

LIFTLINE Manual

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1 Introduction

LIFTLINE is a collection of MATLAB scripts and functions that implement lifting-line theory. It solves the monoplane equation to estimate aerodynamic characteristics of a finite wing. It also provides capabilities to analyze shear force and bending moment along the wing spar.

1.1 Theory

LIFTLINE implements lifting-line theory as described in [1] and [2]. The program estimates the spanwise circulation of a finite wing. Following from this result, the program can compute the lift and vortex-induced drag coefficients. The program can also estimate structural characteristics, such as shear and bending moment along the wing.

Classical lifting-line theory assumes incompressible flow, no wing sweep, and linear airfoil section lift-curve slopes. Currently, LIFTLINE assumes:

1. Incompressible flow
2. No wing sweep (input required to properly draw planform)
3. Linear airfoil section lift-curve slopes
4. Symmetrical loading.

Future versions of LIFTLINE will implement a modified lifting-line theory and relax these assumptions.

1.2 Program Organization

The directory `scripts.aircraft` contains input scripts that define wing geometry and structural loads (e.g., fuel or stores). The directory `scripts.cases` contains input scripts that define case parameters. These parameters may be overridden for certain analyses.

1.3 Concept of Operations

When analyzing a wing of finite span, the primary goals are to determine: spanwise circulation, spanwise lift distribution, lift coefficient, and vortex-induced drag coefficient. Other goals might include finding shear and bending moment for a cantilever wing, determining rolling and yawing moments (for asymmetric loading), or determining the stalling angle of attack. The general procedure for carrying out this analysis using LIFTLINE is outlined below.

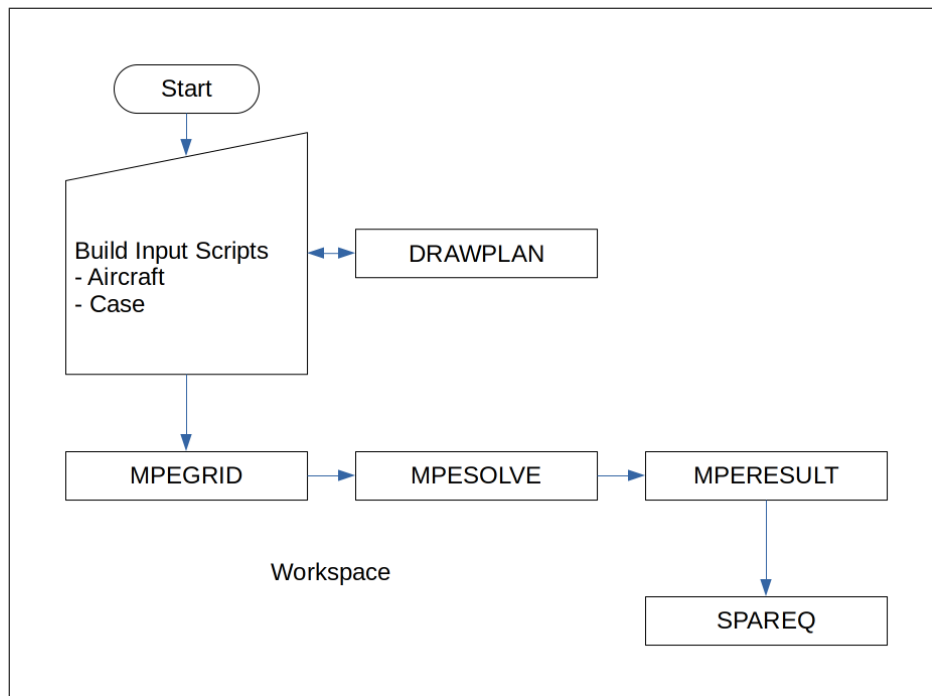
1. Build an input script (`aircraft`) that defines the wing geometry.
2. Plot and verify the planform geometry.

3. Build an input script (case) that defines case parameters.
4. Run a grid convergence analysis, and update the case script.
5. For level flight, find the angle of attack for $L = W$, and update the case script.
6. Run case for a particular angle of attack to get spanwise circulation, spanwise lift distribution, lift coefficient, and vortex-induced drag coefficient.

If desired, the lift and vortex-induced drag coefficients can be computed across a range of angles of attack. The spar shear and bending moment can also be computed.

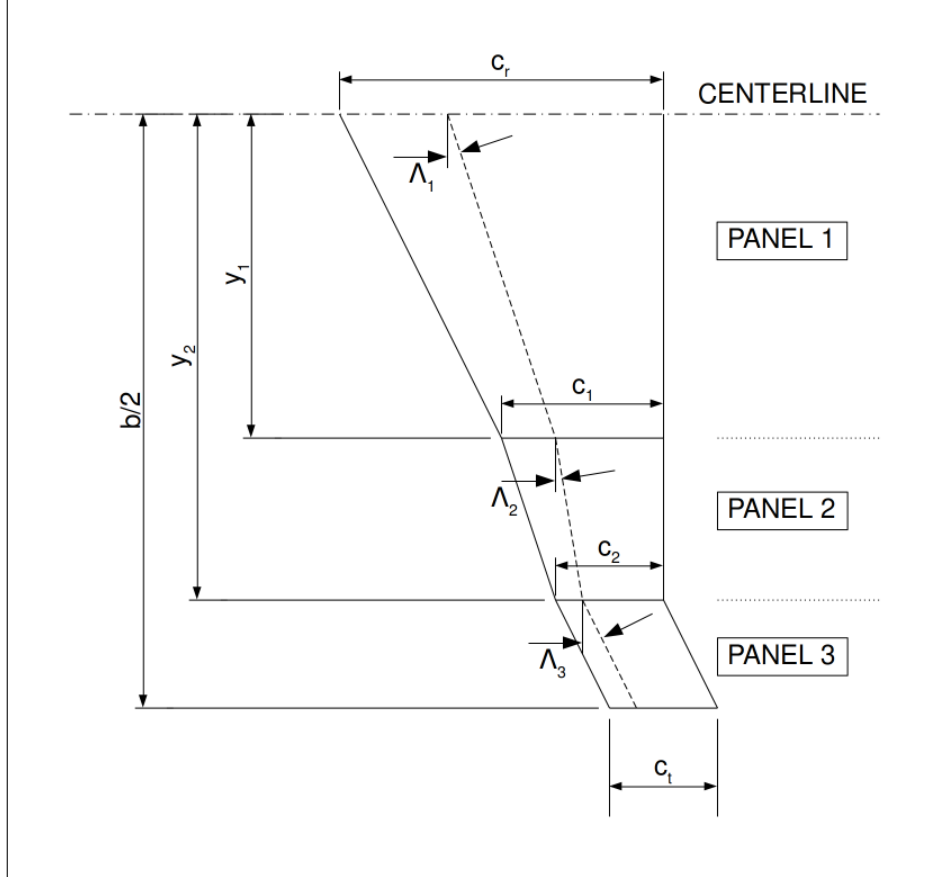
The function LIFTLINEUI provides an interactive user interface, which aids the user in executing many of these tasks.

Advanced users can also call any of the program's library functions directly. In this way, users can build custom analysis routines. The figure below graphically depicts a typical workflow.



2 Definition of Inputs

2.1 Planform Geometry



An M-script in the folder `scripts.aircraft` defines the wing geometry. The required variables are given in the table below. N panels compose the semi-span. For each row of `clalp`, `alpzl`, and `clmax`, the inboard value applies immediately outboard of the panel's inboard breakpoint.

Variable Name	Dimension	Description	Units
ybp	1 x (N+1)	Spanwise coordinates of breakpoints $[0, y_1, y_2, \dots, b/2]$	m
cbp	1 x (N+1)	Chord at breakpoints $[c_r, c_1, c_2, \dots, c_t]$	m
sweepa	1 x N	Sweep angle ($c/4$) of each panel	deg
twista	1 x N	Twist angle at tip of each panel	deg
clalp	N x 2	Airfoil section lift-curve slope, C_{l_α} . Each row corresponds to a panel. Column 1 is the inboard value. Column 2 is the outboard value.	1/deg
alpzl	N x 2	Airfoil section zero lift angle of attack, α_{0l} . Each row corresponds to a panel. Column 1 is the inboard value. Column 2 is the outboard value.	deg
clmax	N x 2	Airfoil section maximum lift coefficient, $C_{l_{max}}$. Each row corresponds to a panel. Column 1 is the inboard value. Column 2 is the outboard value.	-
b	Scalar	Span	m
S	Scalar	Reference wing area	m ²
W	Scalar	Aircraft Weight	N

2.2 Structural Loads

An M-script in the folder `scripts_aircraft` defines structural loads (e.g., fuel or stores). The function SPAREQ (Spar Equilibrium Shear and Bending Moment) optionally uses the inputs `ploads` and `uloads`, as defined below.

Variable Name	Dimension	Description	Units
ploads	R x 2	Point loads applied along the wing. Each row specifies a point load. Column 1, y-values (meters). Column 2, loads. Negative loads values apply in the downward direction. For example, a store would be input as a negative load. Example: $\begin{bmatrix} 1.2 & -50 \\ 2.1 & -25 \end{bmatrix}$	N
uloads	R x 3	Uniform distributed loads applied along the wing. Column 1, inboard y-values (meters). Column 2, outboard y-values (meters). Column 3, load per unit span. Negative loads values apply in the downward direction. For example, a fuel tank would be input as a negative load. Example: $\begin{bmatrix} 1.1 & 2.2 & -10 \\ 2.0 & 4.0 & -20 \end{bmatrix}$	N/m

2.3 Case Parameters

An M-script in the folder `scripts_cases` defines case parameters (i.e., run conditions). Depending on the particular analysis, some of these variables may be overridden or not required.

Variable Name	Dimension	Description	Units
<code>alpha_r</code>	Scalar	Angle of attack (root)	deg
<code>ncoef</code>	Scalar	Number of Fourier sine series coefficients	-
<code>U</code>	Scalar	Free-stream velocity	m/s
<code>rho</code>	Scalar	Density	kg/m ³

3 Using LIFTLINEUI

The function `LIFTLINEUI` provides an interactive user interface, which aids the user in executing some of the program's functions. This section steps through a typical analysis scenario using the function `LIFTLINEUI`. This section assumes that you have installed the software per the README and your MATLAB path is set to the top-level directory of `LIFTLINE`.

1. Build a script in the folder `scripts_aircraft` that defines the planform geometry as described in section 2.1. See the folder `scripts_aircraft` for example M-files.
2. Run the command `drawplan(ybp,cbp,sweepa,twista)`. This plots the planform, and you can verify that the planform geometry looks correct.
3. Build a script in the folder `scripts_cases` that defines the case parameters as described in section 2.3. See the folder `scripts_cases` for example M-files. Note, you can start with a small `ncoef` (for example, `ncoef = 5`) and a nominal angle of attack (for example, `alpha_r = 5`). Appropriate values will be determined as part of this analysis.
4. Run the command `liftlineui`.
5. Choose the option to "Select aircraft script," and select the input script that defines the planform geometry.
6. Choose the option to "Select run conditions script," and select the input script that defines the case parameters.
7. Choose the option to "Continue."
8. Choose the option to execute a "Grid Convergence Analysis."
9. Enter the desired accuracy (significant figures) for the lift coefficient.
10. Enter the desired accuracy (significant figures) for the vortex-induced drag coefficient.

11. The program will output the required `ncoef` to achieve the desired accuracy. Update the case parameters script with this value for `ncoef`.
12. Rerun `liftlineui` using the same procedure, but this time, choose the option to “Find AoA for $L = W$.”
13. The program will output the required `alpha_r` for level flight. Update the case parameters script with this value for `alpha_r`.
14. Rerun `liftlineui` using the same procedure. Choose the option to “Run Case as Defined in Run Conditions Script.” This will output the wing lift coefficient and wing vortex-induced drag coefficient for the particular case. This will also plot the spanwise circulation and spanwise section lift coefficient.
15. Rerun `liftlineui` using the same procedure. Choose the option for “AoA Range.” This will plot the wing lift coefficient and wing drag polar over a range of angles of attack.
16. Rerun `liftlineui` using the same procedure. Choose the option for “Spar Shear and Bending Moment.” This will plot the spanwise shear and bending moment diagrams.¹

4 Function Reference

4.1 Symmetric Solvers

The symmetric solvers (i.e., symmetrical wing loading) use inputs/outputs as defined below. `N` is the number of Fourier sine series coefficients.

¹Currently, this option in `LIFTLINEUI` only considers the aerodynamic wing loading and does not use a structural loads script as described in section 2.2. To include structural loads, the user must call the function `SPAREQ` directly.

Variable Name	Dimension	Description	Units
alpha_r	Scalar	Angle of attack (root)	deg
ncoef	Scalar	Number of Fourier sine series coefficients	-
theta	1 x N	Transformed spanwise coordinate	rad
y	1 x N	Spanwise coordinate	m
c	1 x N	Chord	m
a0	1 x N	Section lift-curve slope	1/rad
alpha	1 x N	Section angle of attack	rad
alpha_zl	1 x N	Section zero lift angle of attack	rad
clmax_vec	1 x N	Section maximum lift coefficient	-
An	1 x N	Fourier coefficients $[A_1, A_3, \dots, A_{2N-1}]$	-
U	Scalar	Free-stream velocity	m/s
Gamma	1 x N	Circulation	m^2/s
CL	Scalar	Wing lift coefficient	-
CDv	Scalar	Wing vortex-induced drag coefficient	-
cl	1 x N	Section lift coefficient	-
V	1 X N	Shear	N
M	1 X N	Bending Moment	N^*m
M.root	1 X N	Root Bending Moment	N^*m

mpegrid Returns vectors of coordinates and section values at N spanwise locations from $0 \leq y \leq b/2$.

```
>> [theta,y,c,a0,alpha,alpha_zl,clmax_vec] = ...
    mpegrid(ybp,cbp,twista,clalp,alpzl,clmax,alpha_r,ncoef)
```

mpesolve Solves the monoplane equation for symmetrical loading.

```
>> An = mpesolve(theta,c,a0,alpha,alpha_zl,b)
```

mperesult Returns aerodynamic results based on the output of MPESOLVE.

```
>> [Gamma,CL,CDv,cl] = mperesult(An,theta,c,U,b,S)
```

spareq Returns shear and bending moment for the cantilever wing. Optional input arguments ploads and uloads specify point loads and uniform distributed loads along the wing, respectively. See section 2.2 for more information on defining structural loads.

```
>> [V,M,M_root] = spareq(Gamma,y,rho,U)
```

```
>> [V,M,M_root] = spareq(Gamma,y,rho,U,ploads,uloads)
```

4.2 Plotting

Use the MATLAB help command to display usage instructions for the plotting functions (e.g., `help mpeplot` for help using the MPEPLOT function).

drawplan Plots the planform.

mpeplot Utility function to plot results.

References

- [1] John J. Bertin and Russell M. Cummings. *Aerodynamics for Engineers*. 5th ed. Upper Saddle River, NJ: Pearson Prentice-Hall, 2009.
- [2] John D. Anderson. *Fundamentals of Aerodynamics*. 5th ed. New York: McGraw-Hill, 2011.