1) Given 3 T/F ques, no of ways to answer

| 1 | 2 | 3 |
|---|---|---|
| 2 | 2 | 2 |

AND $\Rightarrow$ *

$2 \times 2 \times 2 = 8$

Begines fighter Pro

FFF FFT FTF FTT

TFF TFT TTF TTT

Secret of Success

1) Class
2) Ass / HW
3) Contest
4) Viva / MI

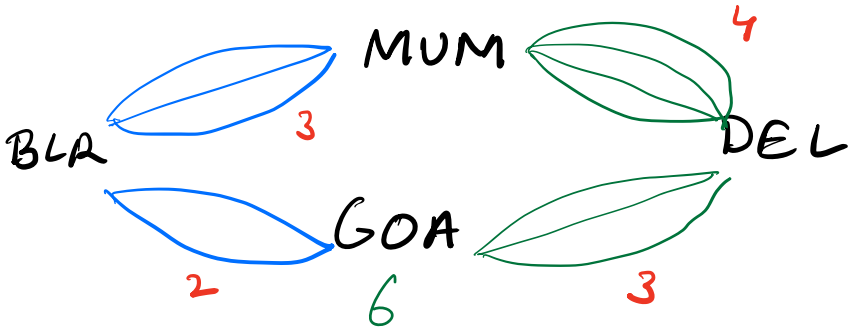
2) 10 boys & 7 girls. How many pairs can be formed

1 B AND 1 G

10 7

$10 \times 7 = 70$

12




ways = 12 + 6 = 18

BLR to DEL via MUM OR BLR to DEL via GOA

AND → *  OR → +

Permutation : arrangement of objects

Q Given 3 distinct characters.

How many ways to arrange them

Let chars be a, b, c

a b c
a c b
b a c
b c a
c a b
c b a

3 2 1

$3 \times 2 \times 1 = 6$

$3!$

Q Given 4 distinct characters.

How many ways to arrange them

4 3 2 1

$4! = 24$

Q Given N distinct characters.
How many ways to arrange them

_ _ _ . . . . . _

ans = $n!$

Q 5 distinct chars, ways to arrange 2 of them

5 4 $\quad 5 \times 4 = 20$

$\frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} \quad \frac{5!}{3!} = \frac{5!}{(5-2)!}$

5 chars, arrange 3

5 4 3

$\frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{5!}{2!} = \frac{5!}{(5-3)!}$

- $n$ objects, $r$ arrange

permutation $\frac{n!}{(n-r)!} = {}^{n}P_{r}$

Combination : selection of objects. x y
y x

5 objects, select 2

5 4 $\quad \frac{5 \times 4}{2} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 3 \times 2 \times 1}$

$= \frac{5!}{2!\ 3!}$

N objects, select r =

$$\frac{{}^{n}P_r}{r!} = \frac{n!}{(n-r)!\ r!} = {}^{n}C_r$$

- ${}^{n}C_r \leq {}^{n}P_r$

Permutation - Care abt order
Combination/ Choose — Dont care about order

## Properties

1) $^{N}C_0 + {}^{N}C_1 + {}^{N}C_2 + \cdots + {}^{N}C_N = 2^N$

Proof:

$^{N}C_0 \Rightarrow$ select 0 items

$^{N}C_1 \Rightarrow$ select 1 items

$\vdots$

$^{N}C_N \Rightarrow$ select N items

Total no of ways of all selections.

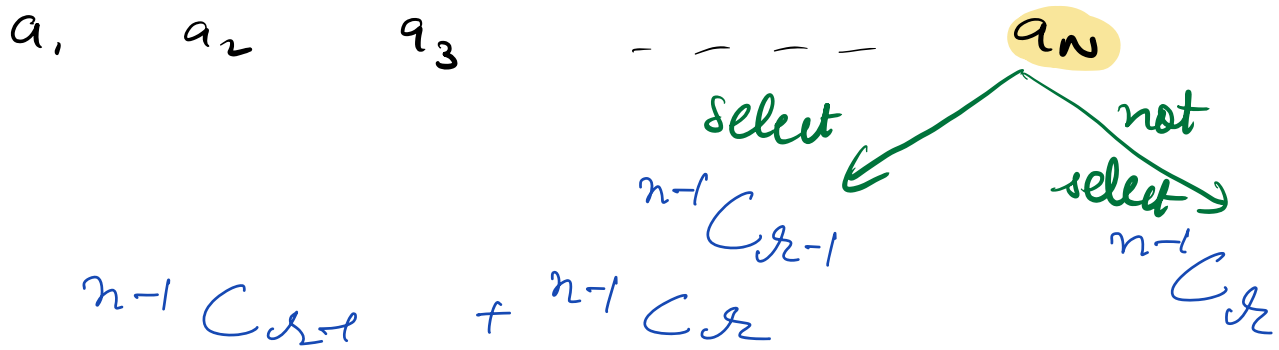
$2 \quad 2 \quad 2 \quad \cdots \quad 2$

$\underbrace{2 \times 2 \times 2 \cdots \times 2}_{2^n}$

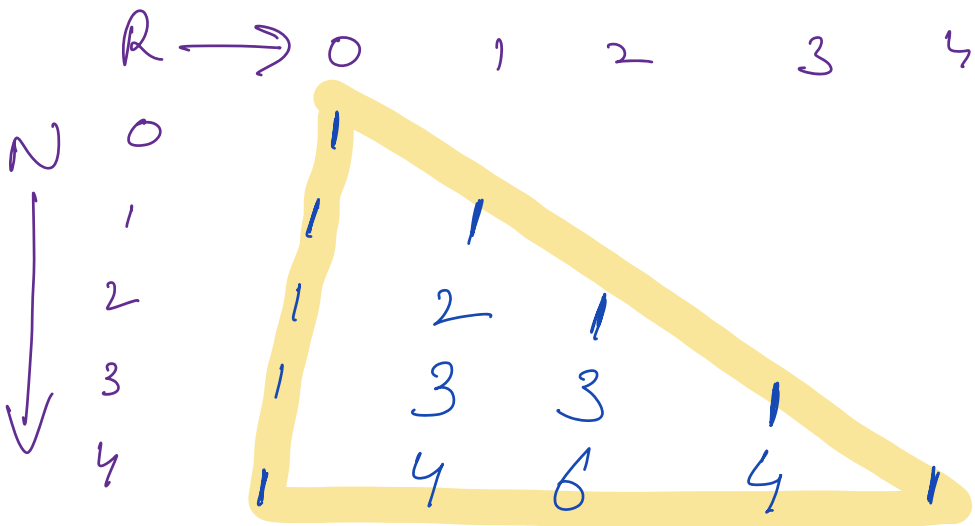
2) Pascals Triangle

$$^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}$$

Proof




$^{n-1}C_{r-1} + {}^{n-1}C_{r}$




top + top-left

$$^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}$$

top top-left

$$^{i}C_{j} = {}^{i-1}C_{j} + {}^{i-1}C_{j-1}$$

Q Find $^{n}C_{r}$ % M

Code `c[N+1][n+1] = {0};` initialize with 0.

```
c[0][0] = 1
for (i=1; i<=N; i++) {
    c[i][0] = c[i][i] = 1
    for (j=1; j<i; j++) {
        c[i][j] = (c[i-1][j] % M
                 + c[i-1][j-1] % M) % M
    }
}
```

**Inverse Modulo** Fermat theorem

$p \Rightarrow$ prime

$(a * x) \% p = 1$

then $(a^{-1}) \% p = x$

$a^{-1} \text{ wrt } p = a^{p-2} \% p$

If $p$ is prime

$a^{-1} = (a^{p-2}) \% p$

pow(a, p-2, p)

$$a^b \% m = \left[(a \% m)^b\right] \% m$$

- If $p$ is prime ${}^nC_r = \frac{n!}{r!\,(n-r)!}$

$$\frac{1}{r!} \% p = \left[(r!)^{p-2}\right] \% p$$

$$= \left[\left[r! \% p\right]^{p-2}\right] \% p$$

$fact\_r = r! \% p$

$fact\_nr = (n-r)! \% p$

${}^nC_r = (n! \% p) \times pow(fact\_r, p-2, p) \times pow(fact\_nr, p-2, p)$

$(a \times b \times c) \% p = ((a \times b) \% p \times c) \% p$

$= \left(\left[(n! \% p) \times pow(fact\_r, p-2, p)\right] \% p \times pow(fact\_nr, p-2, p)\right) \% p$

Q. Excel Column Title

1 → A

2 → B

3 → C

⋮

26 → Z

27 → AA

28 → AB

⋮

N = ?

Obs → this seems like numbers written in base 26.

27 → $1 \times 26^1 + 1 \times 26^0$ → AA

28 → $1 \times 26^1 + 2 \times 26^0$ → AB

1 0

A Z = 52

$1 \times 26^1 + 26 \times 26^0$

1 0

B A =

$2 \times 26^1 + 1 \times 26^0 = 53$

How to get

If num % 26 == 0

ch = Z

else

ch = 'A' + n % 26 − 1

update N = $\frac{N-1}{26}$

Code

```
string ans = " "
while (N != 0) {
    char ch
    if (N % 26 == 0) {
        ch = 'z'
    }
    else {
        ch = 'A' + N % 26 - 1
    }
    ans = ans + ch
    N = (n-1)/26
}
return rev(ans)
```

{done}

TC: $O(\log_{26} n)$

SC: $O(1)$

52 $\frac{52-1}{26} = 1$

2A

$N = 95$

$95 - 1 - 26 \Rightarrow 17$

QC $\frac{95-1}{26} = 3$ $\frac{3-1}{26}$

CQ