Fermi Energies

To determine the Fermi energy at a given temperature, the following cubic equation must be solved for ϵ_f where $n, W_1, W_2, W_3, N_1, N_2, N_3$ are known:

$$n = \frac{N_1}{e^{(W_1-\epsilon_f)/kT}+1} + \frac{N_2}{e^{(W_2-\epsilon_f)/kT}+1} + \frac{N_3}{e^{(W_3-\epsilon_f)/kT}+1}$$

Since $W_1 = 0$, the first term on the right hand side approaches N_1:

$$n = N_1 + \frac{N_2}{e^{(W_2-\epsilon_f)/kT}+1} + \frac{N_3}{e^{(W_3-\epsilon_f)/kT}+1}$$

Which is a quadratic equation that can easily be solved:

$$\epsilon_f(174) = 0.8999[eV]$$
$$\epsilon_f(290) = 0.8476[eV]$$

Solving the original cubic using MATLAB gives identical results. The number of electrons in the energy level W_3:

$$n_3 = \frac{N_3}{e^{(W_3-\epsilon_f(T))/kT}+1}$$

For $T = 174$:

$$\begin{aligned} n_3 &= \frac{2E23}{e^{(1-0.8999)/(8.617E-5\times 174)}+1} \\ &= 2.5044E20[cm^{-3}] \end{aligned}$$

For $T = 290$:

$$\begin{aligned} n_3 &= \frac{2E23}{e^{(1-0.8476)/(8.617E-5\times 290)}+1} \\ &= 4.4598E20[cm^{-3}] \end{aligned}$$

Vacancies can be found from:

$$p_i = N_i(1 - \frac{1}{e^{(W_i-\epsilon_f(T))/kT}+1})$$

State	$T = 174K$	$T = 290K$
$W_1[cm^{-3}]$	0	1.7764E8
$W_2[cm^{-3}]$	2.5083E20	4.4538E20

Table 1: Vacancies in W_1 and W_2