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Stochastic financial analytics for cash flow forecasting

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ABSTRACT

Accurate cash flow forecasting is essential for successful management of firms and it becomes especially critical during uncertain market and credit conditions. Without accurate cash flow forecasting, a firm may fail to meet its short-term obligations and risk bankruptcy. Accurate cash flow forecasting can be limited by a number of factors including changes in macro-economic conditions that influence liquidity in the economy, customer payment behavior that can vary from time to time as well by industry, and dynamics of the particular supply chain itself. We develop stochastic financial analytics for cash flow forecasting for firms by integrating two models: (1) Markov chain model of the aggregate payment behavior across all customers of the firm using accounts receivable aging and; (2) Bayesian model of individual customer payment behavior at the individual invoice level. As the stochastic dynamics of cash flow evolves every day, the forecast can be updated every time an invoice is paid. The proposed model is back-tested using empirical data from a small manufacturing firm and found to differ 3–6% from actual monthly cash flow, and differs approximately 2–4% compared to actual annual cash flow. The forecast accuracy of the proposed stochastic financial analytics model is found to be considerably superior to other techniques commonly used. Furthermore, in computer simulation experiments, the proposed model is found to be largely robust to supply chain dynamics, including when subjected to severe bullwhip effect. The proposed model has been implemented in Excel, which allows it to be easily integrated with the accounts receivable aging data, making it practicable for small and large firms.

1. Introduction

In order to keep businesses running with sufficient working capital and to manage cash flow efficiently, an accurate cash flow forecast is critical. Smaller firms usually incurs higher financing cost due to its credit risk and resource constraints (Baas and Schrooten, 2006). On the other hand, larger firms may find it is difficult to manage its financial assets when it moves to another stage of business life cycle (Mcmahon, 2001). Essentially, cash is the main bloodline of all firms. A firm without profit may be able to survive for a while, but without cash a firm can become insolvent, and risks bankruptcy. Cash flow forecast can serve different purposes such as treasury management, working capital financing, and business valuation.

Besides the aforementioned purposes, the cash flow forecast may be used for various strategic purposes such as controlling a group of subsidiary companies, and for general management (WWCP, 2012). For example, a firm may use variance between the actual cash flow and forecasted cash flow to diagnose underlying problems and respond in a timely manner.

Two widely used techniques for a short term cash flow forecast are the receipts and disbursements forecast technique, and a statistical technique. The former technique is to determine all expected cash inflows and cash outflows over the forecast period whereas the latter technique can be a bit more sophisticated in that it considers historical trends. Such statistical techniques include, for example, simple moving average, exponential smoothing, regression analysis, and distribution model (WWCP, 2012). Many commercial software packages for enterprise resource planning (ERP) and treasury management system (TMS) use these statistical techniques for cash flow forecast.

Several research studies have focused on developing and improving cash flow forecasting techniques. In 1950s, the idea of treating cash as products in inventory management was used to forecast and optimize the cash position (Baumol, 1952; Whitin, 1953). However, the assumption of predetermined cash flow in this body of work may not be realistic in many practical situations. Hence, another model was developed to maintain the cash position and minimize transaction fee (Miller and Orr, 1966). Several techniques have also been developed with a specific focus on construction industry (Bromilow and Henderson, 1977; Evans and

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Kaka, 1998; Hudson, 1978; Kenley and Wilson, 1986; Khosrowshahi, 1991; Miskawi, 1989; Singh and Woon, 1984; Skitmore, 1992, 1998).

Accounts receivable (AR) aging is a report classifying the length of time since invoices have been sent to various customers. This report is a part of an accounting analytics routinely used by many companies to identify irregular payments and closely monitor overdue accounts. A typical AR aging report consists of many customer accounts in the report and each different customer may have different payment behavior. AR aging and related data can be used in many ways for cash flow forecast. The pioneering efforts of Cyert et al. (1962) used Markov Chain for estimating the allowance for doubtful accounts. This was further improved by incorporating exponential smoothing to AR aging for forecasting cash flow (Corcoran, 1978; Cyert et al., 1962). Later, Kuelen et al. (1981) modified the model and improved the accuracy of the forecast by changing the total balance aging to determine probabilities of the next payments by using the accounts receivable aging (Kuelen et al., 1981). These two techniques outperform other common practices such as moving average and exponential smoothing techniques which are still widely used in practice (Beattie, 2011; WWCP, 2012). Another key development in this area is to model cash flow as a stochastic process to predict cash on-hand for short-term financial planning (Pate-Cornell, 1986; Pate-Cornell et al., 1990).

In general, cash flow can be viewed as a stochastic process which is sequence of random variables that depend upon a number of factors including macro-economic conditions that influence liquidity in the economy, customer payment behavior that can vary from time to time as well by the industry, and dynamics of the particular supply chain itself. For example, one of the prominent and widely studied dynamics of supply chain is the bullwhip effect in inventory. Bullwhip effect in inventory is an undesirable phenomenon in forecast-driven distribution channels where the variance of orders from downstream supply chain gets amplified as it propagates upstream as shown in Fig. 1 (Baganha and Cohen 1998; Kahn, 1987; Lee et al., 1997b, 2004; Metters, 1997). Adverse impacts of the bullwhip effect can result in excessive inventory, stock-outs, back-orders, production swing, and low utilization of distribution channels. The majority of past research of the bullwhip effect concentrated on key factors which cause such phenomenon and explanation of its existence (Burbidge, 1971; Lee et al., 1997a; Mason-Jones and Towill, 2000; Sterman, 1989). Such key factors are disorganization, lack of communication, order batching, and price variations. Interest in this area was shifted to its impacts and techniques to reduce it (Chen et al., 1998; Lee et al., 1997a). Several approaches were developed to mathematically quantify the bullwhip effect (Chen et al., 2000; Fiorolli and Fogliatto, 2008; Kim et al., 2006). Tangsucheeva and Prabhu (2013) studied the impact of the inventory bullwhip effect on the corresponding cash flow bullwhip (CFB) in a supply chain as shown in the lower graph of Fig. 1 (Tangsucheeva and Prabhu, 2013). It needs to be emphasized that the cash flow of a firm not only depends on its immediate customers but potentially also on the system dynamics of its supply chain.

There is a need for cash flow models and analytics that more fully utilize the available financial data to improve the accuracy of cash flow forecasts. In this paper, we focus on improving the accuracy of cash flow forecasting technique by modeling individual customer payment behavior for determining payment probability of individual invoices by using historic AR aging data. This individual customer level stochastic model is implemented in Excel, which is the tool used by about 70% of companies for accounting and cash flow forecasts (Fuchs, 2011). This development potentially provides users a practical and convenient forecasting tool without having to dwell on the intricacies cash flow forecasting techniques.

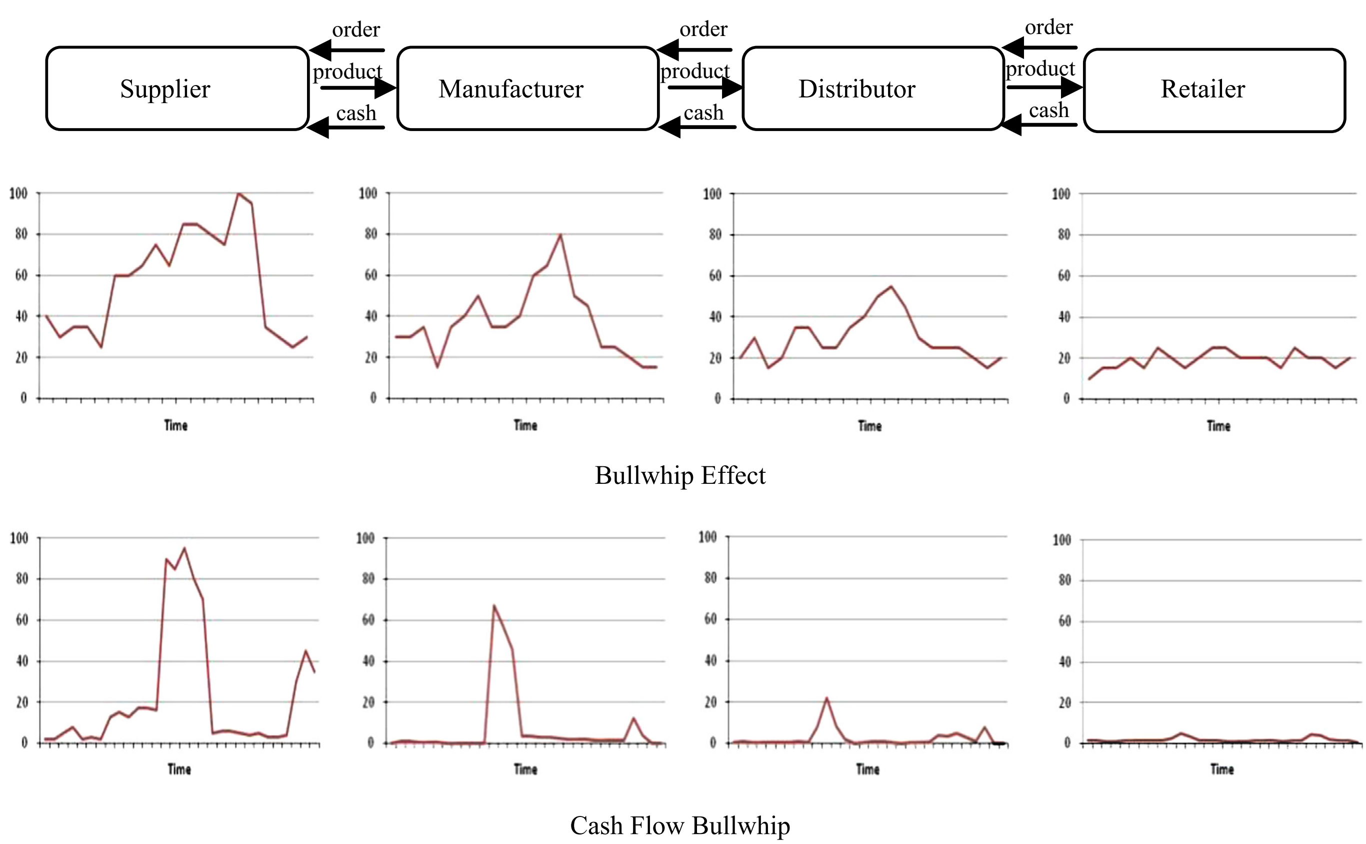

Fig. 1. Bullwhip effect and cash flow bullwhip in the supply chain.

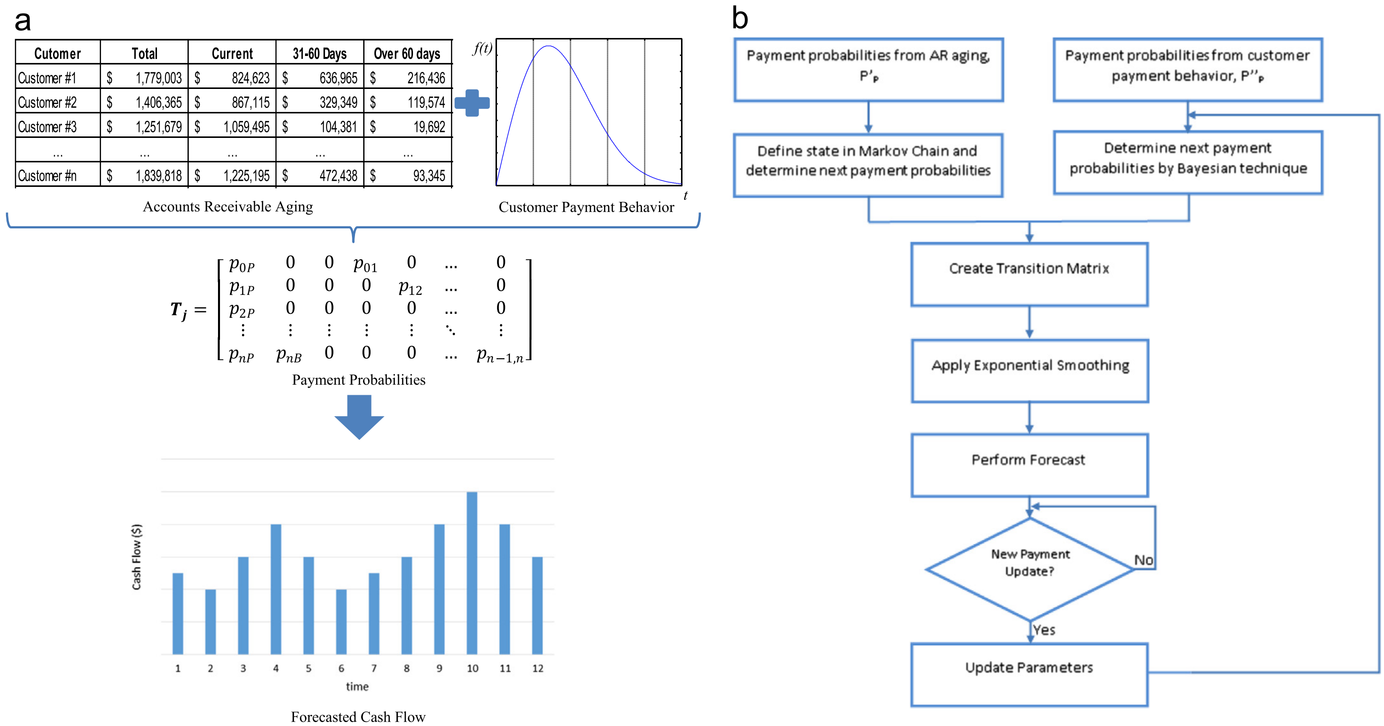

Fig. 2. (a) Model concept and (b) process of the proposed cash flow forecast.

The rest of the paper is organized as follows. The next section presents the stochastic financial analytics model for cash flow forecasting. In Section 3, the proposed model is applied and back-tested for a small manufacturing firm. In Section 4, the proposed model is applied and tested for a 4-tier supply chain using computer simulation models. Additionally, Section 4 also investigates the impact of the bullwhip effect on the cash flow forecasting models. Lastly, Section 5 provides conclusions and directions for future research.

Table 1
State definition.

AR aging	State
0–30 days	0
31–60 days	1
61–90 days	2
…	…
Last AR aging range	n

2. Stochastic financial analytics model

In this section we propose a stochastic analytics model by building on the work done by Corcoran (1978), Cyert et al. (1962) and Pate-Cornell et al. (1990). Based on this model we then suggest a computational algorithm for cash flow forecast.

Since the accounts receivable aging is one of the accounting routines, it is quite convenient to use the data from the accounts receivable aging to perform the cash flow forecast. However, the forecasting accuracy of the model could be improved by explicitly modeling the individual customer payment behavior as the concept of the proposed model shown in Fig. 2(a). Fig. 2(b) illustrates the process of the proposed cash flow forecasting model. It consists of the following six steps: (1) determine payment probabilities for the next period from AR aging, $\boldsymbol{P}'_P$, by estimating the transition probabilities of the Markov chain, (2) determine payment probabilities from customer specific payment behavior, $\boldsymbol{P}''_P$, using a Bayesian model, (3) create a transition matrix, (4) apply exponential smoothing, (5) perform the cash flow forecast, and (6) update the new actual data.

The first step is to define the Markov chain states in order to prepare a transition matrix to calculate the cash flow forecast. To formulate the Markov Chain, the transient states and the absorbing states are defined from the accounts receivable aging. Let $S_t=\{0, 1, 2, \ldots, n\}$ be a set of transient states where 0, 1, 2, and n represent the AR aging states in the Markov Chain. Typically in practice there are 4 states corresponding to 0–30 days, 31–60 days, 61–90 days, and over 120 days, as summarized in Table 1 below.

Additionally, two absorbing states are defined as $S_a=\{P, B\}$ where P and B represent *Paid* and *Bad Debt* states. An absorbing state is a Markov chain state which can be reached from any state. However, once it enters the absorbing state, it cannot leave this state. Table 2 shows an example of the accounts receivable aging and defined states of the Markov chain in parenthesis.

The accounts receivable aging in Table 2 can be converted into the accounts receivable aging matrix ($\boldsymbol{R}$).

$$\boldsymbol{R} = \begin{bmatrix} r_{10} & r_{11} & r_{12} & \cdots & r_{1n} & r_{1B} \\ r_{20} & r_{21} & r_{22} & \cdots & r_{2n} & r_{2B} \\ r_{30} & r_{31} & r_{32} & \cdots & r_{3n} & r_{3B} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{j0} & r_{j1} & r_{j2} & \cdots & r_{jn} & r_{jB} \end{bmatrix} \quad (1)$$

where $r_{j,i}$ is the amount of the accounts receivable aging in period j at *State* i and $r_{j,B}$ is the amount of bad debt in period j.

Fig. 3 shows the state transition diagram and the notation for corresponding transition probabilities. The process starts from *State* 0, indicating the current charges (0–30 days) in the AR aging, and transitions to *State P* if the invoice is *Paid* while in this state; or transitions to *State* 1 (31–60 days) if the invoice is not paid. The probability of an invoice getting paid in State 0 is modeled as the transition probability from *State* 0 to *State P* and denoted by p_{0P}.

Table 2
Accounts receivable aging and bad debt.

Month	Total	Accounts receivable aging (state i)					
(period j)		Current (0)	30 days (1)	60 days (2)	90 days (3)	Over 120 days (4)	Bad debt (B)
November (11)	1,609,405	702,560	623,810	101,470	36,480	127,085	16,270
December (12)	2,655,895	1,418,250	530,080	528,400	75,295	103,870	12,328
January (1)	2,287,070	896,140	505,465	669,020	76,330	140,115	4556
February (2)	2,109,595	829,995	586,160	450,745	66,365	107,700	13,590

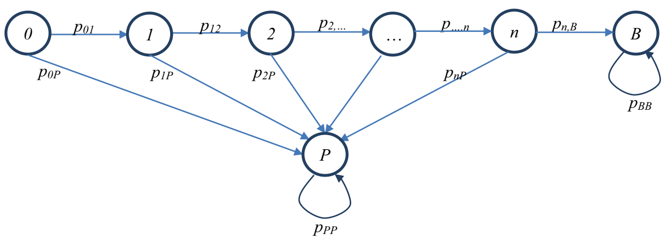

Fig. 3. Markov chain state diagram.

Therefore probability from *State* 0 to *State* 1 can be modeled as $p_{01}=1-p_{0P}$. The process continues to the last state n where it can move to either *State P* (*Paid*) or *B* (*Bad Debt*). Since P and B are the absorbing states, $p_{PP}=1$ and $p_{BB}=1$ in this model.

Then determine the remaining $2(n+1)$ payment probabilities. Let $\boldsymbol{P_P}$ be the $(n+1)\times 2$ matrix of the payment probability which is the transition probability from any transient state n to *State P*.

$$\boldsymbol{P_P}=\beta\boldsymbol{P'_P}+(1-\beta)\boldsymbol{P''_P} \tag{2}$$

where β is a weighting parameter, $\boldsymbol{P'_P}$ is the payment probability matrix from the AR aging and $\boldsymbol{P''_P}$ is the payment probability matrix from the customer payment behavior. Here the payment probability matrix $\boldsymbol{P_P}$ is modeled to consist of two parts: the first part, $\boldsymbol{P'_P}$ models the aggregate payment behavior across all customers of the firm in the recent past. This can be expected to model any macro-economic trend that has influenced payment behavior across customers and other trends across the industry. The second part, $\boldsymbol{P''_P}$ models payment behavior of a specific customer at the individual invoice level based on all know payment history of the customer.

The weighting parameter β can be obtained from back testing using historic data of individual customers. The value of β is selected to provide the most accurate forecasting result. Furthermore, β may vary from customer to customer, and over time. For a new customer, when there is no payment history, β would be set to unity thereby treating the customer as a “typical” customer. However, for a longstanding customer, β could be smaller thereby increasing the weight for customer-specific payment behavior. The proposed model in Eq. (2) can be viewed as a convex combination of Corcoran's model and Pate-Cornell et al.'s model. If $\beta=1$, Eq. (2) becomes Corcoran's model and if $\beta=0$, it becomes Pate-Cornell et al.'s model.

$\boldsymbol{P'_P}$ can be determined from the changes in the accounts receivable aging from the previous period to the current period (Corcoran, 1978).

$$\boldsymbol{P'_P}=\begin{bmatrix} p'_{0P} & 0 \\ p'_{1P} & 0 \\ \vdots & \vdots \\ p'_{nP} & p'_{nB} \end{bmatrix} \tag{3}$$

where

$$p'_{iP}=(r_{j,i}-r_{j+1,i+1})/r_{j,i} \tag{4}$$

$$p'_{nB}=r_{j,B}/r_{j-1,n} \tag{5}$$

where p'_{iP} is the payment probability from *State i* to *State P (Paid)*, p'_{nB} is the transition probability from *State n* to *State B (Bad Debt)*, $r_{j,i}$, $r_{j+1,i+1}$, $r_{j,B}$, and $r_{j-1,n}$ are the elements from the accounts receivable aging matrix ($\boldsymbol{R}$) in Eq. (1).

$\boldsymbol{P'_P}$ models aggregate behavior across all customers of the firm, however, it does not include customer specific payment behavior. Therefore, the second step, taking these factors from distinct customers into account by modeling $\boldsymbol{P''_P}$ can be expected to improve the forecasting accuracy. Assume that information of the customer payment behavior such as the number of days that the invoices have been sent out to customers and the minimum number of days that individual customer makes a payment are available.

$$\boldsymbol{P''_P}=\begin{bmatrix} p''_{0P} & 0 \\ p''_{1P} & 0 \\ \vdots & \vdots \\ p''_{nP} & 0 \end{bmatrix} \tag{6}$$

To compute the probability of a given invoice that will be paid between time t_0 and $t_0+\Delta t$, the distribution of the payment time of each customer can be modeled conveniently as Weibull distribution (Pate-Cornell, 1986; Pate-Cornell et al., 1990).

Fig. 4 shows Weibull distribution with the scale parameter $\lambda=1$ and the shape parameter k=1, 2, and 5. Notice that for the shape parameter k=2, this particular distribution provides a linearly increasing rate which rises to a peak quickly and then decreases over time. This model of individual customer payment behavior can be used to characterize payment lead-time that is used to set payment terms, probability of payment delay beyond the expected lead-time, and probability of bad debt. In this paper, individual customer is modeled using a Weibull distribution with the shape parameter $k=2$

$$f(x;\lambda,\ k)=\begin{cases} \frac{2x}{\lambda^2}e^{-(x/\lambda)^2}, & x\geq 0 \\ 0, & x<0 \end{cases} \tag{7}$$

where $\lambda>0$ is the scale parameter.

The probability that an invoice will be paid during time $t_0+\Delta t$ can be written as

$$\begin{aligned} p''_{iP} &= p[t\leq t_0+\Delta t \mid t\geq t_0] \\ &= \frac{p[t_0\leq\ t\leq t_0+\Delta t]}{p[t\geq t_0]} \\ &= \frac{[F_j(t_0+\Delta t-t_b)-F_j(t_0-t_b)]}{[1-F_j(t_0-t_b)]} \end{aligned}$$

where p''_{iP} is the payment probability from the customer payment behavior of *State i*, t_b is the time a given invoice is billed to the customer, and $F_j(.)$ is the cumulative distribution for the payment time of the accounts receivable. From the cumulative

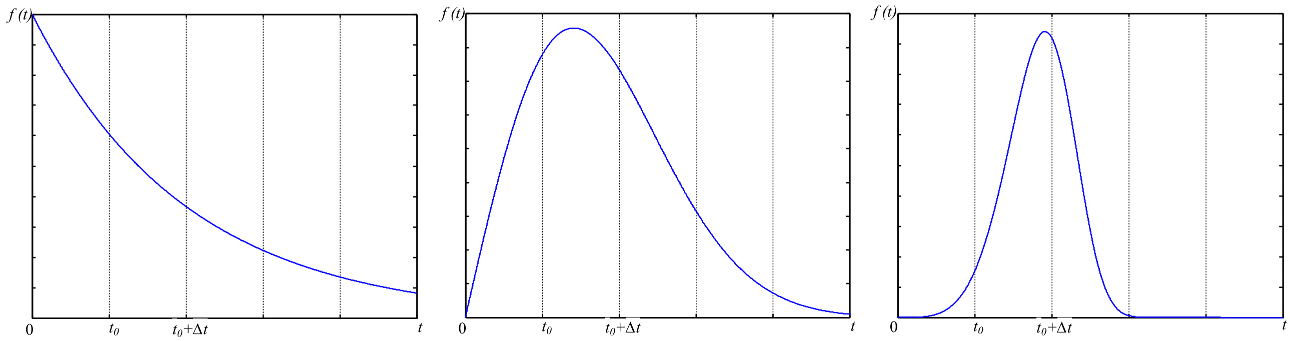

Fig. 4. Weibull distribution with the shape parameter $k=1$, 2, and 5.

distribution function, we can determine three possible cases of the payment probability.

Case I: $t_0-t_b\geq\gamma$

$$p''_{iP}=1-\exp\left\{-\frac{[2(t_0-t_b-\gamma)(\Delta t)+(\Delta t)^2]}{\hat{\lambda}_j^2}\right\}\tag{8}$$

Case II: $t_0-t_b\leq\gamma$ and $t_0-t_b+\Delta t\geq\gamma$

$$p''_{iP}=1-\exp\left\{-\frac{[(t_0-t_b+\Delta t-\gamma)^2]}{\hat{\lambda}_j^2}\right\}\tag{9}$$

Case III: $t_0-t_b+\Delta t\leq\gamma$

$$p''_{iP}=p[t\leq t_0+\Delta t|t\geq t_0,\lambda]=0\tag{10}$$

where γ is the minimum payment time and $\widehat{\lambda}_j$ is the estimate scale parameter which characterizes customer's payment lead time (Pate-Cornell et al., 1990). These model parameters can be estimated periodically or even after invoice payment.

Once the payment probabilities $\boldsymbol{P'_P}$ and $\boldsymbol{P''_P}$ are obtained, $\boldsymbol{P_P}$ can be determined by Eq. (2) and then $\boldsymbol{P_D}$, the $(n+1)$ square matrix of the delay payment probability, can be determined by

$$\boldsymbol{P_D}=\begin{bmatrix}0 & p_{01} & 0 & \dots & 0\\ 0 & 0 & p_{12} & \dots & 0\\ 0 & 0 & 0 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \dots & p_{n-1,n}\end{bmatrix}$$

where

$$p_{i,i+1}=1-p_{iP}\tag{11}$$

$p_{i,i+1}$ is the probability of the amount that will age from *State i* to *State* $i+1$ and p_{iP} is the payment probability from matrix $\boldsymbol{P_P}$. Now it is ready to construct a transition matrix for period j, $\boldsymbol{T_j}$, from the probabilities in step 1 and step 2 as shown in Eq. (12).

$$\boldsymbol{T_j}=\begin{bmatrix}\boldsymbol{P_P} & \boldsymbol{P_D}\end{bmatrix}$$

$$\boldsymbol{T_j}=\begin{bmatrix}p_{0P} & 0 & 0 & p_{01} & 0 & \dots & 0\\ p_{1P} & 0 & 0 & 0 & p_{12} & \dots & 0\\ p_{2P} & 0 & 0 & 0 & 0 & \dots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ p_{nP} & p_{nB} & 0 & 0 & 0 & \dots & p_{n-1,n}\end{bmatrix}\tag{12}$$

Then, we use an exponential smoothing technique to smooth the data and then perform the forecast of the next period cash flow. The formula of the exponential smoothing technique can be written as

$$\overline{\boldsymbol{A_j}}=\alpha\boldsymbol{T_j}+(1-\alpha)\overline{\boldsymbol{A_{j-1}}}\tag{13}$$

where $\overline{\boldsymbol{A_j}}$ is the estimated transition matrix or exponentially smoothed matrix for period j, α is the smoothing factor, $\boldsymbol{T_j}$ is the transition matrix for period j.

The smoothing factor α can be identified by back-testing historic data of a firm to provide the most accurate forecast.

Then, the cash inflow forecast and bad debt can be obtained by (Corcoran, 1978; Tangsucheeva et al., 2013)

$$F_{j+1}=R_j\overline{\boldsymbol{A_j}}$$

$$F_{j+1}=\begin{bmatrix}r_{j0} & r_{j1} & r_{j2} & \dots & r_{jn}\end{bmatrix}\begin{bmatrix}p_{0P} & 0 & 0 & p_{01} & 0 & \dots & 0\\ p_{1P} & 0 & 0 & 0 & p_{12} & \dots & 0\\ p_{2P} & 0 & 0 & 0 & 0 & \dots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ p_{nP} & p_{nB} & 0 & 0 & 0 & \dots & p_{n-1,n}\end{bmatrix}\tag{14}$$

where F_{j+1} is the forecast vector of dollar amount in each state for period $j+1$, R_j is the vector of actual accounts receivable aging report in period j from matrix $\boldsymbol{R}$.

The final step checks the updated actual payment in order to update the payment probabilities and then repeat Step 2 through 6 to dynamically update the cash flow forecast. This individual customer payment forecast is repeated for all customers with outstanding invoices to compute the firm level forecast.

3. Enterprise level application

To illustrate the application of the proposed model, we use AR aging data from a small manufacturing firm in Pennsylvania, which is shown in Table 2 along with states of the Markov chain in parenthesis. For the sake of simplicity, assume that the firm needs a close attention to a particular customer. Based on the past payment history in 5 months of this customer, the average payment time is 25 days and the minimum payment time that this customer pays his invoice is 20 days. This individual customer is modeled by fitting the corresponding AR aging data to identify the Weibull distribution parameters. By using Minitab, given the lowest AD (Anderson–Darling Statistic) $=0.304$ and the significant LRT P value $=0.000$, Weibull distribution with a shape parameter $k=2$ is selected.

We follow the process presented in Section 2. First, define the Markov chain state from the accounts receivable aging as shown in Table 2 and then convert it to a matrix $\boldsymbol{R}$ as shown below.

$$\boldsymbol{R}=\begin{bmatrix}702,560 & 623,810 & 101,470 & 36,480 & 127,085 & 16,270\\ 1,418,250 & 530,080 & 528,400 & 75,295 & 103,870 & 12,328\\ 896,140 & 505,465 & 669,020 & 76,330 & 140,115 & 4,556\\ 829,995 & 586,160 & 450,745 & 66,365 & 107,700 & 13,590\end{bmatrix}$$

Then, determine the first part of the probabilities of the next payment, $\boldsymbol{P}'_{\boldsymbol{P}}$, using Eqs. (3) and (4).

$$p'_{iP} = (r_{j,i} - r_{j+1,\, i+1})/r_{j,i}$$

$$p'_{0P} = (r_{1,0} - r_{2,\,1})/r_{1,0} = (896,140 - 586,160)/896,140 = 0.3459$$

Continue in the same manner to determine the rest of the payment probabilities, the transition probabilities of *State i* to *State P* (*Paid*), for $i=0$, 1, 2, 3, 4. Use Eq. (5) to determine the probability of bad debt.

$$p'_{nB} = r_{j,B}/r_{j-1,n}$$

$$p'_{4B} = r_{2,B}/r_{1,4} = 13,590/140,115 = 0.0970$$

To determine the second part of Eq. (2), $\boldsymbol{P}''_{\boldsymbol{P}}$, the scale parameter λ of the Weibull distribution must be determined first. Let X be the random variable of the payment time in the Weibull distribution. The mean of this distribution for the shape parameter $k=2$ can be determined by

$$\overline{X}(n) = \gamma + \lambda\left(\frac{\sqrt{\pi}}{2}\right)$$

Hence, an estimate of scale parameter λ can be obtained as follows:

$$\hat{\lambda}(n) = \frac{2(\overline{X}(n) - \gamma)}{\sqrt{\pi}}$$

where $\hat{\lambda}(n)$ is an estimate of scale parameter λ, $\overline{X}(n)$ is an average of the n past observations of the payment time of the customer, and γ is the minimum payment time. Based on the given information, the scale parameter of the Weibull distribution is

$$\hat{\lambda}(n) = \frac{2(25-20)}{\sqrt{\pi}} = 5.6419$$

Given that the invoice was sent out to the customer 27 days ago and $\Delta t = 3$ days, $t_0 - t_b = 27$ days $\geq \gamma$ which is 20 days, the probability of receiving the payment from this customer is determined by Eq. (8) as follows:

$$p''_{iP} = 1 - \exp\left\{-\frac{[2(t_0 - t_b - \gamma)(\Delta t) + (\Delta t)^2]}{\hat{\lambda}_j^2}\right\}; \quad t_0 - t_b \geq \gamma$$

$$p''_{0P} = 1 - \exp\left\{-\frac{[2(27-20)(3) + (3)^2]}{5.6419^2}\right\} = 0.7986$$

Hence, the payment probability from *State* 0 to *State P* and the probability of the invoice will be postponed to the next aging can be obtained from Eq. (2) and Eq. (11), respectively.

$$p_{iP} = \beta p'_{iP} + (1-\beta) p''_{iP}$$

$$p_{0P} = 0.2 \times 0.3459 + 0.8 \times 0.7986 = 0.7081$$

$$p_{i,i+1} = 1 - p_{iP}$$

$$p_{01} = 1 - p_{0P} = 1 - 0.7081 = 0.2919$$

The weighting parameter $\beta = 0.2$ is obtained from back testing using historic data of a firm. Once p_{0P} and p_{01} are obtained, next, repeat the processes to determine probabilities for *State* 1, 2, 3, and 4. Once we got all probabilities from all accounts receivable aging of this customer, we can construct the transition matrix $\boldsymbol{T}$. Fig. 5 illustrates an example of transition matrix $\boldsymbol{T}_2$ (for month 2, February), which contains probabilities of cash inflows at each state transition.

Then, the exponential smoothing technique is applied to smooth the data. Assume the exponentially smoothed matrix for January $\overline{\boldsymbol{A}_1}$ is known, the exponentially smoothed matrix for February $\overline{\boldsymbol{A}_2}$ can be determined by Eq. (13). The alpha value of 0.8 is back tested from historic data in order to calculate $\overline{\boldsymbol{A}_2} = 0.8\boldsymbol{T}_2 + (0.2)\overline{\boldsymbol{A}_1}$. Once, the matrix $\overline{\boldsymbol{A}_2}$ is obtained, the estimated vector of dollar amount for March can be determined by $F_3 = R_2\overline{\boldsymbol{A}_2}$, using Eq. (14).

The estimated vector F_3 forecasts that the firm will have \$1,580,481 collected cash from the accounts receivable and \$8358 in bad debt whereas the actual cash collection and bad debt are \$1,540,900 and \$6275, respectively. Fig. 6 illustrates matrix $\overline{\boldsymbol{A}_1}$, matrix $\overline{\boldsymbol{A}_2}$, and the forecasted cash collection as well as bad debt. The difference of the cash collection between the forecast and the actual value of the whole year is 2.31% whereas the difference of the bad debt is 14.79%. (We do not have information of detailed bad debt, so, we will not focus on the bad debt.)

Fig. 7 shows relationship among forecasting accuracy, the smoothing factor, and the weighting parameter β. α and β are back tested from historic data: AR aging and customer specific payment behavior. The selected values are chosen from the ones that provide the most accurate average forecasting accuracy, which in this example, $\alpha = 0.8$ and $\beta = 0.2$.

Fig. 8 shows the sensitivity analysis of how the forecasting accuracy changes over α and β values for this specific customer. First observation is the highest forecasting accuracy of each quarter is in the range of α between 0.7 and 0.9 and in the range of β between 0.1 and 0.3. Since these parameters are quite sensitive to customer payment behavior, for the best of α and β value, they can be varied from industry to industry, customer to customer, and time to time. If the past data shows that α and β do not change with time then these parameters can be estimated

$$\boldsymbol{T}_2 = \begin{bmatrix} 0.7081 & 0 & 0 & 0.2919 & 0 & 0 & 0 \\ 0.8211 & 0 & 0 & 0 & 0.1789 & 0 & 0 \\ 0.9801 & 0 & 0 & 0 & 0 & 0.0199 & 0 \\ 0.8800 & 0 & 0 & 0 & 0 & 0 & 0.1200 \\ 0.8922 & 0.0970 & 0 & 0 & 0 & 0 & 0.0108 \end{bmatrix}$$

Fig. 5. Transition matrix $\boldsymbol{T}_2$.

$$\overline{\boldsymbol{A}_1} = \begin{bmatrix} 0.6010 & 0 & 0 & 0.3984 & 0 & 0 & 0 \\ 0.6638 & 0 & 0 & 0 & 0.3362 & 0 & 0 \\ 0.5892 & 0 & 0 & 0 & 0 & 0.4107 & 0 \\ 0.4400 & 0 & 0 & 0 & 0 & 0 & 0.5600 \\ 0.5439 & 0.0000 & 0 & 0 & 0 & 0 & 0.4561 \end{bmatrix}$$

$$\overline{\boldsymbol{A}_2} = \begin{bmatrix} 0.6867 & 0 & 0 & 0.3133 & 0 & 0 & 0 \\ 0.7896 & 0 & 0 & 0 & 0.2104 & 0 & 0 \\ 0.9019 & 0 & 0 & 0 & 0 & 0.0980 & 0 \\ 0.7920 & 0 & 0 & 0 & 0 & 0 & 0.2080 \\ 0.8225 & 0.0776 & 0 & 0 & 0 & 0 & 0.0998 \end{bmatrix}$$

$$F_3 = [1{,}580{,}481 \quad 8{,}358 \quad 0 \quad 260{,}054 \quad 123{,}305 \quad 44{,}209 \quad 24{,}559]$$

Fig. 6. Exponentially smoothed matrix $\overline{\boldsymbol{A}_1}$ and $\overline{\boldsymbol{A}_2}$, and forecasted vector F_3.

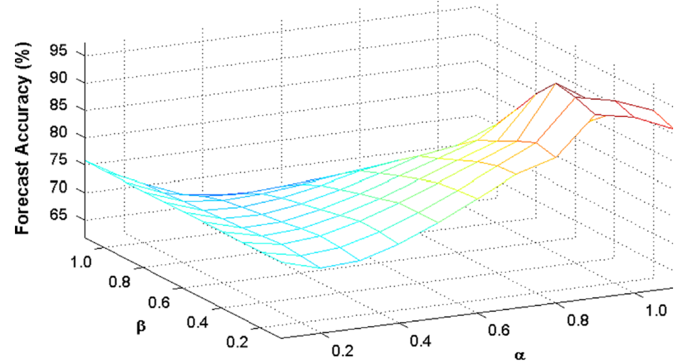

Fig. 7. Forecasting accuracy and β and α value.

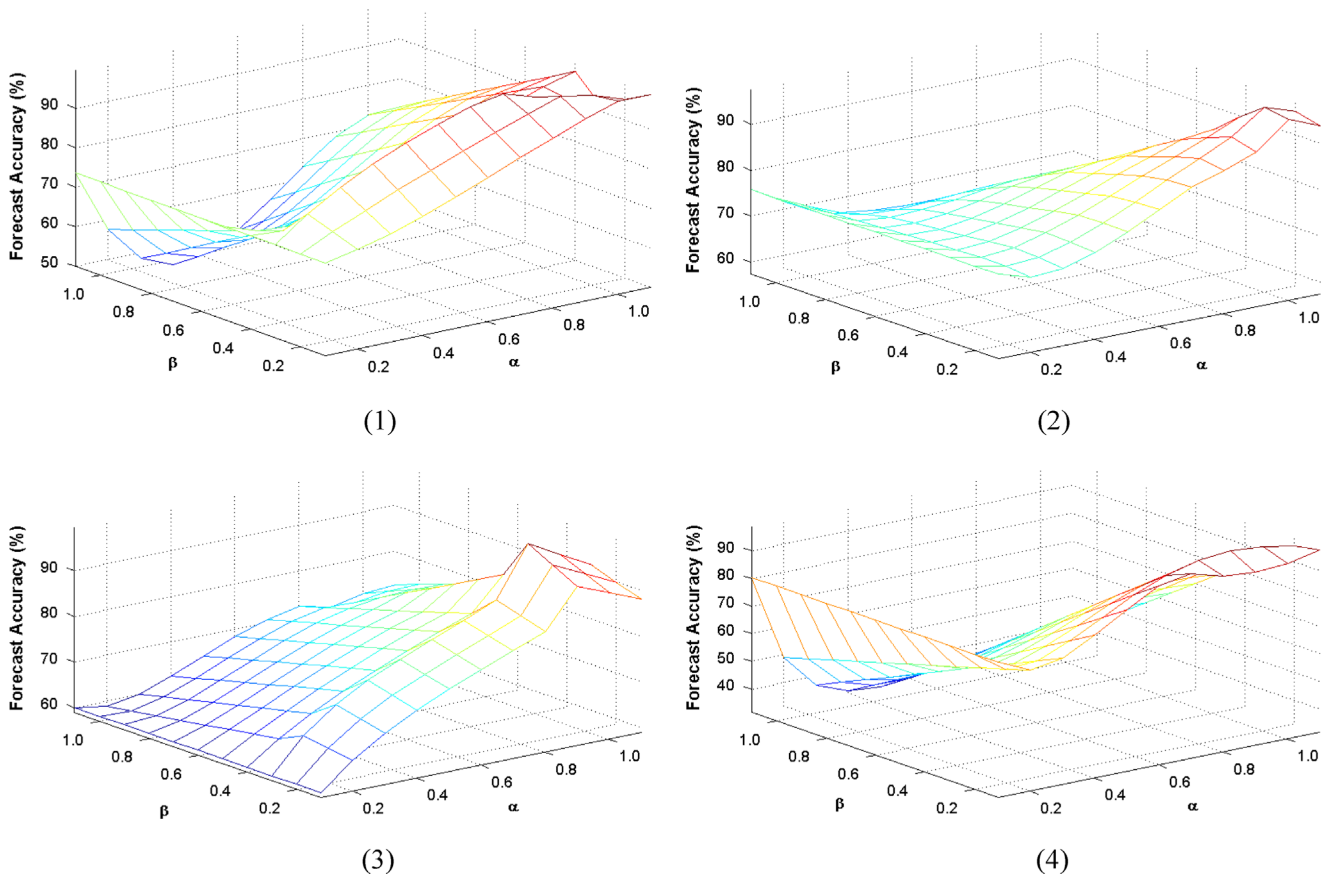

Fig. 8. Sensitivity analysis of α and β for quarter 1, 2, 3, and 4.

Table 3
Cash flow and forecasting difference of customer #1.

Month	Actual cash flow ($)	Proposed model		Corcoran's model		Pate-Cornell's model		Moving average		Exponential smooth	
		Forecast ($)	Difference (%)	Forecast ($)	Difference (%)	Forecast ($)	Difference (%)	Forecast ($)	Difference (%)	Forecast ($)	Difference (%)
1	21,547	21,922	1.74	22,920	6.37	24,890	15.52	31,604	46.67	31,604	46.67
2	21,145	21,045	0.47	25,244	19.39	21,679	2.53	26,576	25.68	23,558	11.41
3	50,890	50,616	0.54	36,974	27.35	45,018	11.54	24,765	51.34	21,627	57.50
4	56,767	59,468	4.76	60,752	7.02	67,564	19.02	31,194	45.05	45,037	20.66
5	43,881	43,506	0.85	50,029	14.01	54,883	25.07	42,934	2.16	54,421	24.02
6	44,776	43,634	2.55	48,354	7.99	42,664	4.72	50,512	12.81	45,989	2.71

infrequently. Second, α value is the exponential smoothing factor of the forecasting. The larger of the α is, the more weight on the most recent data is used to calculate the forecast. When $\alpha \to 1$ provides high forecasting accuracy, it implies that the payment behavior of this customer changes significantly from its past payment. Therefore, using the outdated data or the further past data may decrease the forecasting accuracy. Third, for this customer, the forecasting accuracy is higher when β value decreases. This implies that the payment behavior of this customer is quite different from those of other customers. In other words, when $\beta \to 0$, the payment behavior of this customer is distinct from prevailing industry practice. Since the proposed forecasting model is a convex combination of Corcoran's model and Pate-Cornell et al.'s model, when $\beta \to 0$, the proposed forecasting model relies more on Pate-Cornell et al.'s model which incorporates customer specific payment behavior. On the other hand, when $\beta \to 1$, the proposed model relies more on Corcoran's model which represents the aggregate customer behavior.

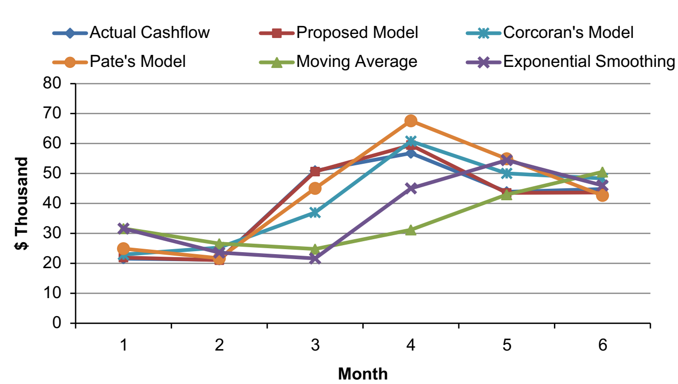

Fig. 9. Cash flow from actual cash flow and various forecasting models.

Let us consider another customer, customer #1, to see how the proposed model performs the forecasting. This customer makes approximately 31 transactions on average per month. The average payment time of this customer is 73 days and the minimum payment time is 48 days. Table 3 shows the actual cash flow compared with the forecasted cash flow, and the percent difference between the actual and the forecasted cash flow determined by five models: (1) the proposed model, (2) Corcoran's model to forecast cash flow from the accounts receivable aging, (3) Pate-Cornell's model, (4) the moving average technique, and (5) the exponential smoothing technique. For the proposed model, we extracted the accounts receivable aging of this customer and followed the process mentioned in Section 2 to forecast the cash flow corresponding to this customer. For Corcoran's model and Pate-Cornell's model, please see detailed explanation in Corcoran (1978) and Pate-Cornell et al. (1990), respectively. For the moving average technique, we use 3 periods moving average to estimate the cash flow. Finally, for the exponential smoothing technique, the smoothing factor is assigned to 0.8.

Fig. 9 illustrates how a customer's actual cash flow changes during month 1 through month 6 compared to 5 other cash flow forecasting models. This figure shows how well the forecasting models can keep track of the actual cash flow. While the cash flows forecasted by the moving average technique and the exponential smoothing technique have some time lag to follow the change of the actual cash flow, Corcoran's model and Pate's model do not have this issue. Thus, they can keep track the actual cash flow better than the former two models. However, they still cannot keep track of the sudden change since the model determines the payment probability based on only previous payment regardless of customer payment behavior. Consequently, when the next payment drastically changes, the estimation from Corcoran's model and Pate's model have big gaps. On the other hand, the proposed model takes the customer payment behavior into account. It considers the average payment time and the minimum payment time of the customer to calculate the payment probability. Hence, the proposed model is able to keep track of the actual cash flow as if they are the same line.

Fig. 10 depicts percent difference between the actual cash flow and the forecasting models. As shown in the figure, the proposed model has the smallest difference among five models followed by Pate's model and Corcoran's model.

Figs. 11 and 12 show forecasting performance of each forecasting model on an average monthly basis and average annually basis, respectively. In Fig. 11, the proposed model performs the best with approximately 3% difference on average while the difference from Corcoran's model and Pate's model are almost 15% on average, and the differences from the moving average and the exponential smoothing technique are approximately 30% on average. The percent difference of the last two models is relatively a lot larger than that of the proposed model since these two models use only the average values from the historical data to smooth out fluctuation. This is why these two models cannot capture the change of the actual cash flow in a timely manner.

The accumulated differences shown in Fig. 12 are, of course, smaller than those in Fig. 11 since the overestimate and underestimate are offset. Still the results shown in Fig. 12 are consistent to those shown in Fig. 11, the proposed model still performs the best among these five methods.

Next, the experiment was conducted to five sample customers over 12 months, who have different payment behavior, to see how the forecasting models perform in different scenarios. In this case, the customers have different number of days in accounts receivable, average payment time, and minimum payment time. Customer #1 represents customers who have an average payment time longer than 60 days with $\alpha = 0.8$ and $\beta = 0.2$, customer #2 and customer #3 represent customers who have an average payment time approximately between 45 and 60 days with $\alpha = 0.6$ and $\beta = 0.3$, customer #4 and customer #5 represent customers who have an average payment time approximately 31–50 days and less than 30 days with $\alpha = 0.9$ and $\beta = 0.1$, respectively.

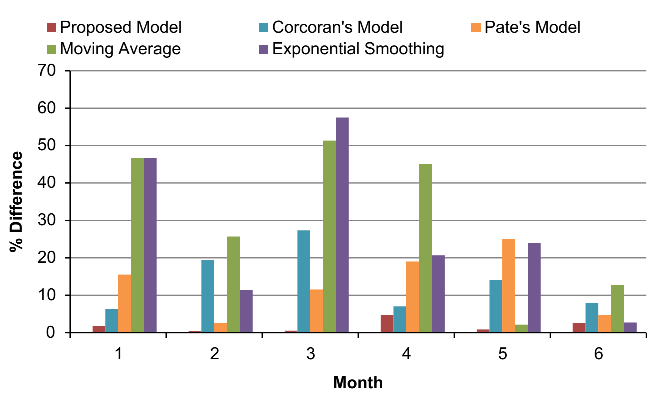

Fig. 10. Percent difference of forecasting models.

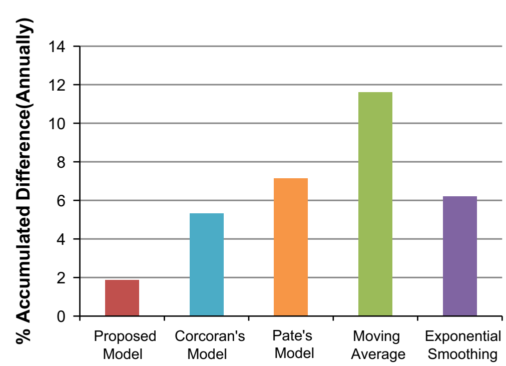

Fig. 12. Accumulated percent annually difference.

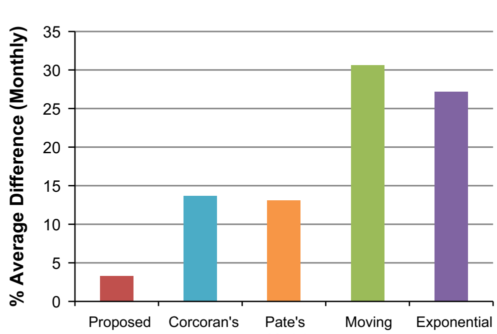

Fig. 11. Average percent monthly difference.

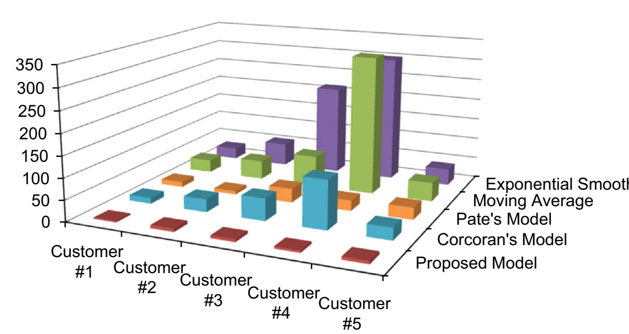

Fig. 13. Forecasting differences (monthly).

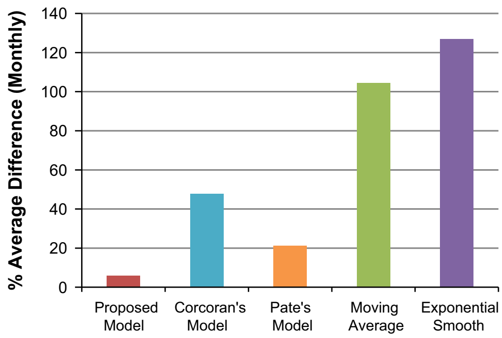

Fig. 14. Average forecasting difference (monthly).

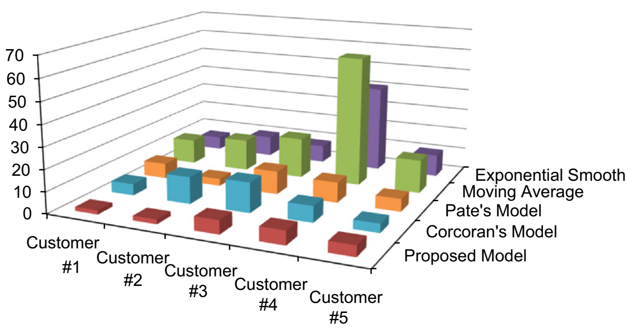

Fig. 15. Forecasting difference (annually).

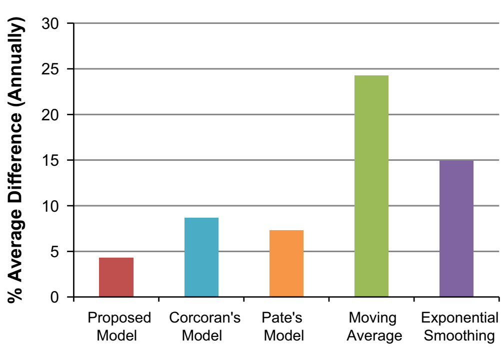

Fig. 16. Average forecasting difference (annually).

Fig. 13 shows the differences of the five forecasting models compared to the actual cash flow of each customer. The proposed model still performs the best by providing the smallest difference among five forecasting models. The percent difference of the proposed model on average from five customers is approximately 6% while the percent differences of the other models are much greater as shown in Fig. 14. The proposed model is able to adjust the parameters relating to the payment behavior so that it can manage to keep the gap of the difference small compared to other models, which do not have this parameters involve in the forecast.

Notice that the difference of the moving average and the exponential smoothing techniques of customer #3 and customer #4 are large because there is so large fluctuation at some points in time that these two models create big gaps between the actual cash flow and the forecasted cash flow.

Figs. 15 and 16 show the performance of the five forecasting techniques in aggregated level (annually). The results here are still consistent with the results of the monthly basis shown in Figs. 13 and 14. The forecasting models perform better because of the aggregated amount offset. The proposed model performs the best among five methods with an average of approximately 4% different from the actual cash flow while Corcoran's model has a difference of approximately 9%, Pate's model has a difference approximately 7%, the moving average technique has a difference of approximately 25%, and the exponential smoothing has a difference of approximately 15%.

4. Supply chain level application

This section discusses how the proposed model performs in the supply chain and how it is impacted by one of the most common problems in supply chain, the bullwhip effect. The following section defines the bullwhip effect (BWE), the inventory bullwhip (IBW), and the cash flow bullwhip (CFB), respectively.

4.1. Bullwhip effect (BWE)

One of the bullwhip effect models that is widely accepted was developed by Chen et al. (2000). The model starts with a simple supply chain, which contains a single manufacturer and a single retailer. The customer demand is in the form of (Kahn, 1987)

$$D_t = d + \rho D_{t-1} + \mu_t \tag{15}$$

where d is a nonnegative constant, ρ is a correlation parameter satisfying $|\rho| < 1$, and μ_t is an independent and identically normally distributed random variable with zero mean and variance σ^2. Assume the lead time L is fixed and the order-up-to policy is applied; hence, the order quantity (q_t) can be written as

$$q_t = y_t - y_{t-1} + D_{t-1} \tag{16}$$

where y_t is the order-up-to point which is estimated from the observed demand as

$$y_t = \hat{D}_t^L + z\hat{\sigma}_{e,t}^L \tag{17}$$

where $\hat{D}_t^L$ is an estimate of the mean lead time demand using a simple moving average, z is a constant to meet a desired service level, and $\hat{\sigma}_{e,t}^L$ is an estimate of the standard deviation of the L period forecast error. The bullwhip effect of a simple supply chain can be obtained from the following (Chen et al., 2000):

$$\frac{Var(q)}{Var(D)} \geq 1 + \left(\frac{2L}{p} + \frac{2L^2}{p^2}\right)(1-\rho^p) \tag{18}$$

The bound is tight when $z=0$.

More detail of this model is discussed in Chen et al. (2000).

4.2. Inventory bullwhip (IBW)

From the previous section, the inventory bullwhip model is extended to see how the bullwhip effect impacts the inventory. For a simple supply chain containing a single retailer and a single manufacturer, the lower bound of the inventory bullwhip can be written as (Tangsucheeva and Prabhu, 2013)

$$\frac{Var(I)}{Var(D)} \geq \frac{Var(q)}{Var(D)} + f(L,p,\rho) \qquad 19)$$

where I is inventory level. The bound is tight when $z=0$.

$$f(L,p,\rho) = \frac{1}{(1-\rho)^2}\begin{bmatrix} \left(\frac{L}{p}\right)^2[2\rho^{p+2} - 2\rho^{p+1} + 2\rho^p - (p+2)\rho^2 + 2\rho + p - 2] \\ -2\left(\frac{L}{p}\right)[\rho^{L+p+2} - \rho^{(L+2)} - \rho^{(p+2)} + \rho^{p+1} - \rho^p + \rho^2 - \rho + 1] \\ +(2\rho^{L+2} - (L+2)\rho^2 + L) \end{bmatrix} \tag{20}$$

More detail of this model has been discussed elsewhere in Tangsucheeva and Prabhu (2013).

4.3. Cash flow bullwhip (CFB)

The model above is further extended for the cash flow bullwhip in order to determine the effect of the bullwhip effect and the inventory bullwhip to the cash flow (Tangsucheeva and Prabhu, 2013). CFB is the variance of the cash conversion cycle (CCC) over the variance of demand. CCC is the most widely used measurement for cash flow. It contains inventory (days inventory outstanding), cash inflow (days sales outstanding), and cash outflow (days payable outstanding) and this is the reason why CCC is used to represent the cash flow.

CFB for a simple supply chain can be obtained from the following formulae (Tangsucheeva and Prabhu, 2013):

Case I: $D \le I$

$$\frac{Var(CCC)}{Var(D)} \cong \frac{(365)^2}{E(D)^2}\left[\left(\frac{s}{c}\right)^2\left(\frac{Var(q)}{Var(D)}+f(L,p,\rho)+\frac{E(I)^2}{E(D)^2}\right)+\left(\frac{Var(q)}{Var(D)}+\frac{E(q)^2}{E(D)^2}\right)\right] \tag{21}$$

Case II: $D > I$

$$\frac{Var(CCC)}{Var(D)} \cong \frac{(365)^2}{E(D)^2}\left[\left(\left(\frac{s}{c}\right)^2+1\right)\left(\frac{Var(q)}{Var(D)}+f(L,p,\rho)+\frac{E(I)^2}{E(D)^2}\right)+\left(\frac{Var(q)}{Var(D)}+\frac{E(q)^2}{E(D)^2}\right)\right] \tag{22}$$

where s is the sales price per unit, c is the unit cost, $E(D)$ is the expected value of demand, and $E(I)$ is the expected value of inventory.

Further information regarding to this model is presented in Tangsucheeva and Prabhu (2013).

4.4. Simulation model

A supply chain simulation is one of a powerful and widely used tool for supply chain analysis. Particularly, in case the analytical model cannot provide a clear analysis or stochastic nature exists in the supply chain. Therefore, in this study, simulation experiments are developed for the purpose of analyzing the cash flow forecasting accuracy at different stages of supply chain and at different intensity of bullwhip effect.

4.4.1. Simulation structure and process

A simulation model of a multi-stage supply chain contains a single supplier, three manufacturers, three distributors, and three retailers. Its structure and product flow of the supply chain are shown in Fig. 17 while its cash flow is in the reverse direction of the product flow. Originally, the simulation model was created for a simple supply chain which contains only a single member in each stage and then more members were added into the model for more practical illustration. The supply chain members in each stage can be adjusted later for more proper use in each supply chain.

A simulation starts from a customer demand obtained from Eq. (15) with a 0.5 demand correlation. The customer demand is produced for 72 months including a 12 months warm up period for each input variable and it is generated separately for each retailer in order to represent three retail stores that have different customer demands. Assume that the initial inventory is twice as much of the customer demand. Each retailer fills the customer demand and periodically observes its inventory level. Excessive demand is backordered and will be filled first when the inventory is replenished. By the end of the observation period t, it places an order to the distributor to replenish its inventory level. The order quantity follows Eq. (16). A fixed lead time L is applied, thus, the order will be received at the start of period $t+L$ and then the process repeats. The supply chain process of the four members – retailers, distributors, manufacturers, and a supplier – are treated in a similar way except that each manufacturer receives orders from three distributors. The process continues like this through the upstream members. The simulation runs for 2160 days (72 months including the warm up period) for each different bullwhip effect value (from BWE=1 through BWE=20) to observe the inventory bullwhip, the cash flow bullwhip, and the cash flow forecast error at all different stages in supply chain. The simulation model focuses on the material flow and the cash flow among supply chain members.

In order to obtain preliminary insights, some known sources of variability are reduced. Therefore, in this simulation, all supply chain members apply the same moving average forecasting method and the same order-up-to inventory policy as well as the same lead time throughout the supply chain. Additional assumption is that the accounts payable and the accounts receivable are 80% of total purchases and sales each period with a 30 days payment term, respectively. However, not all accounts receivable is collected within the pre-determine payment term. The uncollectible amount over 120 days in the accounts receivable aging is written off as a bad debt.

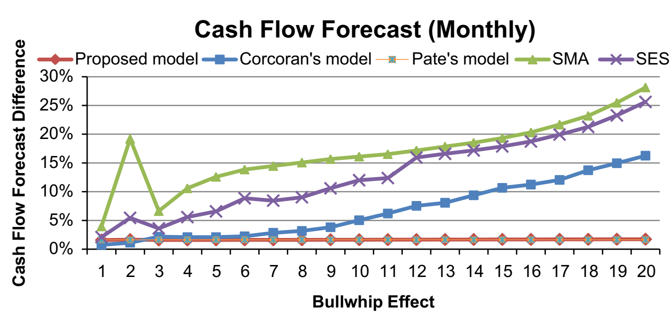

Fig. 18. Comparison among five techniques at a manufacturer stage.

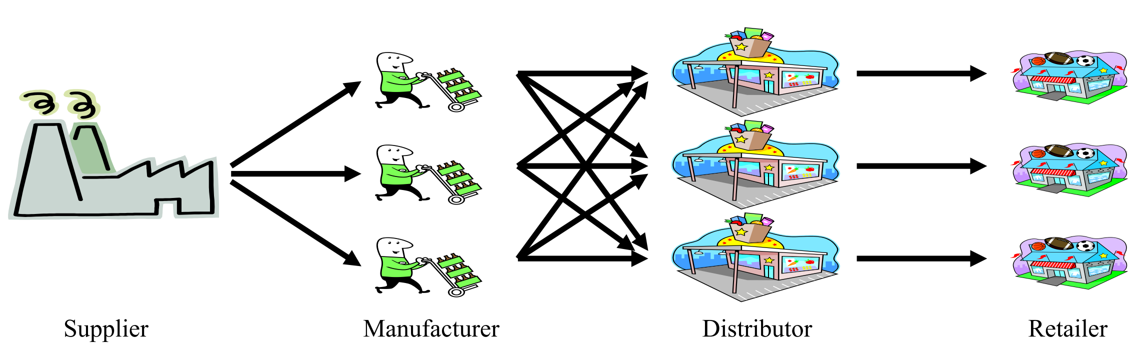

Fig. 17. Structure of a supply chain in this simulation.

4.4.2. Simulation tool

This simulation model is developed by Visual Basic for Applications (VBA) in Microsoft Excel 2007. The three main reasons why we use the VBA in Excel is that, firstly, the data regarding to the inventory and the cash flow is mostly in the tabular form or spreadsheet. Therefore, input and output variables in this simulation are easy to manage in Excel. Secondly, Excel provides adequate capabilities for the required computations, statistical analysis, and chart generation. Lastly, Excel is a very common program used in many companies. Therefore, it is very practical to develop the simulation in Excel. Our simulation model has a friendly user interface, which is easy to use and configure.

Tangsucheeva and Prabhu (2013) shows that an increase in the bullwhip effect leads to an increase in the inventory bullwhip (IBW) and the cash flow bullwhip (CFB). Adverse impacts from these effects include inventory, cost, and cash flow management difficulty. These effects also propagate upstream in supply chain the same way as the bullwhip effect. In other words, the impact of these phenomenon is worse in upstream members, particularly the supplier (Tangsucheeva and Prabhu, 2013).

In this section, five cash flow forecasting models are compared among others on the increase of the bullwhip effect to see how they are impacted as shown in Fig. 18. This experiment was conducted in the simulation model, which contains a supplier, three manufacturers, three distributors, and three retailers as mentioned in Section 4.4.1.

As the bullwhip effect increases, gaps of the differences between the actual cash flow and the forecasted cash flow also increases leading to worse accuracy, except the proposed model. Fig. 18 shows that the result from the proposed model with $\alpha=0.8$ and $\beta=0.2$ and the Pate's model are largely independent of the bullwhip effect. The gap of the difference between the actual cash flow and the forecasted cash flow remains approximately the same. On the other hand, the other three models perform the cash flow forecast based heavily on the historical data only. Therefore, when the data contains large fluctuation, especially when the bullwhip effect is large, it significantly impairs the forecasting accuracy. This is the reason why Corcoran's model, the simple moving average technique (SMA), and the simple exponential smoothing technique (SES), have their cash flow forecasting differences increase.

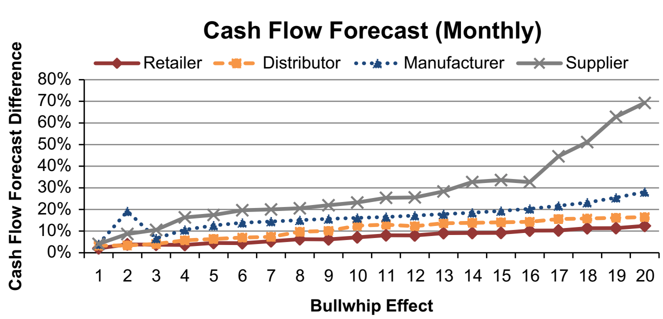

Fig. 19. Cash flow forecast difference at different stages of supply chain (moving average).

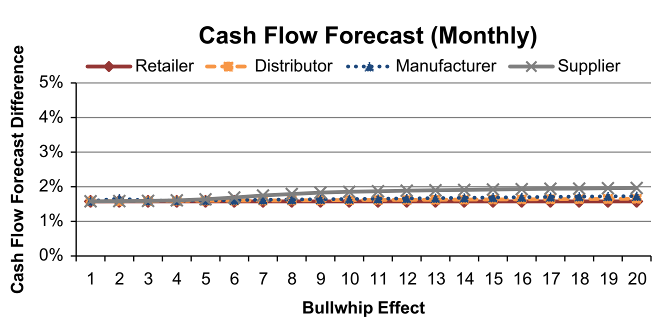

Fig. 20. Cash flow forecast difference at different stages of supply chain (proposed model).

Figs. 19 and 20 break down the impact of the bullwhip effect on the cash flow forecast to each supply chain level and compare the performance of the forecasting techniques between the two models, the moving average and the proposed model with $\alpha=0.8$ and $\beta=0.2$. To be easily distinguishable, the moving average technique is selected to compare with the proposed model since it performs the worst among four models. As shown in Fig. 19, the gaps of the forecasting differences from the moving average technique increase when the bullwhip increases and this inaccuracy continues to amplify when moving upstream toward the supplier. In the worst case scenario, when the bullwhip effect is very high (BW=20), the cash flow forecast of the supplier may approximately result in 70% deviate from the actual cash flow. This effect can cause the supplier a huge amount of interest expense, working capital, opportunity cost, and so forth, meanwhile the forecasted cash flow from the proposed model remains about the same, under 2% for all supply chain levels.

5. Conclusions

The proposed cash flow forecasting model was developed to assist a firm to have accurate results and be convenient to use since the model obtains input data from the accounts receivable aging, which is one of the accounting routine the firm already has. The model is based on the combination of the work done by Cyert et al., Corcoran, and Pate-Cornell et al. The proposed cash flow forecasting for firms is developed by integrating two models: (1) Markov chain model of the aggregate payment behavior across all customers of the firm using accounts receivable aging and; (2) Bayesian model of individual customer specific payment behavior at the individual invoice level.

Actual data from a small manufacturing firm in Pennsylvania was used as the empirical study to evaluate the performance of the forecasting models. The experimental results were generated by the calculation from the analytical models and from a series of simulations. The overall performances of the proposed model were compared with the Corcoran's model and other two common practice models, the moving average and the exponential smoothing techniques, which are widely used in small and medium companies.

In summary, the proposed cash flow forecasting model was demonstrated to be a simple and most accurate forecasting technique for the firm in comparison with other models. The proposed model performs the best with approximately 3–6% different from the actual cash flow and approximately 2–4% difference on an aggregate level. For the supply chain level application, the proposed model was shown to be independent of the bullwhip effect. The forecasting accuracy of the proposed model is high and seems to be largely robust to supply chain dynamics, including when subjected to severe bullwhip effect whereas those of other models become worse when the bullwhip effect is severe. Potential key impact of accurate forecast is efficient management of working capital. By reducing the forecast error from 20% (using simple moving average) to 2% we can reduce the cost of running a business, especially for suppliers who are likely to be SME. In addition, the proposed model was developed in an Excel spreadsheet format which links to the accounts receivable aging. This development provides users a practical and convenient forecasting tool to use even for a person who is not familiar with cash flow forecast.

Regarding the exponential smoothing factor α and the weighting parameter β, these two parameters are back tested from the payment historic data and selected from best values which provide the highest forecasting accuracy. We found that these parameters are quite sensitive and can vary from industry to industry, customer to customer, and time to time. Since the proposed forecasting model is a convex combination of Corcoran's model and Pate-Cornell et al.'s model, when $\beta \to 0$, the proposed forecasting model relies more on Pate-Cornell et al.'s model which incorporates customer specific payment behavior. This implies that the payment behavior of this specific customer is different from the others. A firm needs to pay close attention to any customer that has $\beta \to 0$ because of its distinct from prevailing industry practice, especially when it is an important customer. On the other hand, when $\beta \to 1$, the proposed model relies more on Corcoran's model which represents more aggregate customer behavior. The future direction of this study could be dedicated to the testing of the depth of historic data, how far back of the data we should go, exploit all data or just look one season back in order to determine the weighting parameter β.

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