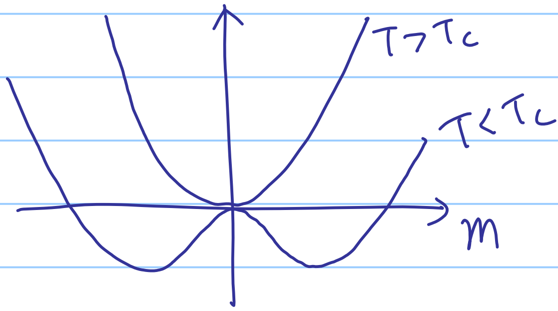
MFT- Continued

Note Title 13/02/2012

Bragg-Williams form of Ising model

$$f = \frac{F}{N} = \frac{1}{2}(T-T_c)m^2 + \frac{T}{12}m^4 + \cdots\cdots$$



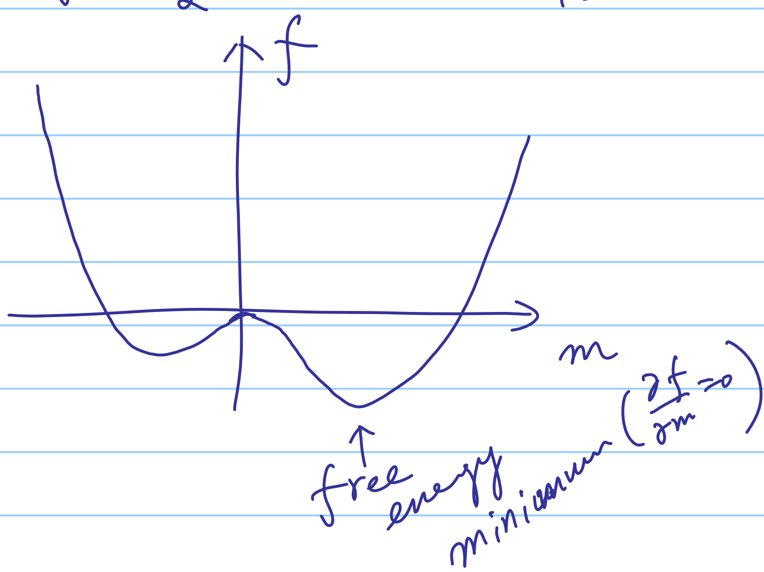
$$m \sim (T_c - T)^{\beta}$$

$$\beta = \frac{1}{2}$$

Add magnetic field (external field) : H

$\rightarrow$ free energy term: $-Hm$

$$f = \frac{1}{2}(T-T_c)m^2 + \frac{T}{12}m^4 - Hm + \cdots$$



$$\frac{\partial f}{\partial m} = \underbrace{(T-T_c)m + \frac{T}{3}m^3 - H = 0}_{\text{eq. state}}$$

At $T = T_c$

$$m^3 = \frac{3H}{T}$$

$$m \propto H^{1/3} \quad (@T_c)$$

$$m \propto H^{1/\delta} \qquad \delta = 3$$

mean field

Susceptibility

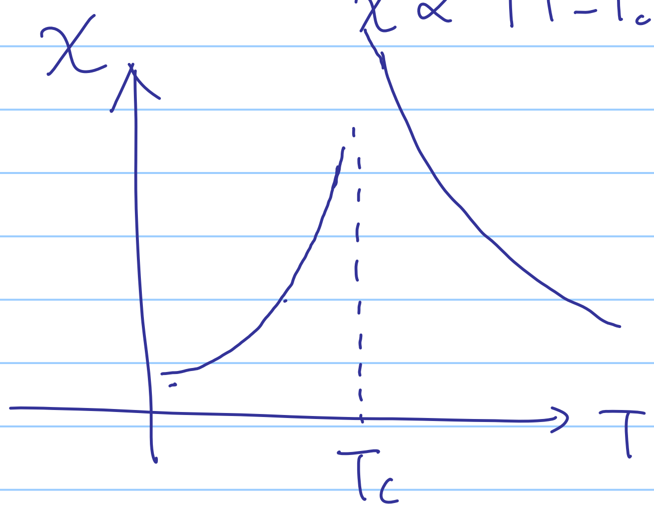
$$\chi = \left.\frac{\partial m}{\partial H}\right|_{H\to 0} \qquad \frac{\partial}{\partial H}\left(\frac{\partial f}{\partial m}\right) = (T-T_c)\underbrace{\frac{\partial m}{\partial H}}_{\chi} + Tm^2\underbrace{\frac{\partial m}{\partial H}}_{\chi} - 1 = 0$$

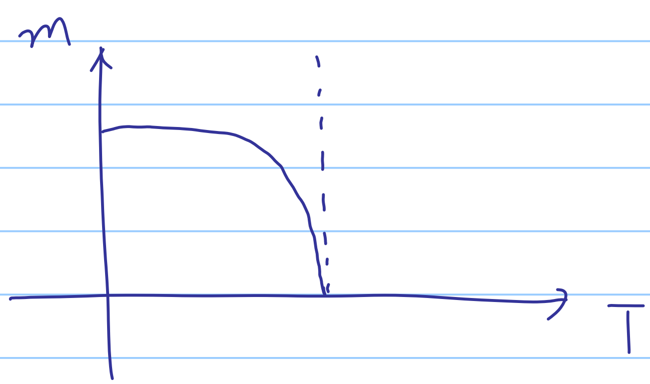
$$\chi = \frac{1}{T - T_c + T_c m^2} \qquad (T \approx T_c)$$

For $T > T_c$: $m = 0$ $\qquad \chi = \frac{1}{T - T_c} = (T - T_c)^{-\gamma}$

$T < T_c$: $m^2 = \frac{3(T_c - T)}{T_c} \Rightarrow \chi = \frac{1}{2(T_c - T)} = \frac{1}{2}(T_c - T)^{-\gamma}$

$$\chi \propto |T - T_c|^{-\gamma} \qquad \gamma = 1$$





Fluctuations are suppressed by order parameter saturation

More critical exponents: ν, η

Landau theory → Free energy can be expanded as a power series in the order parameter

* Only those terms compatible with the symmetry of the system

example: $f = \frac{1}{2}(T - T_c) m^2 + \frac{T}{12} m^4 + \cdots$

Typically, $f = a_0 + a_1 \phi + \underbrace{a_2 \phi^2}_{} + a_3 \phi^3 + \cdots \cdots$

ϕ: order parameter.

$(T - T_c)$ term

$$t \equiv \frac{T - T_c}{T_c}$$

$$a_2 \phi^2 = \tilde{a}_2 t \phi^2$$

Back to Ising model (for correlation fn.)

Non-uniform fluctuations

Allow magnetization variation as a fn. of position

(block spin) $\quad m \to m(x) \quad$ (1D example)

$$\mathcal{H} = -J \sum_x m(x)\, m(x+a)$$

$$m(x)\, m(x+a) = \left\{ -\left[\frac{m(x+a) - m(x)}{a} \right]^2 \cdot a^2 + m(x)^2 + m(x+a)^2 \right\} \frac{1}{2}$$

In Continuum limit $a \to 0$

$$\mathcal{H} = -J \sum_x m(x)^2 + \underbrace{\frac{J}{2} a^2 \sum_x \left(\frac{\partial m}{\partial x}\right)^2}_{\text{Additional term taking into account of fluctuations}} + \text{higher order terms.}$$

With this, Landau MFT (Ising model example)

$$f = a_0 + a_2 \sum_x m(x)^2 + g \sum_x \left(\frac{\partial m}{\partial x}\right)^2$$

Suppose $m(x)$ varies slowly, and small

$$f = a_0 + a_2 \int m(x)^2 dx + g \int \left(\frac{\partial m}{\partial x}\right)^2 dx$$

F.T.
$$\begin{bmatrix} m(x) = \dfrac{1}{2\pi} \displaystyle\int dq\, e^{iqx}\, m(q) \\ m(q) = \displaystyle\int dx\, e^{-iqx}\, m(x) \end{bmatrix}$$

$$f - a_0 = \frac{1}{2\pi}\int dq\,(a_2 + g q^2)\, m(q)\, m(-q)$$

Orstein-Zernike (OZ) assumption.

Independen q contribution to f

$\rightarrow$ Each q: $\underbrace{(a_2 + g q^2)\langle m(q)\, m(-q)\rangle}$

should equally contribute to f

$$(a_2 + g q^2)\langle \underbrace{m(q)\, m(-q)}_{S(q)}\rangle = kT$$

What is $\langle m(q)\, m(-q)\rangle$?

Recall $C(x) = \langle m(x)\, m(0)\rangle - \cancel{\langle m(0)\rangle^2} \to 0$ $\quad T > T_c$

$$S(q) = \langle m(q)\, m(-q)\rangle$$

$$= \frac{kT}{a_2 + g q^2} \xRightarrow{\text{inv. F.T.}} C(x) = e^{-x/\xi}$$

$$\xi = \sqrt{\frac{g}{a_2}}$$

$$a_2 = \tilde{a}_2 (T - T_c)$$

$$\xi \propto |T - T_c|^{-1/2} = |T - T_c|^{-\nu} \qquad \nu = \frac{1}{2}$$