Landau theory

Note Title 14/02/2012

$$S_{nn}(q) = \frac{kT}{a_2 + g q^2} \iff C(x) = e^{-x/\xi}$$

$$a_2 = \tilde{a}_2 t = \tilde{a}_2 \left(\frac{T-T_c}{T_c}\right)$$

$$\xi \propto |T-T_c|^{\nu} \quad \nu = \frac{1}{2} \quad \text{M.F.}$$

correlation length

In general, OZ form: (classical) $C(r) = \dfrac{e^{-r/\xi}}{r^{(d-1)/2}}$ for $r \to \infty$

At $T=T_c$ $\left(\begin{matrix}\xi \to \infty \\ a_2 = 0\end{matrix}\right)$ $C(r) = \dfrac{1}{r^{d-2}}$ $S(q) \sim q^{-2}$

Fisher's modification (@ $T=T_c$)
(M.E. Fisher) $S(q) = q^{-2+\eta}$

$$\Rightarrow C(r) = \frac{1}{r^{d-2+\eta}} \leftarrow$$

η: exponent ($\eta = 0$) For M.F.)

$\underline{\alpha, \beta, \gamma, \delta, \nu, \eta}$

Later learn that $S(q) \propto T \chi(q)$

$$S(q) \propto T \frac{\chi(0)}{\xi^{-2} + q^2}$$

$$S(q) = \underbrace{S_{nn}(q)}_{\substack{\frac{\chi}{\xi^{-2}+q^2} \\ \text{fluctuation}}} + \underbrace{\sum \delta(\vec{q} - \vec{G})}_{\text{order parameter}}$$

From C+L

Table 5.4.2. *Some critical exponents from theory and experiment.*

Exponent	α	β	γ	ν	η
Property	specific heat	order parameter	susceptibility	coherence length	correlation function
Definition	$C \sim t^{-\alpha}$	$\langle\phi\rangle \sim t^{\beta}$	$\chi \sim t^{-\gamma}$	$\xi \sim t^{-\nu}$	$G(q) \sim q^{-2+\eta}$
Mean-field	0	0.5	1	0.5	0
3D theory					
$n = 0$ (SAW)	0.24	0.30	1.16	0.59	
$n = 1$ (Ising)	0.11	0.32	1.24	0.63	0.04
$n = 2$ (xy)	-0.01	0.35	1.32	0.67	0.04
$n = 3$ (Heisenberg)	-0.12	0.36	1.39	0.71	0.04
Experiment					
3D $n = 1$	$0.11^{+.01}_{-.03}$	$0.32^{+.16}_{-.04}$	$1.24^{.16}_{-.04}$	$0.63^{+.04}_{-.04}$	0.03 - 0.06
3D $n = 3$	$0.1^{+.05}_{-.04}$	$0.34^{+.04}_{-.04}$	$1.4^{+.07}_{-.07}$	$0.7^{+.03}_{-.03}$	
2D $n = 1$	$0.0^{+.01}_{-.003}$	$0.3^{+.04}_{-.04}$	$1.82^{+0.7}_{-.07}$	$1.02^{+.07}_{-.07}$	

Experiments on 3D $n = 1$ compiled from liquid-gas, binary fluid, ferromagnetic, and antiferromagnetic transitions.
Experiments on 3D $n = 3$ transitions compiled from some ferromagnetic and antiferromagnetic transitions.
Experiments on 2D $n = 1$ complied from some antiferromagnetic transitions.

How to measure γ, ν, β?

Use $$S(q) = \underbrace{S_{nn}(q)}_{\downarrow \; \frac{\chi}{\xi^{-2}+q^2} \; (\nu \text{ and } \gamma)} + \underbrace{\sum \delta(\vec{q}-\vec{G})}_{\downarrow \; \text{order parameter} \to \beta}$$

Example for β-brass
from: Jens Als-Nielsen, "Neutron Scattering and Spatial Correlation near the critical point": Chap 3 in Vol. 5a of the "Phase Transitions and critical phenomena" series edited by Domb and Lebowitz

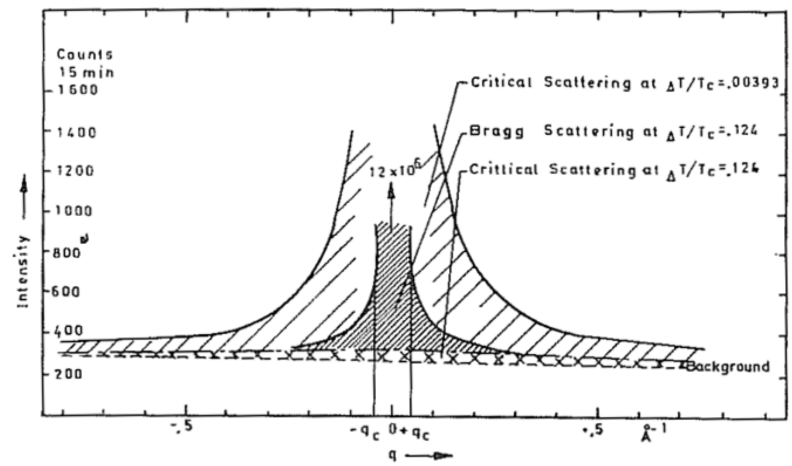
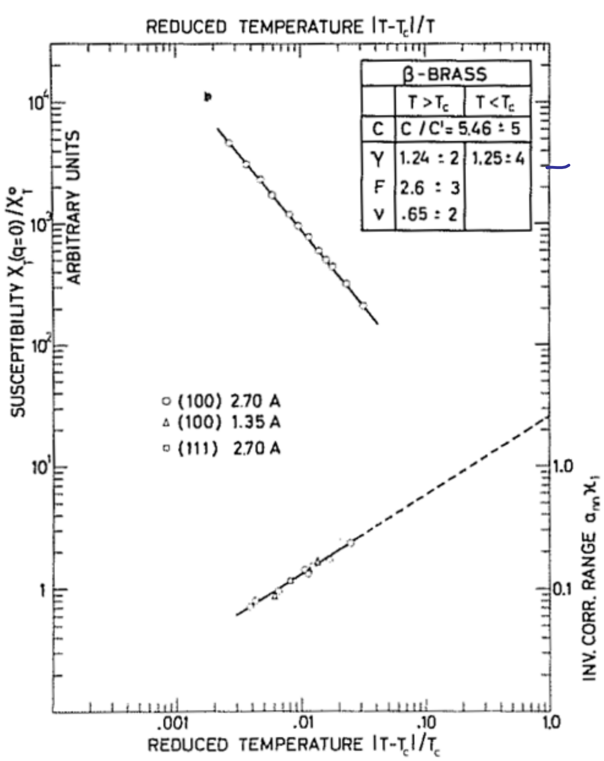


FIG. 4.5. Critical scattering at a fixed temperature below T_c from a Cu^{65}/Zn crystal. The Bragg scattering at $\Delta T/T_c = 0{\cdot}00393$ is much lower than the Bragg scattering shown at $\Delta T/T_c = 0{\cdot}124$, and it is possible to subtract the Bragg scattering in the critical region for $q > 0{\cdot}02$ Å^{-1}.



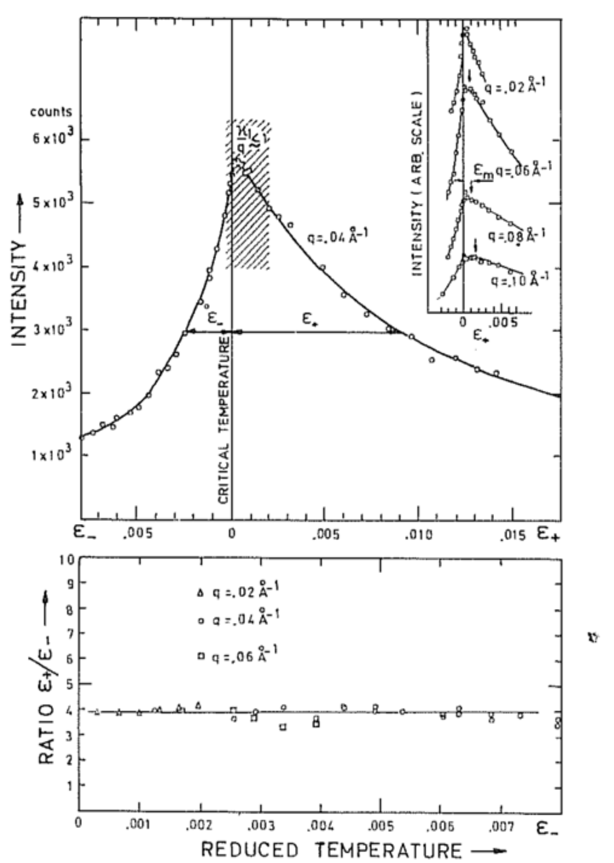
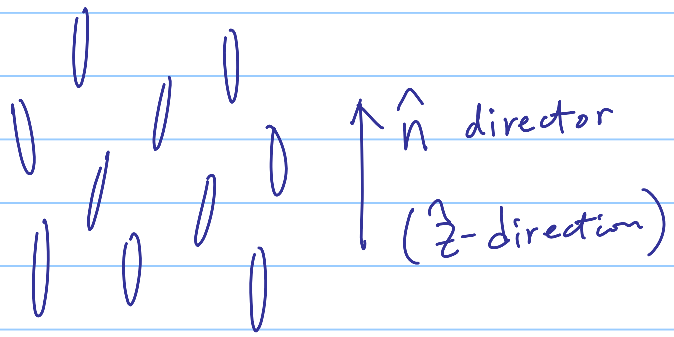


FIG. 4.4. The line shape of critical scattering at fixed temperature is well approximated by a Lorentzian of half-width κ_1 outside the shaded region in which $\kappa_1/q < 1$. The ratio of equi-intensity temperatures in the Lorentzian region is constant, independent of q (lower part). Insert: The flat maximum above T_c is located near T_m given theoretically by $\kappa_1^2(T_m)/q^2 = 0{\cdot}023$. Theoretical T_m values are indicated by arrows.

Nematic–Isotropic transition in L.C (1st order)

Nematic L.C.



Let's define $f(\theta, \varphi)$ as distribution fn. (probability of finding molecule along the (θ, φ) is $f(\theta,\varphi)\,d\Omega$

① $f(\theta, \varphi)$ is indep. of φ

② $f(\theta) = f(\pi - \theta)$

Q] What is the order parameter? $\langle ? \rangle = \int ?\, f(\theta,\varphi)\, d\Omega$

Dipole ? $\langle \cos\theta \rangle = 0$ (Not working)

Next? quadrupole

$$S \equiv \frac{1}{2}\langle (3\cos^2\theta - 1) \rangle = \begin{cases} 1 & \text{for } \theta = 0, \pi \\ 0 & \text{for random orientation} \end{cases}$$

$\langle \cos^2\theta \rangle = \frac{1}{3}$

$\Rightarrow$ Generalization: Traceless, symmetric, 2nd rank tensor

$$Q_{\alpha\beta} = n_\alpha n_\beta - \frac{1}{3}\delta_{\alpha\beta}$$

$\Rightarrow$ diagonalize (for uniaxial nematic)

$$\tilde{Q} = \begin{pmatrix} -S/3 & 0 & 0 \\ 0 & -S/3 & 0 \\ 0 & 0 & 2S/3 \end{pmatrix}$$