Gaussian Pollution Model

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July 10, 2013

Many soot dispersion models use a Gaussian spread model. This model is very similar, utilizing a Gaussian approach (modelling densities rather than tracking individual particles). However, my approach is that wind disperses soot particles in *all* directions, but mostly in the direction of the wind. Also, wind disperses soot a distance proportional to the wind speed.

Approaching this from a different perspective gives us the idea that the pollution at given point is dependent on the speed of the wind at that given area, along with the direction of the wind. So, some small area dA receives some fractional pollution from all of the area around it, depending on the wind velocity and heading.

I believe that this dispersion is both Gaussian in distance (relating to wind velocity) and in angle (relating to wind direction), giving us a two dimensional Gaussian curve. The standard normal curve is as follows (CITE):

$$N(\mu, \sigma, x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

We can extend this into a second dimension by simply multiplying the curve by another normal curve (CITE). In our case, we want the curve to model wind dispersion in a polar coordinate system (r, θ). We can choose two means and two standard deviations, but those are simply parameters that can be reasonably chosen through some experimentation and guessing.

Since, these are Gaussian distributions, we are guaranteed that the area underneath the graphs (in our case, the volume) is 1. The pollution at some specified location is the weighted sum of all pollution, where the weights are determined by wind speed and direction. If we let (a, b) be the point we seek to evaluate, $d_{(x,y)}$ be the distance from the point (x, y) to our point (a, b) and

$\phi_{(x,y)}$ be the angle between our point (a, b) and (x, y), we arrive at a method for determining the pollution density P at some location (x, y) and time t.

$$P(a,b,t+h) = \int_{R^2} N_r(\frac{v(t)}{h}, \sigma_r, d_{x,y}) N_\theta(h(t), \sigma_\theta, \phi_{x,y}) P(x,y,t) dA$$

In this case, $v(t)$ is the velocity of the wind at the current time t, and $h(t)$ is the heading of the wind at the current time. Since we are integrating over all space, we are simply convoluting the "pollution image" in polar space.

However, an issue arises with using this method. Consider we are simulating over some pixel space, say a standard XGA screen (resolution 1280×768). For each of those pixels, every pixel on the screen must be added into a weighted sum to calculate the pollution density for *each pixel*. This results in an $O(n^4)$ algorithm. Most modern computer screens operate at an even higher resolution, making speed an issue.