



# CPSC 425: Computer Vision



**Lecture 16:** Multiview Reconstruction

# Menu for Today

## Topics:

- **Stereo, Optical Flow** recap
- **Multiview Reconstruction**
- **Quiz 5** on Thursday
- Visual **Classification**

## Readings:

- **Today's Lecture:** Szeliski 11.4, 12.3-12.4, 9.3, 5.1-5.2

## Reminders:

- **Assignment 4:** due Thursday
- **Assignment 5:** Scene Recognition with Bag of Words is now available

# 2-view Rigid Matching

- **1D search**, points constrained to lie along epipolar lines



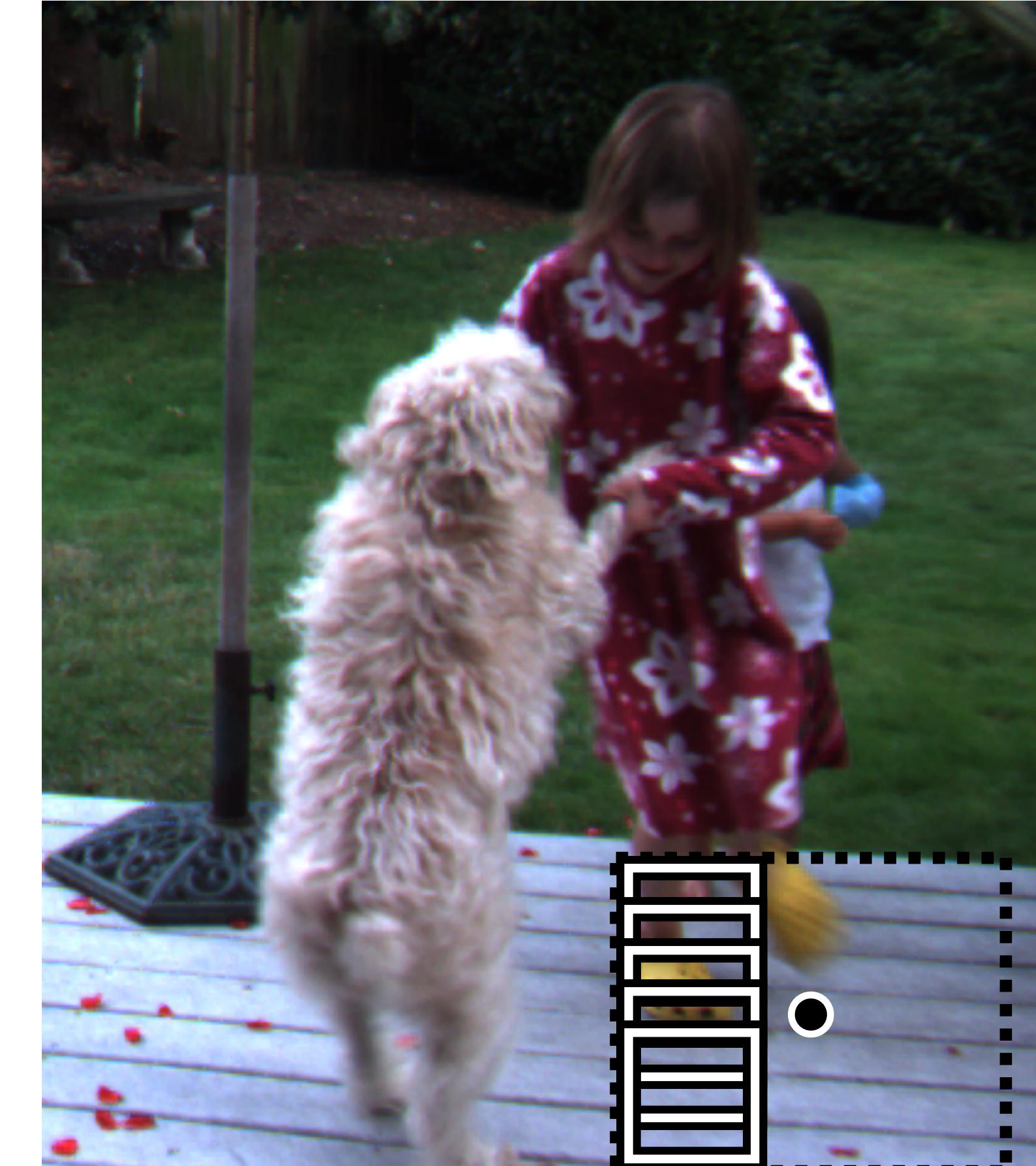
# 2-view Non-Rigid Matching

- **2D search**, points can move anywhere in the image



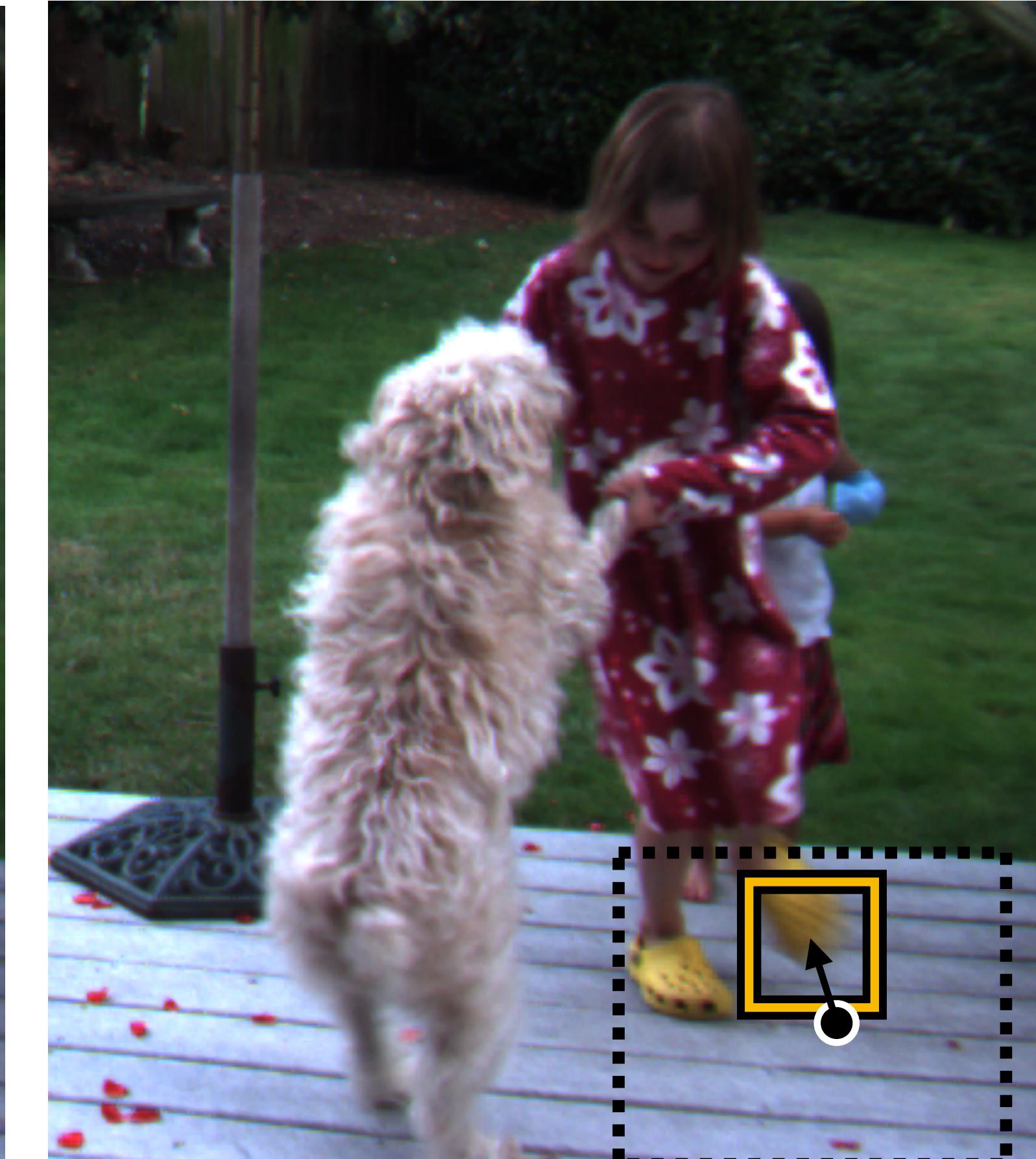
# 2-view Non-Rigid Matching

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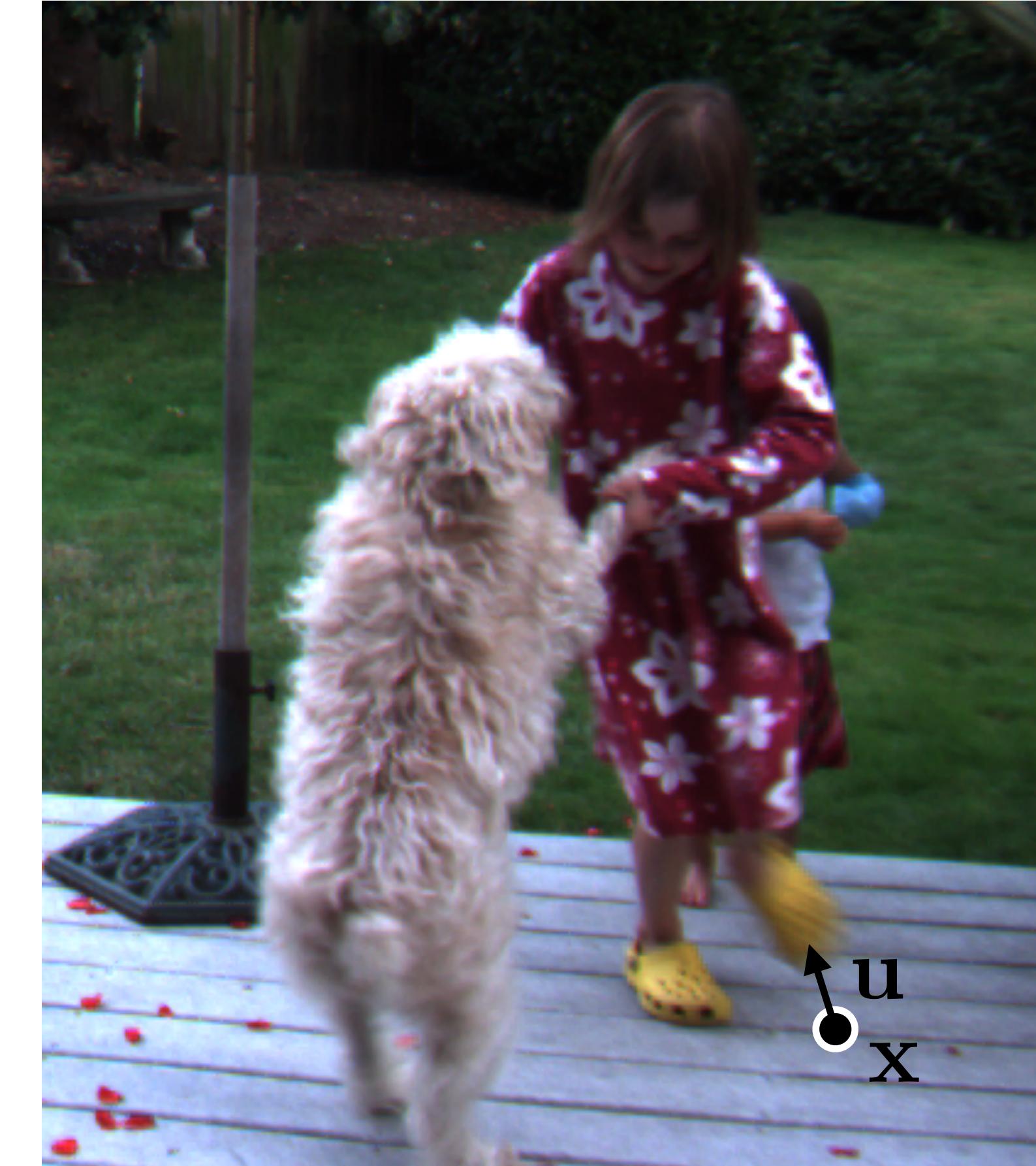
# 2-view Non-Rigid Matching

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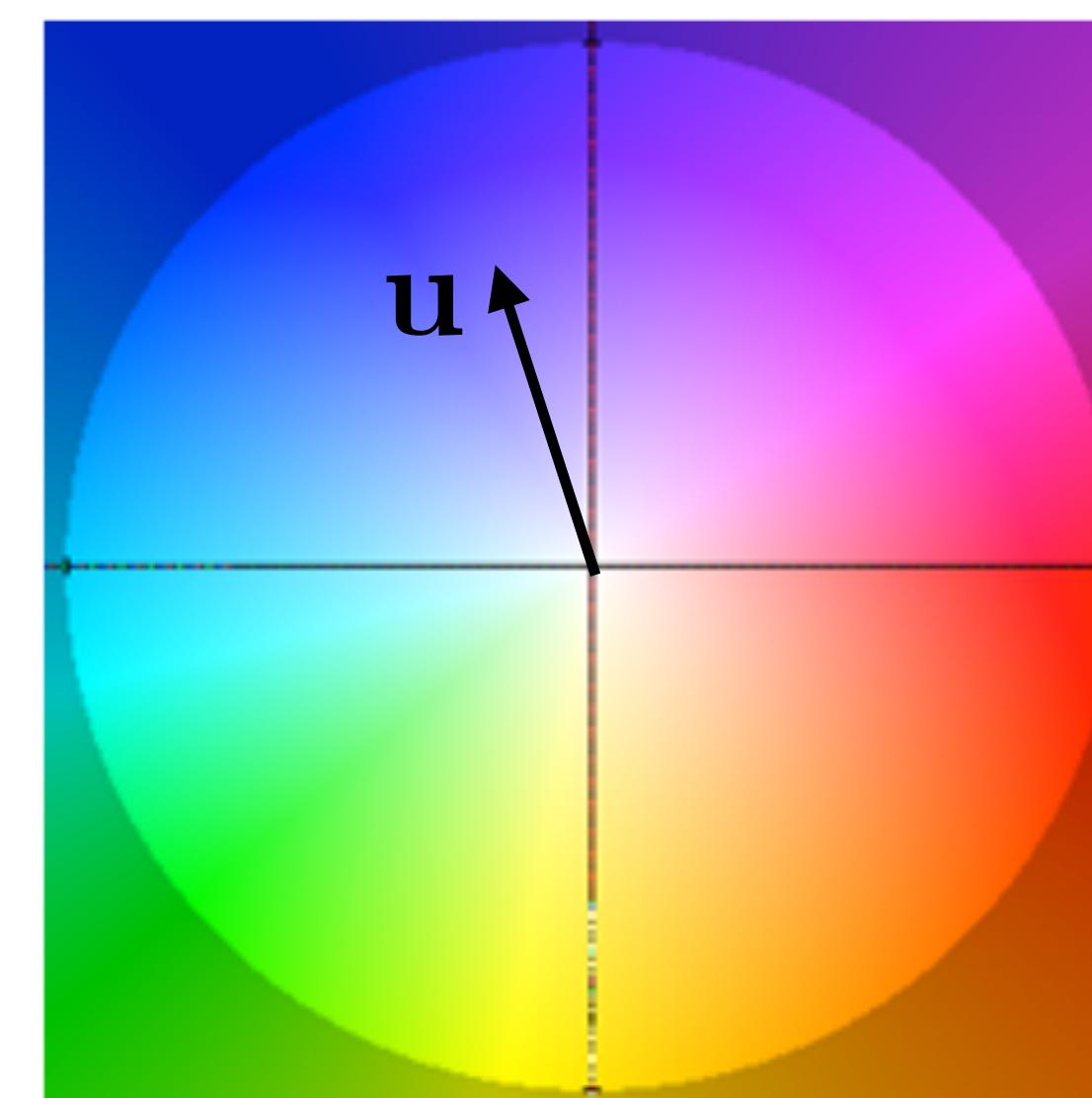
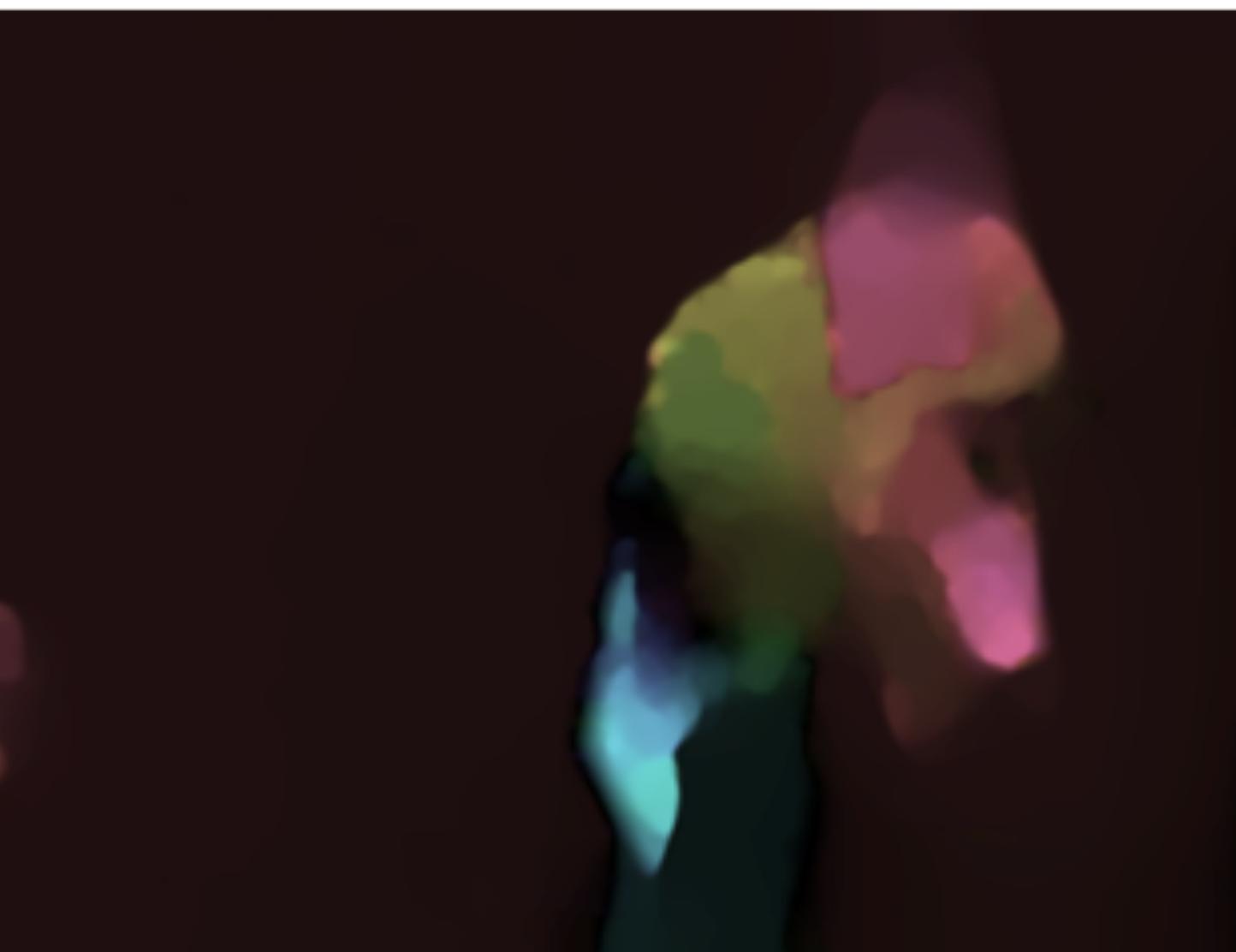


# 2-view Non-Rigid Matching

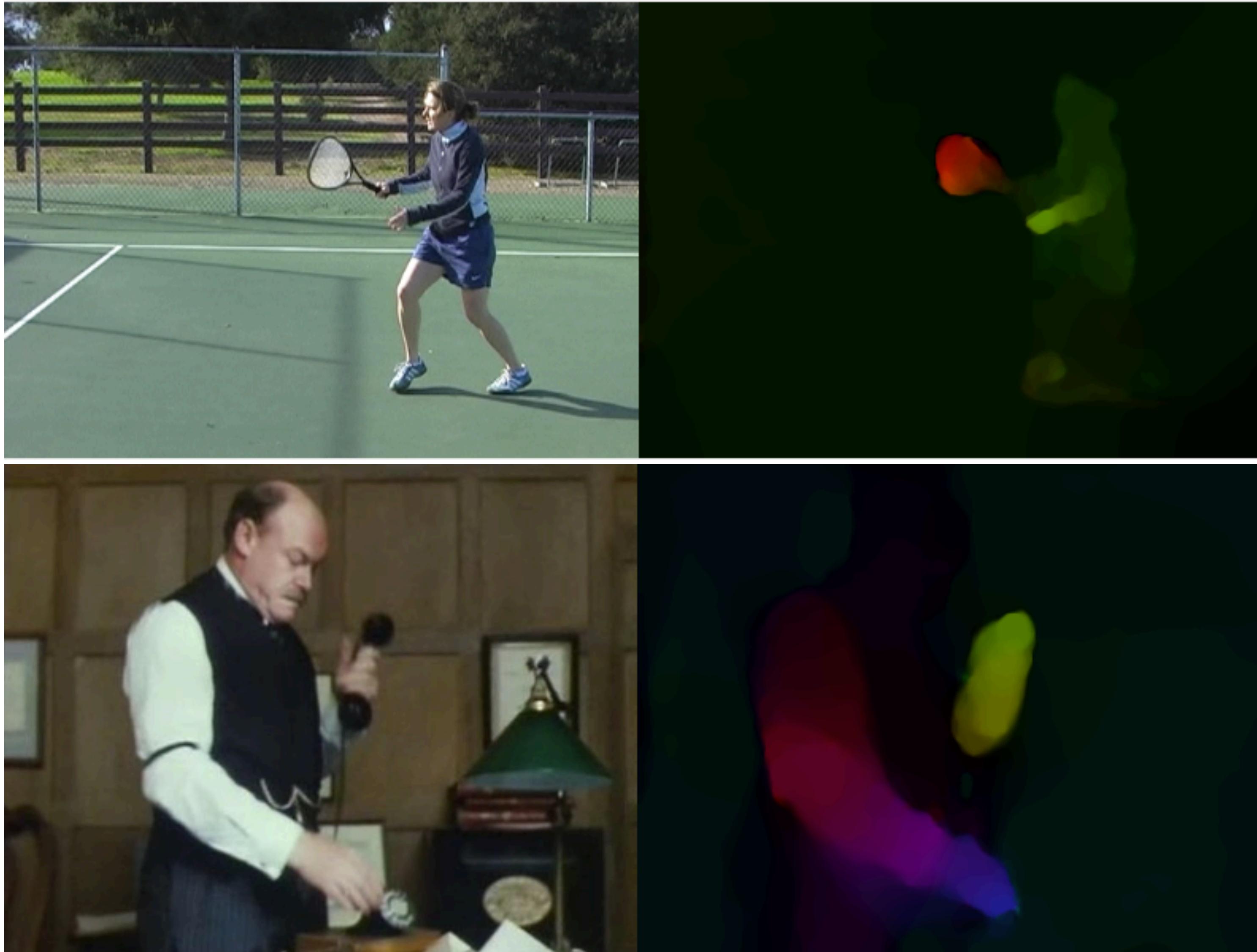
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# Optical Flow: Example I



# Optical Flow: Example 2



# Optical Flow Recap

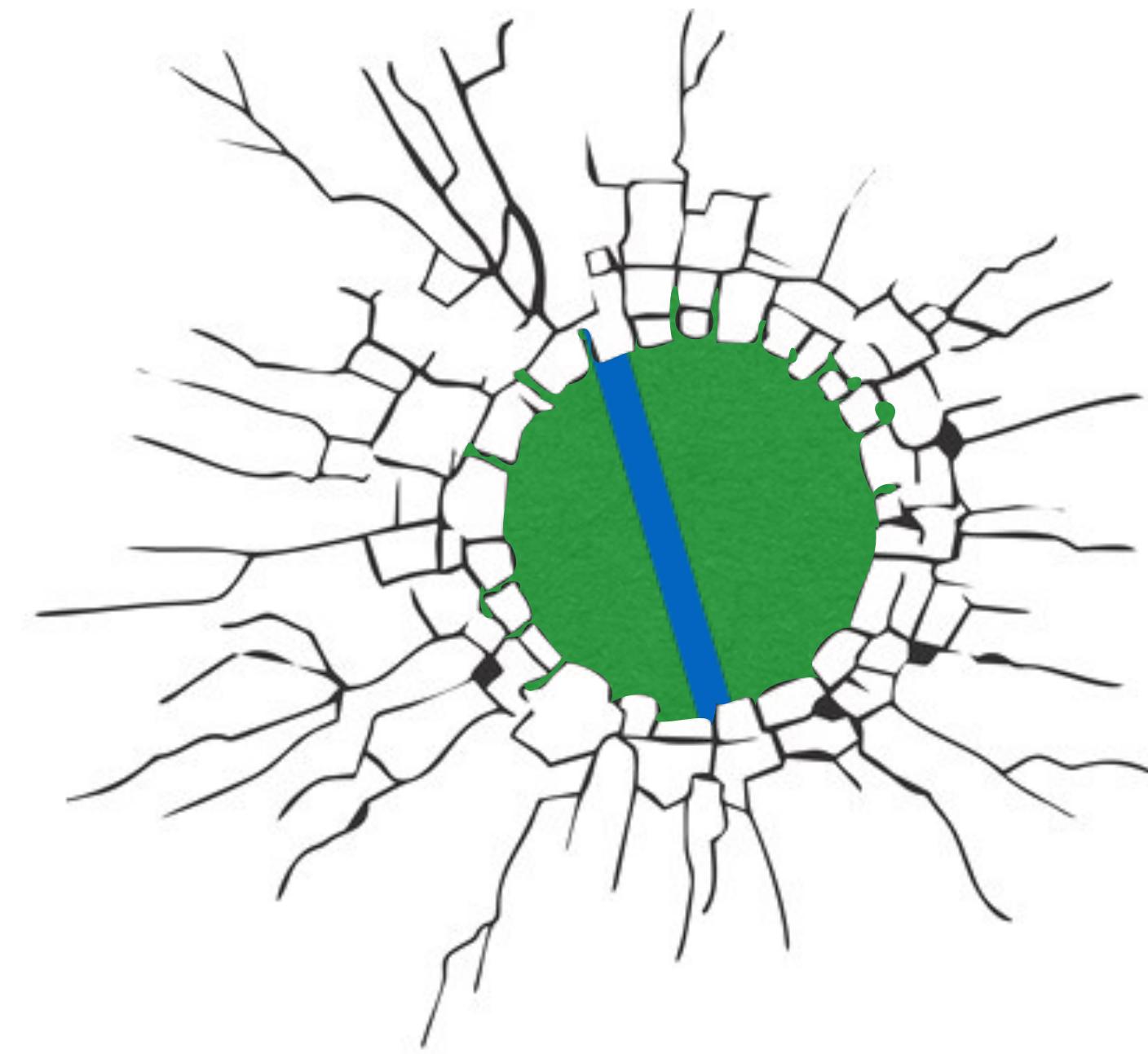
**Optical Flow** the apparent motion of all pixels in an image between a pair of image frames

**Brightness Constancy** a point on an object has the same intensity as it moves (in  $x$ ,  $y$ ,  $t$ )

**Optical Flow Constraint** the derivative of brightness constancy at a point, relates image gradients in  $x$ ,  $y$ ,  $t$  and flow vector  $u, v$

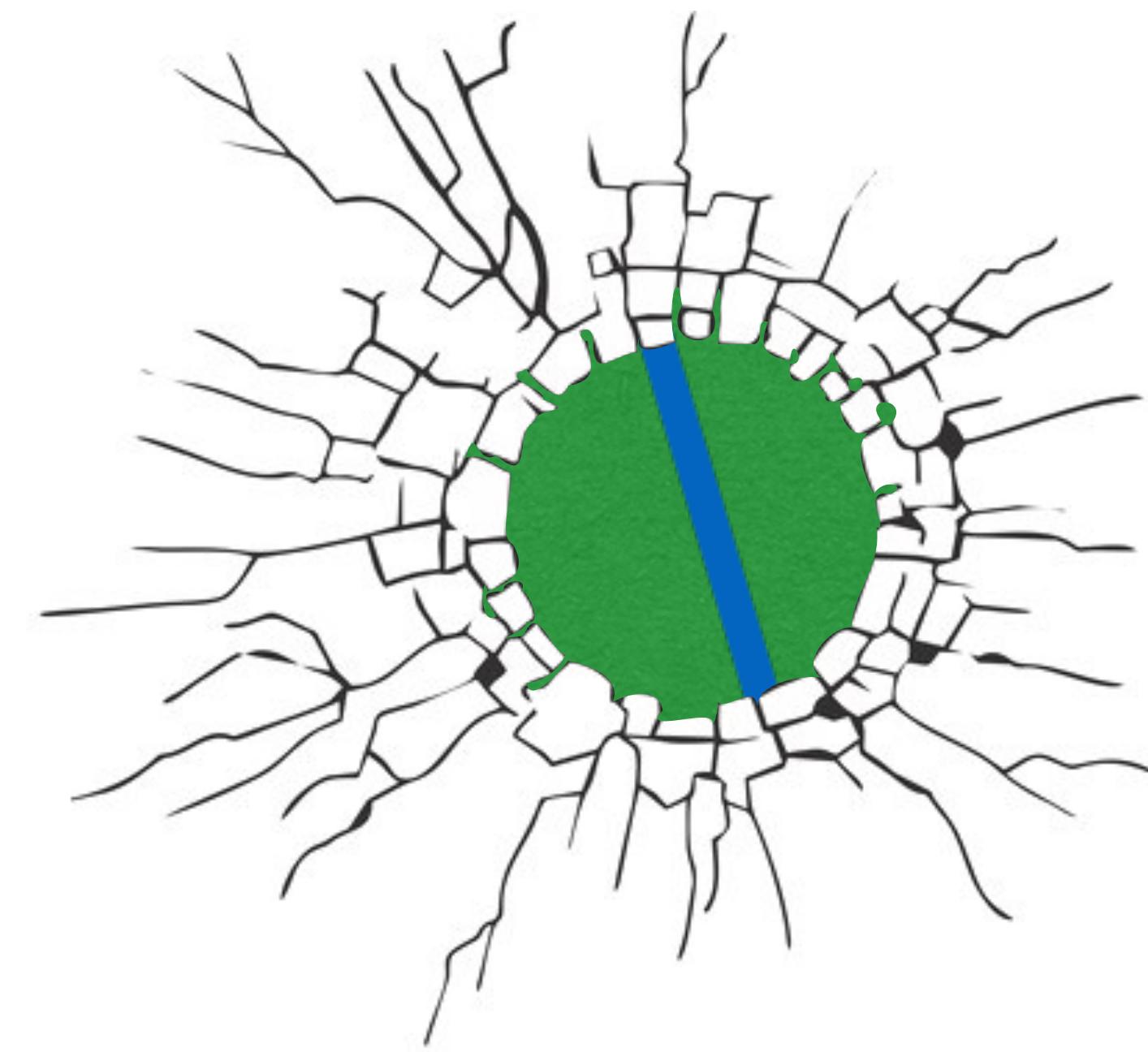
$$I_x u + I_y v + I_t = 0$$

# Aperture Problem



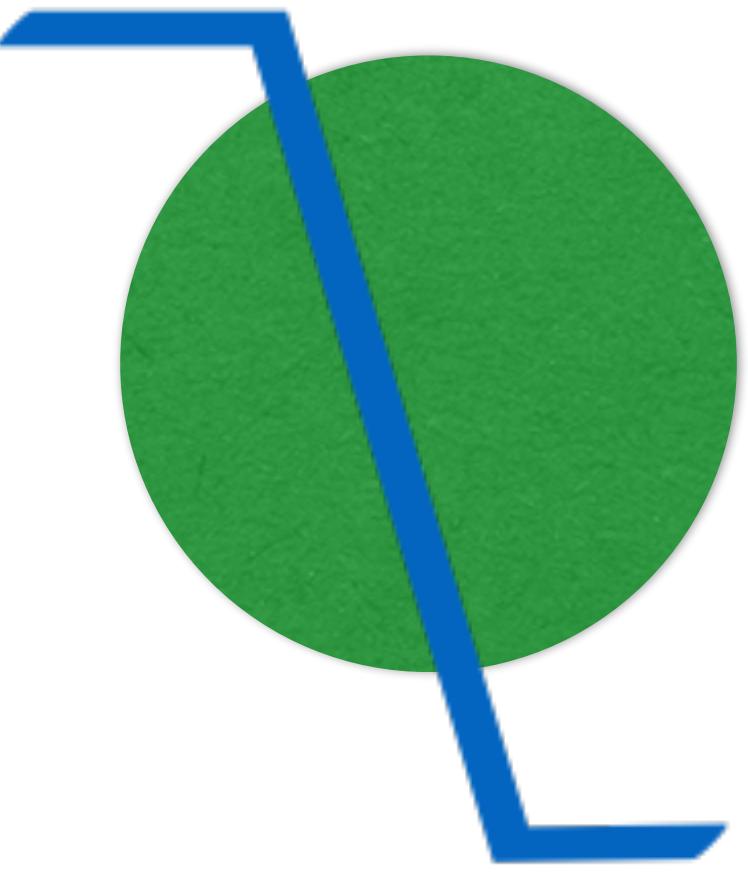
In which direction is the line moving?

# Aperture Problem

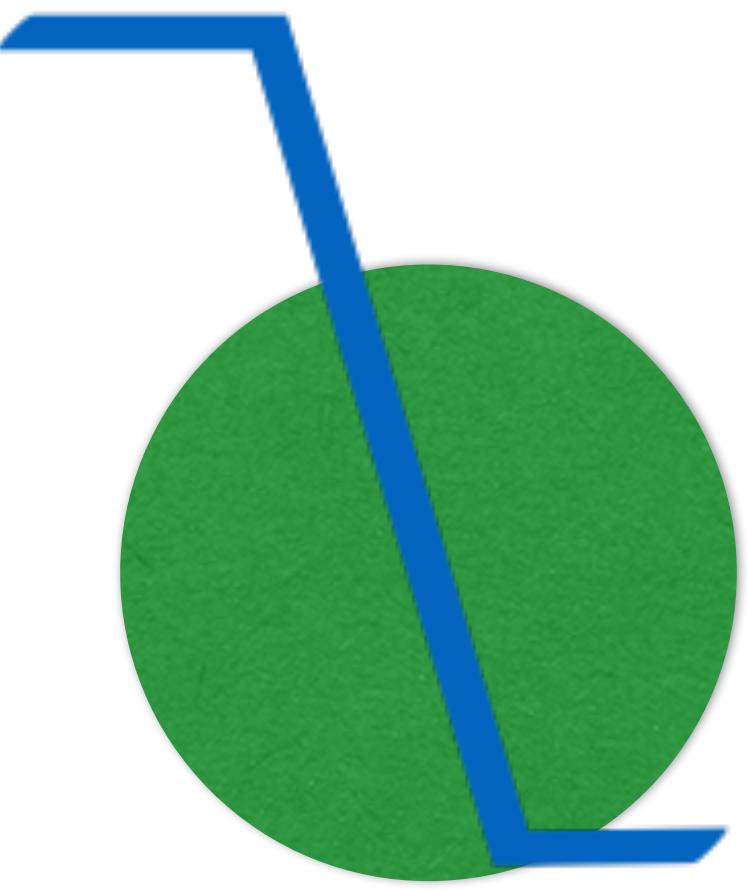


In which direction is the line moving?

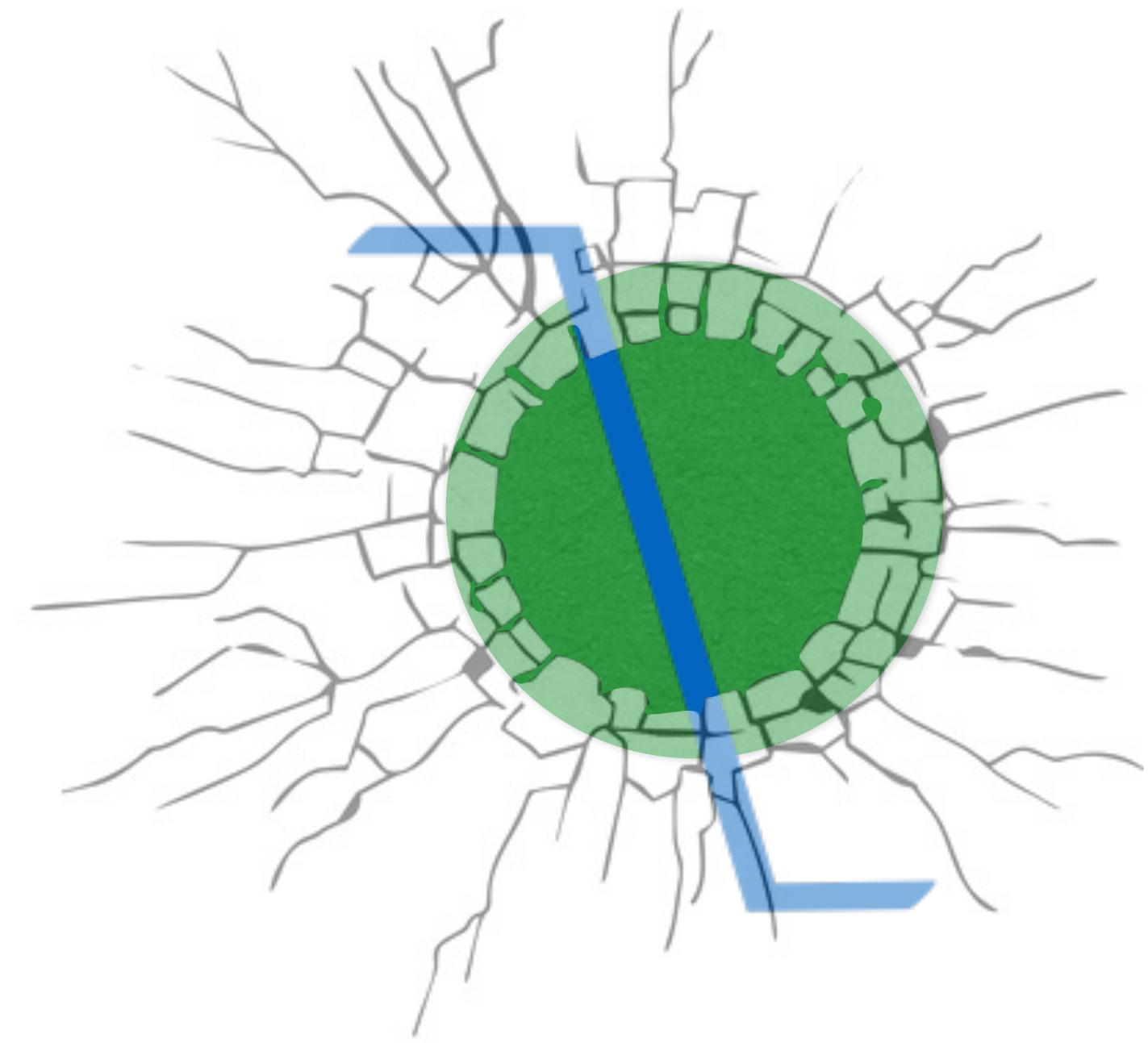
# Aperture Problem



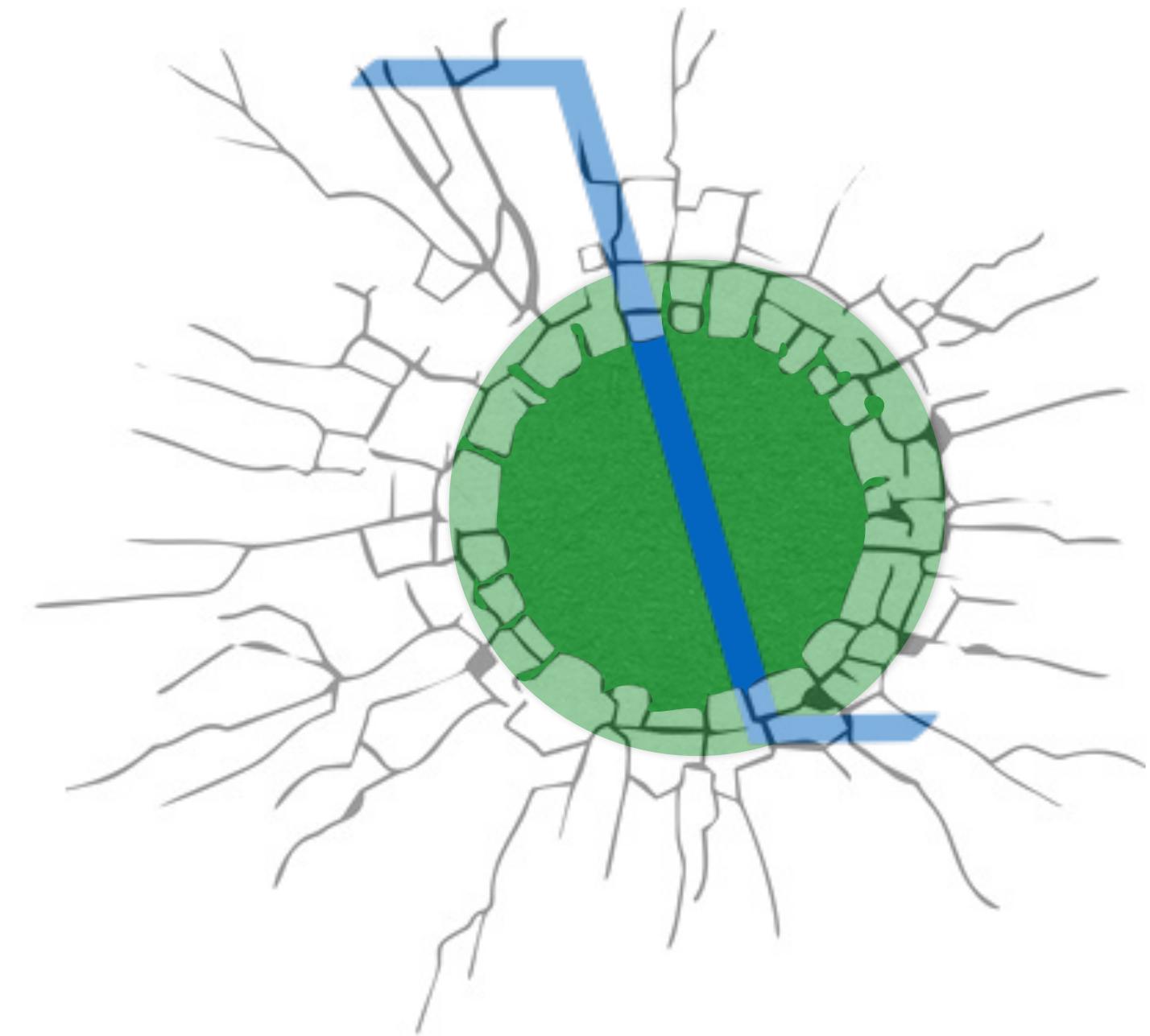
# Aperture Problem



# Aperture Problem



# Aperture Problem



# Optical Flow Algorithms

**Flow Ambiguity** component of velocity parallel to an edge is not defined by the above constraint → aperture problem

**Lucas Kanade** resolves the ambiguity by adding multiple pixels (each with 1D optical flow constraint) → linear system to solve for 2D flow

**Other flow algorithms** (e.g., Horn Schunck) use **priors over the 2D flow field** (e.g., smoothness)

# Multiview + Sparse SFM

- Multiview Image Alignment, Residuals, Error Function
- Structure from Motion (SFM)
- Bundle Adjustment, Pose Estimation, Triangulation

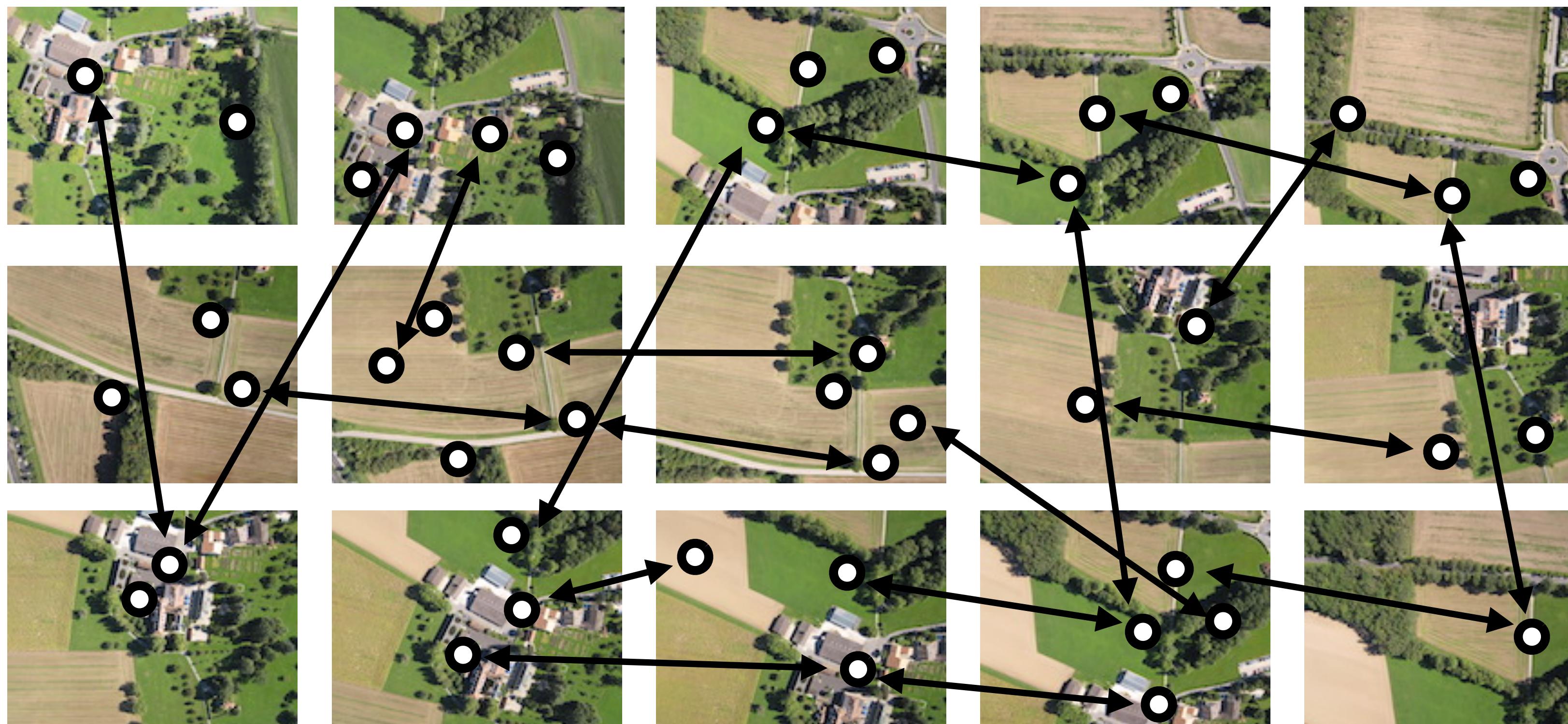
# Multiview Image Alignment

- Align a set of images given a motion model (e.g., planar affine)



# Multiview Image Alignment

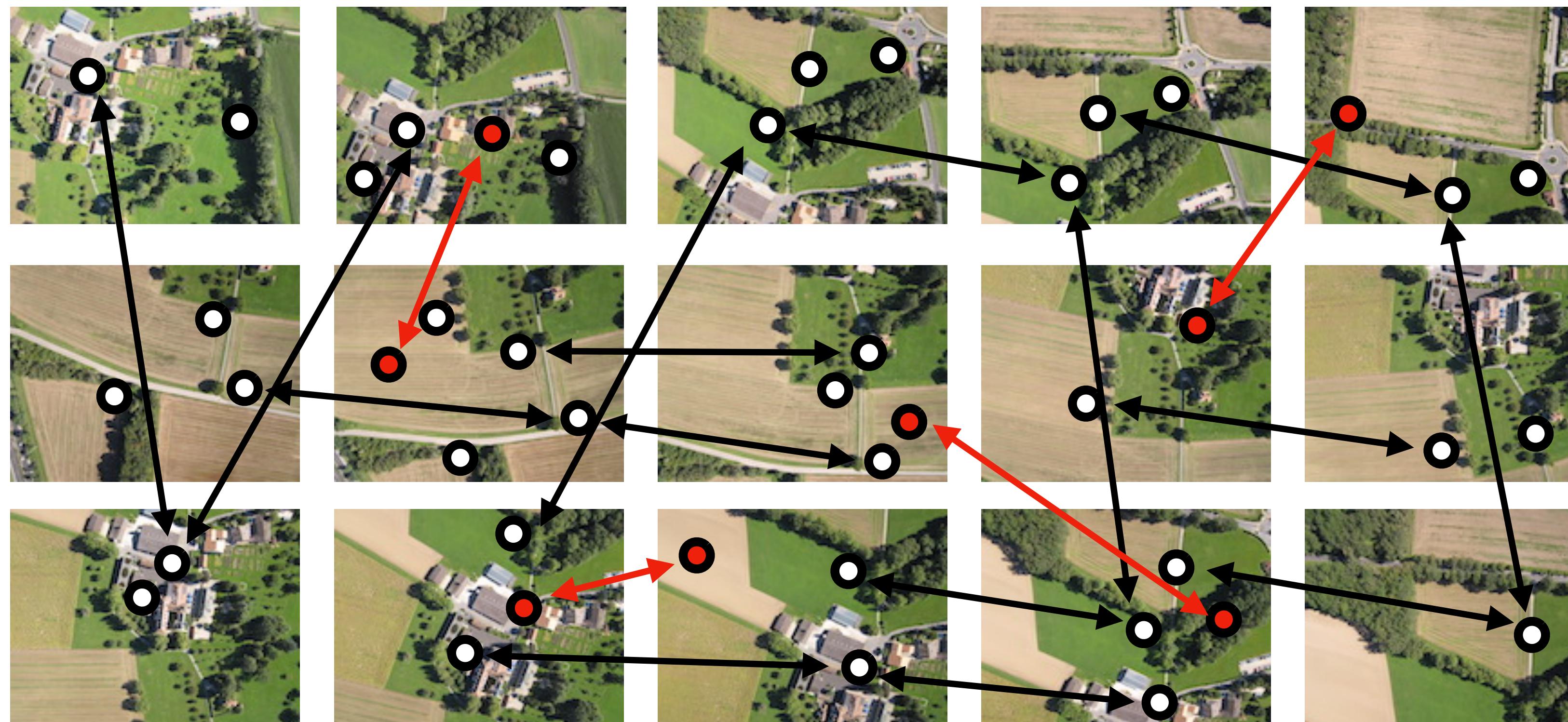
- Align a set of images given a motion model (e.g., planar affine)



Step I: Find all matches between images using SIFT

# Multiview Image Alignment

- Align a set of images given a motion model (e.g., planar affine)

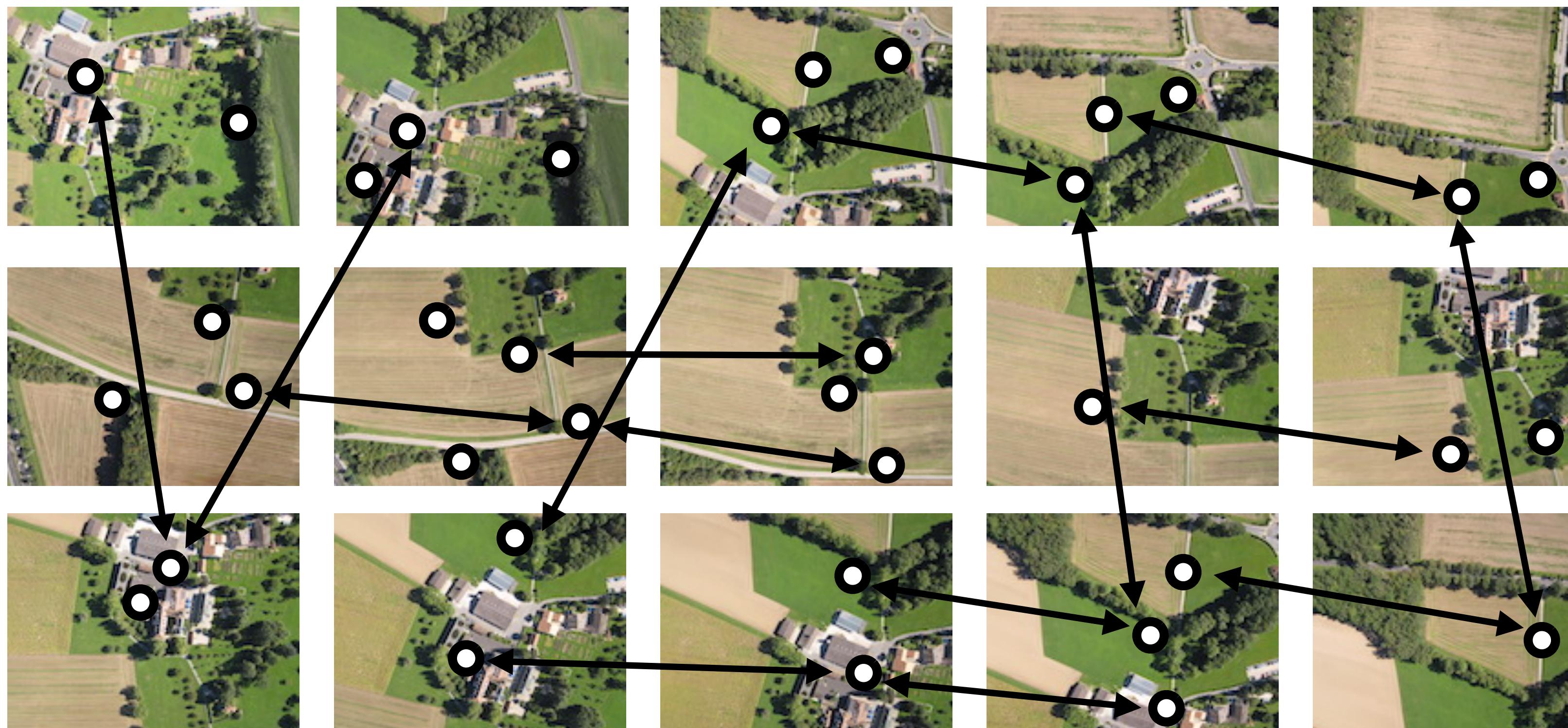


Step 1: Find all matches between images using SIFT

Step 2: Remove incorrect matches using RANSAC

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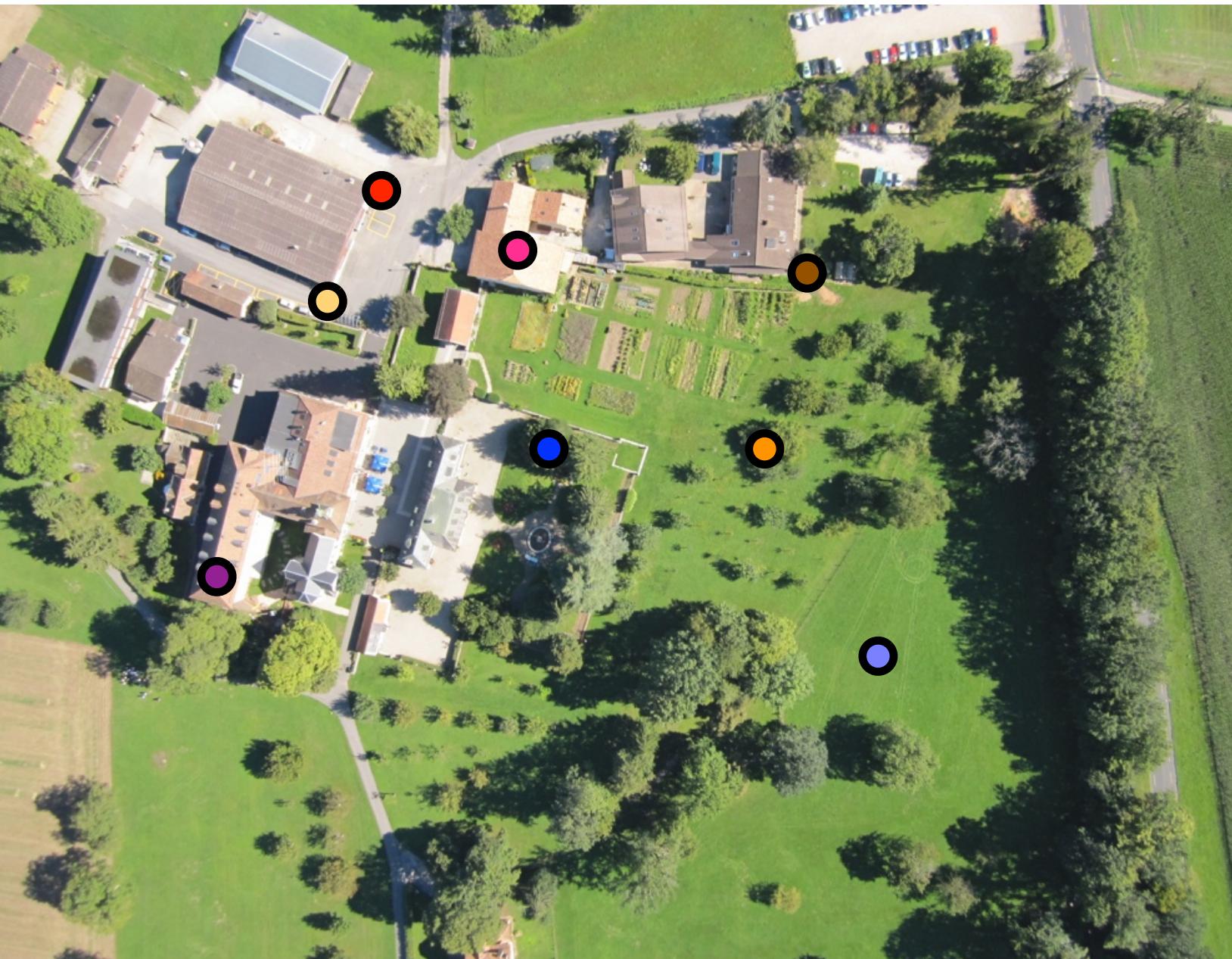
# RANSAC recap

- RANSAC solution for Similarity Transform (2 points)



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4 inliers (red, yellow, orange, brown),  
4 outliers (blue, light blue, purple, pink)

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# RANSAC recap

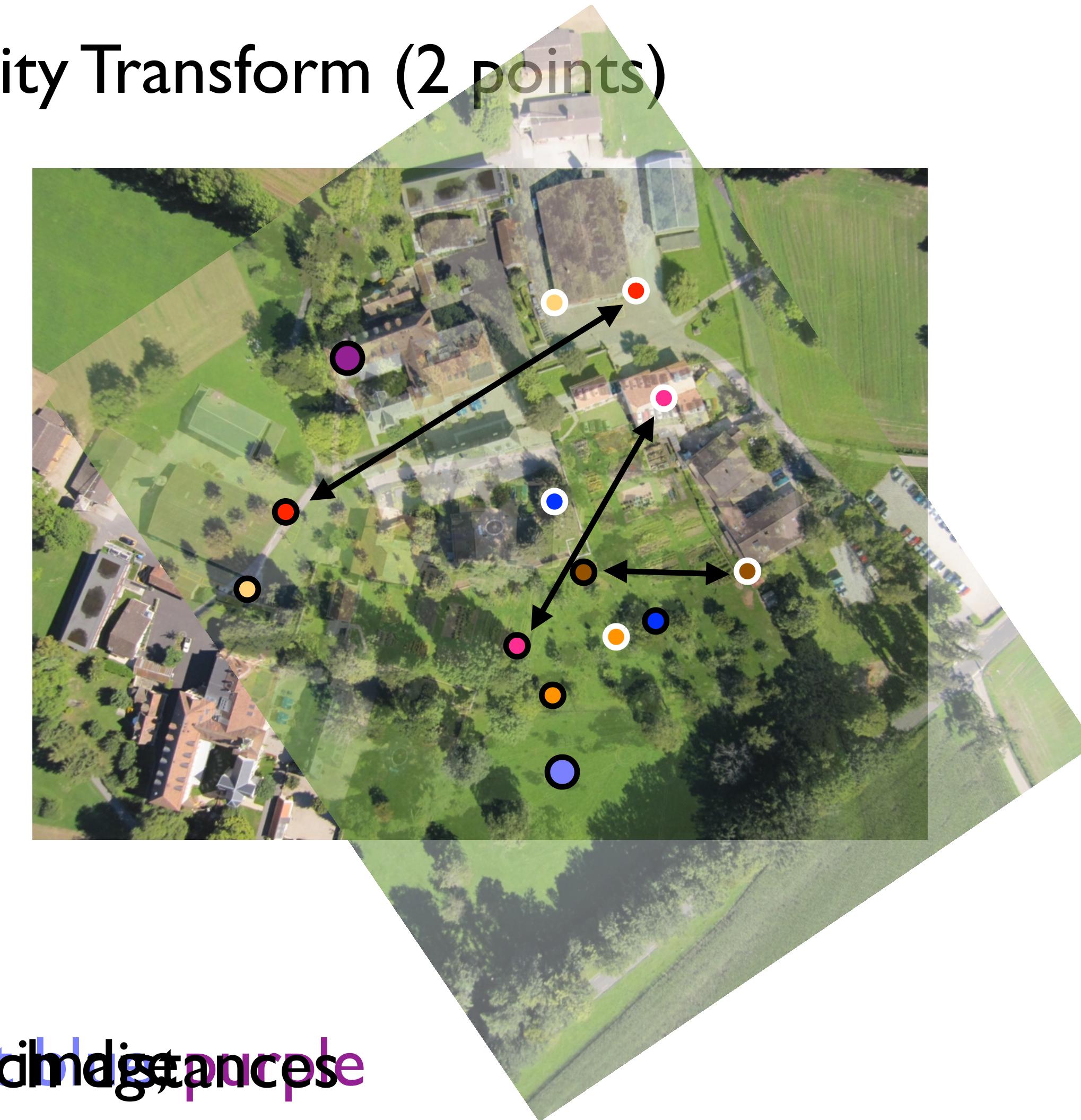
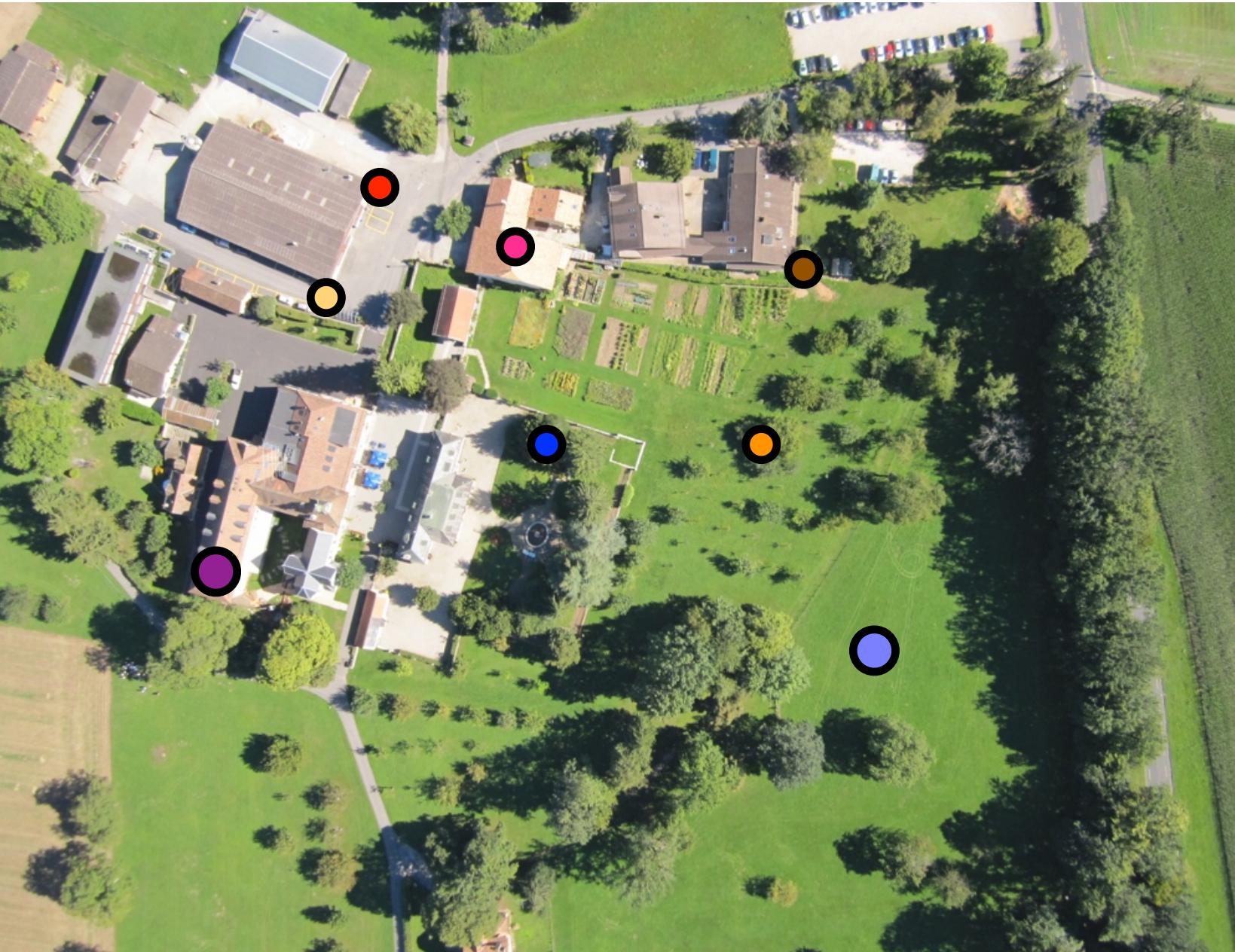
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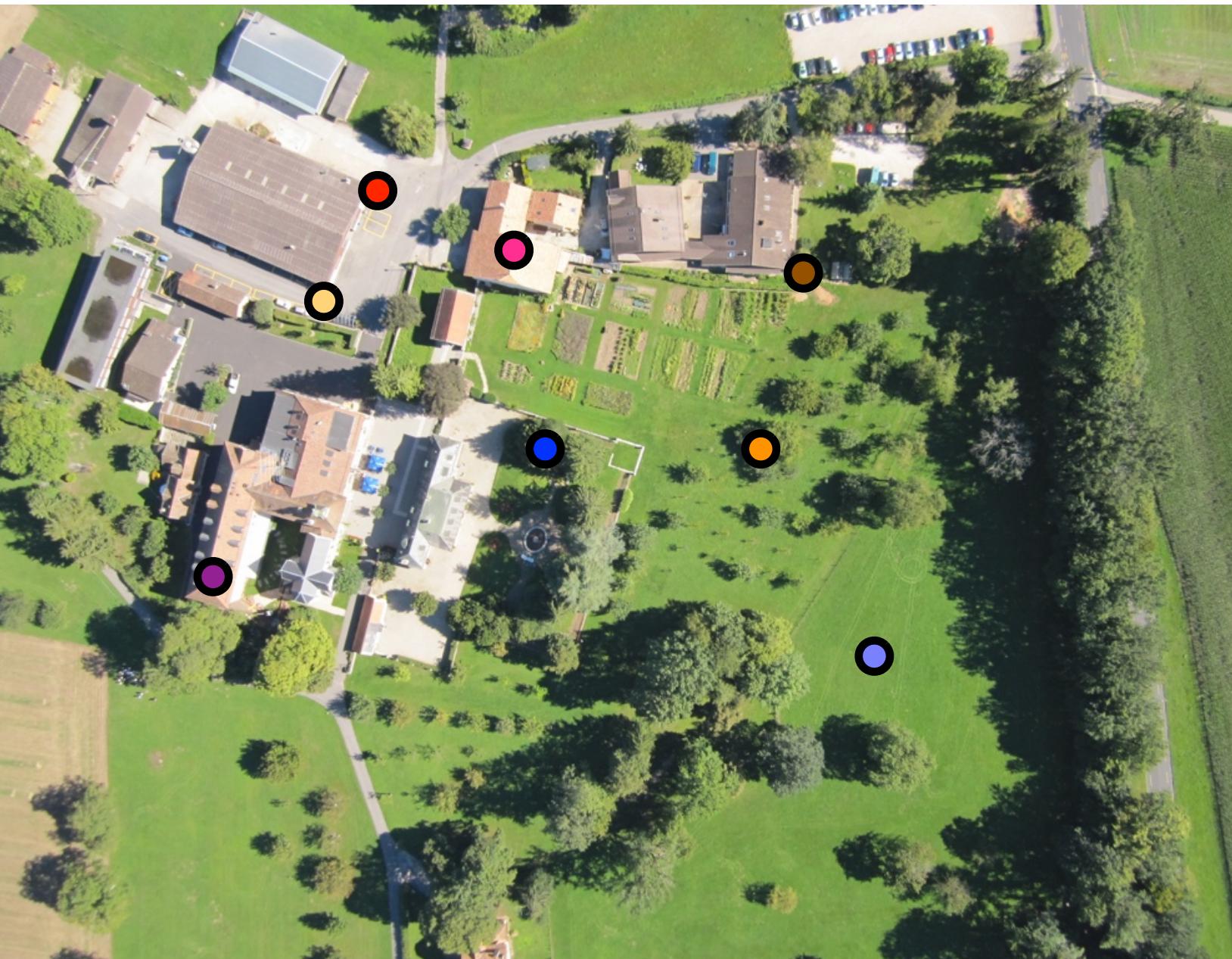


chbeispielhaftigesprinzip

#inliers = 2

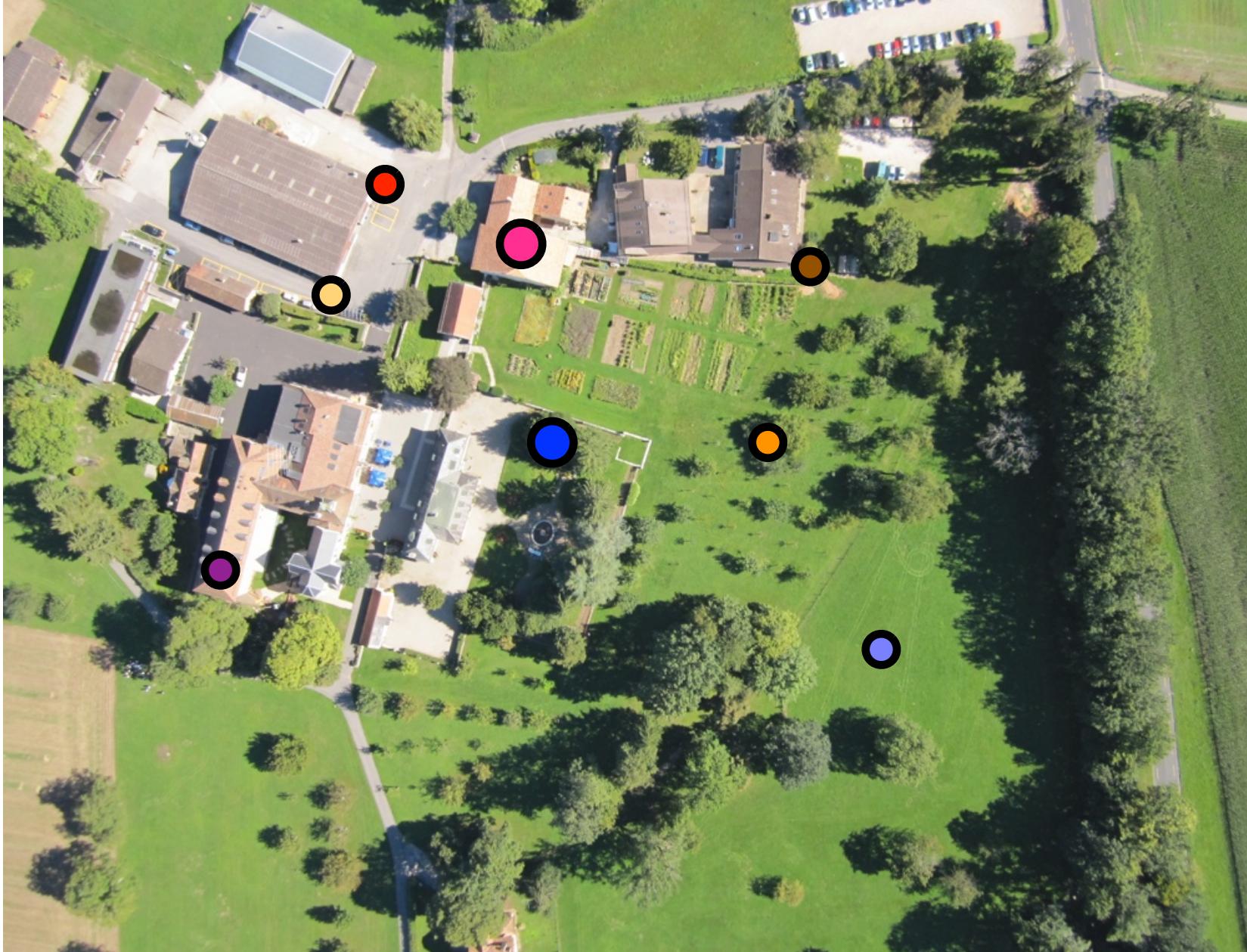
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chebkoosappinkblue

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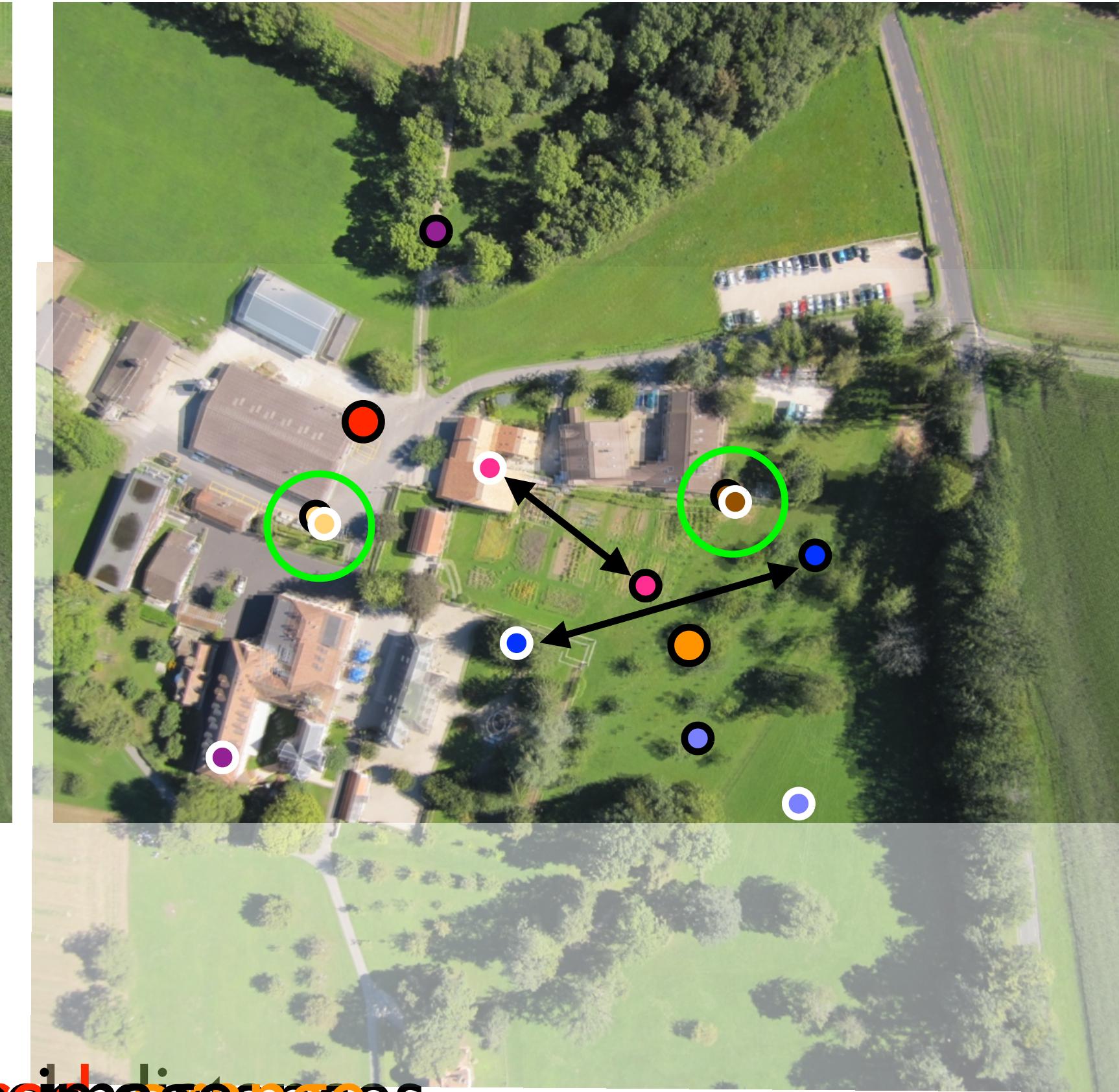
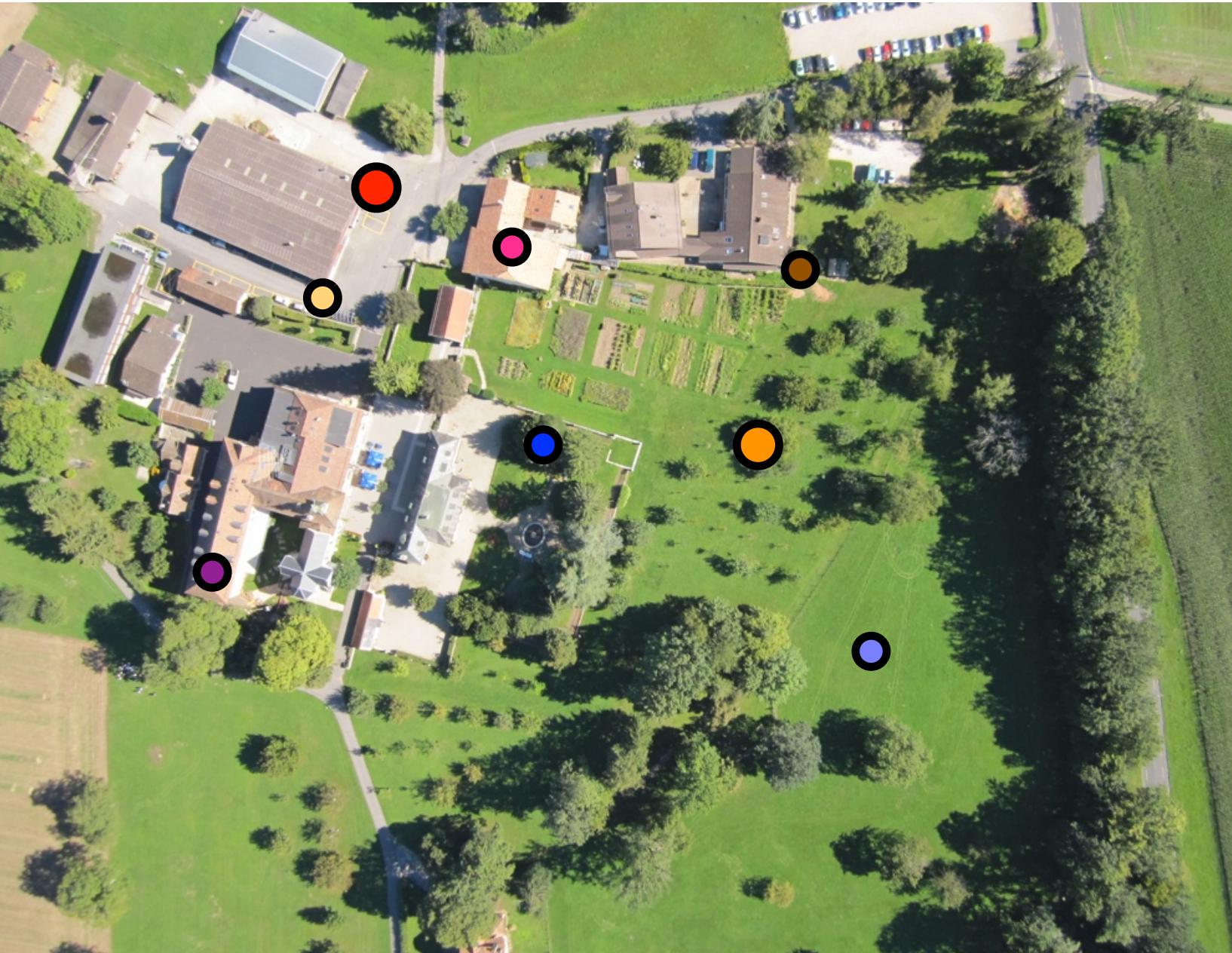
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check sample in images

#inliers = 4

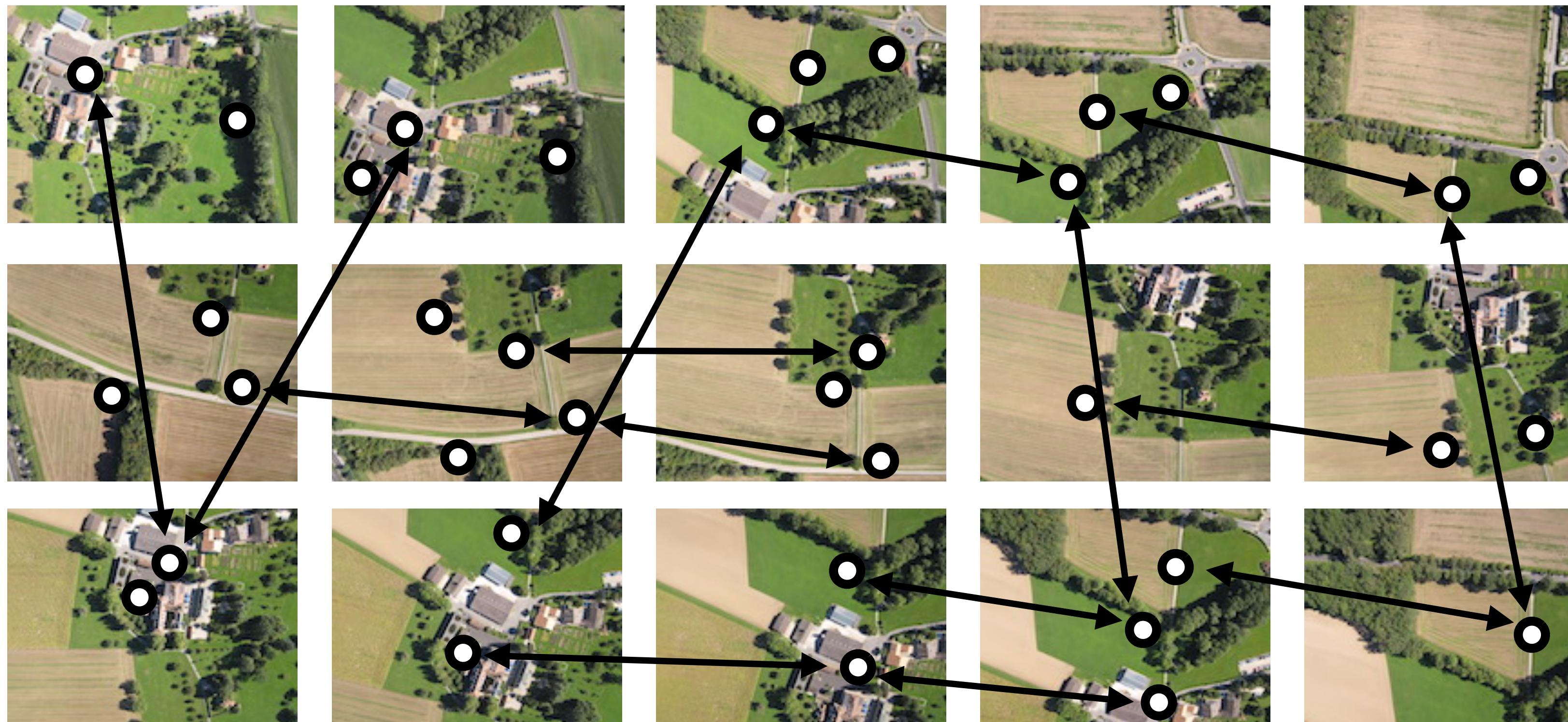
# RANSAC recap

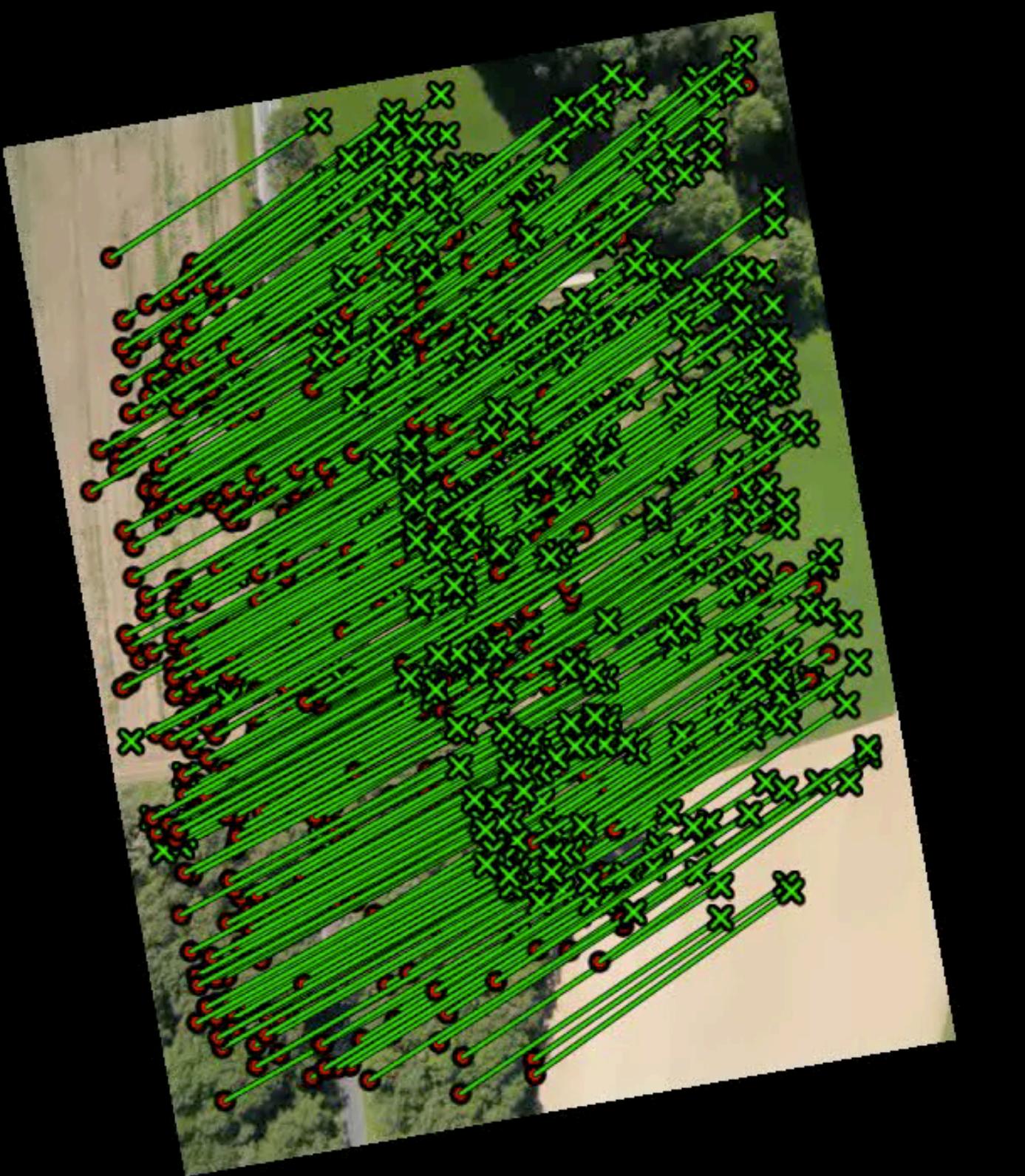
- RANSAC solution for Similarity Transform (2 points)



# Planar Image Alignment

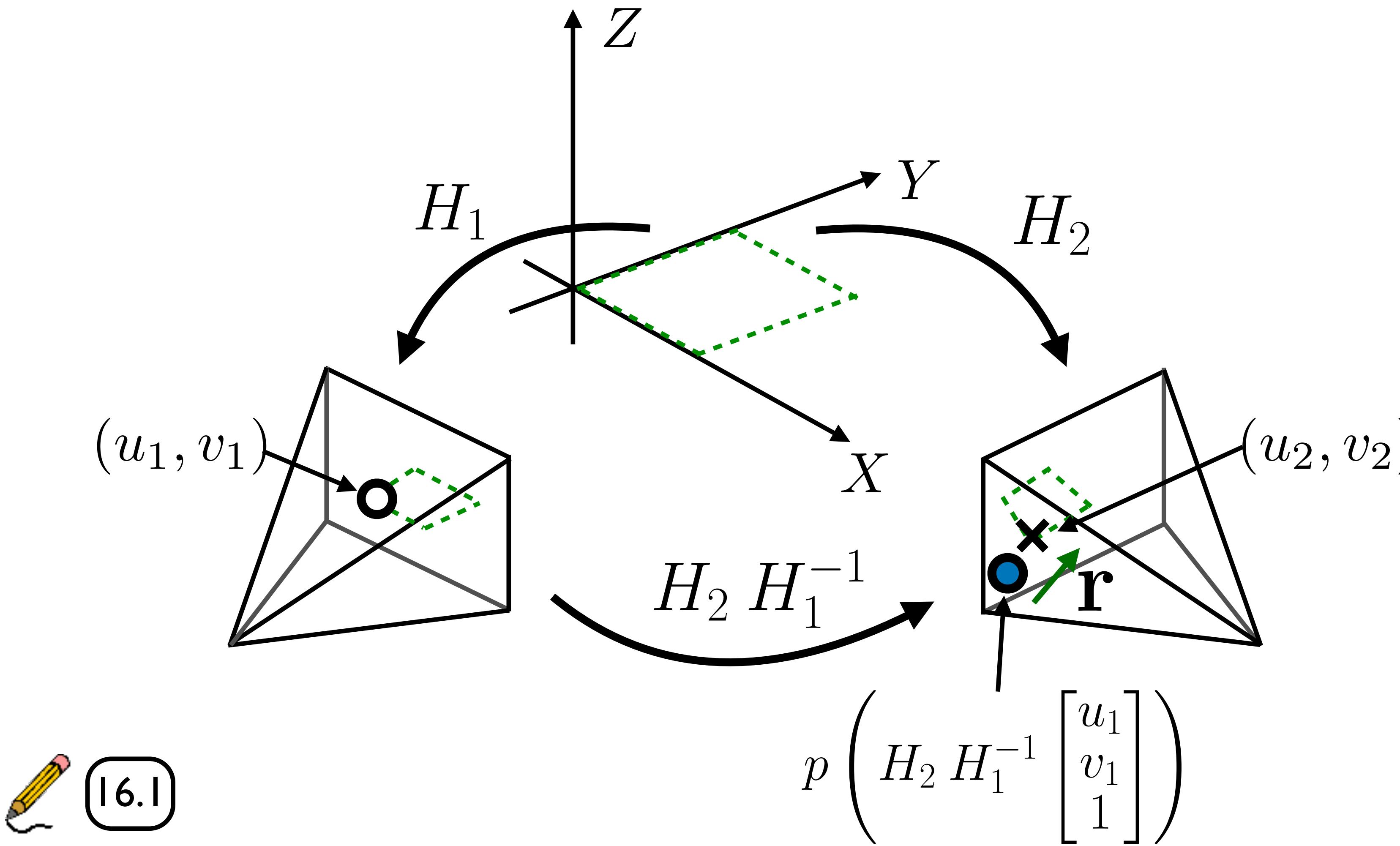
- Given a clean set of correspondences, align all images



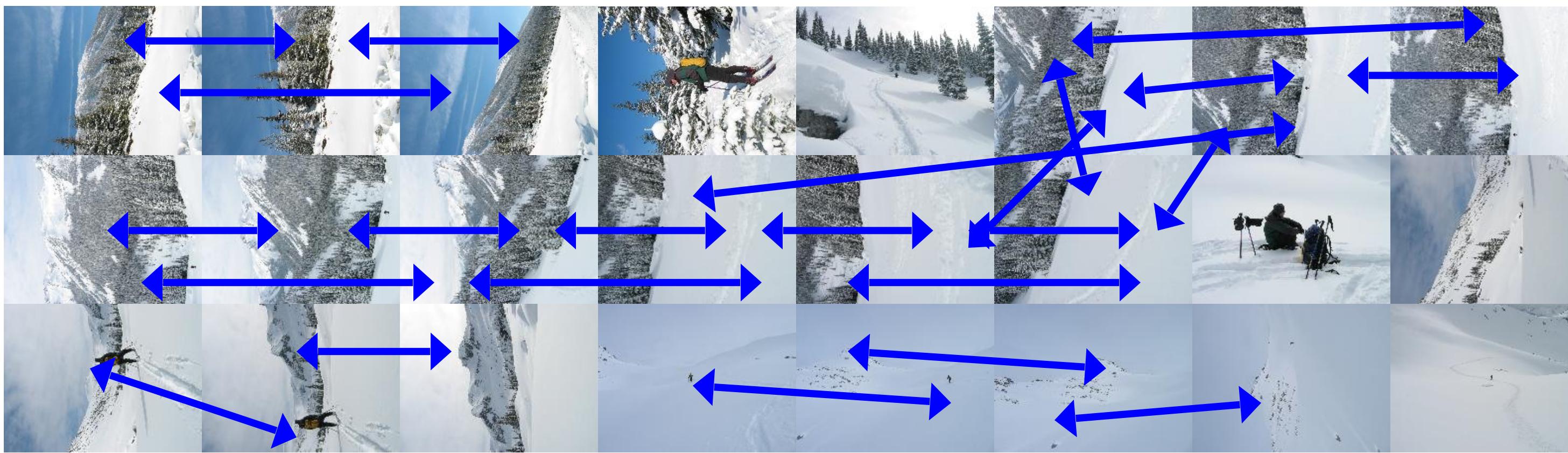


# Multiview Image Alignment

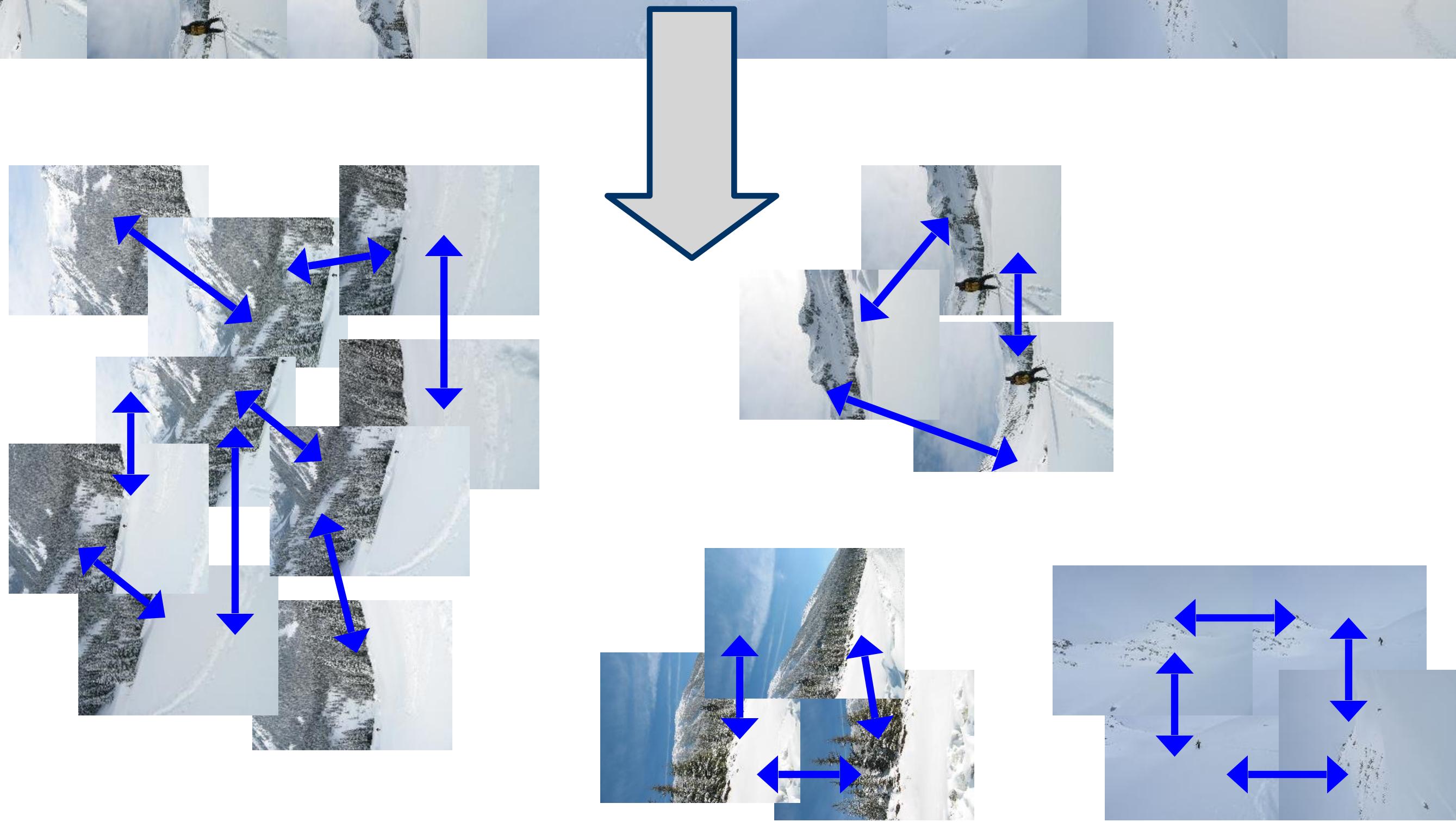
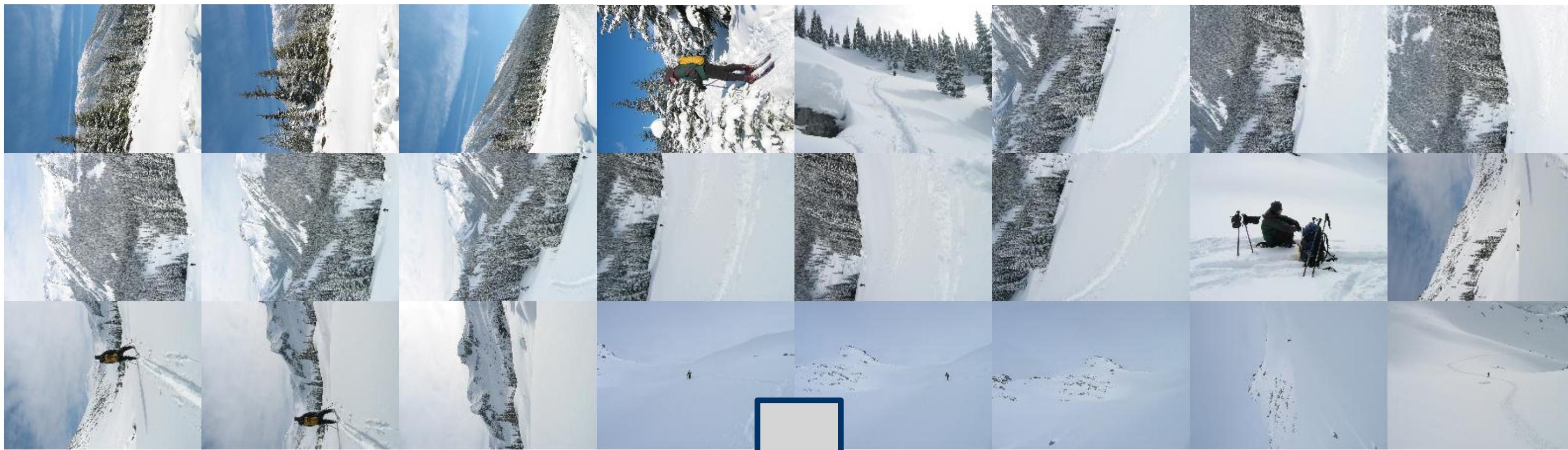
- Residual = vector between observed feature and projection



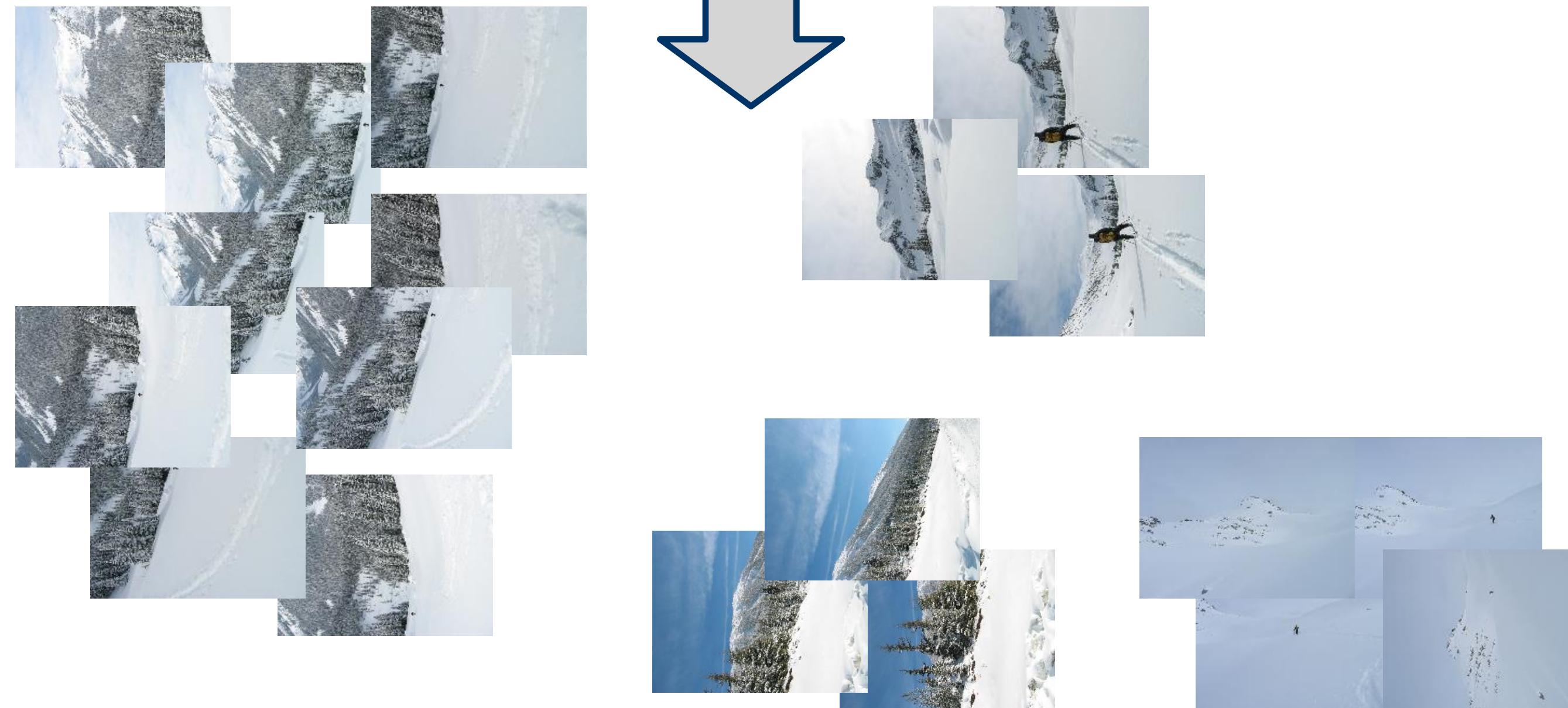
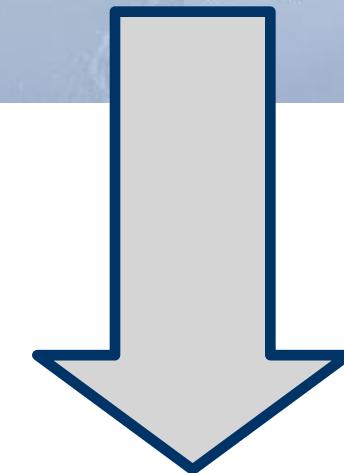
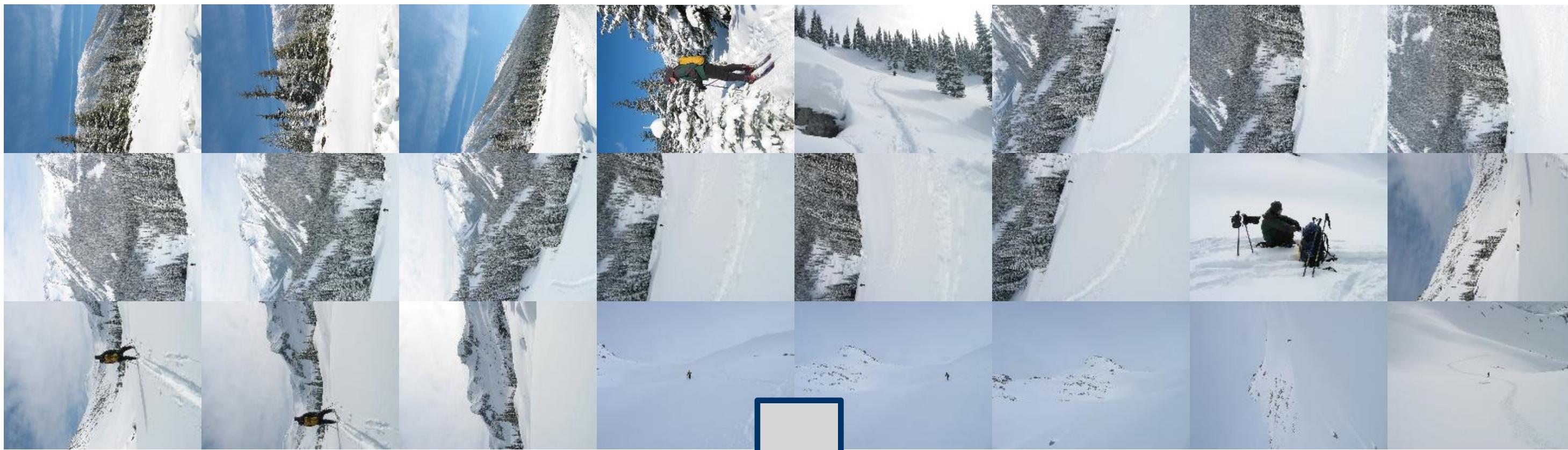
# Panorama Recognition



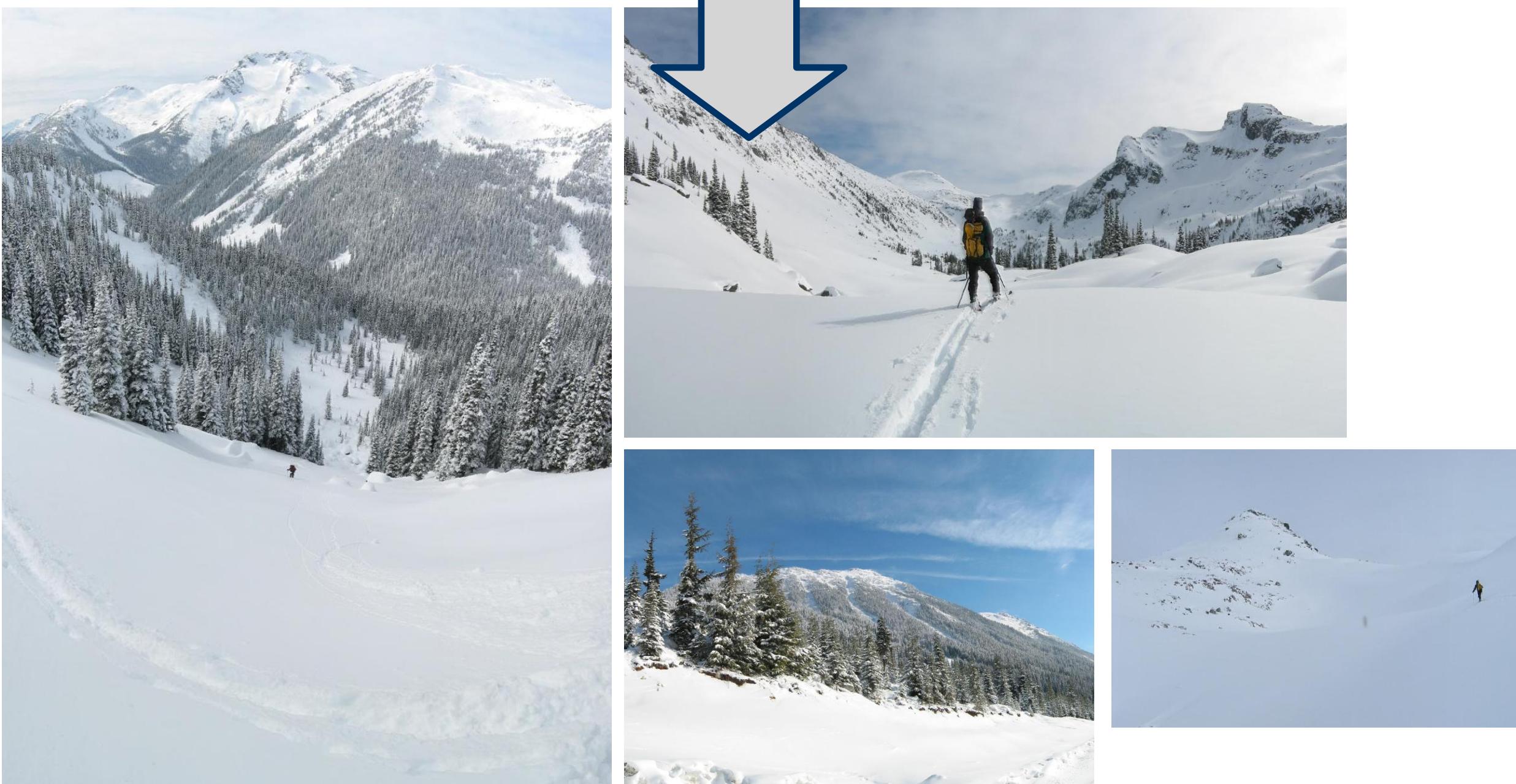
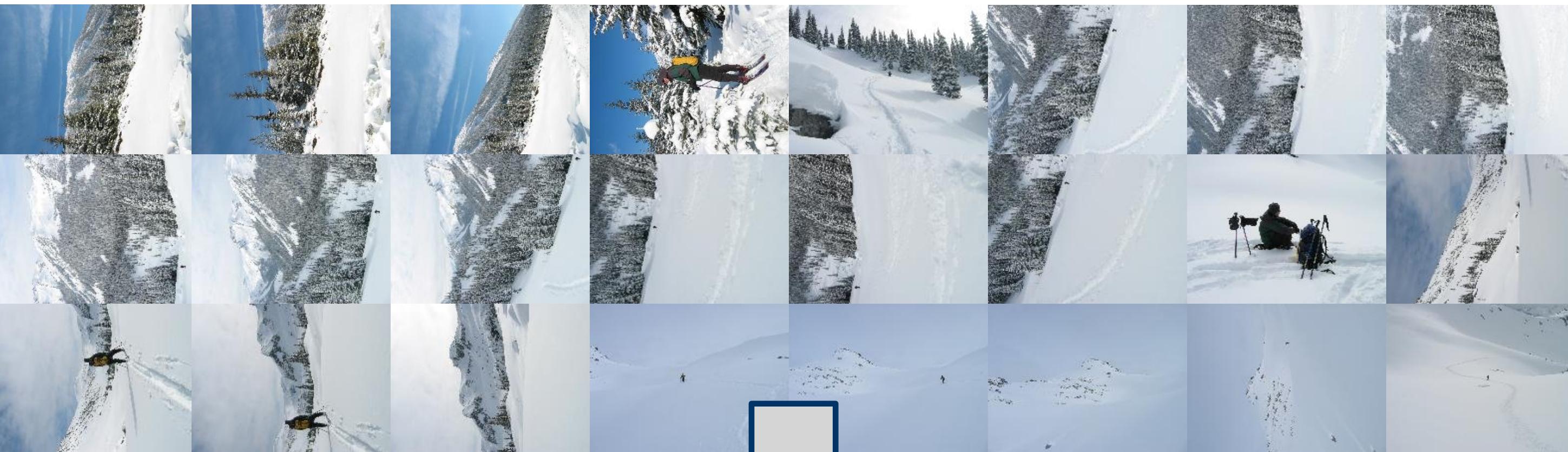
# Panorama Recognition



# Panorama Recognition



# Panorama Recognition





# Panorama Stitching

- We can concatenate pairwise homographies, but over time multiple pairwise mappings accumulate errors
- We use global alignment (bundle adjustment) to close the gap

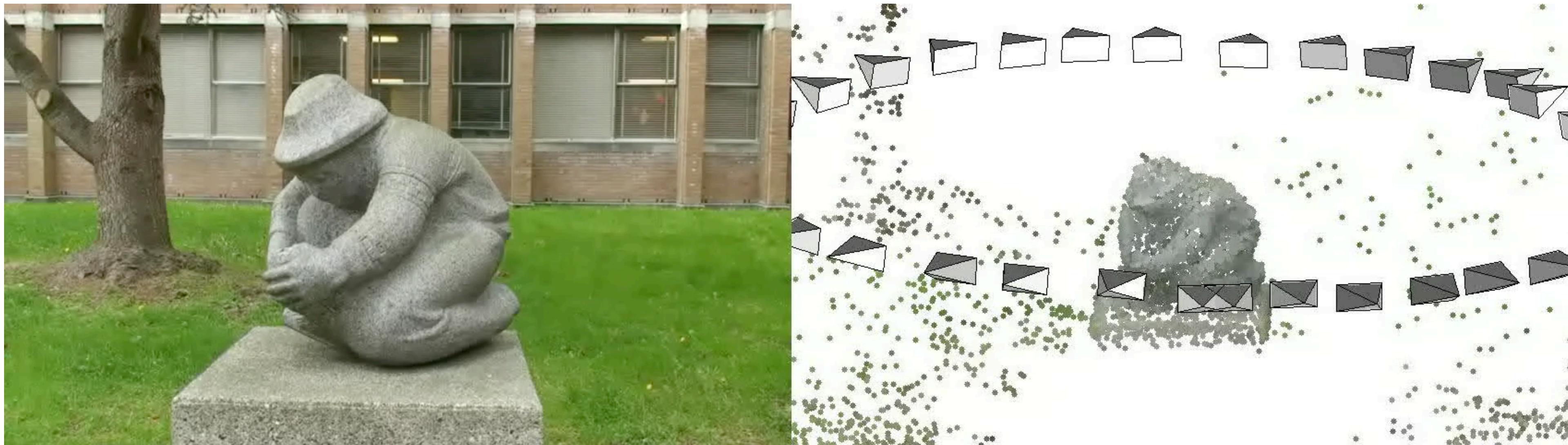


# Structure from Motion



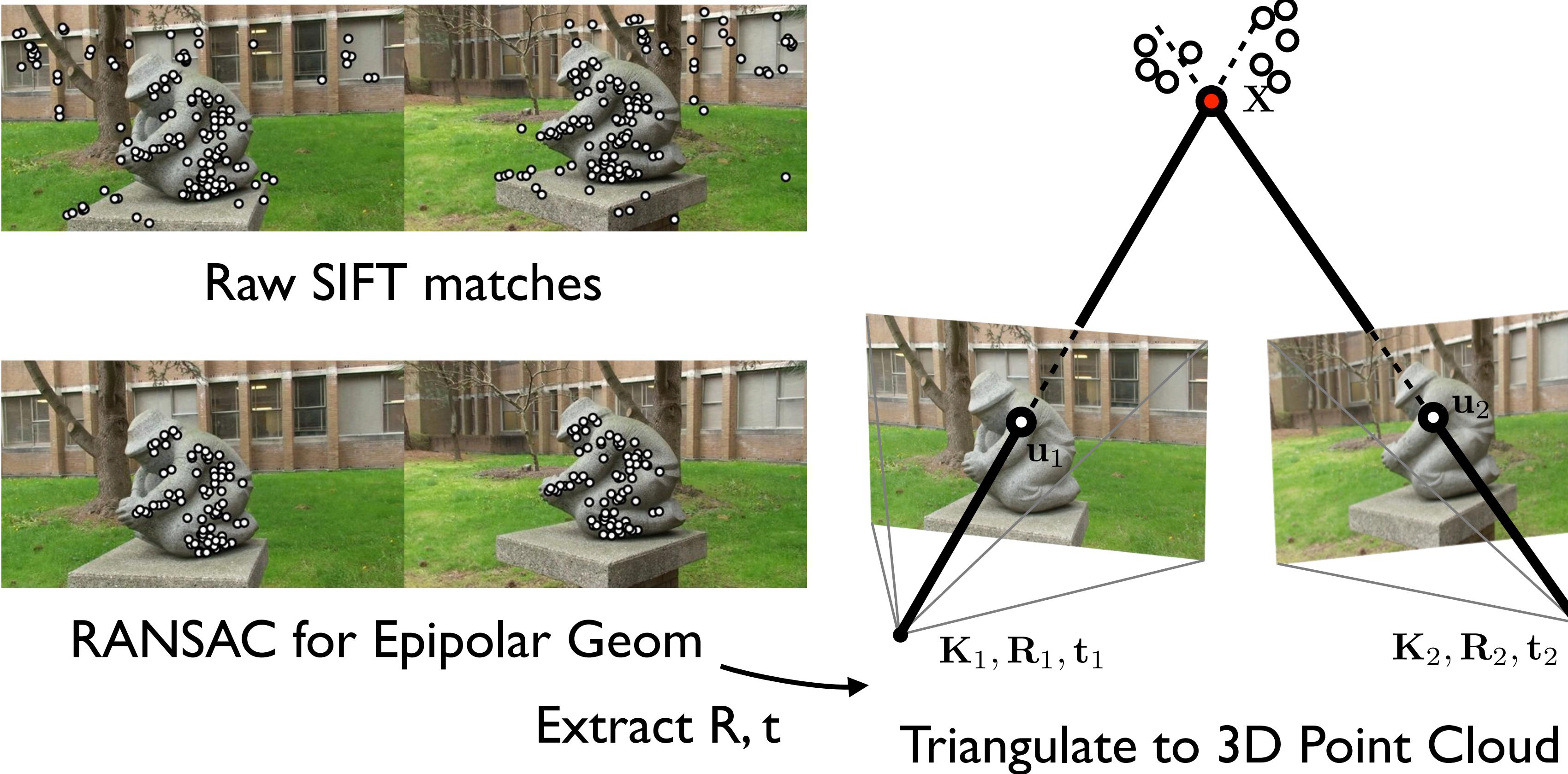
Given an (unordered) set of input images, compute cameras and 3D structure of the scene

# Structure from Motion



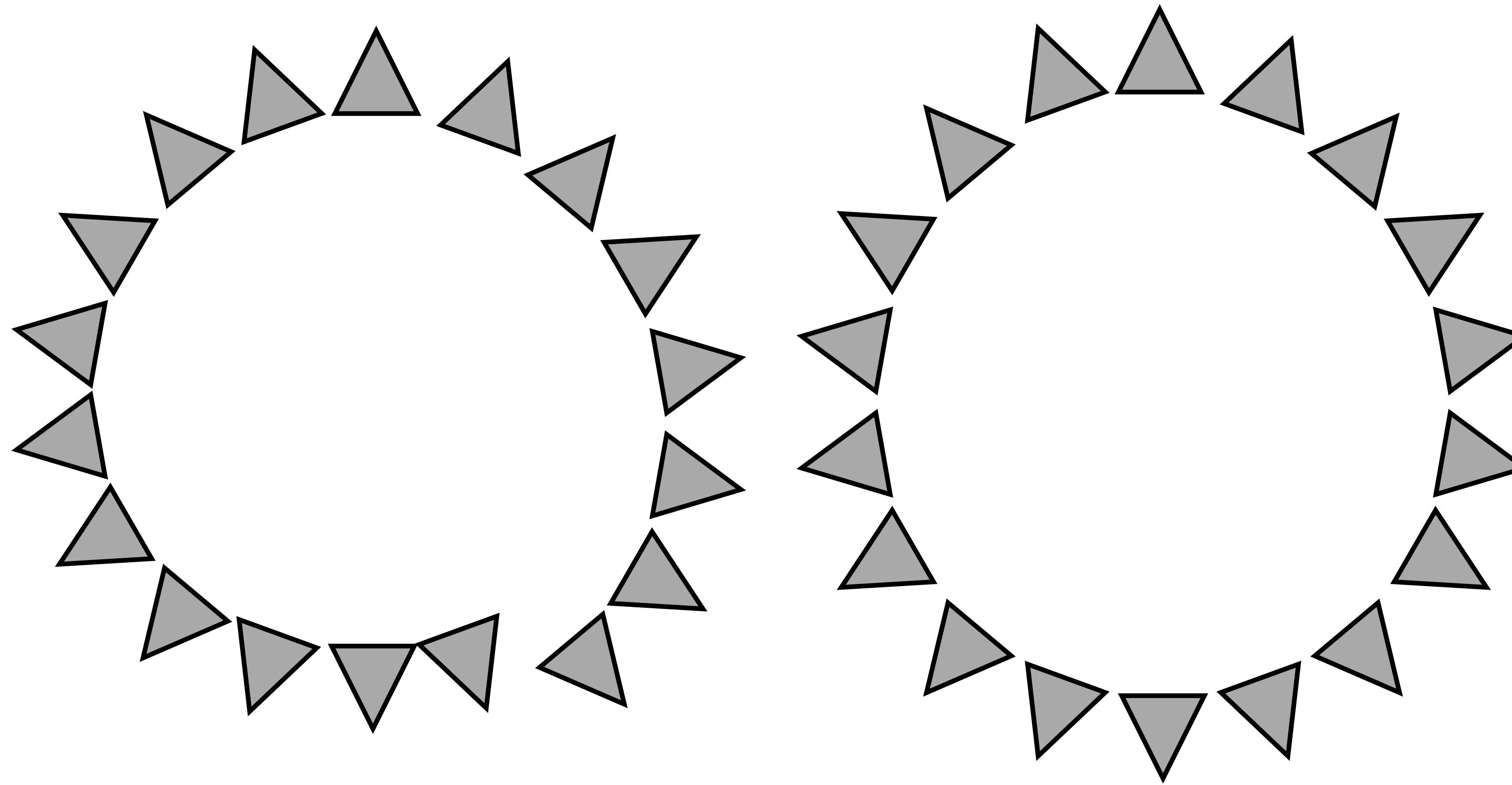
# 2-view Structure from Motion

- We can use the combination of SIFT/RANSAC and triangulation to compute 3D structure from 2 views



# Global Alignment

- Concatenation of pairwise R, t estimates results in drift, e.g.,

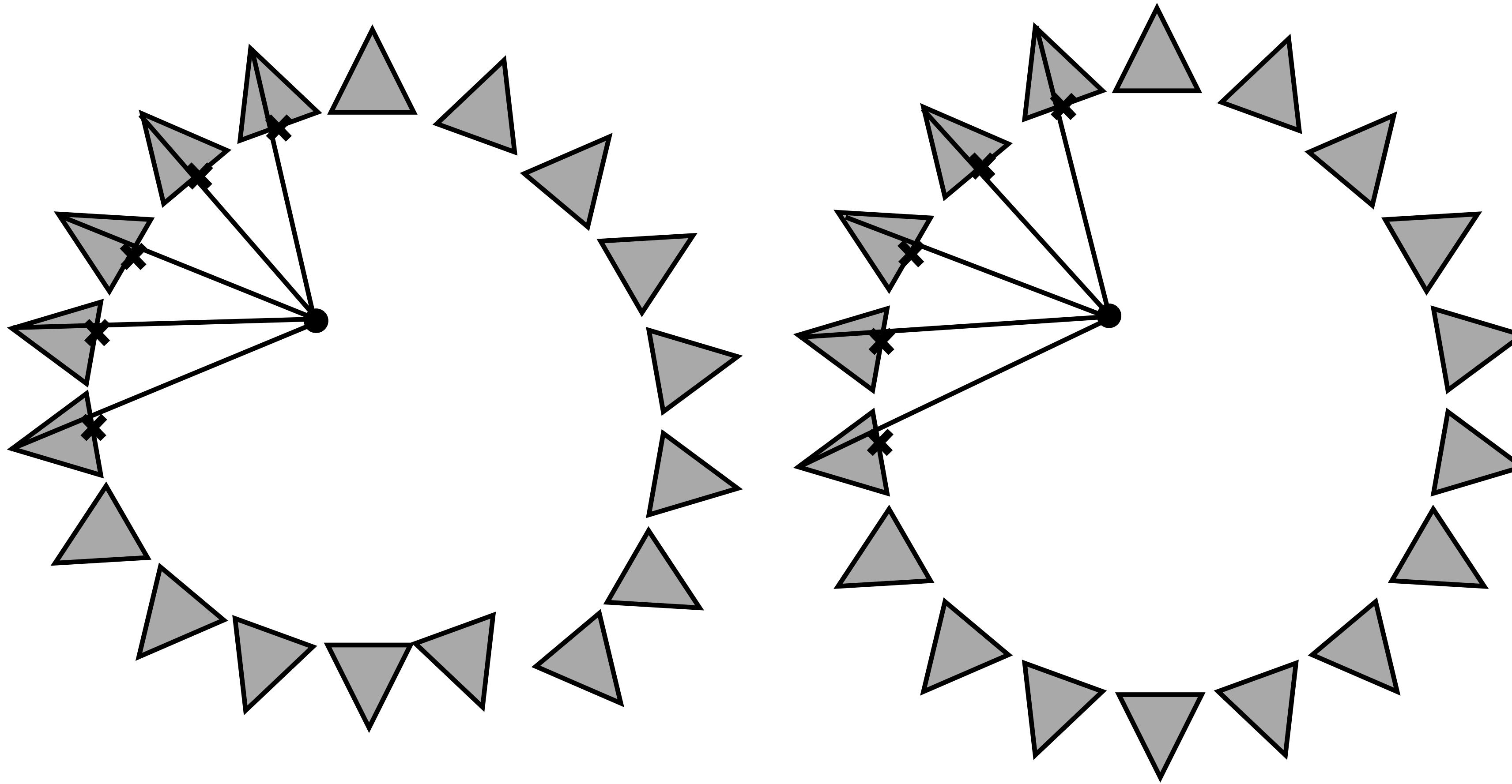


Pairwise alignment

Global alignment

# Global Alignment

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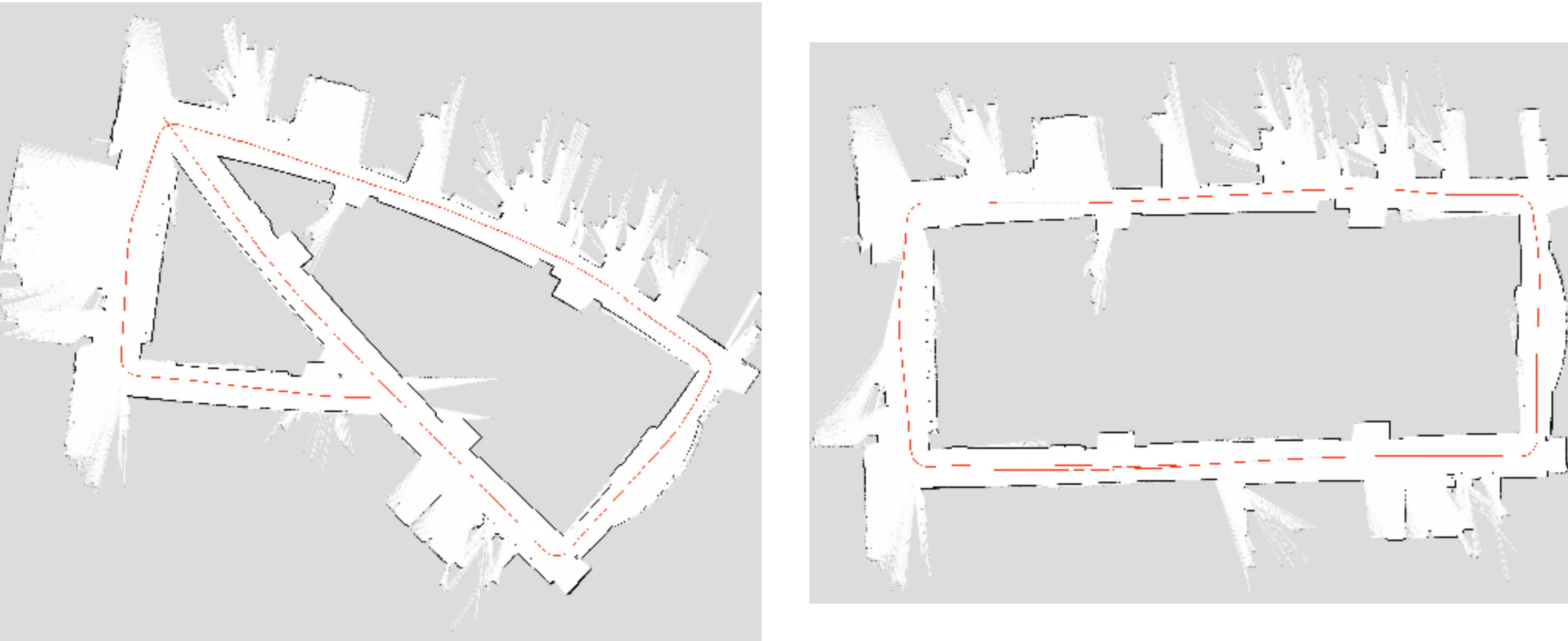


Pairwise alignment

Global alignment

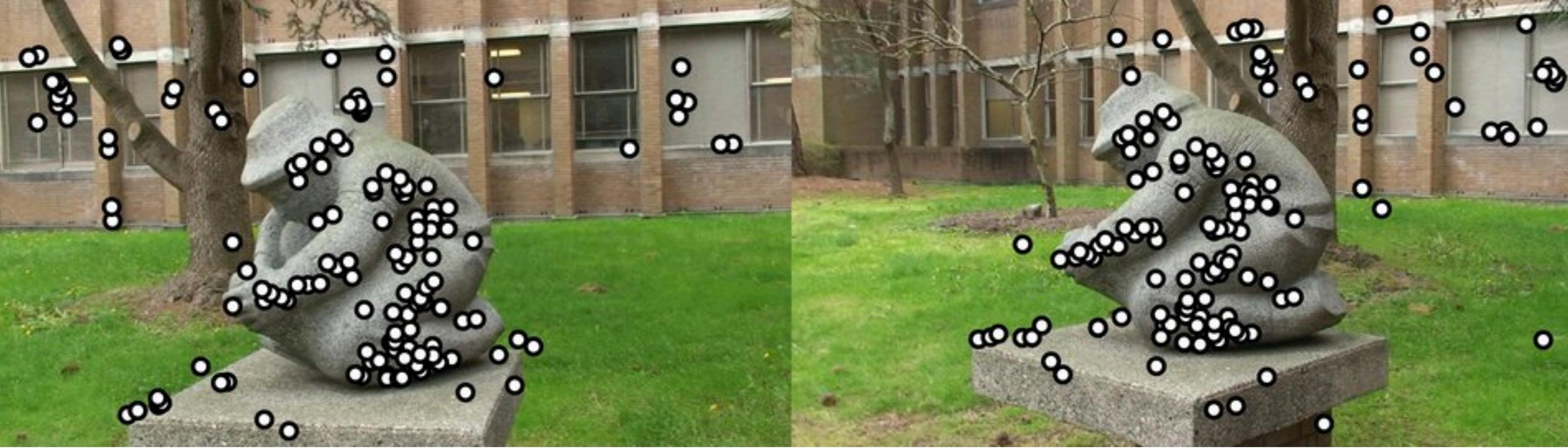
# Global Alignment

- In robotic navigation frame-frame alignment also causes drift



We can use **bundle adjustment** to close the gap

# RANSAC for 3D Matches



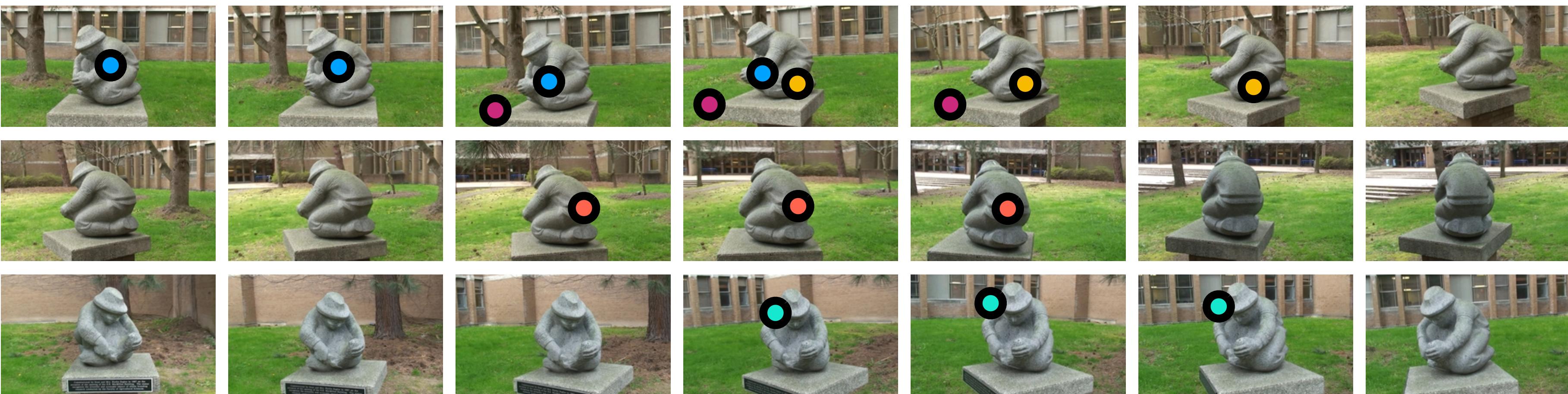
Raw feature matches (after ratio test filtering)



Solved for RANSAC inliers

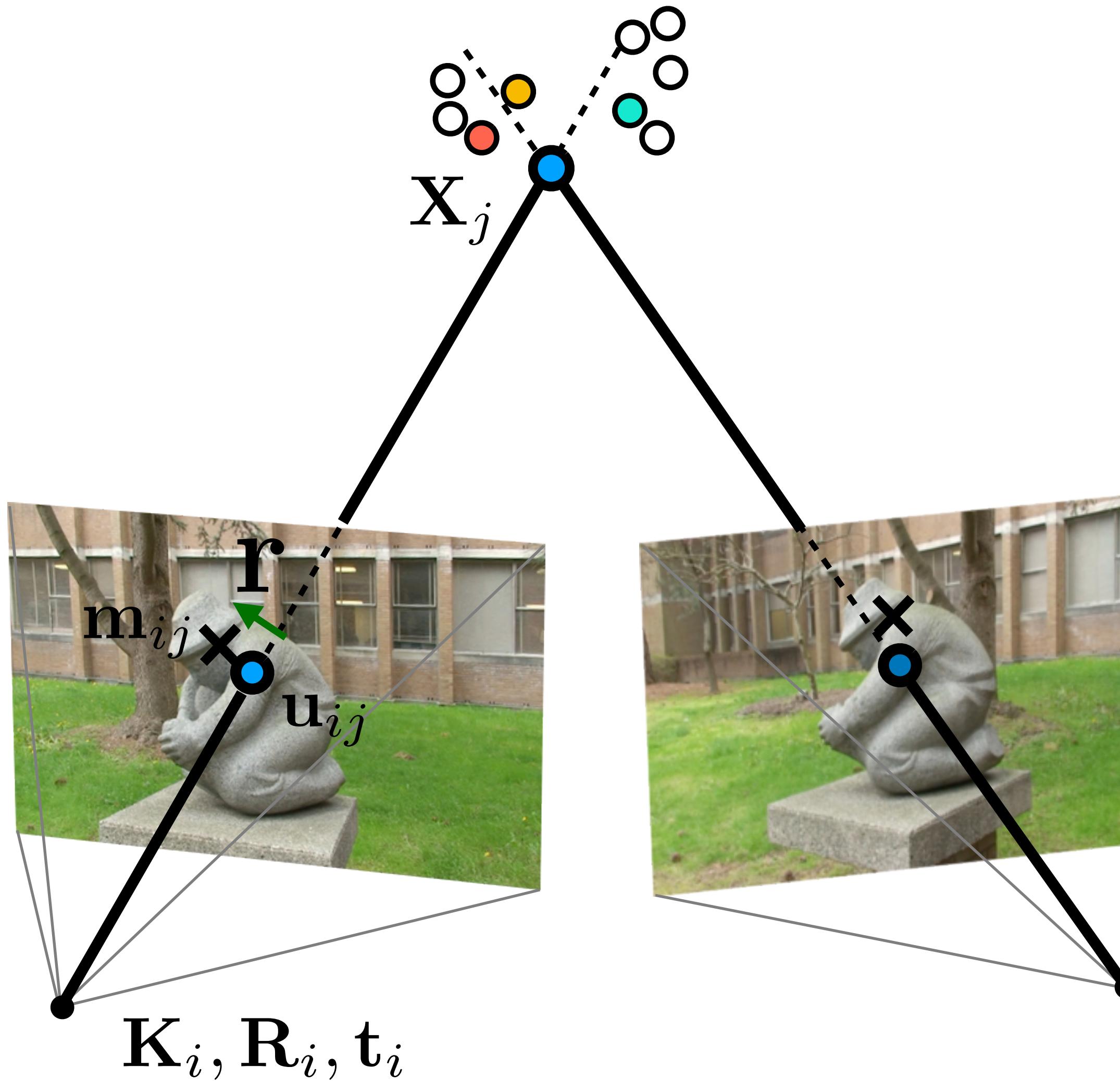
# Feature Tracking

- Form feature tracks by combining pairwise feature matches



- Tracked features become individual 3D points in the reconstruction
- Features matched across 3 or more views provide strong constraints on the 3D reconstruction

# Bundle Adjustment



- Minimise errors projecting 3D points into all images

$$e = \sum_{i \in \text{images}} \sum_{j \in \text{points}} |\mathbf{r}_{ij}(\mathbf{R}_i, \mathbf{t}_i, \mathbf{X}_j)|^2$$

# Bundle Adjustment

- Full bundle adjustment (optimise all cameras and points):

$$e = \sum_{i \in \text{images}} \sum_{j \in \text{points}} |\mathbf{r}_{ij}(\mathbf{R}_i, \mathbf{t}_i, \mathbf{X}_j)|^2$$

- Triangulation (optimise points, fixed cameras):

$$e = \sum_{i \in \text{images}} \sum_{j \in \text{points}} |\mathbf{r}_{ij}(\mathbf{R}_i, \mathbf{t}_i, \mathbf{X}_j)|^2$$

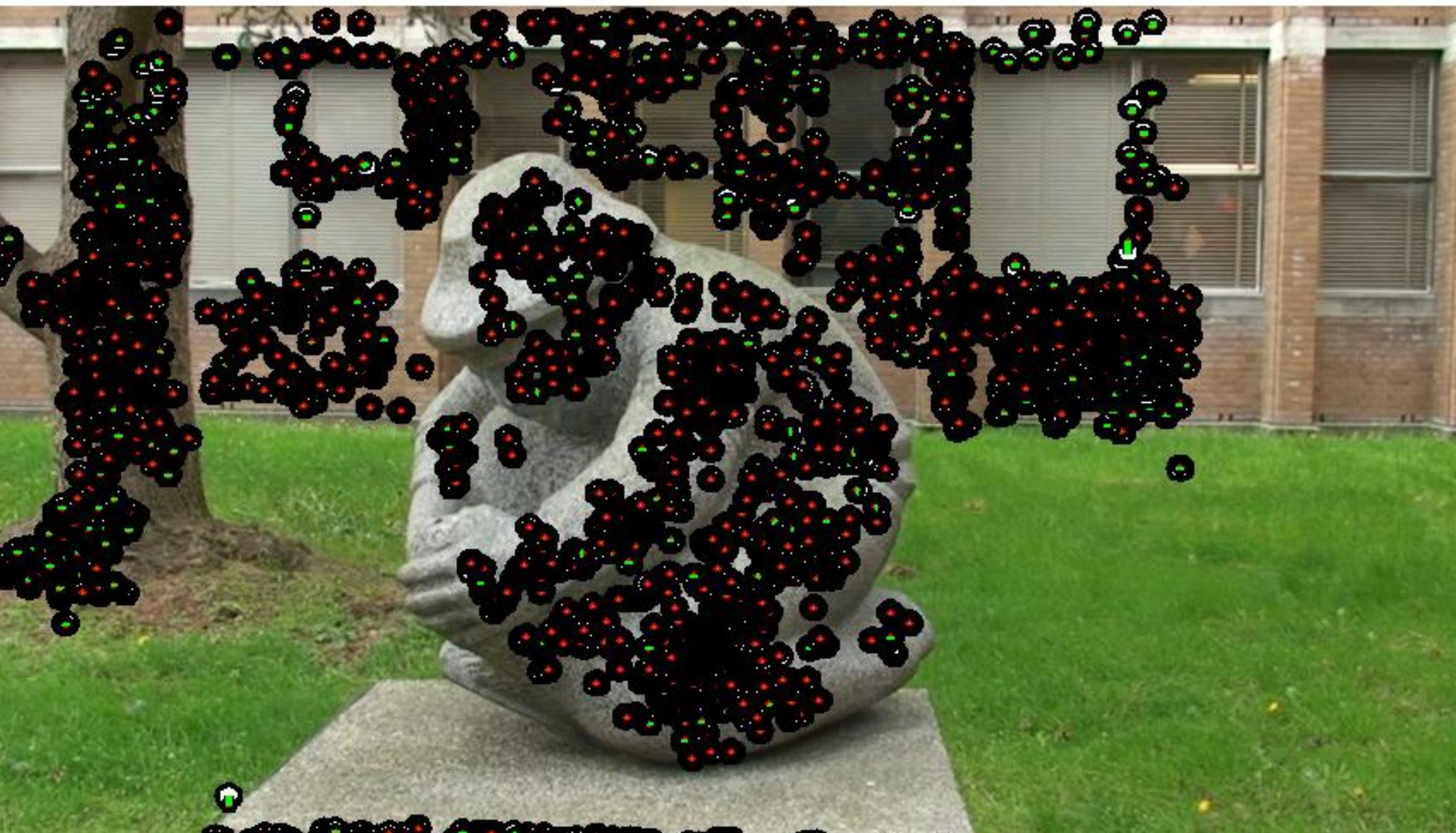
- Pose estimation for camera i:

$$e = \sum_{j \in \text{points}} |\mathbf{r}_{ij}(\mathbf{R}_i, \mathbf{t}_i, \mathbf{X}_j)|^2$$

(optimised parameters are shown in red)

# Bundle Adjustment

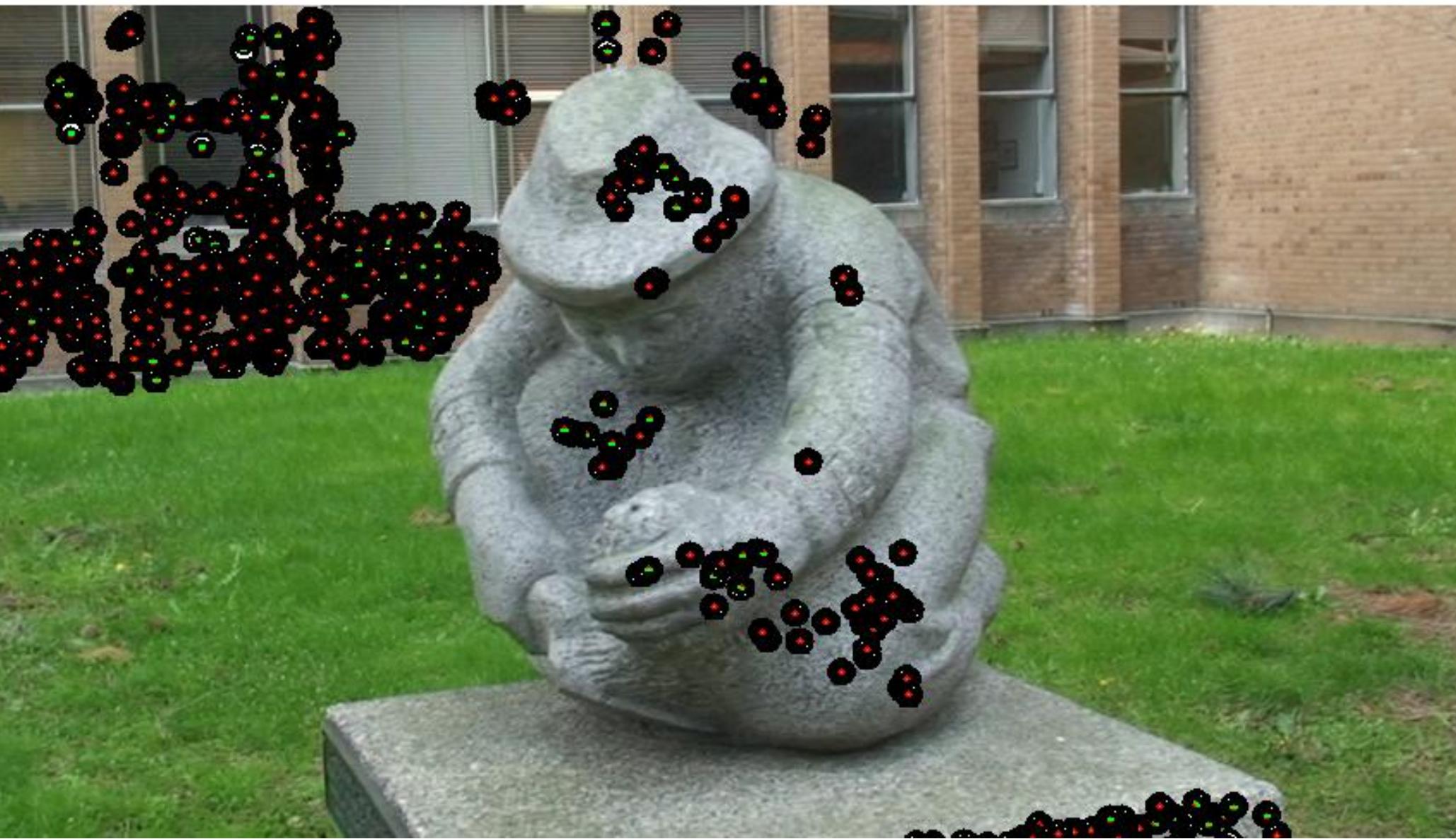
- Initialization with 3 views



Joint optimization of cameras and structure

# Bundle Adjustment

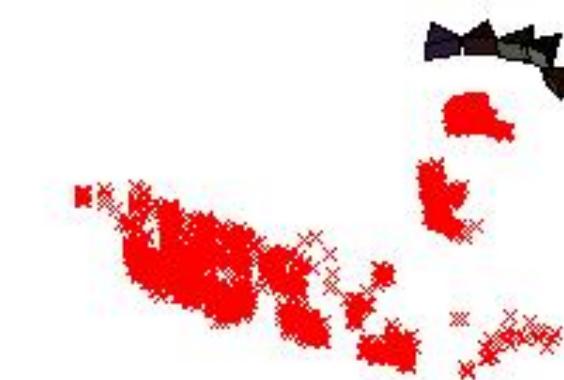
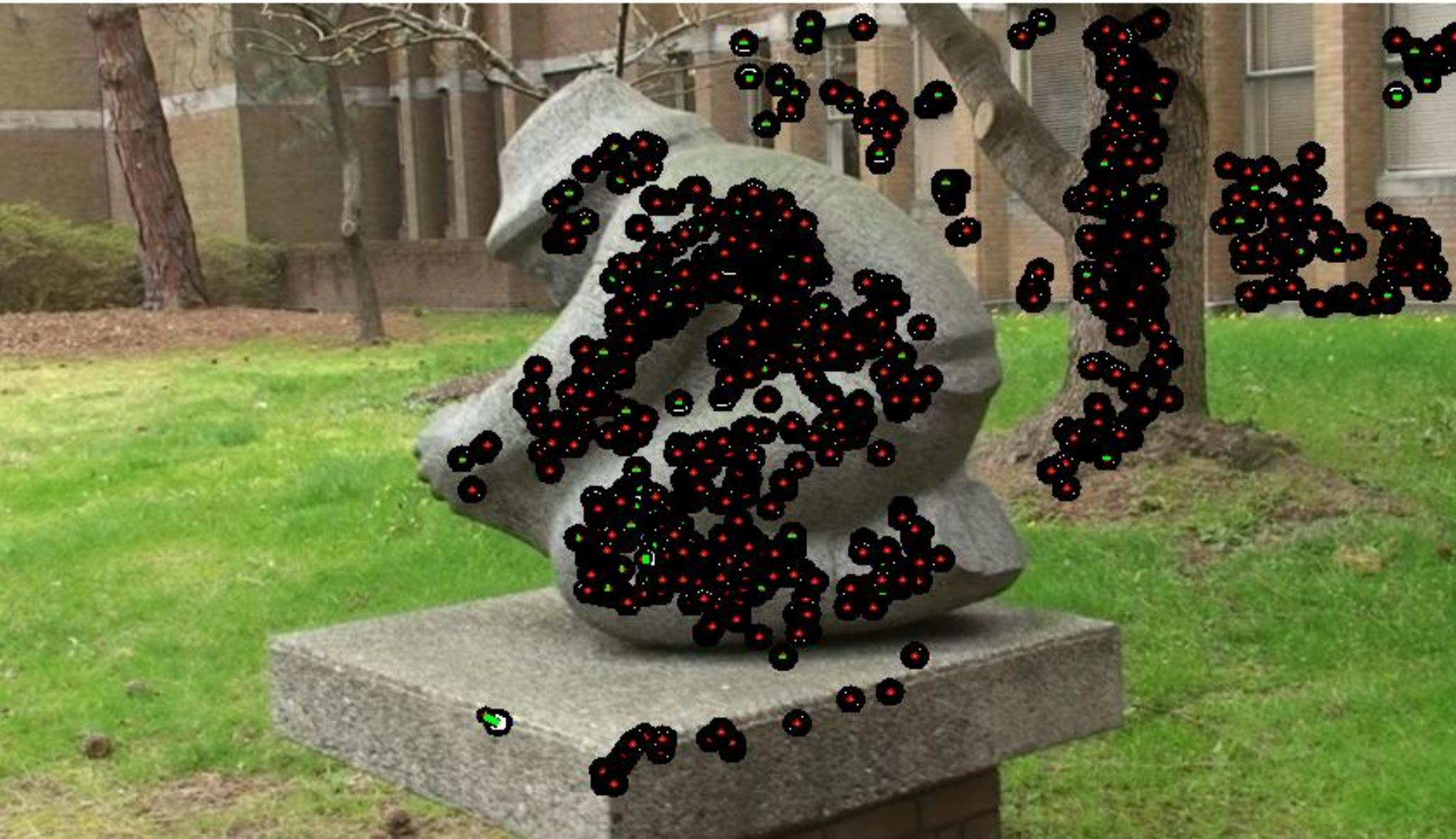
- Add camera 4



Estimate camera pose, add new 3D points, jointly optimize

# Bundle Adjustment

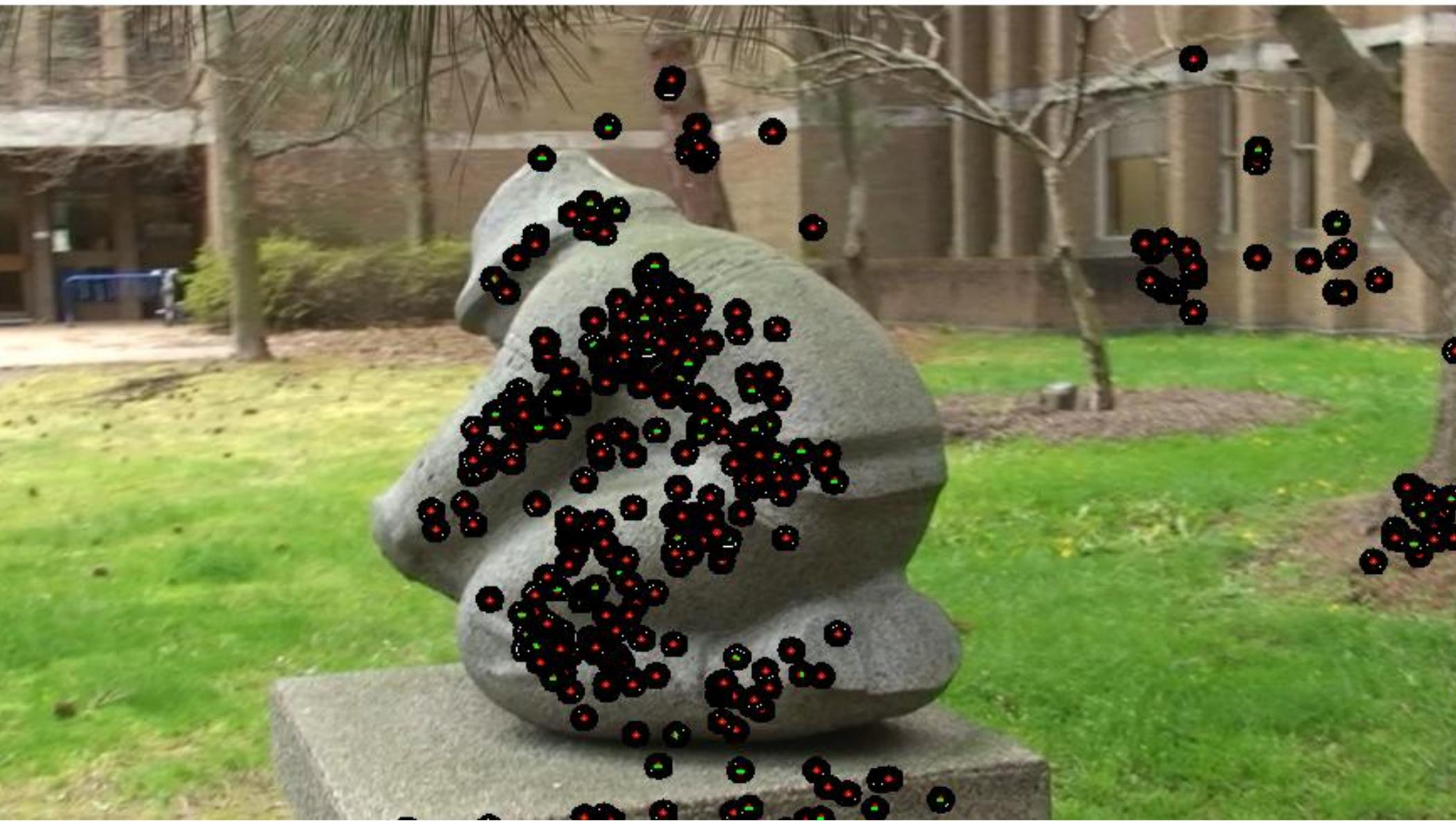
- Add camera 5



Estimate camera pose, add new 3D points, jointly optimize

# Bundle Adjustment

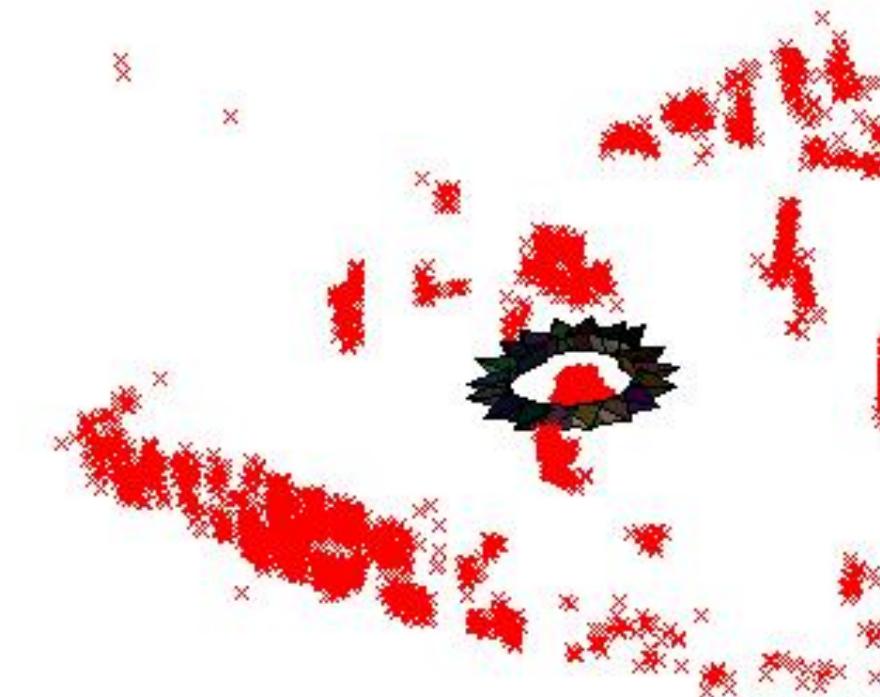
- Add camera 6



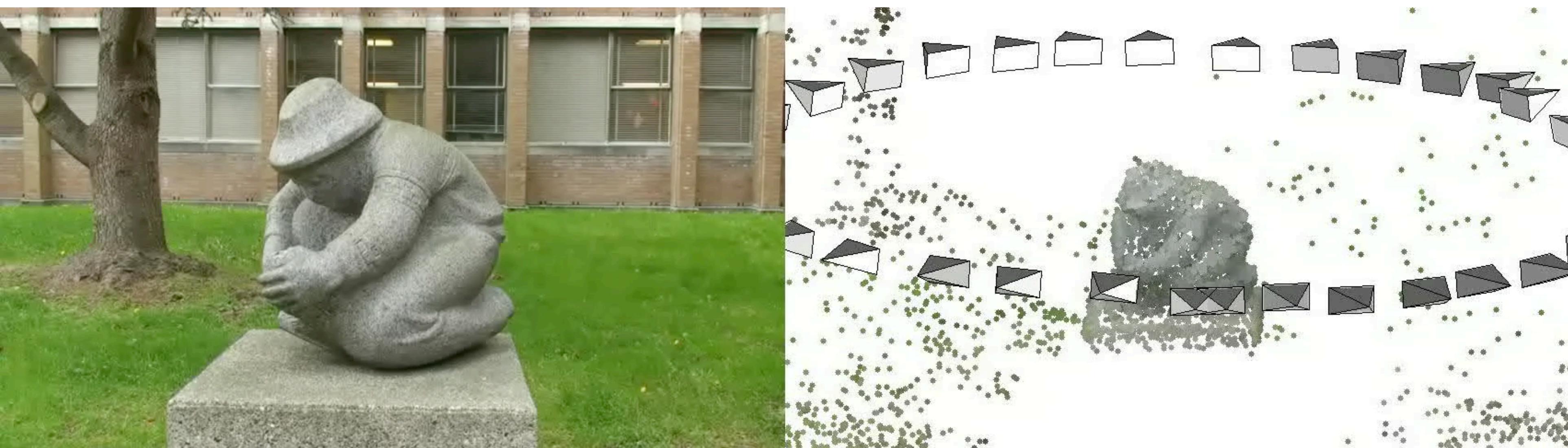
Estimate camera pose, add new 3D points, jointly optimize

# Bundle Adjustment

- Add remaining cameras in same way



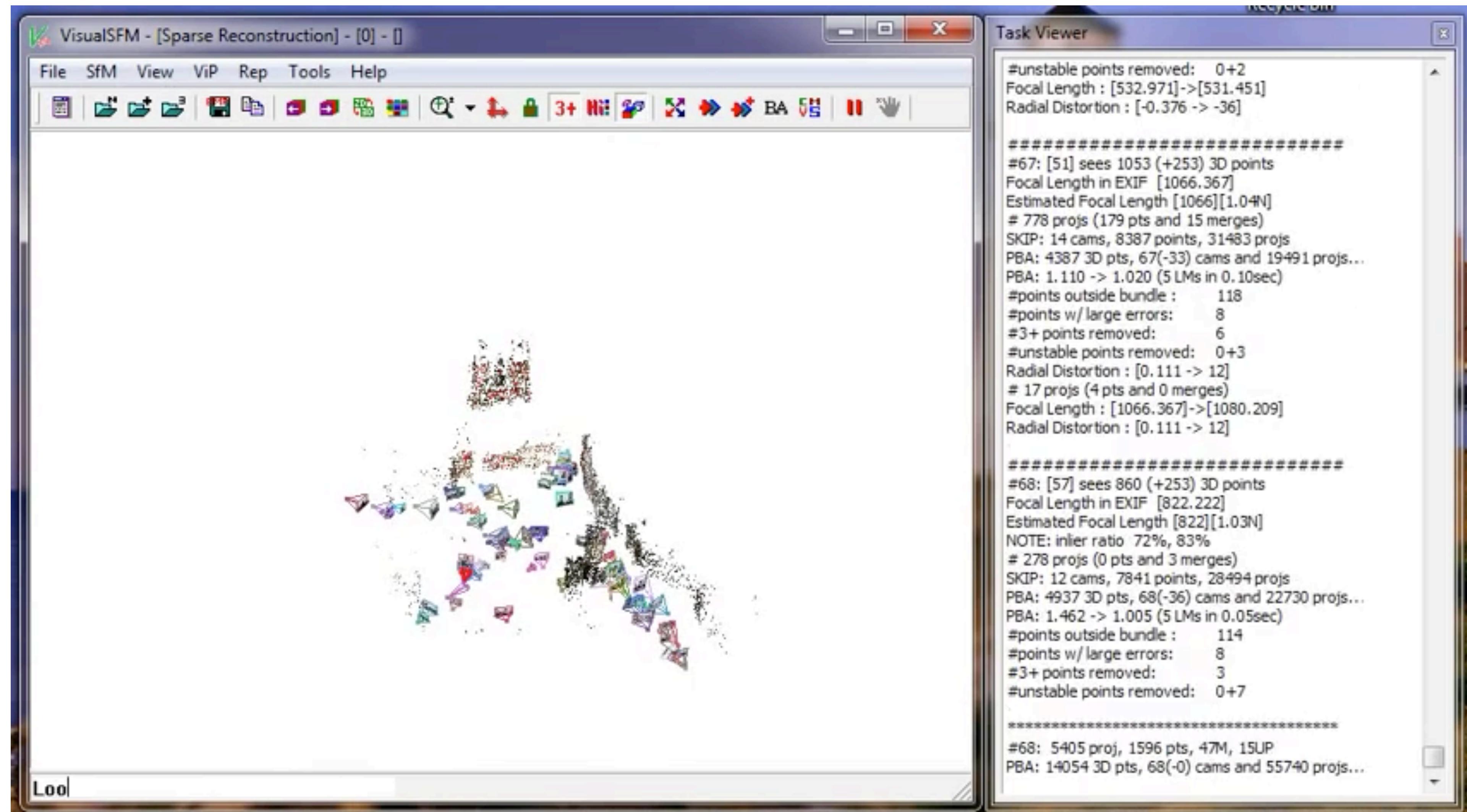
# Structure from Motion



# SFM recap

- Match features, e.g., SIFT, between all views
- Use RANSAC to reject outliers and estimate Epipolar Geometry / Camera matrices
- Form feature tracks by linking multiview matches
- Select an initialization set, e.g., 3 images with lots of matches and good baseline (parallax)
- Jointly optimize cameras  $R, t$  and structure  $X$  for this set
- Repeat for each camera:
  - Estimate pose  $R, t$  by minimising projection errors with existing  $X$
  - Add 3D points corresponding to the new view and optimize
  - Bundle adjust optimizing over all cameras and structure

# Visual SFM



# Application: 3D from Internet Images

- Reconstruct 3D from unordered photo collections



[ Building Rome in a Day, S.Agarwal et al 2009 ]







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