CS 188: Artificial Intelligence
Fall 2010

Lecture 17: Bayes Nets IV
10/22/2010

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Announcements

- Midterm
  - On TUESDAY, check web for details, practice
  - Review sessions TONIGHT 5-7pm, Sunday 12-2pm 105 North Gate
- Written 1
  - Will be in glookup, and returned with solutions in 283E Soda, by Friday noon

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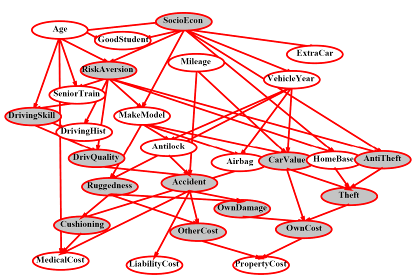
Approximate Inference



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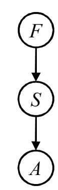
Approximate Inference

- Simulation has a name: sampling
- Sampling is a hot topic in machine learning, and it's really simple
- Basic idea:
  - Draw N samples from a sampling distribution S
  - Compute an approximate posterior probability
  - Show this converges to the true probability P
- Why sample?
  - Learning: get samples from a distribution you don't know
  - Inference: getting a sample is faster than computing the right answer (e.g. with variable elimination)



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Prior Sampling

$P(C)$

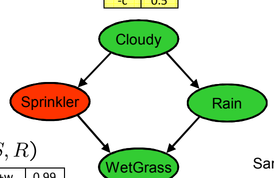
+c	0.5
-c	0.5

$P(S|C)$

+c	+s	0.1
	-s	0.9
-c	+s	0.5
	-s	0.5

$P(R|C)$

+c	+r	0.8
	-r	0.2
-c	+r	0.2
	-r	0.8



$P(W|S,R)$

+s	+r	+w	0.99
		-w	0.01
	-r	+w	0.90
		-w	0.10
-s	+r	+w	0.90
		-w	0.10
	-r	+w	0.01
		-w	0.99

Samples:

+c, -s, +r, +w
-c, +s, -r, +w
…

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Prior Sampling

- This process generates samples with probability:

$$S_{PS}(x_1 \ldots x_n) = \prod_{i=1}^{n} P(x_i|\text{Parents}(X_i)) = P(x_1 \ldots x_n)$$

…i.e. the BN's joint probability

- Let the number of samples of an event be $N_{PS}(x_1 \ldots x_n)$
- Then
$$\begin{aligned} \lim_{N\to\infty} \hat{P}(x_1,\ldots,x_n) &= \lim_{N\to\infty} N_{PS}(x_1,\ldots,x_n)/N \\ &= S_{PS}(x_1,\ldots,x_n) \\ &= P(x_1 \ldots x_n) \end{aligned}$$
- I.e., the sampling procedure is consistent

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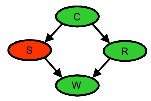
Example

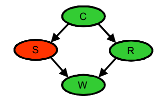
- We'll get a bunch of samples from the BN:
  - +c, -s, +r, +w
  - +c, +s, +r, +w
  - -c, +s, +r, -w
  - +c, -s, +r, +w
  - -c, -s, -r, +w



- If we want to know P(W)
  - We have counts <+w:4, -w:1>
  - Normalize to get P(W) = <+w:0.8, -w:0.2>
  - This will get closer to the true distribution with more samples
  - Can estimate anything else, too
  - What about P(C| +w)? P(C| +r, +w)? P(C| -r, -w)?
  - Fast: can use fewer samples if less time (what's the drawback?)

Rejection Sampling

- Let's say we want P(C)
  - No point keeping all samples around
  - Just tally counts of C as we go



- Let's say we want P(C| +s)
  - Same thing: tally C outcomes, but ignore (reject) samples which don't have S=+s
  - This is called rejection sampling
  - It is also consistent for conditional probabilities (i.e., correct in the limit)

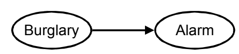
+c, -s, +r, +w
+c, +s, +r, +w
-c, +s, +r, -w
+c, -s, +r, +w
-c, -s, -r, +w

Sampling Example

- There are 2 cups.
  - The first contains 1 penny and 1 quarter
  - The second contains 2 quarters
- Say I pick a cup uniformly at random, then pick a coin randomly from that cup. It's a quarter (yes!). What is the probability that the other coin in that cup is also a quarter?

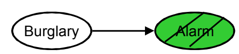
Likelihood Weighting

- Problem with rejection sampling:
  - If evidence is unlikely, you reject a lot of samples
  - You don't exploit your evidence as you sample
  - Consider P(B|+a)



-b, -a
-b, -a
-b, -a
-b, -a
+b, +a

- Idea: fix evidence variables and sample the rest



-b +a
-b, +a
-b, +a
-b, +a
+b, +a

- Problem: sample distribution not consistent!
- Solution: weight by probability of evidence given parents

Likelihood Weighting

$P(C)$

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-c	0.5

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	-s	0.9
-c	+s	0.5
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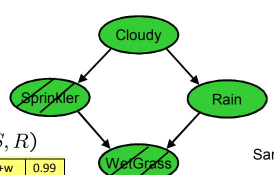

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		-w	0.10
-s	+r	+w	0.90
		-w	0.10
	-r	+w	0.01
		-w	0.99

Samples:

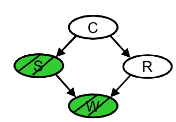
+c, +s, +r, +w
…

$w = 1.0 \times 0.1 \times 0.99$

Likelihood Weighting

- Sampling distribution if z sampled and e fixed evidence

$$S_{WS}(\mathbf{z}, \mathbf{e}) = \prod_{i=1}^{l} P(z_i|\text{Parents}(Z_i))$$



- Now, samples have weights

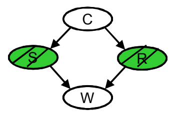
$$w(\mathbf{z}, \mathbf{e}) = \prod_{i=1}^{m} P(e_i|\text{Parents}(E_i))$$

- Together, weighted sampling distribution is consistent

$$S_{\text{WS}}(z, e) \cdot w(z, e) = \prod_{i=1}^{l} P(z_i|\text{Parents}(z_i)) \prod_{i=1}^{m} P(e_i|\text{Parents}(e_i))$$
$$= P(\mathbf{z}, \mathbf{e})$$

Likelihood Weighting

- Likelihood weighting is good
  - We have taken evidence into account as we generate the sample
  - E.g. here, W's value will get picked based on the evidence values of S, R
  - More of our samples will reflect the state of the world suggested by the evidence
- Likelihood weighting doesn't solve all our problems
  - Evidence influences the choice of downstream variables, but not upstream ones (C isn't more likely to get a value matching the evidence)
- We would like to consider evidence when we sample every variable



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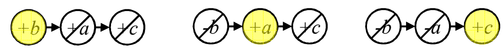
Markov Chain Monte Carlo*

- *Idea*: instead of sampling from scratch, create samples that are each like the last one.
- *Procedure*: resample one variable at a time, conditioned on all the rest, but keep evidence fixed. E.g., for P(b|c):



- *Properties*: Now samples are not independent (in fact they're nearly identical), but sample averages are still consistent estimators!
- *What's the point*: both upstream and downstream variables condition on evidence.

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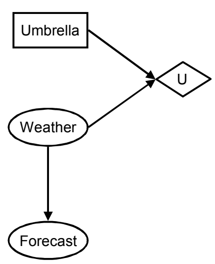
Decision Networks

- MEU: choose the action which maximizes the expected utility given the evidence
- Can directly operationalize this with decision networks
  - Bayes nets with nodes for utility and actions
  - Lets us calculate the expected utility for each action
- New node types:
  - Chance nodes (just like BNs)
  - Actions (rectangles, cannot have parents, act as observed evidence)
  - Utility node (diamond, depends on action and chance nodes)



[DEMO: Ghostbusters]

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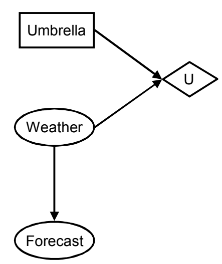
Decision Networks

- Action selection:
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action



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Example: Decision Networks

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}) = \sum_w P(w)U(\mathrm{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

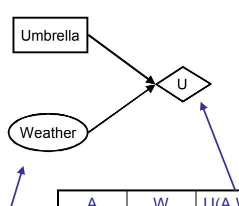
Umbrella = take

$$\mathrm{EU}(\mathrm{take}) = \sum_w P(w)U(\mathrm{take}, w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

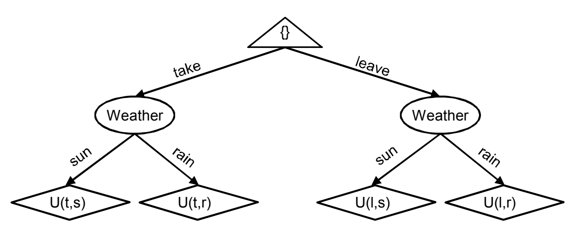
$$\mathrm{MEU}(\phi) = \max_a \mathrm{EU}(a) = 70$$



W	P(W)
sun	0.7
rain	0.3

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decisions as Outcome Trees



- Almost exactly like expectimax / MDPs
- What's changed?

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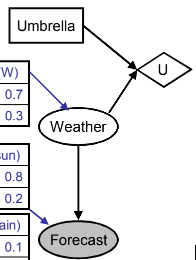
Evidence in Decision Networks



W	P(W)
sun	0.7
rain	0.3

F	P(F|sun)
good	0.8
bad	0.2

F	P(F|rain)
good	0.1
bad	0.9

- Find P(W|F=bad)
  - Select for evidence

W	P(W)
sun	0.7
rain	0.3

$P(W)$

W	P(F=bad|W)
sun	0.2
rain	0.9

$P(bad|W)$

  - First we join P(W) and P(bad|W)
  - Then we normalize

W	P(W,F=bad)
sun	0.14
rain	0.27

$P(W, bad)$

W	P(W | F=bad)
sun	0.34
rain	0.66

$P(W|F = \text{bad})$

Example: Decision Networks

Umbrella = leave

$$\text{EU}(\text{leave}|\text{bad}) = \sum_w P(w|\text{bad})U(\text{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

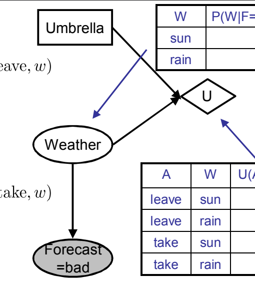
Umbrella = take

$$\text{EU}(\text{take}|\text{bad}) = \sum_w P(w|\text{bad})U(\text{take}, w)$$

$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

Optimal decision = take

$$\text{MEU}(F = \text{bad}) = \max_a \text{EU}(a|\text{bad}) = 53$$



W	P(W|F=bad)
sun	0.34
rain	0.66

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

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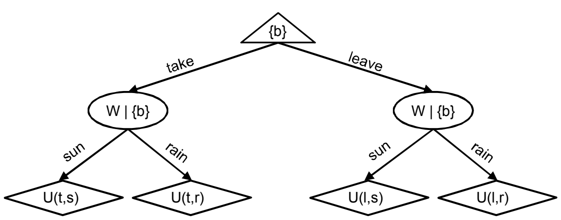
Decisions as Outcome Trees



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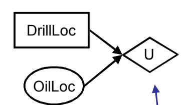
Value of Information

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has MEU = k/2
  - Fair price of drilling rights: k/2
- Question: what's the value of information
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say “oil in a” or “oil in b,” prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2



O	P
a	1/2
b	1/2

D	O	U
a	a	k
a	b	0
b	a	0
b	b	k

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