User Guide

This program is dedicated to calculate the elastic tidal response of a terrestrial planetary body with laterally varying (i.e., 3-D) elastic and density structures in the crust and mantle. The 3-D structures are represented by spherical harmonics in shear modulus (μ)/Lam é parameter (λ)/density (ρ) of one or multiple concentric crust and mantle layers (similar to tomographic models of the Earth's mantle). This program is written in MATLAB scripts and contains the following subroutines (in order of calling sequence):

- **tidal_response.m**: the main routine that calls other subroutines
  - model initialization: body tide mode, input file and directory names, control flags, global variables
  - **model_setup.m**: read in 1-D reference profile and 3-D structures from files and set up nondimensional model parameters and numerical grid
  - **mode_coupling.m**: construct full (up to 2nd order of perturbation) mode coupling hierarchies for specific coupling $(l_{\text{tide}}, m_{\text{tide}}) \otimes (l_1, m_1)$, by calling **vsh_expan.m** or reading from existing VSH expansion repository (in directory ./vsh_expan/)
    - **vsh_expan.m**: perform vector spherical harmonic (VSH) expansions for mode coupling terms ($\mathcal{F}_D$, $\mathcal{B}_D$, $\mathcal{G}_D$ and $\mathcal{H}_D$ in my manuscript) induced by lateral heterogeneities in μ, λ and ρ [$(l_D, m_D) \otimes (l_1, m_1)$], and write expansion coefficients in to files in ./vsh_expan/
      - **phi_func.m**: evaluate ϕ-dependent function and its derivatives
      - **theta_func.m**: evaluate associated Legendre polynomial (θ dependent) and its derivatives for a given harmonic (l, m)
  - **runge_kutta.m**: implement propagator matrix method based on fourth order Runge-Kutta scheme for a specific "child" mode (l_D, m_D), using "parents" solutions at order of perturbation D (refer to Appedix C)

- **create_a_matrix.m**: build **A** matrix in the matrix equation $\frac{d\boldsymbol{X}_{lm}^{D}}{dr} = \boldsymbol{A}_l \boldsymbol{X}_{lm}^{D} - \boldsymbol{F}_{lm}^{D}$
- **net_force.m**: compute total acceleration caused by $-\delta\rho_0 \nabla V_{\text{td}}$

➢ **solver.m**: solve linear equation for tidal response for each eigenstructure (l_1, m_1)

➢ sum up tidal response for each eigenstructure (l_1, m_1) into total response

Miscellaneous:

➢ **compute_mass.m**: compute (non-dim) mass below a given set of radius

➢ **file_sol.m**: matrix equation solution output file name (.mat)

➢ **file_vsh.m**: VSH expansion output file name (.mat)

➢ **nodal_mapping.m**: map quantities from coarser grid onto finer grid

➢ **vis_model.m**: visualize mantle layers and lower/upper boundaries of laterally heterogeneous layers

Program peudocode (see following figure for flow chart):

----- initialization and model setup

=== for each eigenstructure (l_1, m_1):

---------- solve mode coupling $(l_{\text{tide}}, m_{\text{tide}}) \otimes (l_1, m_1)$

====== for order of perturbation from D = 0 to D = Order_end (1 or 2):

---------- set finer numerical grid and assign property values for nodes

========= for each "child" mode (l_D, m_D):

---------- retrieve "child" mode information

---------- propagator matrix method

---------- solve and record tidal response

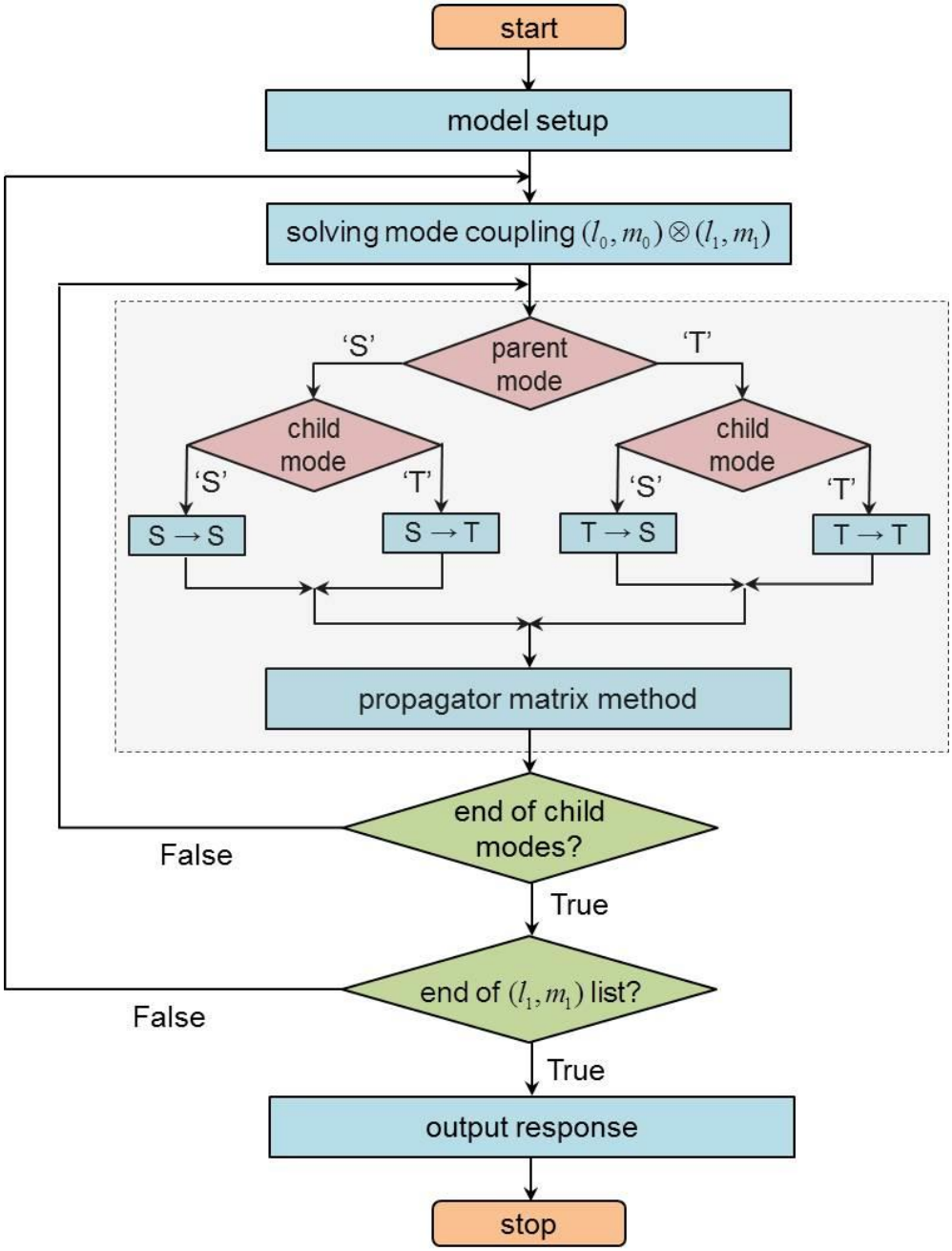
========= end

====== end

=== end

----- obtain total tidal response and write to file

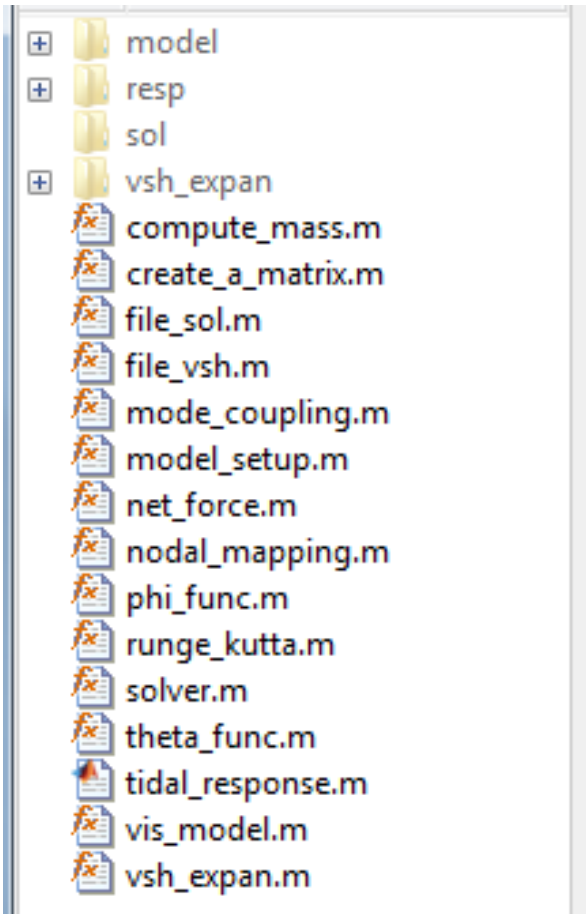
Flow chart of solution procedure of the perturbation method



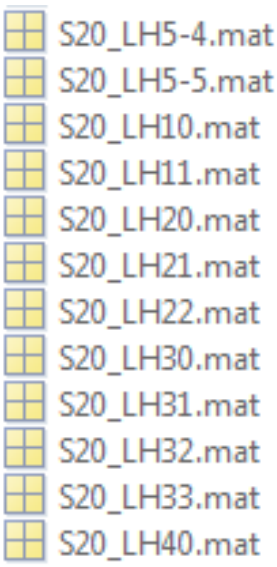
To run the code, simply type

```
>> run tidal_response
```

at the prompt in working directory. The working directory looks like:



- model/: 1-D and 3-D model input files
- resp/: total tidal response output files
- sol/: solution files for each mode in the response (see **solver.m** for quantity in each column)
- vsh_expan/: repository of VSH expansion, each file includes “child” modes information and the associated expansion coefficients. For example, some files in the repository are as follows



each .mat file means a spheroidal (2, 0) mode coupling with harmonic structure of (l_1, m_1).

See **vsh_expan.m** for more details.

Examples:

1. Benchmark case: (1, 1) lateral heterogeneity in different depth ranges of lunar mantle.

Manually set up variables in initialization step in **tidal_response.m**. See snapshot below.

L_tide, M_tide: body tide force component harmonic degree and order, **(2, 0)** for this case

mode_1d, model_3d: 1-D and 3-D mode input file names

casename: case name as prefix of output files

Flag. {IS_D_MU, IS_D_LD, IS_D_RHO}: consider lateral heterogeneity in μ, λ or ρ in the calculation (1) or not (0)

Flag. IS_VIS_MODEL: visualize layered model and layer boundaries of lateral heterogeneity (1) or not (0)

Order_init, Order_end: starting and ending order of perturbation *D*. **Order_init** is always 0, **Order_end** can be 1 or 2.

```
% ================= Manual Settings Start =================
% harmonic (l0,m0) of body tide force component considered...
L_tide = 2;          % l0
M_tide = 0;          % m0

% type in model file names and case name...
model_1d = 'moon.dat';
model_3d = 'test_3D_1.dat';
casename = 'test_1_TD20';

% control flags...
Flag = struct;
Flag.IS_D_MU = 1;        % consider lateral heterogeneity in mu
Flag.IS_D_LD = 0;        % consider lateral heterogeneity in la
Flag.IS_D_RHO = 0;       % consider lateral heterogeneity in rh
Flag.IS_VIS_MODEL = 1;   % plot model for visualization (1) or

% order of perturbation, starting at 0th order, truncating at
Order_init = 0;
Order_end = 2;
% ================= Manual Settings End =================
```

Here, 1-D model is the lunar reference model from Weber et al. (2011) (**'model/moon.dat'**)

	radius (m)	density (kg/m^3)	Vp (m/s)	Vs (m/s)
1	240000.0	8000.0	4300.0	2300.0
2	330000.0	5100.0	4100.0	0.0
3	480000.0	3400.0	7500.0	3200.0
4	999100.0	3400.0	8500.0	4500.0
5	1249100.0	3400.0	7600.0	4400.0
6	1499100.0	3400.0	7800.0	4400.0
7	1697100.0	3300.0	7700.0	4400.0
8	1722100.0	2800.0	5500.0	3200.0
9	1737100.0	2700.0	3200.0	1800.0

We include inner core and outer core in our 1-D model. The 3-D model (**'model/test_3D_1.dat'**)

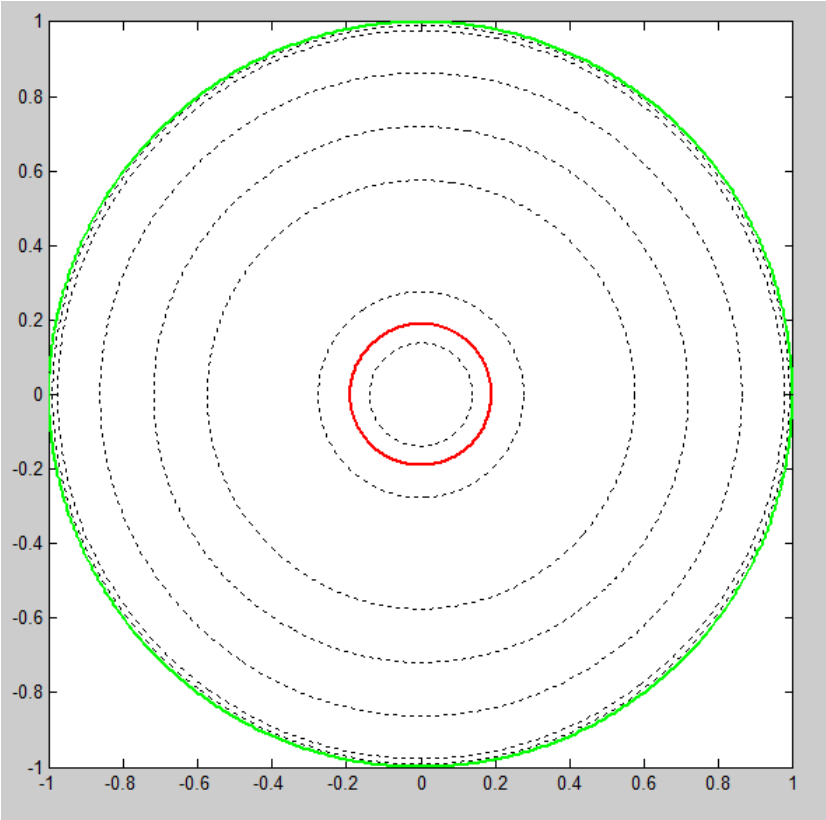
```
1  1   1
2  330000.0   1737100.0
3    1    1    1   0.10   0.10    0.10
```

1st row: number of layers with 3-D structures (**1**), number of harmonics in 3-D structures (**1**)

2nd row: radii of lower (**330000.0**) and upper (**1737100.0**) boundaries of the 3-D structure (in this case, the 3-D structure is thoughout the mantle)

3rd row: indexing of harmonics (**1**), degree (**1**), order (**1**), lateral variabilities (Δ) in μ, λ and ρ (**0.10, 0.10, 0.10**)

If **Flag. IS_VIS_MODEL = 1**, this model can be visualized as



red is lower boundary and green is upper boundary; dashed circles are 1-D property interfaces.

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model_setup.m then stores model information into a 'structure' **M**, with the following quantities. See **model_setup.m** for the meanings of the quantities.

```
         G: 6.6700e-011
       eta: 3.8249                    mu: [9x1 double]
       R_0: 1737100                lambda: [9x1 double]
     RHO_0: 8000                     beta: [9x1 double]
      MU_0: 4.2320e+010              gamma: [9x1 double]
     r_cmb: 0.1900                    mass: [9x1 double]
      r_sf: 1                        L_HET: [9x1 double]
    g0_cmb: 0.0492                 n_depth: 1
     g0_sf: 0.1395                  n_harm: 1
  r_origin: [9x1 double]            r_lower: 0.1900
         N: 9                       r_upper: 1
      l_oc: 2                            L1: 1
     N_MAN: 7                            M1: 1
        NM: [7x1 double]             lv_mu: {[9x1 double]}
         r: [9x1 double]             lv_ld: {[9x1 double]}
       rho: [9x1 double]            lv_rho: {[9x1 double]}
```

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Then, the code loops over different eigenstructures. In this case, there is only 1 harmonic (1, 1) (**M. n_harm = 1, L1 = 1, M1 = 1**).

Then, **mode_coupling.m** is called to solve mode couplings between (**L_tide = 2, M_tide = 0**) and structure (**L1 = 1, M1 = 1**). It returns a structure **MODE**, which contains information of all response modes generated from order of perturbation **Order_init** to **Order_end**. **MODE** looks like, see **mode_coupling.m** for details.

```
    cnt_mu: 14
    cnt_ld: 9
   cnt_rho: [11 11 3]
     modes: [10x4 double]
         N: 10
  mode_mu: [14x13 double]
  mode_ld: [9x9 double]
 mode_rho: {[11x9 double]  [11x9 double]  [3x9 double]}
```

specifically, **MODE.N = 10** means there are 10 distinct modes generated from this coupling (same modes at different *D* are considered different). The modal information is stored in **MODE.modes**, that is

```
0     1     2     0
1    -1     2    -1
1     1     1     1
1     1     3     1
2    -1     3    -2
2     1     0     0
2     1     2     0
2     1     2     2
2     1     4     0
2     1     4     2
```

1st column: order of perturbation *D*, 2nd column: spheroidal (1) or toroidal (-1) mode, 3rd column: harmonic degree, 4th column: harmonic order (see Fig.1 in Qin et al. (2014)).

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For each mode in **MODE.modes**, retieve modal information from **MODE** and store it in **mc**. For example, **mc** of mode [2, 1, 4, 2] is

```
   L0: 2
   M0: 0
   L1: 1
   M1: 1
order: 2
 type: 1
    l: 4
    m: 2
   mu: [1 3 1 0.3372 -2.5288 4.0460 0.1264 0.3372 -3.5403]
rho_1: [1 3 1 -0.1686 -0.1264]
   ld: [1 3 1 0.1686 0.1686]
rho_2: [1 3 1 0.1686 -0.5058]
rho_3: []
```

mc.mu, mc.ld, mc.rho_1, mc.rho_2 and mc.rho_3 are its "parent" mode information and the associated coupling coefficients. **mc.rho_1, mc.rho_2 and mc.rho_3** represent 3 different terms due to density anomalies; **mc.rho_3** is empty, meaning this term won't cause [2, 1, 4, 2] mode.

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Propagator matrix method is then used for solution. Since we only switch on **Flag. IS_D_MU**, only delta_mu effect is considered in the solution process (see **runge_kutta.m** for details). **runge_kutta.m** returns a structure **RK**, which is passed to **solver.m** for tidal response solutions.

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Tidal response is output to file. In this case, **'resp/resp.test_1_TD20.dat'**:

1	0	1	2	0	1.000000e+000	1.000000e+000	1.000000e+000
2	1	-1	2	-1	0.000000e+000	0.000000e+000	-5.565628e-002
3	1	1	1	1	8.736507e-005	0.000000e+000	1.319845e-002
4	1	1	3	1	-1.180999e-002	-8.508381e-003	-4.050898e-003
5	2	-1	3	-2	0.000000e+000	0.000000e+000	5.194421e-004
6	2	1	0	0	-4.416889e-006	0.000000e+000	0.000000e+000
7	2	1	2	0	6.846128e-004	7.352468e-004	8.950954e-004
8	2	1	2	2	7.297619e-005	1.026563e-004	9.877903e-005
9	2	1	4	0	-1.276000e-004	-7.193933e-005	-1.111989e-005
10	2	1	4	2	1.426611e-004	8.043062e-005	1.243242e-005

Here we omit the last three columns in the file. The last three columns in this snapshot are relative responses in radial displacement (*h*), gravitational potential (*k*) and horizontal displacement (*l* or *w*).

2. Lunar crustal thickness variations (see descriptions and results in my manuscript):

Initialization as below

```
% ================= Manual Settings Start =================
% harmonic (l0,m0) of body tide force component considered...
L_tide = 2;          % l0
M_tide = 2;          % m0

% type in model file names and case name...
model_1d = 'ct1.dat';
model_3d = 'ct1_spectrum.dat';
casename = 'crust_TD22';

% control flags...
Flag = struct;
Flag.IS_D_MU = 1;        % consider lateral heterogeneity in mu
Flag.IS_D_LD = 0;        % consider lateral heterogeneity in lamb
Flag.IS_D_RHO = 0;       % consider lateral heterogeneity in rho
Flag.IS_VIS_MODEL = 1;   % plot model for visualization (1) or no

% order of perturbation, starting at 0th order, truncating at mo
Order_init = 0;
Order_end = 2;
% ================= Manual Settings End =================
```

1-D model (**‘model/ct1.dat’**) is slightly different from (**‘model/moon.dat’**) for the topmost layers

1	240000.0	8000.0	4300.0	2300.0
2	330000.0	5100.0	4100.0	0.0
3	480000.0	3400.0	7500.0	3200.0
4	999100.0	3400.0	8500.0	4500.0
5	1249100.0	3400.0	7600.0	4400.0
6	1499100.0	3400.0	7800.0	4400.0
7	1675100.0	3220.0	7700.0	4400.0
8	1703100.0	3220.0	7700.0	4400.0
9	1737100.0	2800.0	5500.0	3200.0

3-D model (**‘model/ct1_spectrum.dat’**) has two layers of lateral heterogeneity below the surface that are adjacent to each other. Each layer has harmonic structure up to (5, 5) (35 harmonics in total)

```
 1   2    35
 2   1675100.0    1703100.0
 3   1    1    -1   -0.09077046 -0.09865199 -0.02192267
 4   2    1    0    0.01592423  0.01730692  0.00384598
 5   3    1    1    -0.1684741  -0.18310258 -0.04068946
 6   4    2    -2   -0.09454566 -0.102755   -0.02283444
 7   5    2    -1   -0.00794353 -0.00863326 -0.0019185
 8   6    2    0    0.08672127  0.09425121  0.02094471
 9   7    2    1    0.06992722  0.07599895  0.01688866
10   8    2    2    -0.03165318 -0.03440161 -0.0076448
11   9    3    -3   -0.03032151 -0.03295431 -0.00732318
12   10   3    -2   0.02854378  0.03102222  0.00689383
13   11   3    -1   -0.00058969 -0.00064089 -0.00014242
14   12   3    0    -0.00843395 -0.00916627 -0.00203695
15   13   3    1    0.0434972   0.04727403  0.01050534
16   14   3    2    0.06828972  0.07421927  0.01649317
17   15   3    3    0.0532422   0.05786518  0.01285893
18   16   4    -4   -0.02806538 -0.03050228 -0.00677829
19   17   4    -3   0.03477055  0.03778965  0.0083977
20   18   4    -2   0.01537852  0.01671382  0.00371418
21   19   4    -1   0.01140483  0.01239511  0.00275447
22   20   4    0    -0.02164852 -0.02352825 -0.0052285
23   21   4    1    0.00624242  0.00678444  0.00150765
24   22   4    2    0.01554561  0.01689542  0.00375454
25   23   4    3    0.02379455  0.02586062  0.0057468
26   24   4    4    0.04179926  0.04542866  0.01009526
27   25   5    -5   -0.00784109 -0.00852192 -0.00189376
28   26   5    -4   0.00275332  0.00299239  0.00066498
29   27   5    -3   0.02706756  0.02941782  0.00653729
30   28   5    -2   0.00940599  0.01022271  0.00227171
31   29   5    -1   0.00695417  0.00755799  0.00167956
32   30   5    0    -0.02577215 -0.02800993 -0.00622443
33   31   5    1    -0.01021321 -0.01110002 -0.00246667
34   32   5    2    0.00402303  0.00437235  0.00097163
35   33   5    3    -0.00974498 -0.01059113 -0.00235358
36   34   5    4    -0.00407043 -0.00442385 -0.00098308
37   35   5    5    0.01715745  0.01864721  0.00414382
38   1703100.0    1737100.0
39   1    1    -1   0.03328657  0.03103993  0.04260849
40   2    1    0    0.01623984  0.02267531  -0.01046268
41   3    1    1    -0.04194814 -0.07323822 0.0878832
42   4    2    -2   0.02474275  0.02154863  0.03799606
43   5    2    -1   0.02017667  0.02479172  0.00102756
44   6    2    0    -0.03494383 -0.02764125 -0.06524428
45   7    2    1    0.13703188  0.18268942  -0.05241405
46   8    2    2    0.07746865  0.09135358  0.01985622
```

47	9	3	-3	0.0410576	0.04955442	0.00580191
48	10	3	-2	0.01631885	0.02388243	-0.01506459
49	11	3	-1	-0.00159178	-7.33e-006	-0.00816609
50	12	3	0	0.05033976	0.06230645	0.00068658
51	13	3	1	0.0528225	0.07605002	-0.04355499
52	14	3	2	-0.00524213	0.00325882	-0.0405149
53	15	3	3	0.02997059	0.04516213	-0.03306339
54	16	4	-4	-0.0013905	-0.00411594	0.0099181
55	17	4	-3	-0.00674806	-0.0018911	-0.02690094
56	18	4	-2	-0.00222311	-0.00046923	-0.00950047
57	19	4	-1	0.01153781	0.01501448	-0.00288786
58	20	4	0	-0.02768377	-0.0382348	0.01609542
59	21	4	1	0.04328485	0.05731504	-0.01493031
60	22	4	2	0.06857202	0.09015899	-0.02099835
61	23	4	3	0.02200247	0.03098176	-0.01525514
62	24	4	4	-0.01835013	-0.01801547	-0.01973868
63	25	5	-5	-0.01186405	-0.01604517	0.00548461
64	26	5	-4	-0.00753626	-0.00887916	-0.0019642
65	27	5	-3	0.00573051	0.01151545	-0.01827281
66	28	5	-2	0.03713048	0.04808354	-0.00831682
67	29	5	-1	-0.0118994	-0.01512304	0.00147638
68	30	5	0	-0.00018392	-0.00274865	0.01045787
69	31	5	1	0.03938023	0.04831171	0.00232098
70	32	5	2	0.05952664	0.07598212	-0.00875174
71	33	5	3	8.068e-005	0.00049867	-0.00165371
72	34	5	4	0.00618405	0.00829002	-0.00255426
73	35	5	5	0.00920521	0.01381095	-0.00990534