

\* We are going to do lot of close approximations.

Q. Given  $10^4$  no's, write an algo to sort these no's.

Namit

Nam Algo

Execution : \* 10 sec  
time  
{ MacBook Pro }



10 sec  
(Python)



\* C++  
8 sec



Antarctica  
(Cold)

Akshay

Aks algo.

15 sec

{ Old windows }



{ MacBook Pro }



\* 9 sec  
(C++)



9 sec



Volcano  
(Hot)



Antarctica  
(Cold)



\* 7.5 sec

\* Execution time is NOT a good factor to compare

depends on S/W, H/W & external factors.

\* for (i = 1; i ≤ N; i++) {

-----

}

⇒ No. of iterations are NOT dependent on H/W, S/W & external factors.

Q. Given  $10^4$  no's, write an algo to sort these no's.

Aman

Tanzeer

# iterations

$$100 \log_2 N$$

$$N/10$$

Which algo is better.

$$N = 2^{32}$$

$$\Rightarrow 100 \log_2 2^{32}$$

$$\Rightarrow 100 \times 32$$

$$\Rightarrow \underline{\underline{3200}}$$

$$\frac{2^{32}}{10}$$

$$\rightarrow \frac{\overset{1024 \approx 10^3}{\uparrow} \left( \frac{10}{2} \right) \cdot 2^{10} \cdot 2^{10} \cdot 2^2}{10}$$

$$\rightarrow \frac{10^3 \cdot 10^3 \cdot 10^3 \cdot 4}{10}$$

$$\rightarrow \underline{\underline{4 \times 10^8}}$$

if  $N \leq 3900$ , Aman's algo is having more no. of iterations than Tanzeer's algo.

$N > 3900$ , Tanzeer's algo is having more no. of iterations than Aman's algo.

$\Rightarrow$  Ind vs  $N^2$ , WC S.F 2019

3 Crore concurrent users.

$\Rightarrow 3 \times 10^7$  users

$\Rightarrow$  We'll see the no. of iterations, for very <sup>large</sup> input values.

# Asymptotic analysis

$\Rightarrow$  Performance of your algorithm for very large input values.  
( $N \rightarrow \infty$ )

1) Big O  $\Rightarrow$  Worst Case

2) Omega }

3) Theta }

# How to find the Big O notation :-

- 1) No. of iterations.
- 2) Neglect all the lower order terms.
- 3) Neglect all constant terms.

Ex Revanti's Algo.

$$\# \text{ of iterations} = N^2 + 10N$$

1)  $N = 100$

$$\begin{aligned}\# \text{ of iterations} &= 100^2 + 10 \cdot 100 \\ &= 10^4 + 10^3\end{aligned}$$

$$\begin{aligned}\% \text{ of } 10N \text{ in total no. of iterations} &= \frac{1000}{10^4 + 10^3} \times 100 \\ &\approx \underline{\underline{10\%}}\end{aligned}$$

2)  $N = 10^5$

$$\begin{aligned}\# \text{ of iterations} &= (10^5)^2 + 10 \cdot 10^5 \\ &= 10^{10} + \underline{\underline{10^6}}\end{aligned}$$

$$\begin{aligned}\% \text{ of } 10N \text{ in total no. of iterations} &= \frac{10^6}{10^{10} + 10^6} \times 100 \\ &\approx \underline{\underline{0.01\%}}\end{aligned}$$

$$3) N = 10^6$$

$$\therefore \text{ of } 10N \ll \underline{\underline{0.01\%}}$$

$\Rightarrow$  NOTE: Contribution of lower order terms is very negligible w.r.t to higher order terms.

$$\Rightarrow \underline{\underline{O(N^2)}}$$

Ex

$$1) 10 \log N \Rightarrow O(\log N)$$

$$2) 10^2 \log N \Rightarrow O(\log N)$$

$$3) 10^4 \log N \Rightarrow O(\log N)$$

$$4) \underline{10^4} N + 10^6 \Rightarrow O(N)$$

$$5) 10^3 N \log N \Rightarrow O(N \log N)$$

$$6) N + 10 \log N \Rightarrow \underline{\underline{O(N)}}$$

$\Rightarrow$  Constants & lower order terms are NOT affecting much the # of iterations when N is very large.

## # Issues with Big O notations

1) Mohan's Algo Mayur's Algo

$$100N$$

$$N^2$$

$$\Rightarrow O(N)$$

$$O(N^2)$$

$$N=10 \quad 1000 > 100$$

$$N=99 \quad 9900 > 9801$$

$$N=101 \quad 10100 < 10201$$

$\Rightarrow N > 100$  : Mohan's Algo is better

$N < 100$  : Mayur's Algo is better

2)

Arshad

Chinmay

$$10N^2 + 5N$$

$$11N^2 + 100N$$

$$\Rightarrow O(N^2)$$

$$\Rightarrow O(N^2)$$

Both algo's are  $O(N^2)$ , we can't compare these 2 algo's based on their Big O notation.

# Importance of Big O:

$\rightarrow$  makes comparisons very smooth.

## # Space Complexity

```
fun (int N) {
```

```
    int n = 10; → 4B
```

```
    int y = n2; → 4B
```

```
    long z = n * y → 8B
```

```
    double pie = 3.14; → 8B
```

```
}
```

int ⇒ 4 Bytes

long ⇒ 8 B

double ⇒ 8 B

Total memory = 24 Bytes.

SC:  $O(1)$  { constant Extra }  
                    Space

Ex

```
fun (int N) {
```

```
    int A[N];
```

```
    - - - - -
```

3

3

0 1 2 3 - - - - - N-1



int(4B)

⇒ N int ⇒ 4N

Extra Space = 4N

(Auxiliary Space) ⇒  $O(N)$  ⇒ SC

E<sub>2</sub>

```
fun (int N) {
```

```
    int x = N; → 4B
```

```
    int y = 100; → 4B
```

```
    long z = x * y; → 8B
```

```
    double pie = 3.14; → 8B
```

```
    int arr[N]; → 4N B
```

```
    bool mat[N][N]; → N2 B
```

3

Extra Space: 24 + 4N + N<sup>2</sup>

⇒ O(N<sup>2</sup>) : SC

E<sub>2</sub>

```
int fun (int N) {
```

```
    int x; // 100 variables
```

```
    ⋮
```

```
}
```

SC: 400 Bytes

: O(1)



Q. Given an Array, find the sum of array elements.

```
int arraySum (int a[], int N) {  
    int sum = 0  
    for (int i = 0; i < N; i++) {  
        sum += a[i]  
    }  
    return sum;  
}
```

4B ←  
4B ←  
3  
3

Extra space : 8B.

: SC: O(1)

⇒ We don't consider the input space in the Space Complexity

⇒ Amount of extra space created is called as S.C

## # Linear Search

```
bool fun(int arr[], int N, int K) {  
    for(i = 0; i < N; i++) {  
        4B ← if(arr[i] == K) {  
            return true;  
        }  
    }  
    return false;  
}
```

10	20	5	60	15	18
----	----	---	----	----	----

↑

K = 10  $\Rightarrow$  iterations = 1

K = 18  $\Rightarrow$  iterations = 6 (N)

T<sub>C</sub> { Worst case : N } O(N) { WC }  
Best case : 1

SC: O(1)

# TLE: Time Limit Exceeded

↳ Code is taking more than expected time.

⇒ Reduce the no. of iterations to overcome TLE.

Polynomial:  $N^1 \mid N^2 \mid N^3 \mid \dots$   
                  ↓          ↓                  ↓  
              Linear Quadratic                  Cubic

Exponential :-  $2^N \mid 3^N \mid \dots$

Constant :  $O(1)$