CS 188: Artificial Intelligence
Fall 2010

Lecture 14: Bayes' Nets
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Probabilistic Models

- Models describe how (a portion of) the world works
- Models are always simplifications
  - May not account for every variable
  - May not account for all interactions between variables
  - "All models are wrong; but some are useful."
    – George E. P. Box
- What do we do with probabilistic models?
  - We (or our agents) need to reason about unknown variables, given evidence
  - Example: explanation (diagnostic reasoning)
  - Example: prediction (causal reasoning)
  - Example: value of information

Model for Ghostbusters

- Reminder: ghost is hidden, sensors are noisy
- T: Top sensor is red
  B: Bottom sensor is red
  G: Ghost is in the top
- Queries:
  P(+g) = ??
  P(+g | +t) = ??
  P(+g | +t, -b) = ??
- Problem: joint distribution too large / complex

Joint Distribution

T	B	G	P(T,B,G)
+t	+b	+g	0.16
+t	+b	¬g	0.16
+t	¬b	+g	0.24
+t	¬b	¬g	0.04
¬t	+b	+g	0.04
¬t	+b	¬g	0.24
¬t	¬b	+g	0.06
¬t	¬b	¬g	0.06

Independence

- Two variables are *independent* if:

$$\forall x, y : P(x, y) = P(x)P(y)$$

  - This says that their joint distribution *factors* into a product two simpler distributions
  - Another form:

$$\forall x, y : P(x|y) = P(x)$$

  - We write: $X \perp\!\!\!\perp Y$
- Independence is a simplifying *modeling assumption*
  - *Empirical* joint distributions: at best "close" to independent
  - What could we assume for {Weather, Traffic, Cavity, Toothache}?

Example: Independence

- N fair, independent coin flips:

$P(X_1)$

h	0.5
t	0.5

$P(X_2)$

h	0.5
t	0.5

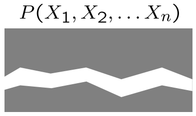
$\dots$

$P(X_n)$

h	0.5
t	0.5

$P(X_1, X_2, \dots X_n)$

2^n



Example: Independence?

$P(T)$

T	P
warm	0.5
cold	0.5

$P_1(T, W)$

T	W	P
warm	sun	0.4
warm	rain	0.1
cold	sun	0.2
cold	rain	0.3

$P_2(T, W)$

T	W	P
warm	sun	0.3
warm	rain	0.2
cold	sun	0.3
cold	rain	0.2

$P(W)$

W	P
sun	0.6
rain	0.4

Conditional Independence

- P(Toothache, Cavity, Catch)
- If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:
  - P(+catch | +toothache, +cavity) = P(+catch | +cavity)
- The same independence holds if I don't have a cavity:
  - P(+catch | +toothache, ¬cavity) = P(+catch| ¬cavity)
- Catch is *conditionally independent* of Toothache given Cavity:
  - P(Catch | Toothache, Cavity) = P(Catch | Cavity)
- Equivalent statements:
  - P(Toothache | Catch , Cavity) = P(Toothache | Cavity)
  - P(Toothache, Catch | Cavity) = P(Toothache | Cavity) P(Catch | Cavity)
  - One can be derived from the other easily

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Conditional Independence

- Unconditional (absolute) independence very rare (why?)
- *Conditional independence* is our most basic and robust form of knowledge about uncertain environments:

$$\forall x, y, z : P(x, y|z) = P(x|z)P(y|z) \qquad X \perp\!\!\!\perp Y|Z$$
$$\forall x, y, z : P(x|z, y) = P(x|z)$$

- What about this domain:
  - Traffic
  - Umbrella
  - Raining
- What about fire, smoke, alarm?

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The Chain Rule

$$P(X_1, X_2, \ldots X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2)\ldots$$

- Trivial decomposition:

$$P(\text{Traffic}, \text{Rain}, \text{Umbrella}) =$$
$$P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain}, \text{Traffic})$$

- With assumption of conditional independence:

$$P(\text{Traffic}, \text{Rain}, \text{Umbrella}) =$$
$$P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain})$$

- Bayes' nets / graphical models help us express conditional independence assumptions

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Ghostbusters Chain Rule

- Each sensor depends only on where the ghost is
- That means, the two sensors are conditionally independent, given the ghost position
- T: Top square is red
  B: Bottom square is red
  G: Ghost is in the top
- Givens:
  P(+g) = 0.5
  P(+t | +g) = 0.8
  P(+t | ¬g) = 0.4
  P(+b | +g) = 0.4
  P(+b | ¬g) = 0.8

P(T,B,G) = P(G) P(T|G) P(B|G)

T	B	G	P(T,B,G)
+t	+b	+g	0.16
+t	+b	¬g	0.16
+t	¬b	+g	0.24
+t	¬b	¬g	0.04
¬t	+b	+g	0.04
¬t	+b	¬g	0.24
¬t	¬b	+g	0.06
¬t	¬b	¬g	0.06

Bayes' Nets: Big Picture

- Two problems with using full joint distribution tables as our probabilistic models:
  - Unless there are only a few variables, the joint is WAY too big to represent explicitly
  - Hard to learn (estimate) anything empirically about more than a few variables at a time
- Bayes' nets: a technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)
  - More properly called graphical models
  - We describe how variables locally interact
  - Local interactions chain together to give global, indirect interactions
  - For about 10 min, we'll be vague about how these interactions are specified

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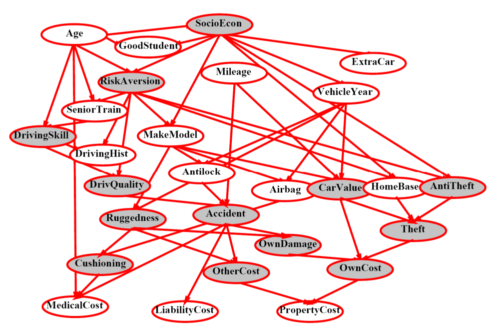
Example Bayes' Net: Insurance



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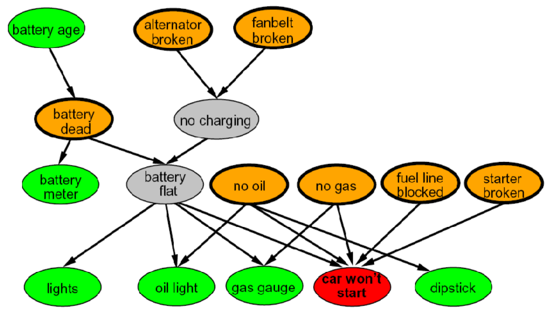
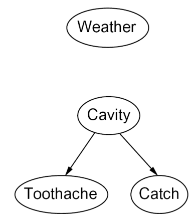
Example Bayes' Net: Car



Graphical Model Notation

- Nodes: variables (with domains)
  - Can be assigned (observed) or unassigned (unobserved)
- Arcs: interactions
  - Similar to CSP constraints
  - Indicate "direct influence" between variables
  - Formally: encode conditional independence (more later)
- For now: imagine that arrows mean direct causation (in general, they don't!)



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Example: Coin Flips

- N independent coin flips

- No interactions between variables: absolute independence

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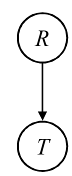
Example: Traffic

- Variables:
  - R: It rains
  - T: There is traffic
- Model 1: independence
- Model 2: rain causes traffic
- Why is an agent using model 2 better?



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Example: Traffic II

- Let's build a causal graphical model
- Variables
  - T: Traffic
  - R: It rains
  - L: Low pressure
  - D: Roof drips
  - B: Ballgame
  - C: Cavity

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Example: Alarm Network

- Variables
  - B: Burglary
  - A: Alarm goes off
  - M: Mary calls
  - J: John calls
  - E: Earthquake!

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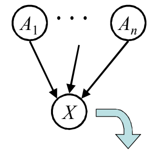
Bayes' Net Semantics

- Let's formalize the semantics of a Bayes' net
- A set of nodes, one per variable X
- A directed, acyclic graph
- A conditional distribution for each node
  - A collection of distributions over X, one for each combination of parents' values

    $$P(X|a_1 \ldots a_n)$$

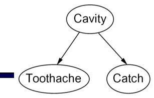
  - CPT: conditional probability table
  - Description of a noisy "causal" process



$$P(X|A_1 \ldots A_n)$$

A Bayes net = Topology (graph) + Local Conditional Probabilities

Probabilities in BNs



- Bayes' nets implicitly encode joint distributions
  - As a product of local conditional distributions
  - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

    $$P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i|parents(X_i))$$

  - Example:

    $$P(+cavity, +catch, \neg toothache)$$

- This lets us reconstruct any entry of the full joint
- Not every BN can represent every joint distribution
  - The topology enforces certain conditional independencies

Example: Coin Flips

$P(X_1)$

h	0.5
t	0.5

$P(X_2)$

h	0.5
t	0.5

$\cdots$

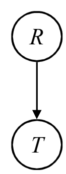
$P(X_n)$

h	0.5
t	0.5

$$P(h, h, t, h) =$$

Only distributions whose variables are absolutely independent can be represented by a Bayes' net with no arcs.

Example: Traffic



$P(R)$

+r	1/4
¬r	3/4

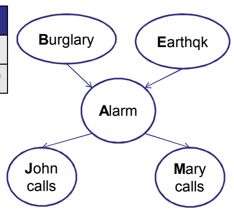
$P(T|R)$

+r	+t	3/4
	¬t	1/4
¬r	+t	1/2
	¬t	1/2

$$P(+r, \neg t) =$$

Example: Alarm Network

B	P(B)
+b	0.001
¬b	0.999



E	P(E)
+e	0.002
¬e	0.998

A	J	P(J|A)
+a	+j	0.9
+a	¬j	0.1
¬a	+j	0.05
¬a	¬j	0.95

A	M	P(M|A)
+a	+m	0.7
+a	¬m	0.3
¬a	+m	0.01
¬a	¬m	0.99

B	E	A	P(A|B,E)
+b	+e	+a	0.95
+b	+e	¬a	0.05
+b	¬e	+a	0.94
+b	¬e	¬a	0.06
¬b	+e	+a	0.29
¬b	+e	¬a	0.71
¬b	¬e	+a	0.001
¬b	¬e	¬a	0.999

Bayes' Nets

- So far: how a Bayes' net encodes a joint distribution
- Next: how to answer queries about that distribution
  - Key idea: conditional independence
  - Last class: assembled BNs using an intuitive notion of conditional independence as causality
  - Today: formalize these ideas
  - Main goal: answer queries about conditional independence and influence
- After that: how to answer numerical queries (inference)