Chapter 4: Directional & Horizontal Drilling

Well Trajectory Design

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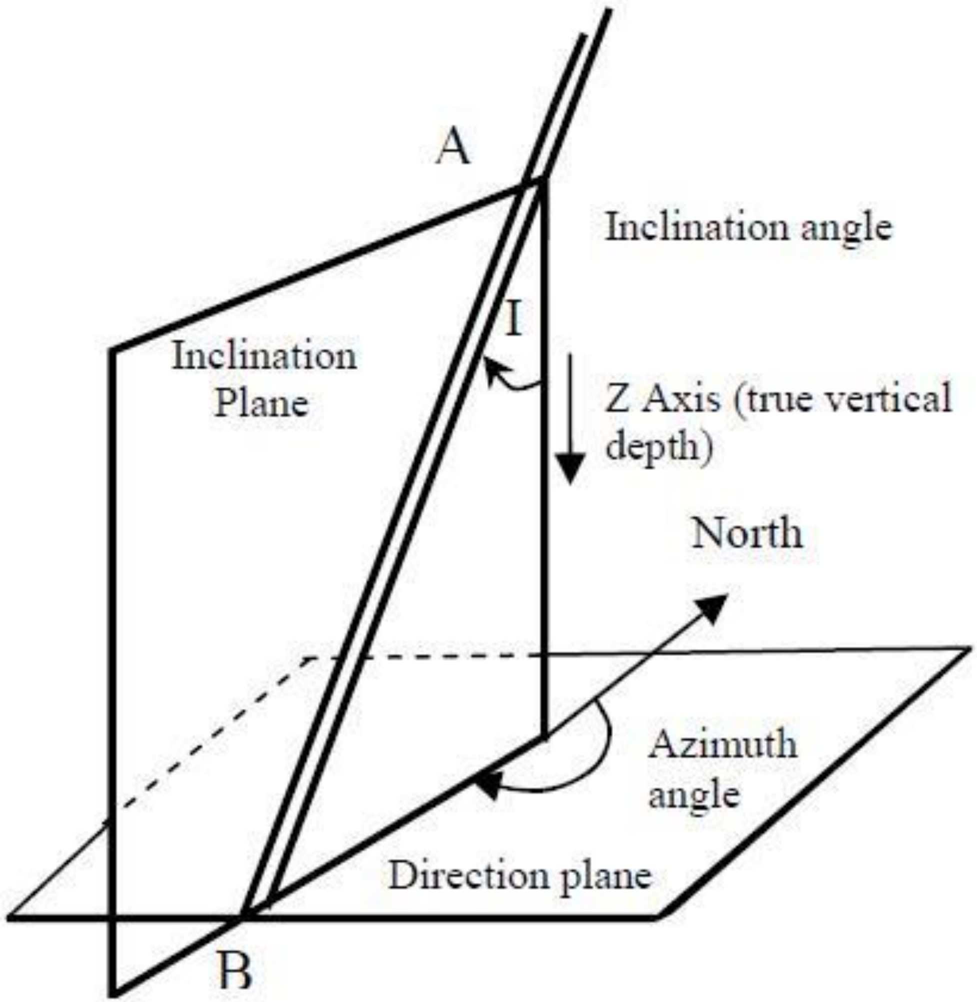
Basic Concepts



Basic Concepts

The distance from rotary table to Point B as measured along the wellbore is called a measured depth.

The vertical distance from rotary table to Point B is called a true vertical depth

The inclination angle ‘I’ is the angle between vertical and the wellbore.

The direction angle ‘A’ is specified as the azimuth between the geographic north and the projection of a wellbore on a horizontal plane.

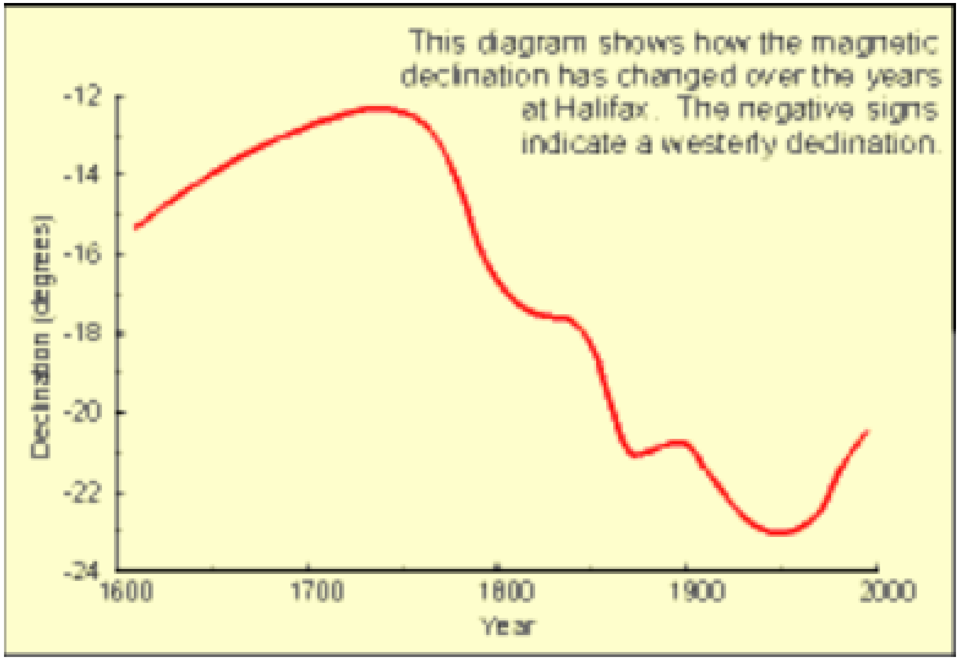
Basic Concepts

Azimuth

- Complex fluid motion in the outer core causes the earth.'s magnetic field to change slowly and unpredictably with time (secular variation)

.• The position of the magnetic poles also change with time .

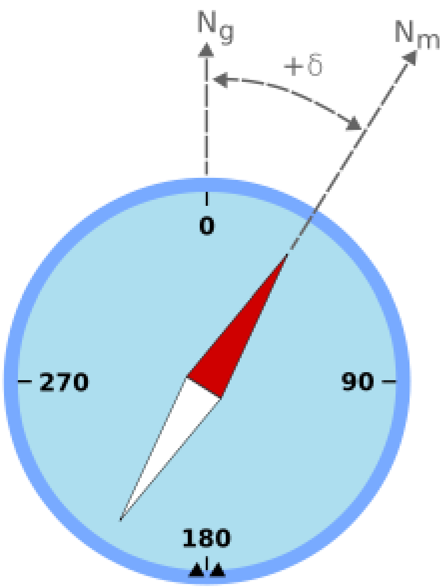
- However, we are able to compensate for this variability by applying a correction (declination) to a magnetic survey which references it to true north



Basic Concepts

Azimuth

The angle between geographic (true) and magnetic north (MN) is called the declination angle. A location where the measurements are taken is called a station. At each station, a measured depth, hole angle, and azimuth are recorded.

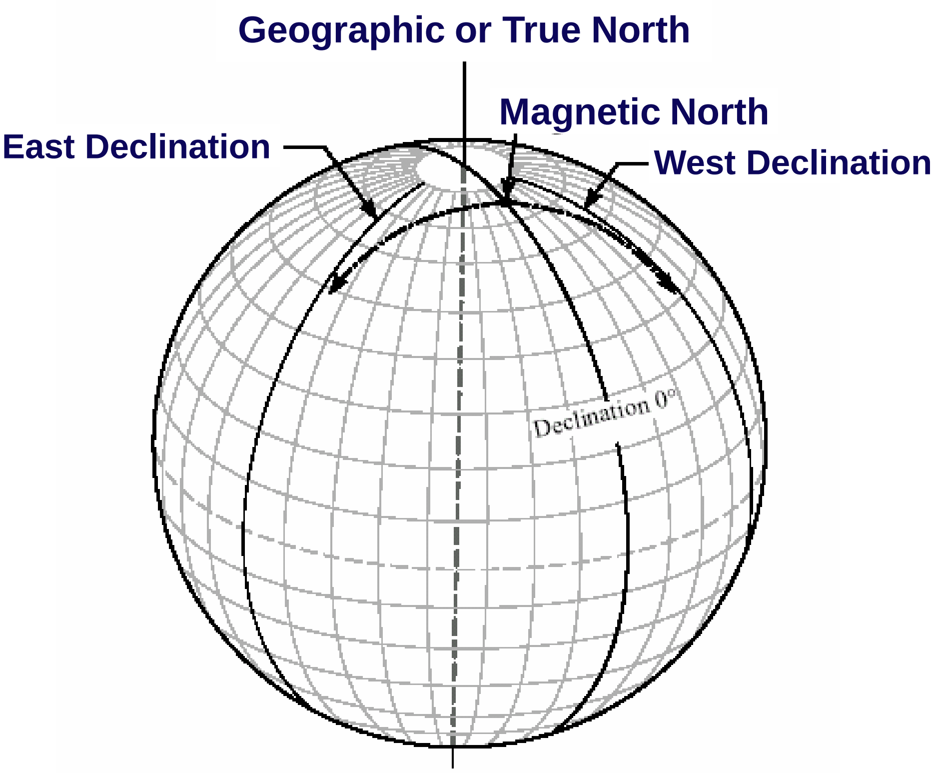


Example of magnetic declination showing a compass needle with a "positive“ variation from geographic north.

Basic Concepts

Azimuth

- True north, or geographic north, is aligned with the spin axis of the Earth
- True north does not move making it a perfect reference
- A survey referenced to true north will be valid today and at any time in the future
- The correction we apply to change a magnetic north direction to a true northdirection is called declination.



Basic Concepts

Azimuth

Applying Declination

- To convert from Magnetic North to True North, Declination must be added:

True Direction = Magnetic Direction + Declination

Important Note:

- East Declination is Positive & West Declination is Negative in both the northern and southern hemispheres

Basic Concepts

Azimuth

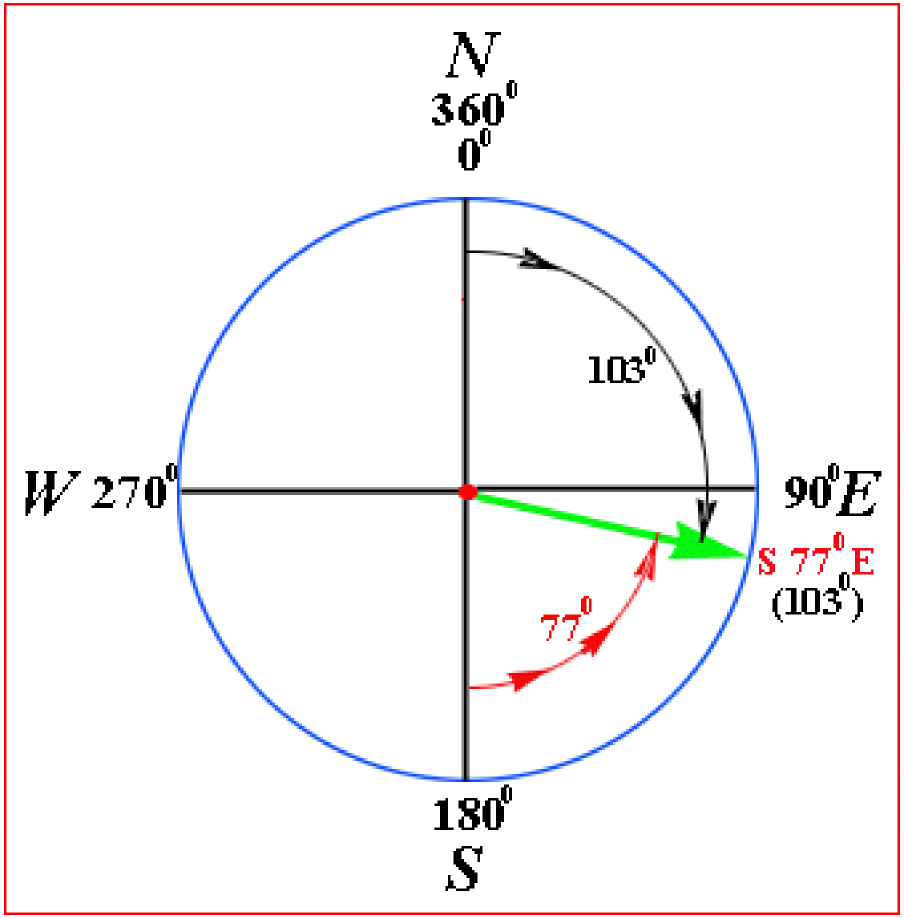
Typically, the measurements are taken at intervals between 30 - 300 ft and even shorter depending on the complexity of well path and the purpose of well bore.

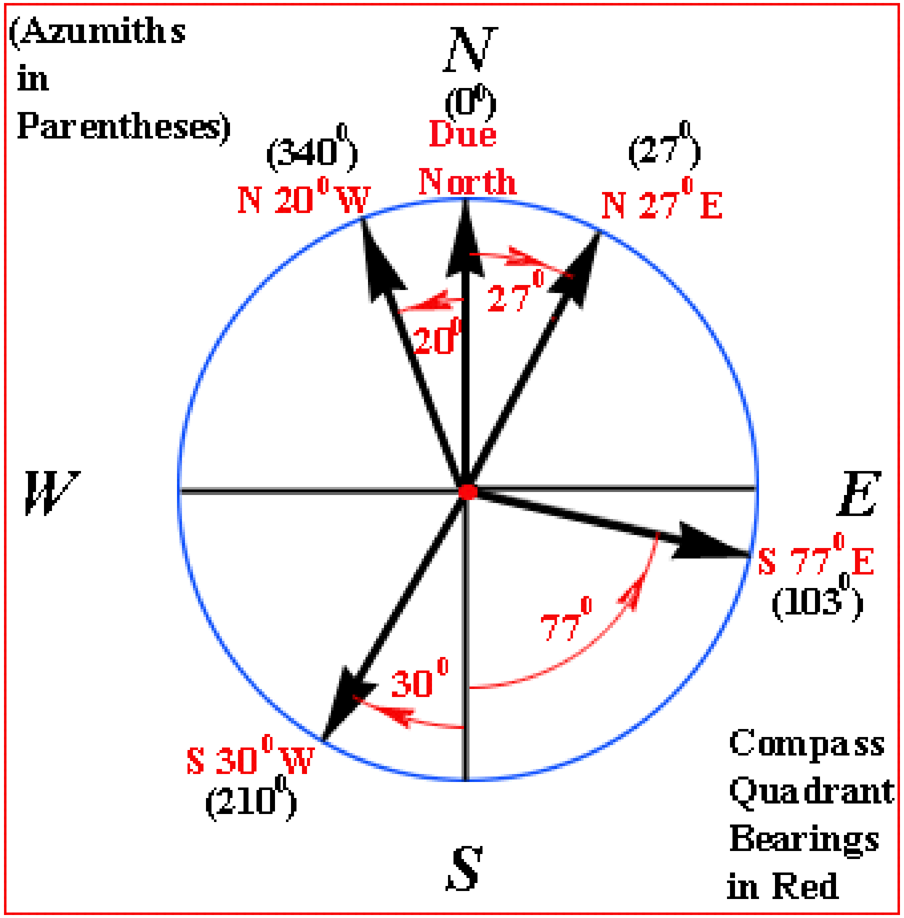
It is a conventional practice to use a 90 deg. quadrant , North, East, South and West (NE-S-W) to report well direction. For example, the azimuth angles A_1 = 27 deg *and* A_2 = 215deg *can also be reported as* A_1 = *N27E and* A_2 = *S35W In other words, if the* hole direction is reported as E26S the azimuth is 116 deg.

Declination angle is the angle between geographic (true) and magnetic north (MN).

Basic Concepts

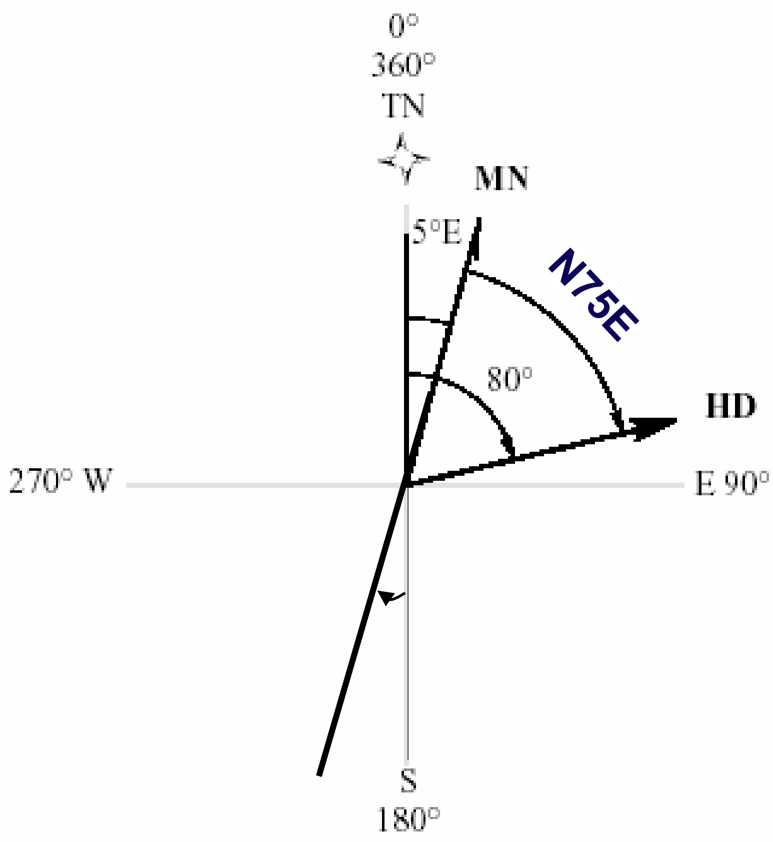
Azimuth



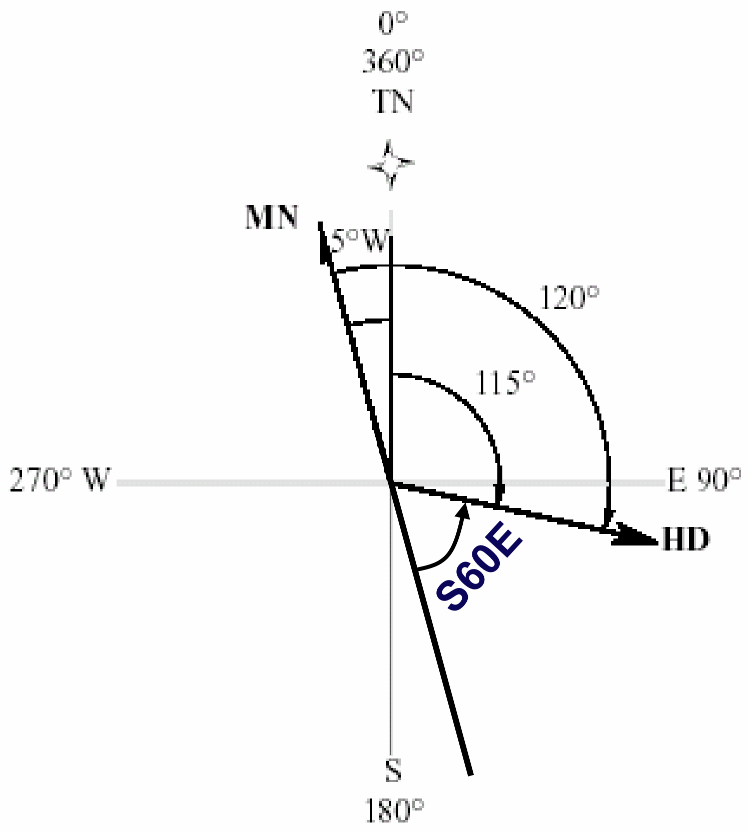


Basic Concepts

Azimuth



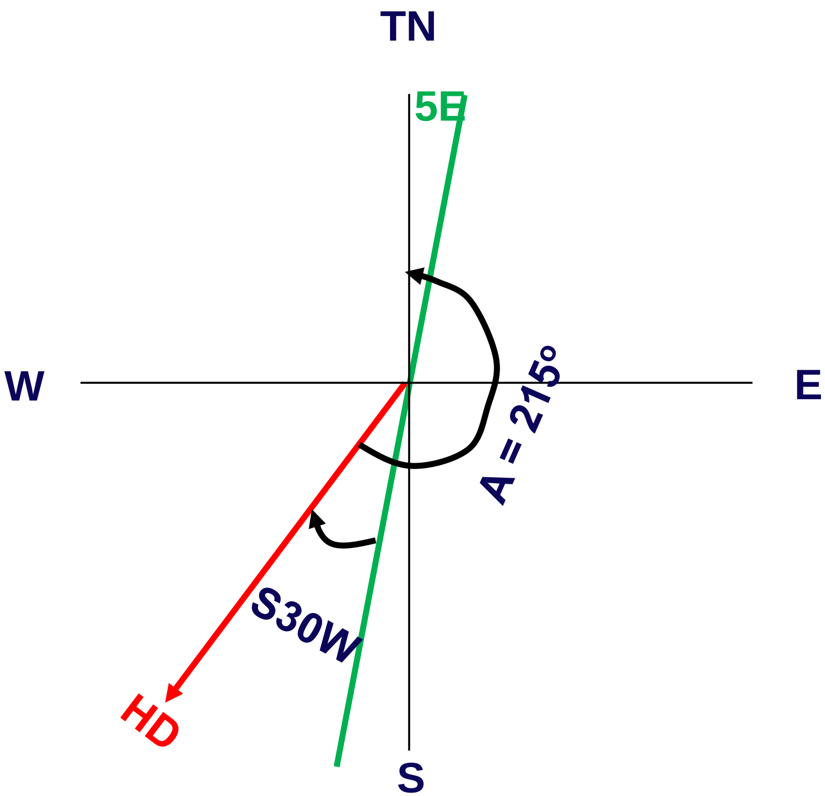
A = Magnetic Direction + Declination
A = 75° + (+5°) = 80°

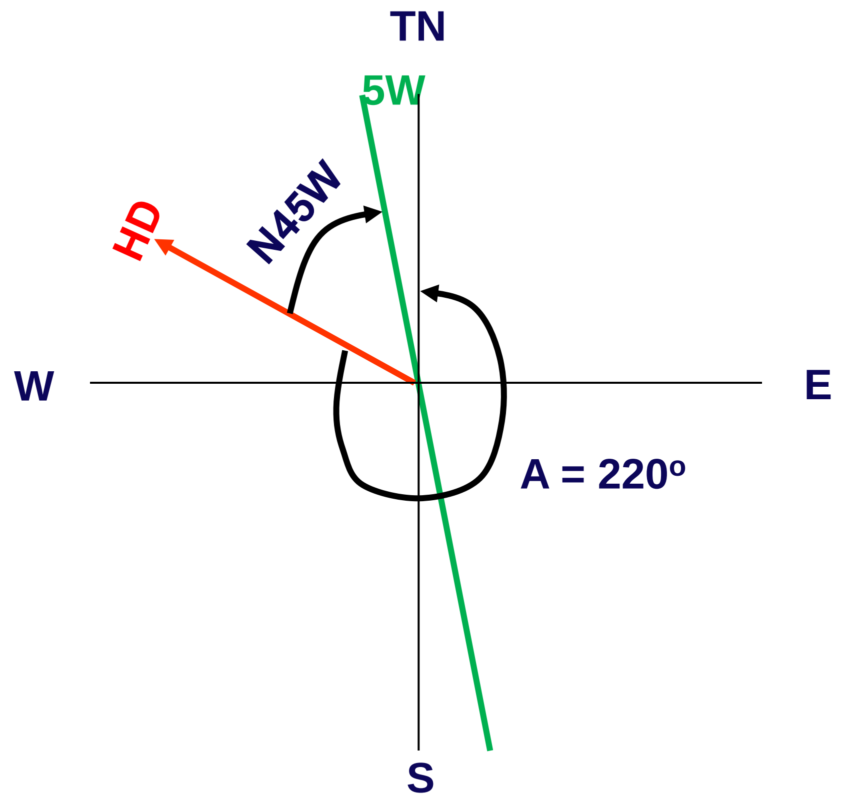


A = Magnetic Direction + Declination
A = 120° + (-5°) = 115°

Basic Concepts

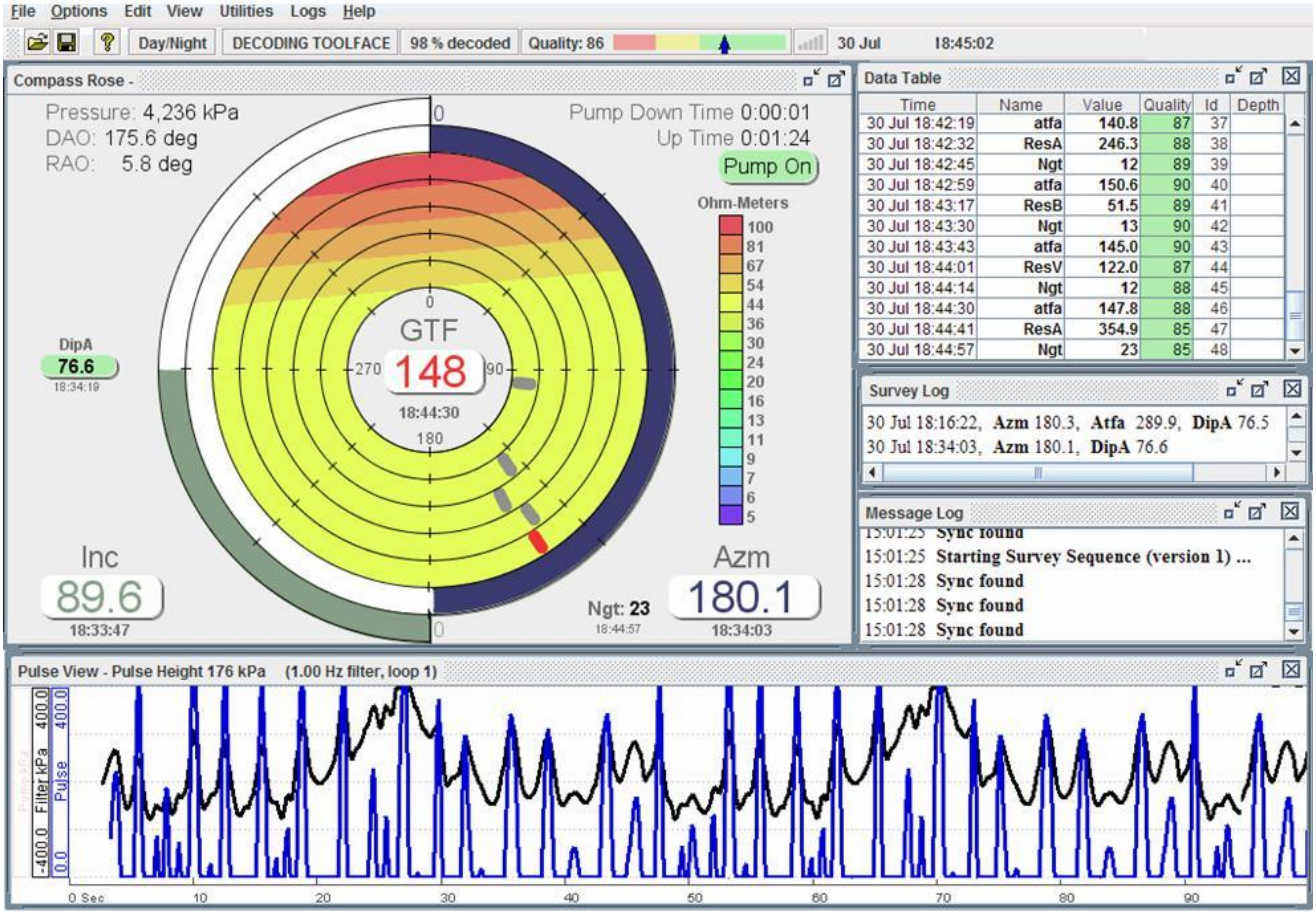
Azimuth





A = Magnetic Direction + Declination
A = 210° + (+5°) = 210°

A = Magnetic Direction + Declination
A = 225° + (-5°) = 220°


File Options Edit View Utilities Logs Help
Day/Night
DECODING TOOLFACE
98 % decoded
Quality: 86
30 Jul 18:45:02
Compass Rose -
Pressure: 4,236 kPa
DAO: 175.6 deg
RAO: 5.8 deg
Pump Down Time 0:00:01
Up Time 0:01:24
Pump On
Ohm-Meters
100 81 67 54 44 36 30 24 20 16 13 11 9 7 6 5
GTF
148
18:44:30
DipA
76.6
18:34:19
Inc
89.6
18:33:47
Ngt: 23
18:44:57
Azm
180.1
18:34:03
Data Table
Time Name Value Quality Id Depth
30 Jul 18:42:19 atfa 140.8 87 37
30 Jul 18:42:32 ResA 246.3 88 38
30 Jul 18:42:45 Ngt 12 89 39
30 Jul 18:42:59 atfa 150.6 90 40
30 Jul 18:43:17 ResB 51.5 89 41
30 Jul 18:43:30 Ngt 13 90 42
30 Jul 18:43:43 atfa 145.0 90 43
30 Jul 18:44:01 ResV 122.0 87 44
30 Jul 18:44:14 Ngt 12 88 45
30 Jul 18:44:30 atfa 147.8 88 46
30 Jul 18:44:41 ResA 354.9 85 47
30 Jul 18:44:57 Ngt 23 85 48
Survey Log
30 Jul 18:16:22, Azm 180.3, Atfa 289.9, DipA 76.5
30 Jul 18:34:03, Azm 180.1, DipA 76.6
Message Log
15:01:25 Starting Survey Sequence (version 1) ...
15:01:28 Sync found
15:01:28 Sync found
15:01:28 Sync found
Pulse View - Pulse Height 176 kPa (1.00 Hz filter, loop 1)
Filter kPa
Pulse
0 Sec 10 20 30 40 50 60 70 80 90

Basic Concepts

Lead Angle

The lead angle is to modify the direction of the target. The lead angle is usually to the left of the target horizontal departure line. The magnitude and direction (left or right) are based on the analysis of forces at the drill bit and local field experience.

Drillpipe rotation results in right hand bit walk and therefore a left lead angle is used to compensate for this tendency.

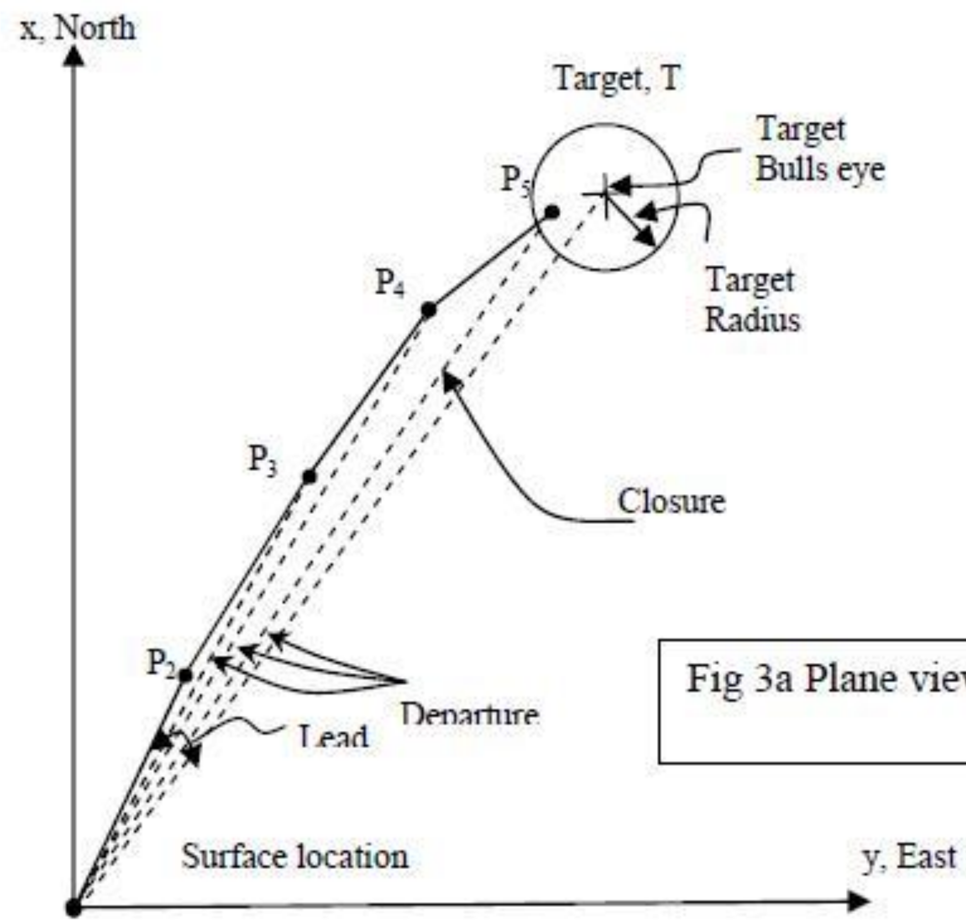


Fig 3a Plane view

Basic Concepts

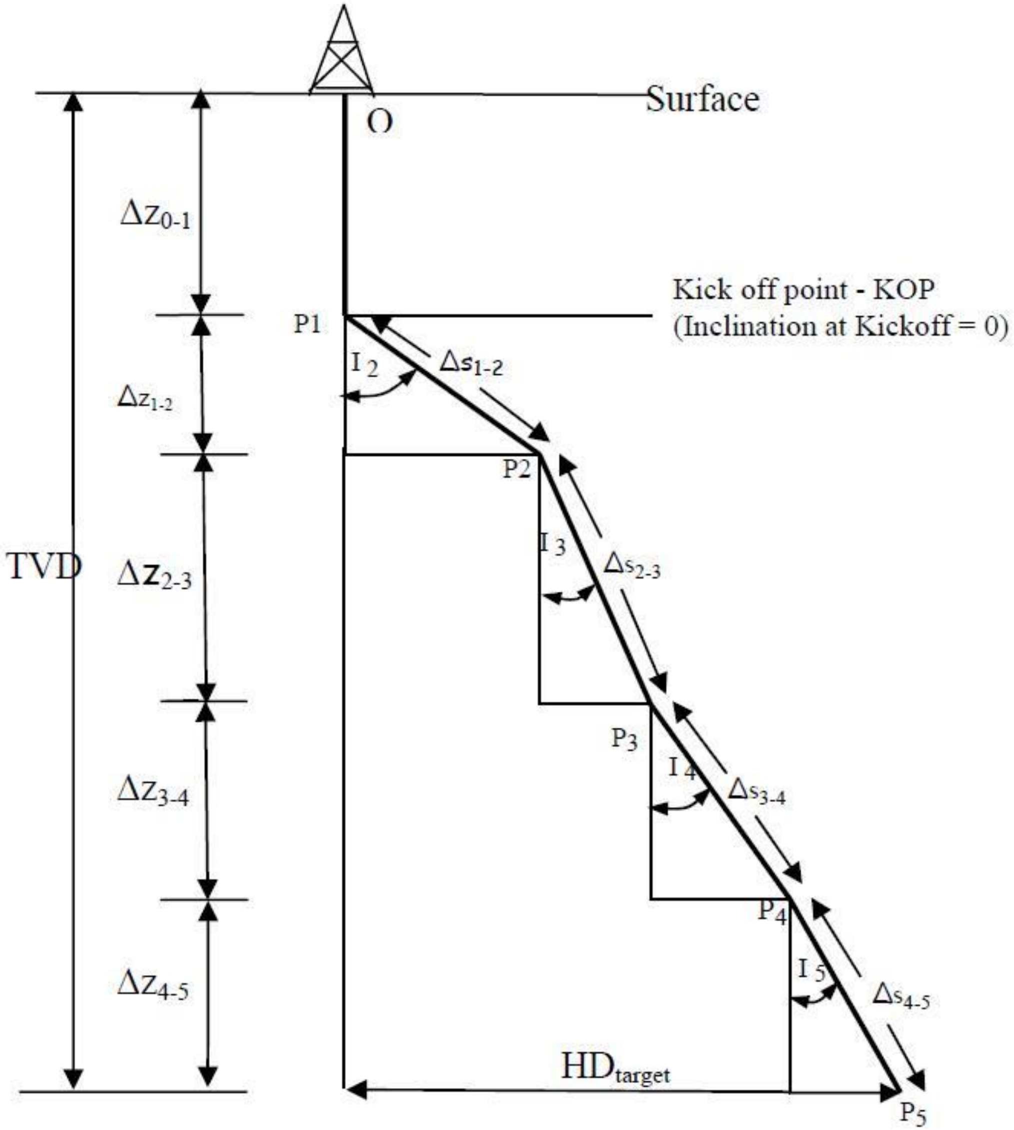
KOP – Build Rate – Turn Rate

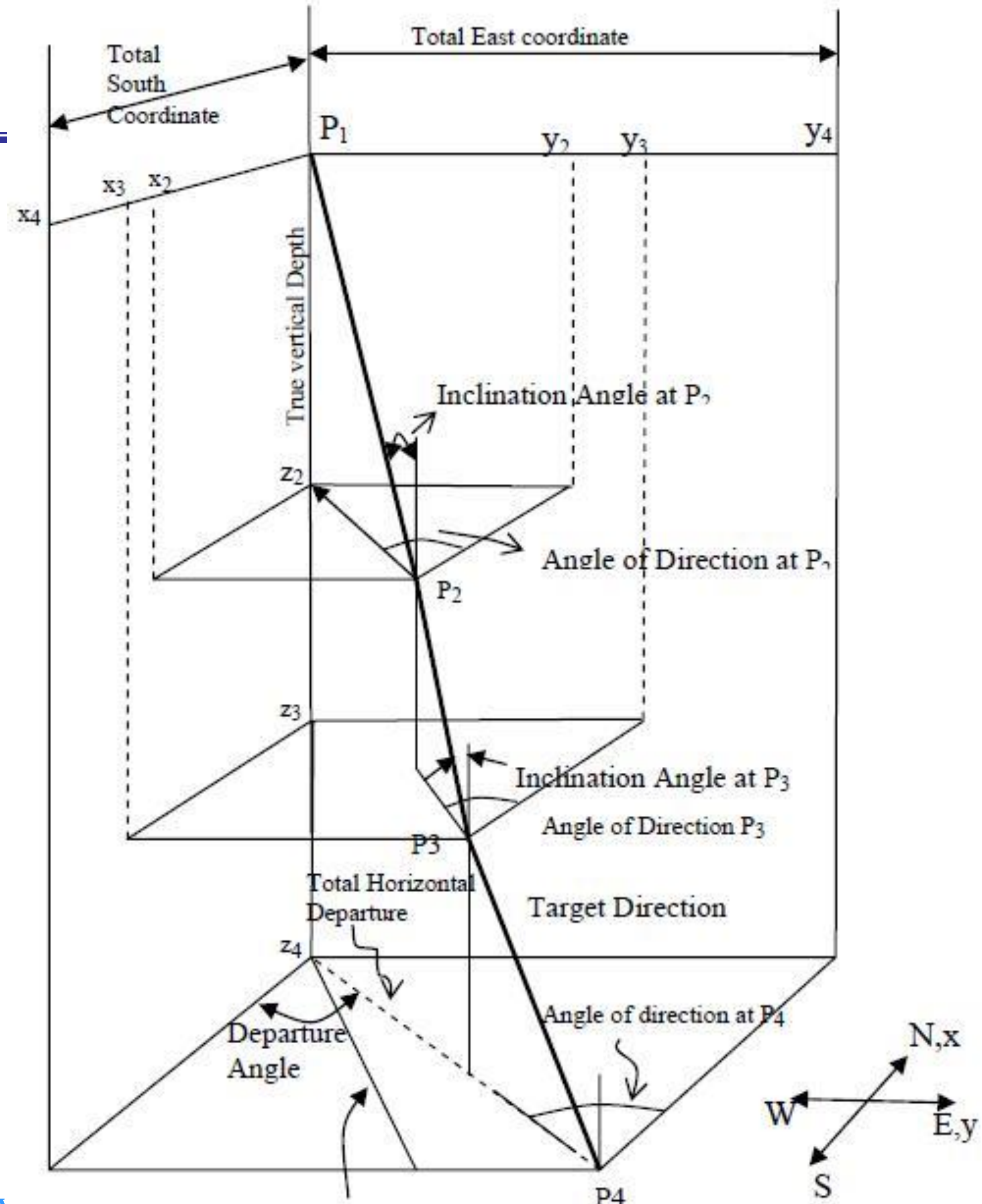
Kick-off point (KOP) is the depth where the well trajectory departs from the vertical in the direction of the target or modified by the lead angle.

The build-rate angle and drop-rate angle, respectively, refer to the incremental increase or decrease in inclination from vertical

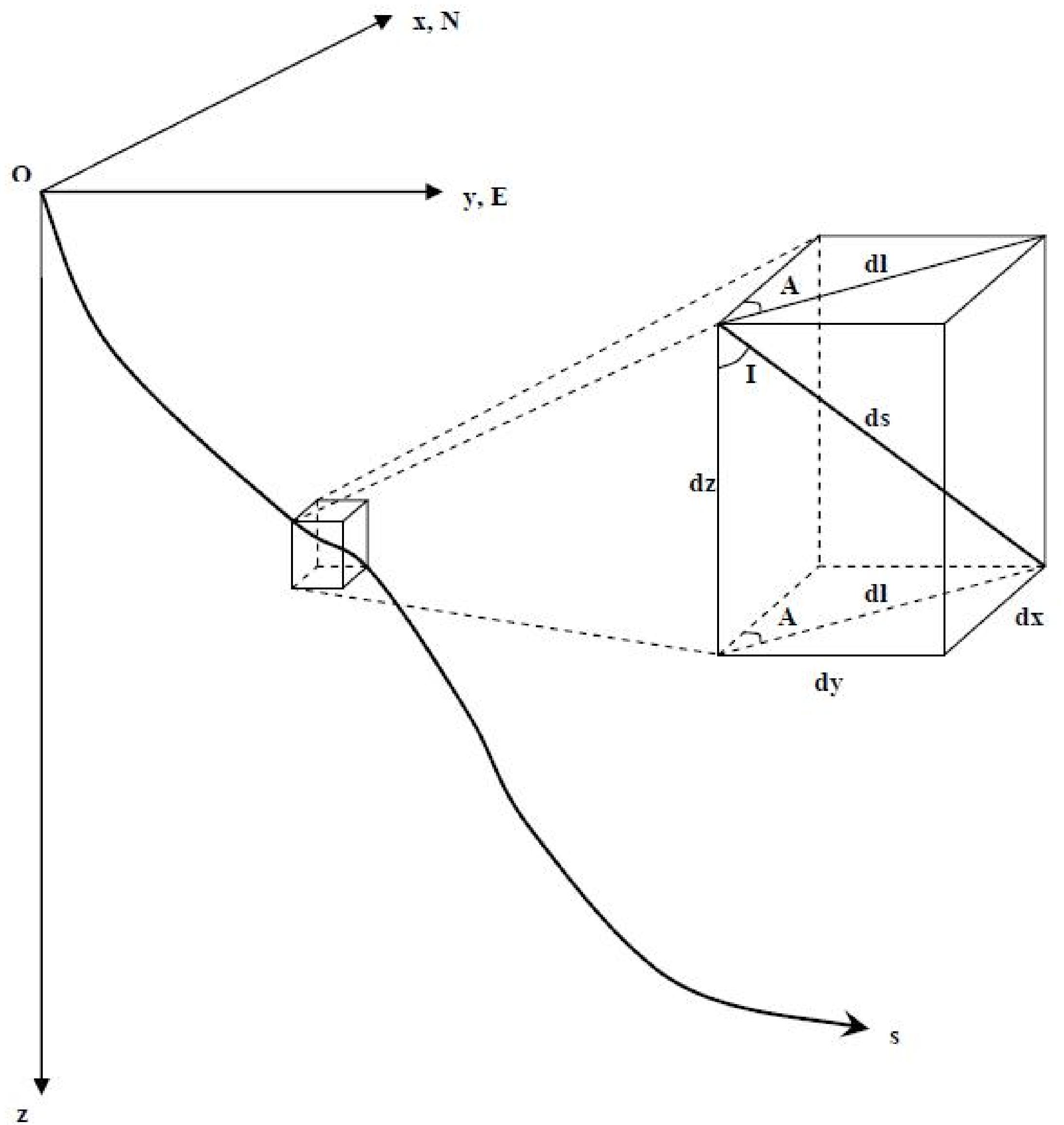
The turn-rate angle is the change in azimuth per 100 feet or meters of hole section.

Basic Concepts




Total East coordinate
Total South Coordinate
P1
y2
y3
y4
x4
x3
x2
True vertical Depth
Inclination Angle at P2
z2
Angle of Direction at P2
P2
z3
Inclination Angle at P3
Angle of Direction P3
P3
Total Horizontal Departure
Target Direction
z4
Departure Angle
Angle of direction at P4
N,x
W
E,y
S
P4

Fundamental Mathematical Formulations



Fundamental Mathematical Formulations

The continuous curve O-s represents the well trajectory (wellbore center line). Let us consider a small element ‘ds’ with the components of dx, dy and dz. Since the element ‘ds’ is small we can approximate it as a straight segment with the inclination angle I and azimuth A. The projection of the small element ‘ds’ on a horizontal plane we denote as ‘dl’. As we move along the well path, in general, both inclination angle and azimuth will change. In other words, hole inclination angle and azimuth are functions of measured depth‘s’.

Fundamental Mathematical Formulations

Definitions

Build rate: rate of change of hole inclination angle along the well path

$$B(s) = \frac{dI(s)}{ds} \qquad (1)$$

Hrizontal turn rate: rate of change of azimuth angle along the projection of well path on a horizontal plane

$$H(s) = \frac{dA(s)}{dl} \qquad (2)$$

Turn rate: rate of change of azimuth along the well path

$$T(s) = \frac{dA(s)}{ds} \qquad (3)$$

Fundamental Mathematical Formulations

Definitions

In directional drilling the build and turn rates are usually expressed in deg/100ft (deg/30m) and care should be exercised when calculations are carried out in a consistent system of units (radians rather than degrees).

The rate of change can be positive or negative depending whether the angles increase or decrease with the measured depth.

For example, the negative build rate indicates that the inclination angle decreases with depth and then it is usually called a drop rate.

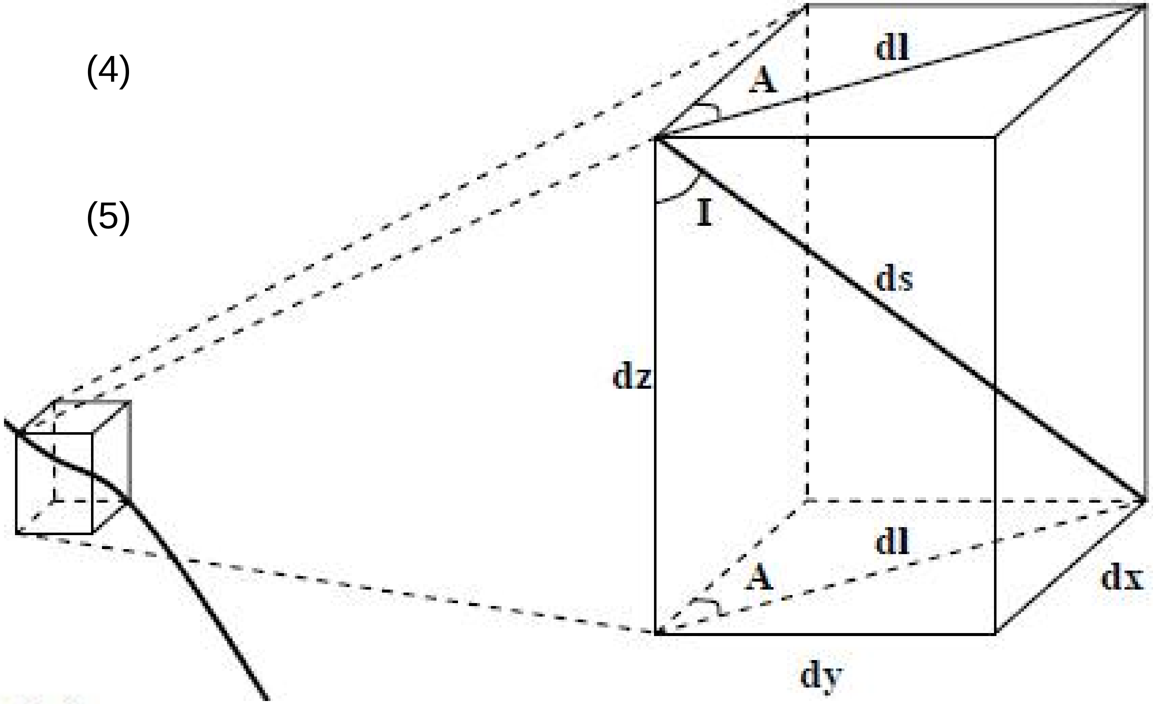
Fundamental Mathematical Formulations

$$\frac{dx}{dl} = \cos A(s) \quad (4)$$

$$\frac{dy}{dl} = \sin A(s) \quad (5)$$

$$\frac{dl}{ds} = \sin I(s) \quad (6)$$

$$\frac{dz}{ds} = \cos I(s) \quad (7)$$



$$\frac{dx}{ds} = \frac{dx}{dl}\frac{dl}{ds} = \sin I(s)\cos A(s) \quad (8)$$

$$\frac{dy}{ds} = \frac{dy}{dl}\frac{dl}{ds} = \sin I(s)\sin A(s) \quad (9)$$

Fundamental Mathematical Formulations

$$\frac{d^2x}{ds^2} = \cos I \cos A \frac{dI}{ds} - \sin I \sin A \frac{dA}{ds} = B \cos I \cos A - T \sin I \sin A \quad (10)$$

$$\frac{d^2y}{ds^2} = \cos I \sin A \frac{dI}{ds} + \sin I \cos A \frac{dA}{ds} = B \cos I \sin A + T \sin I \cos A \quad (11)$$

$$\frac{d^2z}{ds^2} = -\sin I \frac{dI}{ds} = -B \sin I \quad (12)$$

Fundamental Mathematical Formulations

The task of the designer is to calculate the coordinates, of all other points on the well path.

This is usually accomplished in a stepwise manner by selecting the subsequent point on the trajectory a distance Δs *(measured along the trajectory) from the initial point (e.g.* KOP).

To carry out the required calculations assumptions must frequently be made about well path build and turn rates as well hole inclination and directions angles.

Calculations are repeated until a smooth well path is obtained that will reach the target / targets.

Fundamental Mathematical Formulations

Basic Equations

From Eqn. (8) we obtain the difference in 'x' coordinates:

$$\Delta x = x_2 - x_1 = \int_{s_1}^{s_2} \sin I(s) \cos A(s)\, ds \tag{13}$$

From Eqn. (9) the difference in 'y' coordinates:

$$\Delta y = y_2 - y_1 = \int_{s_1}^{s_2} \sin I(s) \sin A(s)\, ds \tag{14}$$

And, from Eqn. (7) the difference in 'z' coordinates:

$$\Delta z = z_2 - z_1 = \int_{s_1}^{s_2} \cos I(s)\, ds \tag{15}$$

Fundamental Mathematical Formulations

2-D Assumption

If a well path is confined to a vertical plane its azimuth ‘A” is constant along the trajectory. Then, Eqn. (13), Eqn. (14) and Eqn. (15) take forms:

$$\Delta x = \cos A \int_{s_1}^{s_2} \sin I(s) ds = \cos A \int_{I_1}^{I_2} \frac{1}{B} \sin I(s) dI \qquad (16a)$$

$$\Delta y = \sin A \int_{s_1}^{s_2} \sin I(s) ds = \sin A \int_{I_1}^{I_2} \frac{1}{B} \sin I(s) dI \qquad (16b)$$

$$\Delta z = \int_{I_1}^{I_2} \frac{1}{B} \cos I(s) dI \qquad (16c)$$

Fundamental Mathematical Formulations

2-D and R Constant Assumptions

If a wellbore segment is a circular arc with the radius *R, the* build rate is constant and equal to *1/R*. Then, Eqns. (16) can be integrated and take forms as below:

$$\Delta x = \frac{\cos A}{B}\left(\cos I_1 - \cos I_2\right) = \xi R \cos A\left(\cos I_1 - \cos I_2\right) \quad (17a)$$

$$\Delta y = \frac{\sin A}{B}\left(\cos I_1 - \cos I_2\right) = \xi R \sin A\left(\cos I_1 - \cos I_2\right) \quad (17b)$$

$$\Delta z = \frac{1}{B}\left(\sin I_2 - \sin I_1\right) = \xi R(\sin I_2 - \sin I_1) \quad (17c)$$

The parameter $\xi = +1$ for positive turn rate and $\xi = -1$ for negative rate. $R = 1/B$

The horizontal departure HD between point 1 and point 2 is

$$HD_{1\text{-}2} = \sqrt{\Delta x^2 + \Delta y^2} = \xi R\left(\cos I_1 - \cos I_2\right) \quad (18)$$

Fundamental Mathematical Formulations

2-D and R Constant Assumptions

Example 1: Consider two points on a curved part of a trajectory located in a vertical plane with the azimuth A = 60deg. The hole inclination angle at Point 1 is I_1 = 65deg. And point 2 is I_2 = 32deg. The drop off rate is 6.5deg/100 ft. (B = −6.5 deg 100 ft). The rectangular coordinates of Point 1 are x_1 = 1650 ft., y_1 = 2858 ft. and z_1 = 4250 . Calculate

a - x, y, z coordinates at Point 2

b - Radius of curvature R

c - Horizontal departure between Point 1 and Point 2

d - Length of the segment Δs

Fundamental Mathematical Formulations

2-D and R Constant Assumptions

$$B = -(6.5)\left(\frac{\pi}{180}\right)\left(\frac{1}{100}\right) = -(1.1339)\left(10^{-3}\right)1/ft$$

Applying Eqns.(17) we obtain the rectangular coordinates as follows:

$$x_2 = 1650 + \frac{\cos 60}{(-1.1339)\left(10^{-3}\right)}(\cos 65 - \cos 32) = 1837.6\,ft$$

$$y_2 = 2858 + \frac{\sin 60}{(-1.1339)\left(10^{-3}\right)}(\cos 65 - \cos 32) = 3182.9\,ft$$

$$z_2 = 4250 + \frac{1}{(-1.1339)\left(10^{-3}\right)}(\sin 32 - \sin 65) = 4581.9\,ft.$$

Fundamental Mathematical Formulations

2-D and R Constant Assumptions

Radius of curvature $R = \dfrac{1}{B} = \dfrac{1}{(1.1339)(10^{-3})} = 882\,ft.$

Horizontal departure between Point 1 and Point 2

$$HD_{1-2} = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(187.9)^2 + (324.9)^2} = 375.2\,ft.$$

For a circular segment in a vertical plane the differential length of the segment is: $ds = RdI$, hence we write:

$$\Delta s = \int_{s_1}^{s_2} ds = -\int_{I_1}^{I_2} RdI = R(I_1 - I_2) = (882)\left(\frac{\pi}{180}\right)(65-32) = 508\,ft.$$

Wellbore Curvature and Dogleg Severity

In directional drilling nomenclature the wellbore curvature is frequently named as a dogleg severity (DLS) and expressed in deg/100ft as mentioned earlier. We know from the course on calculus that the curvature of a 3D curve can be calculated as follows:

$$\kappa(s) = \left[\left(\frac{d^2x}{ds^2} \right)^2 + \left(\frac{d^2y}{ds^2} \right)^2 + \left(\frac{d^2z}{ds^2} \right)^2 \right]^{\frac{1}{2}} \qquad (20)$$

Eqn (20) gives curvature in any consistent system of units e,g. 1/ft , 1/m.

Students, derive this equation!!!

Wellbore Curvature and Dogleg Severity

In practical directional drilling terminology we frequently use the term the dogleg severity rather than curvature and express it in degrees per unit length. If the dogleg severity is expressed in deg/100ft we write:

$$DLS = \frac{18{,}000\kappa(s)}{\pi} \tag{21}$$

Note: the unit of curvature K is 1/ft. And 180 degrees equivalent to π in radian. There for the unit of DLS calculated as equation (21) is degrees/100ft.

The radius of curvature R is defined as the inverse of curvature and we write:

$$R(s) = \kappa(s)^{-1}$$

Wellbore Curvature and Dogleg Severity

Substituting Eqn. (10), Eqn. (11) and Eqn. (12) to Eqn. (20), after some rearrangements, we obtain the wellbore curvature in terms of build rate, turn rate and hole inclination angle as follows.

$$\kappa(s) = \sqrt{B^2 + T^2 \sin^2 I(s)} \qquad (22)$$

Since $T = \frac{dA}{dl}\frac{dl}{ds} = H \sin I(s)$ we can also write the curvature equation in terms of build rate and horizontal turn rate as given below:

$$\kappa(s) = \sqrt{B^2 + H^2 \sin^4 I(s)} \qquad (23)$$

Wellbore Curvature and Dogleg Severity

The overall angle change (dogleg) between two points on wellpath is defined as the angle between the tangent lines at the two points under considerations. The curvature is the rate of over all angle change along the trajectory; therefore, the overall angle change "β " between two neighboring points on the trajectory located Δs apart is obtained by integrating curvature along the trajectory as follows:

$$\beta = \int_{o}^{\Delta s} \kappa(s)\,ds = \int_{0}^{\Delta s} B\,ds \qquad (24)$$

Note that: $B = K(s) = \frac{dt}{ds} = \frac{dI}{ds}$ and $DLS = \frac{18{,}000\kappa(s)}{\pi}$

If there is no change in azimuth along the well path, the dogleg (DL) would be:

$$\beta = DL = \int_{o}^{\Delta s} B\,ds = \int_{I_1}^{I_2} dI = I_2 - I_1 \qquad (25)$$

Wellbore Curvature and Dogleg Severity

Lubinski (1953) was the first to derive the equation for dogleg in a form:

$$\beta = 2\arcsin\sqrt{\sin^2\left(\frac{I_2 - I_1}{2}\right) + \sin I_1 \sin I_2 \sin^2\left(\frac{A_2 - A_1}{2}\right)} \qquad (26)$$

Analysis of Eqn. (26) shows that for $A_1 = A_2$ *Eqn (26) is reduced to Eqn. (25)*

Design of Two Dimensional Well Profiles

Directional Wells

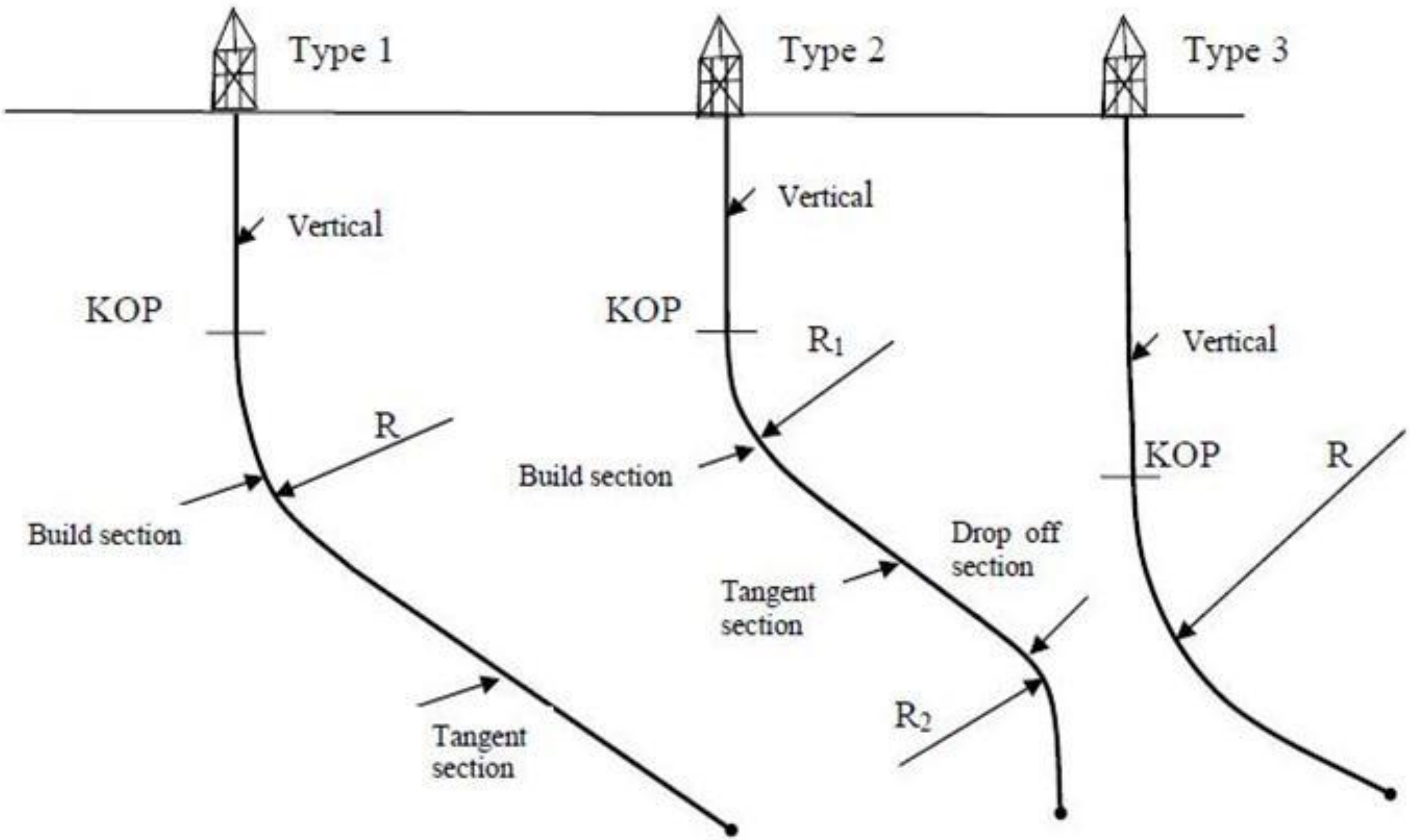
Typically the design of a directional well profile consists of two phases:

1. Firstly a wellpath is constructed to connect target with the surface location and then adjustments are made to account for factors that will eventually influence the final trajectory.

2. After the base well trajectory is calculated the designer needs to make corrections to compensate for the anticipated effects related to drill pipe rotation (bit walk), formation hardness and dip angle, type of drillbits etc.

Design of Two Dimensional Well Profiles

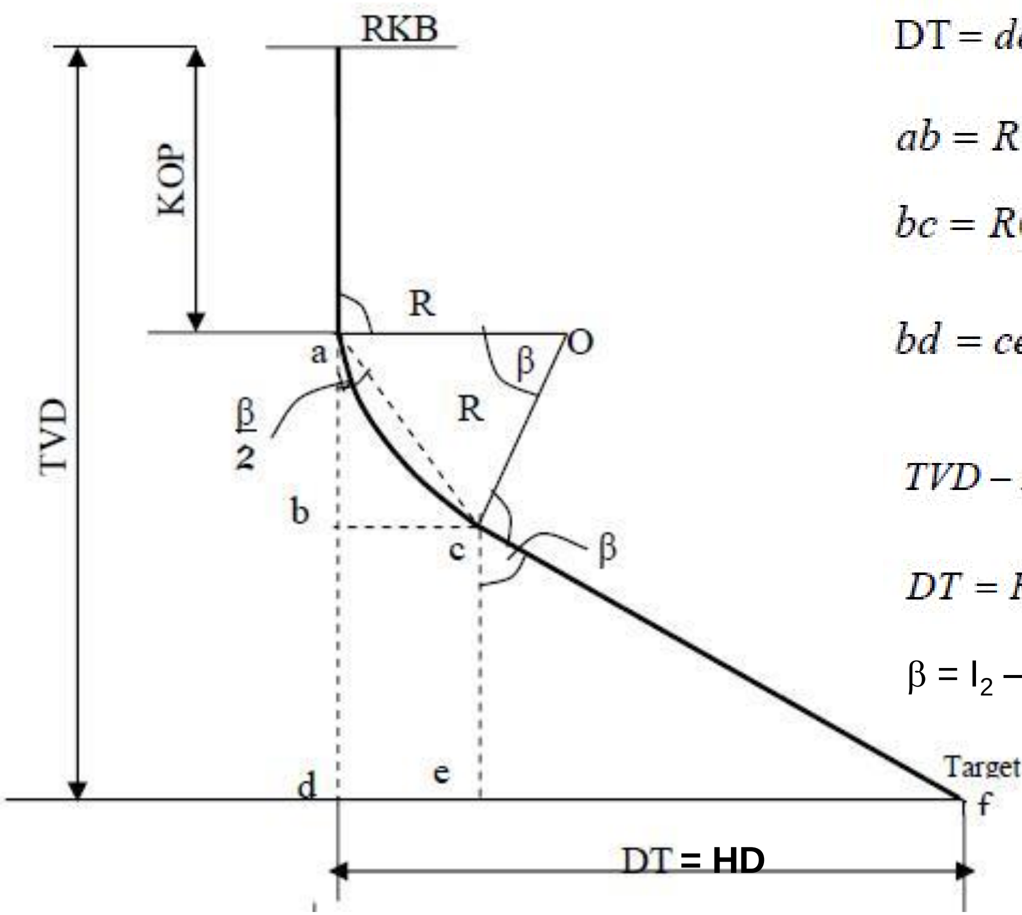
Directional Wells



Design of Two Dimensional Well Profiles

Directional Wells

Type 1: Build and hold (Slant type)



$$TVD - KOP = ab + bd$$

$$DT = de + ef$$

$$ab = R \sin \beta$$

$$bc = R(1 - \cos \beta)$$

$$bd = ce = \frac{ef}{\tan \beta}$$

$$TVD - KOP = r \sin \beta + \frac{ef}{\tan \beta}$$

$$DT = R(1 - \cos \beta) + ef = \text{HD}$$

$$\beta = I_2 - I_1 = I_2$$

Design of Two Dimensional Well Profiles

Directional Wells

$$sin\frac{\beta}{2} = \frac{ac}{2R}$$

$$cos\frac{\beta}{2} = \frac{ab}{ac}$$

$$cos\frac{\beta}{2} = \frac{ab}{2Rsin\frac{\beta}{2}}$$

$$ab = 2Rsin\frac{\beta}{2}cos\frac{\beta}{2}$$

$$ab = Rsin\beta$$

Design of Two Dimensional Well Profiles

Directional Wells

Combining these equations give

$$(TVD - KOP)\sin\beta + (R - DT)\cos\beta = R \tag{27}$$

If target total vertical depth TVD, kick off point depth KOP, horizontal departure DT and radius of build section are given then Eqn (27) can be solved for the dogleg angle β :

$$\beta = \arcsin\left(\frac{R}{\sqrt{(R-DT)^2 + (TVD-KOP)^2}}\right) - \arctan\left(\frac{R-DT}{TVD-KOP}\right) \tag{28}$$

Design of Two Dimensional Well Profiles

Directional Wells

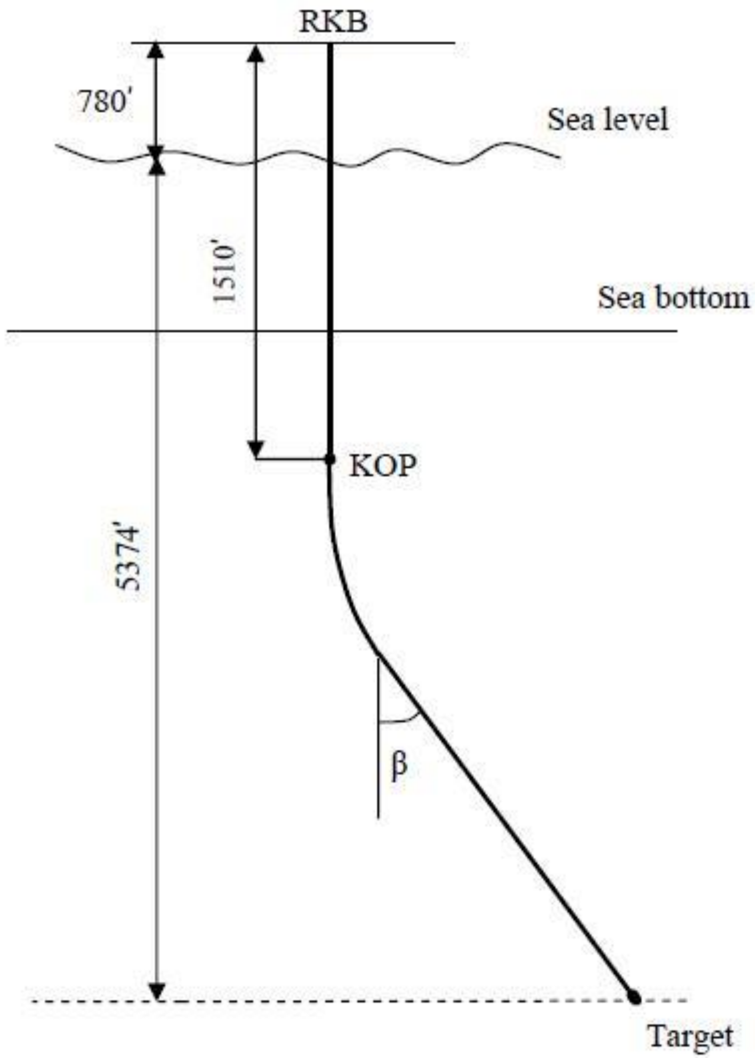
Example 2

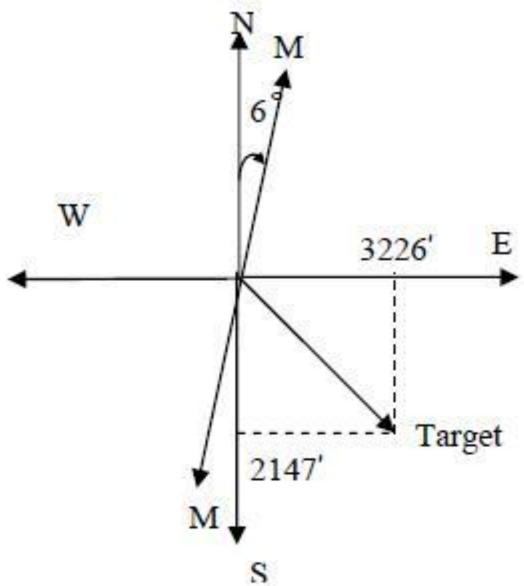
Design trajectory of a slant type offshore well for the conditions as stated below:

- elevation (above sea level) = 780 *ft.* - target depth (sub-sea) = -5374 *ft.* - target south coordinate = 2147 *ft.* - target east coordinate = 3226 *ft.* - declination = 6° E - KOP depth = 1510 *ft.* - build-up rate = 2 deg/100 *ft.*	It is required to find: 1. slant angle 2. vertical depth at the beginning of tangent part 3. departure at the beginning of tangent part 4. measured depth to the target

Design of Two Dimensional Well Profiles

Directional Wells





Design of Two Dimensional Well Profiles

Directional Wells

- target true vertical depth TVD= 780 + 5374= 6154 ft
- target departure DT = $\sqrt{2147^2 + 3226^2} = 3875\,ft$
- target direction = $\arctan\frac{3226}{2147} = S56.35E$ (azimuth 180-56.35=123.65deg)
- radius of curvature: $R = \frac{180}{(0.02)\pi} = 2865\,ft.$
- slant angle:
- $$\beta = \arcsin\left(\frac{2865}{\sqrt{(2865-3875)^2 + (6154-1510)^2}}\right) - \arctan\left(\frac{2865-3875}{6154-1510}\right) = 49.3\deg$$

Design of Two Dimensional Well Profiles

Directional Wells

For further calculations we choose the slant angle = 49deg.

- vertical depth at the beginning of the tangent part

$$VD_2 = 1510 + (2865)(\sin 49) = 3684\,ft.$$

- departure at the beginning of tangent part

$$HD_2 = 2865(1 - \cos 49) = 998\,ft.$$

- measured depth at the beginning of the tangent part

$$s_2 = 1510 + \frac{49}{0.02} = 3977\,ft.$$

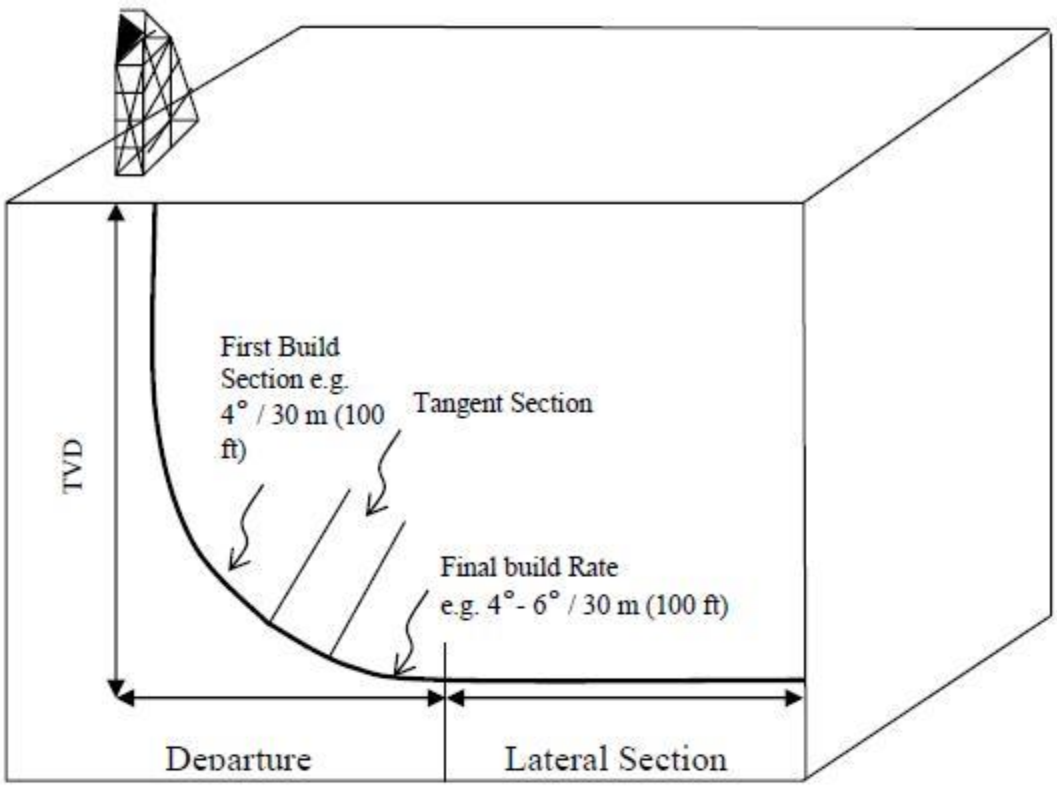
- measured depth at the target

$$s_{target} = 3977 + \frac{6154 - 3684}{\cos 49} = 7769\,ft.$$

Design of Two Dimensional Well Profiles

Horizontal Wells

In practical applications, horizontal wells are high angle wells with the inclination angle in the range of about 80 deg to 100 deg. An ideal horizontal well, as the name indicates, the inclination angle is equal to 90 deg.



Design of Two Dimensional Well Profiles

Horizontal Wells

Typically, horizontal wells are classified by their radius of curvature as follows:

- *long radius* with the radius R = 1000 - 3000 *ft (DLS = 6 – 2 deg/100ft)*
- *medium radius* with the radius R = 200 - 1000 *ft (DLS = 30 – 6 deg/100ft)*
- *short radius* with the radius R = 30 - 200 *ft (DLS = 180 – 30 deg/100ft)*

There are also ultra short radius systems that apply high pressure jetting techniques for turning the well from vertical to horizontal.

Design of Two Dimensional Well Profiles

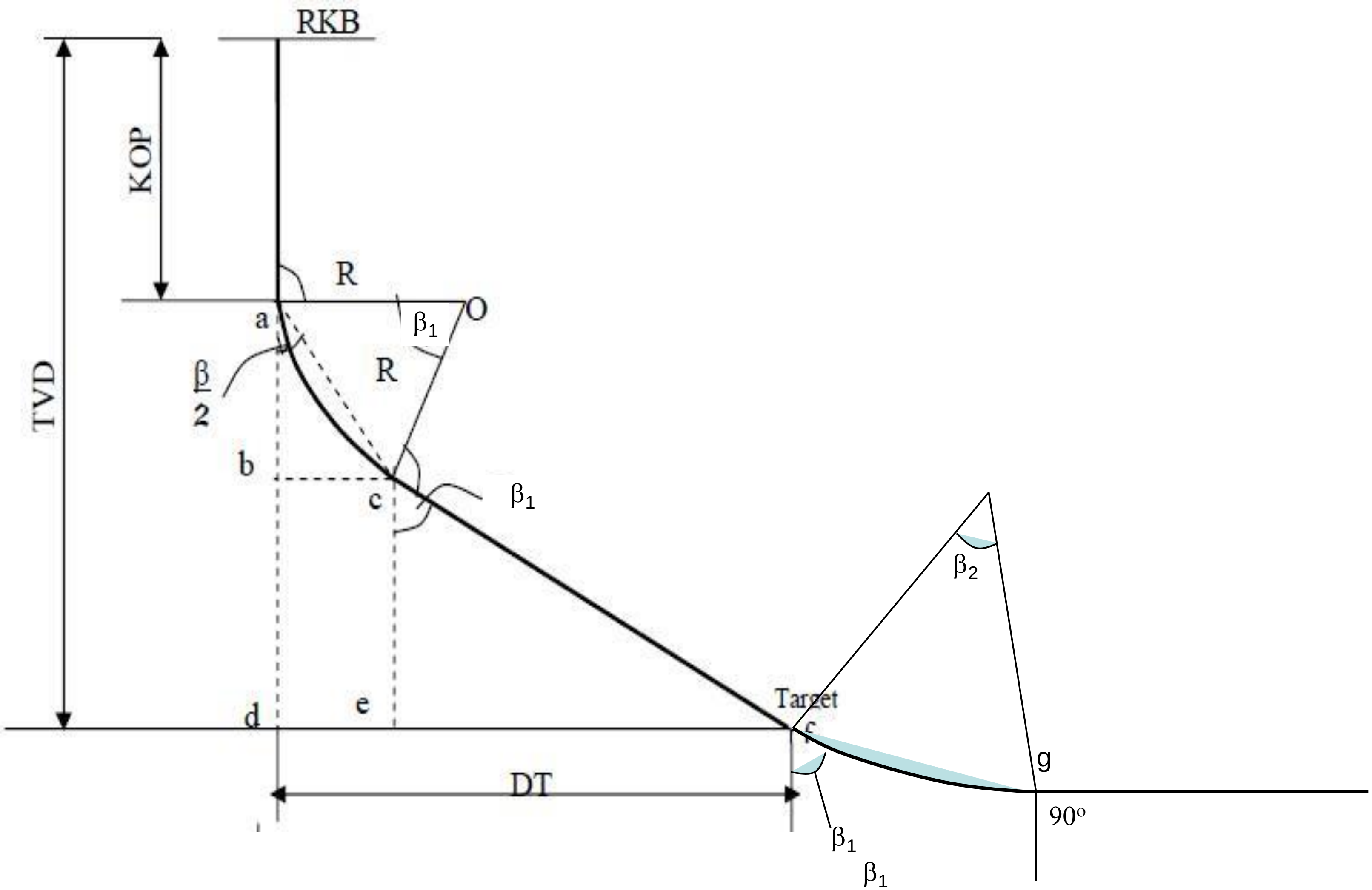
Horizontal Wells

It is desired to design the simple tangent horizontal well profile given:

- KOP = 8206 ft.
- TVD = 9000 ft.
- Tangent length = 120 ft
- Tangent angle = 50 deg
- Target angle = 90 deg. at TVD
- The expected build rate is 8 deg/100 ft.

Design of Two Dimensional Well Profiles

Horizontal Wells



Design of Two Dimensional Well Profiles

Horizontal Wells

Build radius **(Eq. (21))**

$$R = \frac{5729}{B} = \frac{5729}{8} = 716\,ft$$

Height of first build (Eqn. (17c)): Δz_1 = ab = Rsinβ = 716xsin50 = 549 ft

$$\Delta z_1 = \text{ab} = R(\sin I_2 - \sin I_1) = 716(\sin 50 - \sin 0) = 549\,ft.$$

Height of tangent:

$$\Delta z_{tan} = \text{bd} = s_{tan} \cos I_{tan} = (120)(\cos 50) = 77\,ft.$$

Height of second build: Do not use this equation: Δz_2 = ab = Rsinβ

$$\Delta z_2 = \text{dg} = (716)(\sin 90 - \sin 50) = 168\,ft.$$

Design of Two Dimensional Well Profiles

Horizontal Wells

Horizontal displacement of first build **(Eq. (18))**

$$\Delta HD_1 = R(\cos I_1 - \cos I_2) = (716)(\cos 0 - \cos 50) = 256\,ft.$$ Or bc = R(1 – cosβ)

Horizontal displacement of tangent:

$$\Delta HD_{tan} = s_{tan} \sin I_{tan} = (120)(\sin 50) = 92\,ft.$$

Horizontal displacement of second build:

$$\Delta HD_2 = (716)(\cos 50 - \cos 90) = 460\,ft.$$

Length of first build:

$$\Delta s_1 = \frac{100(I_2 - I_1)}{B} = \frac{100(50-0)}{8} = 625\,ft.$$

Length of second build:

$$\Delta s_2 = \frac{100(90-50)}{8} = 500\,ft.$$

Design of Two Dimensional Well Profiles

Horizontal Wells

Measured depths:

At end of first build: 8206 + 625 = 8831ft.

At end of tangent: 8831 + 120 = 8951 ft.

At end of second build: 8951 + 500 = 9451 ft.

Design of Three Dimensional Well Profiles

Introduction

In engineering practice any well trajectory that is not located in a vertical plane is considered as a 3D well.

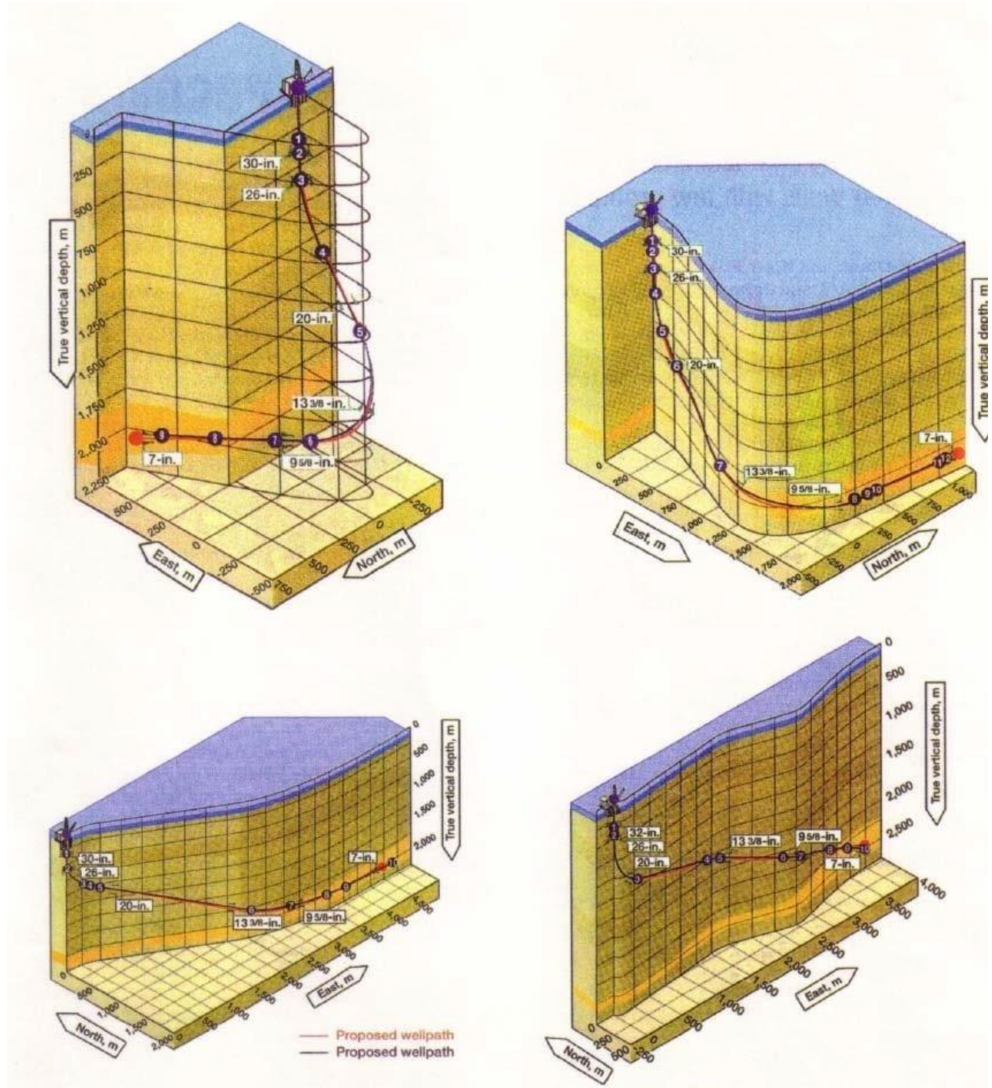


Fig. 2. Examples of designer wells in Gullfaks field.

Design of Three Dimensional Well Profiles

Introduction

In general, the design task is to construct a 3D smooth path that connects the surface or subsurface location to a know target or targets.

In addition to 3D geometrical requirements the designer must also consider other factors related to the drilling process such as drill string mechanical integrity, wellbore stability, cuttings transport, running casing, cementing and perforating operations etc.

For the purpose of the well-path optimization we usually use minimum cost of drilling or minimum drilling time as the optimization criteria.

Design of Three Dimensional Well Profiles

Introduction

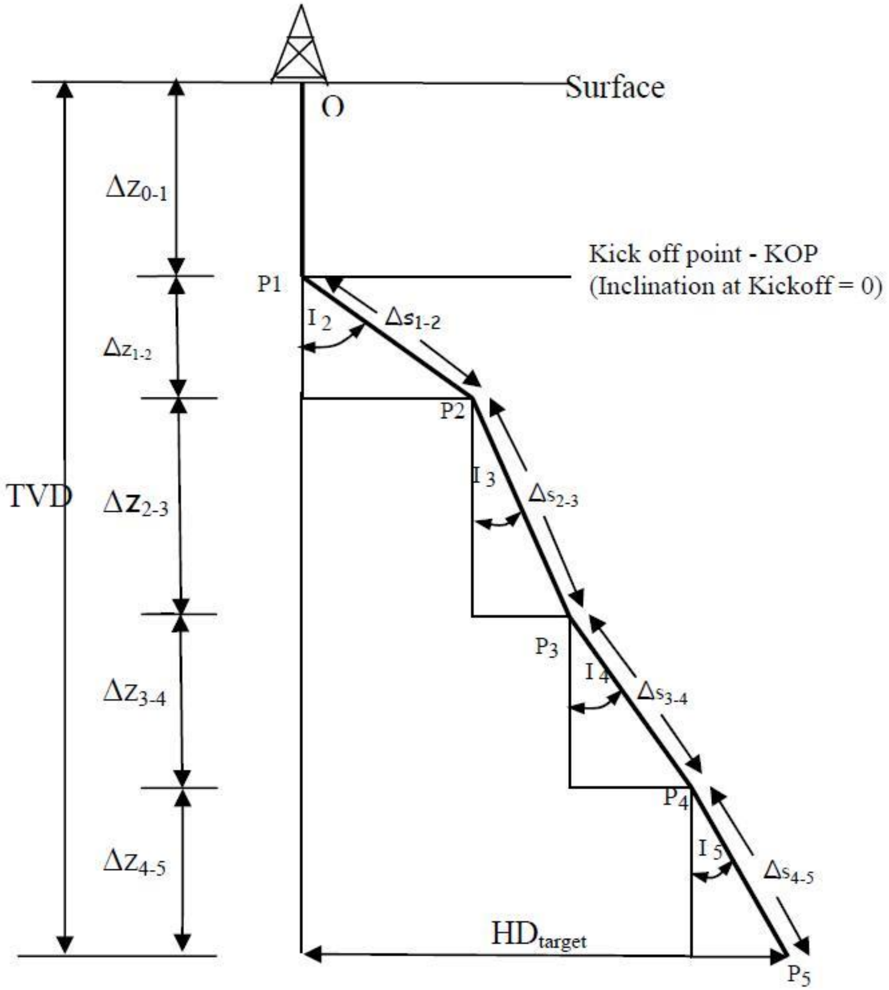
Here we will focus on just the 3D geometrical considerations and discussion of seven methods that are available from SPE literature:

1. Average Angle Method
2. Tangential Method.
3. Balanced Tangential Method.
4. Radius of Curvature.
5. Constant Build and Turn Rate.
6. Constant Curvature and Build Rate (Constant Tool Face).
7. Minimum Curvature

Design of Three Dimensional Well Profiles

The Average Angle Method

One of the earlier methods is the so called Average Angle Method (AAM). In this approach the well is modeled as a series of straight segments in a vertical and horizontal plane. It is assumed that the inclination and azimuth angles are constant and equal to the average value of two subsequent points on the trajectory. With this assumption, Eqn.(13), Eqn(14) and Eqn(15) are written as below:



Design of Three Dimensional Well Profiles

The Average Angle Method

$$x_2 = x_1 + \left(\sin \bar{I} \cos \bar{A}\right)\Delta s \tag{29a}$$

$$y_2 = y_1 + \left(\sin \bar{I} \sin \bar{A}\right)\Delta s \tag{29b}$$

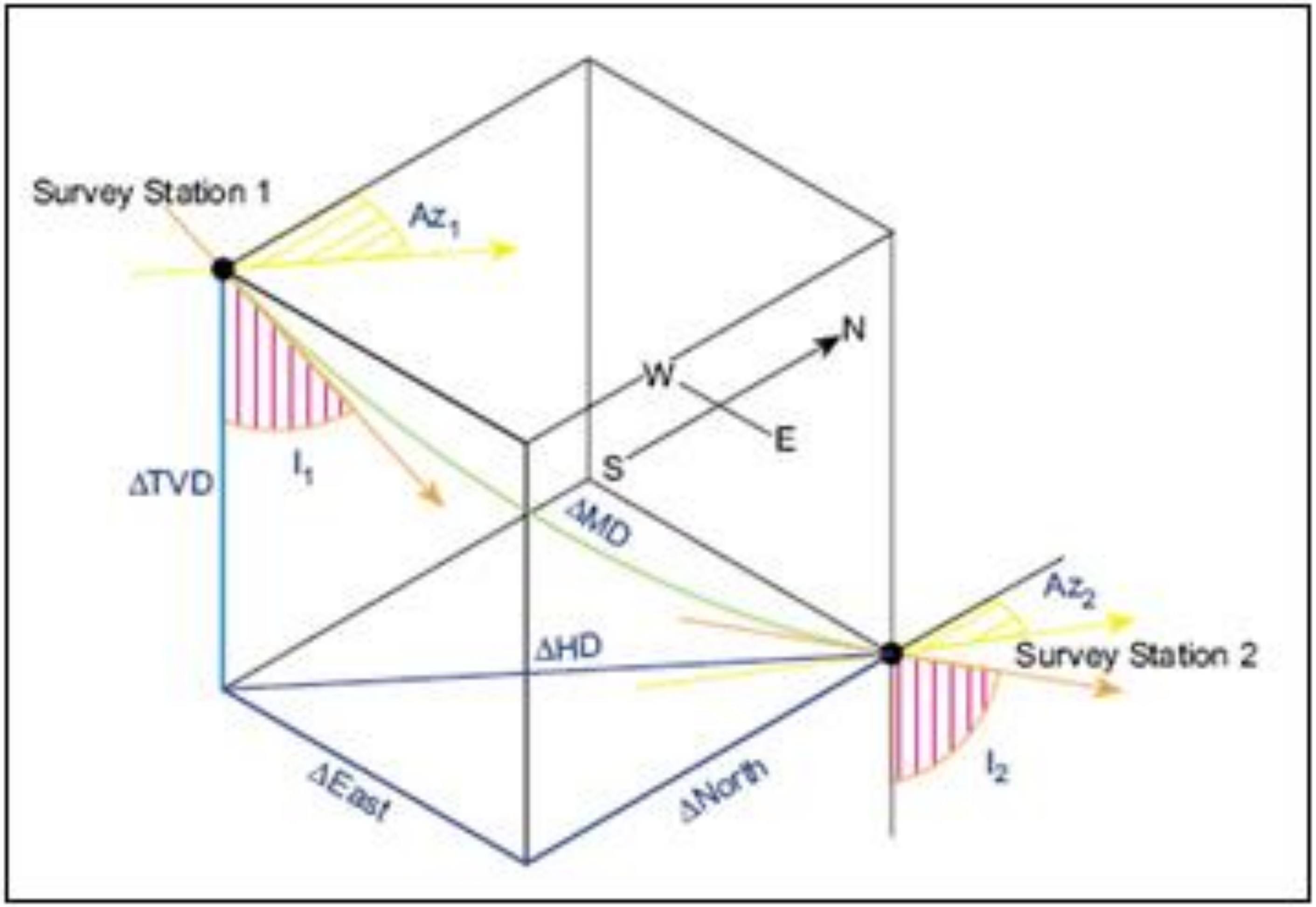
$$z_2 = z_1 + \left(\cos \bar{I}\right)\Delta s \tag{29c}$$

Where the average values of hole inclination and azimuth angles ($\bar{I}$ and $\bar{A}$) are defined as follows:

$$\bar{I} = \frac{I_i + I_{i-1}}{2} \quad \text{and} \quad \bar{A} = \frac{A_i + A_{i-1}}{2}$$

Design of Three Dimensional Well Profiles

The Average Angle Method



Design of Three Dimensional Well Profiles

The Average Angle Method

$$\Delta North = \Delta MD \times \sin\left(\frac{I_1 + I_2}{2}\right) \times \cos\left(\frac{A_{z1} + A_{Z2}}{2}\right)$$

$$\Delta East = \Delta MD \times \sin\left(\frac{I_1 + I_2}{2}\right) \times \sin\left(\frac{A_{z1} + A_{Z2}}{2}\right)$$

$$\Delta TVD = \Delta MD \times \cos\left(\frac{I_1 + I_2}{2}\right)$$

Design of Three Dimensional Well Profiles

The Average Angle Method

Example 4

It is required to calculate the rectangular coordinates of the well from the depth of 8000 ft. to 8400 ft. The kick off point is at 8000 ft. and the rate of build is 1 deg/100ft., using a lead of 10 deg. and a right-hand walk rate of 1 deg/100ft. (turn rate in a horizontal plane). Direction of the target is N30E. Assume the first 200ft. is to set the lead, where the direction is held constant to 8200 ft. and then turns right at a rate of 1 deg/100ft.

Design of Three Dimensional Well Profiles

The Average Angle Method

The origin of the coordinates is set at the top of the hole ($x_0 = 0$, $y_0 = 0$, $z_0 = 0$) and the first segment is vertical, hence the inclination angle $I_1 = 0$ deg. and the azimuth is undetermined. The coordinates of kick off point are: $x_1 = 0$ ft, $y_1 = 0$ ft and $z_1 = 8600$ ft. For the point located at $\Delta s = 100$ ft. from the kick off point we calculate: Note $A_2 = 30$ – lead angle = 30 – 10 = 20°.

$$x_2 = x_1 + \left(\sin \bar{I} \cos \bar{A}\right)\Delta s \qquad x_2 = (100)\sin\left(\frac{1+0}{2}\right)\cos 20 = 0.82\,ft.$$

$$y_2 = y_1 + \left(\sin \bar{I} \sin \bar{A}\right)\Delta s \qquad y_2 = (100)\sin\left(\frac{1+0}{2}\right)\sin 20 = 0.30\,ft.$$

$$z_2 = z_1 + \left(\cos \bar{I}\right)\Delta s \qquad z_2 = 8000 + (160)\cos\left(\frac{1+0}{2}\right) = 8099.99\,ft$$

Design of Three Dimensional Well Profiles

The Average Angle Method

From the depth of 8100 ft. to 8200 ft. the inclination angle is increased to 2 deg. And azimuth is N20E.

$$x_2 = x_1 + \left(\sin \bar{I} \cos \bar{A}\right)\Delta s \qquad x_3 = 0.82 + (100)\sin\left(\frac{1+2}{2}\right)\cos\left(\frac{20+20}{2}\right) = 3.28\,ft.$$

$$y_2 = y_1 + \left(\sin \bar{I} \sin \bar{A}\right)\Delta s \qquad y_3 = 0.30 + (100)\sin\left(\frac{1+2}{2}\right)\sin\left(\frac{20+20}{2}\right) = 1.20\,ft.$$

$$z_2 = z_1 + \left(\cos \bar{I}\right)\Delta s \qquad z_3 = 8099.99 + (100)\cos\left(\frac{1+2}{2}\right) = 8199.96\,ft.$$

Design of Three Dimensional Well Profiles

The Average Angle Method

From 8200 ft to 8300 ft the inclination angle is increased to 3 deg. and azimuth to N21E hence we calculate:

$$x_4 = 3.28 + (100)\sin\left(\frac{2+3}{2}\right)\cos\left(\frac{20+21}{2}\right) = 7.37\,ft.$$

$$y_4 = 1.20 + (100)\sin\left(\frac{2+3}{2}\right)\sin\left(\frac{20+21}{2}\right) = 2.73\,ft.$$

$$z_4 = 8199.96 + (100)\cos\left(\frac{2+3}{2}\right) = 8299.86\,ft.$$

Design of Three Dimensional Well Profiles

The Average Angle Method

From 8300 ft. to 8400 ft. the inclination and azimuth angles increase to 4 deg. and N22E and we get:

$$x_5 = 7.37 + (100)\sin\left(\frac{3+4}{2}\right)\cos\left(\frac{21+22}{2}\right) = 13.05\,ft.$$

$$y_5 = 2.73 + (100)\sin\left(\frac{3+4}{2}\right)\sin\left(\frac{21+22}{2}\right) = 4.97\,ft.$$

$$z_5 = 8299.86 + (100)\cos\left(\frac{3+4}{2}\right) = 8399.67\,ft.$$

Design of Three Dimensional Well Profiles

The Average Angle Method

The total horizontal departure is:

$$HD = \sqrt{(\Sigma\Delta x_i)^2 + (\Sigma\Delta y_i)^2} = \sqrt{(13.05)^2 + (4.97)^2} = 13.96 \text{ ft}$$

$$\text{Departure angle} = \arctan\left(\frac{\Sigma\Delta y_i}{\Sigma\Delta x}\right) = \arctan\left(\frac{4.97}{13.05}\right) = 20.8 \text{ deg}$$

Design of Three Dimensional Well Profiles

Tangential Method

This method treats the inclination angles and azimuths constant and equal to the inclination angles and azimuths at point 2.

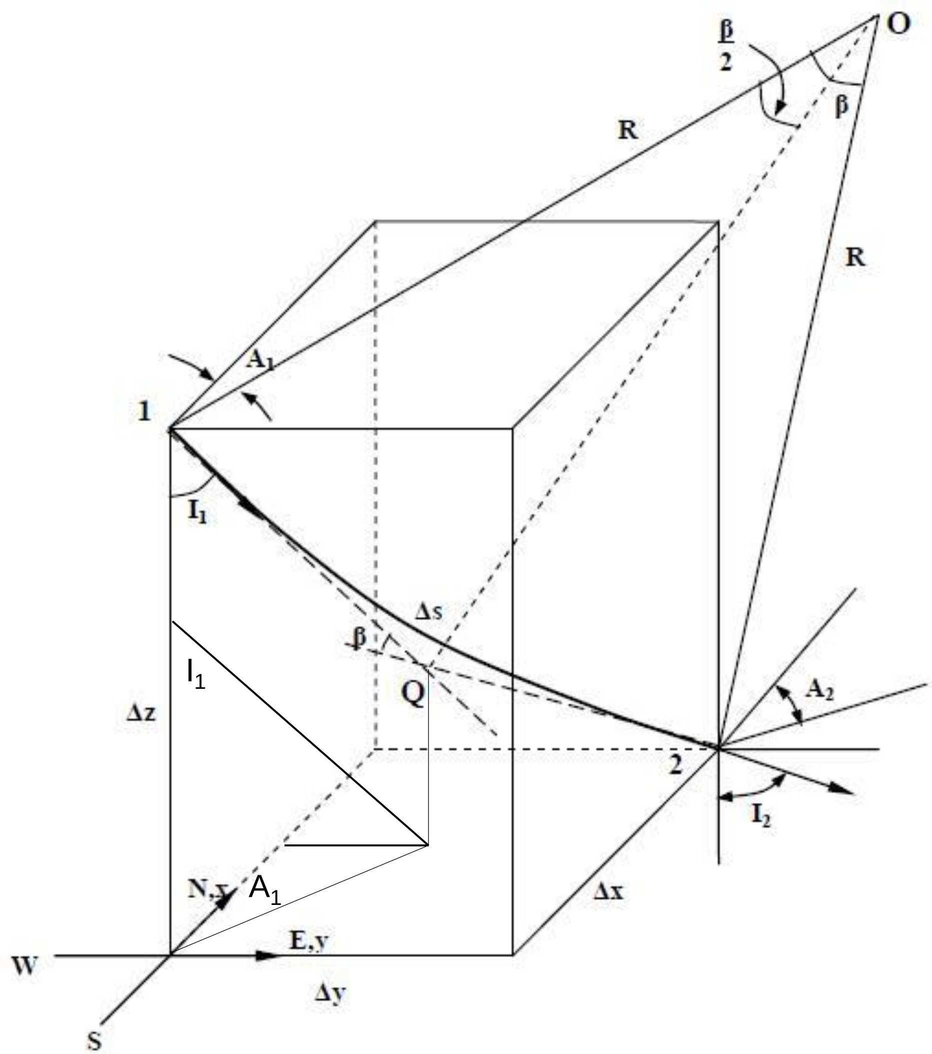
$$x_2 = x_1 + \left(\sin \bar{I} \cos \bar{A}\right)\Delta s \qquad x_2 = x_1 + (\sin I_2 \cos A_2)\Delta s$$

$$y_2 = y_1 + \left(\sin \bar{I} \sin \bar{A}\right)\Delta s \qquad y_2 = y_1 + (\sin I_2 \sin A_2)\Delta s$$

$$z_2 = z_1 + \left(\cos \bar{I}\right)\Delta s \qquad z_2 = z_1 + (\cos I_2)\Delta s$$

Design of Three Dimensional Well Profiles

Balanced Tangential Method



Design of Three Dimensional Well Profiles

Balanced Tangential Method

This method assumes that the actual wellpath can be approximated by two straight line segments of equal length.

$$\Delta x = \frac{\Delta s}{2}\left(\sin I_1 \cos A_1 + \sin I_2 \cos A_2\right)$$

$$\Delta y = \frac{\Delta s}{2}\left(\sin I_1 \sin A_1 + \sin I_2 \sin A_2\right)$$

$$\Delta z = \frac{\Delta s}{2}\left(\cos I_1 + \cos I_2\right)$$

$$North = \frac{MD}{2} \times \left[\sin(I_1) \times \cos(A_{z1}) + \sin(I_2) \times \cos(A_{z2})\right]$$

$$East = \frac{MD}{2} \times \left[\sin(I_1) \times \sin(A_{z1}) + \sin(I_2) \times \sin(A_{z2})\right]$$

$$TVD = \frac{MD}{2} \times \left[\cos(I_1) + \cos(I_2)\right]$$

Design of Three Dimensional Well Profiles

Balanced Tangential Method

<u>Survey 1</u>

Depth = 3500 ft

Inclination = 15 degree (I_1)

Azimuth = 20degree (A_1)

<u>Survey 2</u>

Depth = 3600 ft

Inclination = 25 degree (I_2)

Azimuth = 45 degree (A_2)

Using balanced tangential method to calculate the Δx, Δy, and Δz.

Design of Three Dimensional Well Profiles

Balanced Tangential Method

MD = 3600 – 3500 = 100 ft

$$North = \frac{100}{2} \times [\sin(15) \times \cos(20) + \sin(25) \times \cos(45)]$$

$$East = \frac{100}{2} \times [\sin(15) \times \sin(20) + \sin(25) \times \sin(45)]$$

$$TVD = \frac{100}{2} \times [\cos(15) + \cos(25)]$$

North = 27.1 ft

East = 19.37 ft

TVD = 93.61 ft

Design of Three Dimensional Well Profiles

Radius of Curvature Method

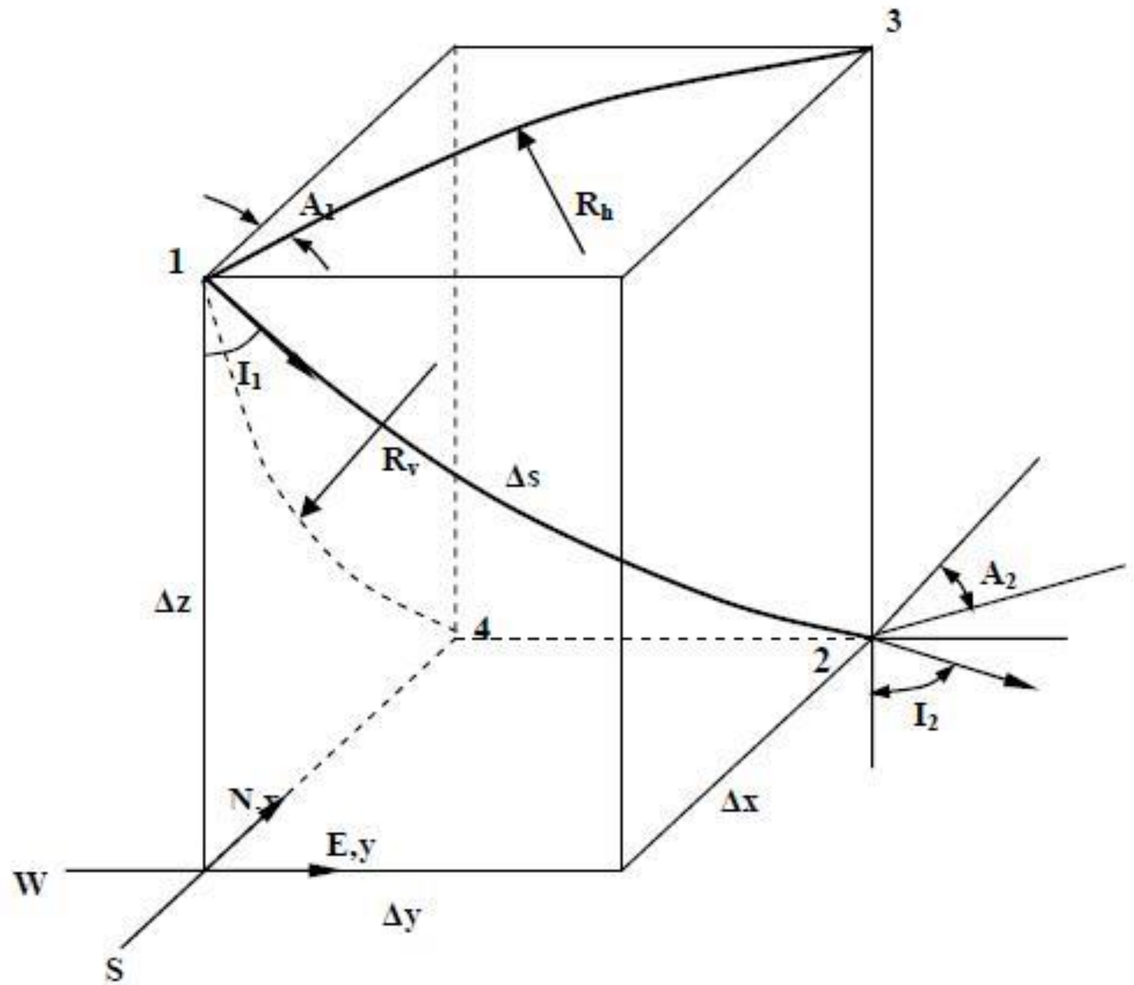
The Radius of Curvature Method (RCM) was originally proposed by Wilson (1968) to replace earlier methods that used a series of straight line segments representing wellbore between surveying stations.

In this method it is assumed that the build rate B and horizontal turn rate H are constant on the trajectory.

This is the most famous method for directional survey calculation.

Design of Three Dimensional Well Profiles

Radius of Curvature Method



B and H are constant imply that the projections on the vertical and horizontal plane have constant curvature.

$R_v = 1/B$

$R_h = 1/H$

Design of Three Dimensional Well Profiles

Radius of Curvature Method

It should be pointed out that even if build and turn rates are constant the wellbore curvature is not constant between the two points on the well path since the hole inclination angle varies between Point 1 and Point 2.

With the assumption that both built and turn rates are constant (B=const and H=const) by integration of Eqn. (13) , Eqn. (14) and Eqn. (15) we obtain the following equations for calculating the desired rectangular trajectory coordinates:

Design of Three Dimensional Well Profiles

Radius of Curvature Method

$$B = \frac{dI}{ds} \qquad T = \frac{dA}{ds} \qquad H = \frac{dA}{dl} \qquad T = \frac{dA}{dl}\frac{dl}{ds} = HsinI$$

$$\Delta x = x_2 - x_1 = \int_{s_1}^{s_2} \sin I(s) \cos A(s)\, ds = \int \frac{1}{T} sinIcosAdA = \int \frac{1}{HsinI} sinIcosAdA = \int \frac{1}{H} cosAdA$$

$$\Delta y = y_2 - y_1 = \int_{s_1}^{s_2} \sin I(s) \sin A(s)\, ds = \int \frac{1}{T} sinIsinAdA = \int \frac{1}{HsinI} sinIsinAdA = \int \frac{1}{H} sinAdA$$

$$\Delta z = z_2 - z_1 = \int_{s_1}^{s_2} \cos I(s) ds = \int \frac{1}{B} cosIdI =$$

Design of Three Dimensional Well Profiles

Radius of Curvature Method

$$\Delta x = \int_{A_1}^{A_2} \frac{1}{H} \cos A(s)\, dA = \frac{1}{H}\left(\sin A_2 - \sin A_1\right) \quad \text{30a}$$

$$\Delta y = \int_{A_1}^{A_2} \frac{1}{H} \sin A(s)\, dA = \frac{1}{H}\left(\cos A_1 - \cos A_2\right) \quad \text{30b}$$

$$\Delta z = \int_{I_1}^{I_2} \frac{1}{B} \cos I(s)\, dI = \frac{1}{B}\left(\sin I_2 - \sin I_1\right) \quad \text{30c}$$

$$North = \frac{MD \times (\cos I_1 - \cos I_2) \times (\sin A_2 - \sin A_1)}{(I_2 - I_1)(A_2 - A_1)} \times \left(\frac{180}{\pi}\right)^2$$

$$East = \frac{MD \times (\cos I_1 - \cos I_2) \times (\cos A_1 - \cos A_2)}{(I_2 - I_1)(A_2 - A_1)} \times \left(\frac{180}{\pi}\right)^2$$

$$TVD = \frac{MD \times (\sin I_2 - \sin I_1)}{I_2 - I_1} \times \left(\frac{180}{\pi}\right)$$

Design of Three Dimensional Well Profiles

Radius of Curvature Method

Example 5

At a certain point P1 on a well path the inclination angle and azimuth are: I_1 = 10.8 deg and A_1 = 36.5 deg. Assuming an increment in the measured depth Δs = 200 ft (distance between Point 1 and Point 2) at Point 2 we need to calculate:

- hole inclination angle I_2 and azimuth A_2
- the increments in x, y, z coordinates Δx, Δy and Δz
- wellbore curvature (dogleg severity)
- Dogleg

Assume build rate B = 5.14 deg/100ft and horizontal turn rate H = 17 deg/100ft

Design of Three Dimensional Well Profiles

Radius of Curvature Method

Solution

Integrating Eqn (1) gives:

$$\int_{o}^{\Delta s} B ds = \int_{I_1}^{I_2} dI \qquad I_2 = I_1 + B\Delta s \qquad \text{(a)}$$

Hence we get:

$$I_2 = 10.8 + \frac{5.14}{100}(200) = 21.08 \text{ deg} \qquad \text{(b)}$$

$$B = \frac{dI}{ds} = \frac{dI}{dl} \times \frac{dl}{ds} = \frac{dI}{dl} sinI \quad \rightarrow \quad dl = \frac{sinI dI}{B}$$

$$\int_{A_1}^{A_2} dA = \int_{l_1}^{l_2} H dl = \frac{H}{B} \int_{I_1}^{I_2} sinI dI \qquad \text{(c)}$$

Design of Three Dimensional Well Profiles

Radius of Curvature Method

Applying Eqn. (2) and Eqn. (6) we get

$$\int_{A_1}^{A_2} dA = \frac{H}{B}\int_{I_1}^{I_2} \sin I \; dI \qquad \text{(c)}$$

And, upon integration:

$$A_2 = A_1 + \frac{H}{B}\left(\cos I_1 - \cos I_2\right) \qquad \text{(d)}$$

The second term in Eqn (d) needs to be expressed in degrees if A_1 is in degrees, hence, we write:

$$A_2 = 36.5 + \frac{17}{5.14}\left(\frac{180}{\pi}\right)\left(\cos 10.8 - \cos 21.08\right) = 45.8 \;\; \text{deg} \qquad \text{(e)}$$

Design of Three Dimensional Well Profiles

Radius of Curvature Method

Now we can calculate the increments in coordinates using Eqn. (30a), Eqn. (30b) and Eqn. (30c) to be: $\Delta x = 41.14\,ft,\ \ \Delta y = 35.95\,ft,\ \ and\ \ \Delta z = 192.03\,ft$

To calculate the wellbore curvature (dogleg severity) we use $K = DLS = \sqrt{B^2 + H^2 sin^4 I}$

$$DLS = \sqrt{(5.14)^2 + (17.0)^2 \sin^4 I(s)} \quad \deg/100\,ft \qquad \text{(f)}$$

Clearly the wellbore curvature is not constant between the two points, but changes with the inclination angle. If we use the average inclination angle $\left(\frac{I_1 + I_2}{2}\right)$ for determining the "average DLS" we obtain:

$$DLS = \sqrt{(5.14)^2 + (17.0)^2 \sin^4 (15.94)} = 5.3\ \deg/100\,ft \qquad \text{(g)}$$

Design of Three Dimensional Well Profiles

Radius of Curvature Method

To calculate the dog leg, we use Lubinski's equation

$$\beta = 2\arcsin\sqrt{\sin^2\left(\frac{I_2 - I_1}{2}\right) + \sin I_1 \sin I_2 \sin^2\left(\frac{A_2 - A_1}{2}\right)} \qquad (26)$$

$$DL = 2\arcsin\sqrt{\sin^2(5.14) + \sin(10.8)\sin(21.08)\sin^2(4.65)} = 10.56\,\text{deg} \qquad (h)$$

Design of Three Dimensional Well Profiles

Minimum Curvature Method

The analytical formulations of minimum curvature method were originally proposed by Taylor and Mason (1971)

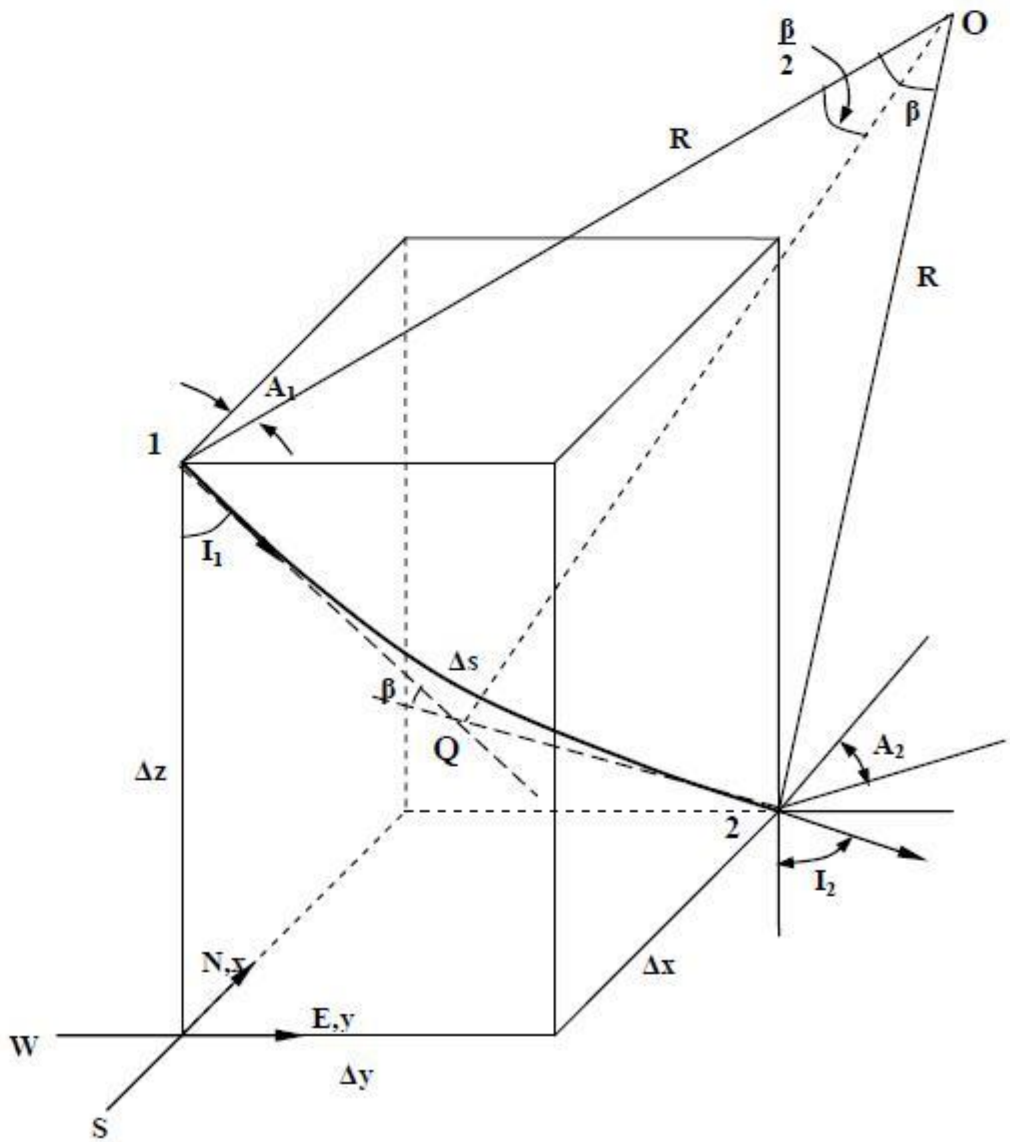
In this method, two successive points on the trajectory are assumed to lie on a circular arc that is located in a plane as schematically depicted in the figure next slide.

In other words, Point 1, Point 2 and Point O in lie on the same plane and the curvature of the segment between Point 1 and Point 2 is constant.

The measured depth between Point 1 and Point 2 is Δs and the radius of circular arc connecting the two points is R. The angle β is called the dogeleg.

Design of Three Dimensional Well Profiles

Minimum Curvature Method



Design of Three Dimensional Well Profiles

Minimum Curvature Method

$$\Delta x = \left(\sin I_1 \cos A_1 + \sin I_2 \cos A_2\right) RF$$

$$\Delta y = \left(\sin I_1 \sin A_1 + \sin I_2 \sin A_2\right) RF$$

$$\Delta z = \left(\cos I_1 + \cos I_2\right) RF$$

where

$$RF = \frac{\Delta s}{\beta} \tan \frac{\beta}{2} \qquad RF = \frac{1Q}{\Delta s / 2}$$

From Eqn(32) we can notice that for small values of dogleg $\beta, RF = \frac{\Delta s}{2}$

Design of Three Dimensional Well Profiles

Minimum Curvature Method

Example 7

Given are the following data:

	Measured depth, ft.	Inclination angle, deg.	Azimuth angle, deg.	x	Coordinates y	z
Point 1	1050	3.15	350	102	99	1044
Point 2	1627	32.11	76	?	?	?

Find the x, y, and z coordinates at Point 2

Design of Three Dimensional Well Profiles

Minimum Curvature Method

Solution

Using Eqn. (26) we find the dogleg β

$$\beta = 2arc\sin\sqrt{\sin^2\left(\frac{32.11-3.15}{2}\right)+\sin(3.15)\sin(32.11)\sin^2\left(\frac{76-350}{2}\right)} = 32.03\deg \quad \text{(a)}$$

From Eqn. (32) the ratio factor is: $RF = \frac{\Delta s}{\beta} tan\frac{\beta}{2}$

$$RF = \frac{577}{32.03}\left(\frac{180}{\pi}\right)\tan\left(\frac{32.03}{2}\right) = 296.1\,ft \qquad \text{(b)}$$

Design of Three Dimensional Well Profiles

Minimum Curvature Method

Hence, applying Eqn. (31a), Eqn. (31b) and Eqn. (31c) we obtain:

$$x_2 = 102 + (\sin 3.15 \cos 350 + \sin 32.11 \cos 76)(296.1) = 156\,ft$$

$$y_2 = 99 + (\sin 3.15 \cos 350 + \sin 32.11 \sin 76)(296.1) = 248\,ft$$

$$z_2 = 1044 + (\cos 3.15 + \cos 32.11)(296.1) = 1590\,ft$$

Design of Three Dimensional Well Profiles

Constant Build and Turn Rate Method

In this method, proposed by Planeix (1971), we assume that build rate *B and turn rate T is* constant along the well trajectory. With this assumption by integrating Eqn. (13) and Eqn. (14) we obtain the trajectory coordinates as follows:

$$\Delta x = \frac{1}{T^2 - B^2}\left[T\left(\sin I_2 \sin A_2 - \sin A_1 \sin A_1\right) + B\left(\cos I_2 \cos A_2 - \cos I_1 \cos I_1\right)\right] \qquad (33)$$

and

$$\Delta y = \frac{1}{T^2 - B^2}\left[B\left(\sin A_2 \cos I_2 - \sin A_1 \cos I_1\right) - T\left(\cos A_2 \sin I_2 - \cos A_1 \sin I_1\right)\right] \qquad (34)$$

$\Delta z = \int_{I_1}^{I_2} \frac{1}{B} \cos I(s)\, dI = \frac{1}{B}\left(\sin I_2 - \sin I_1\right)$: the same as radius curvature method

Design of Three Dimensional Well Profiles

Constant Build and Turn Rate Method

It should be remembered that Eqn (33) and Eqn (34) are valid only for points on the build and turn curves. As for the radius of curvature method, one can consider several special cases such as: *T = 0 and B = const; B = 0 and T = const or perhaps B = 0 and T = 0.* Because the turn rate and horizontal turn rate are functionally related (*T = H sin I) the* solution for *Δx, Δy, Δz, are all the same as the radius of curvature method.*

Here we consider a special case of a wellbore trajectory that is composed of a segment of a circular helix. Because a circular helix has constant curvature and constant inclination angle the build rate is nil (B = 0) and the turn rate T is constant.

Design of Three Dimensional Well Profiles

Constant Build and Turn Rate Method

Example 8

Given are hole inclination and azimuth angles at the two points 100 ft apart on a trajectory composed of a part of circular helix.

$$I_1 = 46.31 \text{ deg} \quad \text{and} \quad I_2 = 46.31 \text{ deg}$$

$$A_1 = 65.5 \text{ deg} \quad \text{and} \quad A_2 = 73.78 \text{ deg}$$

It is required to calculate:

- dogleg severity and dogleg
- pitch and radius of the helix

Design of Three Dimensional Well Profiles

Constant Build and Turn Rate Method

Solution

To calculate the dogleg severity (curvature) we use Eqn. (22). Since $I = const, B = 0$ we get

$$DLS = \sin I \frac{dA}{ds} \qquad \text{(a)}$$

For a circular helix the curvature is constant so integrating the above equation yields

$$DLS = \sin I \frac{\Delta A}{\Delta s} = (1.045)\left(10^{-3}\right) rd/ft = 5.987 \deg/100\,ft \qquad \text{(b)}$$

Hence the turn rate T = 8.28 deg/100 ft (0.001445 rd/ft) and the dogleg is 5.987 deg.

Design of Three Dimensional Well Profiles

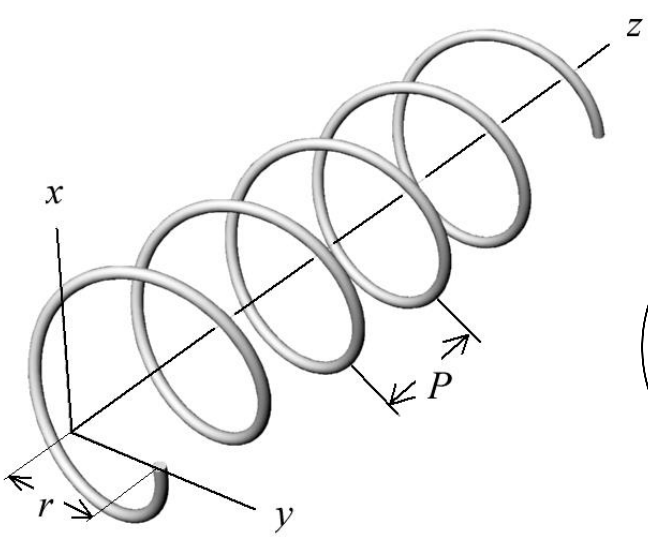
Constant Build and Turn Rate Method

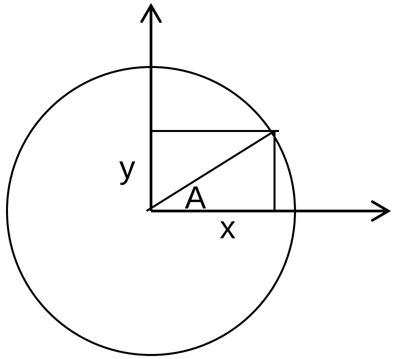
From calculus we know that the *x, y, z* coordinates of circular helix with the radius 'r' and pitch 'p' are given by the following equations:

$$x(s) = r\cos A(s) \quad \text{(c)}$$

$$y(s) = r\sin A(s) \quad \text{(d)}$$

$$z(s) = \frac{p}{2\pi}A(s) \quad \text{(e)}$$





Design of Three Dimensional Well Profiles

Constant Build and Turn Rate Method

Taking the derivatives with respect to ‘s’ we obtain

$$\frac{dx}{ds} = -rT\sin A \tag{f}$$

$$\frac{dy}{ds} = -rT\cos A \tag{g}$$

$$\frac{dz}{ds} = \frac{p}{2\pi}T \tag{h}$$

We also know (Eqn(7) and Eqn(9)) that: $\frac{dz}{ds} = \cos I \; and \; \frac{dy}{ds} = \sin I \cos A$

Design of Three Dimensional Well Profiles

Constant Build and Turn Rate Method

Considering the above equations, we obtain

- the radius of helix

$$r = \frac{\sin I}{T} = \frac{\sin(46.31)}{0.001446} = 500\,ft \qquad \text{(i)}$$

- the pitch of helix

$$p = \frac{2\pi \cos I}{T} = \frac{2\pi \cos(46.31)}{0.0011446} = 3000\,ft \qquad \text{(j)}$$

For practical 3D well path calculations the designer can assume the radius, pitch of helix and the required hole inclination angle and then calculate the turn rate, hole azimuth and the corresponding coordinates x,y and z along the well path.

Design of Three Dimensional Well Profiles

Constant Curvature and Build Rate Method

The constant curvature method was proposed by Guo, Miska and Lee (1992) to produce a well path that can be drilled with constant tool face (will be discussed in the next chapter) and more flexibility in 3D well path trajectory designs. In this method it is assumed that the wellbore curvature (κ ,*DLS) and build rate B are constant along the measured depth s.* This method is also known as Constant Tool Face Method and was presented by Frank J. Schuh (1992).

Again to calculate the coordinates we need to use Eqn.(13), Eqn.(14) and Eqn.(15). To perform the required integrations, we need to determine the inclination and azimuth angles along the trajectory.

Design of Three Dimensional Well Profiles

Constant Curvature and Build Rate Method

Applying Eqn. (1) and Eqn. (3) we write

$$I(s) = I_1 + B(s - s_1) \qquad (35)$$

and

$$A(s) = A_1 + \int_{s_1}^{s_2} T(s)dL \qquad (36)$$

The turn rate in Eqn. (36) can be expressed in terms of dogleg severity and build rate as below

$$T(s) = \frac{\sqrt{DLS^2 - B^2}}{\sin I(s)} \qquad (37)$$

Design of Three Dimensional Well Profiles

Constant Curvature and Build Rate Method

With the assumption that the dogleg severity and built rate are constant integration of Eqn (36) yields:

$$A(s) = A_1 + \frac{\sqrt{DLS^2 - B^2}}{B} \ln\left(\frac{\tan \frac{I(s)}{2}}{\tan \frac{I_1}{2}} \right) \quad (38)$$

It is clear that due to the nonlinear form of Eqn. (38) the integrals of the trajectory equations Eqn. (13) and Eqn. (14) needs to be done numerically. Close form solutions can be obtain for a case of when the well path is a part of a circular helix. For a circular helix the build rate is nil resulting in constant hole inclination angle and constant turn rate as discussed earlier. Much simpler solutions are possible if the average values of turn rates are used piecewise for calculations.

Design of Three Dimensional Well Profiles

Constant Curvature and Build Rate Method

The average values are given by the following equations:

$$\bar{H} = \frac{\sqrt{DLS^2 - B^2}}{B\ \Delta s} \int_{I_1}^{I_2} \frac{dI}{\sin^2 I} = \frac{\sqrt{DLS^2 - B^2}}{B\ \Delta s} (\cot I_1 - \cot I_2) \qquad (39)$$

$$\bar{T} = \frac{\sqrt{DLS^2 - B^2}}{B\ \Delta s} \int_{I_1}^{I_2} \frac{dI}{\sin I} = \frac{\sqrt{DLS^2 - B^2}}{B\ \Delta s} \ln \left(\frac{\tan \frac{I_2}{2}}{\tan \frac{I_1}{2}} \right) \qquad (40)$$

Once the average values are determined one can use the radius of curvature or constant build and turn method to calculate the desired rectangular coordinate x,y and z along the well trajectory.