% ⇒ Modulus Operator

↳ Remainder

n % a ⇒ Remainder when n is divided by a.

10 % 4 = 2 | 13 % 5 = 3

5) 13 (2 → Q
d ← 5, 13 → D
−10
3 → R

13 = 2 * 5 + 3

$$\boxed{\text{Divident} = \text{Quotient} * \text{divisor} + \text{remainder.}}$$

Remainder = Divident − Quotient * divisor

largest multiple of divisor <= Divident.

* 13 % 5 = 13 − 10
  = 3

* 10 % 4 = 10 − x * 4
  = 10 − 8 = 2

Quiz

150 % 11 = 7

11) 150 (13
−11
40
33
7

Quiz $100 \% 7 \Rightarrow 100 - 98 = 2$

Quiz $-43, -42, -75, -32$

$-32 > -42 > -43 > -75$

Quiz $-40 \% 7$

$r = -40 - (\text{largest multiple of } 7 <= -40)$

$= -40 - (-42)$

$= -40 + 42 = \underline{\underline{2}}$

Quiz $-60 \% 9$

$-60 - (\text{largest multiple of } 9 <= -60)$

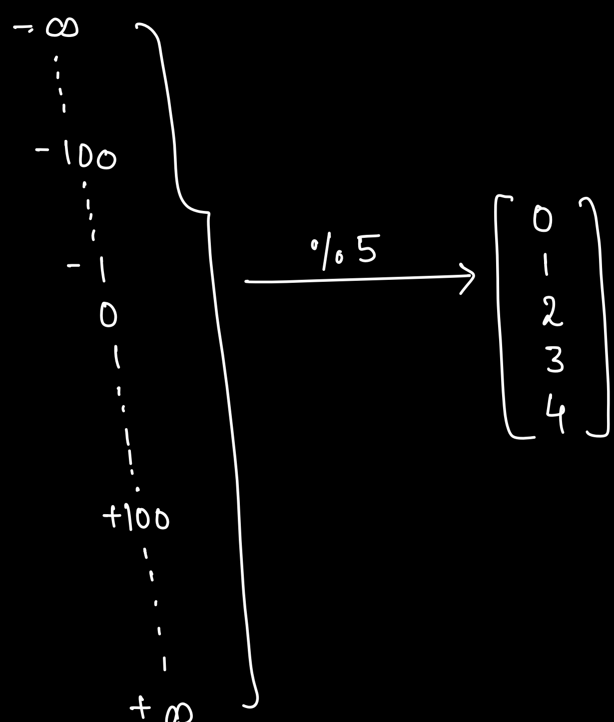
$-60 - (-63) = \underline{\underline{3}}$

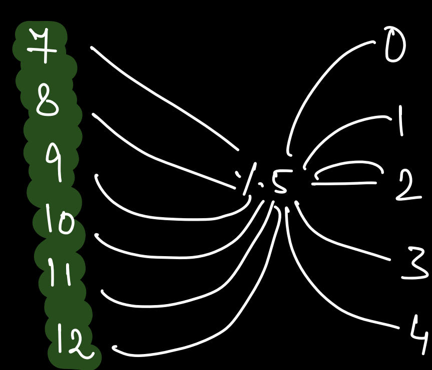
$$\boxed{A \% M \in [0, M-1]}$$

$\underline{\underline{C \mid C{+}{+} \mid Java}} \mid JS$		Python
$-40 \% 7 = -5$	$\xrightarrow{+7}$	2
$-60 \% 9 = -6$	$\xrightarrow{+9}$	3
$-30 \% 4 = -2$	$\xrightarrow{+4}$	2

MODULO Operator restricts the range.

$$A \% M \in [0, M-1]$$





→ Hashing | HashTables | Dict | map
→ Consistent Hashing.

Modulo Arithmetic

1) $(a+b)\%M = (a\%M + b\%M)\%M$

Quiz $(a+b)\%M \in [0, M-1]$

Ex $a = 6,\ b = 8,\ m = 10$

$(6+8)\%10$	$6\%10 + 8\%10$
$14\%10$	$6 + 8$
4	$14 \xrightarrow{\%10} 14\%10 = 4$

$\Rightarrow 14\%5 = 4$

$(14\%5)\%5 = 4\%5 = 4$

$((14\%5)\%5)\%5 = 4$

$$\boxed{a\%M = (a\%M)\%M}$$

2) $(a \times b)\%M = (a\%M \times b\%M)\%M$

+
−
*
/

Advance: − , /

Intermediate: + , *

Q. Implement power function

$pow(a, n, p) \Rightarrow a^n \% P$

$a=2, n=5, P=7 \Rightarrow 2^5 \% 7 = 4$

$a=3, n=4, P=6 \Rightarrow 3^4 \% 6 = 81 \% 6 = 3$

```
int pow(a, n, p){
    int ans = 1;
    for(i=1; i<=n; i++){
        ans = ans*a;
    }
    return ans % P;
}
```

$a^n = \underbrace{a \times a \times a \cdots\cdots a}_{n \text{ times}}$

Ex $a = 10, n = 40, P$

$ans = 10^{40}$? X

```
int pow(a, n, p){
    int ans = 1;
    for(i=1; i<=n; i++){
        ans = (ans*a) % P;
    }
    return ans % P;
}
```

This multiplication can also overflow.

$\rightarrow$ ans = (ans % P * a % P) % P

a = 2 , n = 4 , P = 10

ans = 1

ans = (1 % 10 × 2 % 10) % 10 = 2

ans = (2 % 10 × 2 % 10) % 10 = 4

ans = (4 % 10 × 2 % 10) % 10 = 8

ans = (8 % 10 × 2 % 10) % 10 = (8 × 2) % 10

= 6.

ans = (ans % P * a % P) % P

Eg P = $10^9 + 7$

ans % P ∈ [0, P-1]

a % P ∈ [0, P-1]

ans = ($\underbrace{\text{ans \% P}}_{P-1}$ * $\underbrace{\text{a \% P}}_{P-1}$) % P

$\boxed{\text{ans = ((long) (ans \% P) * a \% P) \% P}}$

TC: O(N)

SC: O(1)

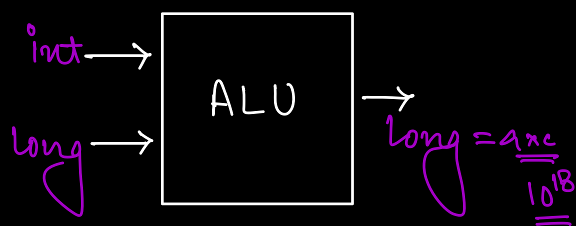
Eg

```
long ans;
int a = 10^9 , b = 10^9
long c = 10^9 , d = 10^9
```

ans = a*b ✗

ans = a×c ✓

ans = c*d ✓



Quiz No. not divisible by 3

231

4562

7821

1026

Quiz

4351

3521

7326 → $(7+3+2+6) = 18 \% 3 = 0$

8236

Divisibility rule for 3.

⇒ If sum of all digits is divisible by 3 then the number will be divisible by 3.

$(4372) \% 3$

$\hookrightarrow (4 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 2 \times 10^0) \% 3$

$[(4 \times 10^3) \% 3 + (3 \times 10^2) \% 3 + (7 \times 10) \% 3 + (2 \times 10^0) \% 3] \% 3$

$(4 \% 3 \times \underbrace{10^3 \% 3}_{1}) \% 3 + (3 \% 3 \times \underbrace{10^2 \% 3}_{1}) \% 3 + (7 \% 3 \times \underbrace{10 \% 3}_{1}) \% 3 + (2 \% 3)] \% 3$

$10 \% 3$
$10^2 \% 3$
$10^3 \% 3$
$\vdots$
$10^n \% 3$
$\Big\} \; 1$

$(4 \% 3 \times 1) \% 3$

$\Rightarrow 4 \% 3$

$\Rightarrow (4 \% 3 + 3 \% 3 + 7 \% 3 + 2 \% 3) \% 3$

$$\boxed{(a \% m + b \% m) \% m \rightarrow (a+b) \% m}$$

$\Rightarrow \underbrace{(4 + 3 + 7 + 2)}_{\text{Sum of digits.}} \% 3$

HW $\Rightarrow$ Divisibility rule of 4, 5 & 8.

Q: Given a number A in form of an Array of size N, & a no. P return $A \% P$

Google.

A :

1	2	3	4	4

P = 4

$A \% P = 12344 \% 4$

$= 0$

Constraints

$$\boxed{\begin{array}{l} 1 \le P \le 10^9 \\ 1 \le N \le 10^5 \end{array}}$$

int $\Rightarrow 10^9$

$\Rightarrow$ 10 digits

long $\Rightarrow 10^{18}$

$\Rightarrow$ 19 digits

A :

1	2	3	4	4

4×10^0

4×10^1

3×10^2

2×10^3

1×10^4

Ex

	0	1	2	3	4	5	6
A :	3	8	4	3	6	8	9

9×10^0

8×10^1

6×10^2

3×10^3

4×10^4

8×10^5

3×10^6

$N-1 \Rightarrow 10^0$

$N-2 \Rightarrow 10^1$

$N-3 \Rightarrow 10^2$

$\vdots$

$1 \Rightarrow 10^{N-2}$

$0 \Rightarrow 10^{N-1}$

$\boxed{i \Rightarrow N-1-i}$

$$A\,\%\,P = (3\times10^6 + 8\times10^5 + 4\times10^4 + 3\times10^3 + 6\times10^2 + 8\times10 + 4)\,\%\,P$$

$$= ((3\times10^6)\,\%\,P + (8\times10^5)\,\%\,P + (4\times10^4)\,\%\,P + (3\times10^3)\,\%\,P + (6\times10^2)\,\%\,P + (8\times10)\,\%\,P + 4\,\%\,P)\,\%\,P$$

$$(3\,\%\,P \times \underbrace{10^6\,\%\,P}_{pow(10,6,P)})\,\%\,P$$

$$(8\,\%\,P \times \underbrace{10^5\,\%\,P}_{pow(10,5,P)})\,\%\,P$$

$$(4\,\%\,P \times \underbrace{10^4\,\%\,P}_{pow(10,4,P)})\,\%\,P$$

$$A: \overset{0}{a_0} \;\overset{1}{a_1}\; \overset{2}{a_2}\; \overset{3}{a_3} \;\cdots\cdots\; \overset{n-2}{a_{n-2}}\; \overset{n-3}{a_{n-1}}$$

$$(a_0 \times 10^{n-1} + a_1 \times 10^{n-2} + a_2 \times 10^{n-3} + \cdots\cdots + a_{n-1} \times 10^0) \% P$$

$$((a_0 \times 10^{n-1}) \% P + (a_1 \times 10^{n-2}) \% P + \cdots\cdots + (a_{n-1} \times 10^0) \% P) \% P$$

$$((a_0 \% P * pow(10, n-1, P)) \% P + (a_1 \% P \times pow(10, n-2, P)) \% P \cdots$$

$$\cdots\cdots + (a_{n-1} \% P * pow(10, 0, P)) \% P \;) \% P$$

$$\Rightarrow \boxed{\sum_{i=0}^{N-1} (a_i \% P * pow(10, n-1-i, P)) \% P}$$

```
ans = 0
for( i= 0; i< N; i++ ) {
    ans = ans + ( A[i] % P * pow(10,N-1-i, P)) % P
}
return ans;
```

$O(N)$

$$\boxed{pow(a, n, P) \Rightarrow a^n \% P}$$

TC: $O(N^2)$

i	
0	$10^{N-1} \% P = (10 \times 10^{N-2}) \% P$
1	$10^{N-2} \% P = (10 \times 10^{N-3}) \% P$
2	$10^{N-3} \% P$
3	$10^{N-4} \% P$
⋮	⋮
$N-2$	$10^{1} \% P$
$N-1$	$10^{0} \% P$

i

$N-1 \Rightarrow 10^0 \% P = 1$

$N-2 \Rightarrow 10^1 \% P = (10 \times 10^0) \% P = (10 \% P \times 10^0 \% P) \% P$

$N-3 \Rightarrow 10^2 \% P \Rightarrow (10 \times 10^1) \% P = (10 \% P \times 10^1 \% P) \% P$

$N-4 \Rightarrow 10^3 \% P \Rightarrow (10 \times 10^2) \% P = (10 \% P \times 10^2 \% P) \% P$

$N-5 \Rightarrow 10^4 \% P \Rightarrow (10 \times 10^3) \% P = (10 \% P \times 10^3 \% P) \% P$

```
r = 1, ans = 0
for ( i = N-1 ; i >= 0 ; i-- ) {
    ans =  ans + (A[i] % P * r ) % P
    r = (r x 10) % P
}
return ans;
```

TC: O(N)

SC: O(1)

— * —