corVis: An R Package for Visualising Associations and Conditional Associations

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Abstract Correlation matrix displays are important tools to explore multivariate datasets. These displays with other measures of association can summarize interesting patterns to an analyst and assist them in framing questions while performing exploratory data analysis. In this paper, we present new visualisation techniques to visualise association between all the variable pairs in a dataset in a single plot, which is something existing displays lack. Also, we propse new methods to visualise relationship among variable pairs using conditioning. We use different layouts like matrix or linear for our displays. We use seriation in our displays which helps in highlighting interesting patterns easily. The R package corVis provides an implementation.

Section 1: Introduction

Correlation matrix display is a popular tool to visually explore correlations among variables while performing Exploratory Data Analysis (EDA) on a multivariate dataset. Popularized by Friendly (2002) as corrgram, these displays are produced by first calculating the correlation among the variables and then plotting these calculated values in a matrix display. With effective ordering techniques, these displays quickly highlight variables which are highly correlated and an analyst interested in building a predictive model could use these displays to remove correlated variables and avoid multicollinearity.

The correlation displays are generally used with one of the Pearson's, Spearman's or Kendall's correlation coefficient and are therefore limited to quantitative variables. An analyst can use one-hot encoding of the qualitative variables in order to use these displays but will need to deal with the high dimensions as a result of the encoding. In addition to the dimensionality problem, it is not easy to assess the overall correlation when using the one-hot encoding. The existing methods to quickly explore association among qualitative variables in a dataset include using proportions or counts with different graphical displays like boxplots or barplots. Using association measures for qualitative pairs similar to correlation for quantitative pairs will help in summarizing the relationship, which then can be displayed like the correlation displays.

Tukey and Tukey introduced scagnostics which are measures for scatterplots (Tukey and Tukey, 1985). Along with scagnostics, they proposed a scagnostics scatterplot matrix which is a visual display to explore and compare these measures for all the variable pairs in a dataset. By comparing multiple measures at once, the unusual variable pairs could be identified and looked at in more detail. In a similar manner, a display comparing association measures will help in finding interesting variable pairs. Many association measures have been proposed to summarize different types of relationships. The most commonly used measure is Pearson's correlation coefficient which captures any linear trend present between the variables. Other popular measures include Kendall's or Spearman's rank correlation coefficient which are non-parametric measures and looks for monotonic relationship. Distance correlation (Székely et al., 2007) is an important measure useful in exploring non-linear relationships. The information theory measure maximal information coefficient (MIC) (Reshef et al., 2011) is capable of summarizing complex relationships. With effective displaying techniques, the multiple measures of association provide a comparison tool that assist an analyst to reveal structure present in the data.

Small multiples (or Trellis display) is a simple yet powerful approach to compare partitions of data and understand multidimensional datasets (Tufte, 1986). The display is produced by splitting the data into groups by a conditioning variable and then plotting the data for each group. Such displays allow analysts to quickly infer about the impact of the conditioning variable. A similar idea applied to displays of association measures (correlation plot) will help uncover underlying patterns in the data. One such pattern is Simpson's paradox which can be detected by comparing Pearson's correlation for data at overall level versus individual levels of the conditioning variable.

In this paper, we propose extensions of the correlation plot and new visualizations which look at variables of mixed type, multiple association measures and conditional associations. These displays are implemented in the R package corVis. The next section provides a review of existing packages which deal with correlation displays and a quick background on association measures and the packages used for calculating them. Then we describe our approach to calculate the association measures, followed by visualizations of associations and conditional associations. We conclude with a summary and future work.

Section 2: Background

In this section we provide a brief review of existing packages used for correlation displays and association measure calculation.

Section 2.1: Literature Review on Correlation Displays

According to Hills (1969), "the first and sometimes only impression gained by looking at a large correlation matrix is its largeness". To overcome this, Murdoch and Chow (1996) proposed a display for large correlation matrices which uses a matrix layout of ellipses where the parameters of the ellipses are scaled to the correlation values. Friendly (2002) expanded on this idea by rendering correlation values as shaded squares, bars, ellipses, or circular 'pac-man' symbols. The variables in the matrix displays were optionally ordered using the angular ordering of the first two eigen vectors of the correlation matrix. The ordering places highly-correlated pairs of variables nearby, making it easier to quickly identify groups of variables with high mutual correlation.

Nowadays, there are many R packages devoted to correlation visualisation. Table 1 provides a summary, listing the displays offered, and whether these extend to factor variables or mixed numeric-factor pairs.

The R package **corrplot** (Wei and Simko, 2021) provides an implementation of the methods in Friendly (2002). The package **corrr** (Kuhn et al., 2020) organises correlations as tidy data, so leveraging the data manipulation and visualisation tools of the **tidyverse** (Wickham et al., 2019). In addition to various matrix displays, the package offers network displays where line-thickness encodes correlation magnitude, with a filtering option to discard low-correlation edges.

The package **corrgrapher** (Morgen and Biecek, 2020) uses a network plot for exploring correlations, where the nodes close to each other have high correlation magnitude, edge thickness encodes the absolute correlation value and edge color indicates the sign of correlation. The package also handles mixed type variables by using association measures obtained as transformations of *p*-values obtained from Pearson's correlation test in the case of two numeric variables, Kruskal's test for numerical and factor variables, and a chi-squared test for two categorical variables.

The package linkspotter (Samba, 2020) offers a variety of association measures (distance correlation, MIC, maximum normalized mutual information) in addition to correlation, where the measure used depends on whether the variables are both numerical, categorical or mixed. The results are visualized in a network plot, which may be packaged into an interactive shiny application.

Our own package corVis offers a variety of displays, and has new features not available elsewhere, in particular simultaneous display of multiple association measures, and association displays stratified by levels of a grouping variable. This will be described in the following sections.

There have been other extensions to correlation displays which are useful when dealing with high dimensional datasets. Hills (1969) proposed a QQ plot of the z-transform of the entries of the correlation matrix to discover correlation coefficients too large to come from a normal distribution with mean zero. Buja et al. (2016) proposed Association Navigator which is an interactive visualization tool for large correlation matrices with upto 2000 variables. The R package scorrplot (Gerber, 2022) produces an interactive scatterplot for exploring pairwise correlations in a large dataset by projecting variables as points and encoding the correlations as space between these points. The package provides a functionality to update variable of interest which creates tour of the correlation space between different projections of the data.

The R package correlationfunnel offers a novel display which assists in feature selection in a setting with a single response and many predictor variables. All numeric variables including the response are binned. All (now categorical) variables in the resulting dataset are one-hot encoded and Pearson's correlation calculated with the response categories. The correlations are visualised in a dot-plot display, where predictors are ordered by maximum correlation magnitude. Correlations between one-hot encoded variables are challenging to interpret, especially as the number of levels increase. In corVis we offer a similar dot-plot display, but showing multiple correlation or association measures, or alternatively measures stratified by a grouping variable.

Section 2.2: Literature Review on Association Measures

An association measure can be defined as a numerical summary quantifying the relationship between two or more variables. For example, Pearson's correlation coefficient summarizes the strength and direction of the linear relationship present between two numeric variables and is in the range [-1,1]. Kendall's or Spearman's rank correlation coefficient are other popular measures which assess montonic relationship among two numeric variables and are in the range [-1,1]. As these measures are limited

Table 1: List of the R packages dealing with correlation or correlation displays with information on whether the plots display multiple measures, conditional display of measures and mixed variables in a single plot

Package	Display	CalculateCorrelation	MixedVariables	MultipleMeasures	Condit
corrplot	heatmap				
corrr	heatmap/network	Yes			
corrgrapher	network	Yes			
linkspotter	network	Yes	Yes		
correlation	heatmap/network	Yes			
corVis	heatmap/matrix/linear	Yes	Yes	Yes	Yes

to linear or monotonic relationships, there's a need to use association measures which are able to capture complex relationships. In addition to association measures for numeric variables, association measures for ordinal, nominal and mixed variable pairs are useful in exploring a multivariate dataset. We now give an overview of available association measures.

For a pair of numeric variables, various measures of association have been proposed in literature. The distance correlation coefficient (Székely et al., 2007) is an association measure which looks for the non-linear association between two numeric variables and summarizes it in [0,1]. Similarly, MIC (Reshef et al., 2011) is capable of summarizing non-linear as well as periodic relationships between numeric variables and is in range [0,1].

Agresti (2010) provides an overview of the association measures which are used for exploring association between ordinal variables. Kendall's tau-b (Kendall, 1945) is an association measure useful in summarizing the relationship between two ordinal variables in the range [-1,1]. It is a relatively stable measure with respect to the changes in categories of any variable i.e. if two categories are merged to make a single category. The polychoric correlation (Olsson, 1979) measures the correlation between two ordinal variables by assuming two normally distributed latent variables for a contingency table of two ordinal variables and summarizes the association in [-1,1].

The association measures for the case of nominal pair of variables should be invariant to the order in which the categories appear. Pearson's contingency coefficient uses the χ^2 value from the Pearson's χ^2 test for independence and is a useful measure to summarize the association between two nominal variables in [0,1]. Another measure for nominal variable pair is the Uncertainty coefficient (Theil, 1970) measuring the proportion of uncertainty in one variable which is explained by the other variable.

The canonical correlation is a measure useful in exploring association among mixed variables. The goal of the canonical correlation analysis is to maximize the association between the low-dimensional projections of two sets of variables (Härdle and Simar, 2019). Each of these measures are are consistent with respect to the order of the categories of the nominal variable.

Section 3: Introducing corVis

corVis is an R package which calculates measures of association for every variable pair in a dataset and helps in visualising these associations in different ways. Most of the existing correlation displays are limited to numeric pairs of variables. This package extends these displays to every variable pair.

We use multiple association measures in a single display for different variable pairs which serves as a comparison tool while exploring association in a dataset and assist in identifying unusual variable pairs. These multiple measures can be displayed in a scatterplot matrix similar to what Tukey and Tukey (1985) proposed. They suggested that scatterplot matrix of the scagnostics measures, which are measures summarizing a scatterplot, can be used to identify unusual scatterplots or variable pairs. Wilkinson et al. (2005) used this idea with their graph-theoretic scagnostic measures to highlight unusual scatterplots. Similarly, Kuhn et al. (2013) have used this idea in a predictive modeling context. They have produced a scatterplot matrix of the measures between the response and continuous predictors such as Pearson's correlation coefficient, pseudo- R^2 from the locally weighted regression model, MIC and Spearman's rank correlation coefficient to explore the predictor importance during feature selection step. These displays show the importance of comparing multiple association measures at once for different variable pairs.

We also introduce conditional association displays which are useful in uncovering conditional structure present in the data. Conditional displays (also called small multiple displays or trellis displays) are visualisations for the subsets of data produced by dividing the data by a partitioning 'pairwise_1d_compare'

'pairwise_1d_plot'

Function

Usage

Input

'calc_assoc'
 Calculation
 'calc_assoc_by'
 Calculation
 'association_heatmap'
 'pairwise_2d_plot'

Usage

Input

- Dataframe, - Types of association measures, - NA handler

- Dataframe, - Name of the grouping variable, - Types of association measure dataframe for lower and upper triangle of the responsible of the

Table 2: List of the available functions in **corVis** package with input arguments and outputs

variable and then plotting them. Popularised by Tufte (1986) and Becker et al. (1996), these displays are efficient for discovering differences among the groups in the data.

Visualization Visualization

Efficient seriation techniques have been included to order and highlight variables with high value for an association measure. These ordered association and conditional association displays are useful in finding interesting patterns, such as Simpson's Paradox. These new displays also help an analyst to quickly discover any unusual variable pair(s) in the dataset.

Table 2 provides a list of the functions available in the package which are useful for calulating association measures among variable pairs and visualising these associations using novel displays.

Section 4: corVis: Calculating Association

This section describes the calculation of association measures in our package corVis. The package provides a collection of various measures of association which quantifies the relationship between two variables. The association measures available in the package are not limited to numeric variables and are used with nominal, ordinal and mixed variable pairs as well. Table 3 lists different functions provided in the package to calculate measures of association. The funName represents the function name used to calculate measure(s) of associations in this package. The typeX and typeY columns provide the information on types of variables which can be used with the corresponding functions. The X or Y variable can be anyone out of numeric, nominal, ordinal or any. The from column corresponds to the package functions used to calculate the association measures by the function under funName. The symmetric column represents if the measure is symmetric i.e. if the value of measure is same regardless of the order of variables. The last column provides the range of values for these measures. The function tbl_easy can be used to calculate association measures available in the R package correlation which can use different variable types. The highlighted functions in 3 calculate the association measures which have been implemented in this package.

For numeric pairs of variables, this package provides a range of association measures. The popular correlation coefficients like Pearson's or Spearman's or Kendall's are calculated using tbl_cor function. The measures such as distance correlation or MIC which assess more complex relationship are calculated using tbl_dcor or tbl_mine respectively. The association measures available in the package for the ordinal pairs of variables are polychoric correlation and Kendall's coefficients which are calculated using tbl_polycor or tbl_tau respectively. For nominal pairs of variables, the functions like tbl_gkTau, tbl_gkLambda, tbl_gkGamma, tbl_uncertainty, tbl_chi, tbl_cancor are used for exploring association among the variables. These measures are consistent with respect to the order of the nominal variable which some of the existing measures lack.

The function tbl_cancor implemented in the package calculates canonical correlation for mixed pairs of variables. We calculate canonical correlation between a numeric variable and a nominal variable by first converting the nominal variable into dummy variables and then calculating the correlation between the continuous variable and set of dummy variables.

Calculating association for a single type of variable pairs

We introduce a method which creates a tibble structure for the variable pairs in a dataset along with calculated association measure. The package contains various functions (shown in Table 3) for different association measures in the form tbl_* to calculate them. For example, a user might be interested in calculating distance correlation for numeric pair of variables in a dataset. This can be done by using tbl_dcor.

```
df <- penguins
distance <- tbl_dcor(df)</pre>
```

Table 3: List of the functions available in the package for calculating different association measures along with the packages used for calculation.

funName	typeX	typeY	from	symmetric	range
tbl_cor	numerical	numerical	stats::cor	TRUE	[-1,1]
tbl_dcor	numerical	numerical	energy::dcor2d	TRUE	[0,1]
tbl_mine	numerical	numerical	minerva::mine	TRUE	[0,1]
tbl_polycor	ordinal	ordinal	polycor::polychor	TRUE	[-1,1]
tbl_tau	ordinal	ordinal	DescTools::KendalTauA,B,C,W	TRUE	[-1,1]
tbl_gkTau	nominal	nominal	DescTools::GoodmanKruskalTau	FALSE	[0,1]
tbl_gkGamma	nominal	nominal	DescTools::GoodmanKruskalGamma	TRUE	[0,1]
tbl_uncertainty	nominal	nominal	DescTools::UncertCoef	TRUE	[0,1]
tbl_chi	nominal	nominal	DescTools::ContCoef	TRUE	[0,1]
tbl_cancor	nominal	nominal	corVis	TRUE	[0,1]
tbl_cancor	nominal	numerical	corVis	TRUE	[0,1]
tbl_nmi	any	any	corVis	TRUE	[0,1]
tbl_easy	any	any	correlation::correlation	TRUE	[-1,1]

head(distance)

Similarly, one can use tbl_nmi to calculate normalised mutual information for numeric, nominal and mixed pair of variables.

```
nmi <- tbl_nmi(df)</pre>
head(nmi)
#> # A tibble: 6 x 4
                      measure measure_type
  <dbl> <chr>
 Х
#> <chr>
          species 0.507 nmi
#> 1 island
#> 2 bill_length_mm species 0.353
                              nmi
nmi
#> 4 flipper_length_mm species 0.343
                              nmi
nmi
#> 6 sex
                 species 0.0000854 nmi
```

The tibble output for the functions mentioned in Table 3 has the following structure:

- x and y representing a pair of variables
- measure representing the calculated value for association measure
- measure_type representing the association measure calculated for x and y pair.

The variable pairs in the output are unique pairs and a subset of all the variable pairs of a dataset where $x \neq y$. As explained earlier, the measure_type represents the association measure calculated for a specific type of variable pair

Calculating association measures for whole dataset

calc_assoc can be used to calculate association measures for all the variable pairs in the dataset at once in a tibble structure. In addition to tibble structure, the output also has pairwise and data.frame class which are important class attributes for producing visual summaries in this package.

The function calc_assoc has a types argument which is basically a tibble of the association measure to be calculated for different variable pairs. The default tibble of measures is default_assoc() which calculates Pearson's correlation if both the variables are numeric, Kendall's tau-b if both the variables are ordinal, canonical correlation if one is factor and other is numeric and canonical correlation for the rest of the variable pairs.

```
default_measures <- default_assoc()</pre>
default_measures
#> # A tibble: 4 x 4
#>
   funName
               typeX
                        typeY
                                argList
#>
    <chr>
               <chr>
                                st>
                        <chr>
#> 1 tbl cor
               numeric numeric <NULL>
#> 2 tbl_tau
               ordered ordered <NULL>
#> 3 tbl_cancor factor numeric <NULL>
#> 4 tbl_cancor other other
penguin_assoc <- calc_assoc(df,types = default_assoc())</pre>
glimpse(penguin_assoc)
#> Rows: 28
#> Columns: 4
#> $ x
                  <chr> "island", "bill_length_mm", "bill_depth_mm", "flipper_len~
                  <chr> "species", "species", "species", "species", "species", "s
#> $ y
#> $ measure
                 <dbl> 0.81328762, 0.84131393, 0.82447508, 0.88217284, 0.8183348~
#> $ measure_type <chr> "cancor", "cancor", "cancor", "cancor", "cancor", "cancor"
class(penguin_assoc)
#> [1] "pairwise"
                    "tbl_df"
                                 "tbl"
                                              "data.frame"
```

An analyst can update these measures using the update_assoc function where one can specify a tbl_* function to calculate association measure depending on the variable pair in the dataset and a method if it calculates more than one measure.

```
updated_assoc <- update_assoc(default=default_assoc(),</pre>
                              num_pair = "tbl_cor",
                              num_pair_argList = "spearman",
                              mixed_pair = "tbl_cancor",
                              other_pair = "tbl_nmi")
updated_assoc
#> # A tibble: 4 x 4
#>
   funName
               typeX
                                argList
                        typeY
                               st>
#>
    <chr>
               <chr>
                       <chr>
#> 1 tbl_cor
               numeric numeric <chr [1]>
#> 2 tbl_tau
               ordered ordered <NULL>
#> 3 tbl_cancor factor numeric <NULL>
#> 4 tbl_nmi
               other
                       other
                               <NULL>
```

If a user is interested in calculating multiple association measures for a type of variable pair, it can be done by using the calc_assoc and update_assoc together for calculating different association measures and then merging the output tibbles.

```
updated_penguin_assoc <- calc_assoc(df, types = updated_assoc)</pre>
head(updated_penguin_assoc)
#> # A tibble: 6 x 4
#>
                              measure measure_type
#>
   <chr>
                     <chr>
                              <dbl> <chr>
#> 1 island
                     species 0.507
                                     nmi
#> 2 bill_length_mm
                     species 0.841
                                     cancor
#> 3 bill_depth_mm
                     species 0.824
                                     cancor
#> 4 flipper_length_mm species 0.882
                                     cancor
cancor
#> 6 sex
                     species 0.0000854 nmi
```

Calculating conditional association

calc_assoc_by is used to calculate association measures for all the variable pairs at different levels of a categorical variable. This helps in exploring the conditional associations and find out the differences between the groups of the conditioning variable. The output of this function is a tibble structure with pairwise and data.frame as additional class attributes. The calc_assoc_by function has a by argument which is used for the grouping variable and it needs to be categorical.

```
penguin_assoc_by <- calc_assoc_by(df,by = "sex")</pre>
```

The calc_assoc_by function also has a types argument which can be updated similarly to calc_assoc.

```
updated_assoc <- update_assoc(num_pair = "tbl_cor",</pre>
                            num_pair_argList = "spearman",
                            mixed_pair = "tbl_cancor",
                            other_pair = "tbl_nmi")
updated_penguin_assoc_by <- calc_assoc_by(df,by = "sex", types = updated_assoc)</pre>
head(updated_penguin_assoc_by)
#> # A tibble: 6 x 5
#>
                             measure measure_type by
  Х
#>
   <chr>
                     <chr>
                            <dbl> <chr> <fct>
#> 1 island
                   species 0.502 nmi
                                               female
#> 2 bill_length_mm species 0.885 cancor
                                               female
#> 3 bill_depth_mm
                    species 0.900 cancor
                                               female
#> 4 flipper_length_mm species 0.914 cancor
                                               female
                    species 0.911 cancor
#> 5 body_mass_g
                                                female
#> 6 year
                     species 0.0457 cancor
                                                female
```

By default, the function calculates the association measures for all the variable pairs at different levels of the grouping variable and the pairwise association measures for the ungrouped data (overall). This behavior can be changed by setting include.overall to FALSE.

```
penguin_assoc_by <- calc_assoc_by(df,by = "sex",include.overall = FALSE)</pre>
```

The tibble output for calc_assoc_by has the similar structure as calc_assoc with an additional by column representing the levels of the categorical variable used in the function. The x and y variables in the output are repeated for every level of by variable. In order to have multiple by variables, the function calc_assoc_by is used multiple times with a different by variable each time and then the multiple outputs are binded row wise. For calculating multiple measures for a specific variable type, one can use update_assoc with calc_assoc_by and then can merge these multiple tibble outputs.

Section 5: corVis: Visualising Association

We propose novel visualisations to display association for every variable pair in a dataset in a single plot and show multiple bivariate measures of association simultaneously to find out interesting patterns. Efficient seriation techniques have been included to order and highlight interesting relationships. These ordered association and conditional association displays help find interesting patterns in the dataset. While designing these displays we considered matrix-type, linear and network-based layouts. A matrix-type layout simplifies lookup, and different measures may be displayed on the upper and lower diagonal. Linear layouts are more space-efficient than matrix plots, but lookup is more challenging. Variable pairs are usually ordered by relevance (usually difference in measures of association or across the factor levels), and less relevant pairs can be omitted.

Association Matrix plot

Figure 1 shows the display for every variable pair in the penguins dataset from the palmerpenguins package. Table 4 provides a brief summary on the variables in this dataset.

It shows a high positive Pearson's correlation among flipper_length and body_mass, flipper_length and bill_length, and bill_length and bodymass. There seems to be a strong negative Pearson's correlation between flipper_length and bill_depth, and bill_depth and body_mass.

The plot also shows that there is a low canonical correlation between species and sex, and island and sex of the penguins suggesting low association, which traditional correlation matrix display

Variable	Description
species	Penguin species (Adelie, Chinstrap or Gentoo)
island	Islands in Palmer Archipelago, Antarctica (Biscoe, Dream or Torgersen)
bill_length_mm	Bill length of penguin (millimeters)
bill_depth_mm	Bill length of penguin (millimeters)
flipper_length_mm	Flipper length of penguin (millimeters)
body_mass_g	Body mass of penguin (grams)
sex	Sex of the penguin (female, male)
year	Year of the study

Table 4: Variable description of the **penguins** dataset

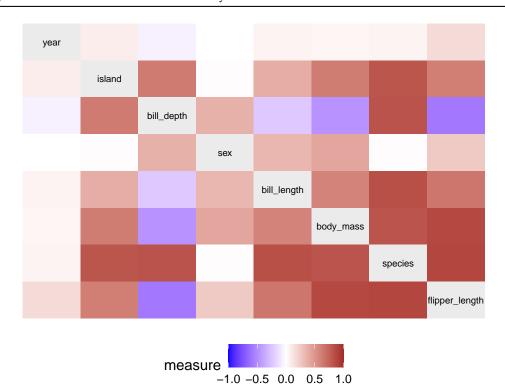


Figure 1: Association matrix display for penguins data showing Pearson's correlation for numeric variable pairs, canonical correlation for mixed variable pairs and categorical variable pairs.

would omit as they are limited to numeric variable pairs only. There is a high canonical correlation between island and species suggesting a stronger association among these two variables.

The high canonical correlation between bill_length and species, body_mass and species, and flipper_length and species suggests that the body measurements of a penguin depends on their species. The variables in the display are ordered using average linkage clustering method to find out highly associated variables quickly.

Multiple Association Measures Plot

We can also calculate multiple association measures for all the variable pairs in the dataset and compare them. This will help in finding out pairs of variables with a high difference among different measures and one can investigate these bivariate relationships in more detail. The pairwise_summary_plot function can be used to compare various measures using the matrix layout. It plots multiple measures among the variable pairs as bars, where each bar represents one measure of association. Figure 2 shows a matrix layout comparing Pearson's and Spearman's correlation coefficient for the numeric variable pairs in penguins data.

In addition to matrix layout, we can also use linear layouts for comparing multiple measures. Figure 3 shows a linear layout comparing multiple association measures for all the variable pairs in the penguins data. Linear layouts seems to be more suitable when comparing high number of association

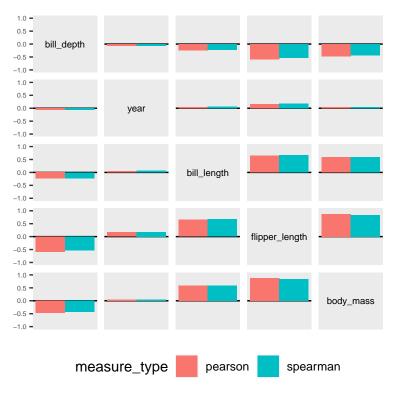


Figure 2: Matrix display comparing Pearson's and Spearman's correlation coefficient. All the variable pairs have similar values for both correlations.

measures.

Conditional Association Plot

The package includes a function calc_assoc_by which calculates the pairwise association at different levels of a categorical conditioning variable. This helps in finding out interesting variable triples which can be explored further prior to modeling. Figure 4 shows a conditional association plot for the penguins data. Each cell corresponding to a variable pair shows three bars which correspond to the association measure (Pearson's correlation for numeric pair and Normalized mutual information for other combination of variables) calculated at the levels of conditioning variable island. The dashed line represents the overall association measure. The plot shows that there is a high value for normalised mutual information between bill_length_mm and species for the penguins which lived in Biscoe island compared to the penguins which lived in Dream island. It can also be seen that the cell corresponding to variable pair flipper_length_mm and bill_depth_mm has a high negative overall Pearson's correlation and for the penguins which lived in Biscoe island but positive correlation for penguins which lived in Dream and Torgersen island. This is an instance of Simpson's paradox which can be taken into account during the modeling step.

We can also use linear layouts for displaying conditional association. Figure 5 shows a funnel-like linear display for conditional association measures with all the variable pairs on the y-axis, the value of association measure on x-axis and color of the points representing the level of the grouping variable. The linear layout becomes more useful over the matrix layout when the number of variables and number of levels of grouping variable are high.

Bibliography

- A. Agresti. Analysis of ordinal categorical data, volume 656. John Wiley & Sons, 2010. [p3]
- R. A. Becker, W. S. Cleveland, and M.-J. Shyu. The visual design and control of trellis display. *Journal of Computational and Graphical Statistics*, 5(2):123–155, 1996. doi: 10.1080/10618600.1996.10474701. URL https://www.tandfonline.com/doi/abs/10.1080/10618600.1996.10474701. [p4]
- A. Buja, A. M. Krieger, and E. I. George. A visualization tool for mining large correlation tables: The association navigator., 2016. [p2]

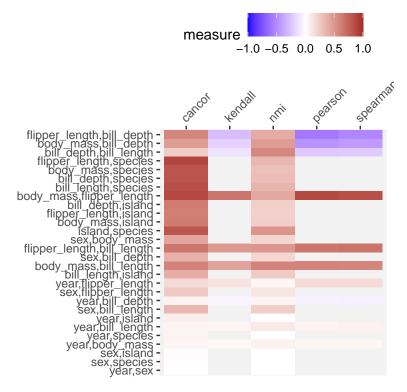


Figure 3: Comparing multiple association measures using a linear layout. The display has variable pairs on the Y-axis and association measures on the X-axis. The cell corresponding to a variable pair and an association measure has been colored grey showing that the measure is not defined for corresponding pair.

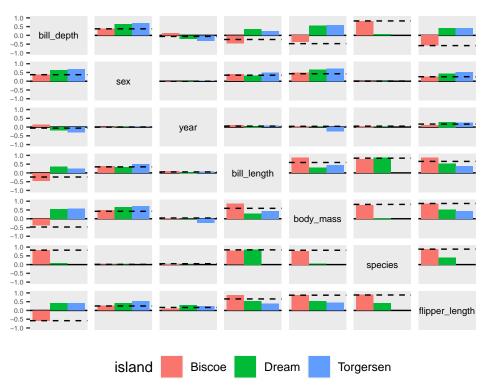


Figure 4: Conditional Association plot for penguins data showing Pearson's correlation for numeric pairs and normalised mutual information for categorical or mixed pairs. The bars in each cell represent the value for association measure colored by the conditioning variable 'island'. The dashed line in each cell represents overall value of the association measure.

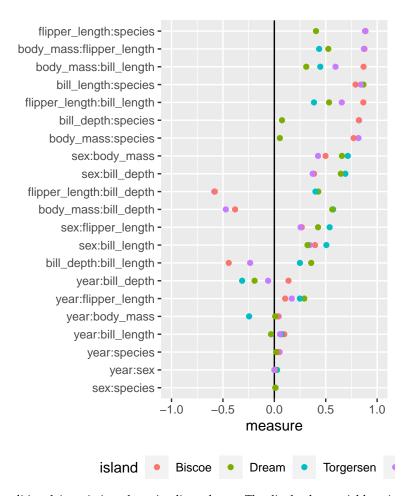


Figure 5: Conditional Association plot using linear layout. The display has variable pairs on the Y-axis and the value of association measures on the X-axis. The points corresponding to every variable pair represents the value of association measure for different levels of the conditioning variable and the overall value of association measure.

- M. Friendly. Corrgrams: Exploratory displays for correlation matrices. *The American Statistician*, 56(4): 316–324, 2002. [p1, 2]
- S. Gerber. scorr: s-CorrPlot: Visualizing Correlation, 2022. URL http://mckennapsean.com/scorrplot/.
 R package version 1.0. [p2]
- W. K. Härdle and L. Simar. Applied multivariate statistical analysis. Springer Nature, 2019. [p3]
- M. Hills. On looking at large correlation matrices. Biometrika, 56(2):249–253, 1969. [p2]
- M. G. Kendall. The treatment of ties in ranking problems. Biometrika, 33(3):239-251, 1945. [p3]
- M. Kuhn, K. Johnson, et al. *Applied predictive modeling*, volume 26. Springer, 2013. [p3]
- M. Kuhn, S. Jackson, and J. Cimentada. *corrr: Correlations in R*, 2020. URL https://CRAN.R-project.org/package=corrr. R package version 0.4.3. [p2]
- P. Morgen and P. Biecek. corrgrapher: Explore Correlations Between Variables in a Machine Learning Model, 2020. URL https://CRAN.R-project.org/package=corrgrapher. R package version 1.0.4. [p2]
- D. J. Murdoch and E. Chow. A graphical display of large correlation matrices. The American Statistician, 50(2):178–180, 1996. [p2]
- U. Olsson. Maximum likelihood estimation of the polychoric correlation coefficient. *Psychometrika*, 44 (4):443–460, 1979. [p3]
- D. N. Reshef, Y. A. Reshef, H. K. Finucane, S. R. Grossman, G. McVean, P. J. Turnbaugh, E. S. Lander, M. Mitzenmacher, and P. C. Sabeti. Detecting novel associations in large data sets. *science*, 334(6062): 1518–1524, 2011. [p1, 3]
- A. Samba. linkspotter: Bivariate Correlations Calculation and Visualization, 2020. URL https://CRAN.R-project.org/package=linkspotter. R package version 1.3.0. [p2]
- G. J. Székely, M. L. Rizzo, and N. K. Bakirov. Measuring and testing dependence by correlation of distances. *The annals of statistics*, 35(6):2769–2794, 2007. [p1, 3]
- H. Theil. On the estimation of relationships involving qualitative variables. *American Journal of Sociology*, 76(1):103–154, 1970. [p3]
- E. R. Tufte. *The Visual Display of Quantitative Information*. Graphics Press, USA, 1986. ISBN 096139210X. [p1, 4]
- J. W. Tukey and P. A. Tukey. Computer graphics and exploratory data analysis: An introduction. In *Proceedings of the sixth annual conference and exposition: computer graphics*, volume 85, pages 773–785, 1985. [p1, 3]
- T. Wei and V. Simko. *R package 'corrplot': Visualization of a Correlation Matrix*, 2021. URL https://github.com/taiyun/corrplot. (Version 0.92). [p2]
- H. Wickham, M. Averick, J. Bryan, W. Chang, L. D. McGowan, R. François, G. Grolemund, A. Hayes, L. Henry, J. Hester, M. Kuhn, T. L. Pedersen, E. Miller, S. M. Bache, K. Müller, J. Ooms, D. Robinson, D. P. Seidel, V. Spinu, K. Takahashi, D. Vaughan, C. Wilke, K. Woo, and H. Yutani. Welcome to the tidyverse. *Journal of Open Source Software*, 4(43):1686, 2019. doi: 10.21105/joss.01686. [p2]
- L. Wilkinson, A. Anand, and R. Grossman. Graph-theoretic scagnostics. In *Information Visualization, IEEE Symposium on*, pages 21–21. IEEE Computer Society, 2005. [p3]

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