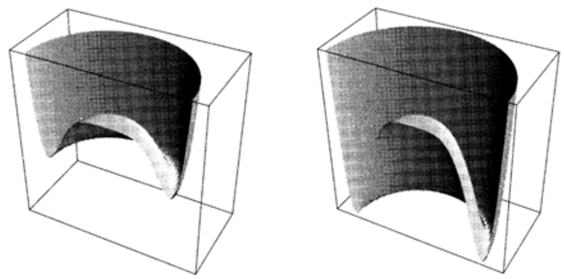
Recap

- For continuous symmetry (e.g. XY model)

$$\chi_{\parallel} \quad \text{(longitudinal fluctuation)}$$

$$\chi_{\perp} = \begin{cases} 0 & (T > T_c) \\ \infty & (T < T_c) \end{cases} \quad \text{(transverse fluctuation)}$$

  - Infinitesimal amount of field is required to rotate the direction of magnetization (recall the wine bottle free energy)



For T < T_c (broken symmetry)

- Now, consider spatially non-uniform deviation from the equilibrium magnetization
  - This will now cost non-zero, but small energy proportional to $(\nabla m)^2$
  - This energy will be smaller for long-wavelength fluctuations
  - O(n) symmetry should have an excitation spectrum with modes of arbitrarily small energy. – One for each transverse direction : Goldstone modes ➔ gapless energy spectrum

Correlation function

- Consider long. and trans. fluctuations explicitly.

$$\vec{\mathbf{m}} = m(\hat{\mathbf{n}} + \phi_{\|}\hat{\mathbf{n}} + \phi_{\perp})$$

$$\Delta f = \frac{m^2}{2}\int d^2r\left[(\nabla\phi_{\|})^2 + (\nabla\phi_{\perp})^2 + 2r\phi_{\|}^2\right]$$

$$S_{\|}(\mathbf{q}) = \frac{m^{-2}}{2r + q^2}$$

$$S_{\perp}(\mathbf{q}) = \frac{m^{-2}}{q^2}$$

Emergence of rigidity

$$\Delta f = long.\ part + \frac{1}{2}\int d^2rR(\nabla\phi_{\perp})^2$$

- R: (= m^2) rigidity, spin-wave stiffness (ρ_s), etc..
- Physical meaning: restoring force in response to transverse fluctuation
- Another example: solid (next week)

$$F_{el} = \frac{1}{2}\int d^3r\left[Bu_{kk}^2 + 2\mu u_{ij}u_{ij}\right]$$

Note Title

2D XY-model

$$\vec{S} = (S_x, S_y)$$

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

If the direction of the spin varies smoothly

$$\cos(\theta_i - \theta_j) \sim 1 - \frac{1}{2}(\theta_i - \theta_j)^2 + \cdots$$

$\sim \frac{\partial \theta}{\partial x}$ (for N.N.)

Continuum limit $\mathcal{H} = E_0 + \frac{J}{2} \int d^2\vec{r}\, (\nabla\theta)^2$

Order parameter correl. fn.

$$\langle \vec{S}(\vec{r}) \cdot \vec{S}(\vec{0}) \rangle = \langle \cos[\theta(\vec{r}) - \theta(\vec{0})] \rangle = \mathrm{Re} \langle e^{i[\theta(\vec{r}) - \theta(\vec{0})]} \rangle$$

$$= e^{-\frac{1}{2}\langle [\theta(\vec{r}) - \theta(\vec{0})]^2 \rangle}$$

Calculation $\langle\ \rangle$ with $\mathcal{H}$ $\rightarrow$ functional integral $\left(\begin{matrix} \text{C+L} \\ 5.2 \end{matrix}\right)$

.....

$$\langle (\theta(\vec{r}) - \theta(\vec{0}))^2 \rangle = \frac{k_B T}{\pi J} \log\left(\frac{r}{L}\right) \qquad r \gg L$$

$\leftarrow$ cutoff lengthscale

$$C(\vec{r}) = \langle S(\vec{r}) \cdot S(0) \rangle = e^{-\frac{k_B T}{2\pi J} \log\left(\frac{r}{L}\right)} \propto r^{-\eta}$$

$$\eta \equiv \frac{k_B T}{2\pi J}$$

① There is no long-range order

② η depends on T & J

③ Transition from exp. decaying corr. fn. to algebraic decay @ $T_c \equiv T_{KT}$

Kosterlitz – Thouless Transition (B-K-T)

$\Rightarrow$ Vortex unbinding transition

Another example of xy-order parameter; Supercond.

Cooper pair wave fn: $\psi_j(\vec{r}) = \frac{1}{\sqrt{V}} a_j(\vec{r}) e^{-i\phi_j(\vec{r})}$

Averaging over ξ^3 (coherence length)

$\psi(\vec{r}) = \frac{1}{\xi^3} \sum \psi_j(\vec{r}_j) = \sqrt{n_s}\, e^{i\phi(\vec{r})}$; non-zero if phase-coherent

$|\psi|^2 = n_s$; superfluid density

Phase choice ; Gauge of vector potential

$$\psi'(\vec{r}) = \psi(\vec{r}) \exp\left(\frac{2\pi i \Lambda(\vec{r})}{\Phi_0}\right)$$

$$\vec{A}' = \vec{A} + \vec{\nabla}\Lambda$$

local gauge

Local gauge should be the same $\rightarrow$ broken symmetry.