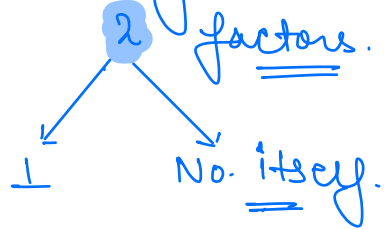Q: Given a number N, check whether it is a prime no. or not.

Quiz   10, 11, 23, 2, 25, 27, 31

Prime Number :- Any +ve number that has exactly 2 factors.

1   No. itself.

Q: Is ① a prime no? ⇒ NO

↓
only 1 factor
i.e ①

⇒ ① is neither prime nor composite.

```
bool isPrime ( int N) {
    int count = 0 // No. of factors.
    for (int i = 1; i <= N; i++) {
        if ( N % i == 0) {
            count ++
        }
    }
    if ( count == 2 ) return true;
    return false;
}
```

\# of iterations $\Rightarrow$ N $\{ i \in [1, N] \}$

* Assumption : 10^8 iterations $|$ sec

  i) $N = 10^9 \Rightarrow 10$ sec

  ii) $N = 10^{18} \Rightarrow \dfrac{10^{18}}{10^8}$ sec $= 10^{10}$ sec

  $\simeq$ 317 years.

today $\longrightarrow$ kids $\longrightarrow$ 2nd gen $\longrightarrow$ 3rd gen $\rightarrow$ 4th gen

$\downarrow$

$\times$ Shoaib
    Malik.
    retired

\# Observations :-

  Let's say there are 3 +ve no's :

  a, b & N

  $a \times b = N \Rightarrow b = N/a$

  $\Rightarrow \{a, b\}$ are factors of N.

  $\Rightarrow \{a, \frac{N}{a}\}$ are factors of N.

* If a is a factor the $\frac{N}{a}$ is also a factor of N.

* factors appears in pair.

Quiz No. of factors of N=24. ⟹ 8

N=24

i	N/i
1	24
2	12
3	8
4	6
6	4
8	3
12	2
24	1

N=100

i	N/i
1	100
2	50
4	25
5	20
10	10
20	5
25	4
50	2
100	1

$$i <= \frac{N}{i}$$

↓

$$(i)_{max} = \frac{N}{i}$$

$$i * i = N$$

$$i = \sqrt{N} \implies i \in [1, \sqrt{N}]$$

Quiz $a <= 57$

$a_{max} = 57$

```
bool isPrime ( int N) {
    int count = 0 // No. of factors.
    for(i = 1 ; i <= √N ; i++) {
        if( N % i == 0) {
            if( i * i == N) count += 1;
            else  count += 2;
        }
    }
    if(count == 2) return true
    return false;
}
```

N = 100

i	N|i	C = 0
1	100	2
2	50	4
3 x		
4	25	6
5	20	8
10	10	10

⟹ factors of 100 :-

1, 2, 4, 5, 10, 20, 25,
50, 100

9

# of iterations = √N

i ∈ [1, √N]

* Assumption : 10^8 iterations / sec

i) $N = 10^9 \Rightarrow$ # of iterations

$$\Rightarrow \sqrt{10^9} = 10^4$$

$$\Rightarrow 0.5 \text{ sec}$$

ii) $N = 10^{18} \Rightarrow$ # of iterations $= 10^9$

$$\Rightarrow 10 \text{ sec}$$

317 yrs $>>> 10$ sec

Q: $1 + 2 + 3 + 4 + 5 + \ldots\ldots + 99 + 100$

Carl Fredrich Gauss $\Rightarrow 4^{th}$ Standard

$$S = 1 + 2 + 3 + 4 + \ldots\ldots + 99 + 100$$
$$S = 100 + 99 + 98 + 97 + \ldots\ldots + 2 + 1$$

$$2S = 101 + 101 + 101 + \ldots\ldots\ldots + 101$$

$$2S = 101 \times 100$$

$$S = \frac{101 \times 100}{2} = 101 \times 50 = 5050$$

Sum of first N natural no's.

$$S = 1 + 2 + 3 + \cdots \cdots + (N-1) + N$$

$$S = N + (N-1) + (N-2) + \cdots \cdots 2 + 1$$

$$2S = (N+1) + (N+1) + \cdots \cdots + (N+1)$$

$$2S = N(N+1)$$

$$\boxed{S = \frac{N(N+1)}{2}}$$

Q. Given N, How many times we need to **divide** N by 2 till it becomes 1.

integer division

$$N = 7 \xrightarrow{\ /2\ } 3 \xrightarrow{\ /2\ } 1 \Rightarrow 2$$

$$N = 1(2^0) \longrightarrow 0$$

$$N = 2(2^1) \xrightarrow{\ /2\ } 1$$

$$N = 3 \xrightarrow{\ /2\ } 1 \Rightarrow 1$$

$$N = 4(2^2) \longrightarrow 2$$

$$N = 8(2^3) \longrightarrow 3$$

$$N = 15 \xrightarrow{\ /2\ } 7 \xrightarrow{\ /2\ } 3 \xrightarrow{\ /2\ } 1 \Rightarrow 3$$

$$N = 16(2^4) \longrightarrow 4$$

Quiz   N = 27 ⟹ 4

$$27 \xrightarrow{/2} 13 \xrightarrow{/2} 6 \xrightarrow{/2} 3 \xrightarrow{/2} ①$$

ans = Highest power of 2 ↑ = N

> $\log_2 N$ ⟹ No. of times we need to divide N by 2 so that it becomes 1.

$$\log_2 4 = 2 \qquad \log_2 1024 = \underline{10}$$
$$\log_2 11 = 3 \qquad \qquad 2^{10}$$

Q. Given a **perfect square**, find the square
Amazon root of the given no.

Quiz   49 is a perfect square

$$n * n = 49$$
$$n^2 = 49 \Rightarrow 7 \times 7 = 49$$
$$\Rightarrow n = 7$$

$$24 \Rightarrow n \times n = 24 \quad \times$$
$$35 \Rightarrow n \times n = 35 \quad \times$$

$$N = 25 \longrightarrow 5$$
$$N = 100 \longrightarrow 10$$
$$N = 36 \longrightarrow 6$$

Idea 1:-

$$\boxed{\sqrt{N} \in [1, N]}$$

```
int sqrt(N) {
    for(i = 1; i <= N; i++) {
        if(i×i == N)
            return i;
```

$$\underline{\underline{?}}$$

$$\underline{\underline{?}}$$

# of iterations = $\sqrt{N}$

N = 36

		i×i
i = 1	⟹	1×1
i = 2	⟹	2×2
i = 3	⟹	3×3
i = 4	⟹	4×4
i = 5	⟹	5×5
i = 6	⟹	6×6 == 36

$$\Longrightarrow \underline{\underline{6}} \text{ iterations}$$

↳ True

$\underline{\underline{N}}$

$\sqrt{N} \in [1, N]$

$N = 100 : \sqrt{N} \in [1, 100]$

1) $50 \Rightarrow 50 \times 50 > 100$

$1, 2, \cdots 49, 50, 51, 52, \cdots 100 \Rightarrow [1, 49]$

2) $25 \Rightarrow 25 \times 25 > 100$

$1, 2, \cdots 23, 24, 25, 26, \cdots 49 \Rightarrow [1, 24]$

3) $12 \Rightarrow 12 \times 12 > 100$

$1, 2, \cdots 11, 12, 13, \cdots 24 \Rightarrow [1, 11]$

4) $6 \Rightarrow 6 \times 6 < 100$

$1, 2, \cdots 5, 6, 7, \cdots 11 \Rightarrow [7, 11]$

5) $9 \Rightarrow 9 \times 9 < 100 \Rightarrow [10, 11]$

$7, 8, 9, 10, 11$

6) $10 \Rightarrow 10 \times 10 == 100$

$\Rightarrow \sqrt{100} = \underline{10}$

$\Rightarrow$ In every step, the size of the range is becoming half.

No. of steps $= \log_2 N$

Compare $\sqrt{N}$ & $\log N$

	$\sqrt{N}$	$\log_2 N$
$N = 1024 = 2^{10}$	$2^5 = 32$	10
$N = 2^{32}$	2^{16}	32
$N = 2^{64}$	2^{32}	64

$$2^{10} = 1024 \approx 1000 \approx 10^3$$

$$2^{32} = \underbrace{2^{10}}_{10^3} \cdot \underbrace{2^{10}}_{10^3} \cdot \underbrace{2^{10}}_{10^3} \cdot 2^2 \approx 2^2 \times 10^9$$

$$\boxed{\sqrt{N} >>> \log_2 N}$$

$$\boxed{\log_2 2^x = x}$$

$\Rightarrow$ BINARY SEARCH

$$\boxed{\log_2 2^{100} = 100}$$

——— ✱ ———