CS 188: Artificial Intelligence
Fall 2010

Lecture 18: Decision Diagrams
10/28/2010

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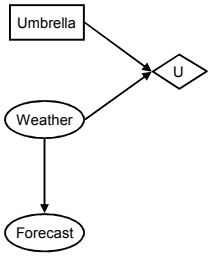
Value of Information

Decision Networks

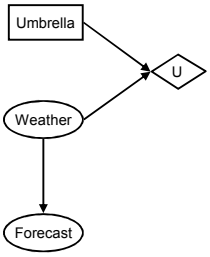
- MEU: choose the action which maximizes the expected utility given the evidence
- Can directly operationalize this with decision networks
  - Bayes nets with nodes for utility and actions
  - Lets us calculate the expected utility for each action
- New node types:
  - Chance nodes (just like BNs)
  - Actions (rectangles, cannot have parents, act as observed evidence)
  - Utility node (diamond, depends on action and chance nodes)



[DEMO: Ghostbusters]

Decision Networks

- Action selection:
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action



Example: Decision Networks

Umbrella = leave

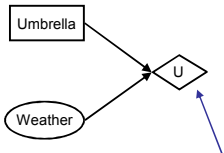
$$\text{EU}(\text{leave}) = \sum_w P(w)U(\text{leave}, w)$$
$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

Umbrella = take

$$\text{EU}(\text{take}) = \sum_w P(w)U(\text{take}, w)$$
$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

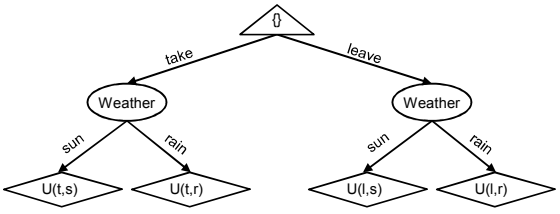
$$\text{MEU}(\emptyset) = \max_a \text{EU}(a) = 70$$



W	P(W)
sun	0.7
rain	0.3

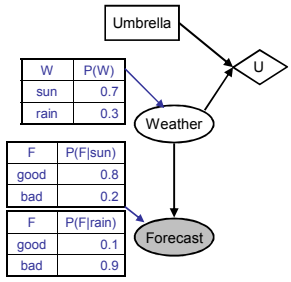
A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decisions as Outcome Trees



- Almost exactly like expectimax / MDPs
- What's changed?

Evidence in Decision Networks



W	P(W)
sun	0.7
rain	0.3

F	P(F|sun)
good	0.8
bad	0.2

F	P(F|rain)
good	0.1
bad	0.9

- Find P(W|F=bad)
  - Select for evidence

W	P(W)
sun	0.7
rain	0.3

$P(W)$

W	P(F=bad|W)
sun	0.2
rain	0.9

$P(bad|W)$

  - First we join P(W) and P(bad|W)
  - Then we normalize

W	P(W,F=bad)
sun	0.14
rain	0.27

$P(W, bad)$

W	P(W | F=bad)
sun	0.34
rain	0.66

$P(W|F = \text{bad})$

Example: Decision Networks

Umbrella = leave

$$\text{EU}(\text{leave}|\text{bad}) = \sum_w P(w|\text{bad})U(\text{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

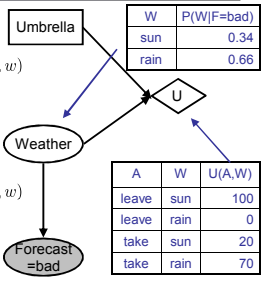
Umbrella = take

$$\text{EU}(\text{take}|\text{bad}) = \sum_w P(w|\text{bad})U(\text{take}, w)$$

$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

Optimal decision = take

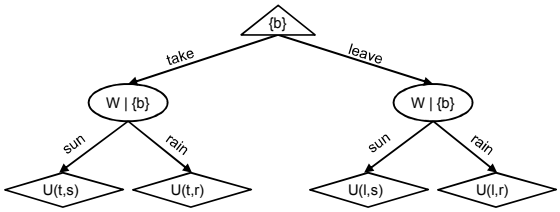
$$\text{MEU}(F = \text{bad}) = \max_a \text{EU}(a|\text{bad}) = 53$$



W	P(W|F=bad)
sun	0.34
rain	0.66

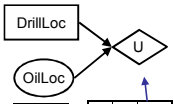
A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decisions as Outcome Trees



Value of Information

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has EU = k/2, MEU = k/2
- Question: what's the value of information of O?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say "oil in a" or "oil in b," prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2



O	P
a	1/2
b	1/2

D	O	U
a	a	k
a	b	0
b	a	0
b	b	k

Value of Information

- Assume we have evidence E=e. Value if we act now:

$$\text{MEU}(e) = \max_a \sum_s P(s|e)\, U(s, a)$$

- Assume we see that E' = e'. Value if we act then:

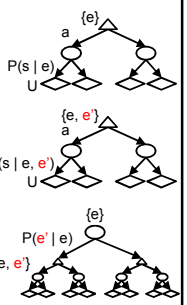
$$\text{MEU}(e, e') = \max_a \sum_s P(s|e, e')\, U(s, a)$$

- BUT E' is a random variable whose value is unknown, so we don't know what e' will be
- Expected value if E' is revealed and then we act:

$$\text{MEU}(e, E') = \sum_{e'} P(e'|e)\text{MEU}(e, e')$$

- Value of information: how much MEU goes up by revealing E' first then acting, over acting now:

$$\text{VPI}(E'|e) = \text{MEU}(e, E') - \text{MEU}(e)$$



VPI Example: Weather

MEU with no evidence

$$\text{MEU}(\emptyset) = \max_a \text{EU}(a) = 70$$

MEU if forecast is bad

$$\text{MEU}(F = \text{bad}) = \max_a \text{EU}(a|\text{bad}) = 53$$

MEU if forecast is good

$$\text{MEU}(F = \text{good}) = \max_a \text{EU}(a|\text{good}) = 95$$

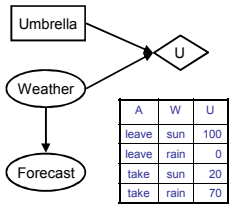
Forecast distribution

F	P(F)
good	0.59
bad	0.41

$$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$

$$77.8 - 70 = 7.8$$

$$\text{VPI}(E|e') = \left(\sum_{e'} P(e'|e)\text{MEU}(e, e')\right) - \text{MEU}(e)$$



A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

VPI Properties

- Nonnegative

$$\forall E', e : \text{VPI}(E'|e) \geq 0$$

- Nonadditive ---consider, e.g., obtaining E_j twice

$$\text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e)$$

- Order-independent

$$\text{VPI}(E_j, E_k|e) = \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j)$$
$$= \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k)$$

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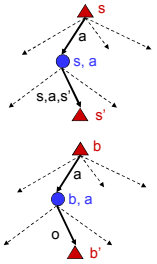
Quick VPI Questions

- The soup of the day is either clam chowder or split pea, but you wouldn't order either one. What's the value of knowing which it is?
- There are two kinds of plastic forks at a picnic. It must be that one is slightly better. What's the value of knowing which?
- You're playing the lottery. The prize will be $0 or $100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?

POMDPs

- MDPs have:
  - States S
  - Actions A
  - Transition fn P(s'|s,a) (or T(s,a,s'))
  - Rewards R(s,a,s')
- POMDPs add:
  - Observations O
  - Observation function P(o|s) (or O(s,o))
- POMDPs are MDPs over belief states b (distributions over S)
- We'll be able to say more in a few lectures



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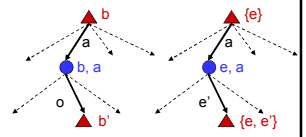
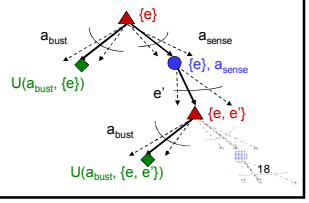
Example: Ghostbusters

- In (static) Ghostbusters:
  - Belief state determined by evidence to date {e}
  - Tree really over evidence sets
  - Probabilistic reasoning needed to predict new evidence given past evidence
- Solving POMDPs
  - One way: use truncated expectimax to compute approximate value of actions
  - What if you only considered busting or one sense followed by a bust?
  - You get a VPI-based agent!





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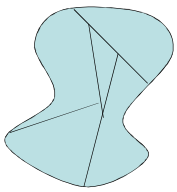
More Generally

- General solutions map belief functions to actions
  - Can divide regions of belief space (set of belief functions) into policy regions (gets complex quickly)
  - Can build approximate policies using discretization methods
  - Can factor belief functions in various ways
- Overall, POMDPs are very (actually PSACE-) hard
- Most real problems are POMDPs, but we can rarely solve then in general!



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