Chapter1. THE GENESIS OF FOURIER ANALYSIS

陈斯杰

2015 年 12 月 31 日

摘要

This book begins the talk of Fourier Analysis with two physical problem: vibrating string and heat conduction. In this review, I am going to illustrate these two examples in detail by supplementing the background knowledge.

1 The Vibrating String

1.1 Simple harmonic motion 简谐运动

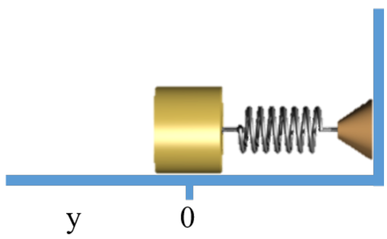

图 1: 简谐运动谐振子

Newton's law produces a 2nd order ODE

$$F = -ky(t) \Rightarrow -ky(t) = my''(t) \tag{1}$$

Let $c = \sqrt{\frac{k}{m}}$, we get a neater form:

$$y''(t) + c^2 y(t) = 0 \tag{2}$$

Equation(2) is a linear homogeneous 2nd-order differential equation(二阶常系数线性方程). For a general case $y'' + py' + qy = 0$, we first solve the characteristic equation $\lambda^2 + p\lambda + q = 0$ and get the characteristic roots: λ_1, λ_2.

Characteristic Roots	Linear Independent Sol. Pair	General Sol.
$\lambda_1, \lambda_2 \in \mathbb{R}$ *and* $\lambda_1 \neq \lambda_2$	$y_1 = e^{\lambda_1 x}, y_2 = e^{\lambda_2 x}$	$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$
$\lambda_1, \lambda_2 \in \mathbb{R}$ *and* $\lambda_1 = \lambda_2$	$y_1 = e^{\lambda_1 x}, y_2 = x e^{\lambda_1 x}$	$y = (C_1 + C_2 x) e^{\lambda_1 x}$
$\lambda_1, \lambda_2 = \alpha \pm i\beta$	$y_1 = e^{\alpha x} cos\beta x, y_2 = e^{\alpha x} sin\beta x$	$y = e^{\alpha x}(C_1 cos\beta x + C_2 sin\beta x)$