Q1 Given N +ve elements, calc number of pairs i,j st $(ar[i]+ar[j]) \% M = 0$

i != j and (i,j) is the same as (j,i)

	0	1	2	3	4	5	
Eg-	4	7	6	5	8	3	M=3

Ans ⇒ (0,3) (1,3) (0,4) (1,4) (2,5)

ans = 5

Brute force: Check for all pairs

TC: $O(n^2)$

sum ⇒ $[0, 2m-2]$

Idea: $(a+b) \% M = 0$

$\Rightarrow (a\%M + b\%M) \% M = 0$

a%m	b%m → [?]
1	m-1
2	m-2
3	m-3
⋮	⋮
M-1	1

$0 \longrightarrow 0$

if $M \% 2 = 0$ $\quad M/2 \longrightarrow M/2$

	0	1	2	3	4	5	6	7	8	9	10	11
Eg -	6	7	5	11	19	20	9	15	14	13	12	23

$M = 5$

1	2	0	1	4	0	4	0	4	3	2	3

How many ways to form pairs of 0?

(2,5) (2,7) (5,7) $\Rightarrow$ $\frac{3(3-1)}{2} = 3$

freq 1 to 4

1	2	3	4
2	2	2	3

1 match with 4 $\rightarrow$ $2 \times 3 = 6$

2 match with 3 $\rightarrow$ $2 \times 2 = 4$

3 match with 2 $\rightarrow$

will you count (3, 2) $\Rightarrow$ NOO

total $= 3 + 10 = 13$

$M=6$

0	1	2	3	4	5	6	7	8	9	10	11	12
2	3	4	8	6	15	5	12	17	7	18	10	9
2	3	4	2	0	3	5	0	5	1	0	4	3

%6	0	1	2	3	4	5
	3	1	2	3	2	2

0 match with 0 $\quad \frac{3(3-1)}{2} = 3$

1 match with 5 $\quad 1 \times 2 = 2$

2 match with 4 $\quad 2 \times 2 = 4$

3 match with 3 $\quad \frac{3(3-1)}{2} = 3$

How to count (3,3) $\Rightarrow \frac{f(f-1)}{2}$

4 match with 2

will you count ? NO

3

total $\Rightarrow$ 12

$\frac{m-1}{2}$

$5 \rightarrow 2$

$6 \rightarrow 2$

$7 \rightarrow 3$

8 → 3

Code

1) Create freq hashmap hm.

ans = 0

// first handle 0 case

f = hm[0]

ans += $\frac{f(f-1)}{2}$

// now handle if M even, then M/2, M/2

```
if (M % 2 == 0) {
    x = hm[M/2]
    ans += x(x-1)/2
}
```

// now loop on the rest.

```
for (i=1 ; i <= (m-1)/2 ; i++) {
    ans += hm[i] * hm[m-i]
}
```

return ans.

TC: $O(N)$

SC: $O(M)$

$$\frac{5 \times 4}{2}$$

$$\frac{5(5+1)}{2} \qquad \frac{10(10-1)}{2}$$

$$\frac{n(n+1)}{2}$$

Q2 Trailing zeros in N!

N	1	2	3	4	5	6	----	10
fact	1	2	6	24	120	720		3628800

$fact(10) = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

$[1, 10] \Longrightarrow 2$

$fa(30) = 1 \times \ldots 5 \times \ldots 10 \times \ldots 15 \times \ldots 20 \times \ldots 25 \times \ldots 30$

$(1, 30) \Longrightarrow 6 + 1 = 7$

Count of multiples of 5 from 1 to n? $n/5$

Count of multiples of 25 from 1 to n? $n/25$

ans = # multiples of 5 +
# multiples of 5^2 +

_ __ __ ___ _

_ __ __ __ +

multiples of 5^k

Code

```
ans = 0
for (x = 5 ; x ≤ N ; x = x*5 ) {
        ans += n/x
}
return ans
```

TC $O(\log_5 n)$

SC: $O(1)$

x
5
25
125

ans = 6 + 1

Properties

1) $(a^b)\ \%\ m$

$$= \left[(a\ \%\ m)^b\right]\ \%\ m$$

$(3247)\ \%\ 5$

$= \left[(3247\ \%\ 5)^9\right]\ \%\ 5$

$(2^9)\ \%\ 5$

```
int pow (a, b, m) {
    if (a == 1)
        return 1 % m
    if (b == 0)
        return 1
    a = a % m
    p = pow (a, b/2, m)
    if (b % 2 == 0)
        return (p x p) % m
    else
    return (p x p x a) % m
}
```

$$x^6 = x^3 \times x^3$$

$$x^{13} = x^6 \times x^6 \times x$$

- Modular inverse $\Rightarrow$ a^{-1}

$x = inv(a, M)$

$(a * x) \% M = 1 \Rightarrow x \; [1, m-1]$

Greatest Common Divisor

Given a, M, a^{-1} exist if $gcd(a, m) = 1$

$a = 7 \quad M = 10$

$(7 \times 1) \% 10 = 7$

$(7 \times 2) \% 10 = 4$

$(7 \times 3) \% 10 = 1$

a^{-1} wrt $M = 3$

3 wr 5

$3 \times 1 \% 5 = 3$

$3 \times 2 \% 5 = 1$

Code given A, M & $gcd(a, m) = 1$. $b = a^{-1} \% M$

```
for (i=1 ; i < M ; i++) {
    if ( (a*i) % M == 1 ) {
        return i
    }
}
```

TC: $O(M)$

SC: $O(1)$

Extendend Euclidean : Inverse in $O(\log M)$

(v v tricky)

if M is prime

Fermat's Little Theorem

$(a^{m-1}) \% m = 1$

The most popular.

$\Rightarrow (a * a^{m-2}) \% m = 1$

Hence a^{-1} wrt $m \implies (a^{m-2}) \% m$

pow(a, m-2, m)

TC : $\log(m)$

inverse of 3 wrt 5

$(3^{5-2}) \% 5$

{done}

$3^3 \% 5 = 27 \% 5$

$= 2$

$(10/5) \% 7$

$\left[10 \% 7 \times inv(5, 7)\right] \% 7$

$\frac{10 \times 9}{2} \Rightarrow \frac{10(10-1)}{2}$

$\frac{35 \quad 35-1}{2}$

$n \qquad \frac{n(n-1)}{2}$

$12 \% 5$

$12 -$ biggest multiple of 5 ≤ 12

$$12 - 10 = 2$$

$$12 \% 5 \Rightarrow 2$$