Signal processing presentation

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Contents

The transfer function of a digital filter is expressed as follows:

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \ldots + b_{n-1} z^{-(n-1)} + b_n z^{-n}}{a_0 + a_1 z^{-1} + \ldots + a_{m-1} z^{-(m-1)} + a_m z^{-m}} = \frac{\sum_{i=0}^{n} b_i z^{-i}}{\sum_{j=0}^{m} a_j b^{-j}}$$

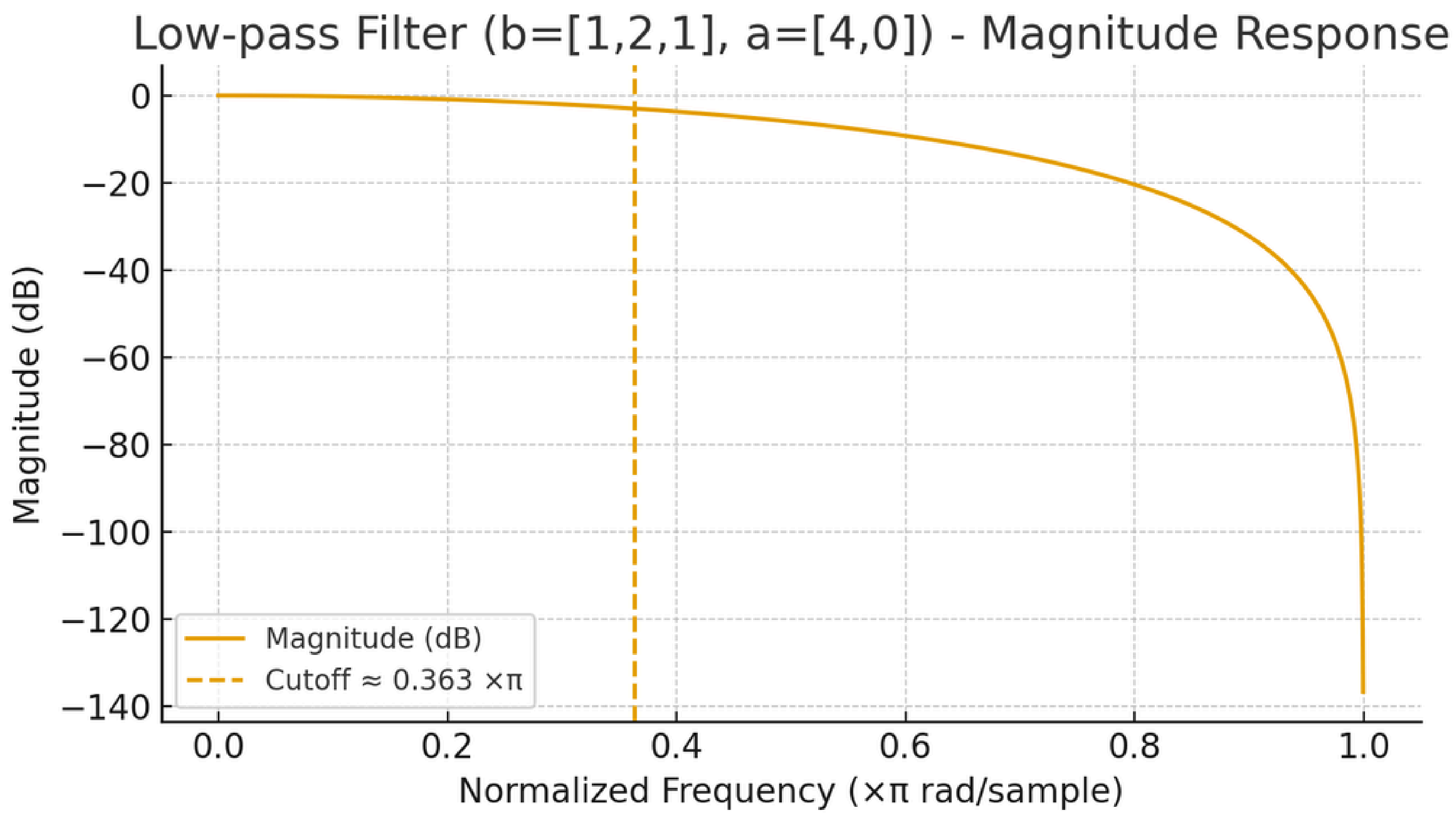
1) 저역통과: b = [1, 2, 1], a = [4, 0]

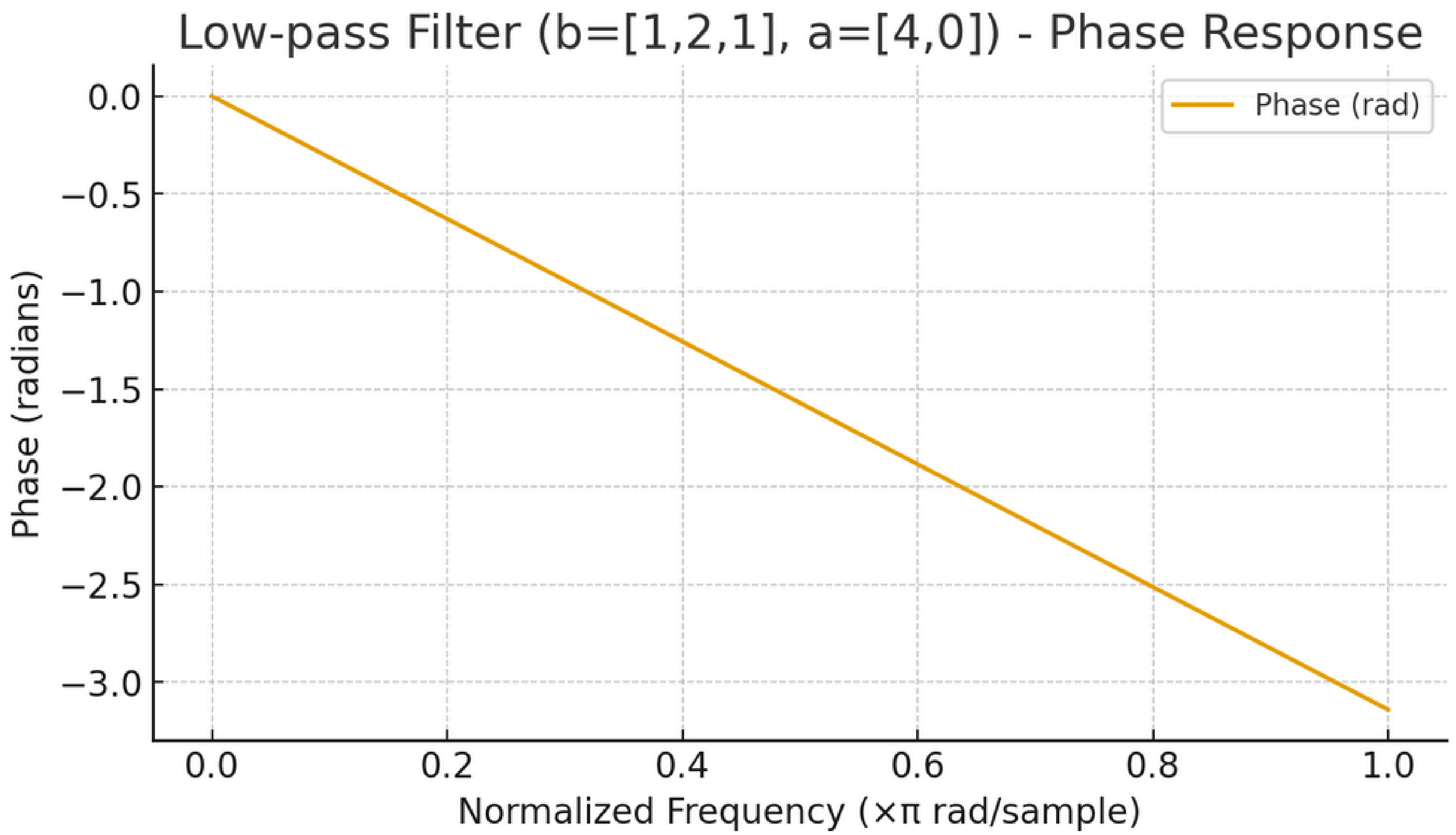
- **차분방정식**: $y[n] = \frac{1}{4}\{x[n] + 2x[n-1] + x[n-2]\}$
  → 최근 3샘플을 **부드럽게 평균(삼각 창)** → 저주파(느린 변화) 통과.
- **주파수 응답(해석)**

$$H_{LP}(e^{j\omega}) = \frac{1 + 2e^{-j\omega} + e^{-j2\omega}}{4} = e^{-j\omega}\cos^2\left(\frac{\omega}{2}\right)$$

  - 크기 $|H| = \cos^2(\omega/2)$:
    $\omega = 0$에서 1(=0 dB), $\omega = \pi$에서 0(=−∞ dB).
  - 위상 $\angle H \approx -\omega$: **선형 위상**, 군지연 ≈ 1 sample.
- **차단 주파수(−3 dB)**: $\cos^2(\omega_c/2) = 10^{-3/20}$ →
  $\omega_c \approx 0.363\,\pi$ (그림의 점선).
  → Hz로는 $f_c \approx 0.363 \times \frac{F_s}{2} \approx 0.1815\,F_s$.

Given a low-pass filter with b=1,2,1 and a=4,0, plot its frequency response (with the cut-off frequency showed in the figure) and its phase response. What's <u>Response</u>?





Problem 1

The transfer function of a digital filter is expressed as follows:

$$H(z)=\frac{B(z)}{A(z)}=\frac{b_0+b_1z^{-1}+\ldots+b_{n-1}z^{-(n-1)}+b_nz^{-n}}{a_0+a_1z^{-1}+\ldots+a_{m-1}z^{-(m-1)}+a_mz^{-m}}=\frac{\sum_{i=0}^{n}b_iz^{-i}}{\sum_{j=0}^{m}a_jb^{-j}}$$

2) 고역통과: b = [−1, 2, −1], a = [4, 0]

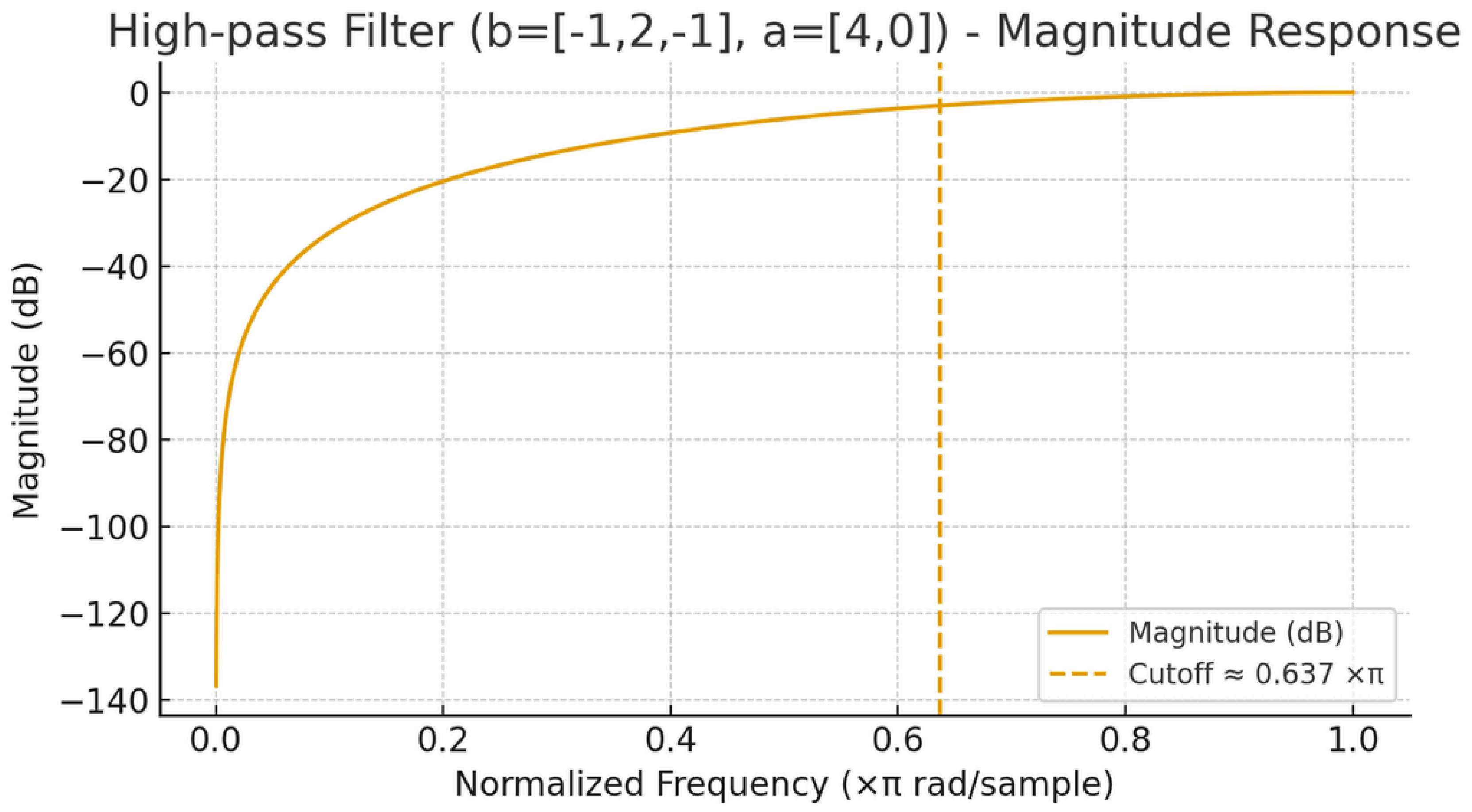
- 차분방정식: $y[n]=\frac{1}{4}\{-x[n]+2x[n-1]-x[n-2]\}$
  → **이차 차분(이산 미분)**에 가까움 → 고주파(급격한 변화) 통과.
- 주파수 응답(해석)

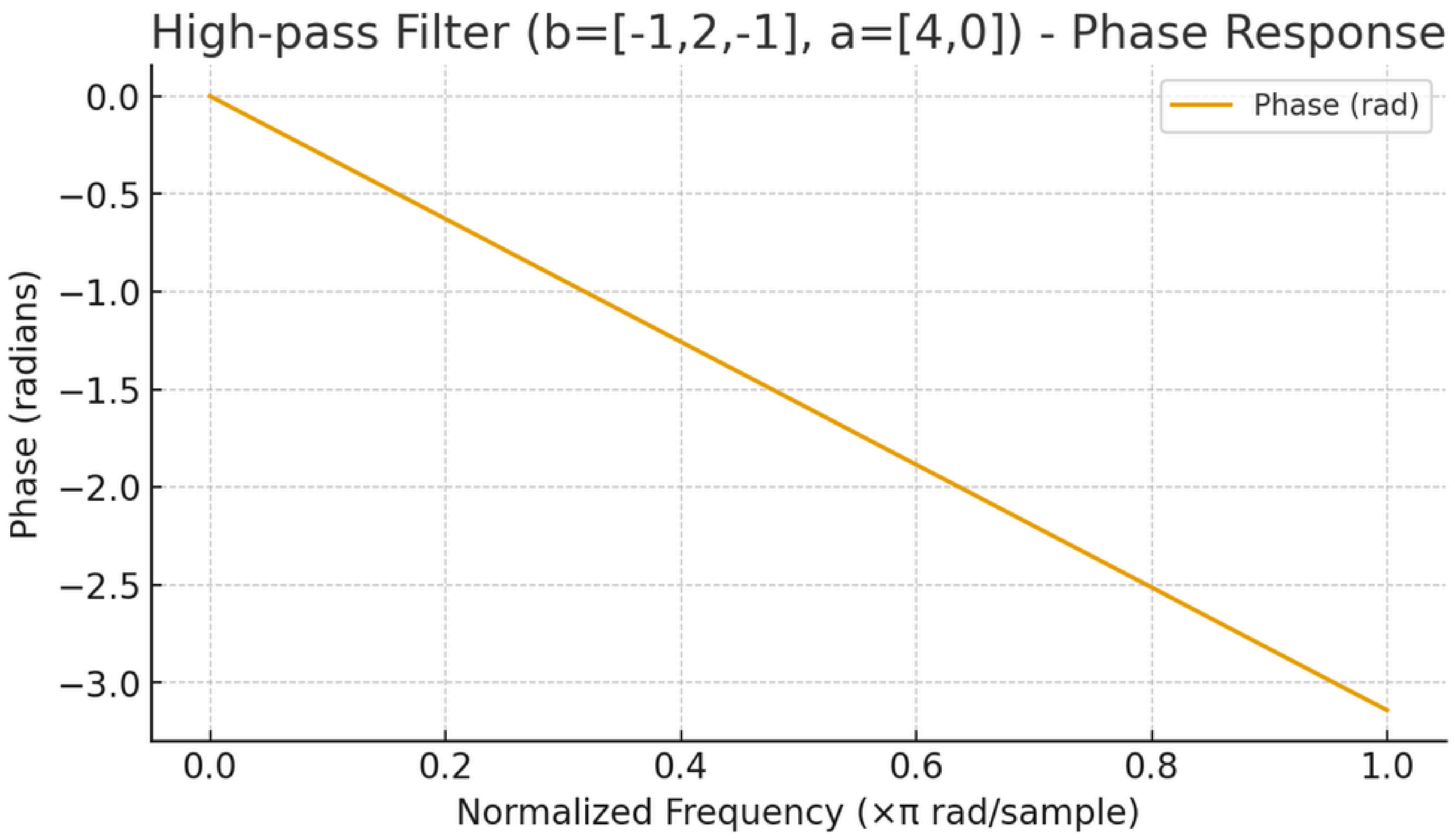
$$H_{HP}(e^{j\omega})=\frac{-1+2e^{-j\omega}-e^{-j2\omega}}{4}=-e^{-j\omega}\sin^2\left(\frac{\omega}{2}\right)$$

  - 크기 $|H|=\sin^2(\omega/2)$:
    $\omega=0$에서 0(=−∞ dB), $\omega=\pi$에서 1(=0 dB).
  - 위상은 $-\omega$에 **상수 π**가 더해진 형태지만(부호 − 때문),
    unwrap하면 그림처럼 거의 직선으로 보입니다(모듈로 2π).
- **차단 주파수(−3 dB)**: $\sin^2(\omega_c/2)=10^{-3/20}$ →
  $\omega_c\approx 0.637\,\pi$ (그림의 점선).
  → Hz로는 $f_c\approx 0.637\times\frac{F_s}{2}\approx 0.3185\,F_s$.

> 참고: 두 필터의 차단 주파수는 **서로 보완되어**
> $0.363\pi+0.637\pi=\pi$가 됩니다.

Given a high-pass filter with b=-1,2,-1 and a=4,0, plot is frequency response (with the cut-off frequency showed in the figure) and its phase response. What's <u>Response</u>?





In result the solved math

- 저역통과 응답:

$$|H_{LP}(e^{j\omega})| = \cos^2\left(\frac{\omega}{2}\right)$$

- 고역통과 응답:

$$|H_{HP}(e^{j\omega})| = \sin^2\left(\frac{\omega}{2}\right)$$

여기서 잘 알려진 항등식이 있죠:

$$\cos^2\left(\frac{\omega}{2}\right) + \sin^2\left(\frac{\omega}{2}\right) = 1$$

즉,

$$|H_{LP}(e^{j\omega})| + |H_{HP}(e^{j\omega})| = 1$$

***To recap**

지금의 LPF/HPF 쌍은 수학적으로 보완 필터 (complementary filters).

- **둘의 제곱 크기 합은 항상 1 → 따라서 저역과 고역을 더하면 중간 주파수 대역이 정확히 이어짐**
- **결국 두 개를 합치면 전 대역(All-pass) 응답이 되며, 위상은 단순히 $-\omega$ 지연만 남음**

"1"이라는 건 통과 이득(=0 dB) 에 해당.
따라서 두 응답을 합치면 전 주파수 대역을 모두 커버하게 됨.
직관적으로는 LPF + HPF = All-pass Filter (위상은 지연만 있음) 이 된다고 볼 수 있음.

The mathematical expression of the 1-D CZP(Circular Zone Plate) signal is as follows:

$$z(x) = C_w \cdot \cos\left(\pi \frac{x^2}{T}\right) + C_{offset}$$

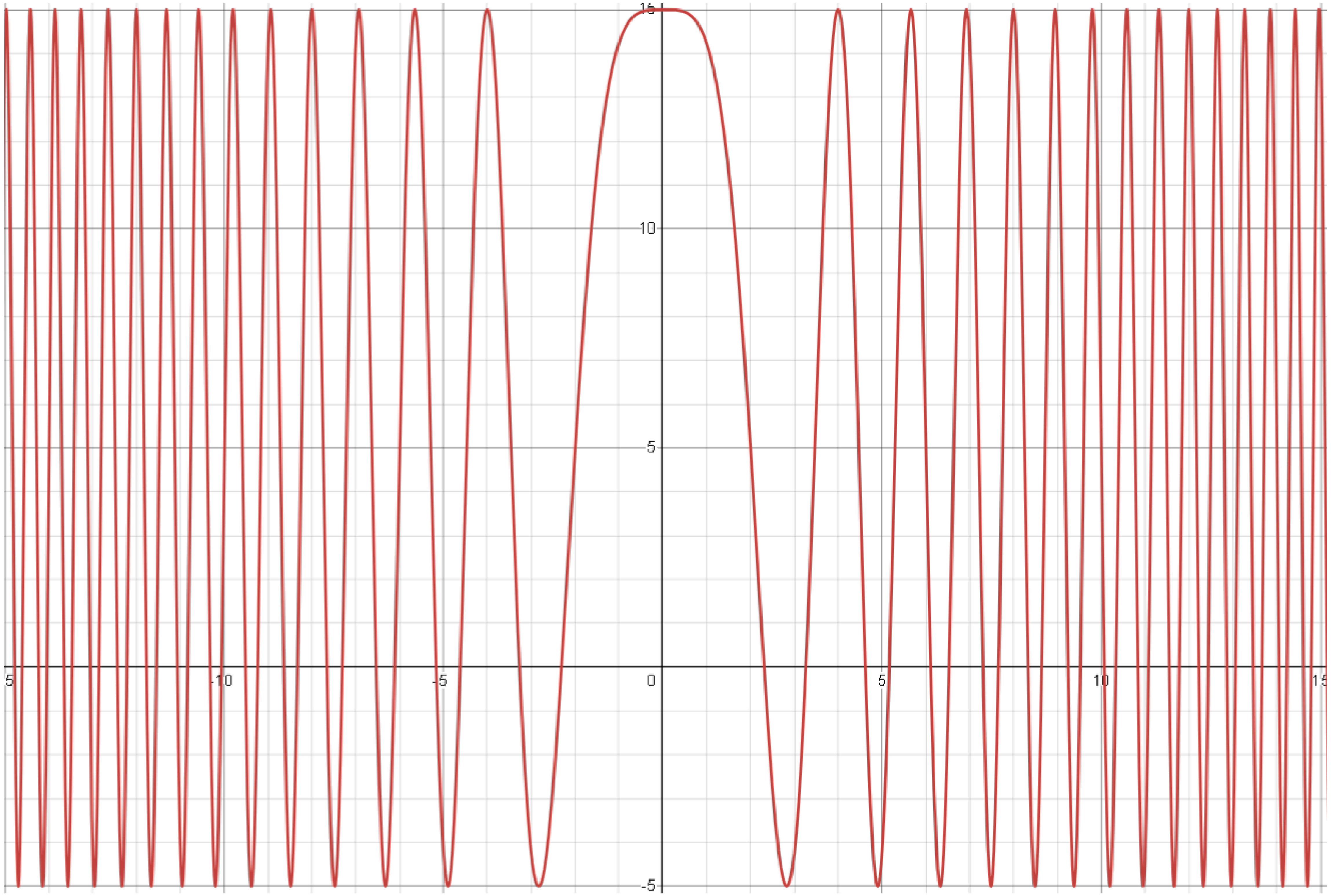
$$\cos\left(\pi \frac{x^2}{T}\right) \rightarrow \left(\pi \frac{x^2}{T}\right) = 2\pi$$

Plot the 1-D CZP signal (C_w, C_{offset}, and T can be freely determined)

C_w = 10, C_{offset} = 5, T = 8

→ x = 4

$$z(x) = 10 \cdot \cos\left(\pi \frac{x^2}{8}\right) + 5$$



The mathematical expression of the 2-D CZP signal is as follows:

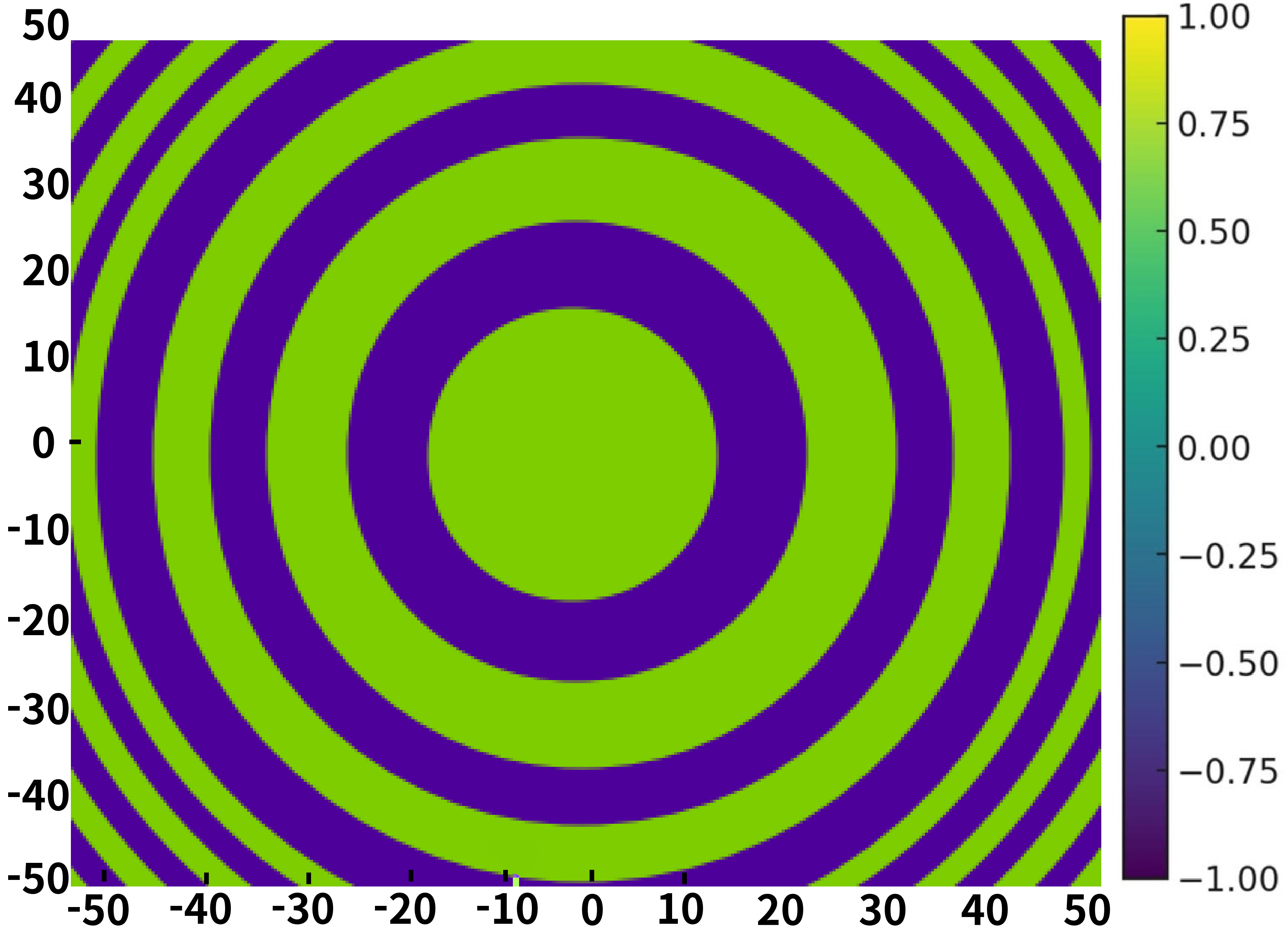
$$z(x,y) = C_w \cdot \cos\left(\pi \frac{x^2 + y^2}{T}\right) + C_{offset}$$

① Plot the 2-D CZP signal (C_w, C_{offset}, and T can be freely determined).

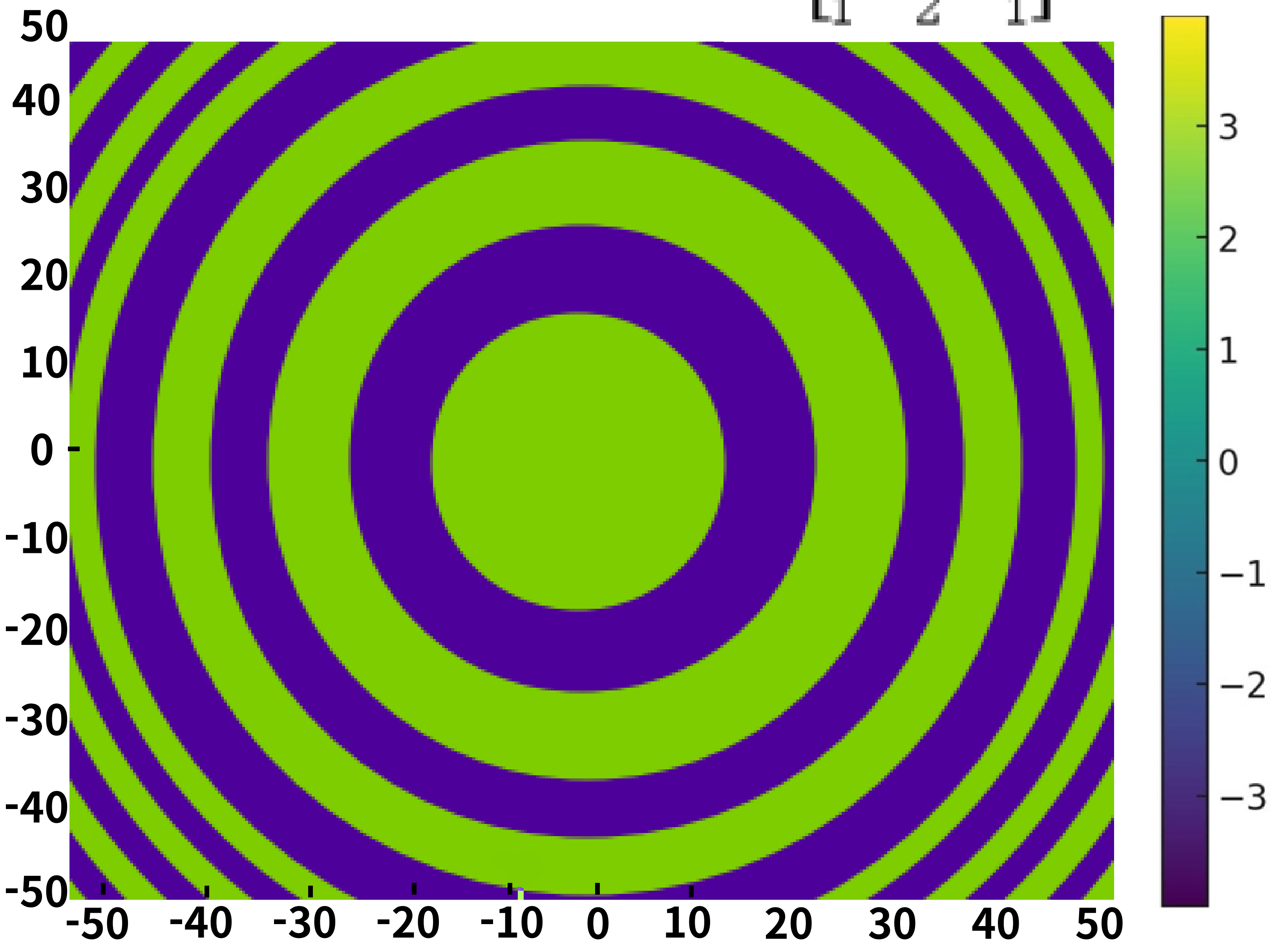
$C_w = 1$, $C_{offset} = 0$, $T = 72$

② Given a low-pass filter represented by the matrix $\frac{1}{4}\cdot\begin{bmatrix}1 & 2 & 1\\2 & 4 & 2\\1 & 2 & 1\end{bmatrix}$, plot the result of filtering the CZP signal.↵

$C_w = 1, C_{offset} = 0, T = 72$

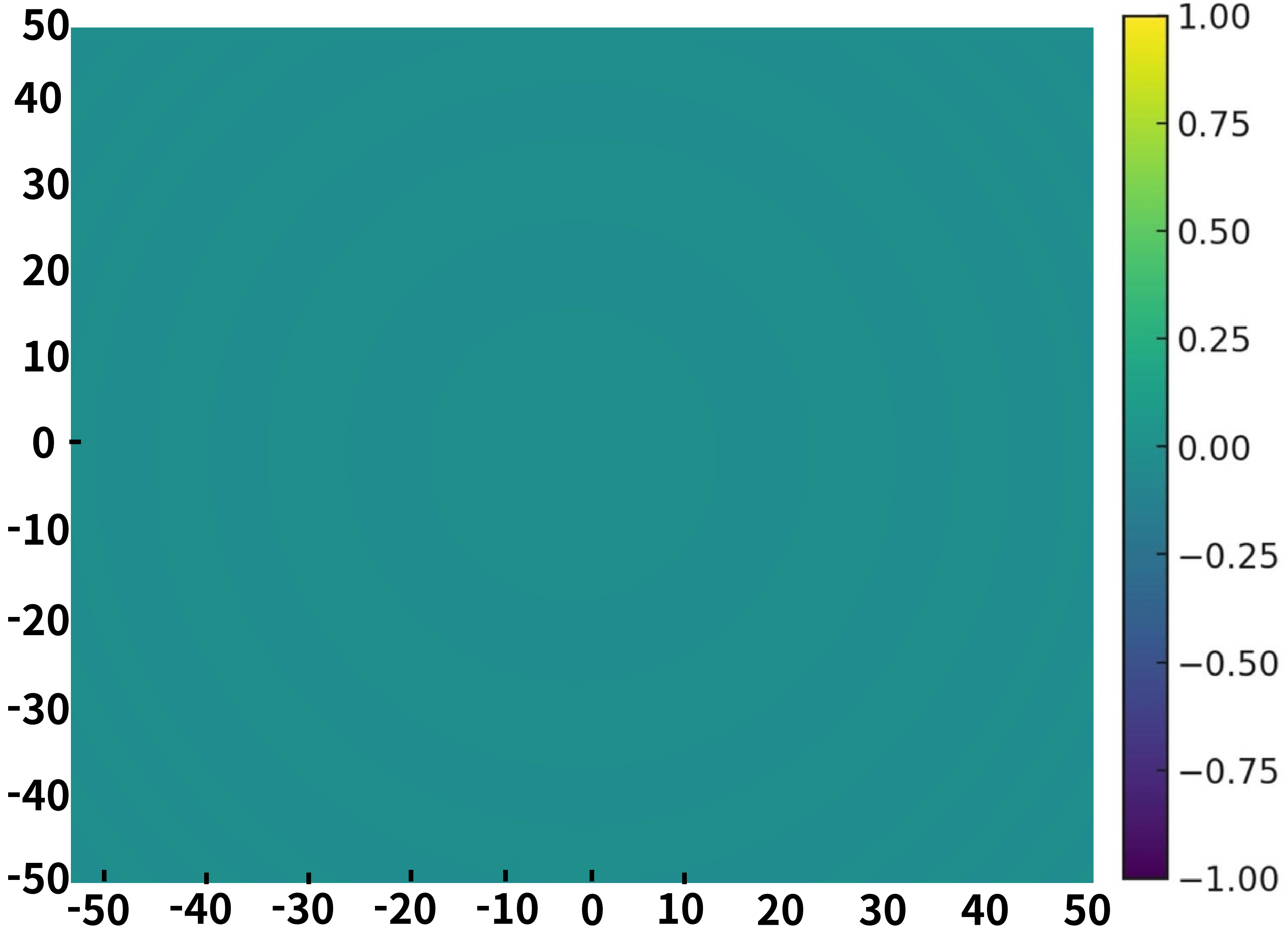


low-pass filtered($\frac{1}{4}\cdot\begin{bmatrix}1 & 2 & 1\\2 & 4 & 2\\1 & 2 & 1\end{bmatrix}$)

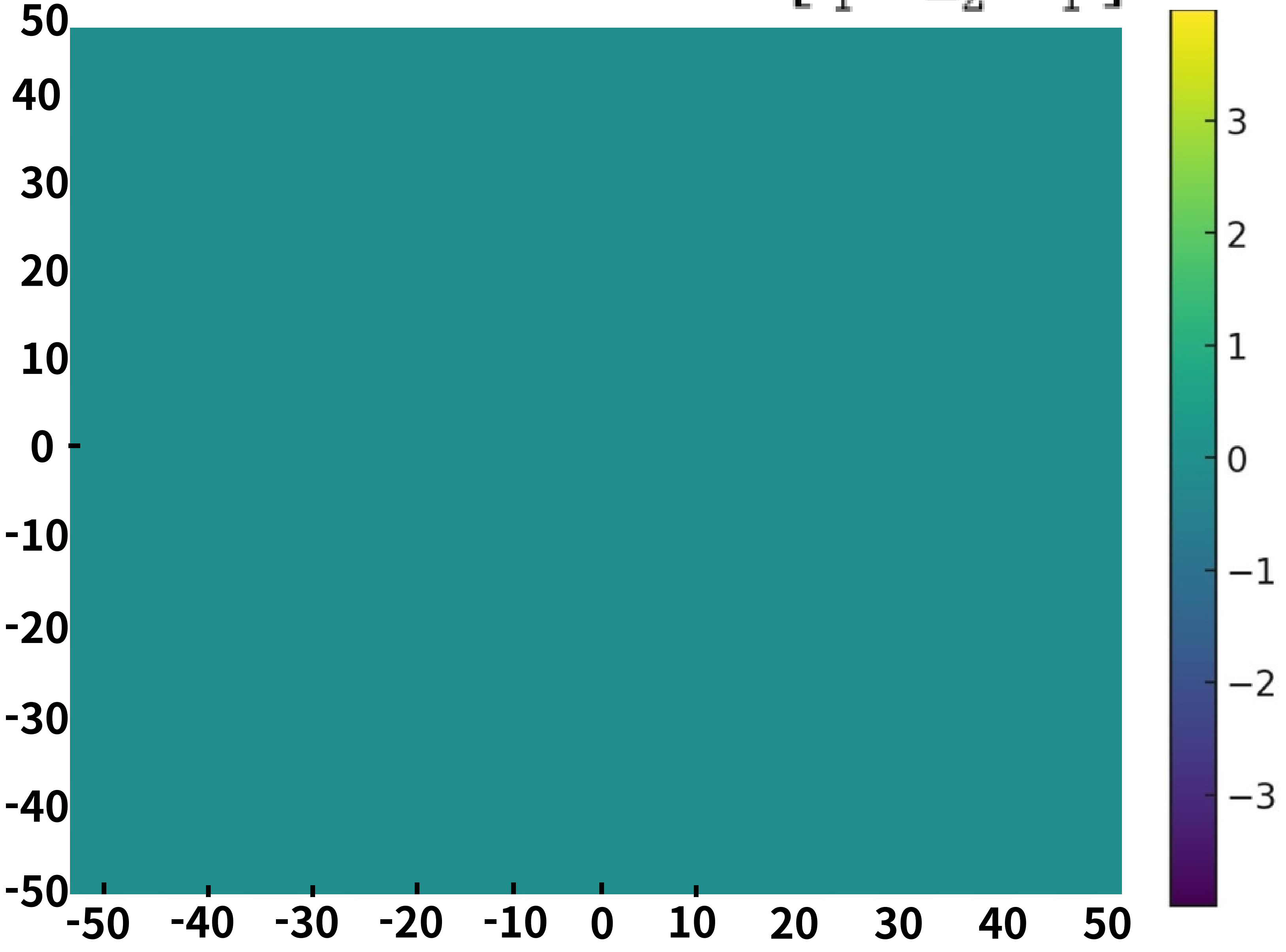


③ Given a high-pass filter represented by the matrix $\frac{1}{4}\cdot\begin{bmatrix}1 & -2 & 1\\ -2 & 4 & -2\\ 1 & -2 & 1\end{bmatrix}$, plot the result of filtering the CZP signal.

$C_w = 1$, $C_{offset} = 0$, $T = 72$



high-pass filtered($\frac{1}{4}\cdot\begin{bmatrix}1 & -2 & 1\\ -2 & 4 & -2\\ 1 & -2 & 1\end{bmatrix}$)



References

1. Goodman, J. W. (2005). Introduction to Fourier Optics (3rd ed.). Roberts and Company Publishers.

2. Cao, Q., & Jahns, J. (2004). Comprehensive focusing analysis of various Fresnel zone plates. Journal of the Optical Society of America A, 21(4), 563–571.

3. Chatgpt 주파수 응답 그래프

감사합니다!