GCD: Greatest Common Divisor OR

HCF: Highest Common Factor

biggest number that divides both a & b

$gcd(a,b)$

$gcd(5,8) = 1$

$gcd(0,17) = 17$

$gcd(20,32) = 4$

$gcd(0,-5) = 5$

$gcd(-8,-18)$

$\Rightarrow gcd(8,18) = 2$

Properties

- $gcd(a,b) = gcd(b,a)$
- $gcd(a,b) = gcd(|a|,|b|)$
- $gcd(0,x) = |x|$

Special property Say $gcd(a,b) = x$ $a > b$

then $gcd(a-b, b) = x$

$(a-b) \% x = (a \% x - b \% x + x) \% x$

$a \% x = 0$, $b \% x = 0$

$(3-6) \% 8 = (3 - 6 + 8) \% 8 = 5$ $\quad 3 - 6 = -3 \neq 0$

Eg $\gcd(23,5) = \gcd(18,5) = \gcd(13,5)$
$= \gcd(8,5) = \gcd(3,5)$

$\gcd(23,5) = \gcd(23\%5, 5)$

Given $A, B > 0$ $\quad a > b$

$\gcd(a,b) = \gcd(a-b, b)$

10, 8 $\quad = \gcd(a-2b, b)$

8, 2 $\quad = \gcd(a-3b, b)$

2, 0

$\vdots$

$= \gcd(a-xb, b)$

subtracting the max divisor

$= \gcd(a\%b, b)$

as $a\%b < b$, write this as $\gcd(b, a\%b)$

$$\gcd(a,b) = \gcd(b, a\%b)$$

```
int gcd ( int a, int b) {
    if (b == 0) return a
    if (a == 0) return b
    return gcd (b, a %. b)
}
```

Euclidian GCD algorithm.

- What is TC ?

$gcd(a,b) = gcd(b, a \%. b)$

Now, compare $a \%. b$ & $a/2$

$b < a/2$	$b = a/2$	$b > a/2$
$a \%. b < b < a/2$	$a \%. b < b$	$2b > a$
$a \%. b < a/2$	$a \%. b < a/2$	$*-1$ $2b - a > 0$
		$a - 2b < 0$
		$2a - 2b < a$
		$2(a-b) < a$
		$a - b < a/2$

$a - b \geq a \%. b$

Now $a \% b < a - b$

Because we will subtract atleast once.

Hence $a \% b \leq a - b < a/2$

Hence each time, we are halving the bigger number

$\Rightarrow N \rightarrow N/2 \rightarrow N/4 \rightarrow N/8 \ldots\ldots \rightarrow 1$

$\log_2 N$

Hence $TC = O(\log_2 a)$

Q) Given N elements, calc gcd of entire array.

Eg - {6, 12, 15} ans = 3

~~Ø~~ 4 {8, 16, 12, 10} ans = 2

- Idea : Keep taking gcd till the end. initial value = 0

```
int gcd_arr (int ar[], int N) {
    int ans = 0
    for (i=0; i<n; i++) {
        ans = gcd (max(ans, ar[i]),
                   min(ans, ar[i]))
    }
    return ans
}
```

TC:

$n \log(\max(arr))$

Q2 Given an array, check if there is a subsequence with gcd = 1

dont consider empty.

Eg - {4, 6, 3, 8} ⇒ 17

{2, 4, 6} ⇒

- Brute force - Check for all subseq
- Idea - What is gcd(x, 1) 1

This means if some numbers have gcd = 1, taking more numbers will be gcd = 1

- Final Idea ⇒ If there is a subseq with gcd = 1 ⟶ gcd of array = 1

Code

```
bool check_gcd_one (int arr[], int N) {
    gcd_of_array = gcd_arr(A, N)
    if ( gcd_of_array == 1)
        return true
    else return false
}
```

Q3 Given N array elements, we have to delete 1 elem such that gcd of remaining is max.

	0	1	2	3	4
Eg -	24	16	18	30	15

ans = 3 (delete 16)

- Brute force: Try deleting all elem one by one & calc the GCD.

TC: $O(n^2 \log(\text{max\_of\_array}))$

Assume N=7

	0	1	2	3	4	5	6

Delete	gcd
0	gcd [1,6]
1	gcd (gcd [0,0], gcd [2,6])
2	gcd (gcd [0,1], gcd [3,6])
3	gcd (gcd [0,2], gcd [4,6])
4	gcd (gcd [0,3], gcd [5,6])
5	gcd (gcd [0,4], gcd [6,6])
6	gcd [0,5]

Remember Prefix & Suffix Max 

We can do same for gcd also.

Pfgcd [i] = gcd of all elem [0, i]

Sfgcd [i] = gcd of all elem [i, n-1]

Code

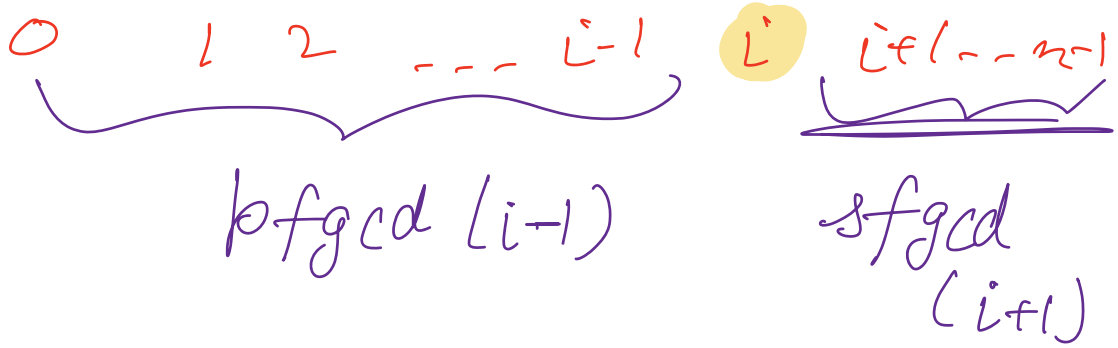
```
pfgcd[N] , sfgcd[N]

pfgcd[0] = a[0]
for(i=1 ; i<N ; i++){
    pfgcd[i] = gcd( pfgcd[i-1] , a[i])
}

sfgcd[n-1] = a[n-1]
for( i=n-2 ; i>=0 ; i--){
    sfgcd[i] = gcd( sfgcd[i+1] , a[i])
}

// Now try deleting every elem.
```

gcd ↳ max, min handling



```
ans = 0
for (i=0; i<n; i++){
    //delete ith
    left_gcd = pfgcd(i-1)

    right_gcd = sfgcd(i+1)

    TODO edge cases for i=0, n-1

    ans = max(ans, gcd(left_gcd,
                       right_gcd))
}
```

TC: $n \log(max)$ SC: $O(N)$

Gcd first argument has to be max.

Q4 PubG

N players, each with health A[i].
Player i attacks player j

$A_i \geq A_j$ j dies

$A_j > A_i \rightarrow A_j = A_j - A_i$

Find min health of last surviving player

0 1

A: [10, 6] ⇒ 2

1 attacks 0 → 4, 6

0 attacks 1 → 4, 2

2, 2

2, 0

Obs: less health player should attack big health player

[9, 6, 15] ⇒

ans = 3

9, 6, 9
9, 6, 3
6, 6, 3
6, 3, 3
3, 3, 3
3, 0, 3
0, 0, 3

done

ans ⇒ gcd of the whole array.

TC: O(n log(max))

0 1 2 } --- 10

1 2 3 } --- 10 0

$a = 7x \qquad b = 8x$

$a^{m-1} \;\%\; m = 1$

$a^{2(m-1)} \;\%\; m = 1$

$a^{3(m-1)} \;\%\; m = 1$

$a^{b} \;\%\; m$

$= a^{b\%(m-1)} \;\%\; m$

$$a^{x(m-1)} \equiv_m 1$$

$$a^{x(m-1)+3} \equiv_m$$

$$= a^3 \equiv_m$$

$$a^{b!} \equiv_m$$

$$= \left(a^{(b! \equiv_{m-1})}\right) \equiv_m$$