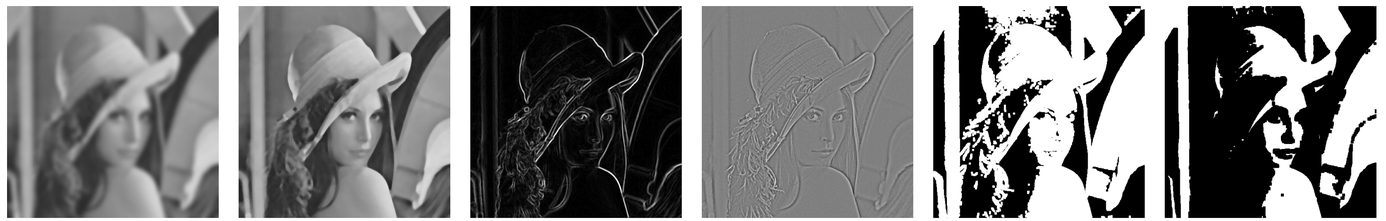
THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 5: Image Filtering (final)

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today

Topics:

- **Linear Filtering** recap
- Efficient convolution, Fourier aside
- **Quiz 1**
- **Non-linear** Filters: Median, ReLU, Bilateral Filter

Readings:

- **Today's** Lecture: Szeliski 3.3-3.4, Forsyth & Ponce (2nd ed.) 4.4

Reminders:

- **Assignment 1:** Image Filtering and Hybrid Images due **September 28th**

New TA **Minglong** + Updated **Office Hours**

Bereket Guta

Mon 3-4, In Person

Rayat Hossain

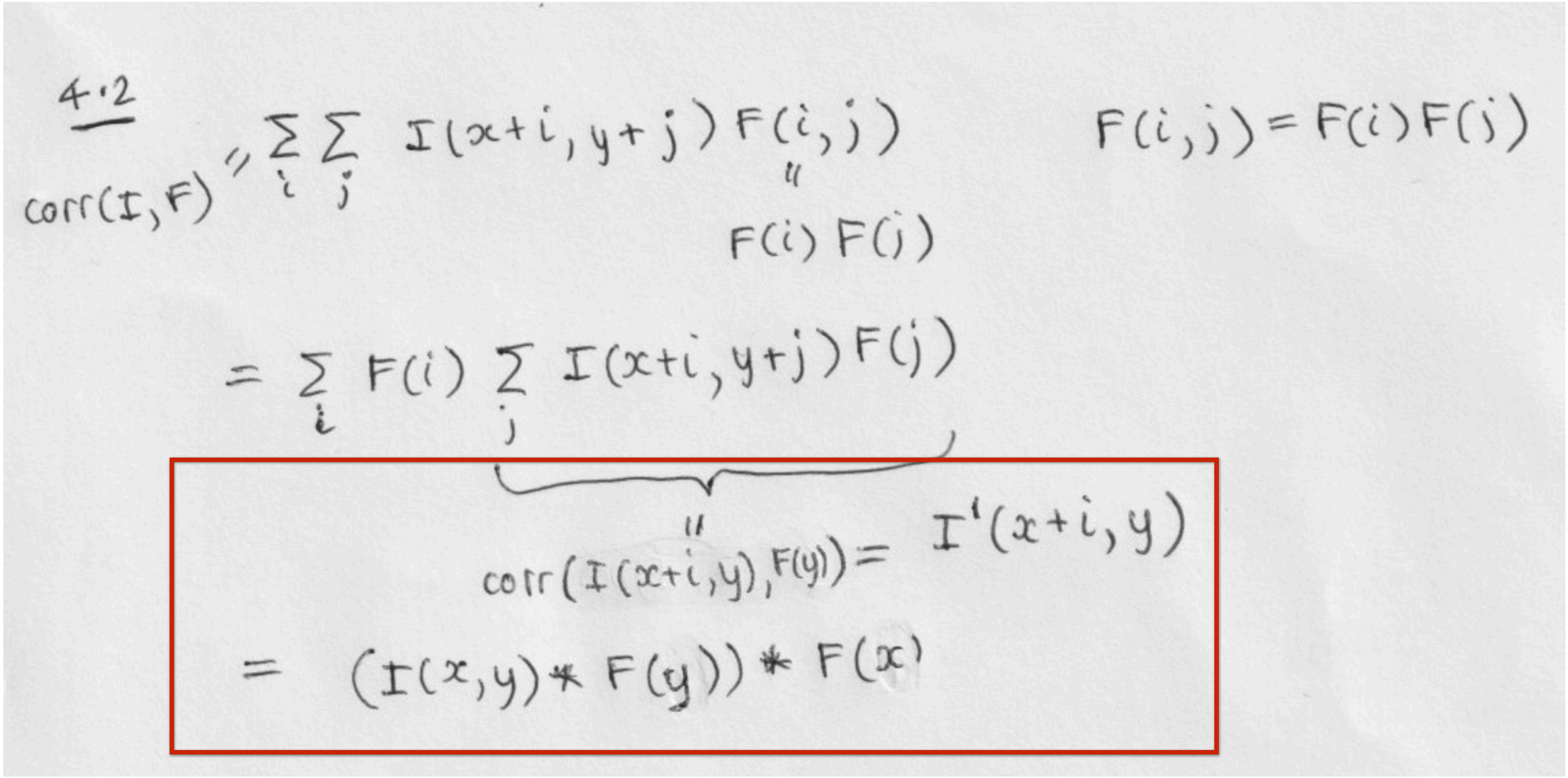
Wed 3-4, In Person

Minglong Li

Fri 11-12, Zoom

See **Canvas** for Links and Locations

Minor Error in Last week's notes



Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

$$I'(X,Y) = \sum_{j=-k}^{k} \sum_{i=-k}^{k} F(i,j) I(X+i, Y+j)$$

$I'(X,Y)$: output

$F(i,j)$: filter

$I(X+i, Y+j)$: image (signal)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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1	1	1
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image $I(X,Y)$

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0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0								

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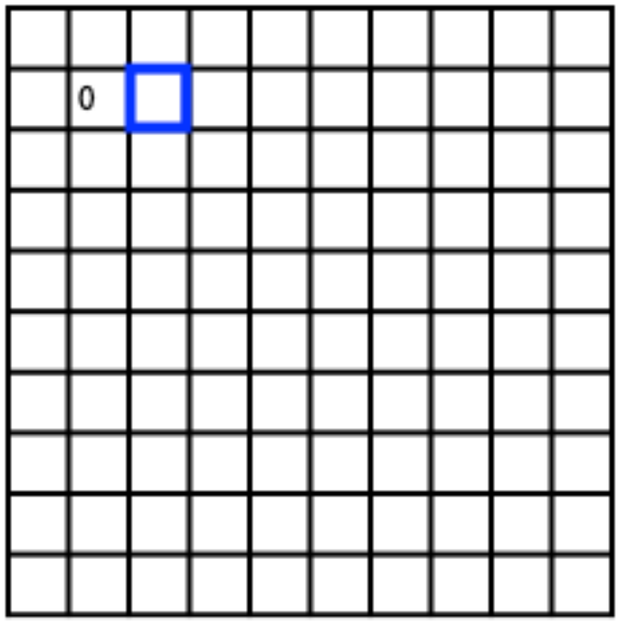
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$



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output | filter | image (signal)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10							

$$I'(X,Y) = \sum_{j=-k}^{k} \sum_{i=-k}^{k} F(i,j) I(X+i, Y+j)$$

$I'(X,Y)$: output

$F(i,j)$: filter

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0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10							

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0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20						

$$I'(X,Y) = \sum_{j=-k}^{k} \sum_{i=-k}^{k} F(i,j) I(X+i, Y+j)$$

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image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20						

$$I'(X,Y) = \sum_{j=-k}^{k} \sum_{i=-k}^{k} F(i,j) I(X+i, Y+j)$$

$I'(X,Y)$: output

$F(i,j)$: filter

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filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30					

$$I'(X,Y) = \sum_{j=-k}^{k} \sum_{i=-k}^{k} F(i,j) I(X+i, Y+j)$$

$I'(X,Y)$: output

$F(i,j)$: filter

$I(X+i, Y+j)$: image (signal)

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Linear Filter **Example**

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filter

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1	1	1
1	1	1
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image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

$$I'(X,Y) = \sum_{j=-k}^{k} \sum_{i=-k}^{k} F(i,j) I(X+i, Y+j)$$

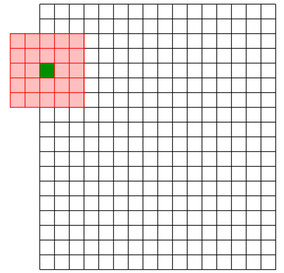
$I'(X,Y)$: output

$F(i,j)$: filter

$I(X+i, Y+j)$: image (signal)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filters: **Boundary** Effects



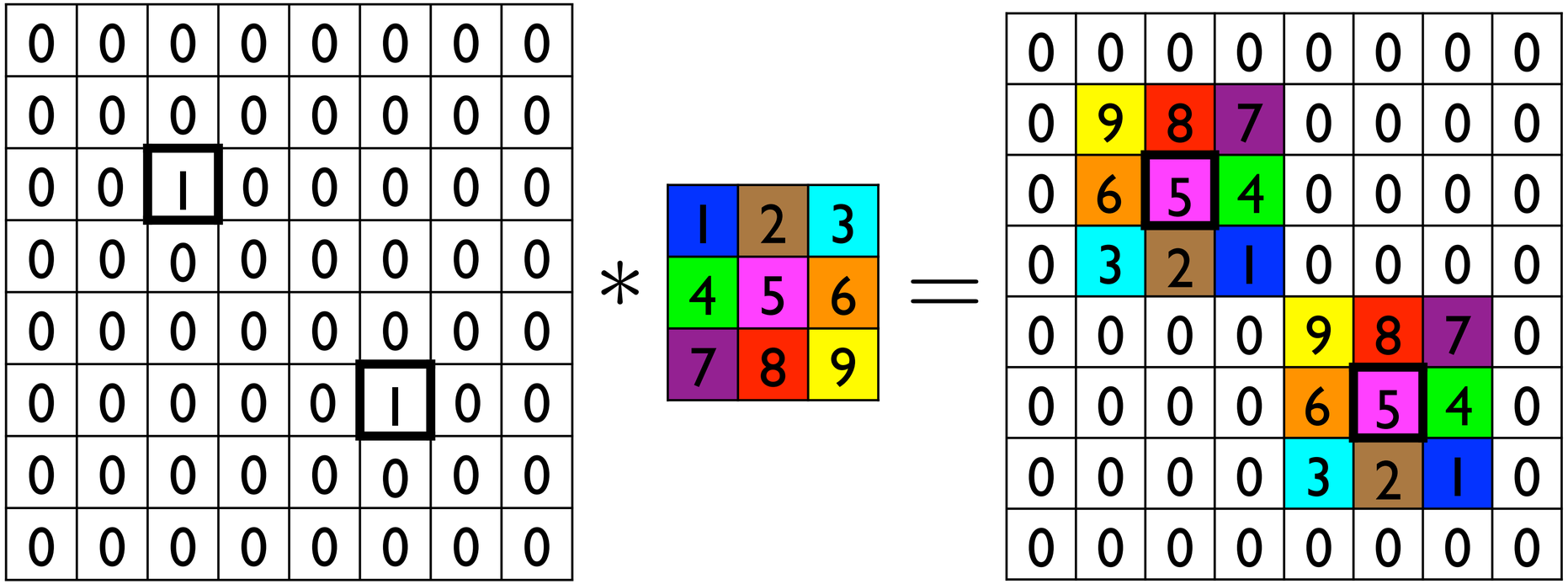
Three standard ways to deal with boundaries:

1. **Ignore these locations:** Make the computation undefined for the top and bottom k rows and the leftmost and rightmost k columns

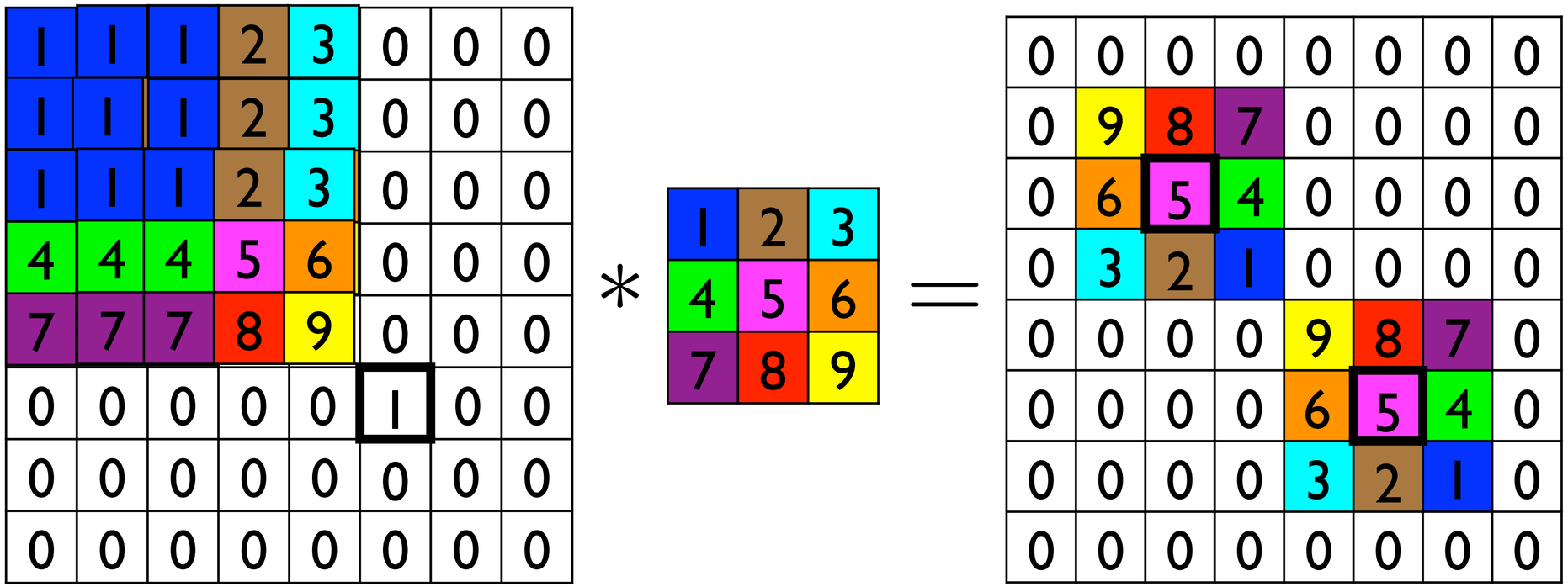
2. **Pad the image with zeros**: Return zero whenever a value of I is required at some position outside the defined limits of X and Y

3. **Assume periodicity**: The top row wraps around to the bottom row; the leftmost column wraps around to the rightmost column

Point Spread Function



Point Spread Function



- The point spread function is the correlation kernel rotated by 180° (= the convolution kernel)

Lecture 4: Re-cap

Linear filtering (one interpretation):

— new pixels are a weighted sum of original pixel values

— “filter” defines weights

Linear filtering (another interpretation):

— each pixel creates a scaled copy of point spread function in its location

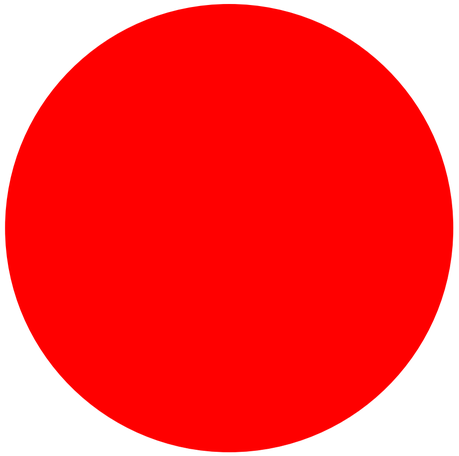
— “filter” specifies the point spread function

Low-pass Filtering = “Smoothing”

Box Filter

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

Pillbox Filter



Gaussian Filter

$$\frac{1}{256}\begin{bmatrix}1 & 4 & 6 & 4 & 1\\ 4 & 16 & 24 & 16 & 4\\ 6 & 24 & 36 & 24 & 6\\ 4 & 16 & 24 & 16 & 4\\ 1 & 4 & 6 & 4 & 1\end{bmatrix}$$

All of these filters are **Low-pass Filters**

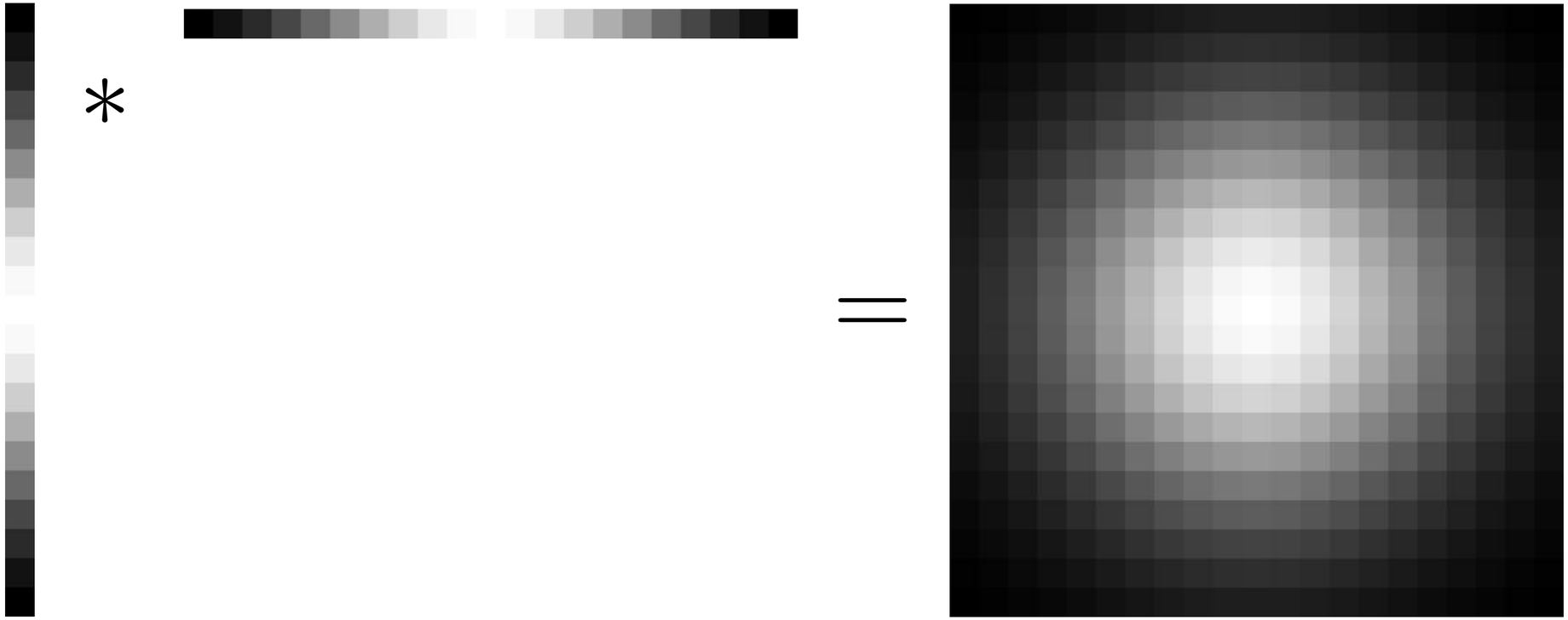
Low-pass filter: Low pass filter filters out all of the high frequency content of the image, only low frequencies remain

Example: Separable Filter

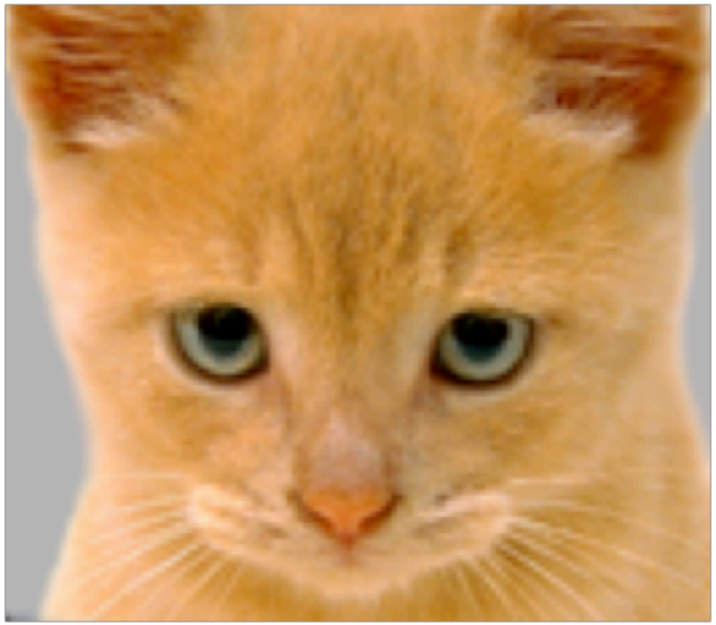
$$\frac{1}{16}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array} \otimes \frac{1}{16}\begin{array}{|c|}\hline 1 \\ \hline 4 \\ \hline 6 \\ \hline 4 \\ \hline 1 \\ \hline \end{array} = \frac{1}{256}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 6 & 24 & 36 & 24 & 6 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array}$$

Gaussian Blur

- 2D Gaussian filter can be thought of as an **outer product** or **convolution** of row and column filters

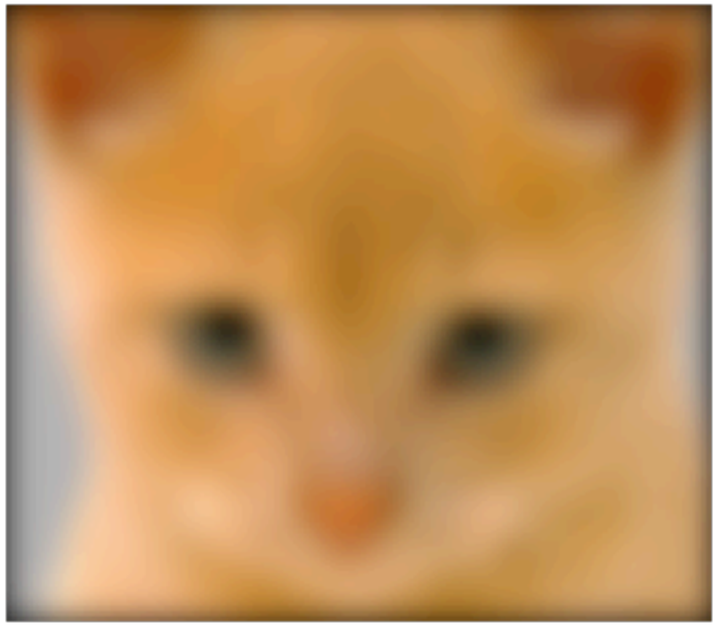


Assignment 1: **Low/High Pass** Filtering



Original

$$I(x, y)$$



Low-Pass Filter

$$I(x, y) * g(x, y)$$

High-Pass Filter

$$I(x, y) - I(x, y) * g(x, y)$$

Next: Please get your **iClickers** —

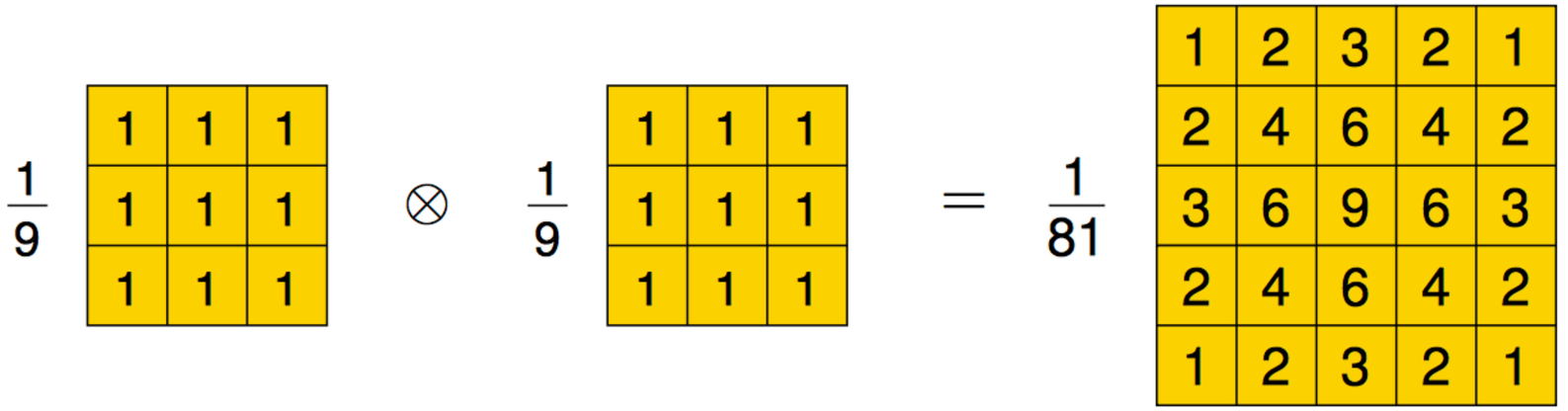
Quiz 1: **6** questions

Advanced Convolution Topics

- Multiple filters
- Fourier transforms

Multiple Filters: Two Box Filters

filter = boxfilter(3)
signal.correlate2d(filter, filter,′ full′)



3x3 **Box** 3x3 **Box**

Multiple Filters: Two Box Filters

filter = boxfilter(3)
temp = signal.correlate2d(filter, filter,′ full′)
signal.correlate2d(filter, temp,′ full′)

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} \otimes \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} \otimes \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} = \frac{1}{729}\begin{bmatrix}1&3&6&7&6&3&1\\3&9&18&21&18&9&3\\6&18&36&42&36&18&6\\7&21&42&49&42&21&7\\6&18&36&42&36&18&6\\3&9&18&21&18&9&3\\1&3&6&7&6&3&1\end{bmatrix}$$

3x3 **Box** 3x3 **Box** 3x3 **Box**

Gaussian: An Additional Property

Let $\otimes$ denote convolution. Let $G_{\sigma_1}(x)$ and $G_{\sigma_2}(x)$ be be two 1D Gaussians

$$G_{\sigma_1}(x) \otimes G_{\sigma_2}(x) = G_{\sqrt{\sigma_1^2+\sigma_2^2}}(x)$$

Convolution of two Gaussians is another Gaussian

Special case: Convolving with $G_{\sigma}(x)$ twice is equivalent to $G_{\sqrt{2}\sigma}(x)$

What follows is for fun

(you will **NOT** be tested on this)

Convolution using **Fourier Transforms**

[Szeliski 3.4]

Convolution **Theorem**:

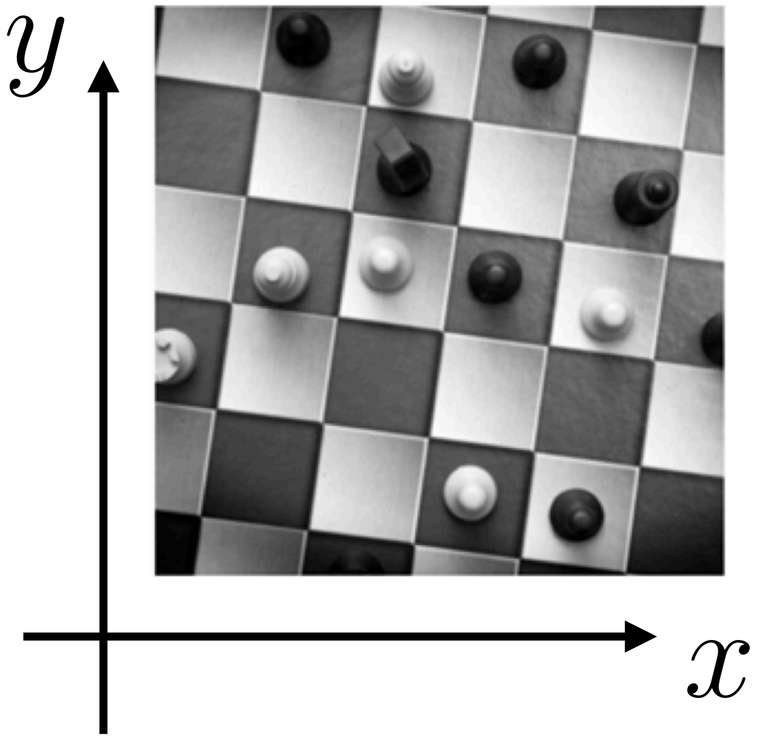
Let $$i'(x,y) = f(x,y) \otimes i(x,y)$$

then $$\mathcal{I}'(w_x, w_y) = \mathcal{F}(w_x, w_y)\ \mathcal{I}(w_x, w_y)$$

where $\mathcal{I}'(w_x, w_y)$, $\mathcal{F}(w_x, w_y)$, and $\mathcal{I}(w_x, w_y)$ are Fourier transforms of $i'(x,y)$, $f(x,y)$ and $i(x,y)$

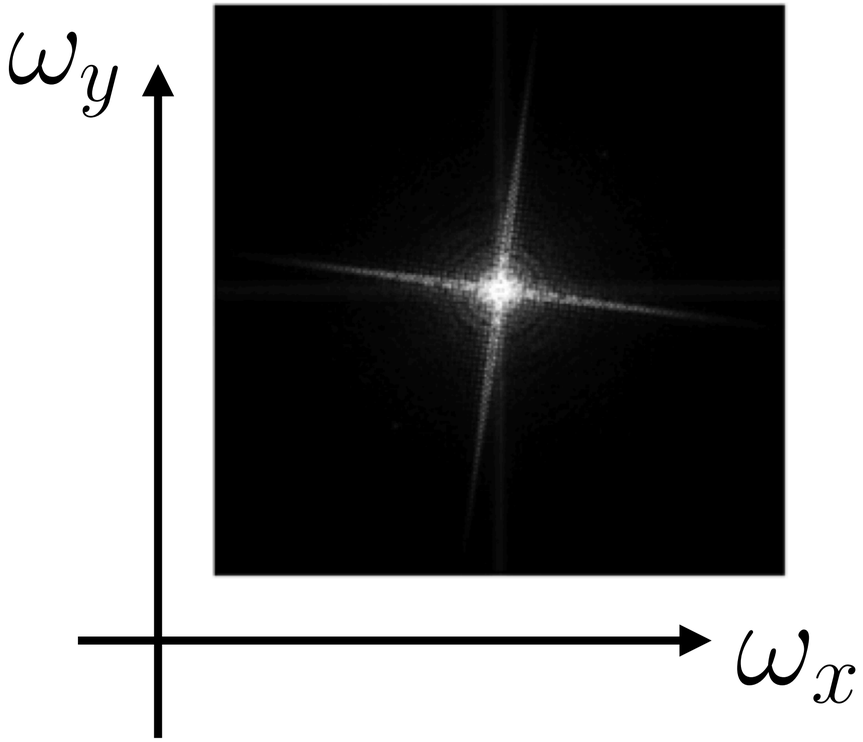
At the expense of two **Fourier** transforms and one inverse Fourier transform, convolution can be reduced to (complex) multiplication

2D Fourier Transforms: Images



$f(x, y)$

Image



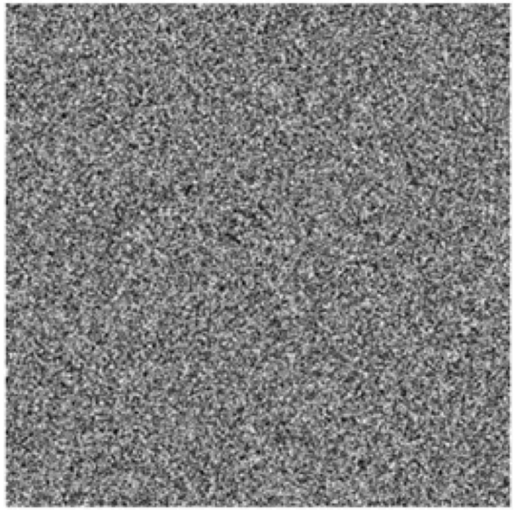
$F(\omega_x, \omega_y)$

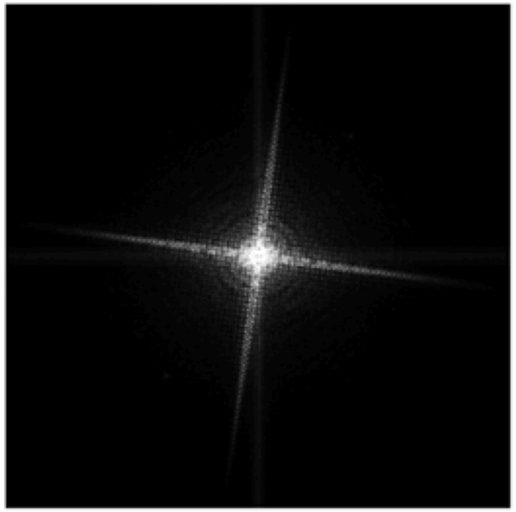
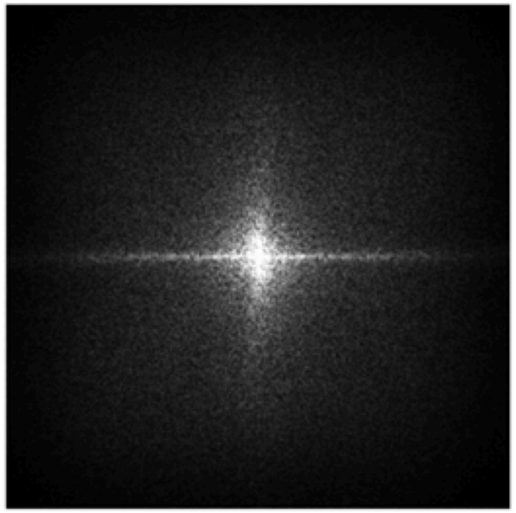
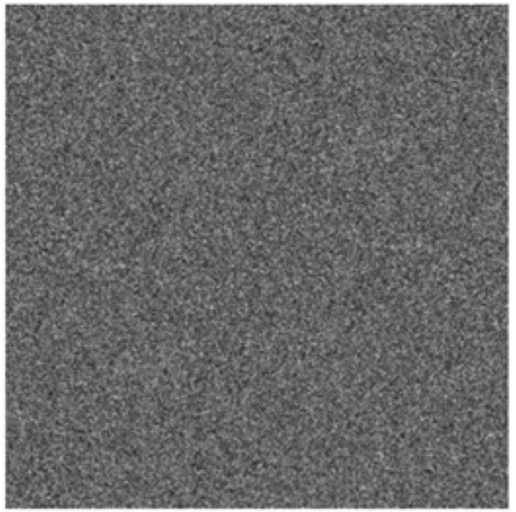
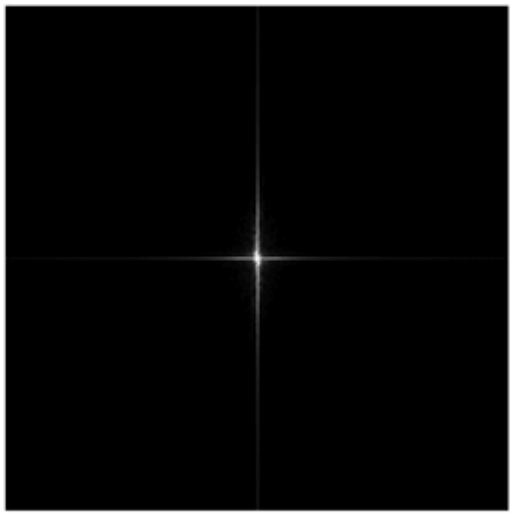
Fourier Transform

5.2

2D Fourier Transforms: Images





Aside: You will not be tested on this …

Image

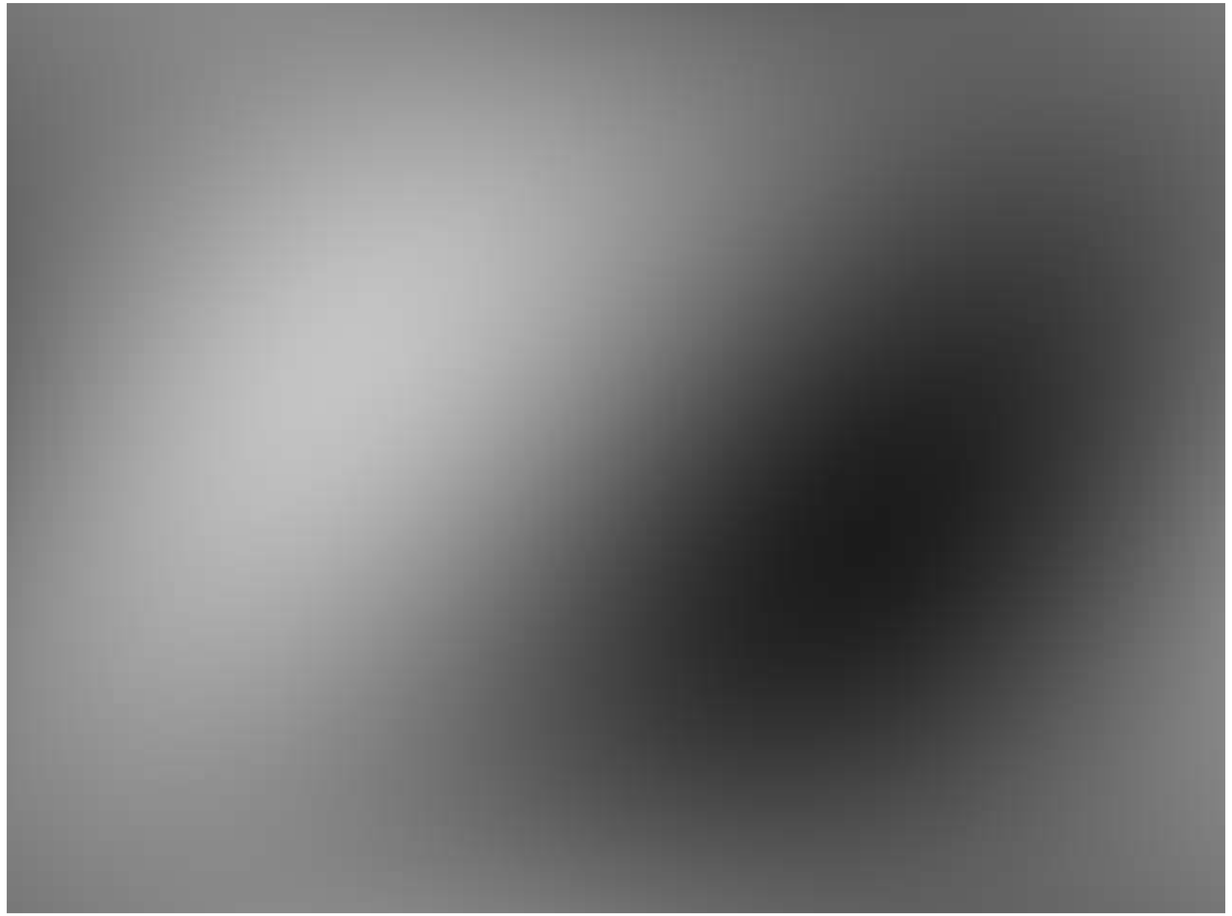
https://photo.stackexchange.com/questions/40401/what-does-frequency-mean-in-an-image/40410#40410

Aside: You will not be tested on this …

First (lowest) frequency, a.k.a. average

https://photo.stackexchange.com/questions/40401/what-does-frequency-mean-in-an-image/40410#40410

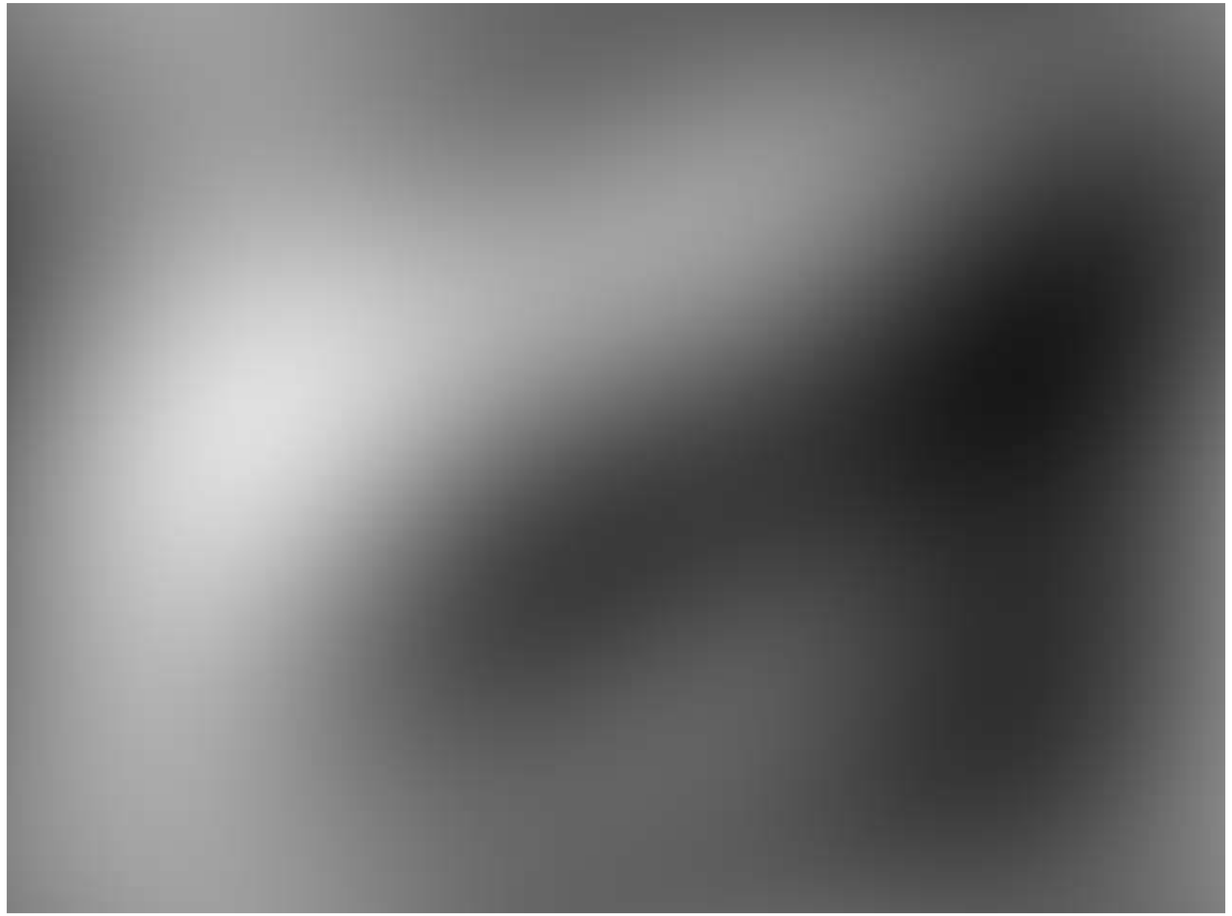
Aside: You will not be tested on this …



+ **Second** frequency

https://photo.stackexchange.com/questions/40401/what-does-frequency-mean-in-an-image/40410#40410

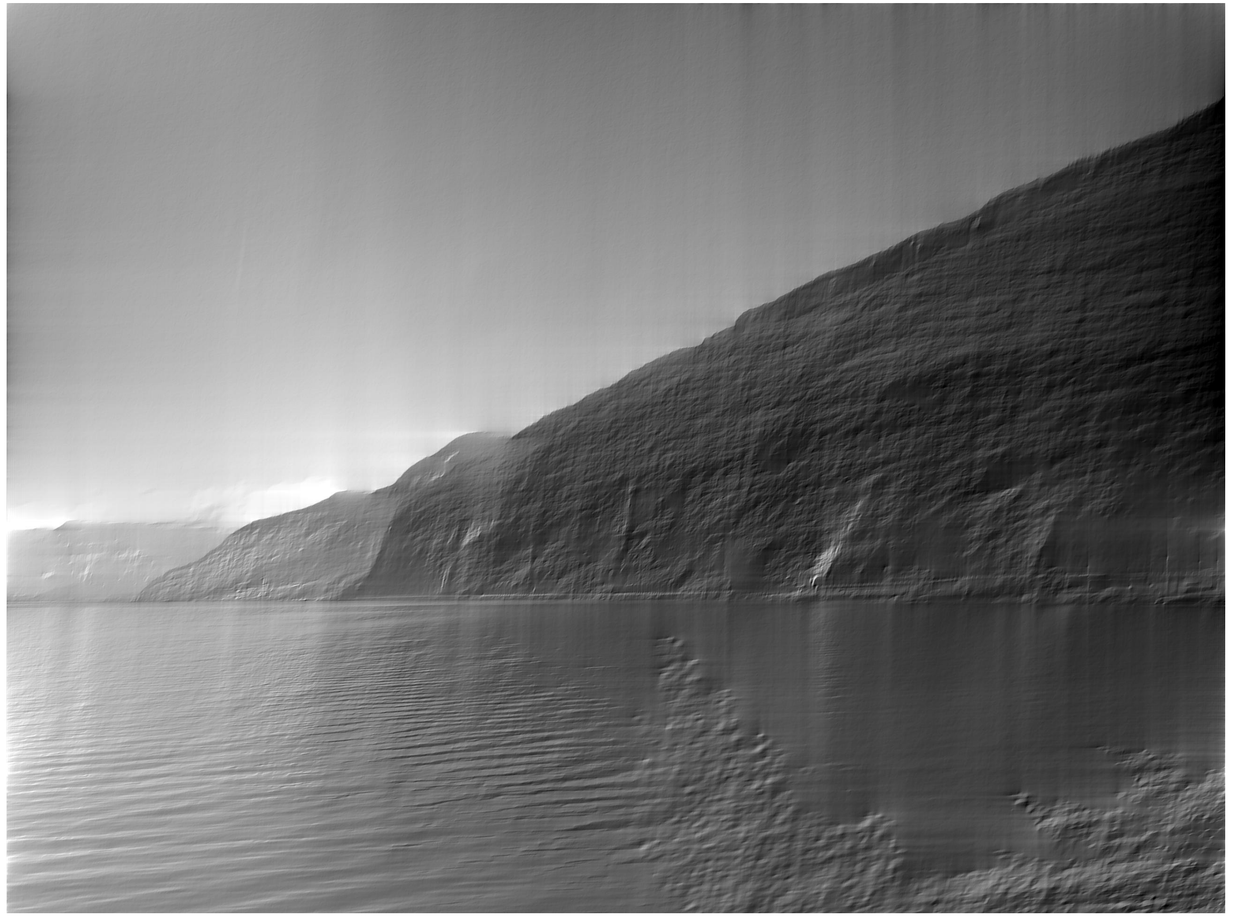
Aside: You will not be tested on this …



\+ **Third** frequency

https://photo.stackexchange.com/questions/40401/what-does-frequency-mean-in-an-image/40410#40410

Aside: You will not be tested on this …



+ **50%** of frequencies

https://photo.stackexchange.com/questions/40401/what-does-frequency-mean-in-an-image/40410#40410

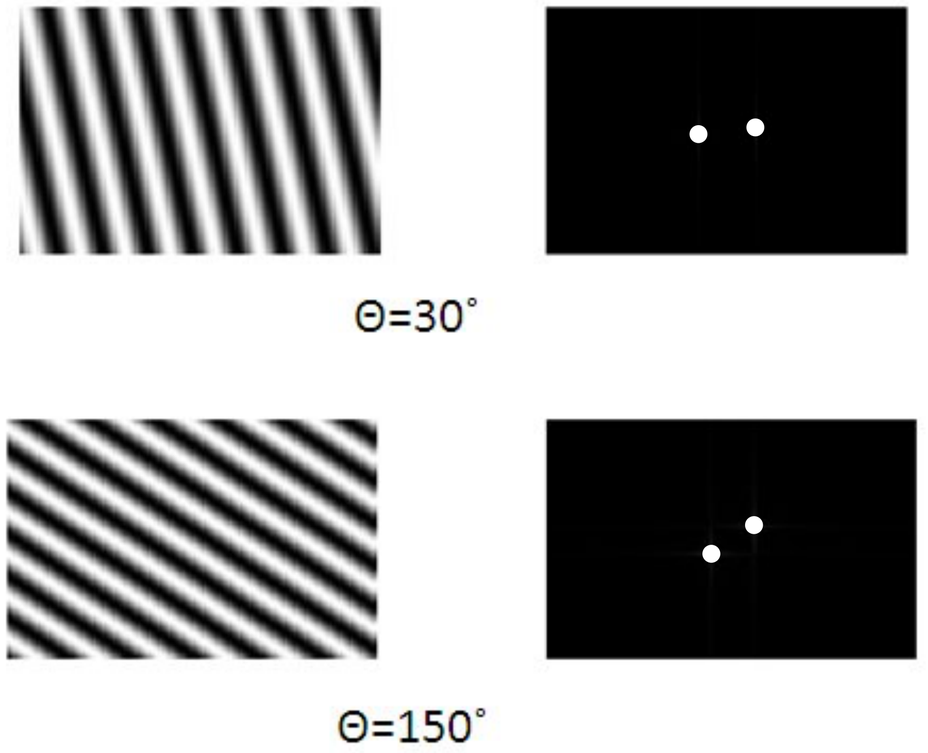
Aside: You will not be tested on this …

https://photo.stackexchange.com/questions/40401/what-does-frequency-mean-in-an-image/40410#40410

Aside: You will not be tested on this …

What are “frequencies” in an image?

Spatial frequency

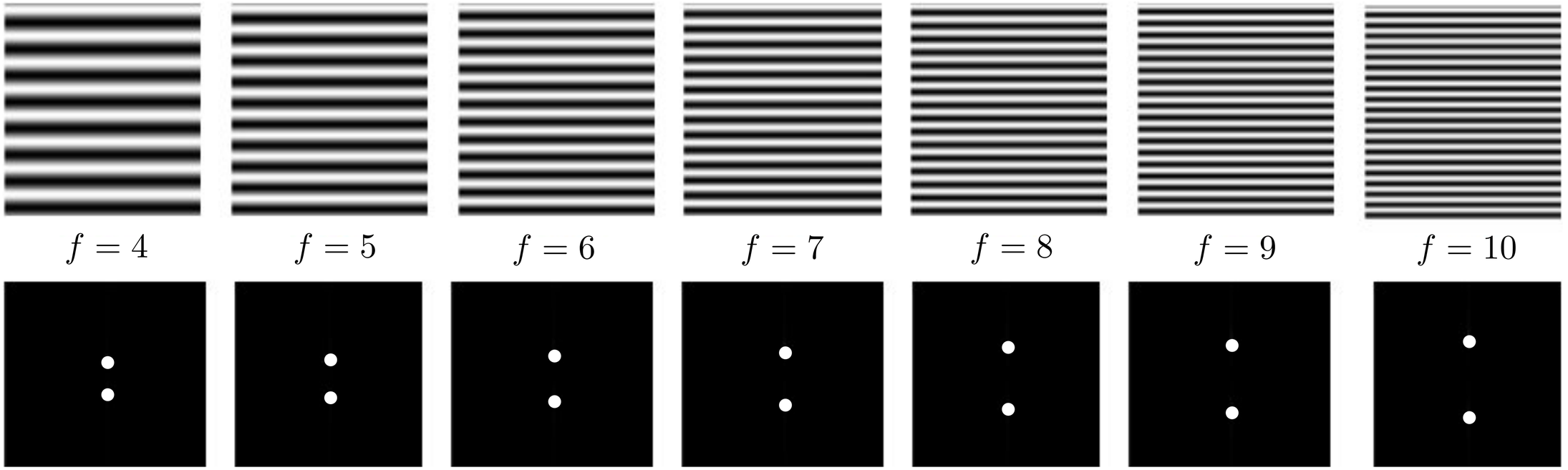


Points (actually point pairs) in Fourier Space correspond to sinusoids with a given frequency and direction

Aside: You will not be tested on this …

What are “frequencies” in an image?

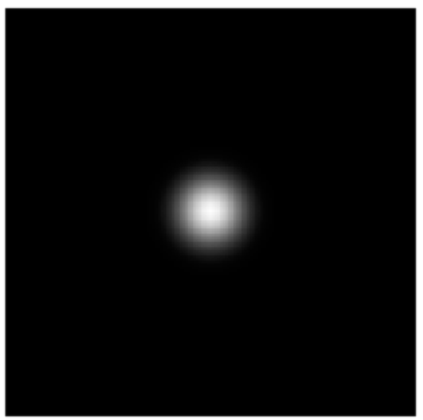
Spatial frequency

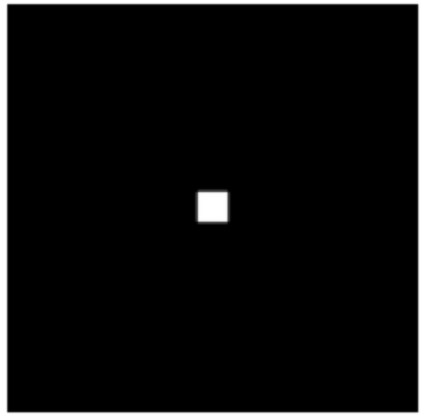


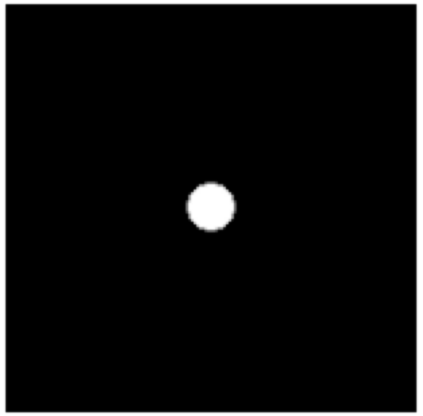
Observation: low frequencies close to the center

2D Fourier Transforms: Kernels

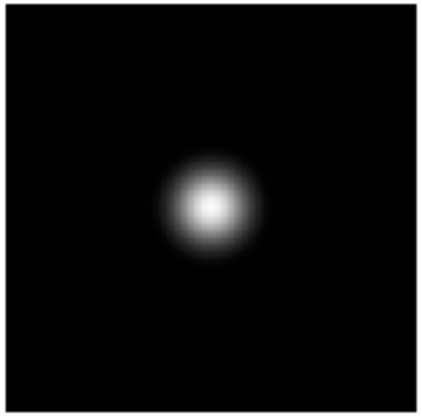
$f(x, y)$

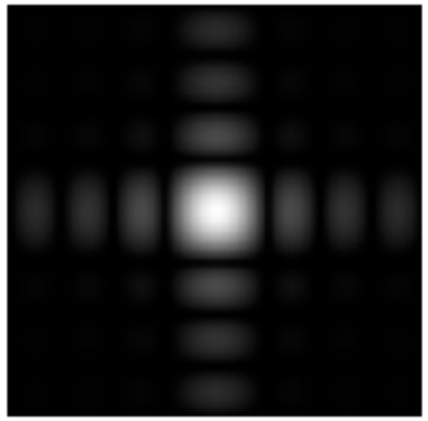


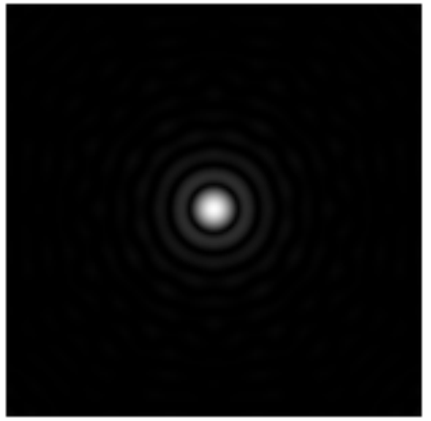


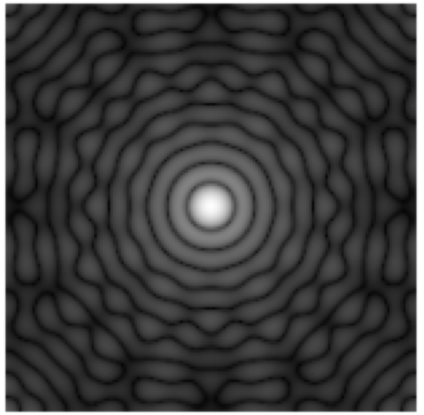


$F(\omega_x, \omega_y)$









Compress power 0.5
to exaggerate lobes
(just for visualization)

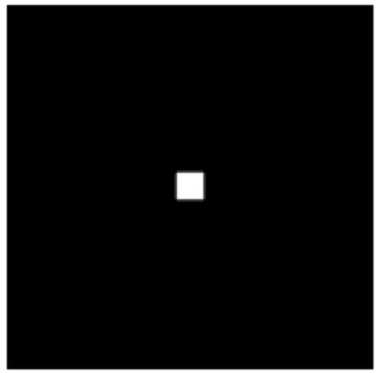
Convolution using **Fourier Transforms**

Convolution **Theorem**: $$i'(x,y) = f(x,y) \otimes i(x,y)$$

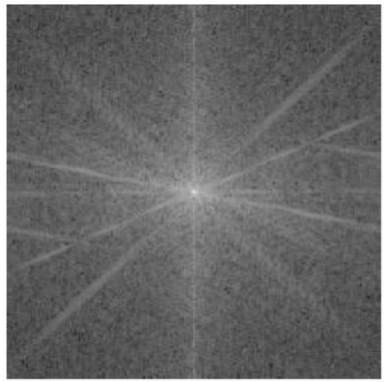
$$\mathcal{I}'(w_x, w_y) = \mathcal{F}(w_x, w_y)\ \mathcal{I}(w_x, w_y)$$

Image

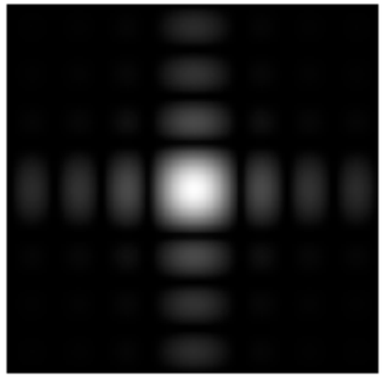
Convolve



FFT



Multiply

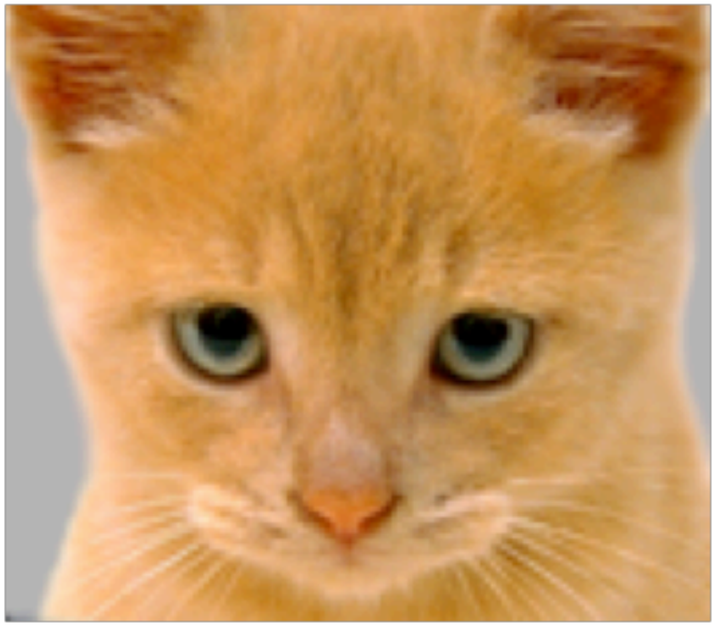


Inverse FFT

What preceded was for fun

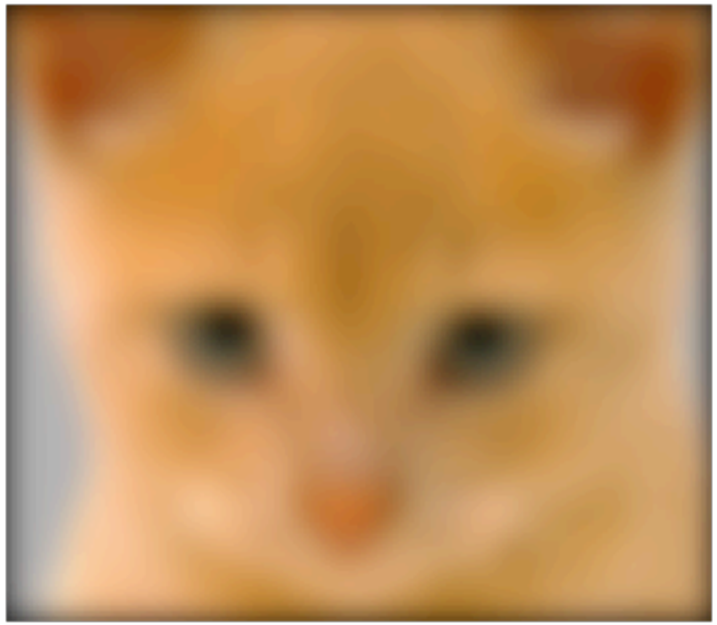
(you will **NOT** be tested on it)

Assignment 1: **Low/High Pass** Filtering



Original

$$I(x, y)$$



Low-Pass Filter

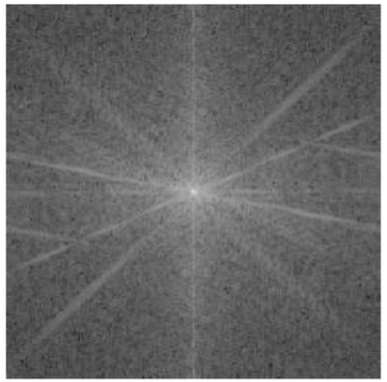
$$I(x, y) * g(x, y)$$

High-Pass Filter

$$I(x, y) - I(x, y) * g(x, y)$$

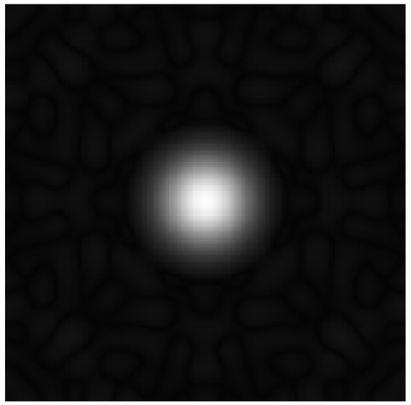
Aside: You will not be tested on this …

image



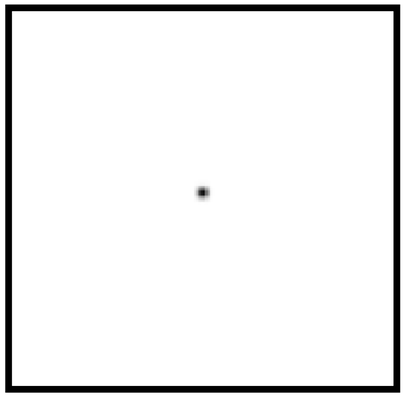
FFT (Mag)

complex
element-wise
multiplication



Low pass

filtered **image**



High pass

filtered **image**

Convolution using **Fourier Transforms**

General implementation of **convolution**:

At each pixel, (X, Y), there are $m \times m$ multiplications

There are $n \times n$ pixels in (X, Y)

Total: $m^2 \times n^2$ multiplications

Convolution if FFT space:

Cost of FFT/IFFT for image: $\mathcal{O}(n^2 \log n)$

Cost of FFT/IFFT for filter: $\mathcal{O}(m^2 \log m)$

Worthwhile if image and kernel are **both** large

Non-linear Filters

We've seen that **linear filters** can perform a variety of image transformations

— shifting

— smoothing

— sharpening

In some applications, better performance can be obtained by using **non-linear filters**.

For example, the median filter selects the **median** value from each pixel's neighborhood.

Non-linear Filtering

- Example: Median filter

“shot” noise

gaussian blurred

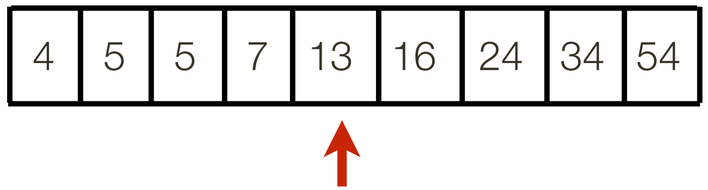
median filtered

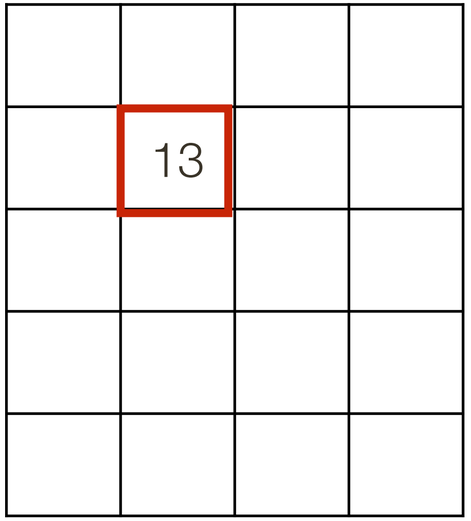
Median Filter

Take the **median value** of the pixels under the filter:

5	13	5	221
4	16	7	34
24	54	34	23
23	75	89	123
54	25	67	12

Image





Output

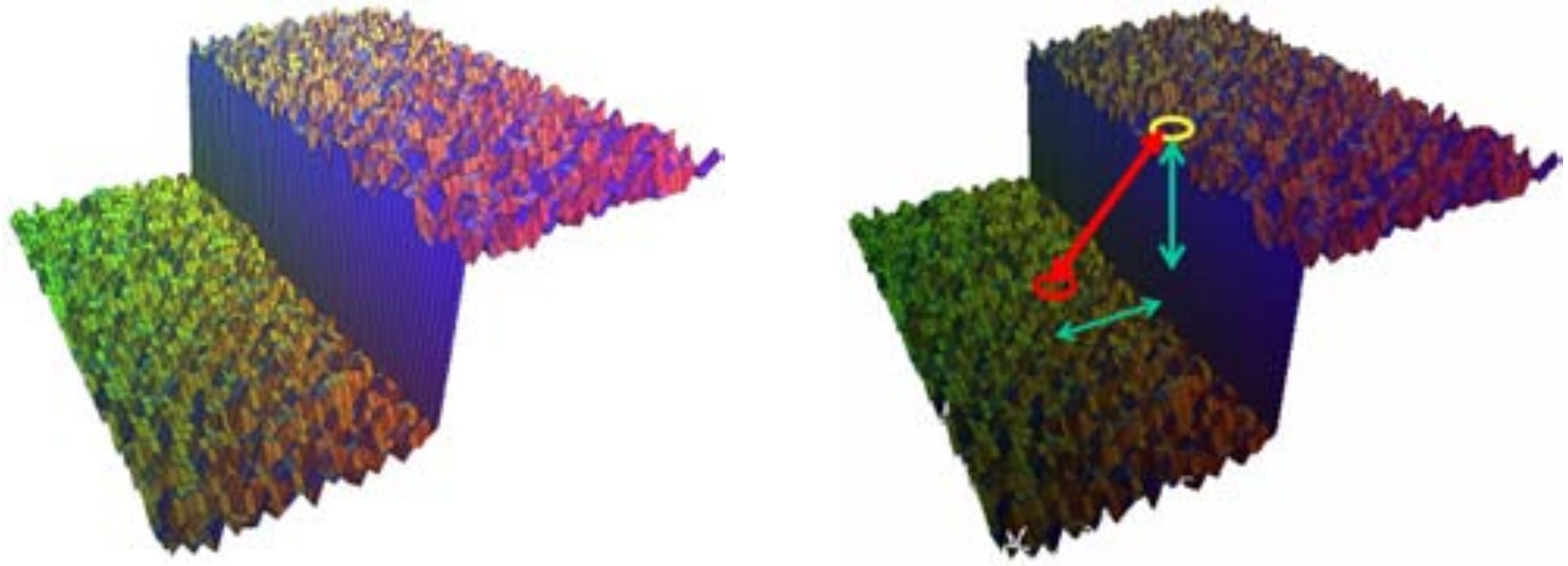
Median Filter

Effective at reducing certain kinds of noise, such as impulse noise (a.k.a ‘salt and pepper’ noise or ’shot’ noise)

The median filter forces points with distinct values to be more like their neighbors

Image credit: https://en.wikipedia.org/wiki/Median_filter#/media/File:Medianfilterp.png

Bilateral Filter



Suppose we want to smooth a noisy step function

A Gaussian kernel performs a weighted average of points over a spatial neighbourhood..

But this averages points both at the top and bottom of the step — blurring

Bilateral Filter idea: look at distances in **range** (value) as well as **space** x,y

Bilateral Filter

An edge-preserving non-linear filter

Like a Gaussian filter:

— The filter weights depend on spatial distance from the center pixel
— Pixels nearby (in space) should have greater influence than pixels far away

Unlike a Gaussian filter:

— The filter weights also depend on range distance from the center pixel
— Pixels with similar brightness value should have greater influence than pixels with dissimilar brightness value

Bilateral Filter

Gaussian filter: weights of neighbor at a spatial offset (x, y) away from the center pixel $I(X, Y)$ given by:

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

(with appropriate normalization)

Bilateral Filter

Gaussian filter: weights of neighbor at a spatial offset (x, y) away from the center pixel $I(X, Y)$ given by:

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

(with appropriate normalization)

Bilateral filter: weights of neighbor at a spatial offset (x, y) away from the center pixel $I(X, Y)$ given by a product:

$$\exp^{-\frac{x^2+y^2}{2\sigma_d^2}} \exp^{-\frac{(I(X+x,Y+y)-I(X,Y))^2}{2\sigma_r^2}}$$

(with appropriate normalization)

Bilateral Filter

Gaussian filter: weights of neighbor at a spatial offset (x, y) away from the center pixel $I(X, Y)$ given by:

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

(with appropriate normalization)

Bilateral filter: weights of neighbor at a spatial offset (x, y) away from the center pixel $I(X, Y)$ given by a product:

domain kernel	$\exp^{-\frac{x^2+y^2}{2\sigma_d^2}}$	$\exp^{-\frac{(I(X+x,Y+y)-I(X,Y))^2}{2\sigma_r^2}}$	**range** kernel

(with appropriate normalization)

Bilateral Filter

image $I(X, Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

Normalised

image $I(X, Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 0.45$

0.08	0.12	0.08
0.12	0.20	0.12
0.08	0.12	0.08

Bilateral Filter

image $I(X, Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

Normalised

image $I(X, Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 0.45$

0.08	0.12	0.08
0.12	0.20	0.12
0.08	0.12	0.08

Range Kernel

$\sigma_r = 0.45$

0.98	0.98	0.2
1	1	0.1
0.98	1	0.1

(differences based on **centre pixel**)

Bilateral Filter

image $I(X, Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

Normalised

image $I(X, Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 0.45$

0.08	0.12	0.08
0.12	0.20	0.12
0.08	0.12	0.08

Range Kernel

$\sigma_r = 0.45$

0.98	0.98	0.2
1	1	0.1
0.98	1	0.1

(differences based on **centre pixel**)

multiply

Range * **Domain** Kernel

0.08	0.12	0.02
0.12	0.20	0.01
0.08	0.12	0.01

Bilateral Filter

image $I(X, Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

Normalised

image $I(X, Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 0.45$

0.08	0.12	0.08
0.12	0.20	0.12
0.08	0.12	0.08

Range Kernel

$\sigma_r = 0.45$

0.98	0.98	0.2
1	1	0.1
0.98	1	0.1

(differences based on **centre pixel**)

multiply

0.08	0.12	0.02
0.12	0.20	0.01
0.08	0.12	0.01

Range * **Domain** Kernel

sum to 1

0.11	0.16	0.03
0.16	0.26	0.01
0.11	0.16	0.01

Bilateral Filter

image $I(X, Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

Normalised

image $I(X, Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 0.45$

0.08	0.12	0.08
0.12	0.20	0.12
0.08	0.12	0.08

Range Kernel

$\sigma_r = 0.45$

0.98	0.98	0.2
1	1	0.1
0.98	1	0.1

(differences based on **centre pixel**)

multiply

Range * **Domain** Kernel

0.08	0.12	0.02
0.12	0.20	0.01
0.08	0.12	0.01

$$\sum \begin{bmatrix} 0.11 & 0.16 & 0.03 \\ 0.16 & 0.26 & 0.01 \\ 0.11 & 0.16 & 0.01 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0.9 \\ 0.1 & 0.1 & 1 \\ 0 & 0.1 & 1 \end{bmatrix} = 0.1$$

Bilateral Filter

Bilateral Filter

image $I(X,Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

Normalised

image $I(X,Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 0.45$

$$\sum \begin{array}{|c|c|c|}\hline 0.08 & 0.12 & 0.08 \\ \hline 0.12 & 0.20 & 0.12 \\ \hline 0.08 & 0.12 & 0.08 \\ \hline \end{array} \times \begin{array}{|c|c|c|}\hline 0 & 0 & 0.9 \\ \hline 0.1 & 0.1 & 1 \\ \hline 0 & 0.1 & 1 \\ \hline \end{array} = 0.3$$

Gaussian Filter (only)

Range Kernel

$\sigma_r = 0.45$

0.98	0.98	0.2
1	1	0.1
0.98	1	0.1

(differences based on **centre pixel**)

multiply

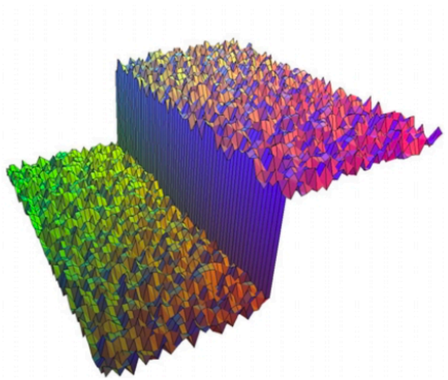
Range * **Domain** Kernel

0.08	0.12	0.02
0.12	0.20	0.01
0.08	0.12	0.01

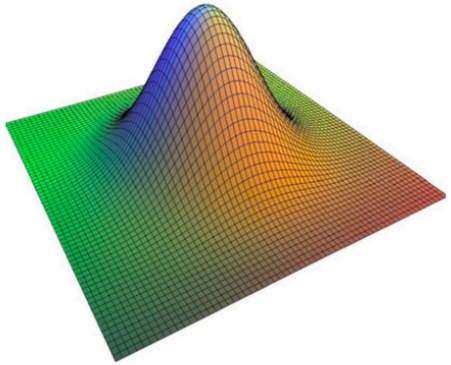
$$\sum \begin{array}{|c|c|c|}\hline 0.11 & 0.16 & 0.03 \\ \hline 0.16 & 0.26 & 0.01 \\ \hline 0.11 & 0.16 & 0.01 \\ \hline \end{array} \times \begin{array}{|c|c|c|}\hline 0 & 0 & 0.9 \\ \hline 0.1 & 0.1 & 1 \\ \hline 0 & 0.1 & 1 \\ \hline \end{array} = 0.1$$

Bilateral Filter

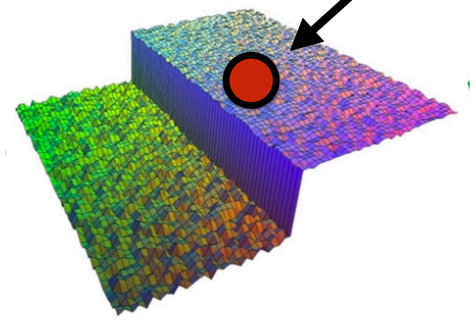
Bilateral Filter



Input

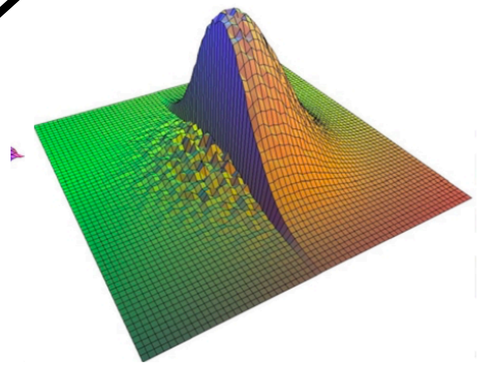


Domain Kernel



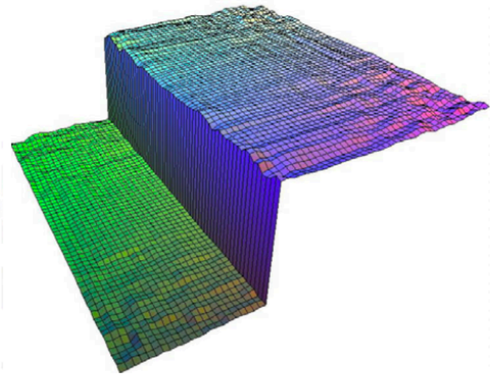
Range Kernel Influence

This example:
weights for point
on top of edge



Bilateral Filter

(domain * range)



Output

Images from: Durand and Dorsey, 2002

Bilateral Filter Application: Denoising

Noisy Image

Gaussian Filter

Bilateral Filter

Slide Credit: Alexander Wong

Bilateral Filter Application: Cartooning

Original Image

After 5 iterations of **Bilateral** Filter

Slide Credit: Alexander Wong

Bilateral Filter Application: Flash Photography

Non-flash images taken under low light conditions often suffer from excessive **noise** and **blur**

But there are problems with **flash images**:
— colour is often unnatural
— there may be strong shadows or specularities

Idea: Combine flash and non-flash images to achieve better exposure and colour balance, and to reduce noise

Bilateral Filter Application: Flash Photography

System using ‘joint’ or ‘cross’ bilateral filtering:

Flash | No-Flash | Detail Transfer with Denoising

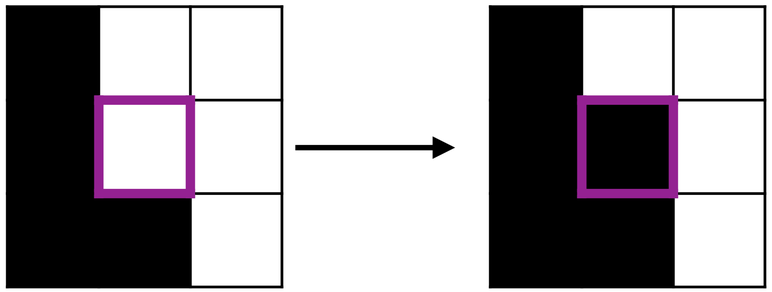
‘**Joint’ or ‘Cross’ bilateral**: Range kernel is computed using a separate guidance image instead of the input image

Figure Credit: Petschnigg et al., 2004

Morphology

- Non-linear binary image operations

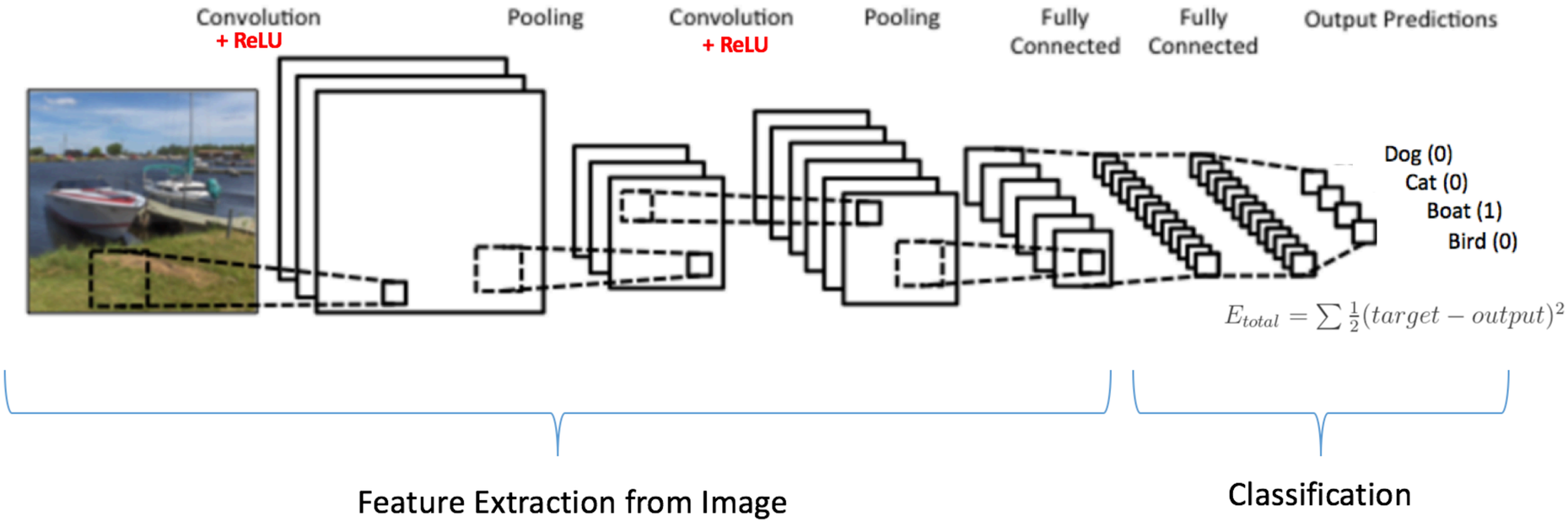
original dilate erode majority open close



Threshold function in local structuring element

close(.) = erode(dilate(.)) etc., see Szeliski 3.3.2

Aside: Linear Filter with ReLU



9	3	5	-8
-6	2	-3	1
1	3	4	1
3	-4	5	1

9	3	5	0
0	2	0	1
1	3	4	1
3	0	5	1

Result of: Linear Image Filtering After Non-linear ReLU

Summary

We covered two three **non-linear filters**: Median, Bilateral, ReLU

The **median filter** is a non-linear filter that selects the median in the neighbourhood

The **bilateral filter** is a non-linear filter that considers both spatial distance and range (intensity) distance, and has edge-preserving properties

Speeding-up Convolution can be achieved using separable filters or Fourier Transforms if the filter and image are both large

Fourier Transforms give us a way to think about image processing operations in Frequency Space, e.g., low pass filter = removing high frequency components

Menu for Today

Topics:

— **Linear Filtering** recap
— Efficient convolution, Fourier aside
— **Quiz 1**

— **Non-linear** Filters:
Median, ReLU, Bilateral Filter

Readings:

— **Today's** Lecture: Szeliski 3.3-3.4, Forsyth & Ponce (2nd ed.) 4.4

Reminders:

— **Assignment 1:** Image Filtering and Hybrid Images due **September 28th**