



CPSC 425: Computer Vision

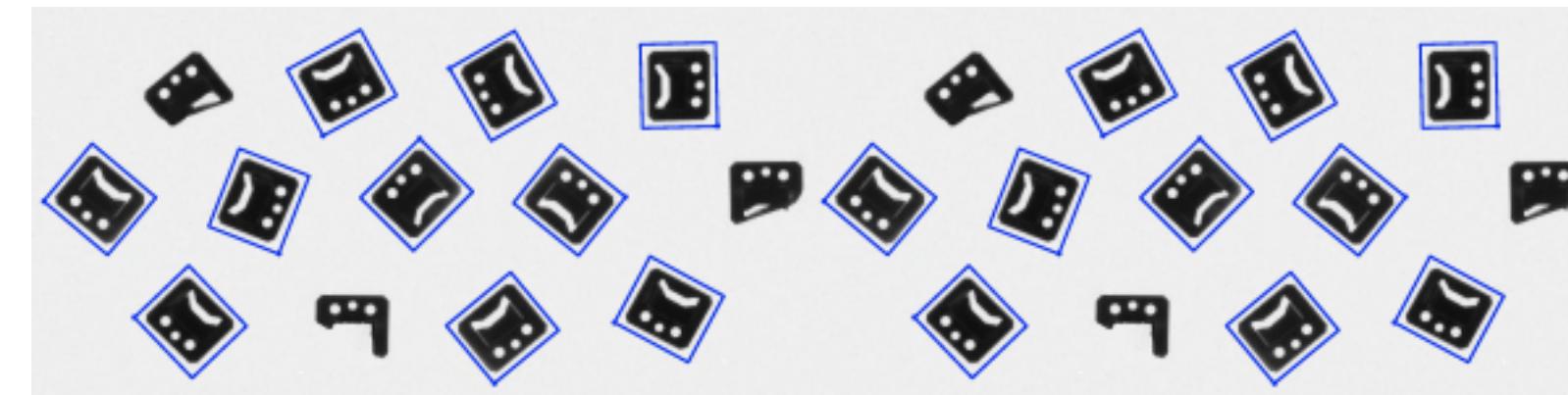


Image Credit: https://docs.adaptive-vision.com/4.7/studio/machine_vision_guide/TemplateMatching.html

Lecture 8: Scaled Representations

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little and Fred Tung**)

Menu for Today

Topics:

- **Digital Imaging** Pipeline
- **Scaled** Representations
- Template **Matching**
- Normalised **Correlation**

Readings:

- **Today's** Lecture: Szeliski 2.3, 3.5, Forsyth & Ponce (2nd ed.) 4.5 - 4.7

Reminders:

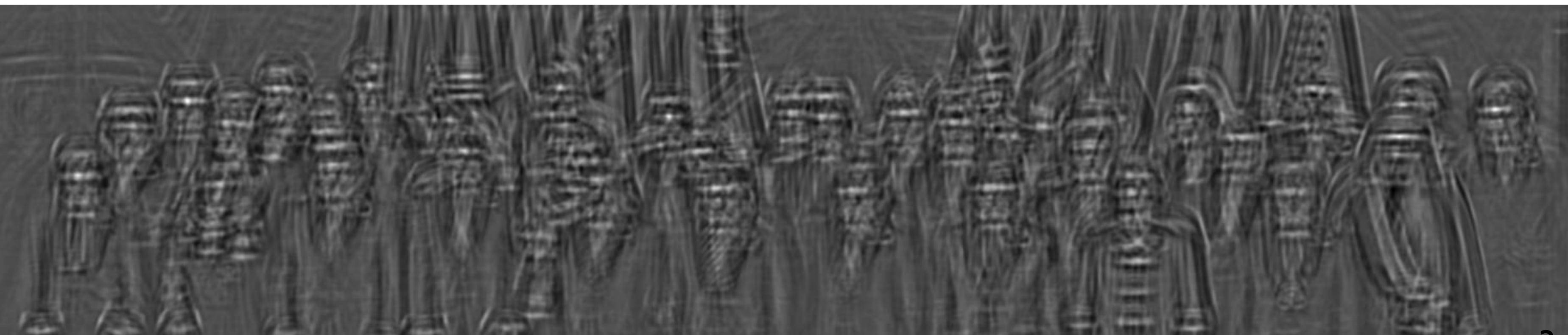
- **Assignment 1:** due **Thursday** September 28th
- **Assignment 2:** Scaled Representations, Face Detection and Image Blending available now

Template Matching

- Convolve image with template, find local maxima



* → Non-max suppress →



Template Matching

- Convolve image with template, find local maxima



* → Non-max suppress →

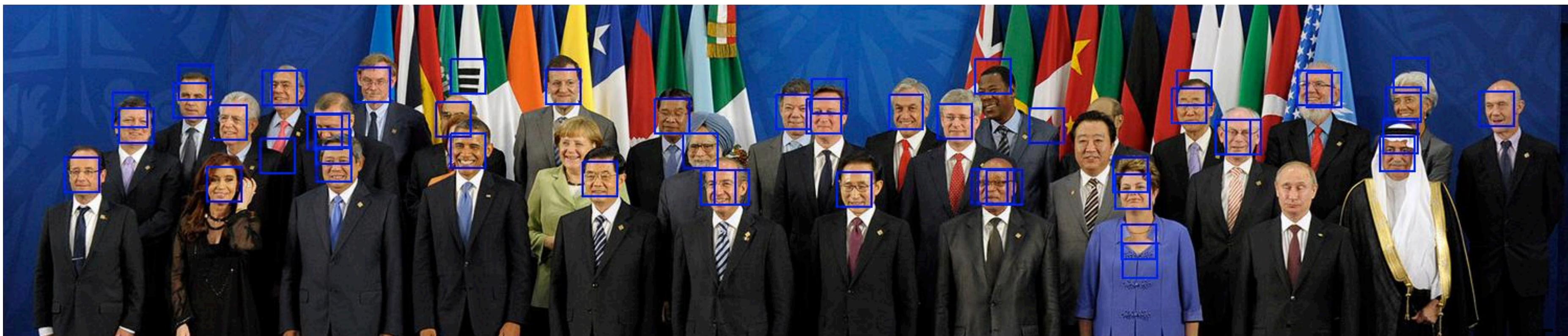


Template Matching

- Convolve image with template, find local maxima



* → Non-max suppress →
+ threshold



Detection Performance

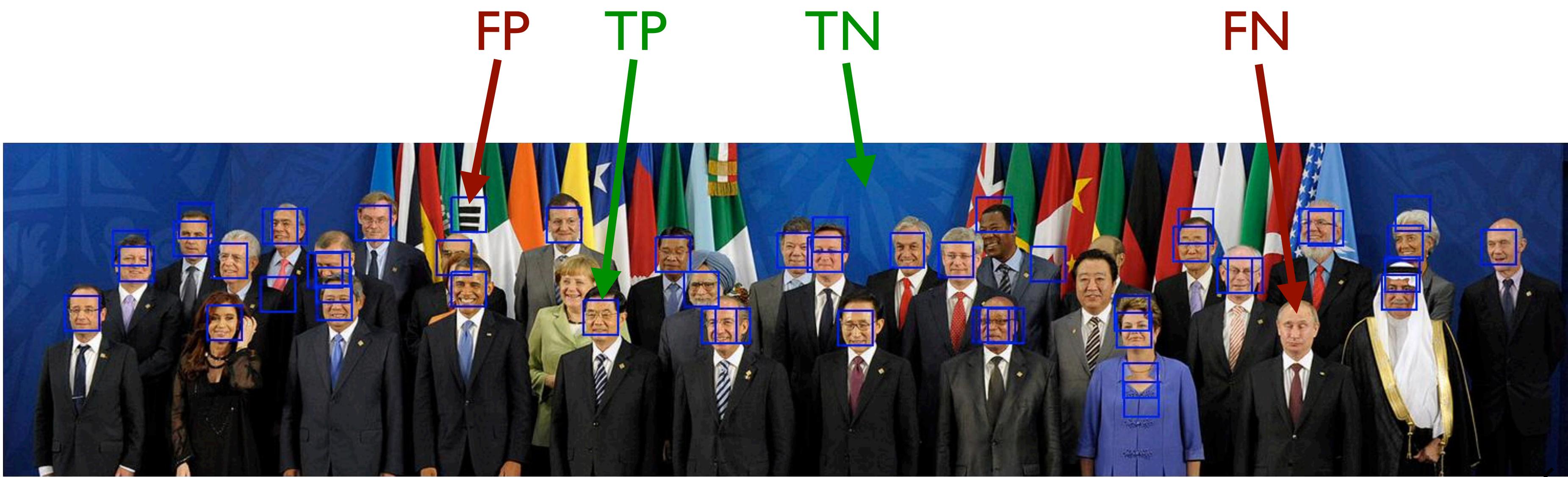
- Types of error in detection:

TP = True positive (true face and detected)

FP = False positive (not face and detected)

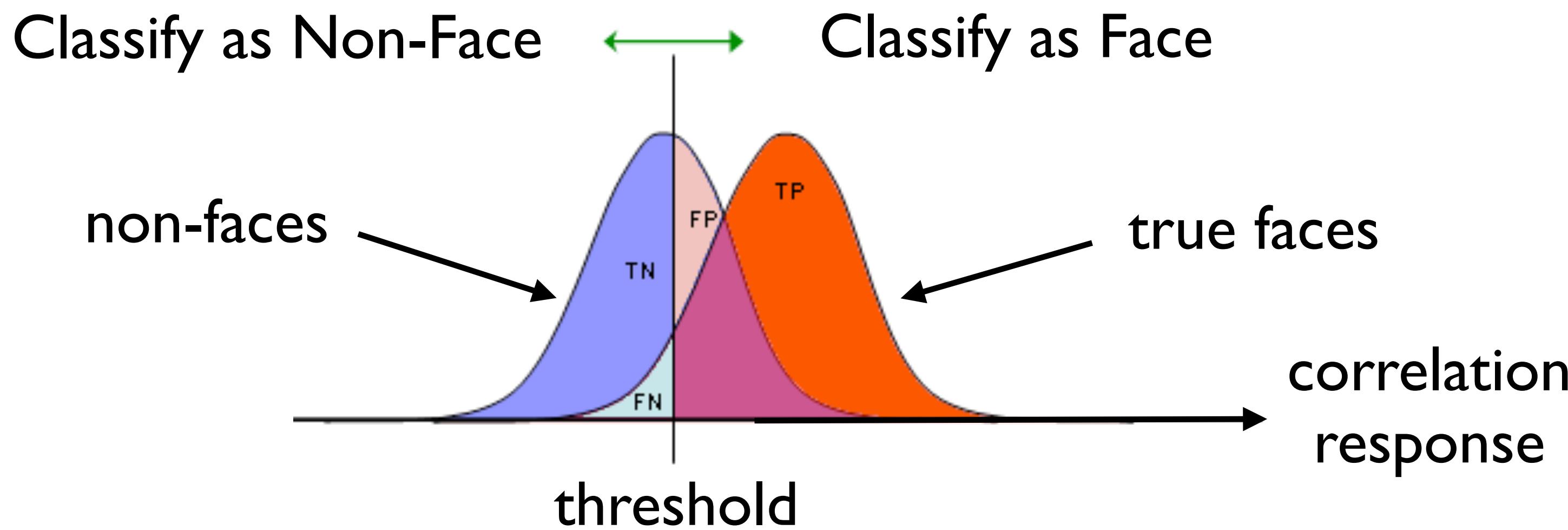
TN = True negative (not face and no detection)

FN = False negative (true face and not detected)



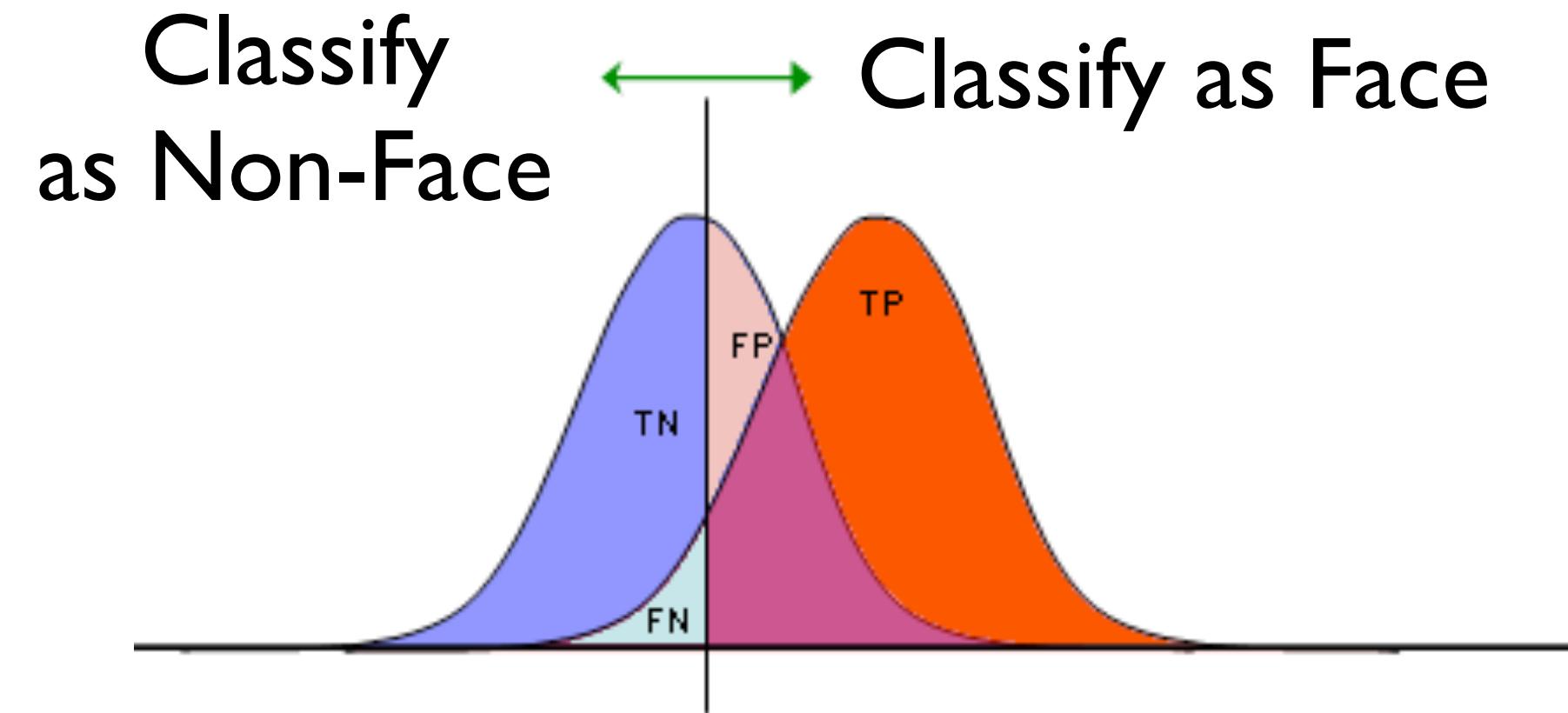
Detection Performance

- Depending on where we set the threshold, we can tradeoff between true positives and false positives

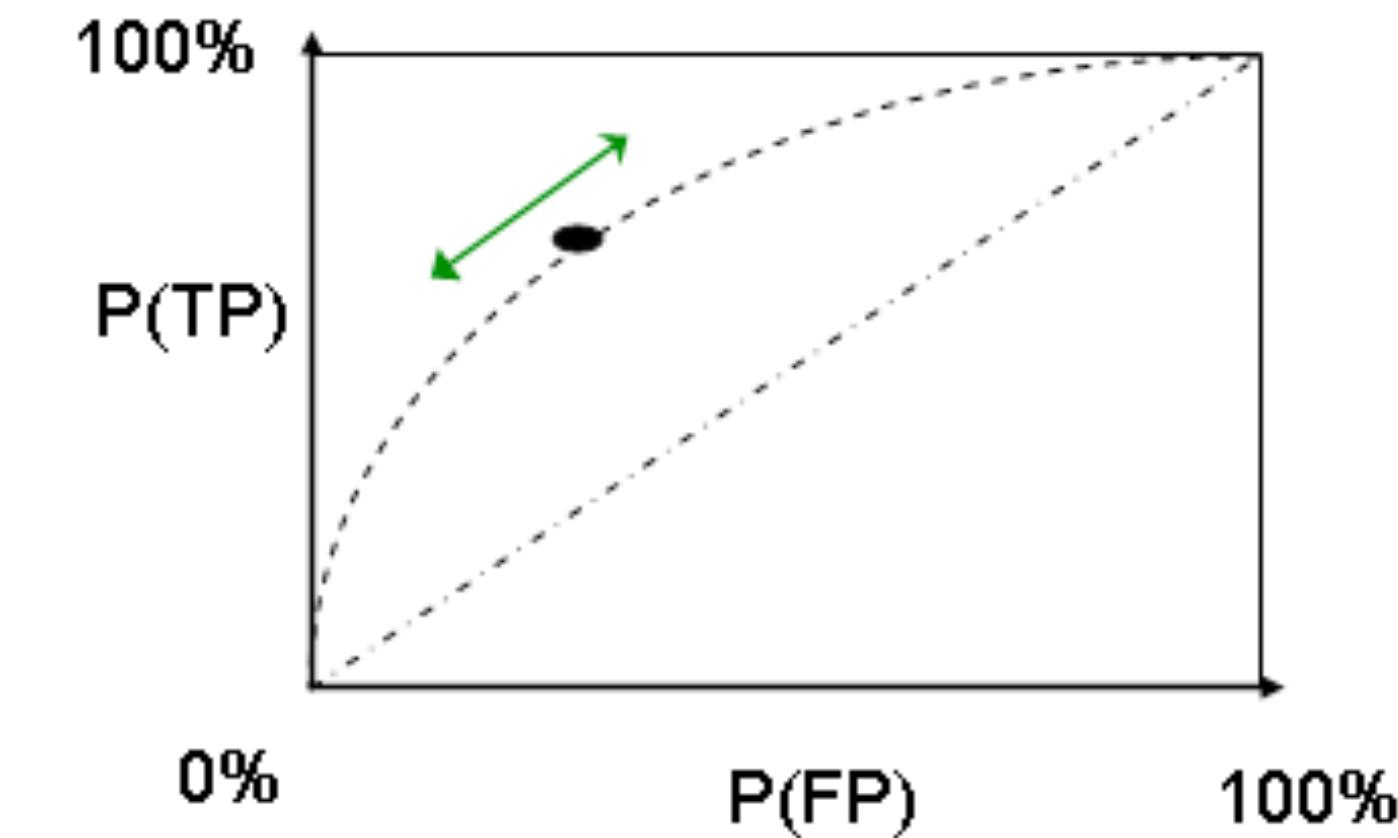


ROC Curves

- Note that we can easily get 100% true positives (if we are prepared to get 100% false positives as well!)
- It is a tradeoff between **true positive rate (TP)** and **false positive rate (FP)**
- We can plot a curve of all TP rates vs FP rates by varying the classifier threshold
- This is a **Receiver Operating Characteristic (ROC)** curve



red = actual faces, blue = actual non-faces



Multi-Scale Template Matching

Problem: Make template matching robust to changes in 2D (spatial) scale.

Key Idea(s): Build a scaled representation: the Gaussian image pyramid

Alternatives:

- use multiple sizes for each given template
- ignore the issue of 2D (spatial) scale

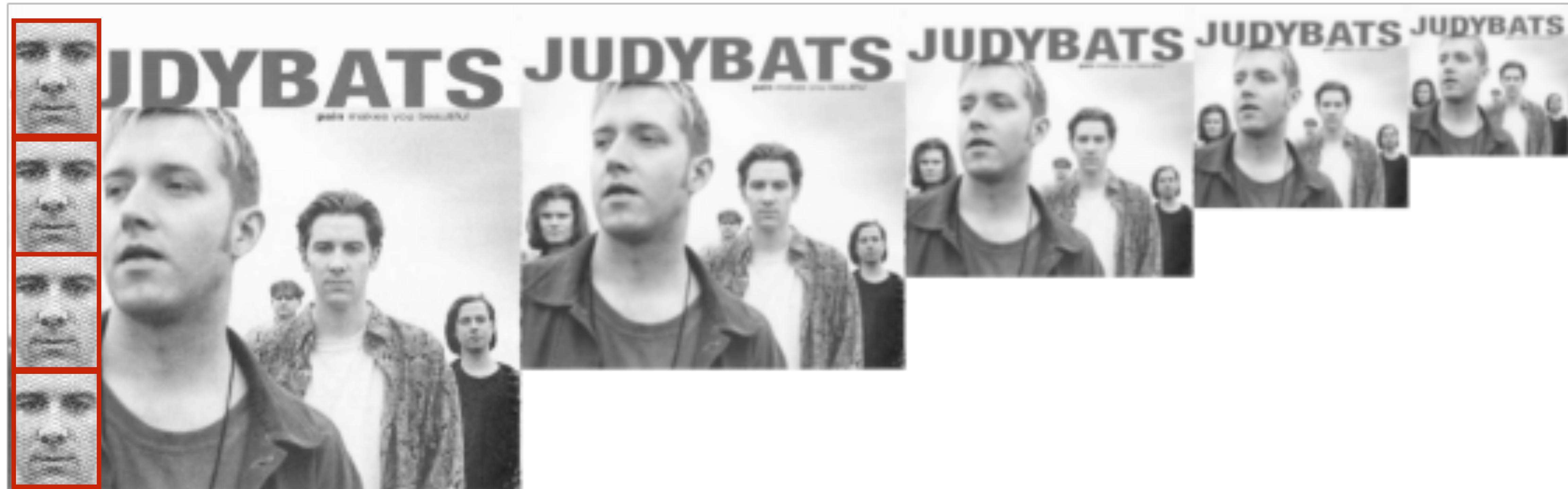
Theory: Sampling theory allows us to build image pyramids in a principled way

“Gotchas:”

- template matching remains sensitive to 2D orientation, 3D pose and illumination

Multi-Scale Template Matching

Correlation with a **fixed-sized template** only detects faces at **specific scales**



Multi-Scale Template Matching

Solution: form a Gaussian Pyramid and convolve with the template at each scale



Shrinking the Image

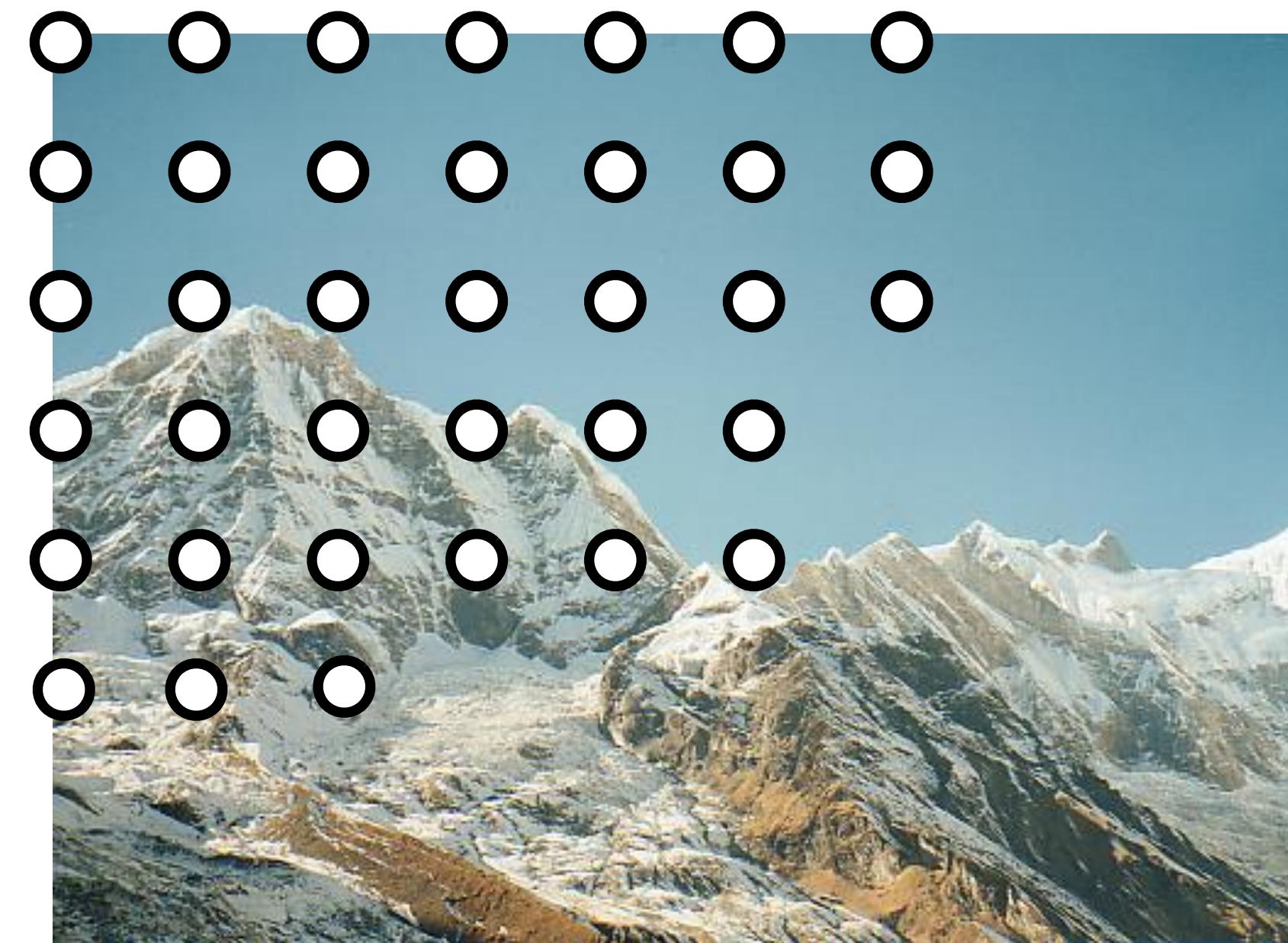
We can't shrink an image simply by taking every second pixel



Why?

Aliasing Example

- Sampling every 5th pixel, with and without low pass filtering



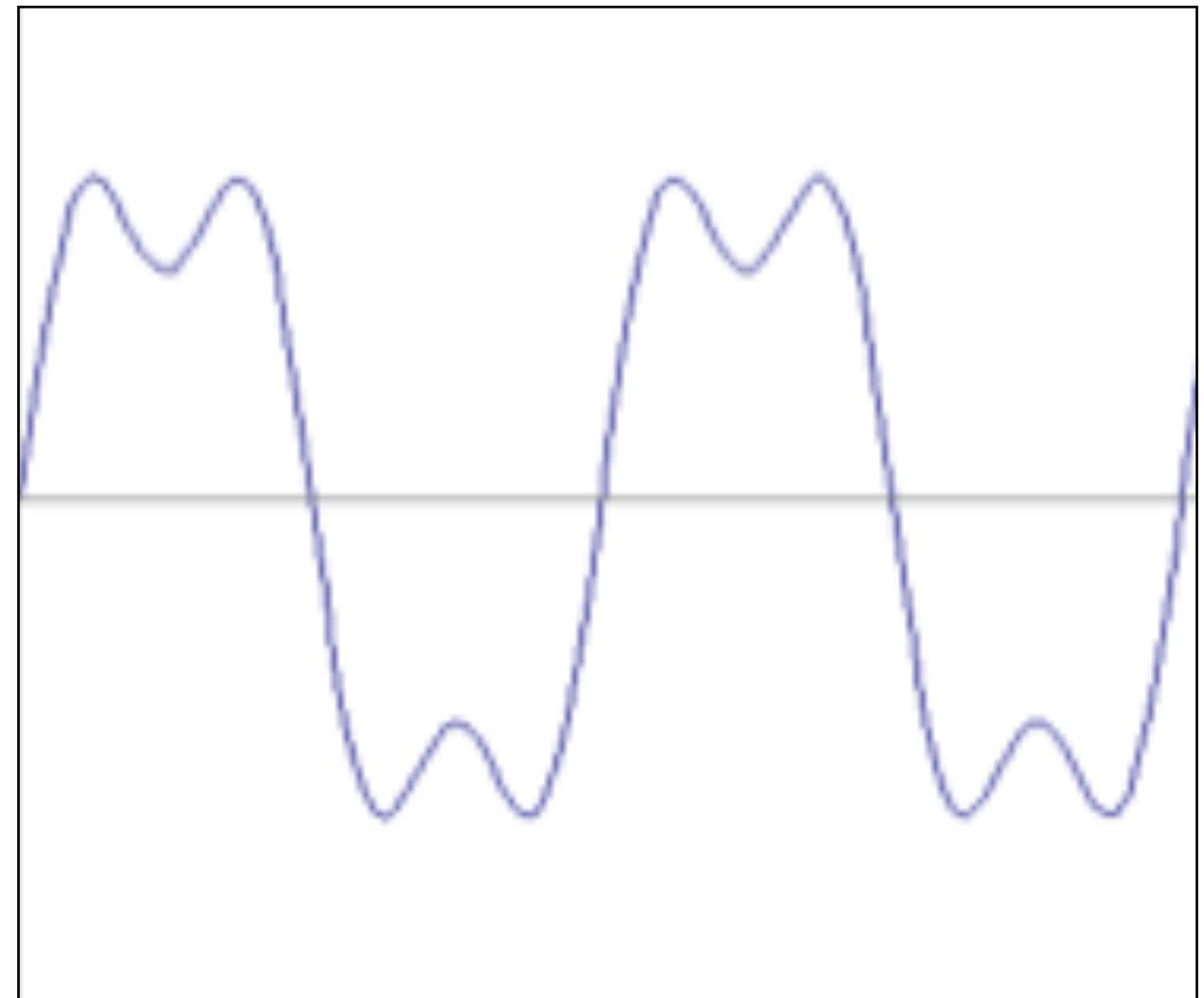
No filtering



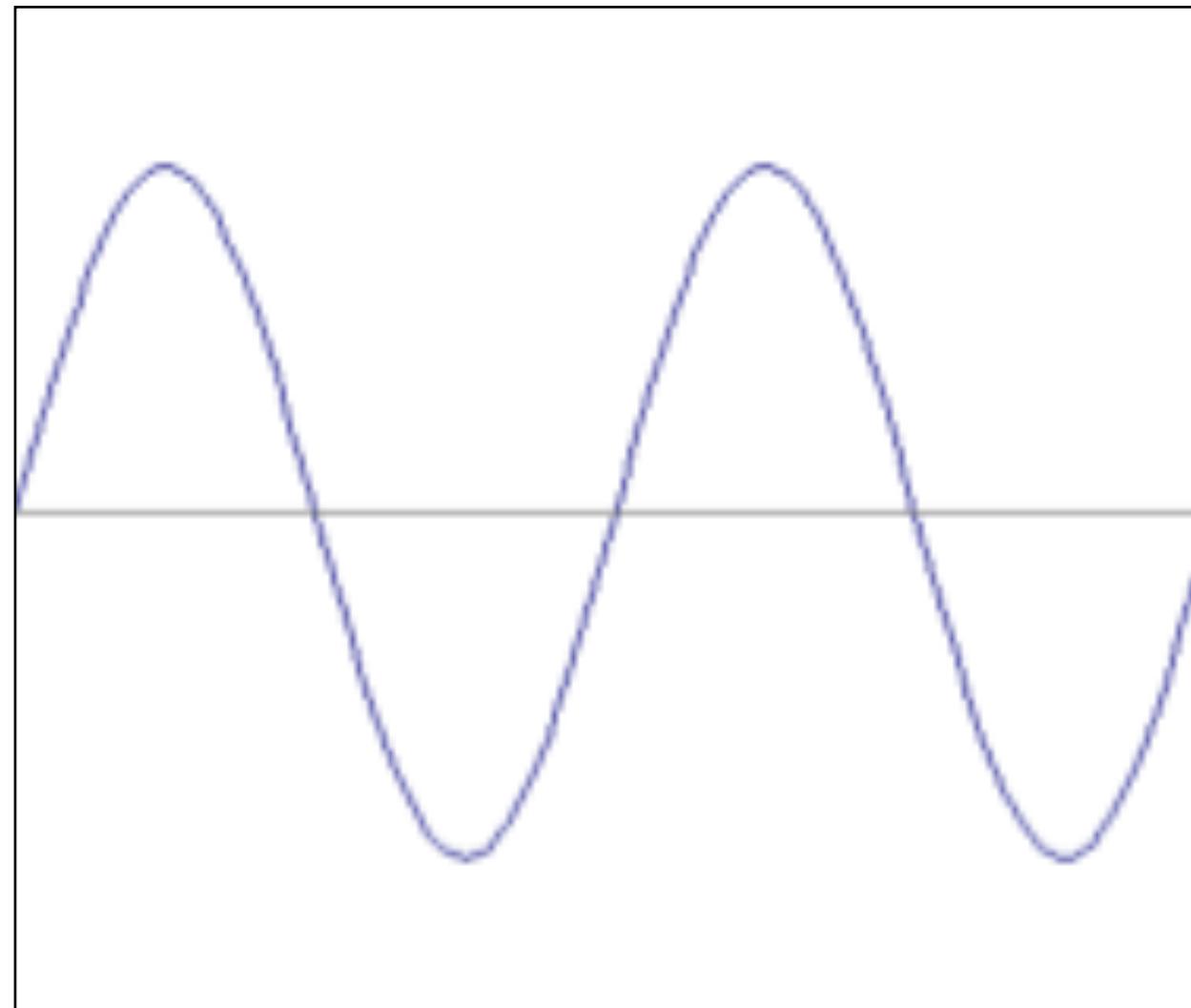
Gaussian Blur $\sigma = 3.0$

Recall: Fourier Representation

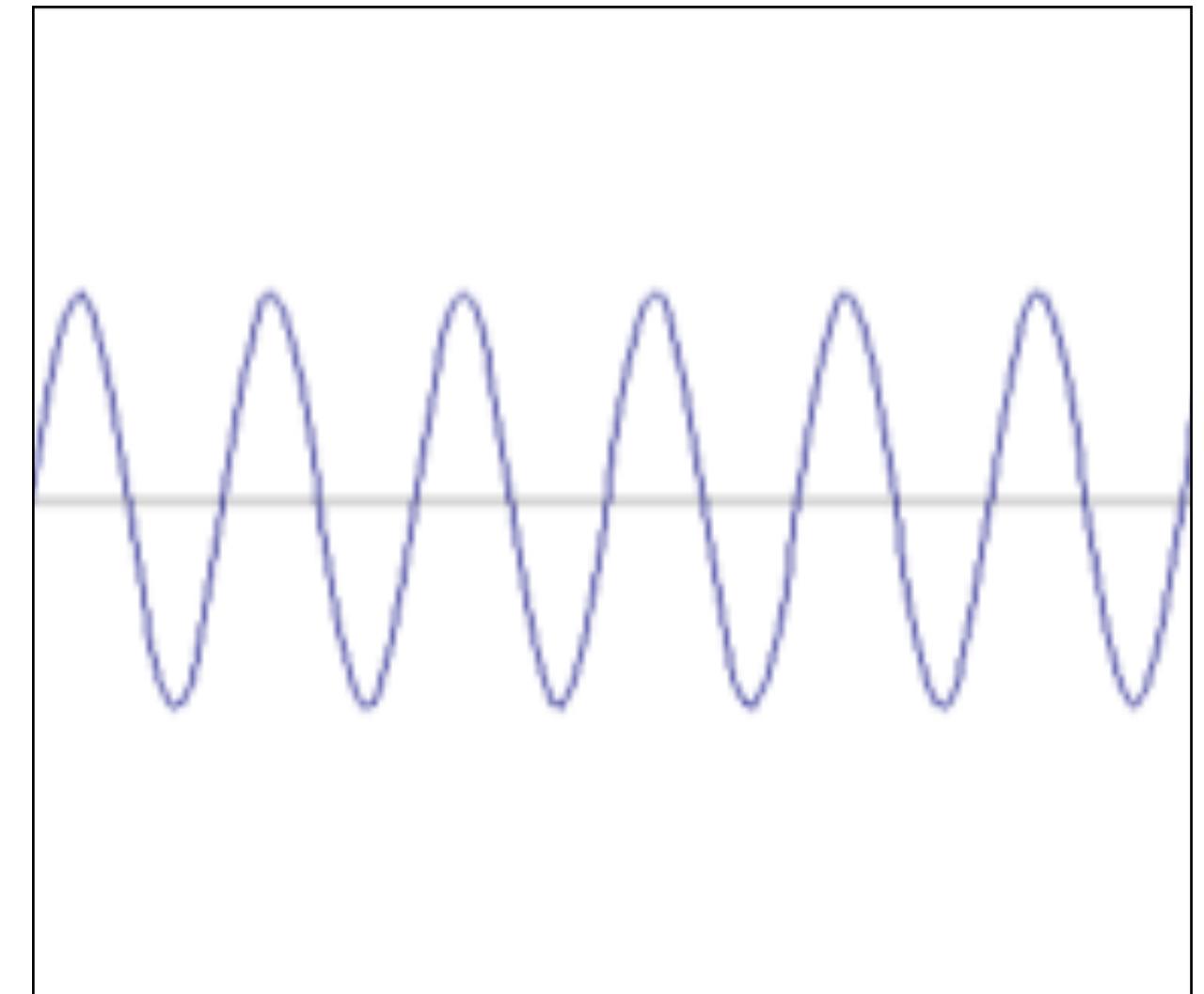
Any signal can be written as a sum of sinusoidal functions



=



+



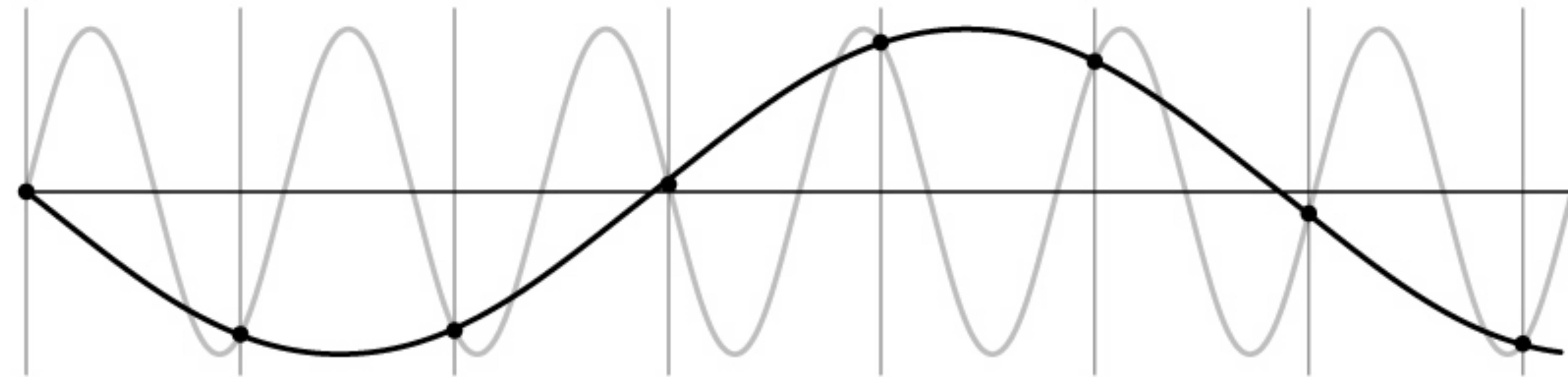
$$f(x) = \sin(2\pi x) + \frac{1}{3} \sin(2\pi 3x)$$

$$\sin(2\pi x)$$

$$\frac{1}{3} \sin(2\pi 3x)$$

Recall: Aliasing

Signal has been sampled too infrequently — result = **Aliasing**

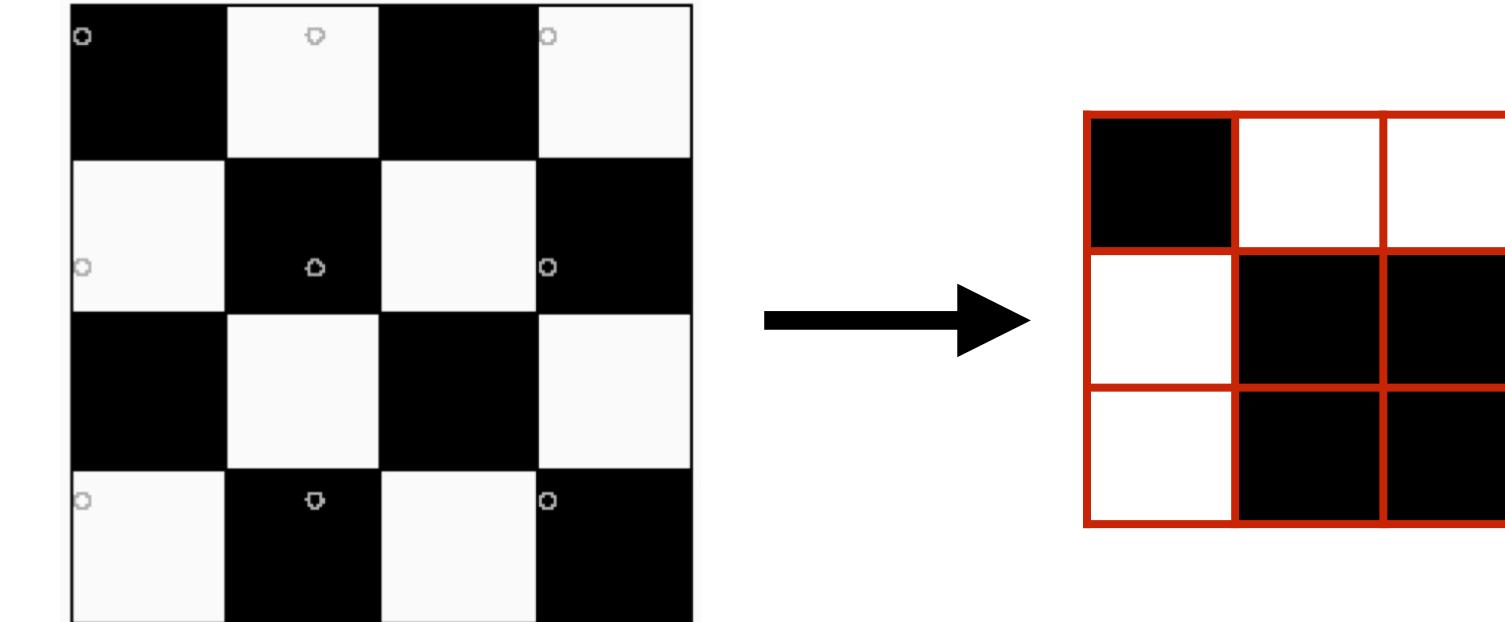
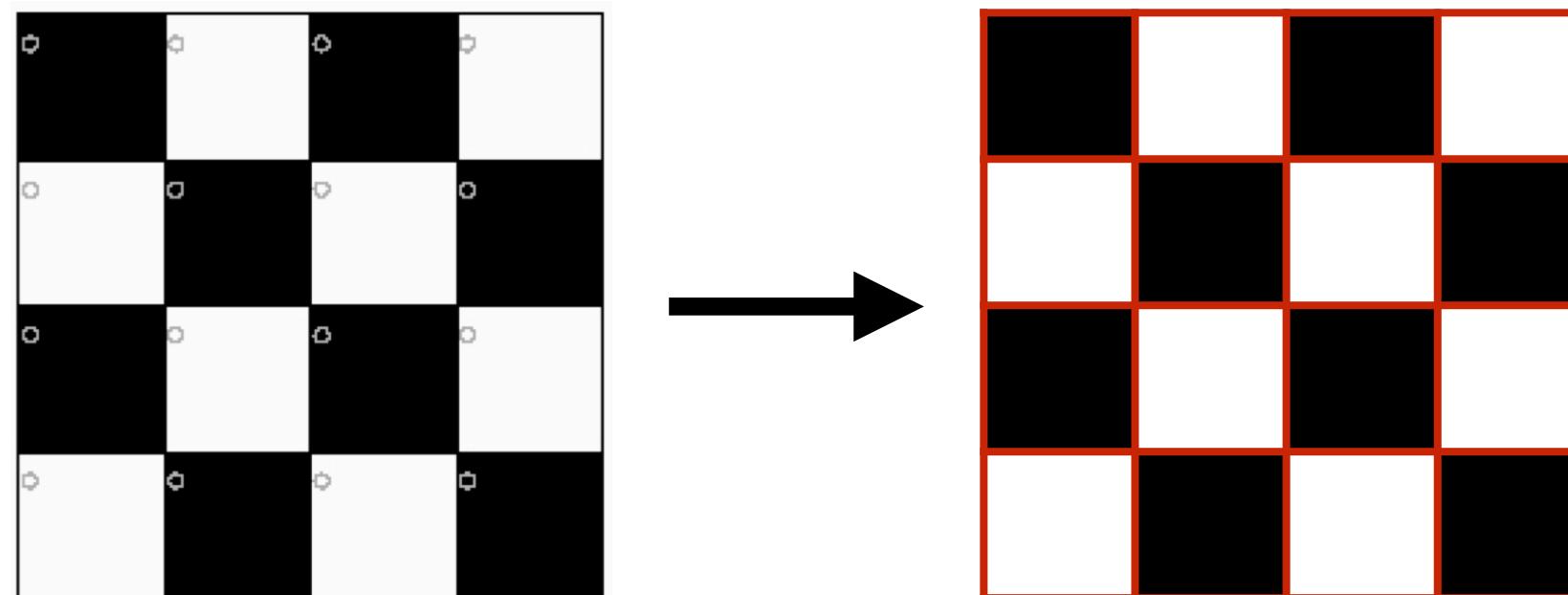


Nyquist Sampling

To avoid aliasing a signal must be sampled at twice the maximum frequency:

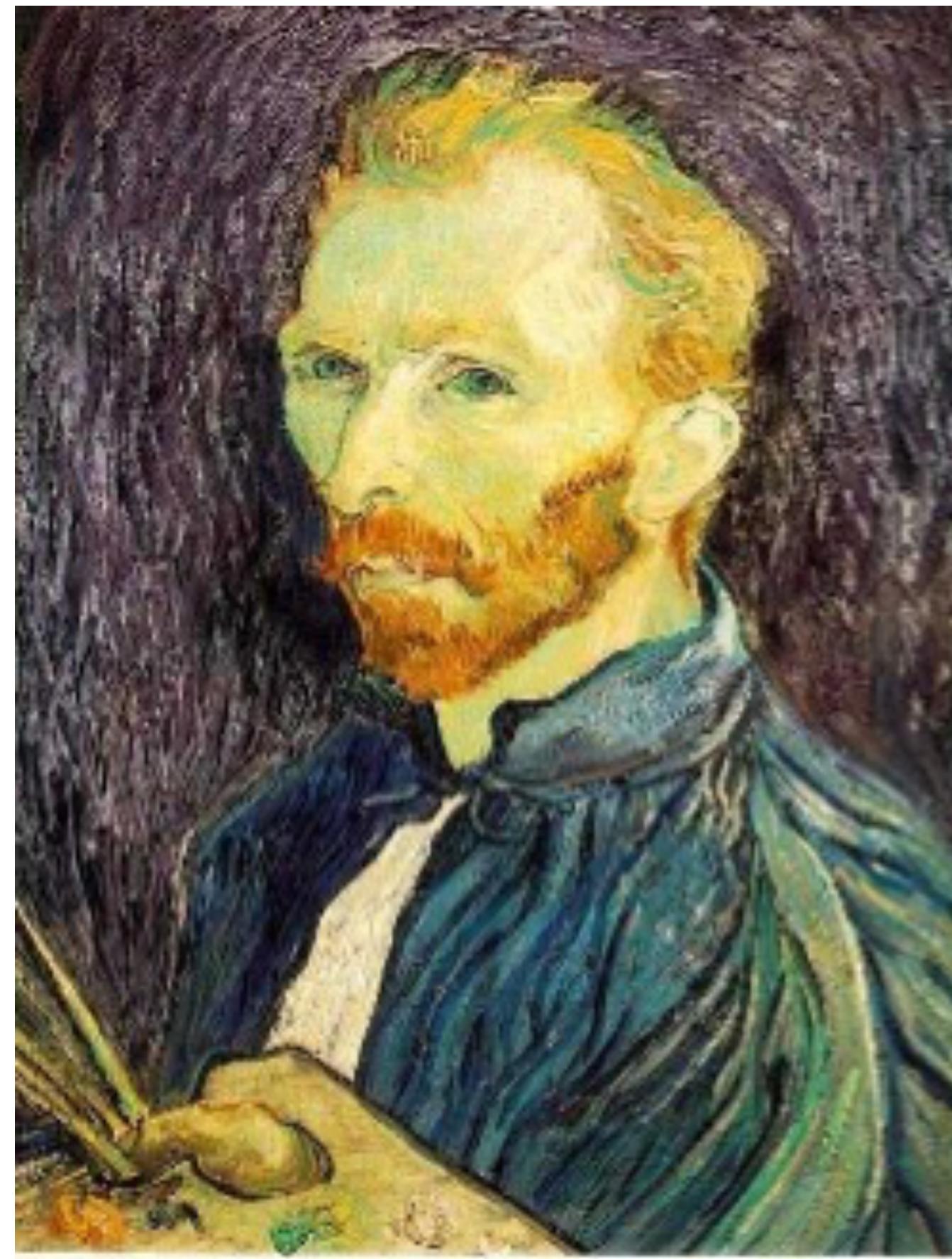
$$f_s > 2 \times f_{max}$$

For Images: We need to sample the underlying continuous signal **at least once per pixel** to avoid aliasing (assuming a correctly sampled image)



undersampling = aliasing

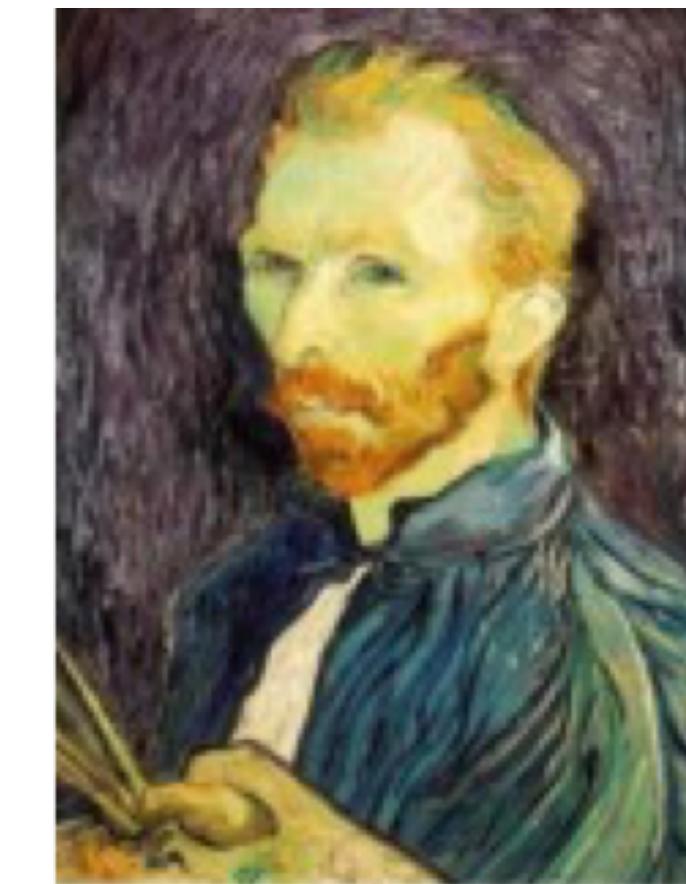
Template Matching: Sub-sample with Gaussian Pre-filtering



1/2

Apply a smoothing filter first, then throw away half the rows and columns

Gaussian filter
delete even rows
delete even
columns



1/4

Gaussian filter
delete even rows
delete even
columns



1/8

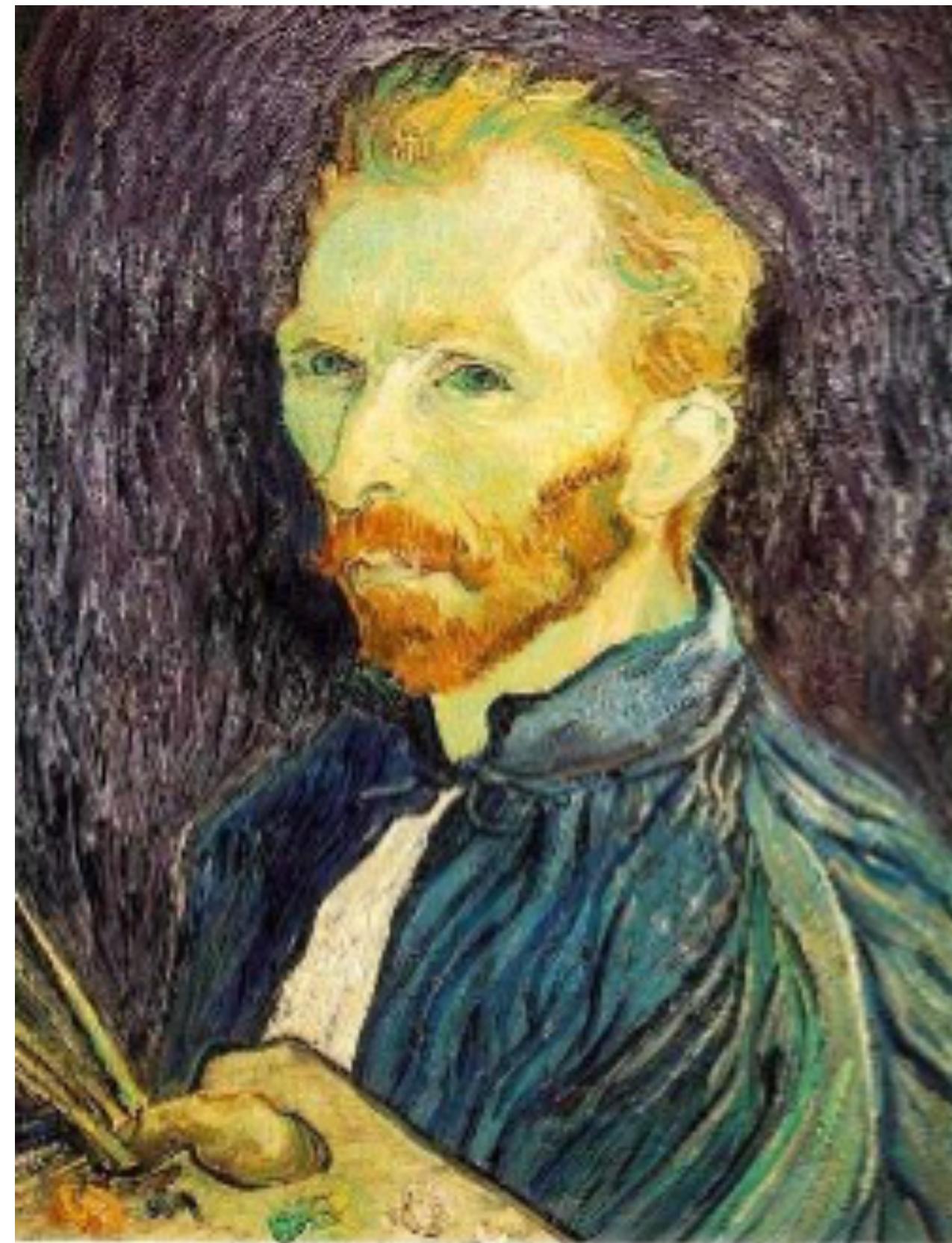
Gaussian Pre-filtering

Question: How much smoothing is needed to avoid aliasing?

Answer: Smoothing should be sufficient to ensure that the resulting image is band limited “enough” to ensure we can sample every other pixel.

Practically: For every image reduction of 0.5, smooth by $\sigma = 1$

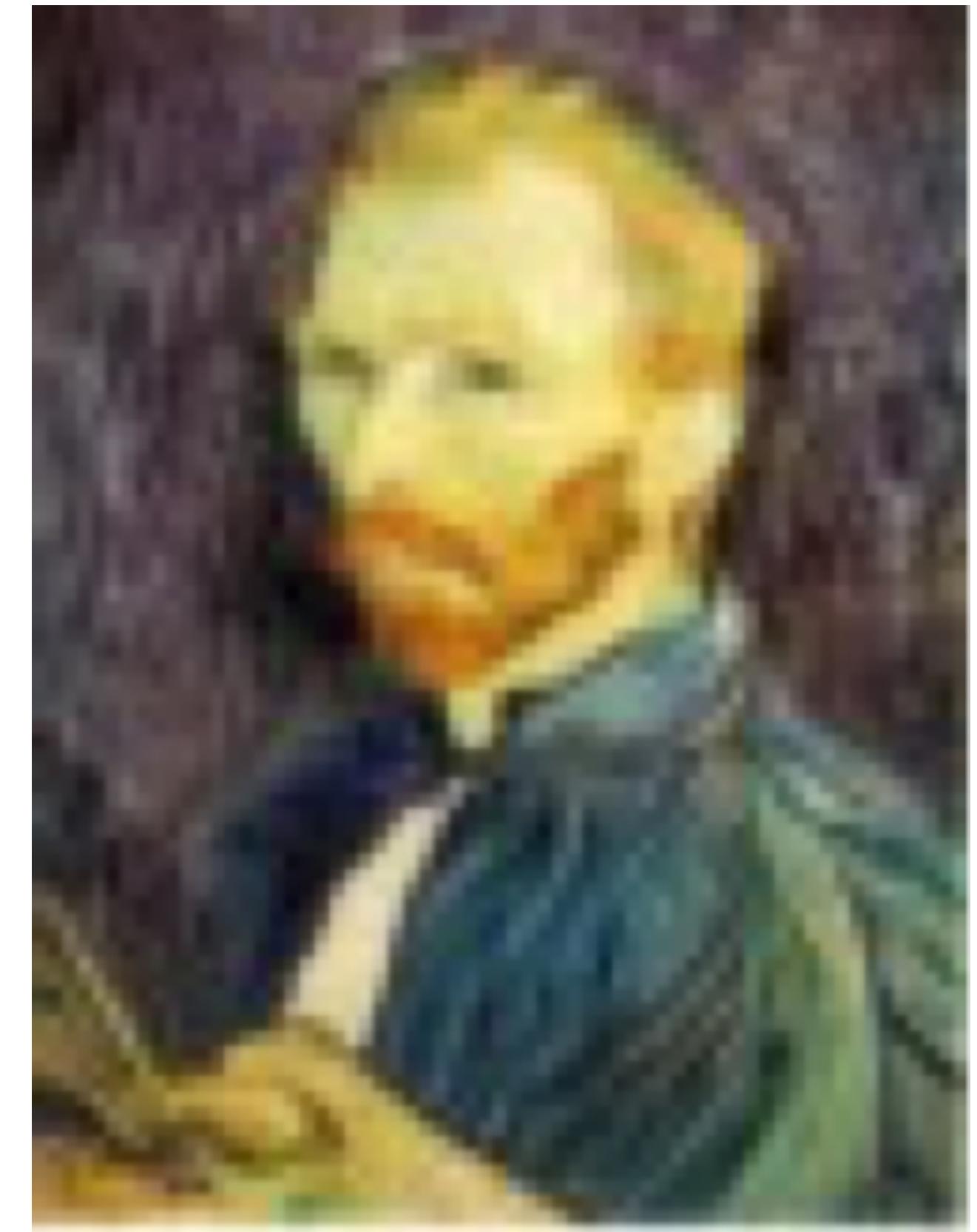
Template Matching: Sub-sample with Gaussian Pre-filtering



1/2

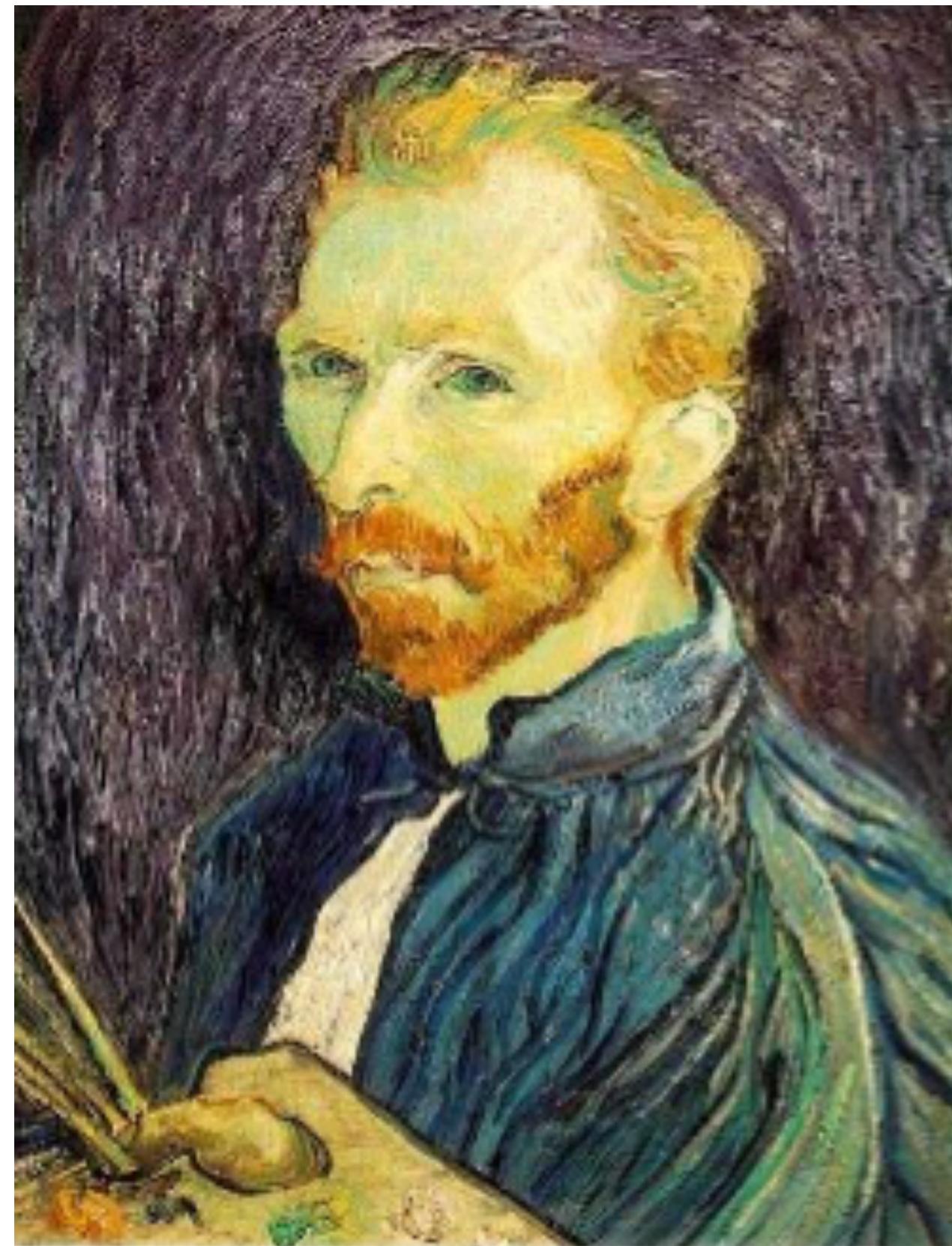


1/4 (2x zoom)



1/8 (4x zoom)

Template Matching: Sub-sample with NO Pre-filtering



1/2



1/4 (2x zoom)



1/8 (4x zoom)

Image Pyramid

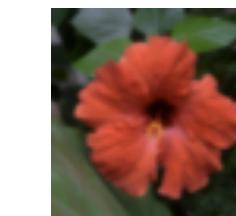
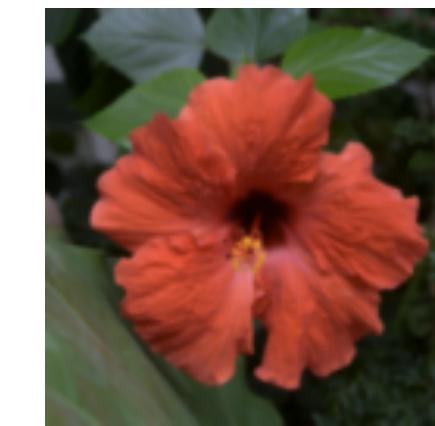
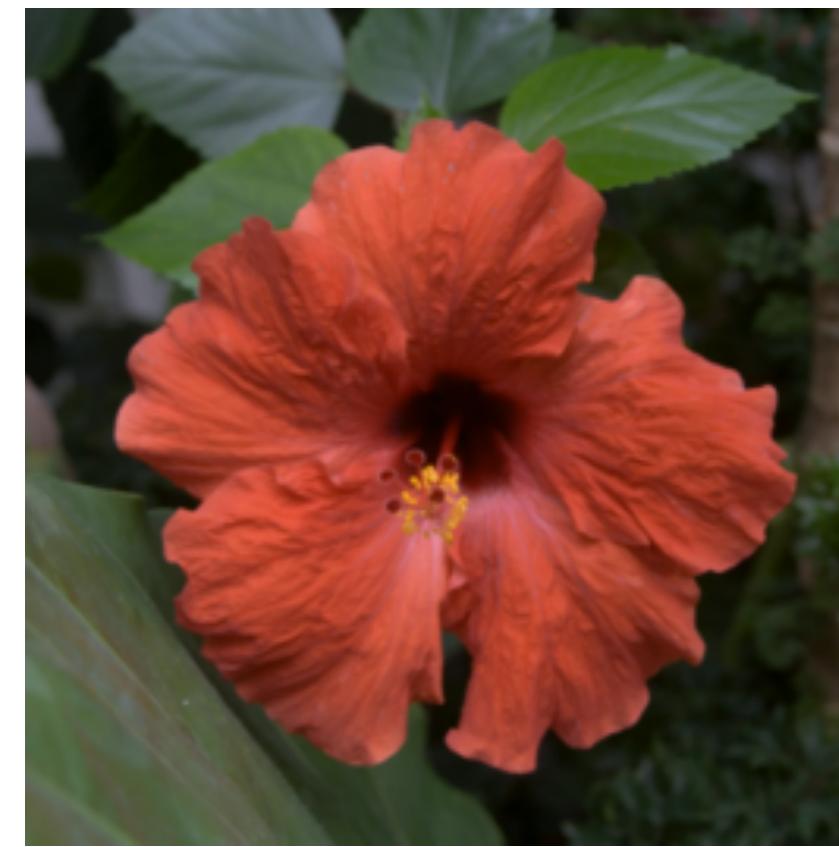
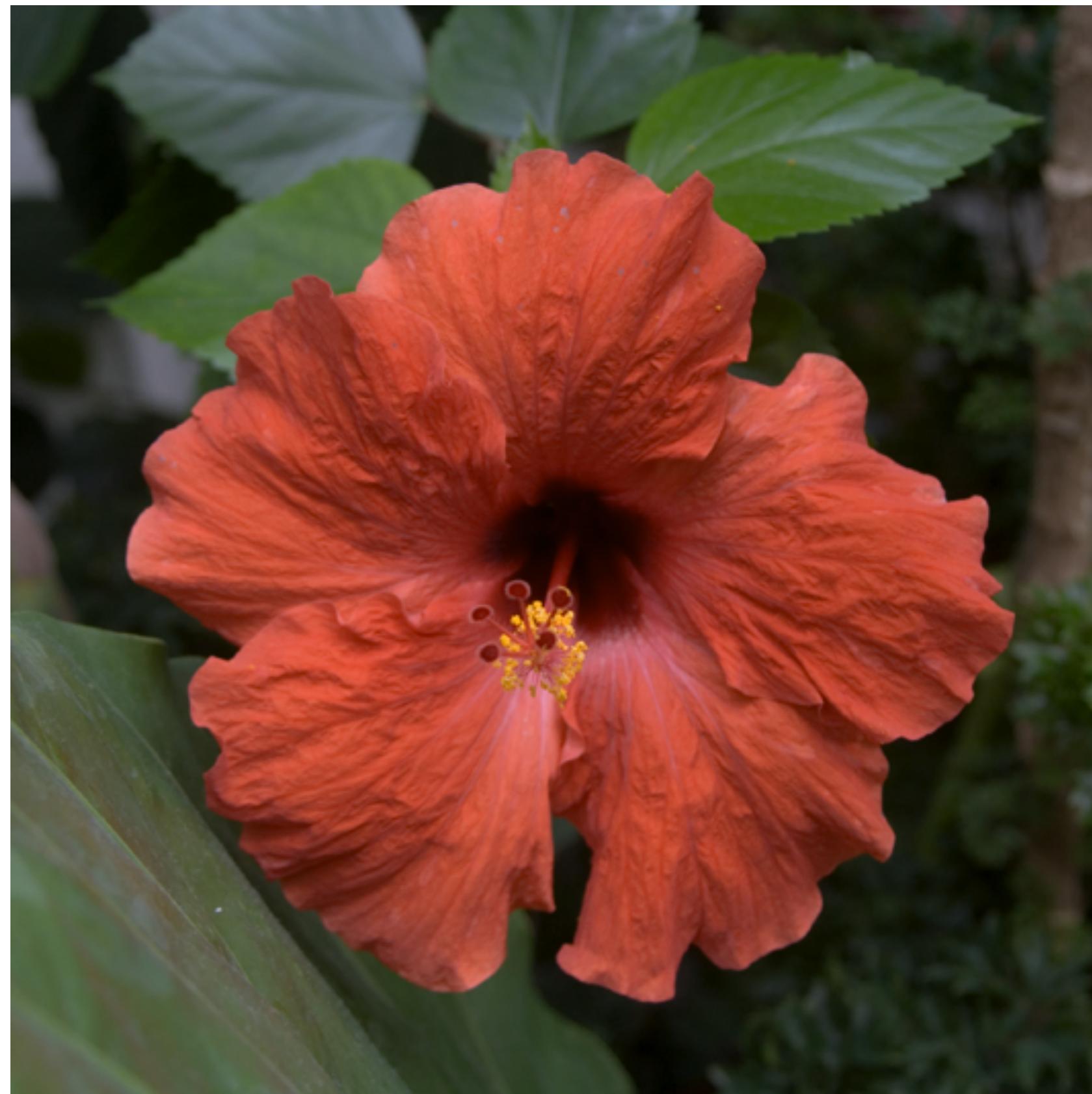


An **image pyramid** is an efficient way to represent an image at multiple scales

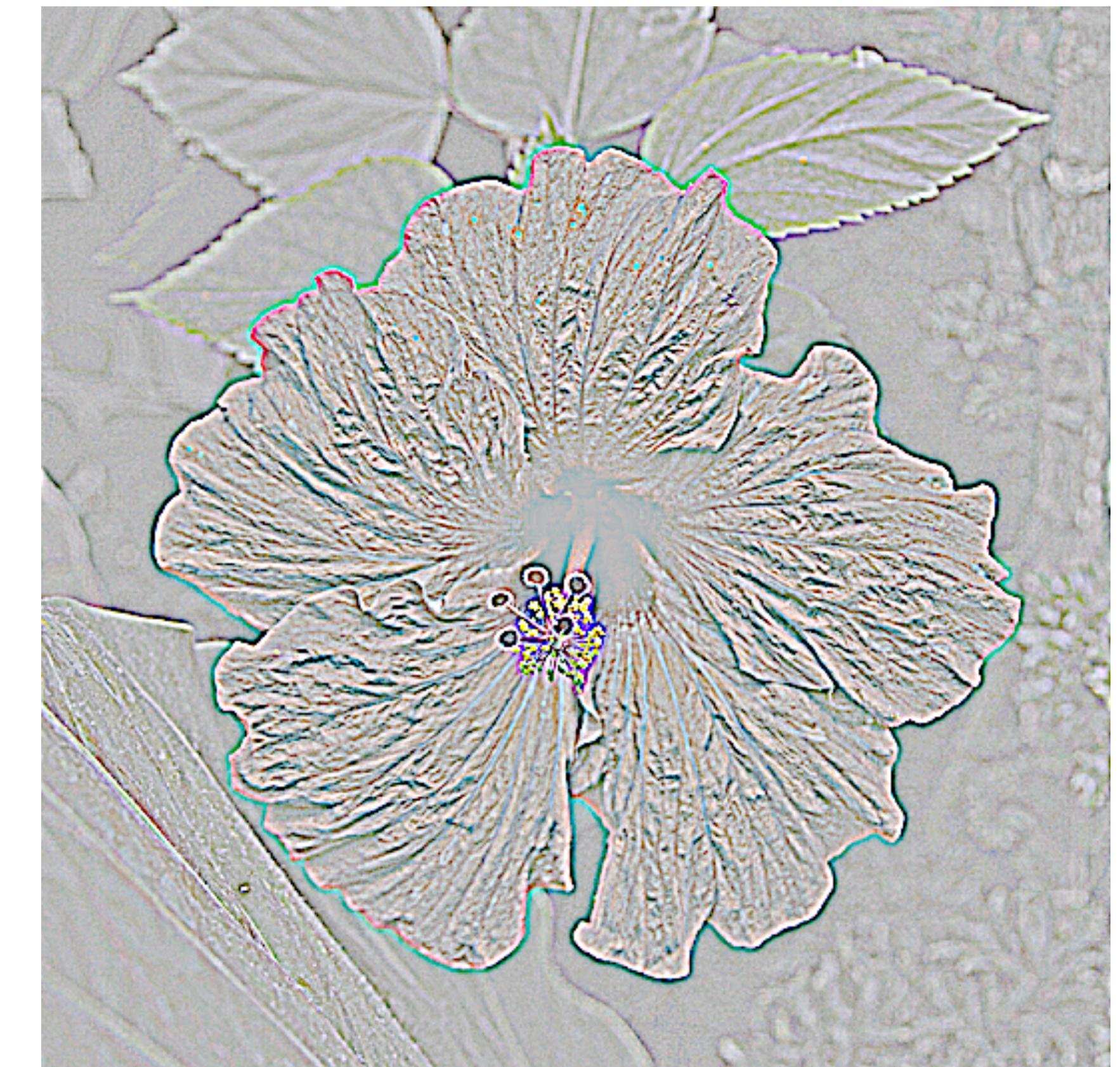
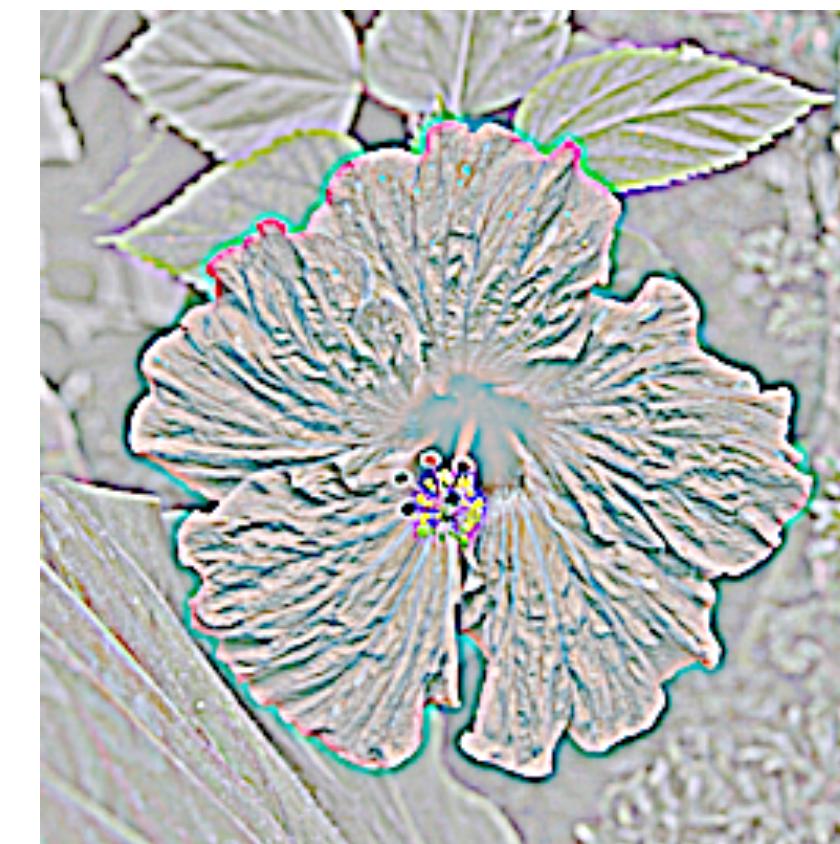
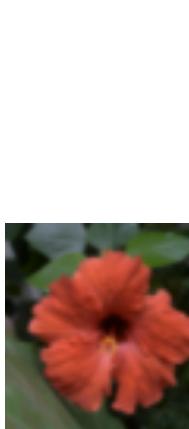
In a **Gaussian pyramid**, each layer is smoothed by a Gaussian filter and resampled to get the next layer, taking advantage of the fact that

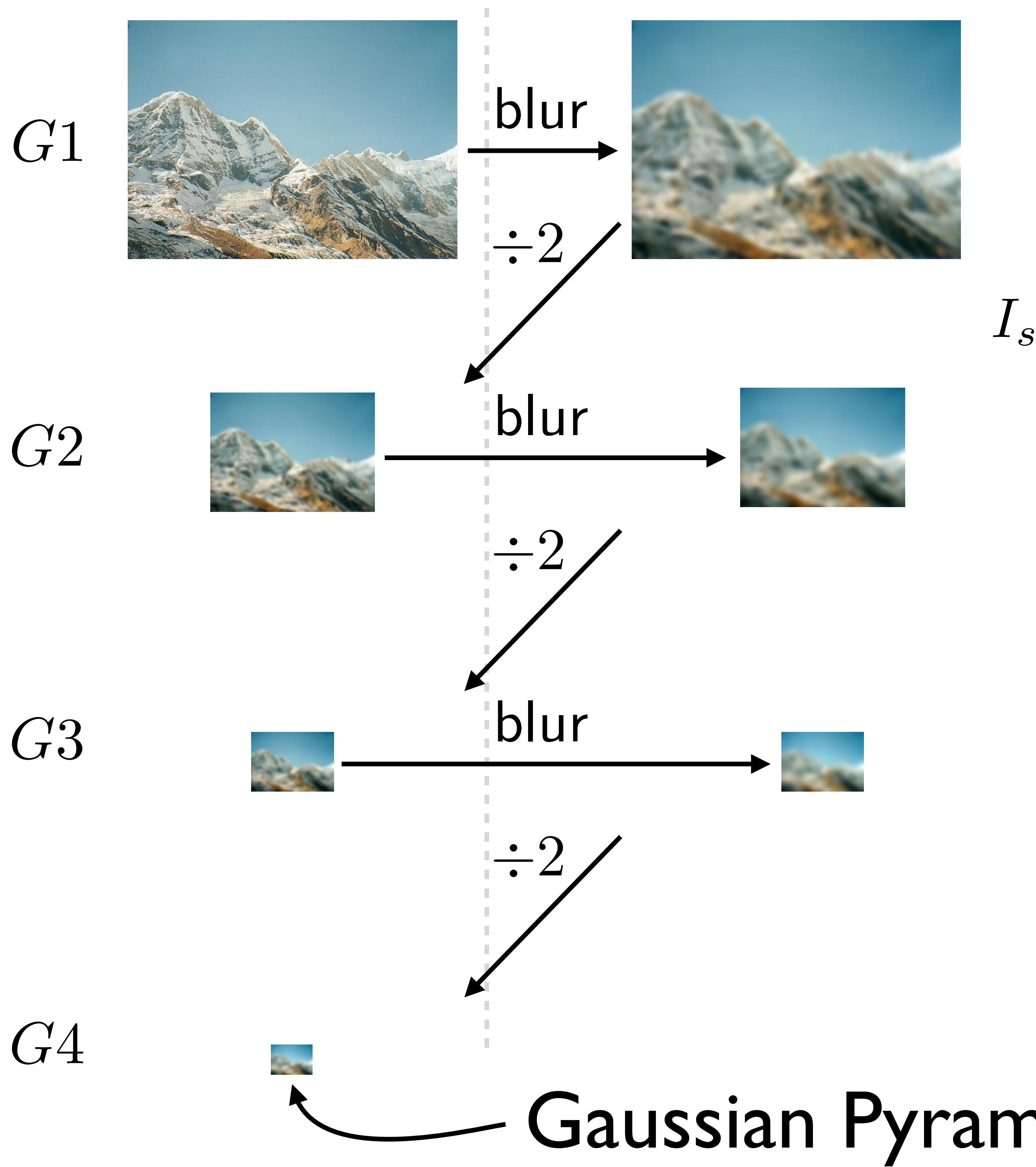
$$G_{\sigma_1}(x) \otimes G_{\sigma_2}(x) = G_{\sqrt{\sigma_1^2 + \sigma_2^2}}(x)$$

Gaussian vs Laplacian Pyramid



Shown in opposite
order for space

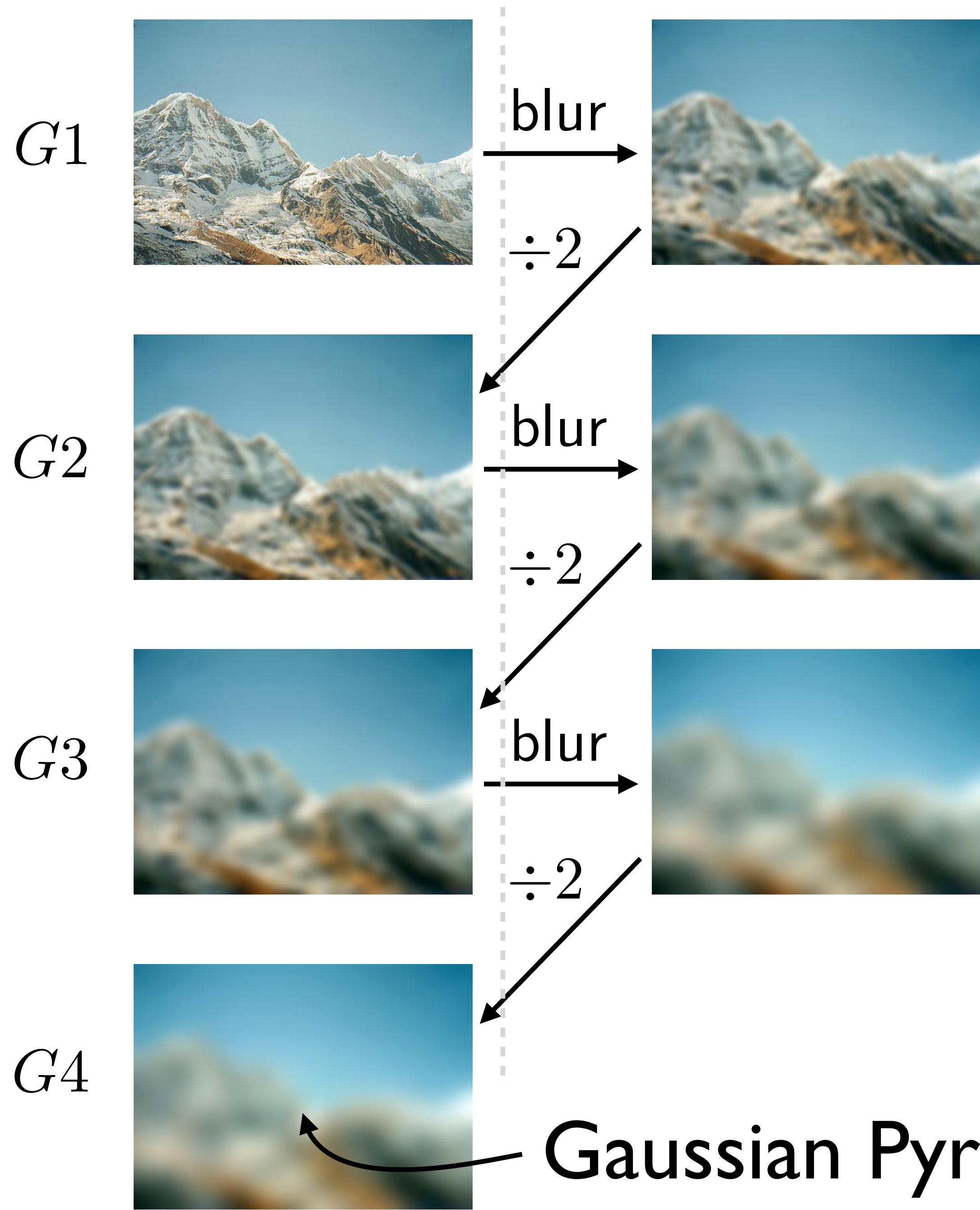




Blur with a Gaussian kernel, then select every 2nd pixel

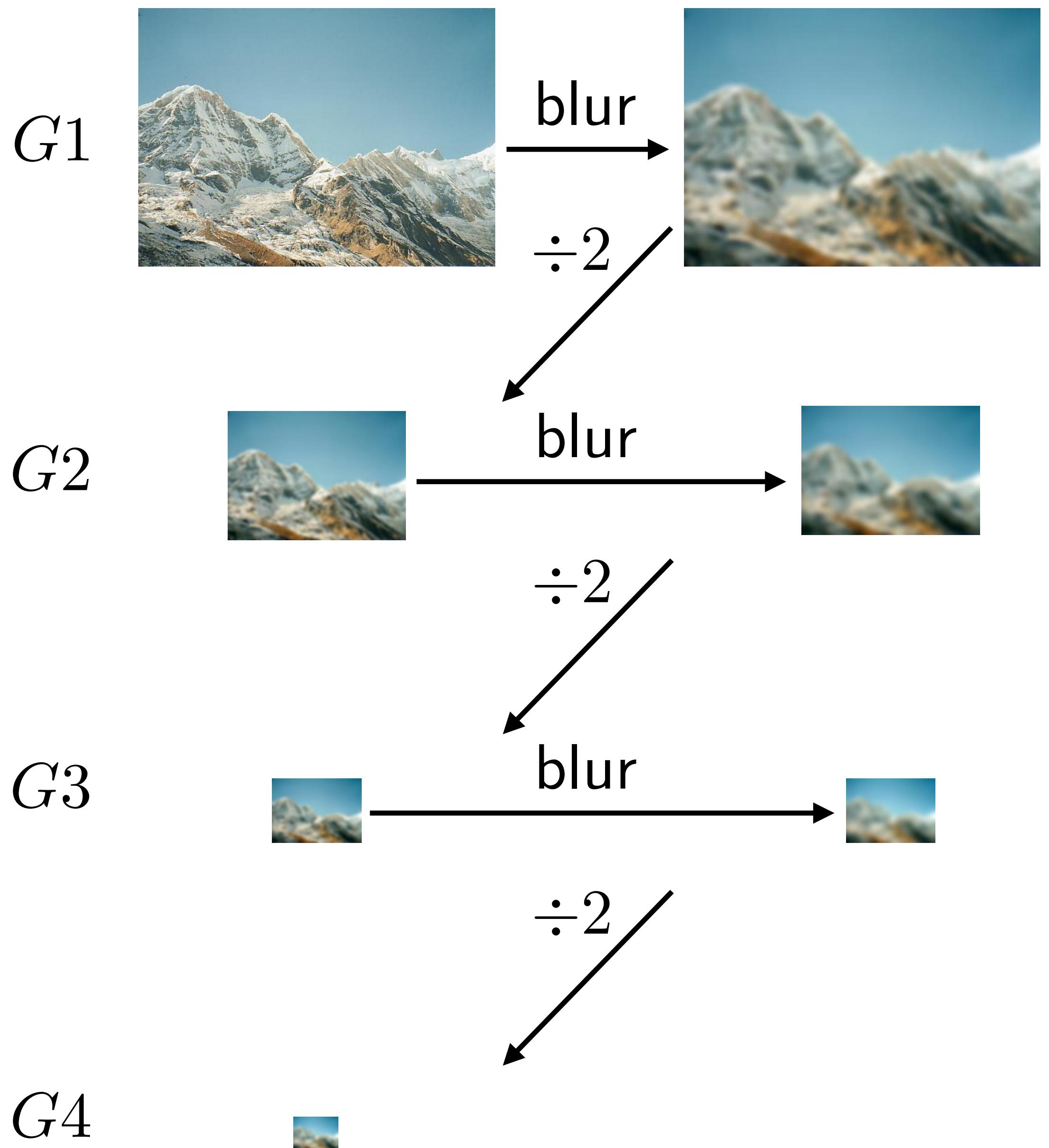
$$I_s(x, y) = I(x, y) * g_\sigma(x, y)$$

Gaussian Pyramid

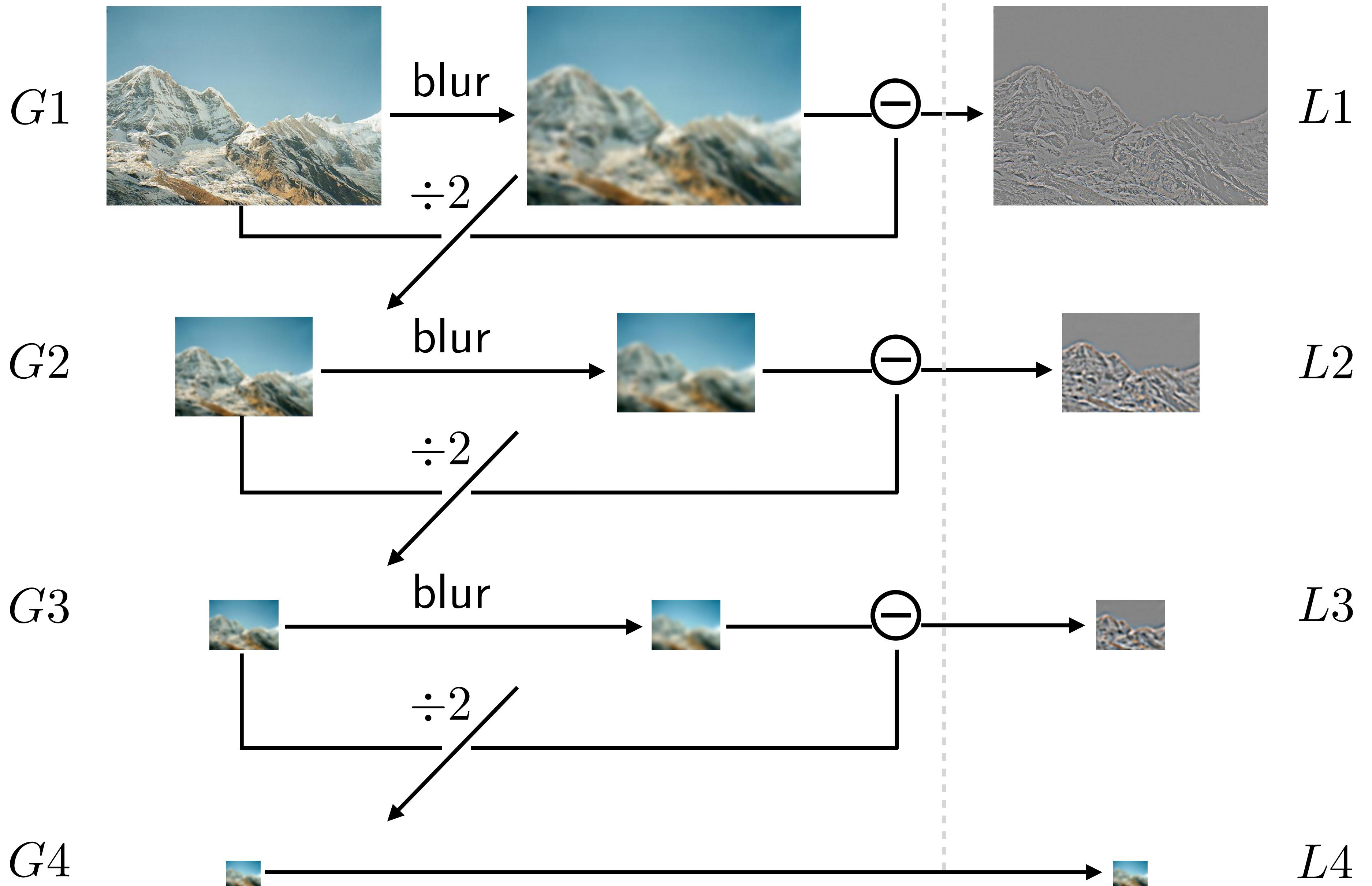


Blur with a Gaussian kernel, then select every 2nd pixel

$$I_s(x, y) = I(x, y) * g_\sigma(x, y)$$

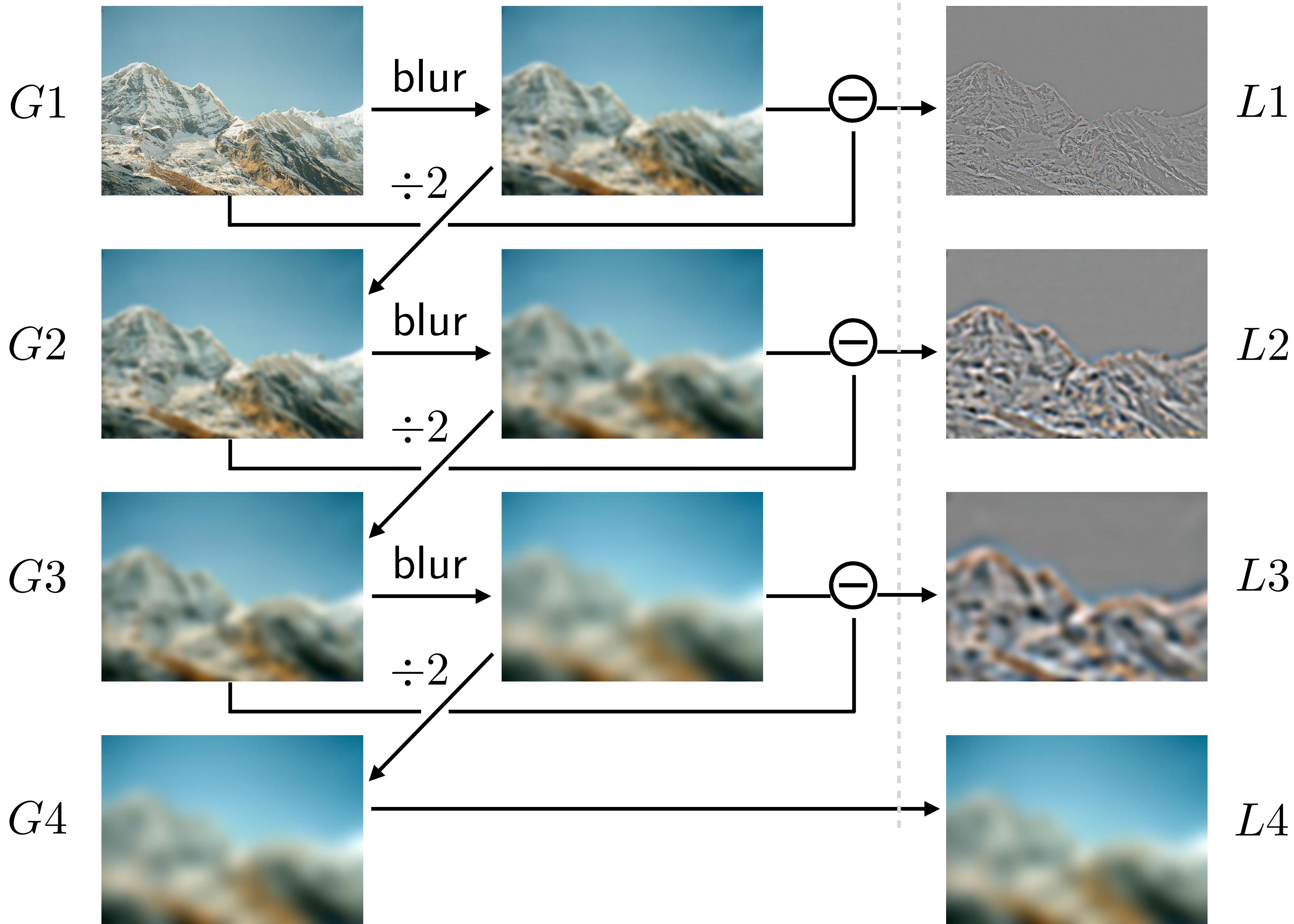


Gaussian Pyramid

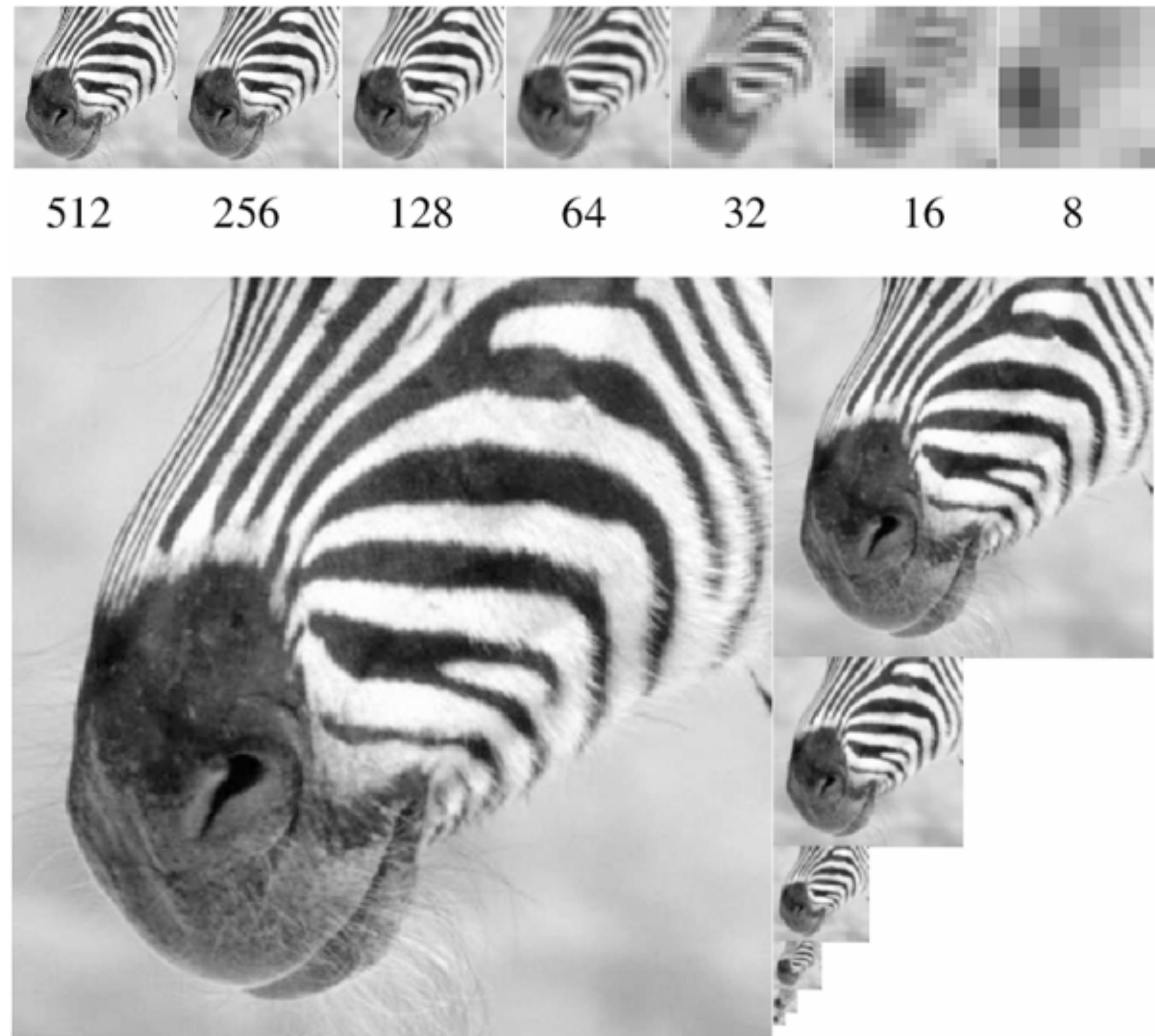


Gaussian Pyramid

Laplacian Pyramid



Gaussian Pyramid



What happens to the details?

- They get smoothed out as we move to higher levels

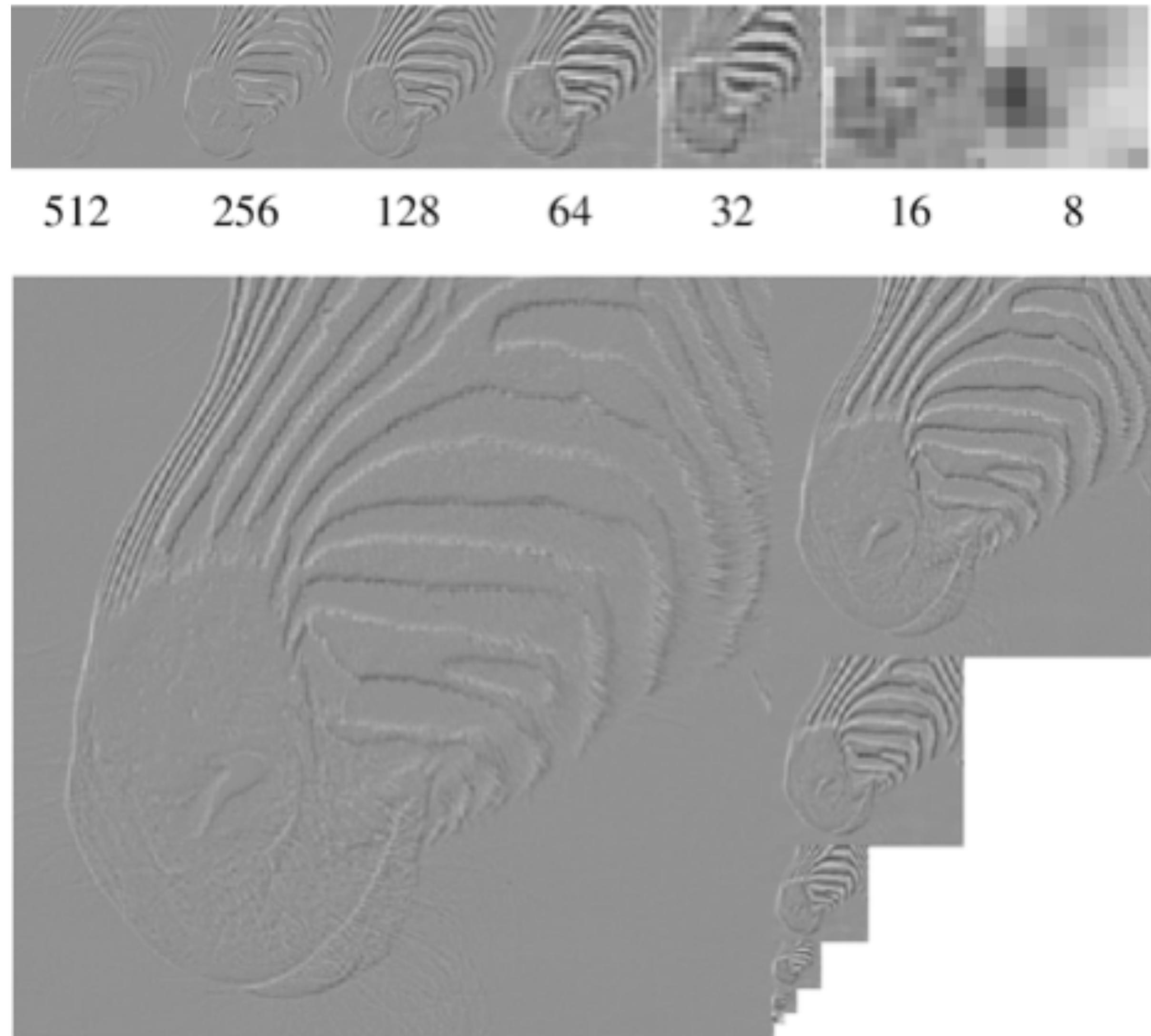
What is preserved at the higher levels?

- Mostly large uniform regions in the original image

How would you reconstruct the original image from the image at the upper level?

- That's not possible

Laplacian Pyramid



At each level, retain the residuals instead of the blurred images themselves.

Why is it called Laplacian Pyramid?

Can we reconstruct the original image using the pyramid?

— Yes we can!

What do we need to store to be able to reconstruct the original image?

Laplacian Pyramid

Building a **Laplacian** pyramid:

- Create a Gaussian pyramid
- Take the difference between one Gaussian pyramid level and the next

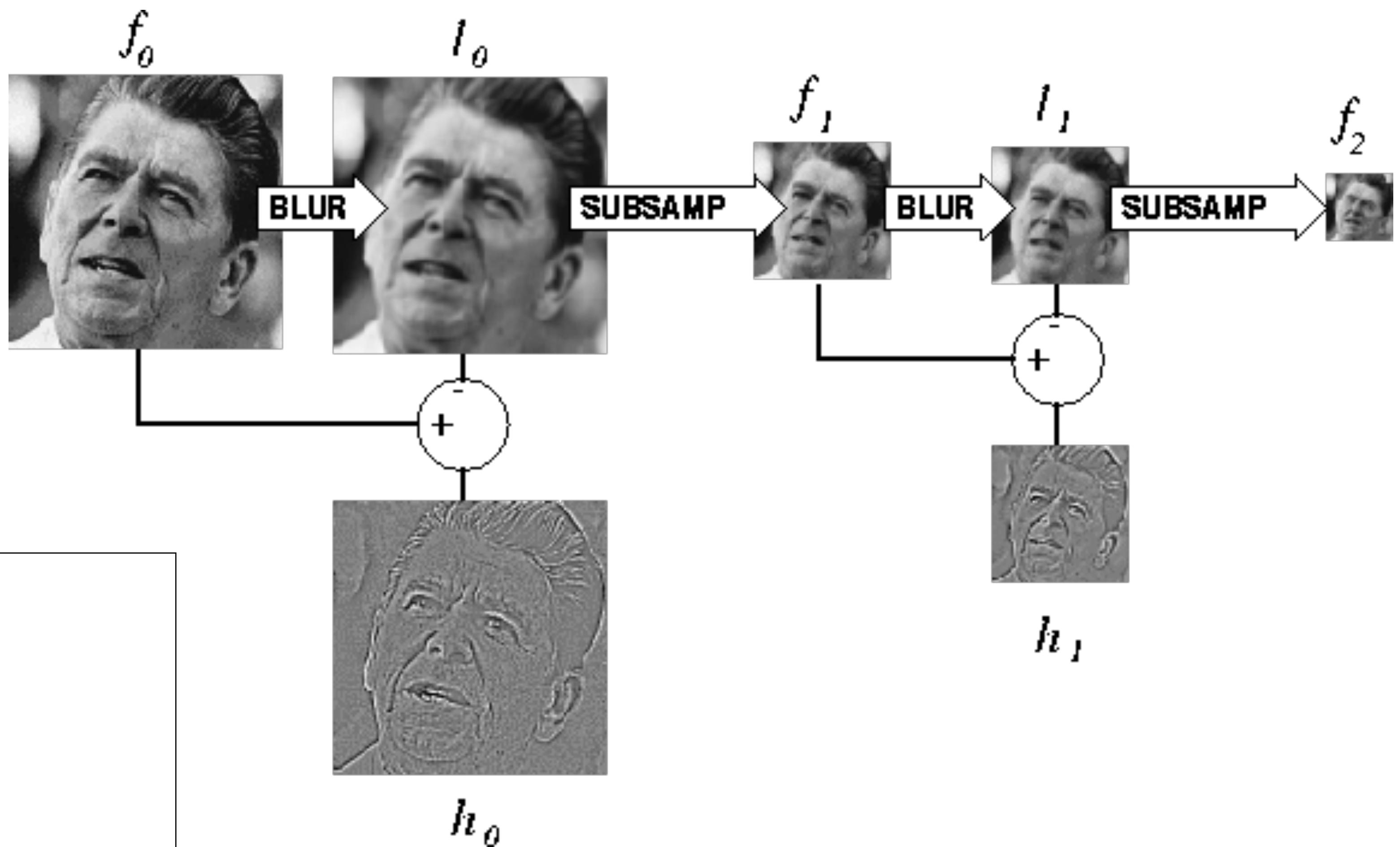
Properties

- Computes a Laplacian / Difference-of-Gaussian (DoG) function of the image at multiple scales
- It is a band pass filter – each level represents a different band of spatial frequencies

Constructing a Laplacian Pyramid

Algorithm

```
repeat:  
    filter  
    compute residual  
    subsample  
until min resolution reached
```



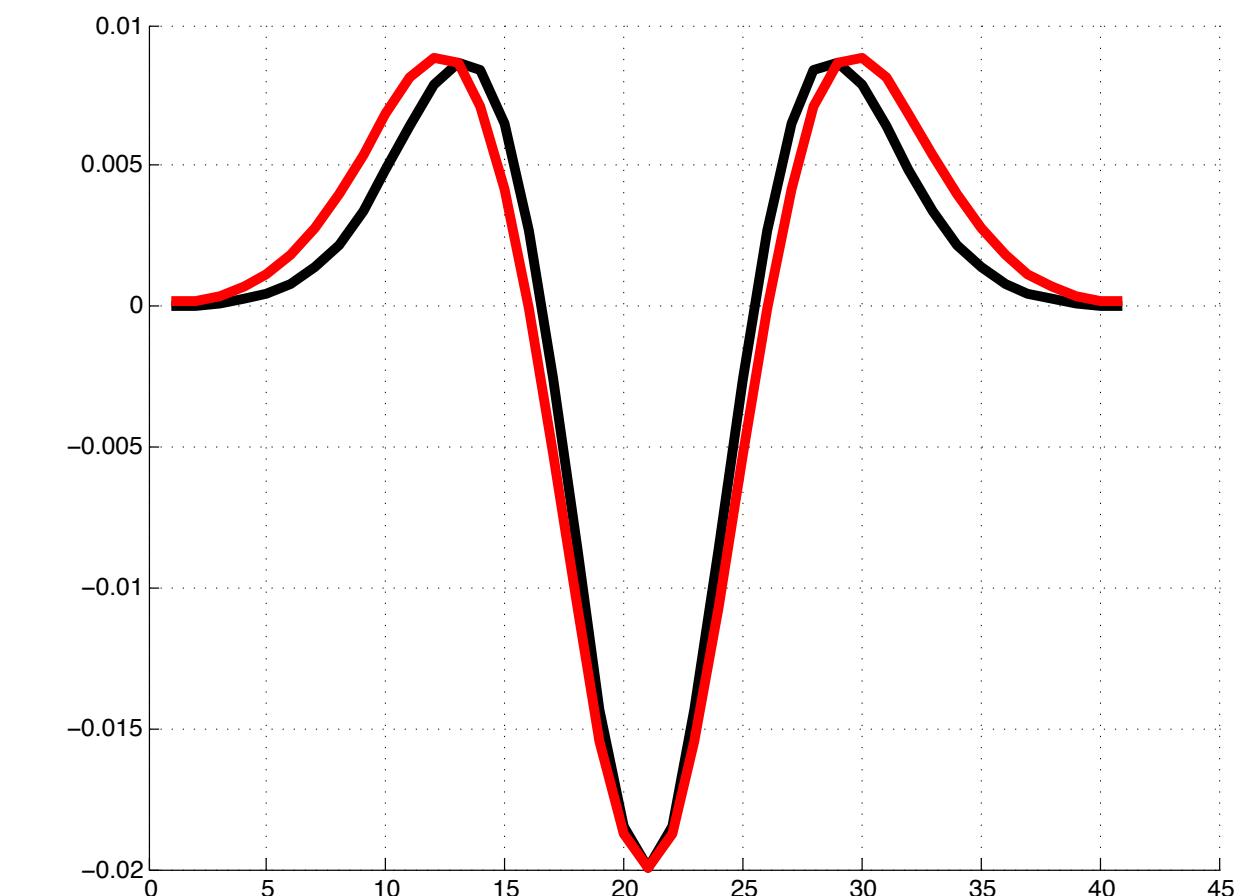
Why Laplacian Pyramid?



8.I

$$\text{red} = [1 \ -2 \ 1] * g(x; 5.0)$$

$$\text{black} = g(x; 5.0) - g(x; 4.0)$$



- Laplacian/DoG = centre-surround filter



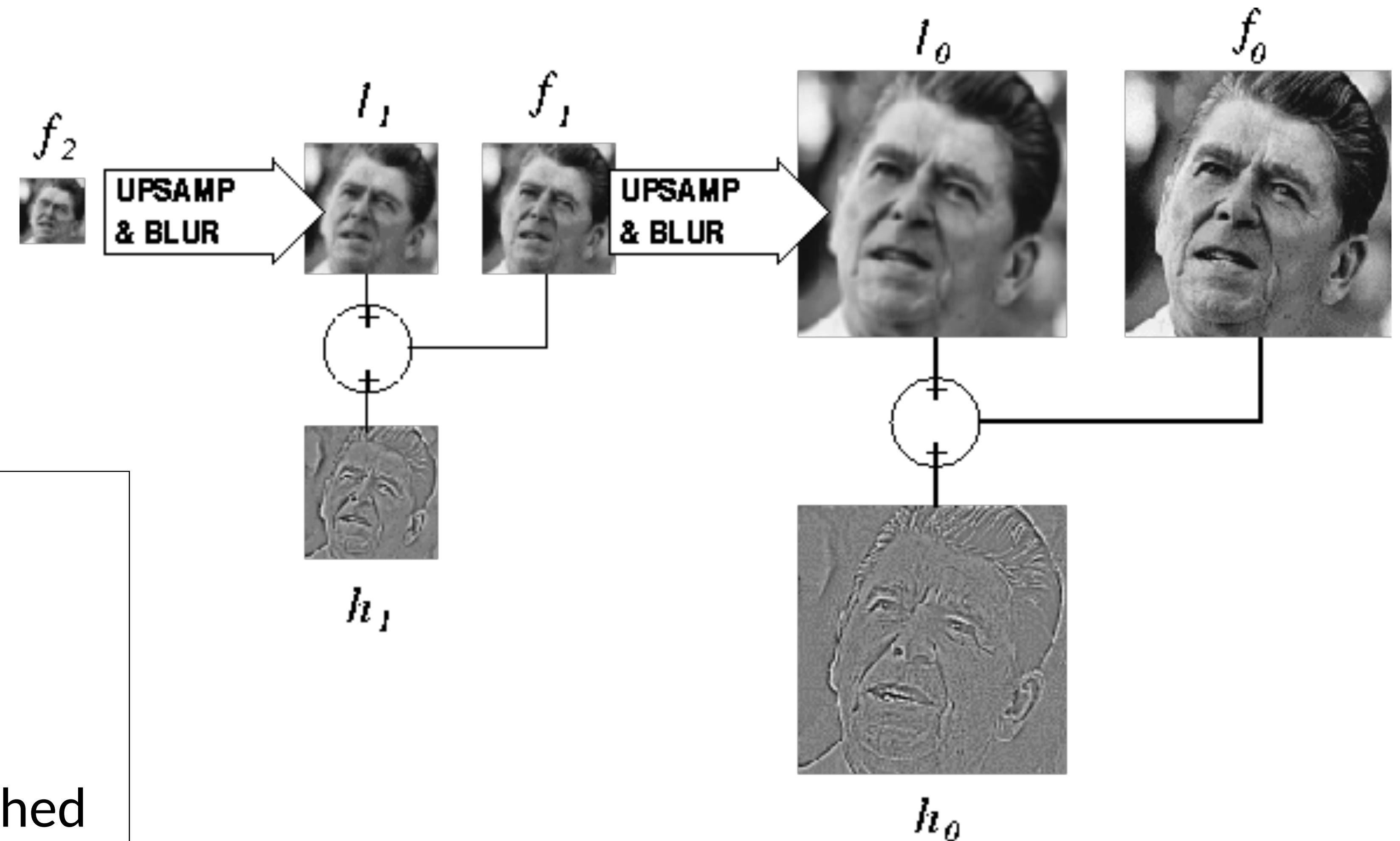
$$* \quad \begin{matrix} & & \\ & \blacksquare & \\ & & \end{matrix} =$$

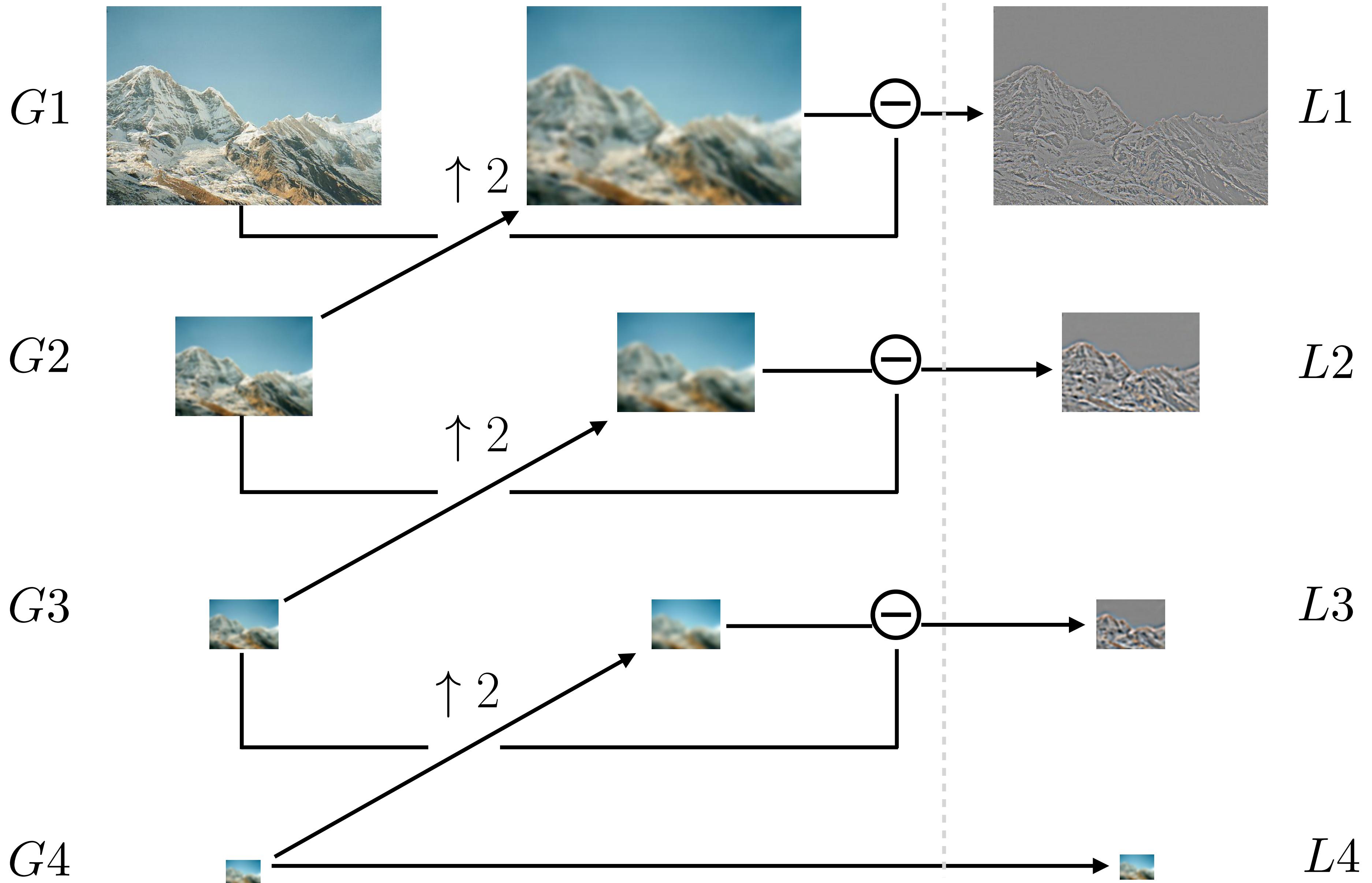


Reconstructing the Original Image

Algorithm

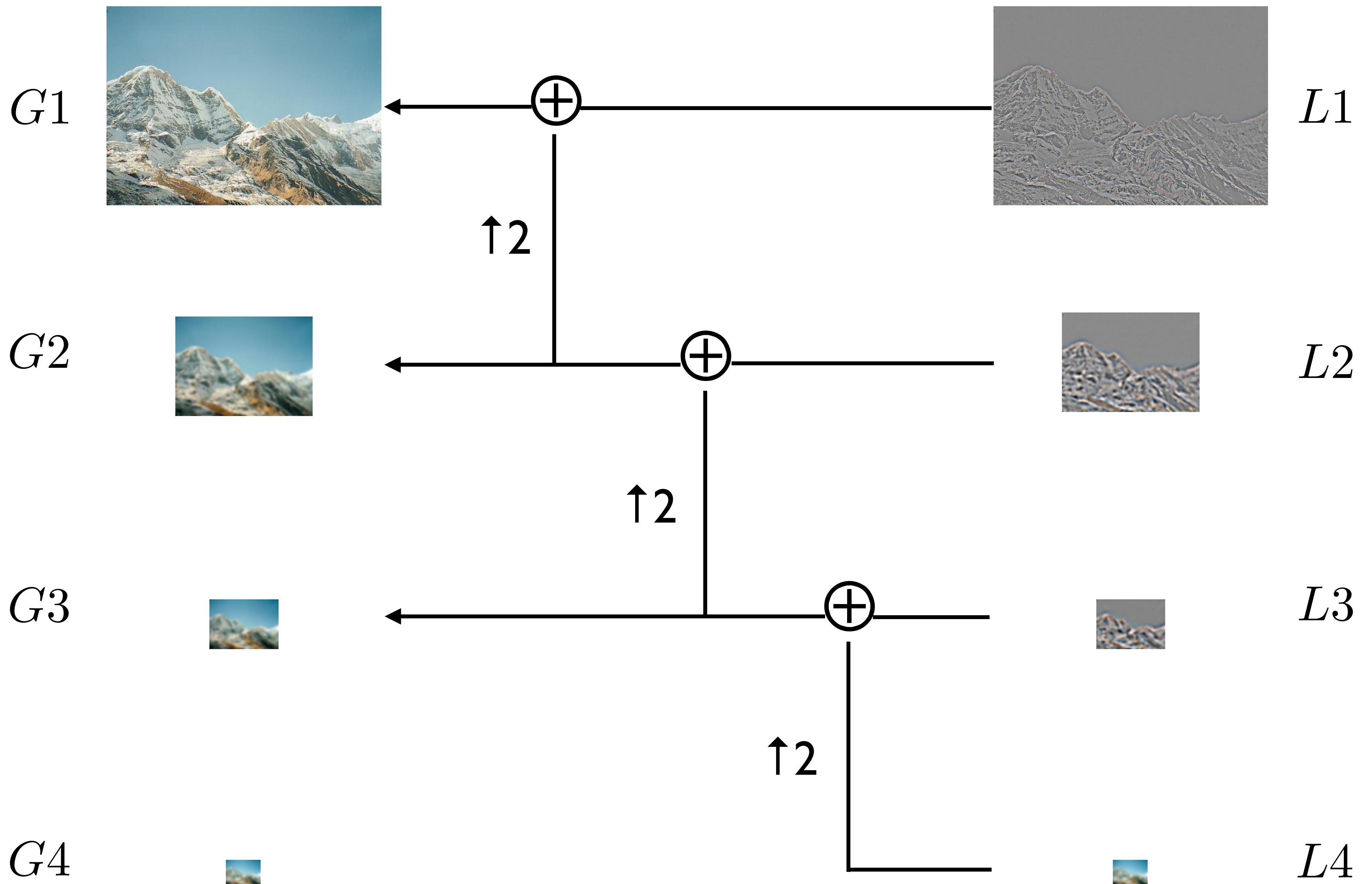
```
repeat:  
    upsample  
    sum with residual  
until orig resolution reached
```





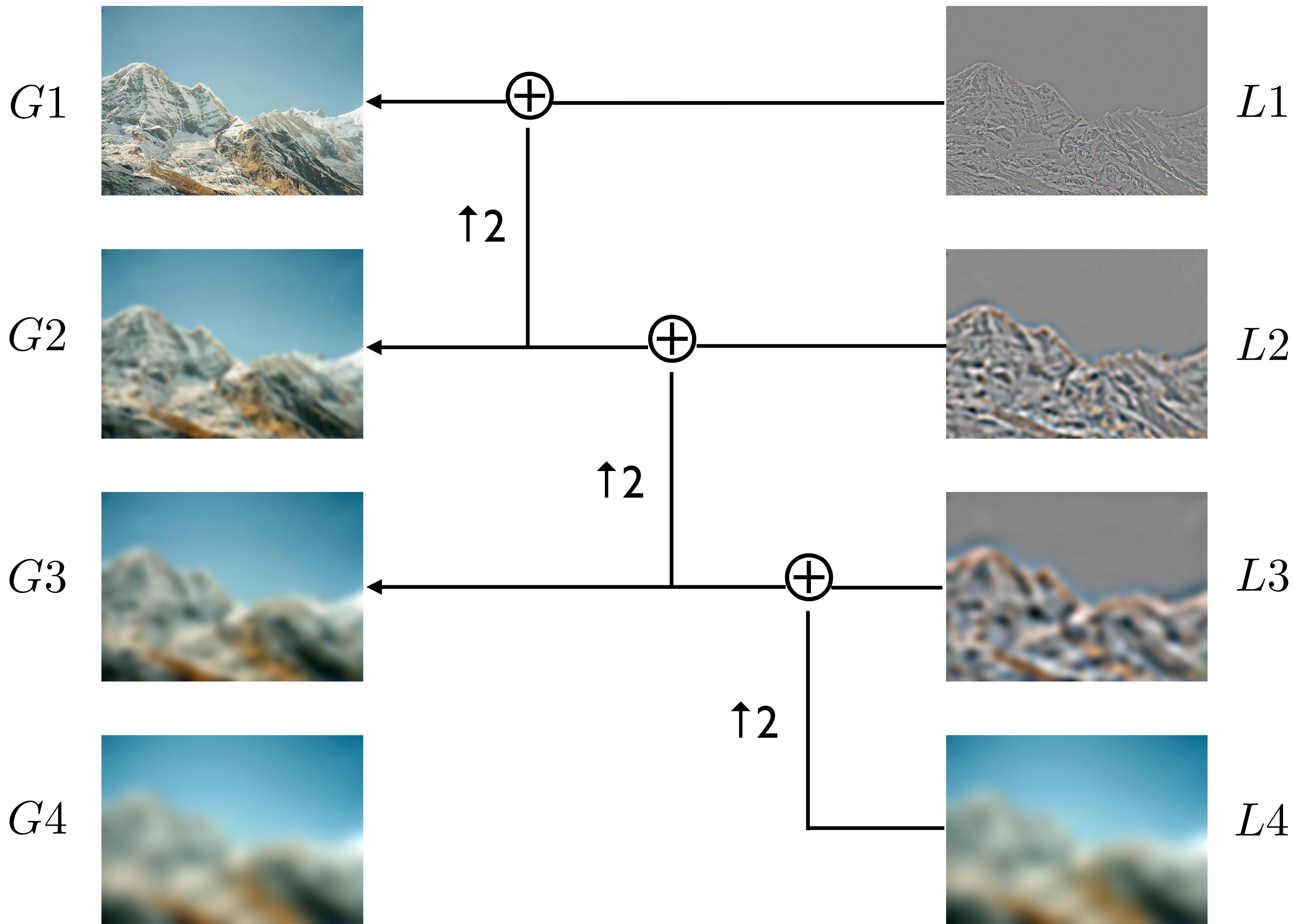
Gaussian Pyramid

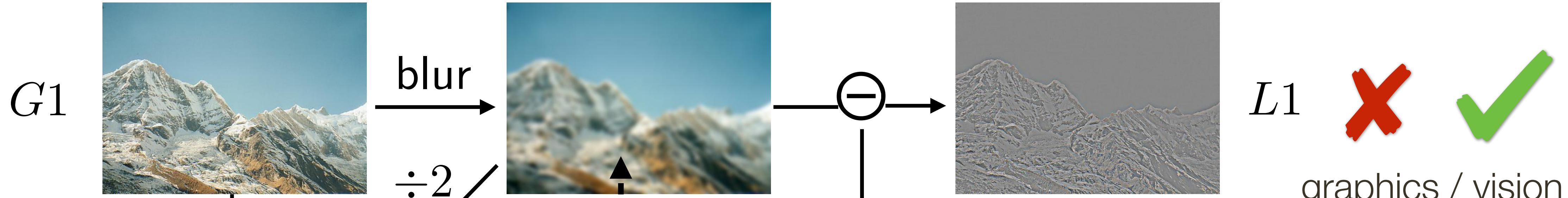
Laplacian Pyramid



Gaussian Pyramid

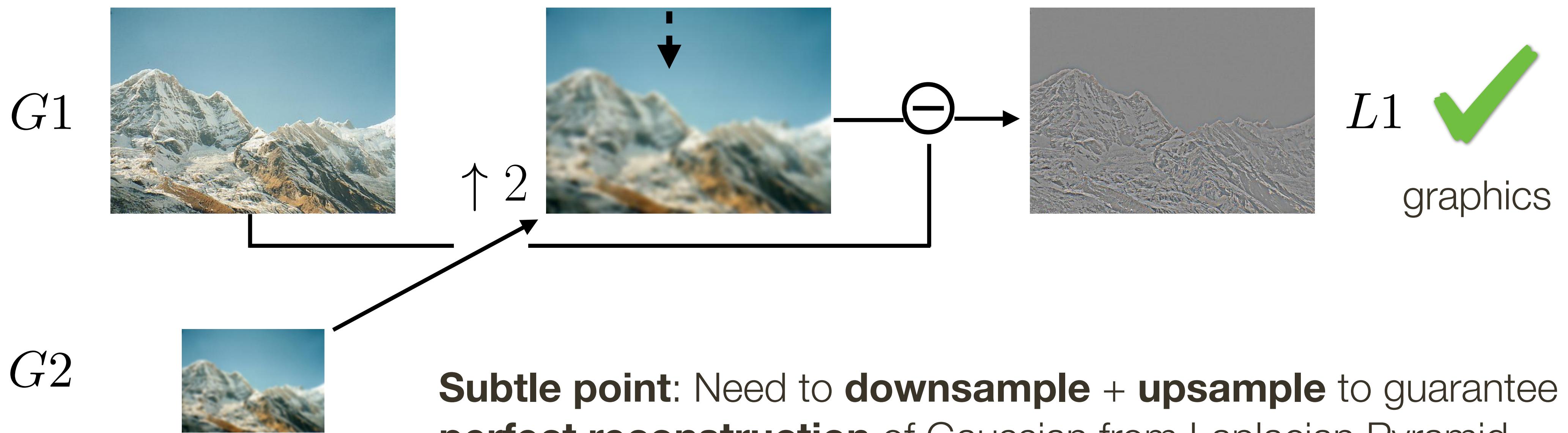
Laplacian Pyramid



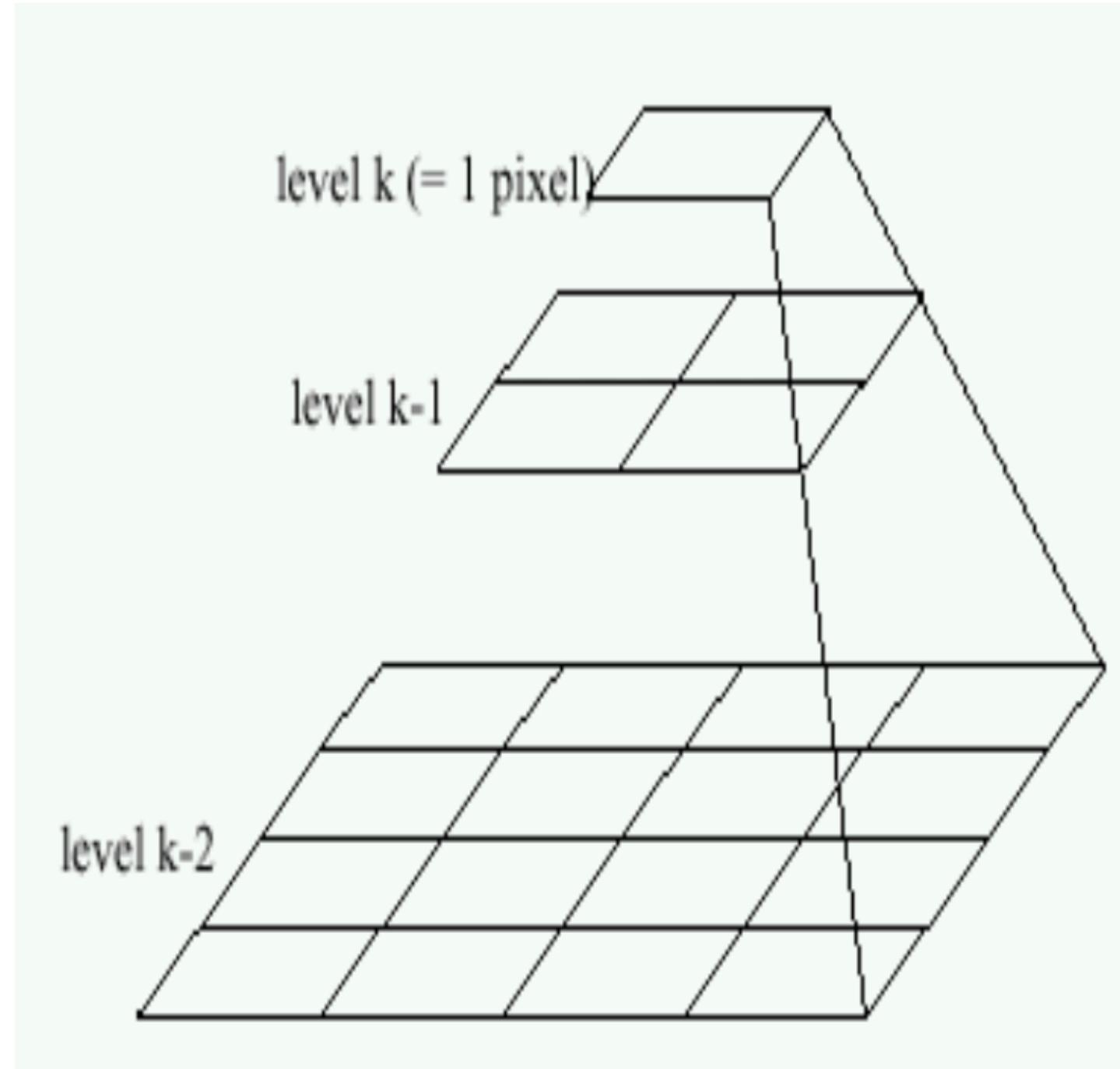


G_2

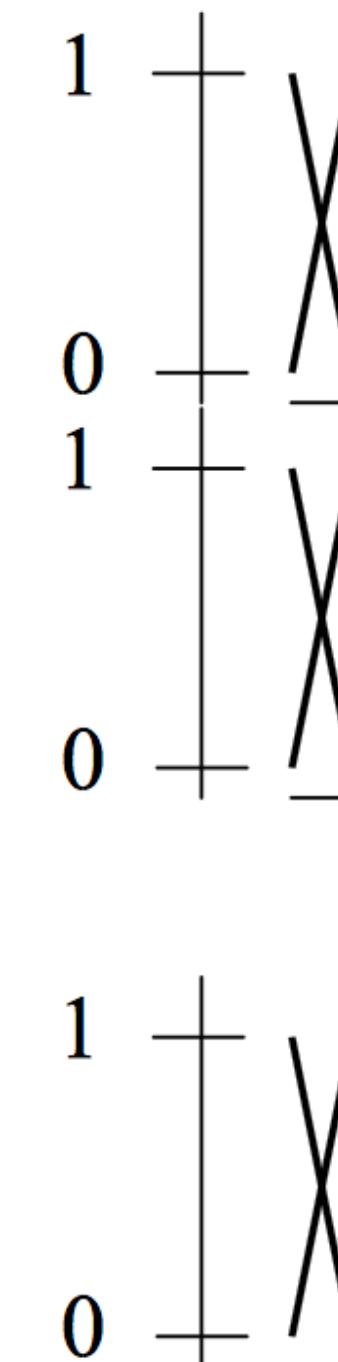
These images are theoretically the same (Nyquist) but in practice slightly different due to imperfect filtering/interpolation and edge effects



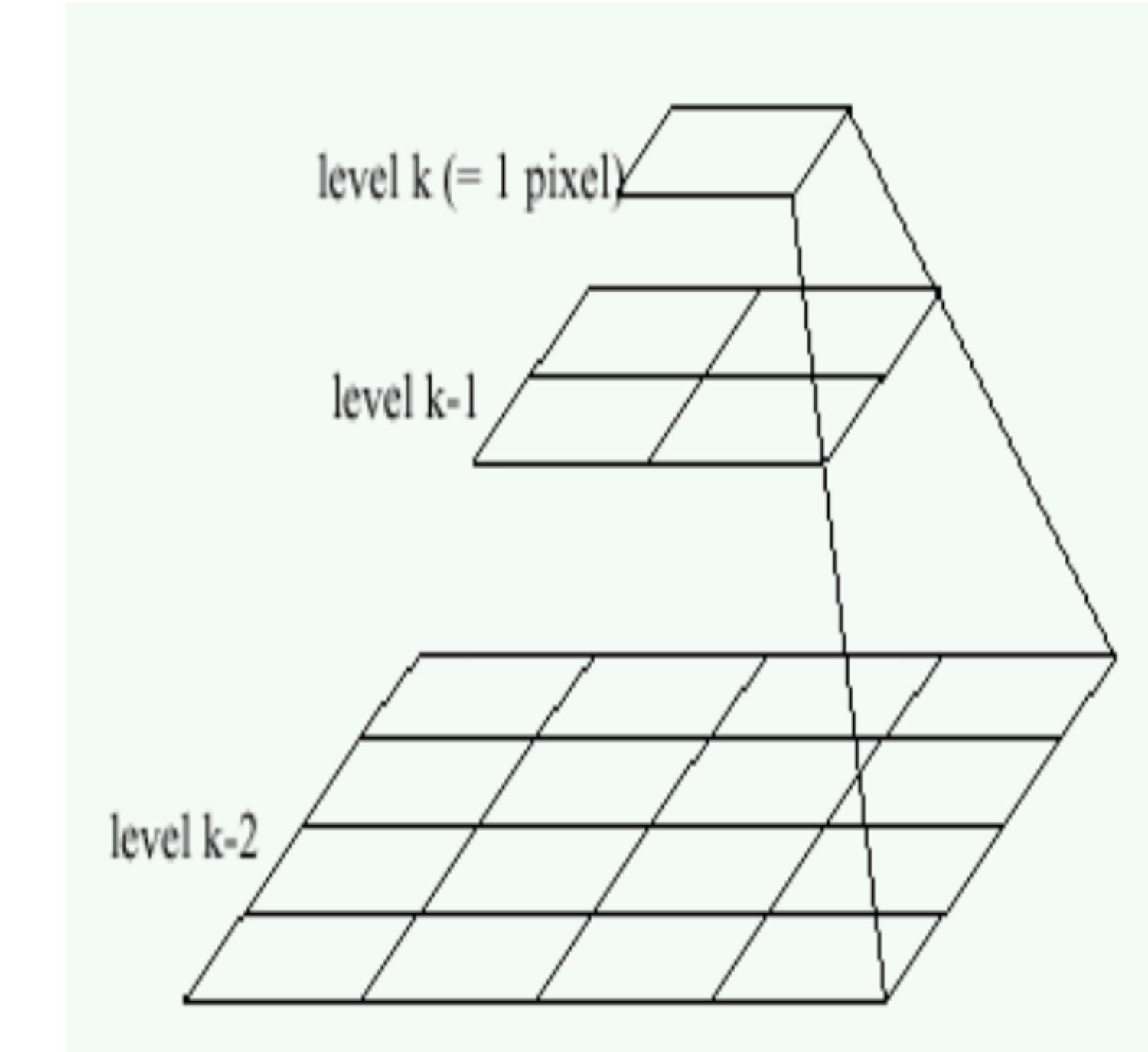
Application: Image Blending



Left pyramid



blend

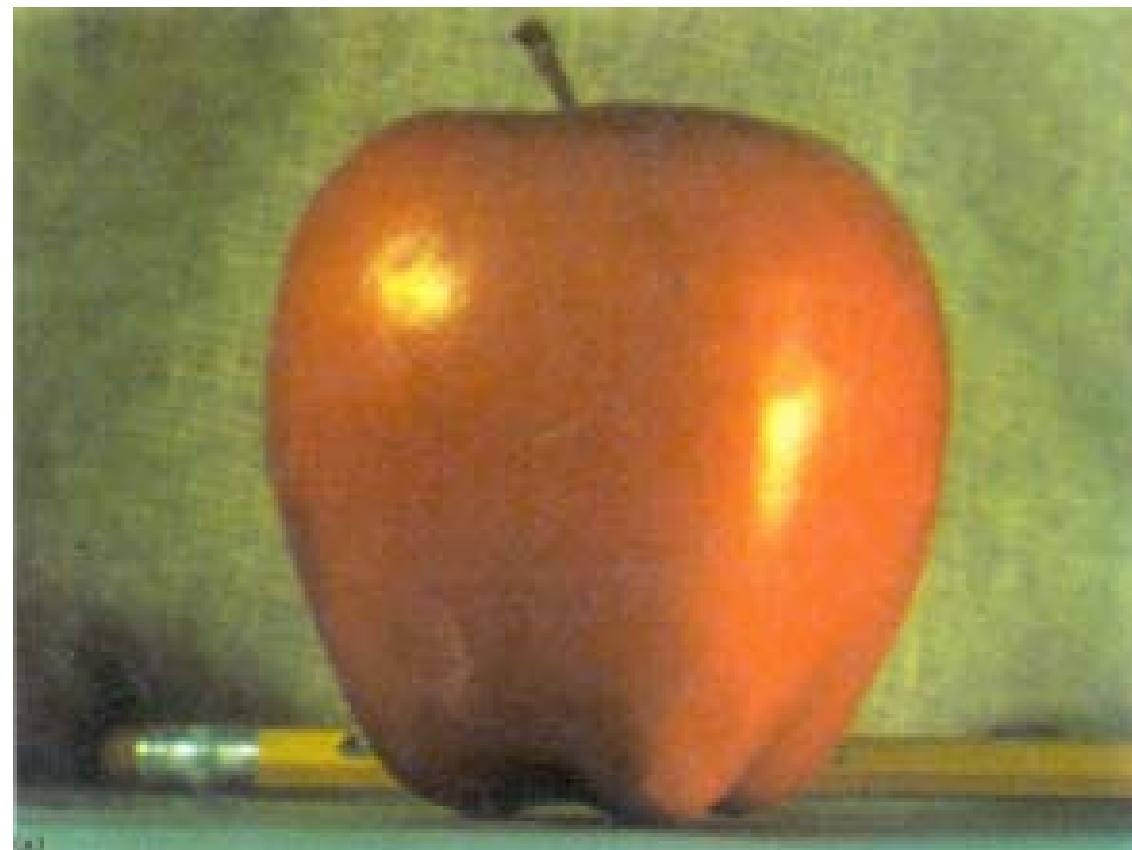


Right pyramid

Burt and Adelson, “A multiresolution spline with application to image mosaics,” ACM Transactions on Graphics, 1983, Vol.2, pp.217-236.

Pyramid Blending

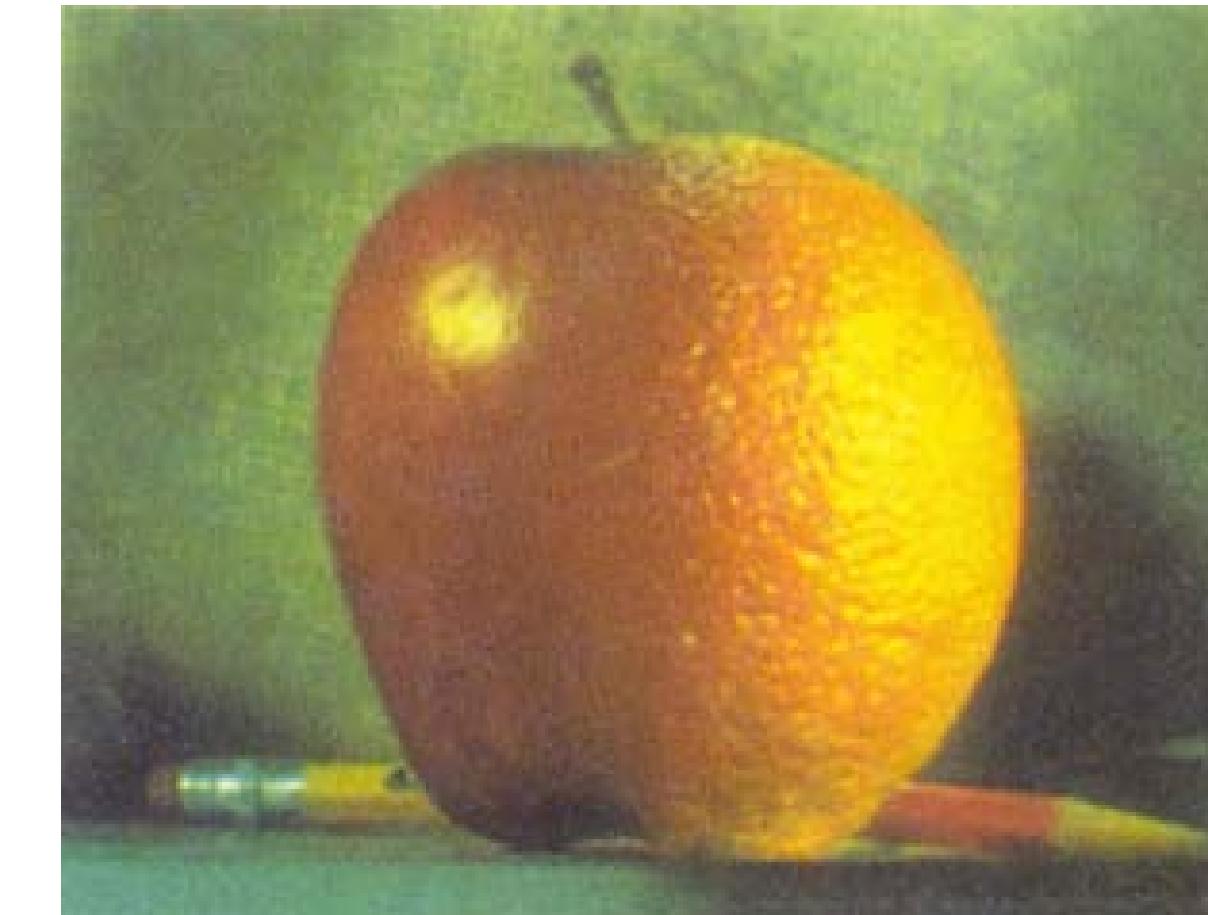
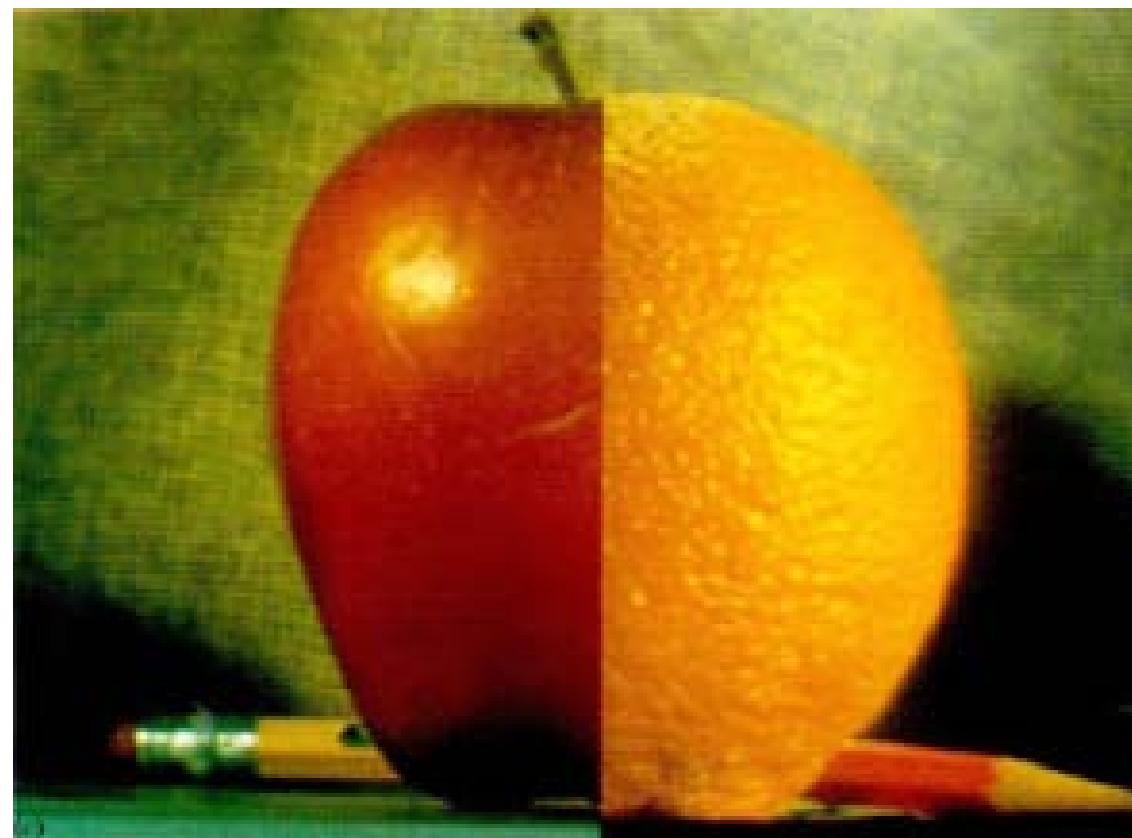
- Smooth low frequencies, whilst preserving high frequency detail



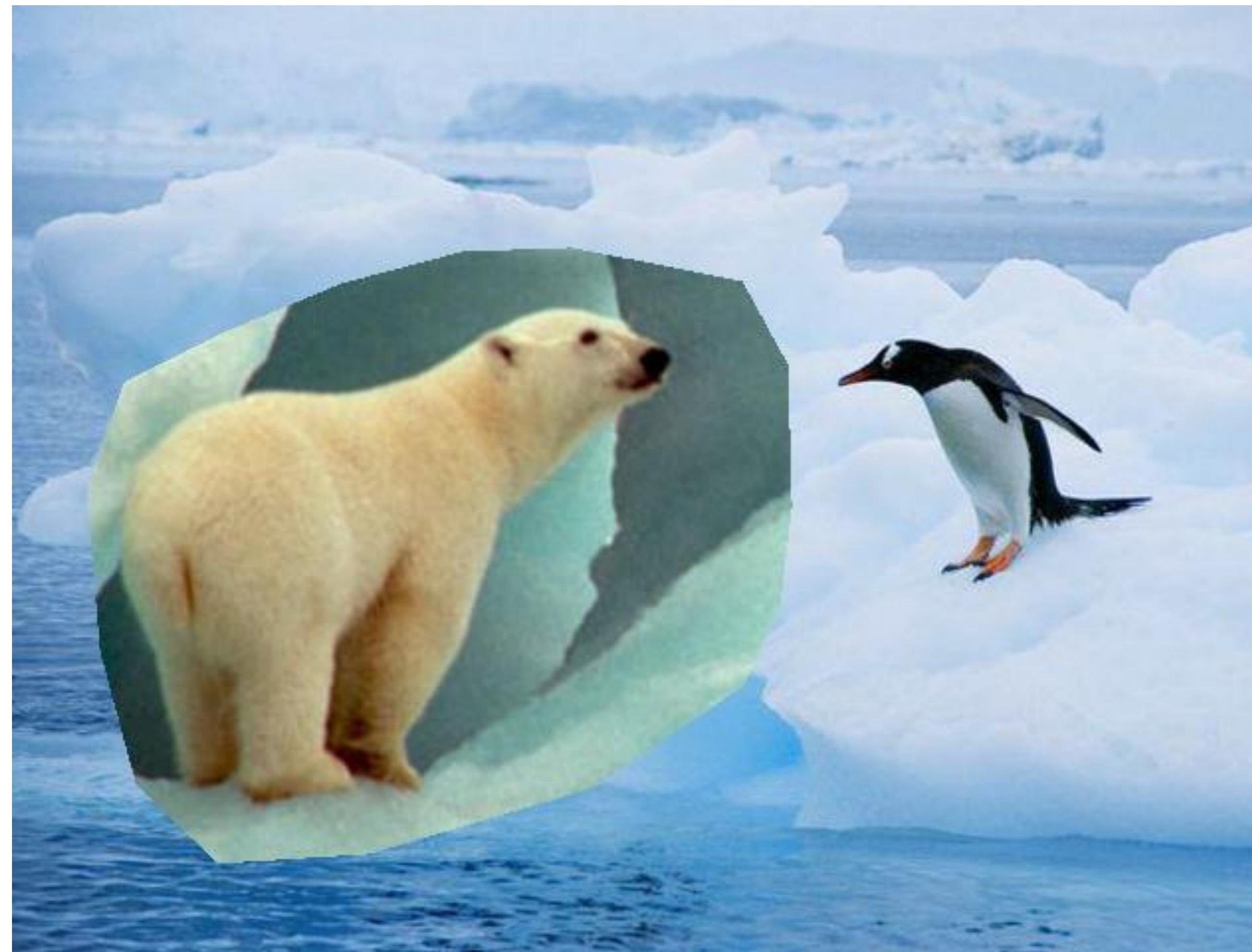
(a)



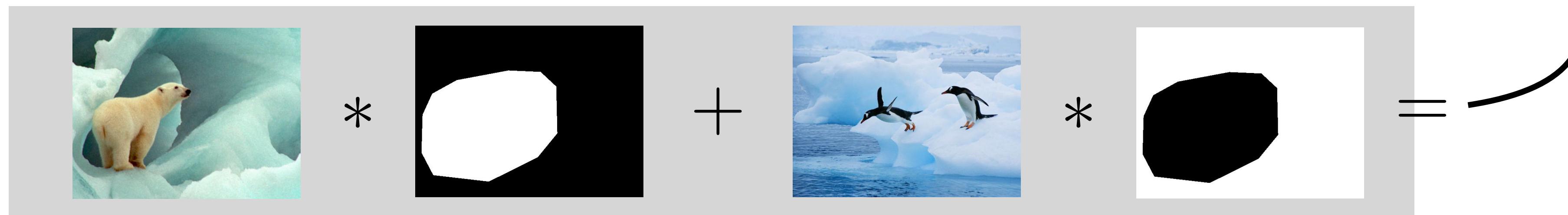
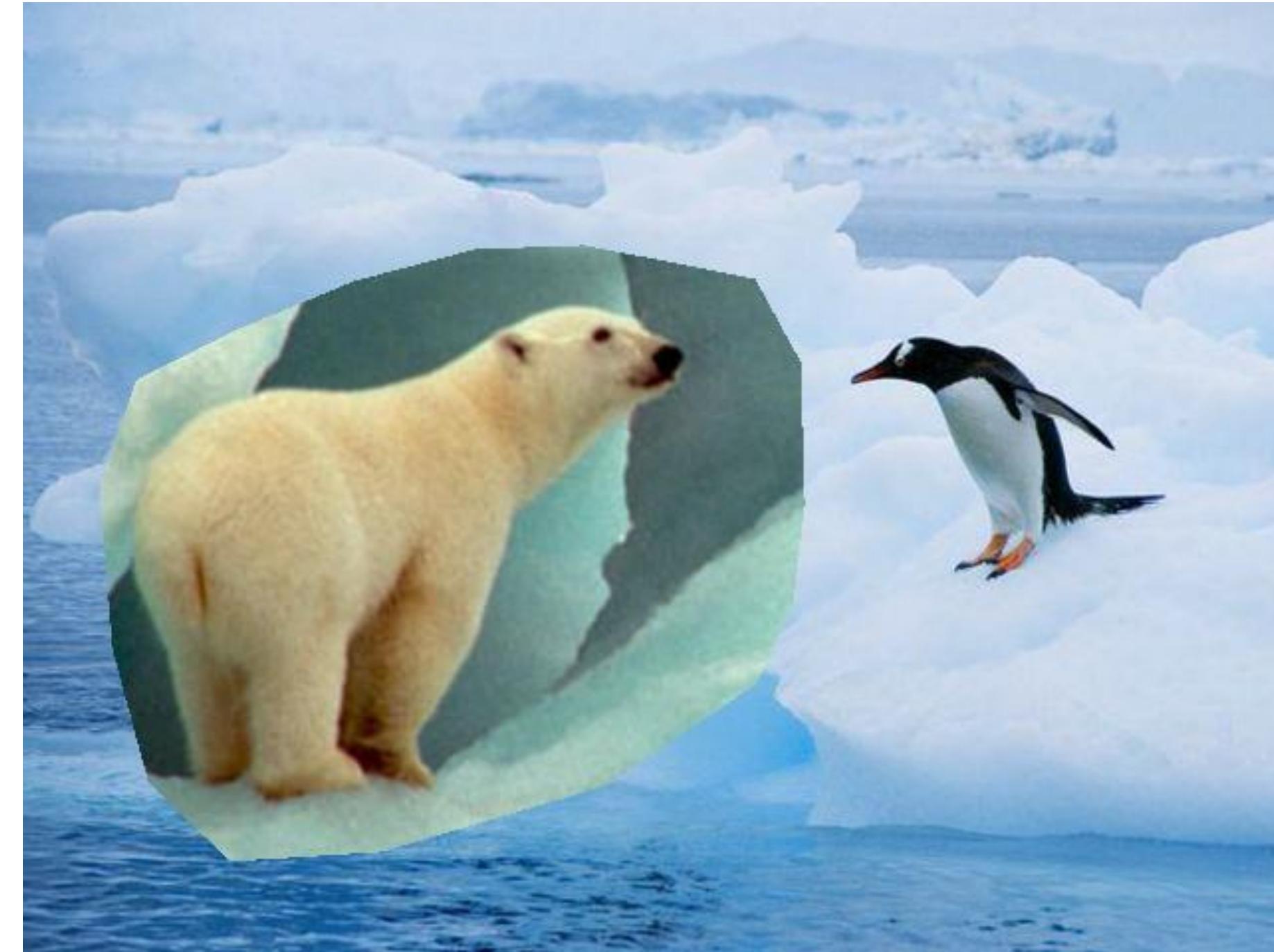
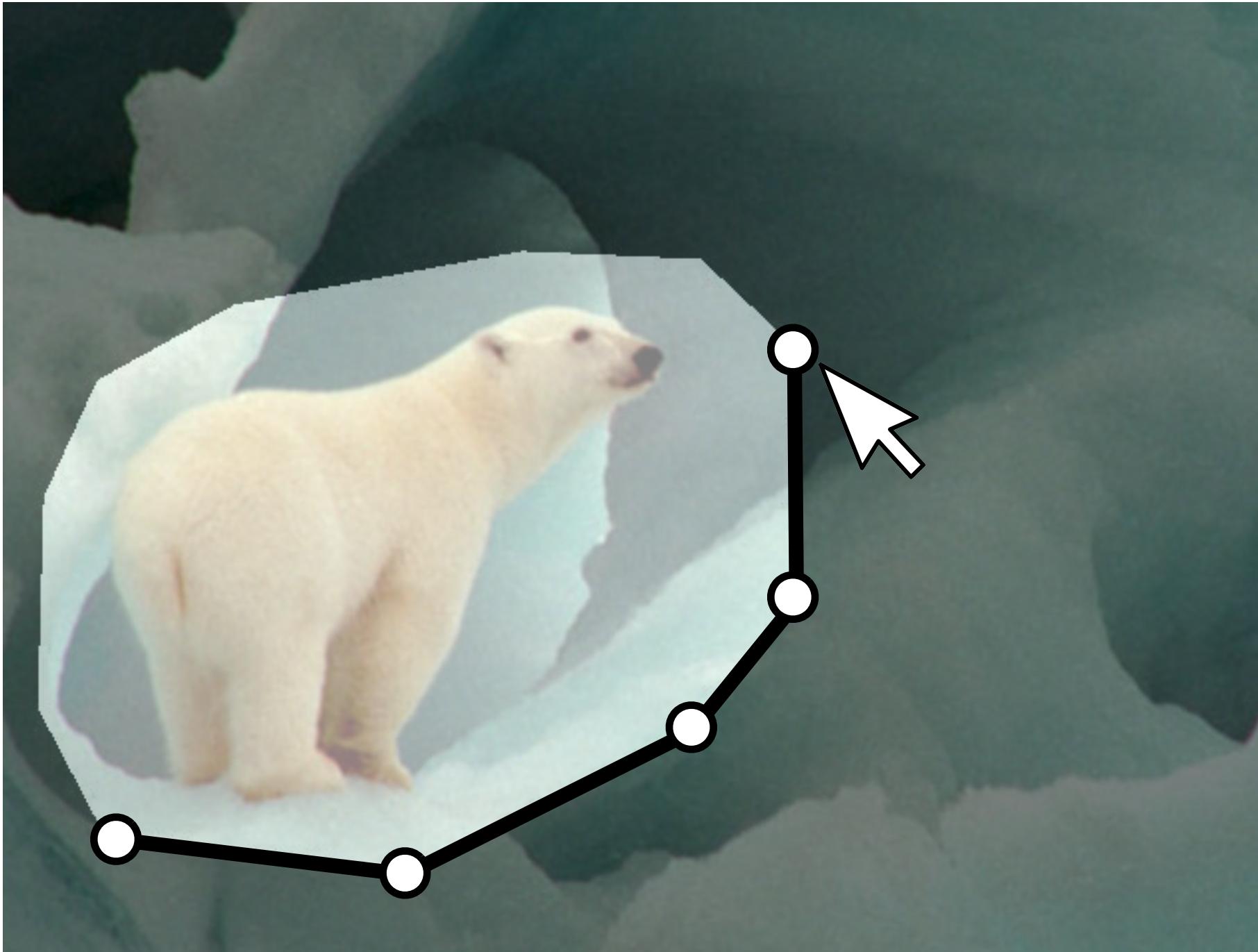
(b)



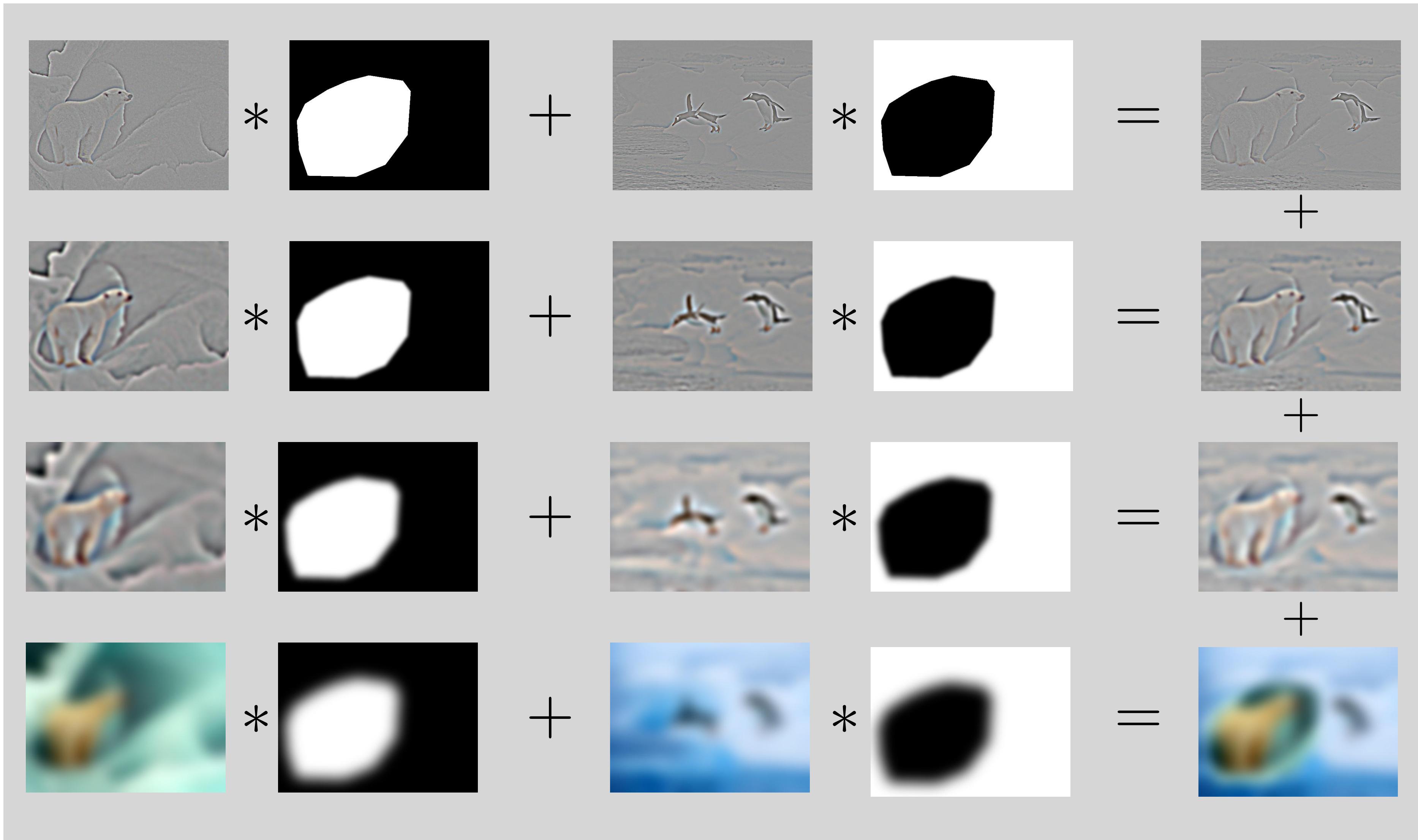
Pyramid Blending



Pyramid Blending



Step 1: Specify an Image Mask



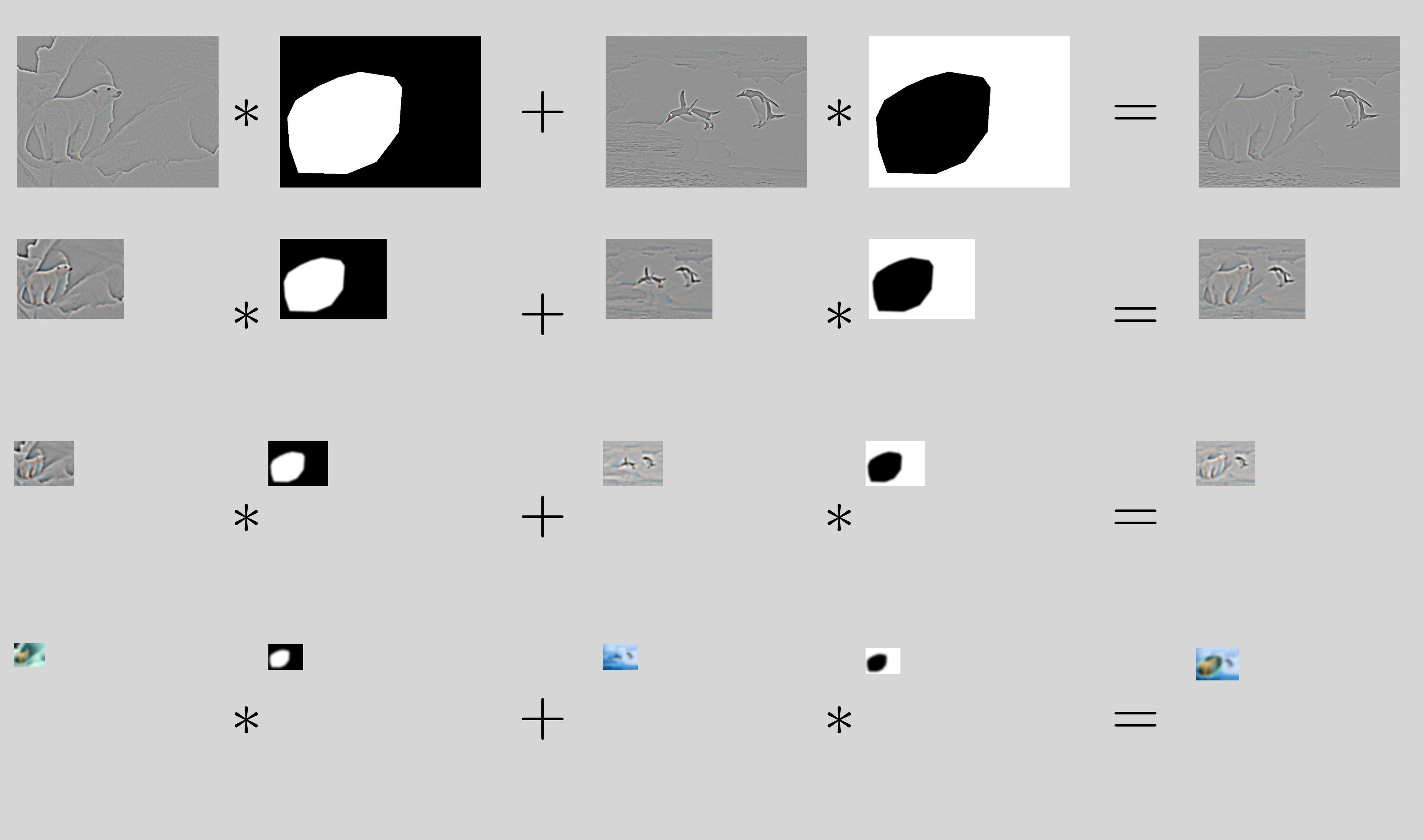
Step 2: blend lower frequency bands over
larger spatial ranges, high frequency bands
over small spatial ranges



Application: Image Blending

Algorithm:

1. Build Laplacian pyramid LA and LB from images A and B
2. Build a Gaussian pyramid GR from mask image R (the mask defines which image pixels should be coming from A or B)
3. From a combined (blended) Laplacian pyramid LS, using nodes of GR as weights: $LS(i,j) = GR(i,j) * LA(i,j) + (1-GR(i,j)) * LB(i,j)$
4. Reconstruct the final blended image from LS



Polar Bear
Laplacian
Pyramid

Mask
Gaussian
Pyramid

Penguin
Laplacian
Pyramid

I - Mask
Gaussian
Pyramid

Result
Pyramid

$$\begin{array}{ccccccccc} \text{Image 1} & * & \text{Mask 1} & + & \text{Image 2} & * & \text{Mask 2} & = & \text{Result 1} \\ \text{Image 1} & * & \text{Mask 1} & + & \text{Image 2} & * & \text{Mask 2} & = & \text{Result 2} \\ \text{Image 1} & * & \text{Mask 1} & + & \text{Image 2} & * & \text{Mask 2} & = & \text{Result 3} \\ \text{Image 1} & * & \text{Mask 1} & + & \text{Image 2} & * & \text{Mask 2} & = & \text{Result 4} \end{array}$$

Reconstruct Result

$$\begin{array}{c} \text{Image 1} * \text{Mask 1} + \text{Image 2} * \text{Mask 2} = \text{Result 1} \\ \text{Image 1} * \text{Mask 1} + \text{Image 2} * \text{Mask 2} = \text{Result 2} \\ \text{Image 1} * \text{Mask 1} + \text{Image 2} * \text{Mask 2} = \text{Result 3} \\ \text{Image 1} * \text{Mask 1} + \text{Image 2} * \text{Mask 2} = \text{Result 4} \\ \text{Image 1} * \text{Mask 1} + \text{Image 2} * \text{Mask 2} = \text{Result 5} \end{array}$$



Reconstruct
Result

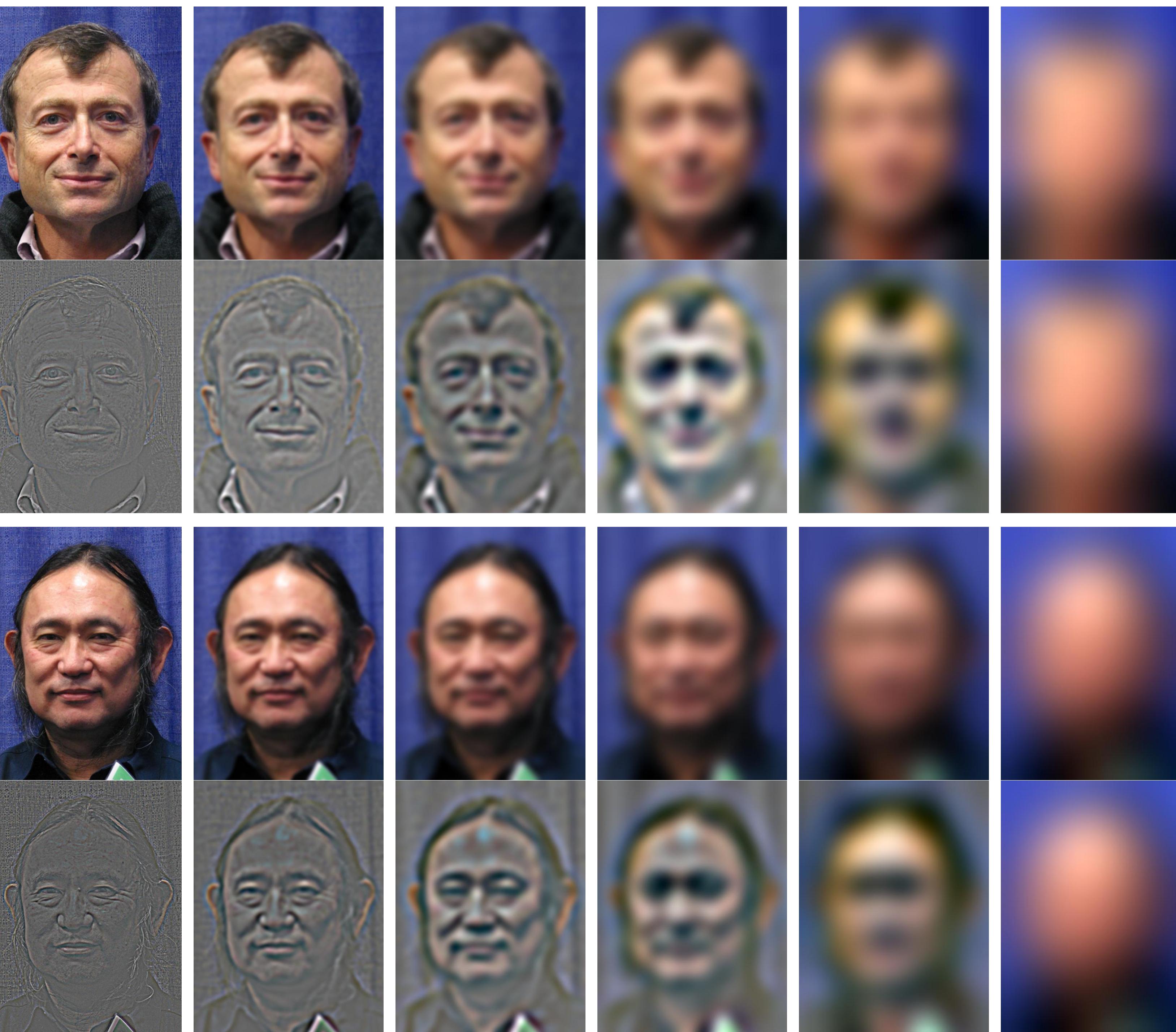




[Jim Kajiya, Andries van Dam] 48



[Jim Kajiya, Andries van Dam] 49





Alpha blend with sharp fall-off



Alpha blend with gradual fall-off



Pyramid Blend

Summary: Scaled Representations

Gaussian Pyramid

- Each level represents a **low-pass** filtered image at a different scale
- Generated by successive Gaussian blurring and downsampling
- Useful for image resizing, sampling

Laplacian Pyramid

- Each level is a **band-pass** image at a different scale
- Generated by differences between successive levels of a Gaussian Pyramid
- Used for pyramid blending, feature extraction etc.

Recap: Multi-Scale Template Matching

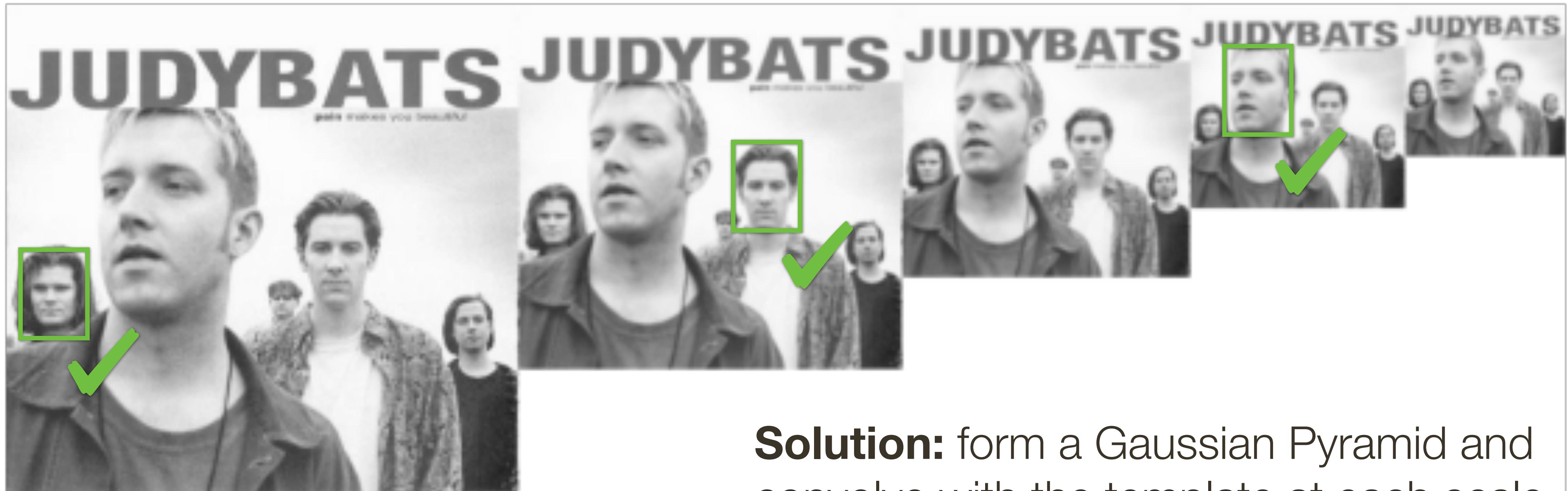
Correlation with a **fixed-sized image** only detects faces at **specific scales**



= Template

Recap: Multi-Scale Template Matching

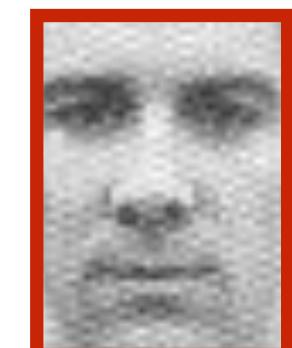
Correlation with a **fixed-sized image** only detects faces at **specific scales**



Solution: form a Gaussian Pyramid and convolve with the template at each scale



Q. Why **scale** the **image** and not the **template**?



= Template

Improving Template Matching

Consider the problem of finding images of an elephant using a template

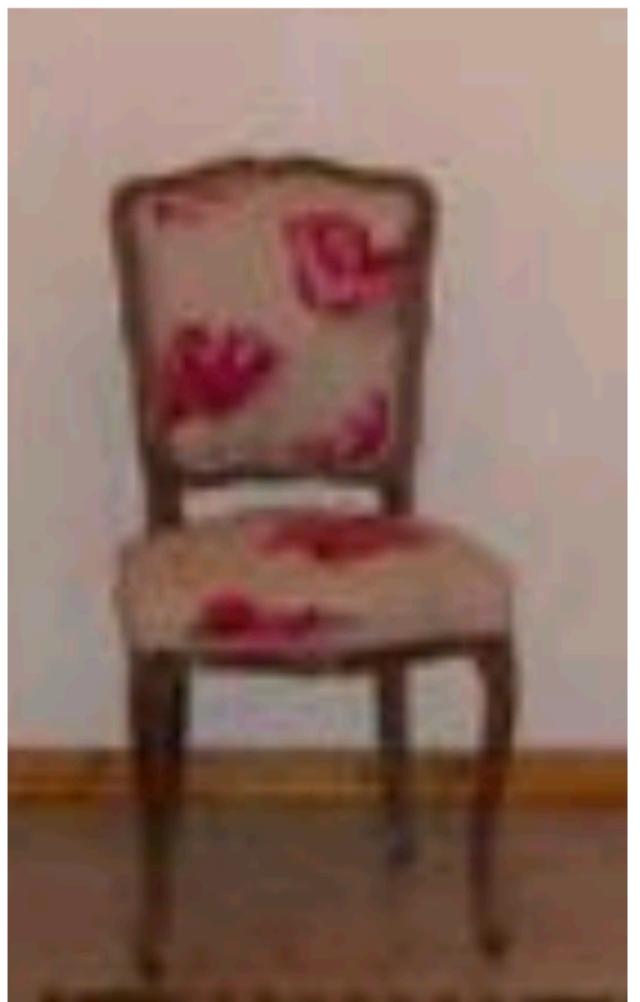
An elephant looks different from different viewpoints

- from above (as in an aerial photograph or satellite image)
- head on
- sideways (i.e., in profile)
- rear on

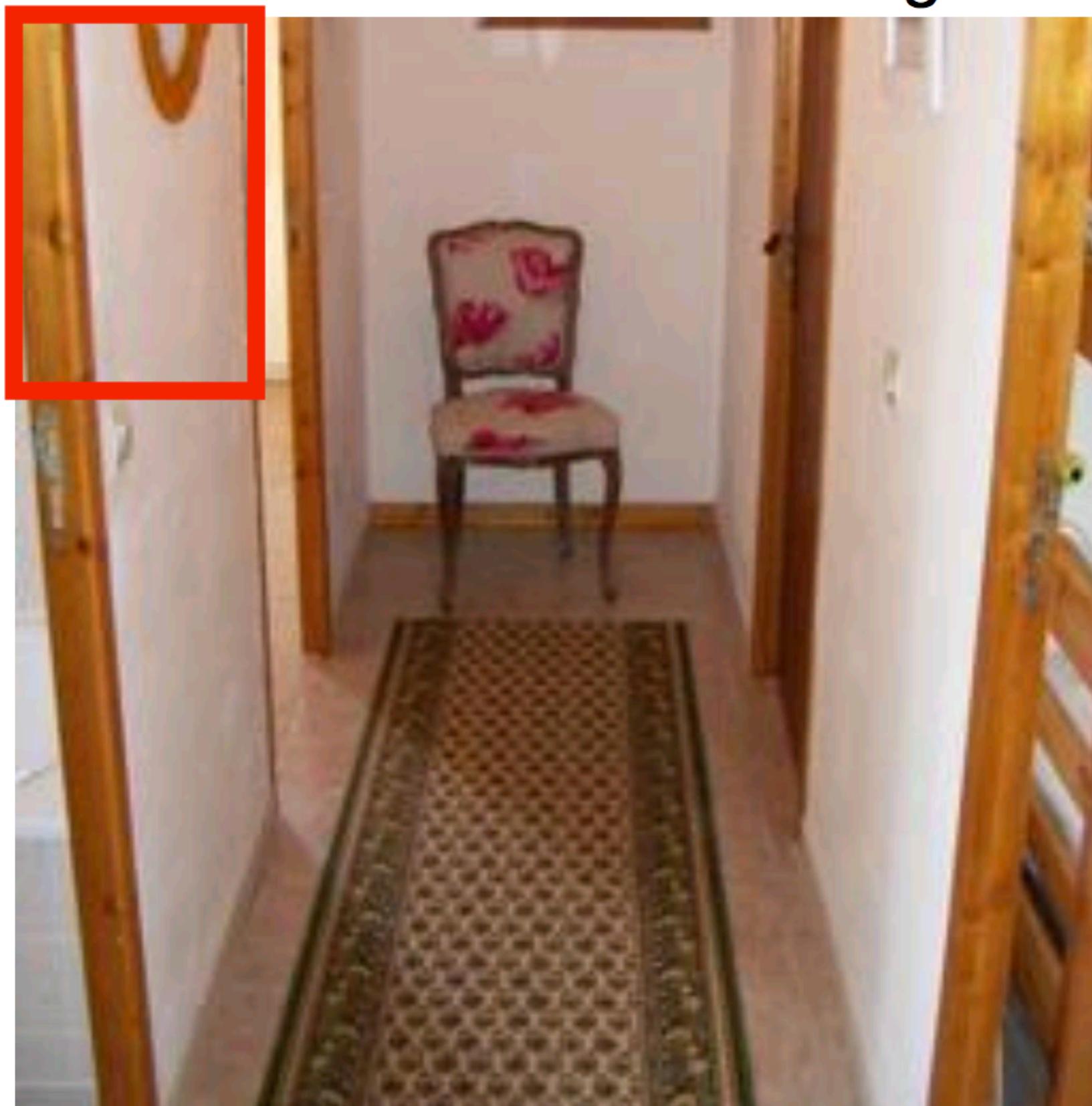
What happens if parts of an elephant are obscured from view by trees, rocks, other elephants?

Improving Template Matching

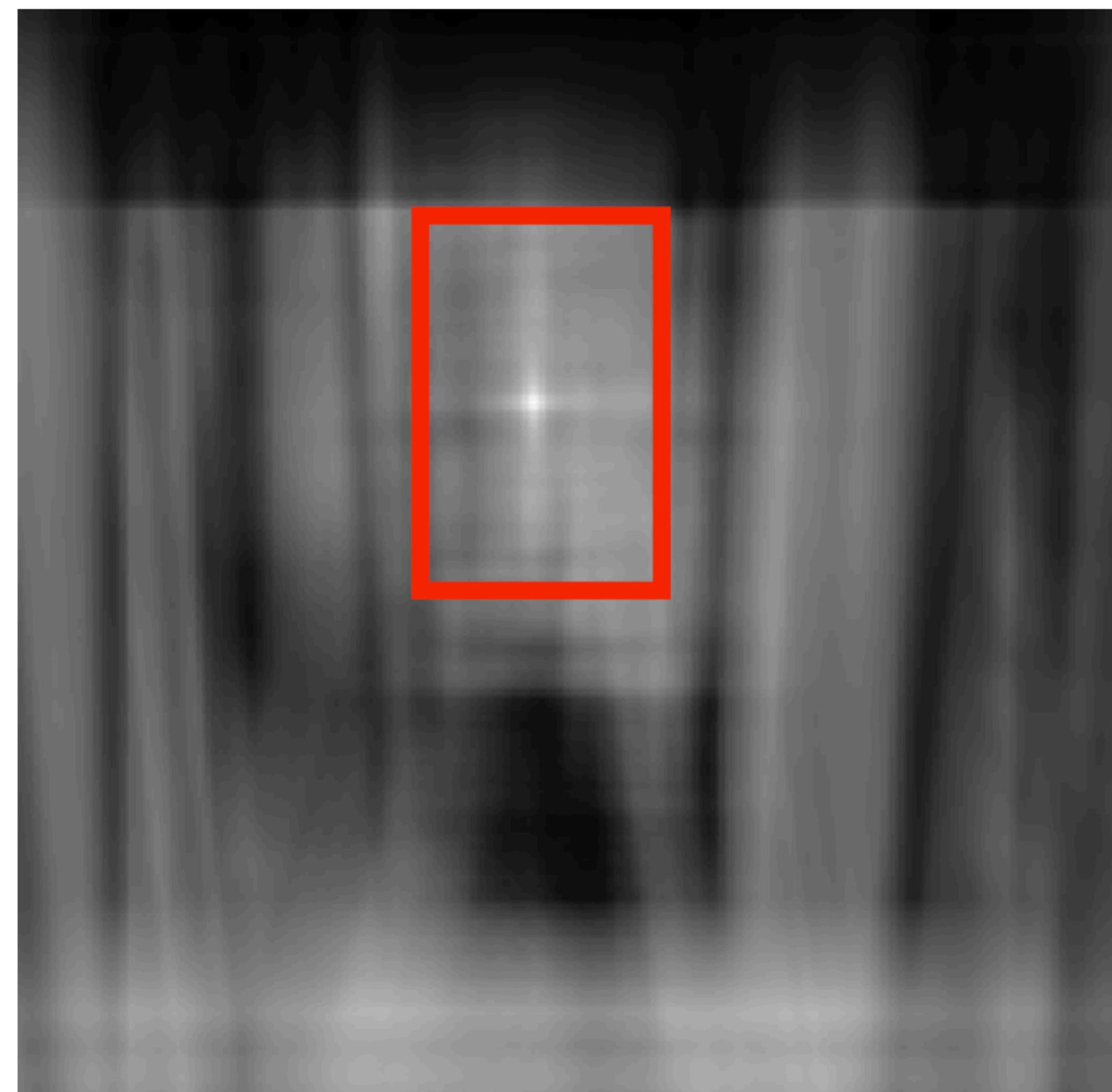
This is a chair



Find the chair in this image



Output of normalized correlation

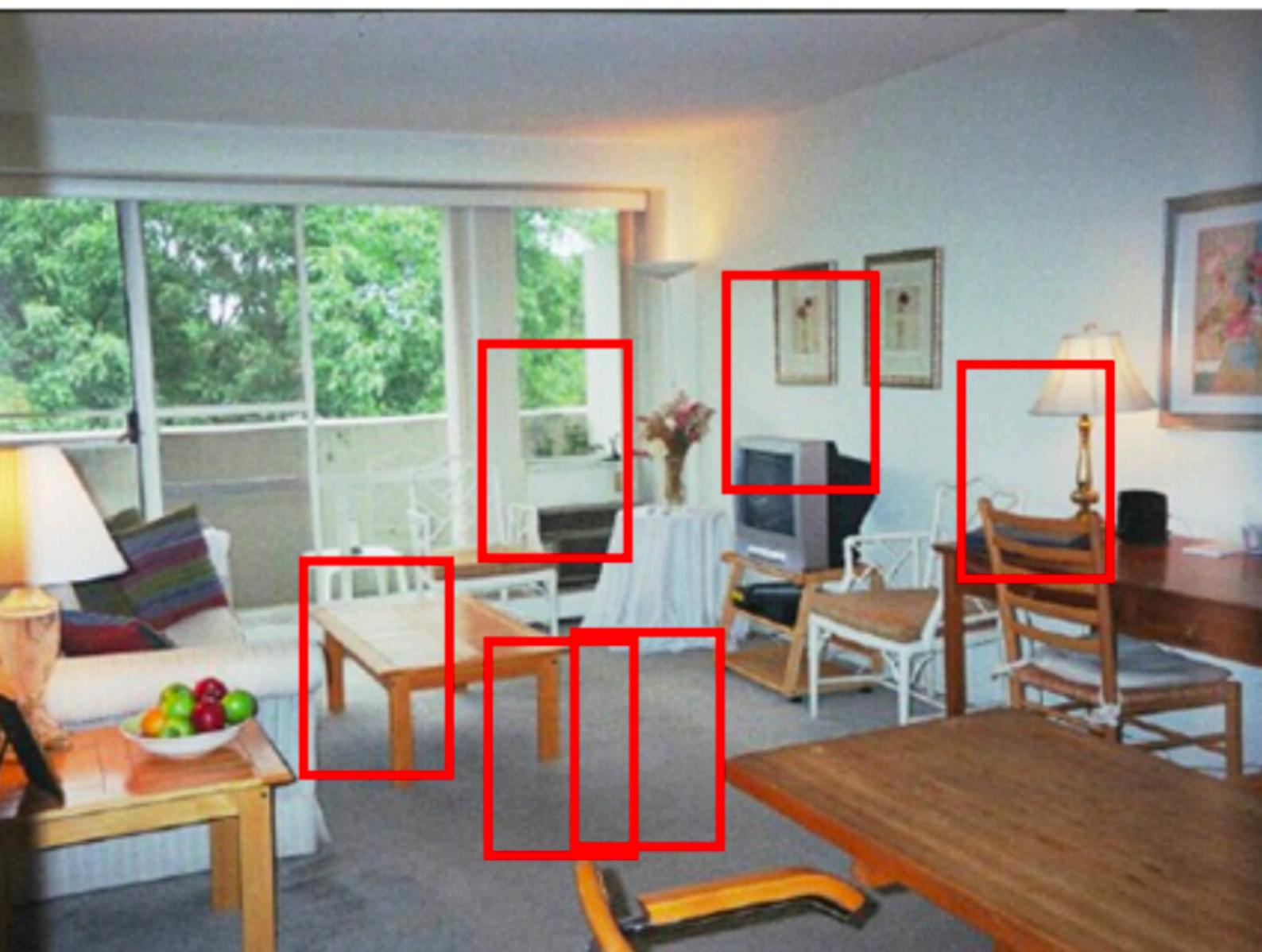


Slide Credit: Li Fei-Fei, Rob Fergus, and Antonio Torralba

Improving Template Matching



Find the chair in this image



Pretty much garbage
Simple template matching is not going to make it

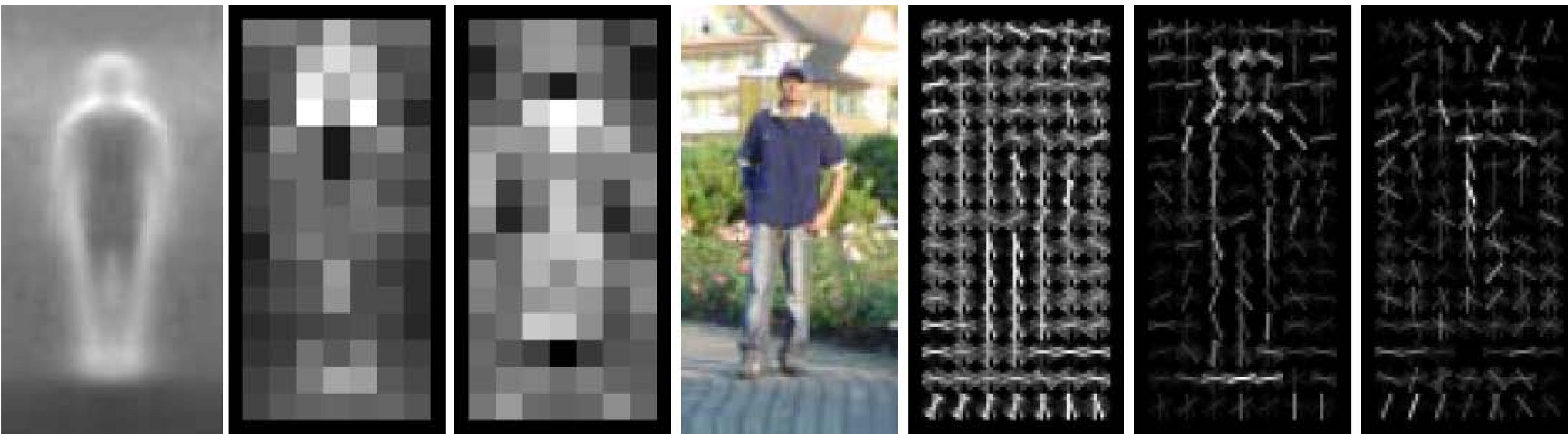
Improving Template Matching

Improved detection algorithms make use of **image features**

These can be **hand coded** or **learned**

Template Matching with HOG

- Template matching can be improved by using better features, e.g., Histograms of Gradients (HOG) [Dalal Triggs 2005]
- The authors use a Learning-based approach (Support Vector Machine) to find an optimally weighted template



avg grad

SVM weights

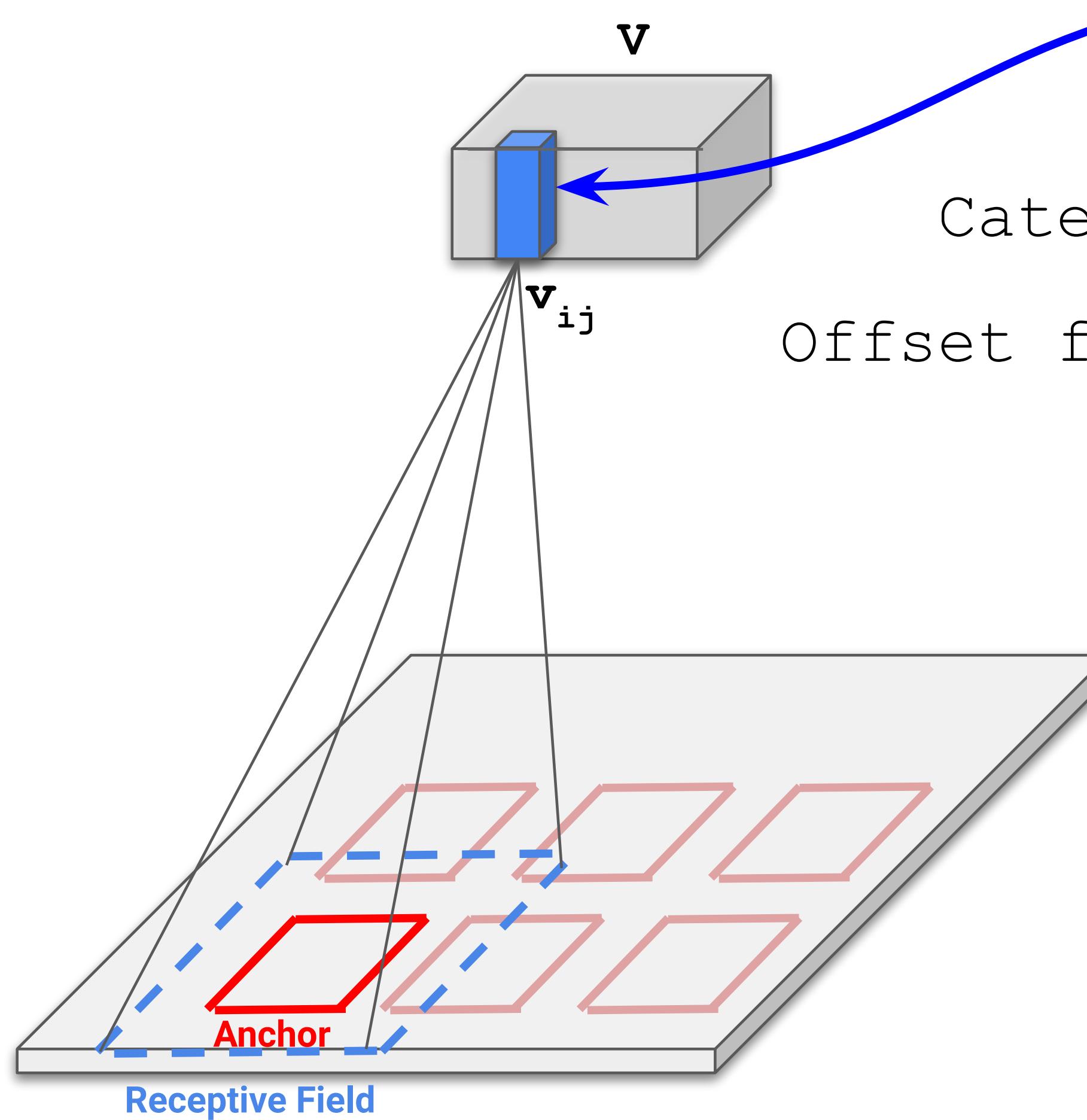
+

HOG

weighted HOG

-

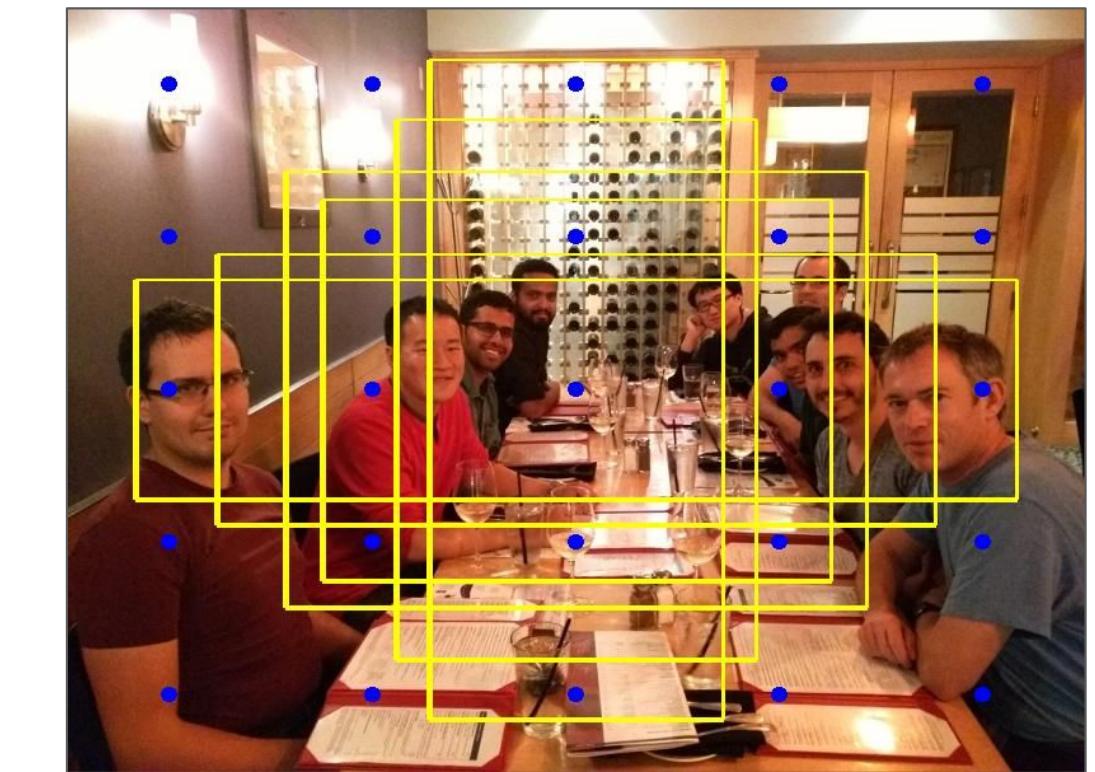
Convnet Object Detection



Think of each feature vector v_{ij} as corresponding to a sliding window (anchor).

$$\text{Category score} = \text{SoftMax}(W^{\text{cls}} \cdot v_{ij})$$

$$\text{Offset from anchor} = W^{\text{loc}} \cdot v_{ij}$$



- Convnet based object detectors resemble sliding window template matching in feature space
- Architectures typically involve multiple scales and aspect ratios, and regress to a 2D offset in addition to category scores

Summary

Template matching as (normalized) correlation. Template matching is not robust to changes in:

- 2D spatial scale and 2D orientation
- 3D pose and viewing direction
- illumination

Scaled representations facilitate

- template matching at multiple scales
- efficient search for image-to-image correspondences
- image analysis at multiple levels of detail

A **Gaussian pyramid** reduces artifacts introduced when sub-sampling to coarser scales

Menu for Today

Topics:

- **Digital Imaging** Pipeline
- **Scaled** Representations
- Template **Matching**
- Normalised **Correlation**

Readings:

- **Today's** Lecture: Szeliski 2.3, 3.5, Forsyth & Ponce (2nd ed.) 4.5 - 4.7

Reminders:

- **Assignment 1:** due **Thursday** September 28th
- **Assignment 2:** Scaled Representations, Face Detection and Image Blending available now