{1}{R} = \frac{d\theta}{ds}$$

For small angles:

$$\frac{dy}{dx} = \tan\theta = \theta$$

Giving:

$$\frac{1}{R} = \frac{d}{dx}\frac{dy}{dx}$$