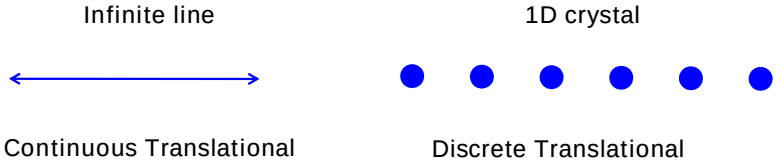
Scheduling

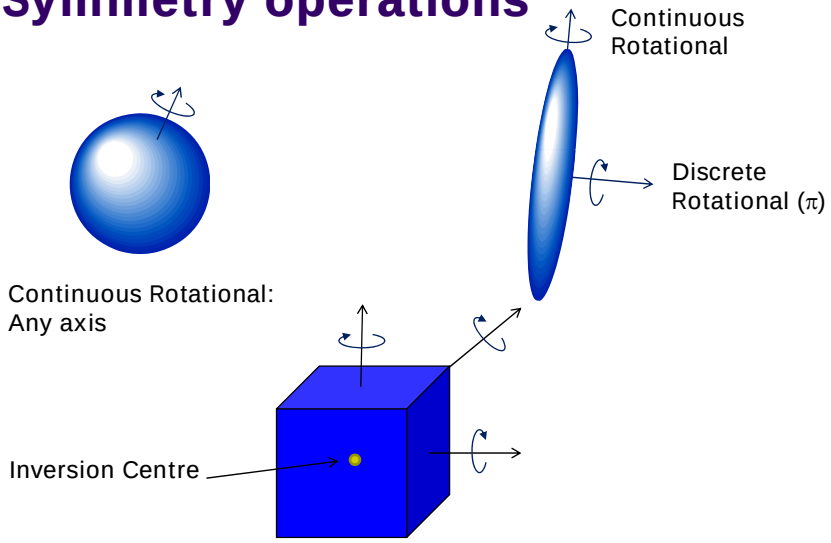
- Lectures: M3-4, T11-12 (MP1115)
- Office hours: MWF4-5 (MP084)
- No lecture on Jan. 17th (next Tuesday)
- Make-up: 10-11 (2 hours) on Jan. 24th (MP606)

Broken symmetry

- What is symmetry?
  - A system possesses a particular symmetry if the Hamiltonian is invariant w.r.t. the transformations associated with elements of a symmetry group.
  - A system "looks the same" after a symmetry operation.
    - Rotation, Translation, Mirror plane, Inversion, etc...
    - Symmetry can be either discrete or continuous



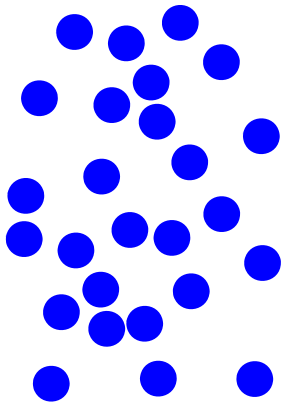
Symmetry operations



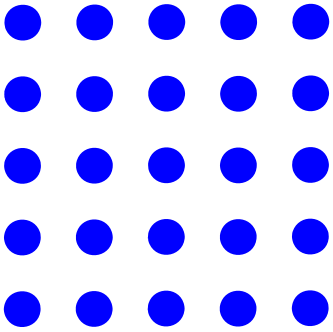
Phase transition and broken symmetry

- Example of H_2O
- Phase transition
  - Free energy: $F = E - TS$
  - High temperature: entropy wins
  - Low temperature: energy wins
- How about symmetry?
  - High temperature (liquid): fully symmetric (both rotational and translational)
  - Low temperature (crystal): Only *discrete* translational and rotational symmetry

But…



Liquid

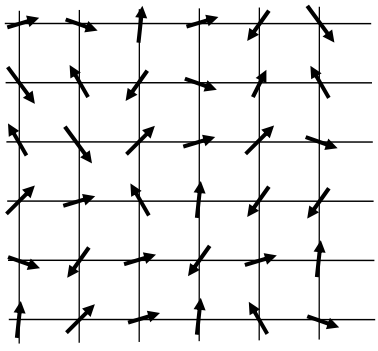


Crystal Lattice

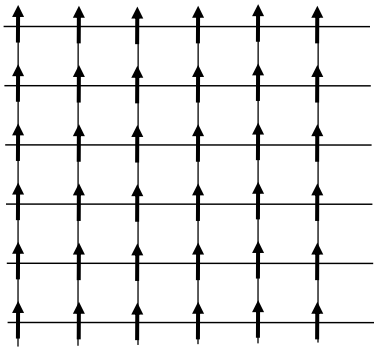
Symmetry of Time averaged density or probability density

Order parameter

- Another example: ferromagnet



T > Tc

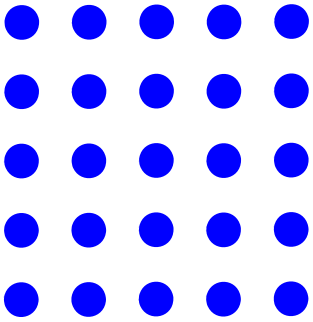


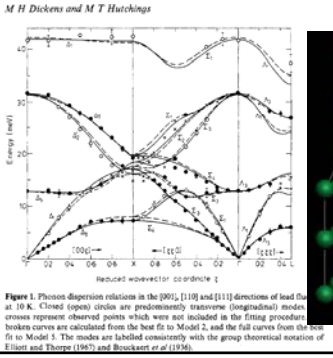
T < Tc

$M = 0$ Order parameter $M = \sum m_i$ $M > 0$

Emergent properties

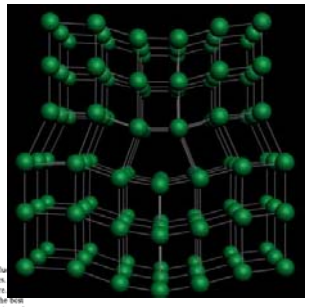
- Example of a crystal
  - Crystal is rigid, elastic modulus is non-zero
  - Acoustic phonon (shear mode)
  - Defects (dislocation)





M H Dickens and M T Hutchings

Figure 1. Phonon dispersion relations in the [001], [110] and [111] directions of lead fluo... at 10 K. Closed (open) circles are predominantly transverse (longitudinal) modes. ... crosses represent observed points which were not included in the fitting procedure. ... broken curves are calculated from the best fit to Model 2, and the full curves from the best fit to Model 5. The modes are labelled consistently with the group theoretical notation of Elliott and Thorpe (1967) and Brouckaert *et al* (1936).



In general

- Broken symmetry phase has
  - Non-zero order parameter
  - Generalized rigidity
  - Goldstone mode (zero energy mode)
  - Defects

Table 1.2.1. *Properties of some representative broken-symmetry phases.*

The Ising magnet has a broken discrete symmetry and no new rigidity or modes.

	Fluid	Nematic	Smectic-*A*	Crystal	Heisenberg magnet	Superfluid	Ising magnet
Broken symmetry	none	rotational	1D translation	3D translation	rotational	phase	up-down
New order	none	orientational	1D periodic density	3D periodic density	spin	condensate wave function	spin
Rigidity	none	rotational elastic constant	layer modulus	shear modulus	spin-wave stiffness	superfluid density	–
New modes	none	diffusive orientational	second sound – undulation	shear sound	spin wave	second sound	–
Defects	none	disclinations, hedgehogs	dislocations	dislocations	hedgehog	vortices	domain walls