# LIFTLINE Manual

Christopher C. Chinske July 27, 2020

### 1 Introduction

LIFTLINE is a collection of MATLAB scripts and functions that implement lifting-line theory. It solves the monoplane equation to estimate aerodynamic characteristics of a finite wing. It also provides capabilities to analyze shear force and bending moment along the wing spar.

#### 1.1 Theory

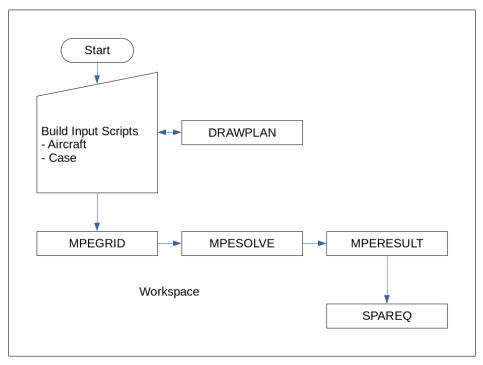
LIFTLINE implements lifting-line theory as described in [1] and [2]. The program estimates the spanwise circulation of a finite wing. Following from this result, the program can compute the lift and vortex-induced drag coefficients. The program can also estimate structural characteristics, such as shear and bending moment along the wing.

Classical lifting-line theory assumes incompressible flow, no wing sweep, and linear airfoil section lift-curve slopes. Currently, LIFTLINE assumes:

- 1. Incompressible flow
- 2. No wing sweep (input required to properly draw planform)
- 3. Linear airfoil section lift-curve slopes
- 4. Symmetrical loading.

Future versions of LIFTLINE will implement a modified lifting-line theory and relax these assumptions.

### 1.2 Graphical Workflow



#### 1.3 LIFTLINEUI

The function LIFTLINEUI provides an interactive interface that guides the user through common analyses. These analyses are described below.

- 1. Grid Convergence Analysis. Find the number of Fourier sine series coefficients required to meet specified error requirements.
- 2. Find AoA for L = W. Find the angle of attack that generates lift equal to the weight of the aircraft specified in the aircraft script.
- 3. Run Case. Run MPEGRID, MPESOLVE, and MPERESULT using a specified case script.
- 4. AoA Range. Find CL and CDv over a range of angle of attack.
- 5. Spar Shear and Bending Moment. Compute and plot shear and bending moment along the wing.

When building a new aircraft script, it's generally recommended to run a grid convergence analysis at a small, positive angle of attack (e.g., 4 deg). Update the case script as necessary. Then, if considering an aircraft in steady flight, find the angle of attack for L=W. Again, update the case script as necessary.

## 2 Definition of Inputs and Outputs

## 2.1 Basic Configuration Geometry Inputs

N panels compose the semi-span. For each row of clalp, alpzl, and clmax, the inboard value applies immediately outboard of the panel's inboard breakpoint.

Variable Name	Dimension	Description	Units
ybp	1 x (N+1)	Spanwise coordinates of breakpoints [0,	m
		$y_1, y_2,, b/2$	
cbp	1 x (N+1)	Chord at breakpoints $[c_r, c_1, c_2,, c_t]$	m
sweepa	1 x N	Sweep angle $(c/4)$ of each panel	deg
twista	1 x N	Twist angle at tip of each panel	deg
clalp	N x 2	Airfoil section lift-curve slope, $C_{l_{\alpha}}$ . Each	1/deg
		row corresponds to a panel. Column 1 is	
		the inboard value. Column 2 is the out-	
		board value.	
alpzl	N x 2	Airfoil section zero lift angle of attack,	deg
		$\alpha_{0l}$ . Each row corresponds to a panel.	
		Column 1 is the inboard value. Column 2	
		is the outboard value.	
clmax	N x 2	Airfoil section maximum lift coefficient,	-
		$C_{l_{max}}$ . Each row corresponds to a panel.	
		Column 1 is the inboard value. Column 2	
		is the outboard value.	
b	Scalar	Span	m
S	Scalar	Reference wing area	m^2
W	Scalar	Weight	N

## 2.2 MPEGRID, MPESOLVE, and MPERESULT

In addition to subsets of the basic configuration geometry inputs, the functions MPEGRID, MPESOLVE, and MPERESULT use inputs/outputs as defined below. N is the number of Fourier sine series coefficients.

Variable Name	Dimension	Description	Units
alpha_r	Scalar	Angle of attack (root)	deg
ncoef	Scalar	Number of Fourier sine series coefficients	-
theta	1 x N	Transformed spanwise coordinate	rad
У	1 x N	Spanwise coordinate	m
С	1 x N	Chord	m
a0	1 x N	Section lift-curve slope	1/rad
alpha	1 x N	Section angle of attack	rad
alpha_zl	1 x N	Section zero lift angle of attack	rad
clmax_vec	1 x N	Section maximum lift coefficient	-
An	1 x N	Fourier coefficients $[A_1, A_3,, A_{2N-1}]$	-
U	Scalar	Free-stream velocity	m/s
Gamma	1 x N	Circulation	m^2/s
CL	Scalar	Wing lift coefficient	-
CDv	Scalar	Wing vortex-induced drag coefficient	-
cl	1 x N	Section lift coefficient	-

## 2.3 SPAREQ

The function SPAREQ (Spar Equilibrium Shear and Bending Moment) optionally uses the inputs ploads and uloads, as defined below. Other inputs/outputs are also defined below.

Variable Name	Dimension	Description	${f Units}$
ploads	R x 2	Point loads applied along the wing. Each	N
		row specifies a point load. Column 1, y-	
		values (meters). Column 2, loads. Nega-	
		tive loads values apply in the downward	
		direction. For example, a store would	
		be input as a negative load. Example:	
		$\begin{bmatrix} 1.2 & -50 \end{bmatrix}$	
		2.1 -25	
uloads	R x 3	Uniform distributed loads applied along	N/m
		the wing. Column 1, inboard y-values	
		(meters). Column 2, outboard y-values	
		(meters). Column 3, load per unit span.	
		Negative loads values apply in the down-	
		ward direction. For example, a fuel tank	
		would be input as a negative load. Exam-	
		ple: $\begin{bmatrix} 1.1 & 2.2 & -10 \\ 2.2 & 4.2 & 22 \end{bmatrix}$	
		$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix}$	
rho	Scalar	Density	$kg/m^3$
V	1 X N	Shear	N
M	1 X N	Bending Moment	N*m
M_root	1 X N	Root Bending Moment	N*m

# References

- [1] John J. Bertin and Russell M. Cummings. *Aerodynamics for Engineers*. 5th ed. Upper Saddle River, NJ: Pearson Prentice-Hall, 2009.
- [2] John D. Anderson. Fundamentals of Aerodynamcs. 5th ed. New York: McGraw-Hill, 2011.