CS 188: Artificial Intelligence
Fall 2010

Lecture 23: Perceptrons and More!

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Errors, and What to Do

- Examples of errors

Dear GlobalSCAPE Customer,

GlobalSCAPE has partnered with ScanSoft to offer you the latest version of OmniPage Pro, for just $99.99* - the regular list price is $499! The most common question we've received about this offer is - Is this genuine? We would like to assure you that this offer is authorized by ScanSoft, is genuine and valid. You can get the . . .

. . . To receive your $30 Amazon.com promotional certificate, click through to

http://www.amazon.com/apparel

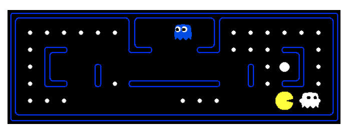
and see the prominent link for the $30 offer. All details are there. We hope you enjoyed receiving this message. However, if you'd rather not receive future e-mails announcing new store launches, please click . . .

What to Do About Errors

- Problem: there's still spam in your inbox
- Need more features – words aren't enough!
  - Have you emailed the sender before?
  - Have 1K other people just gotten the same email?
  - Is the sending information consistent?
  - Is the email in ALL CAPS?
  - Do inline URLs point where they say they point?
  - Does the email address you by (your) name?
- Naïve Bayes models can incorporate a variety of features, but tend to do best in homogeneous cases (e.g. all features are word occurrences)

Later On…

Web Search



Decision Problems

Classification: Feature Vectors

x $\quad f(x)$ $\quad y$

Hello,

Do you want free printr cartriges? Why pay more when you can get them ABSOLUTELY FREE! Just

```
# free      : 2
YOUR_NAME   : 0
MISSPELLED  : 2
FROM_FRIEND : 0
...
```

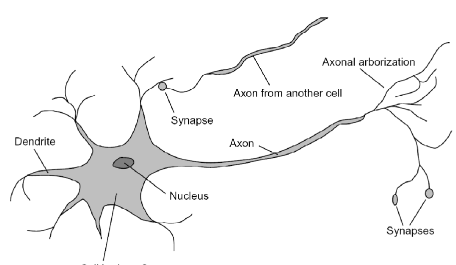
SPAM or +

```
PIXEL-7,12  : 1
PIXEL-7,13  : 0
...
NUM_LOOPS   : 1
...
```

"2"

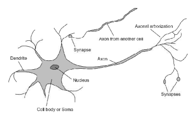
Some (Simplified) Biology

- Very loose inspiration: human neurons



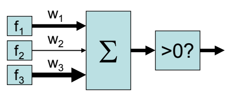
Linear Classifiers

- Inputs are feature values
- Each feature has a weight
- Sum is the activation



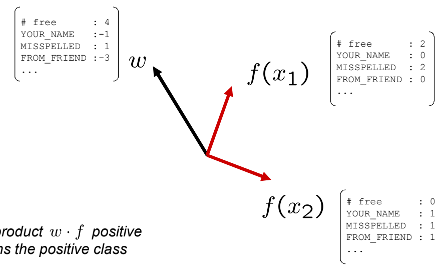
$$\text{activation}_w(x) = \sum_i w_i \cdot f_i(x) = w \cdot f(x)$$

- If the activation is:
  - Positive, output +1
  - Negative, output -1



Classification: Weights

- Binary case: compare features to a weight vector
- Learning: figure out the weight vector from examples



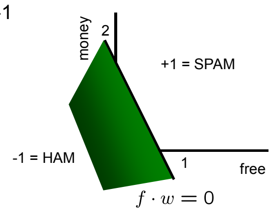
Dot product $w \cdot f$ positive means the positive class

Binary Decision Rule

- In the space of feature vectors
  - Examples are points
  - Any weight vector is a hyperplane
  - One side corresponds to Y=+1
  - Other corresponds to Y=-1

w

```
BIAS  : -3
free  :  4
money :  2
...
```

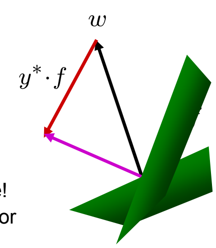


Learning: Binary Perceptron

- Start with weights = 0
- For each training instance:
  - Classify with current weights

$$y = \begin{cases} +1 & \text{if } w \cdot f(x) \geq 0 \\ -1 & \text{if } w \cdot f(x) < 0 \end{cases}$$

  - If correct (i.e., y=y*), no change!
  - If wrong: adjust the weight vector by adding or subtracting the feature vector. Subtract if y* is -1.



$$w = w + y^* \cdot f$$

[Demo]

Multiclass Decision Rule

- If we have multiple classes:
  - A weight vector for each class:

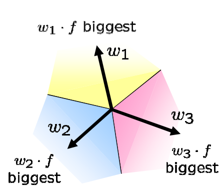
$$w_y$$

  - Score (activation) of a class y:

$$w_y \cdot f(x)$$

  - Prediction highest score wins

$$y = \arg\max_y \; w_y \cdot f(x)$$



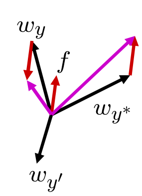
Binary = multiclass where the negative class has weight zero

Learning: Multiclass Perceptron

- Start with all weights = 0
- Pick up training examples one by one
- Predict with current weights

$$y = \arg\max_y \; w_y \cdot f(x)$$

- If correct, no change!
- If wrong: lower score of wrong answer, raise score of right answer



$$w_y = w_y - f(x)$$

$$w_{y^*} = w_{y^*} + f(x)$$

Example: Multiclass Perceptron

"win the vote"

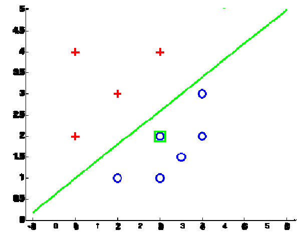
"win the election"

"win the game"

w_{SPORTS}	$w_{POLITICS}$	w_{TECH}
BIAS : 1	BIAS : 0	BIAS : 0
win : 0	win : 0	win : 0
game : 0	game : 0	game : 0
vote : 0	vote : 0	vote : 0
the : 0	the : 0	the : 0
...	...	...

Examples: Perceptron

- Separable Case



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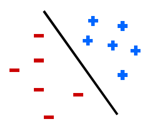
Properties of Perceptrons

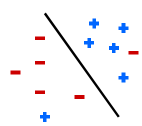
- Separability: some parameters get the training set perfectly correct
- Convergence: if the training is separable, perceptron will eventually converge (binary case)
- Mistake Bound: the maximum number of mistakes (binary case) related to the *margin* or degree of separability

$$\text{mistakes} < \frac{k}{\delta^2}$$

Separable



Non-Separable



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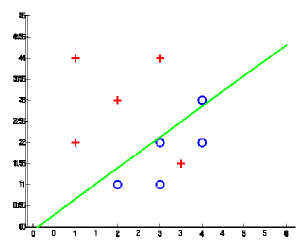
Examples: Perceptron

- Non-Separable Case



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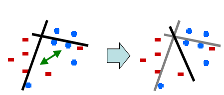
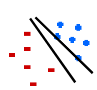
Problems with the Perceptron

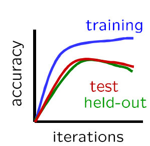
- Noise: if the data isn't separable, weights might thrash
  - Averaging weight vectors over time can help (averaged perceptron)



- Mediocre generalization: finds a "barely" separating solution



- Overtraining: test / held-out accuracy usually rises, then falls
  - Overtraining is a kind of overfitting



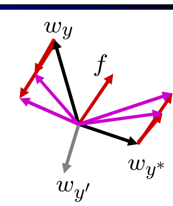
Fixing the Perceptron

- Idea: adjust the weight update to mitigate these effects
- MIRA*: choose an update size that fixes the current mistake…
- … but, minimizes the change to w

$$\min_w \frac{1}{2}\sum_y ||w_y - w'_y||^2$$

$$w_{y^*} \cdot f(x) \geq w_y \cdot f(x) + 1$$

- The +1 helps to generalize



Guessed y instead of y^* on example x with features $f(x)$

$$w_y = w'_y - \tau f(x)$$
$$w_{y^*} = w'_{y^*} + \tau f(x)$$

* Margin Infused Relaxed Algorithm

Minimum Correcting Update

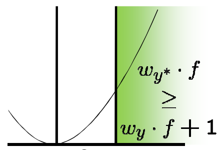
$$\min_w \frac{1}{2}\sum_y ||w_y - w'_y||^2$$
$$w_{y^*} \cdot f \geq w_y \cdot f + 1$$

$$\min_\tau ||\tau f||^2$$
$$w_{y^*} \cdot f \geq w_y \cdot f + 1$$

$$(w'_{y^*} + \tau f) \cdot f = (w'_y - \tau f) \cdot f + 1$$

$$\tau = \frac{(w'_y - w'_{y^*}) \cdot f + 1}{2 f \cdot f}$$

$$\boxed{\begin{aligned} w_y &= w'_y - \tau f(x) \\ w_{y^*} &= w'_{y^*} + \tau f(x) \end{aligned}}$$



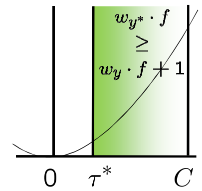
min not τ=0, or would not have made an error, so min will be where equality holds

Maximum Step Size

- In practice, it's also bad to make updates that are too large
  - Example may be labeled incorrectly
  - You may not have enough features
  - Solution: cap the maximum possible value of τ with some constant C

$$\tau^* = \min\left(\frac{(w'_y - w'_{y^*}) \cdot f + 1}{2 f \cdot f}, C\right)$$

  - Corresponds to an optimization that assumes non-separable data
  - Usually converges faster than perceptron
  - Usually better, especially on noisy data



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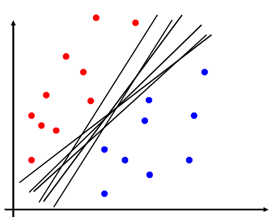
Linear Separators

- Which of these linear separators is optimal?



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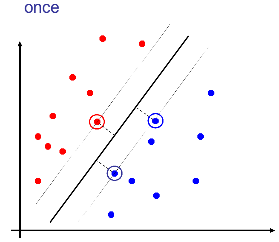
Support Vector Machines

- Maximizing the margin: good according to intuition, theory, practice
- Only support vectors matter; other training examples are ignorable
- Support vector machines (SVMs) find the separator with max margin
- Basically, SVMs are MIRA where you optimize over all examples at once



MIRA

$$\min_w \frac{1}{2}||w - w'||^2$$
$$w_{y^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

SVM

$$\min_w \frac{1}{2}||w||^2$$
$$\forall i, y \; w_{y^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

Classification: Comparison

- Naïve Bayes
  - Builds a model training data
  - Gives prediction probabilities
  - Strong assumptions about feature independence
  - One pass through data (counting)
- Perceptrons / MIRA:
  - Makes less assumptions about data
  - Mistake-driven learning
  - Multiple passes through data (prediction)
  - Often more accurate

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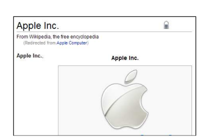
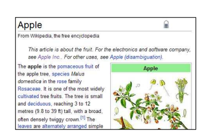
Extension: Web Search

x = "Apple Computers"

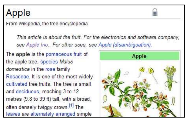
- Information retrieval:
  - Given information needs, produce information
  - Includes, e.g. web search, question answering, and classic IR
- Web search: not exactly classification, but rather ranking





Feature-Based Ranking

x = "Apple Computers"

$f(x,$ 

$) = [0.3\ 5\ 0\ 0\ \ldots]$

$f(x,$ 

$) = [0.8\ 4\ 2\ 1\ \ldots]$

Perceptron for Ranking

- Inputs x
- Candidates y
- Many feature vectors: $f(x, y)$
- One weight vector: w
  - Prediction:

    $y = \arg\max_y \ w \cdot f(x, y)$

  - Update (if wrong):

    $w = w + f(x, y^*) - f(x, y)$