TC-2

1) Time Complexity & Space Complexity.
2) Asymptotic analysis (Big O notation)
3) Big O meaning
4) TLE (Time Limit Exceeded)

No. of iterations.

Quiz-1 No. of times we need to divide N by 2 to make it 1.

$$N \to \frac{N}{2} \to \frac{N}{4} \to \frac{N}{8} \to \frac{N}{16} \to \cdots \cdots \to 1$$

$$\log_2 N$$

* Quiz-2 [3, 10] ⇒ 3, 4, 5, 6, 7, 8, 9, 10

[a,b] ⇒ b-a+1
(a,b) ⇒ b-a-1

Arithmetic Progression (AP)

4, 7, 10, 13, 16, 19, ...

a = 4

3 3 3 3

3 → Common Diff (d)

$a \rightarrow$ first term of AP
$d \rightarrow$ common difference
$N \rightarrow$ no. of terms

$$\text{Sum} = \frac{N}{2}\left(2a + (N-1)d\right)$$

$\Rightarrow$ Sum $= 1 + 2 + 3 + 4 + \ldots\ldots N$

$a = 1$
$d = 1$
$N = N$

$$\text{Sum} = \frac{N}{2}\left(2 \cdot 1 + (N-1) \cdot 1\right)$$

$$= \frac{N}{2}(N+1) = \frac{N(N+1)}{2}$$

* Geometric Progression (GP)

$5, 10, 20, 40, 80, 160, \ldots\ldots$

$\frac{10}{5}$ $\frac{20}{10}$ $\frac{40}{20}$ $\frac{80}{40}$ $\frac{160}{80}$

Every 2 consecutive terms have the **common ratio**.

$$a, ar, ar^2, ar^3, \ldots\ldots ar^{n-1}$$

$a \rightarrow$ first term of GP
$r \rightarrow$ common ratio
$N \rightarrow$ no. of terms

$$\text{Sum} = \frac{a(r^N - 1)}{(r-1)}, \quad r \neq 1$$

Logarithmic Basics (log)

$\log_2 N$ ⇒ No. of times we need to divide N by 2 to make it 1

$\log_x N$ ⇒ No. of times we need to divide N by x to make it 1

$\log_2 2^5 = 5$, $\log_{10} 10^7 = 7$

$$\log_a a^x = x$$

Calculate the No. of iterations.

Quiz-3

```
S = 0
for(i = 1 ; i <= N; i++){
        S = S+i;
}
return S;
```

$i \in [1, N]$

# of iterations = $N - 1 + 1$

$= N$

⇒ $O(N)$

Quiz-4

```
for(i = 1 ; i <= N; i++){
    print(i)
}
```

$i \in [1, N] \Rightarrow N$

```
for(i = 1 ; i <= M ; i++){
    print(i)
}
```

$i \in [1, M] \Rightarrow M$

# of iterations = $N+M \Rightarrow O(N+M)$

Quiz-5

```
s = 0
for(i = 1 ; i <= N; i = i+2){
    s = s+i
}
return s;
```

i

1 ⇒ i = i+2
3 ⇒ i = i+2
5 ⇒ i = i+2
7 ⇒ i = i+2
9 ⋮ ⋮
⋮

⇒ (i) is taking only odd values.

How many odd no's are there from 1 to N

N = 6 ⇒ 1, 3, 5 ⇒ 3
N = 7 ⇒ 1, 3, 5, 7 ⇒ 4
N = 8 ⇒ 1, 3, 5, 7 ⇒ 4
N = 10 ⇒ 1, 3, 5, 7, 9 ⇒ 5

$\frac{(N+1)}{2} \Rightarrow O(N)$

Quiz-6

```
s = 0
for(i = 0; i <= 100; i++){
    ------
}
```

$i \in [0, 100] \Rightarrow 100 - 0 + 1 = 101 \Rightarrow 101 \times N^0 \Rightarrow O(1)$

Quiz-7

```
for( i = 1; i*i <= N; i++){
    ......
}
```

$i <= Sqrt(N)$

$\Rightarrow i * i <= N$

$\Rightarrow i^2 <= N \Rightarrow i <= \sqrt{N}$

$\Rightarrow i \in [1, \sqrt{N}] \Rightarrow \sqrt{N}$

$\Rightarrow Sqrt(N) \Rightarrow O(\sqrt{N})$

Quiz-8

```
i = N
while( i > 1 ){
    i = i / 2;
}
```

i_{Before}	i_{after}	iteration
N	$\frac{N}{2}$	1
$\frac{N}{2}$	$\frac{N}{4}$ $\left(\frac{N}{2^2}\right)$	2
$\frac{N}{4}$	$\frac{N}{8}$ $\left(\frac{N}{2^3}\right)$	3
$\frac{N}{8}$	$\frac{N}{16}$	4
.....	.....	.....
1	$\frac{N}{2^K}$	K

$$\frac{N}{2^k} = 1 \Rightarrow N = 2^k$$

$$\log_2 N = \log_2 2^k \rightarrow k$$

$$\boxed{\log_2 N = k} \Rightarrow O(\log N)$$

Quiz-1

```
for( i = 1 ; i <= N ; i = i x 2 ){
      .....
}
```

i_{before}	i_{after}	iteration
1	2^1	1
2	$4 : 2^2$	2
4	$8 : 2^3$	3
8	$16 : 2^4$	4
16	32	5
⋮	⋮	⋮
	2^k	k

$$2^k = N$$

$$\log_2 2^k = \log N$$

$$k = \log N \Rightarrow O(\log N)$$

Quiz-9

```
for( i = 0 ; i <= N ; i = i*2 ){
    ......
}
```

⇒ Infinite iteration

Quiz-10

```
for( i = 1 ; i <= 10 ; i++ ){
    for( j = 1 ; j <= N ; j++ ){
        ---
    }
}
```

3 → N → 10

⇒ 10*N iterations

i	j	iterations
1	[1, N]	N
2	[1, N]	N
3	[1, N]	N
...		...
10	[1, N]	N

10N iterations

⇒ O(N)

Quiz-11

```
for( i = 1 ; i <= N ; i++ ){
    for( j = 1 ; j <= N ; j++ ){
        .....
    }
}
```

$\Rightarrow N \times N$ iterations

$\Rightarrow N^2$ iterations $\Rightarrow O(N^2)$

Quiz-12

```
for( i = 1 ; i <= N ; i++ ){
    for( j = 1 ; j <= N ; j = j*2 ){
        .....
    }
}
```

↳ $\log N$

$i = 1$, $\log N$ iterations

$i = 2$, $\log N$ iterations

$i = 3$, $\log N$ iterations

⋮

$i = N$, $\log N$ iterations

$N * \log N$

↳ $O(N \log N)$

Quiz-13

```
for( i = 1 ; i <= 2^N ; i++ ){
    .....
}
```

$i \in [1, 2^N] \Rightarrow 2^N$ iterations

$\Rightarrow O(2^N)$

Quiz-14

```
for(i=1; i<=N; i++){
    for(j=1; j<=2^i; j++){
        .....
    }
}
```

i	j	iterations
1	$[1, 2^1]$	2
2	$[1, 2^2]$	2^2
3	$[1, 2^3]$	2^3
4	$[1, 2^4]$	2^4
⋮	⋮	⋮
N	$[1, 2^N]$	2^N

$$\#\ \text{of iterations} = \underbrace{2^1 + 2^2 + 2^3 + 2^4 + \cdots + 2^N}_{\text{GP}}$$

$a \to 2$
$r \to 2$
$\#\text{terms} \to N$

$$= \frac{2(2^N - 1)}{2 - 1}$$

$$= 2(2^N - 1)$$

$$= 2^1 \cdot 2^N - 2$$

$$= 2^{N+1} - 2 \Rightarrow O(2^N)$$

How to find the Big O notation :-

1) No. of iterations.
2) Neglect all the lower order terms.
3) Neglect all constant terms.

Ex # of iterations = $4N^2 + 3N + 1$

$= O(N^2)$

Quiz-15 # of iterations = $4N^2 + 3N + 10^6$

$= O(N^2)$

Quiz-16 # of iterations = $N\sqrt{N} + 4\log N + 31N\log N$

$N = 2^{32}$

$N\sqrt{N} = 2^{32} \cdot \sqrt{2^{32}} = 2^{32} \cdot 2^{16} = 2^{48}$

$N\log N = 2^{32} \cdot \log_2 2^{32} = 2^{32} \cdot 32 = 2^{32} \cdot 2^5$

$= 2^{37}$

$N\sqrt{N} > N\log N$

$\Rightarrow O(N\sqrt{N})$

#

$$O(1) < \log N < \sqrt{N} < N < N\log N < N\sqrt{N} < N^2 < N^3 < 2^N < N! < N^N$$

$N = 32$

1) Ajai
2) Praveen
3) Gaurav Pandey

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