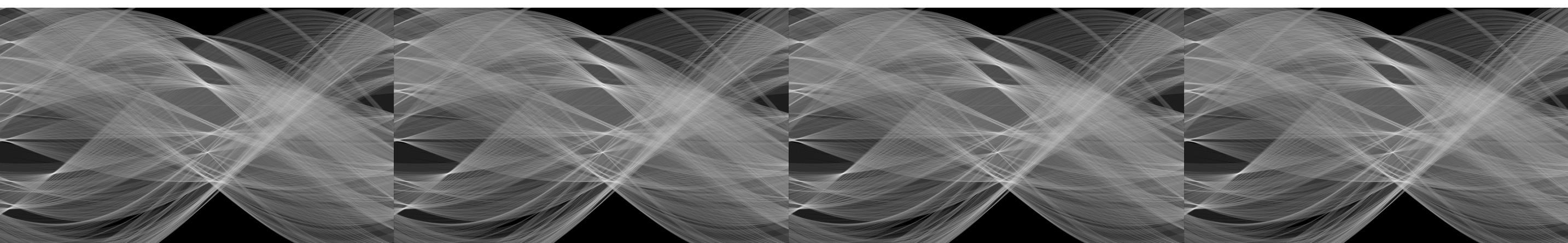




# CPSC 425: Computer Vision



**Image Credit:** Ioannis (Yannis) Gkioulekas (CMU)

## Lecture 14: Stereo

# Menu for Today

## Topics:

- 3D Correspondence, **Epipolar** Geometry
- **Stereo** Vision
- **Quiz 4**

## Readings:

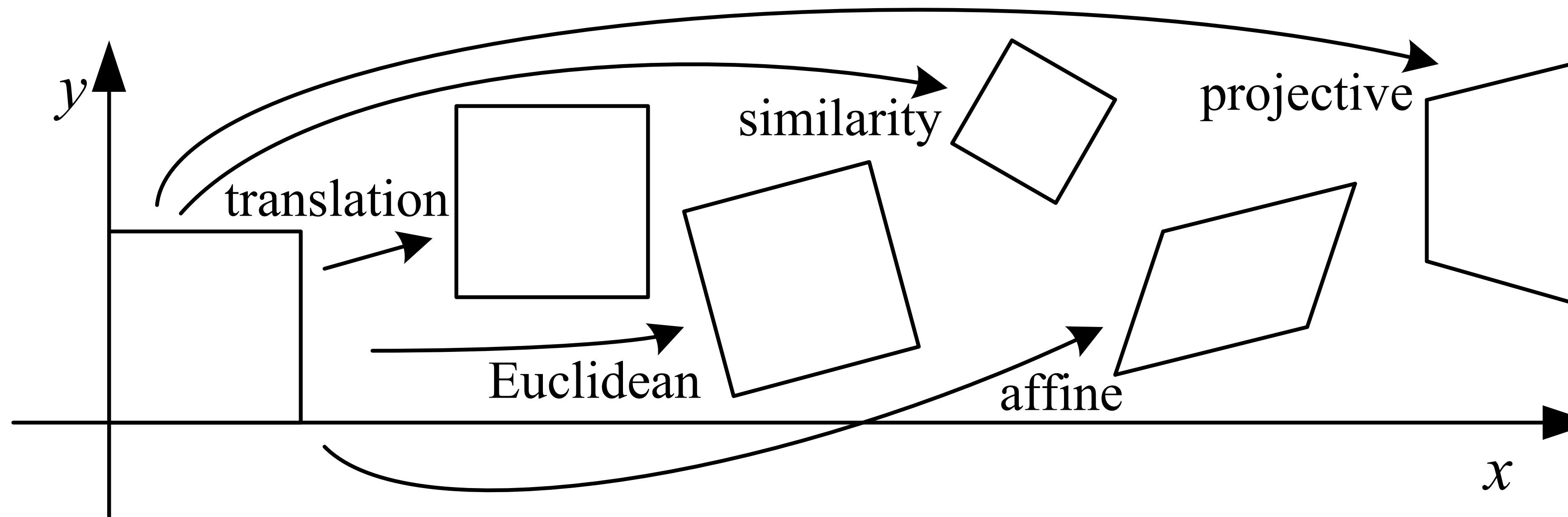
- **Today's** Lecture: Szeliski 12.1, 12.3-12.4, 9.3

## Reminders:

- **Assignment 4:** RANSAC and Panoramas due **November 9th**

# Recap: 2D Transformations

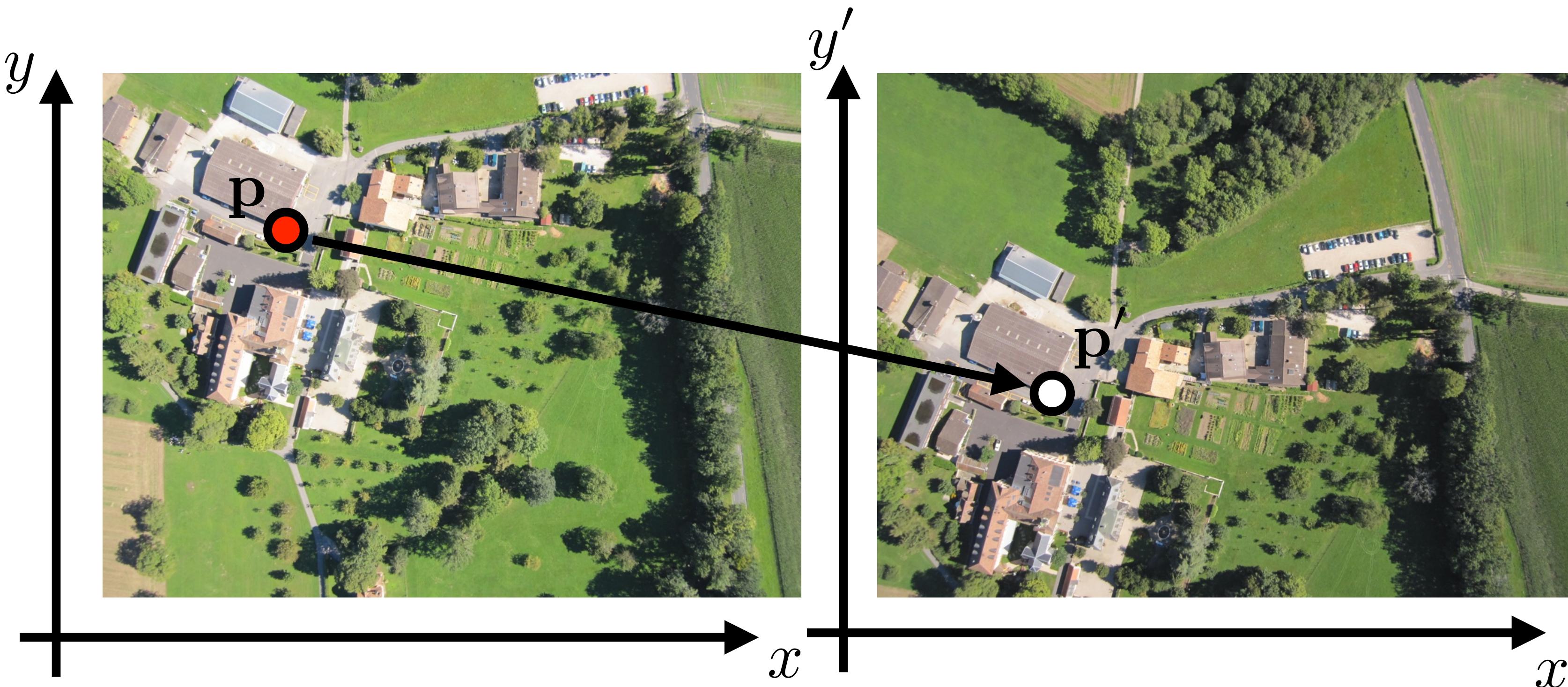
- We will look at a family that can be represented by  $3 \times 3$  matrices



This group represents perspective projections  
of **planar surfaces** in the world

# Recap: 2D Transformations

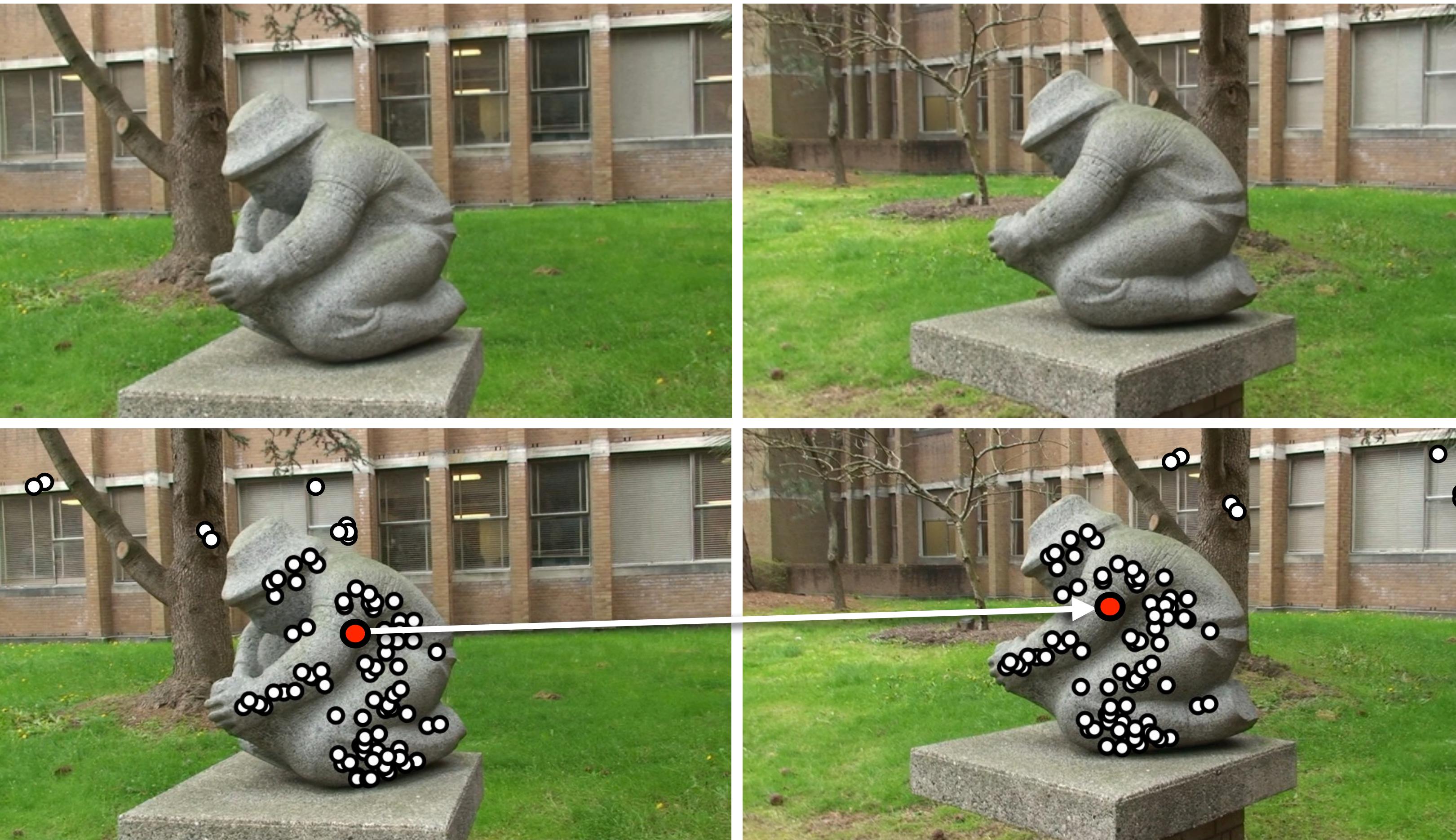
- 3x3 matrix defines one-to-one mapping between views



$$\begin{bmatrix} x'_1 \\ y'_1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

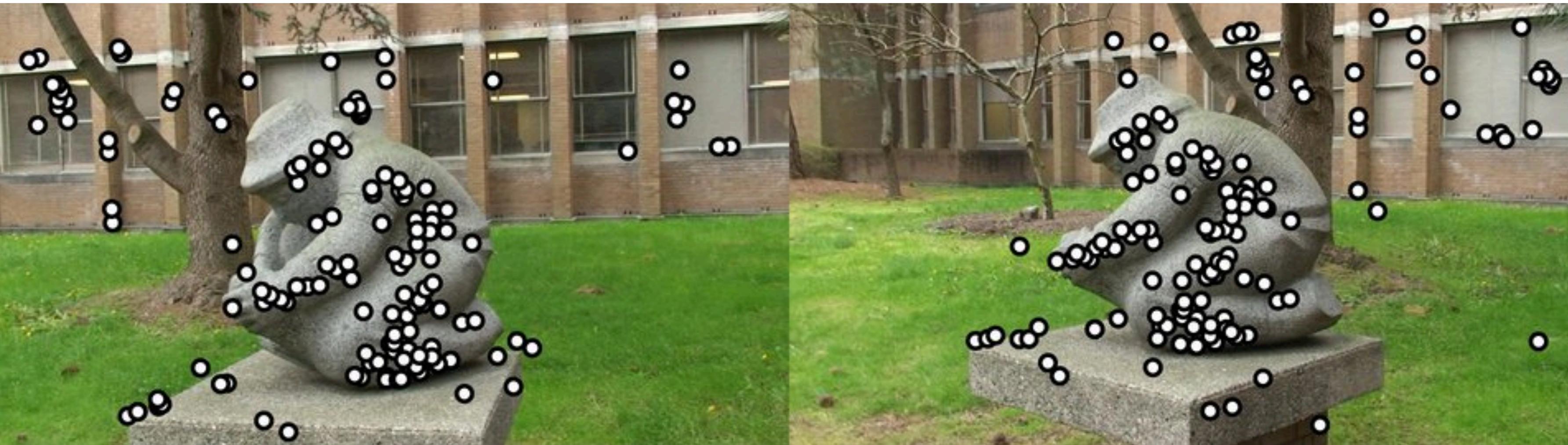
# Correspondences in 3D

- Find all matches between views



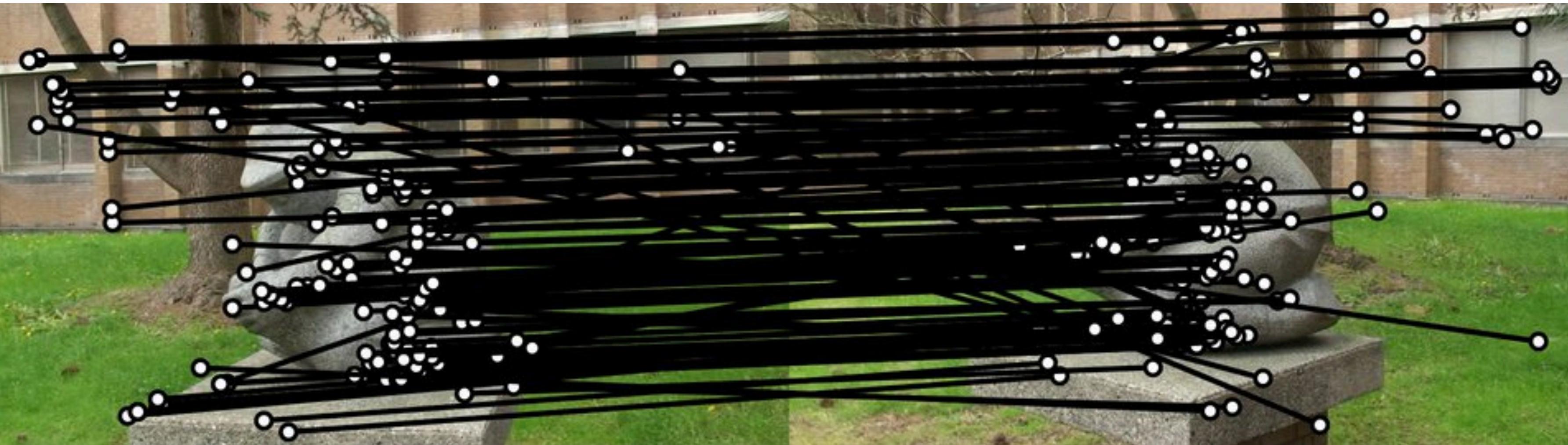
# Correspondences in 3D

- Find subset of matches that are consistent with a geometric transformation



# Correspondences in 3D

- Find subset of matches that are consistent with a geometric transformation



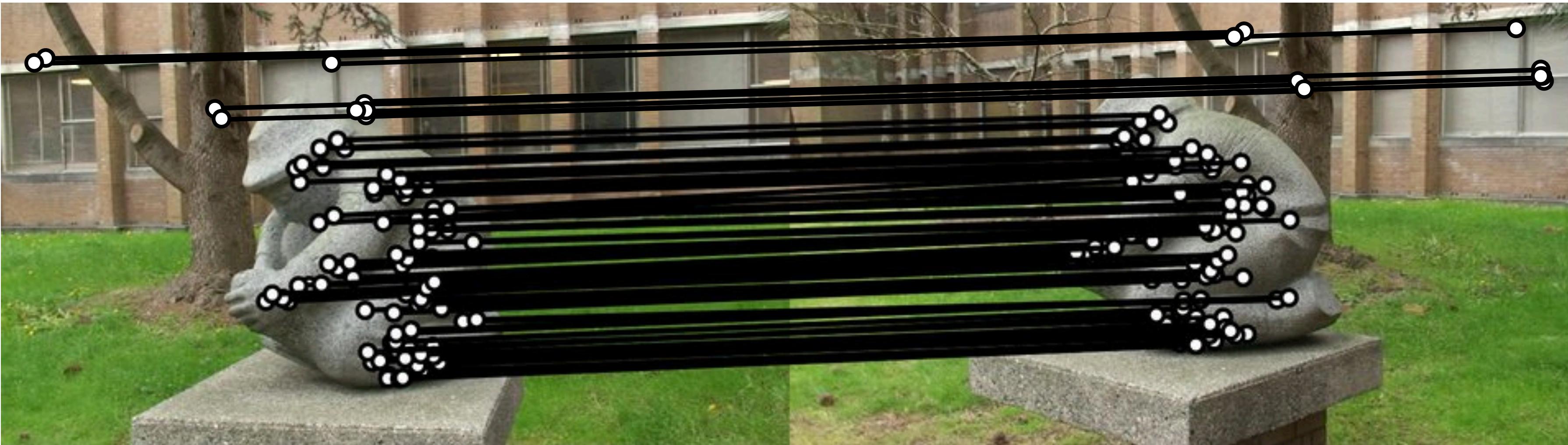
# Correspondences in 3D

- Find subset of matches that are consistent with a geometric transformation



# Correspondences in 3D

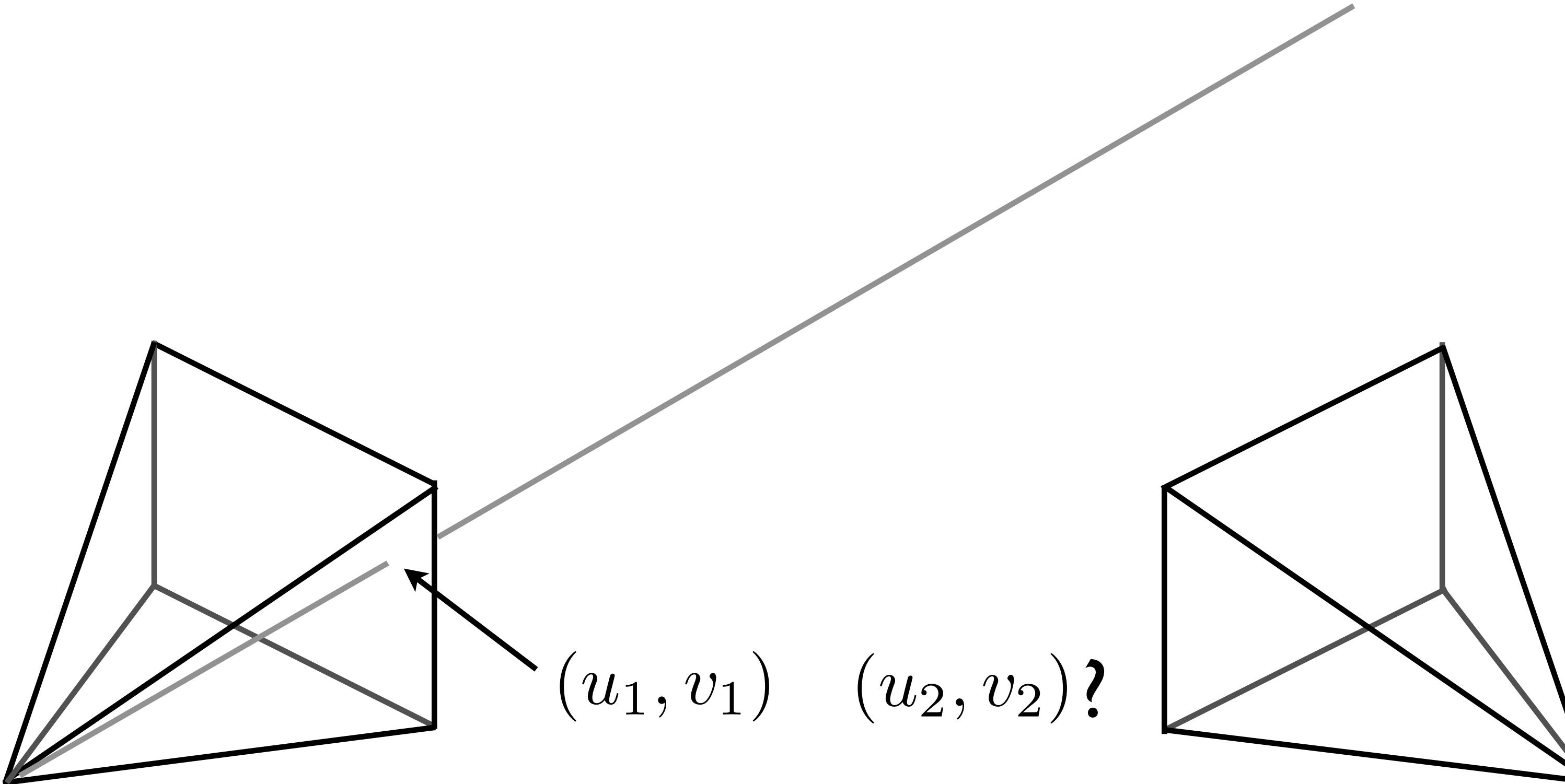
- Find subset of matches that are consistent with a geometric transformation



Consistent matches can be used for subsequent stages,  
e.g., 3D reconstruction, object recognition etc.

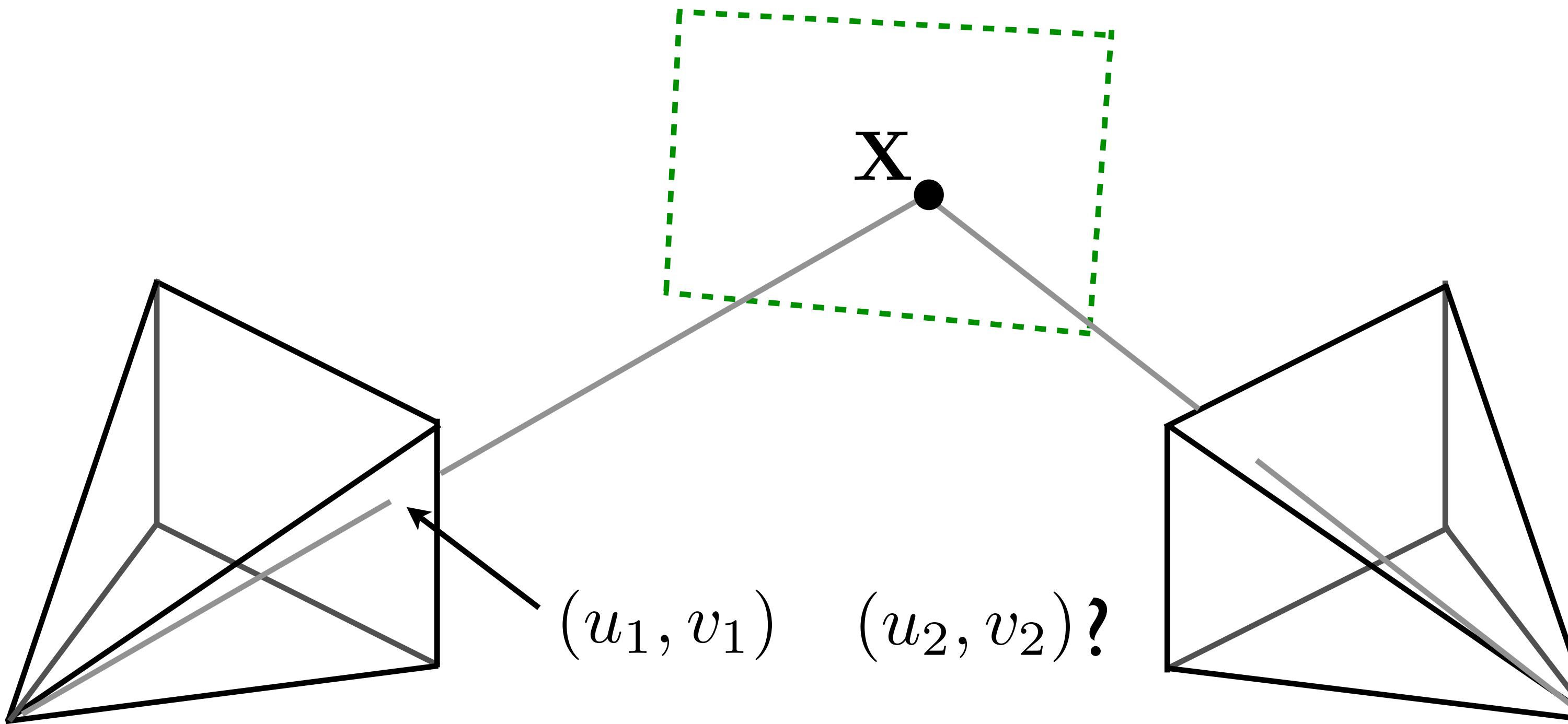
# 2-view Geometry

- How do we transfer points between 2 views?



# 2-view Geometry

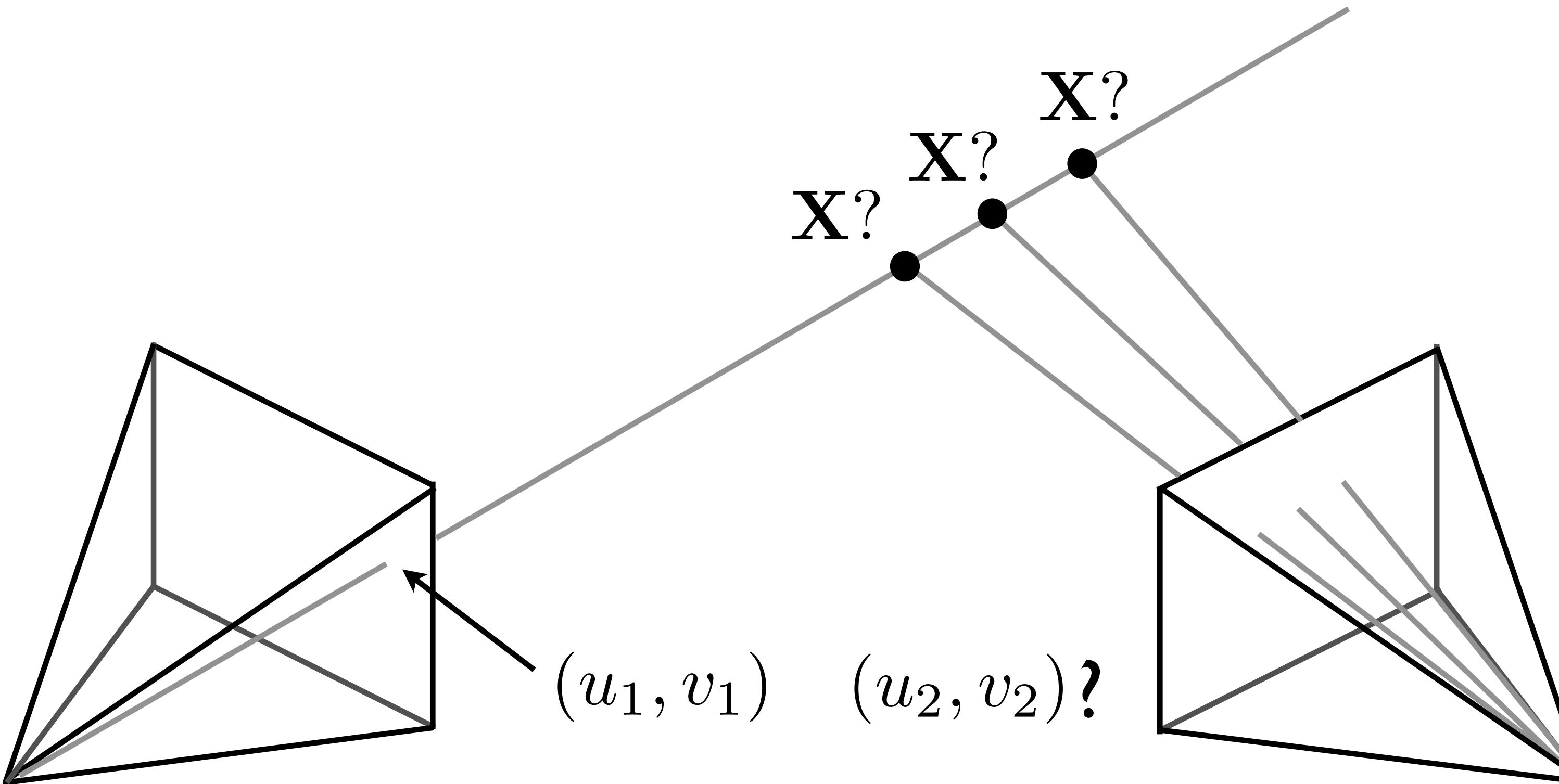
- How do we transfer points between 2 views? (planar case)



Planar case: one-to-one mapping via plane (Homography)

# 2-view Geometry

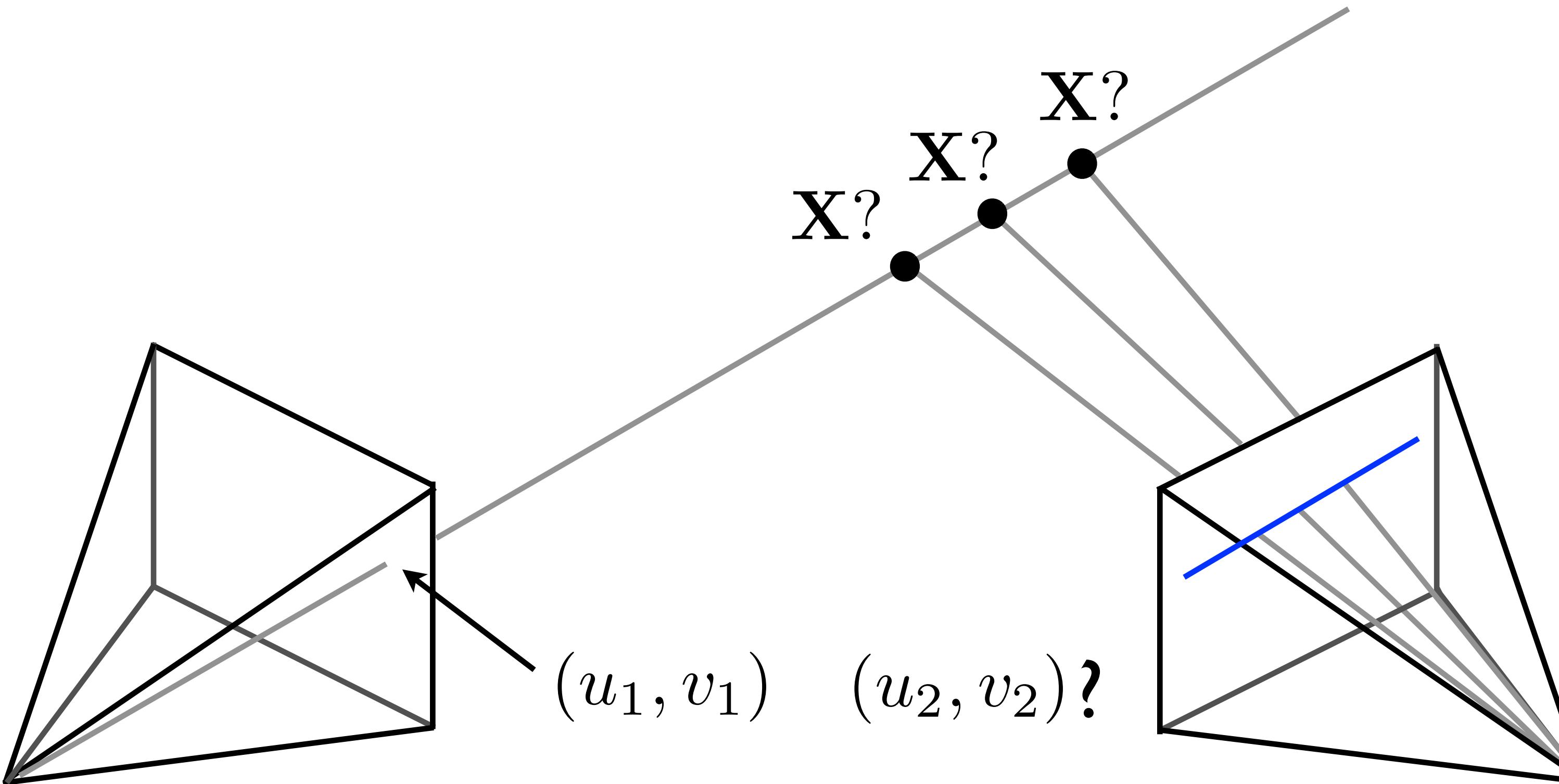
- How do we transfer points between 2 views? (non-planar)



Non-planar case: depends on the depth of the 3D point

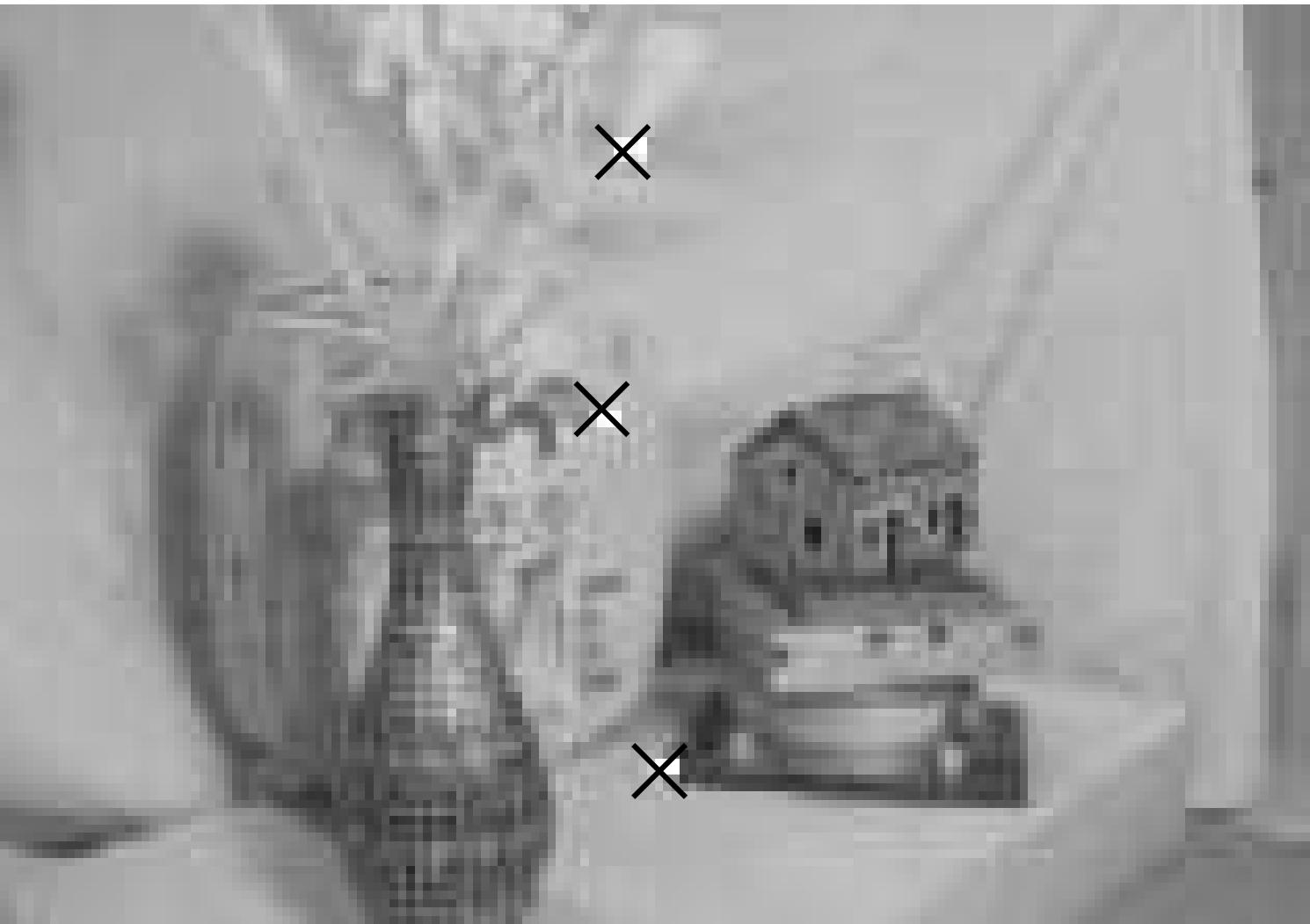
# Epipolar Line

- How do we transfer points between 2 views? (non-planar)

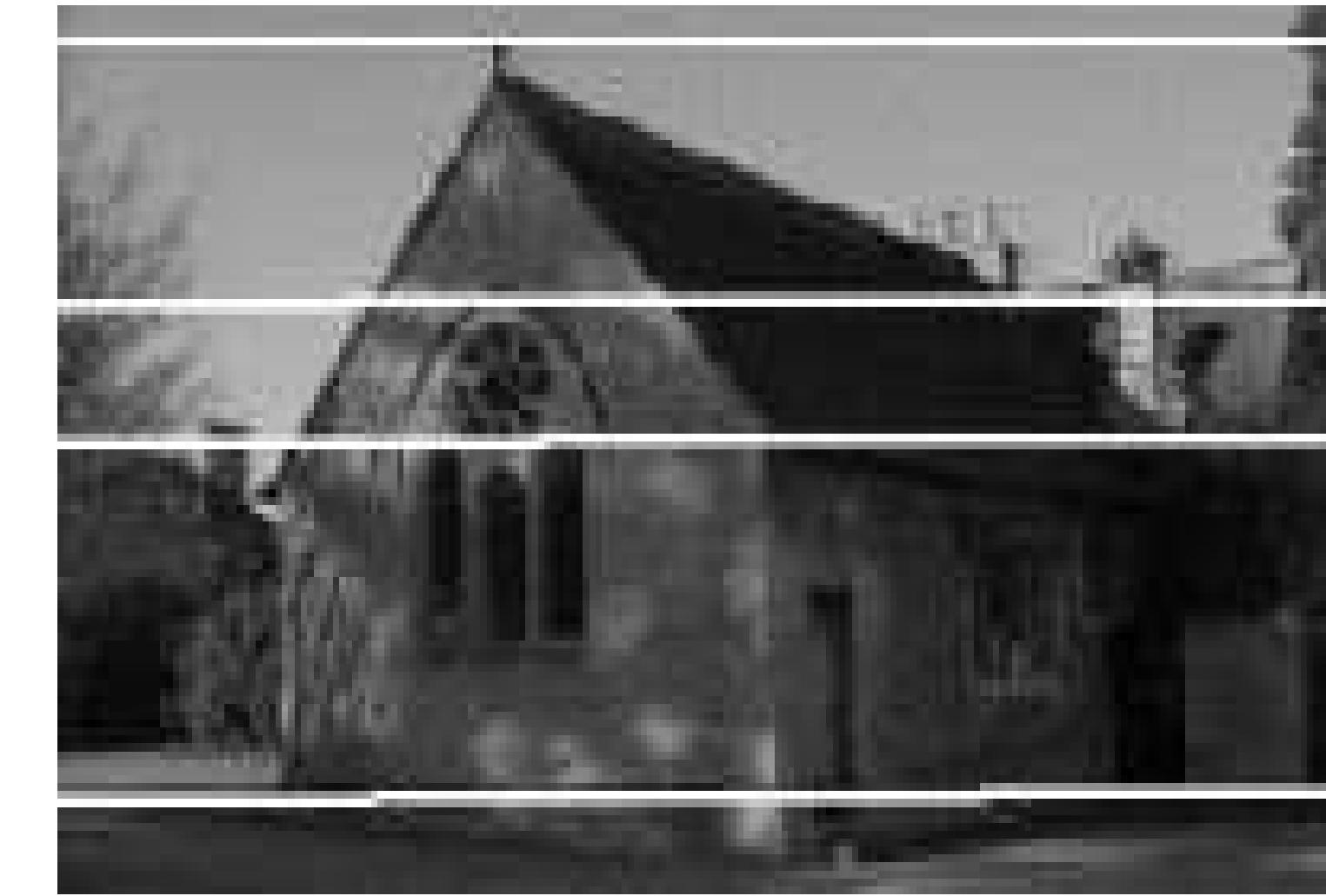


A point in image 1 gives a **line** in image 2

# Epipolar Lines

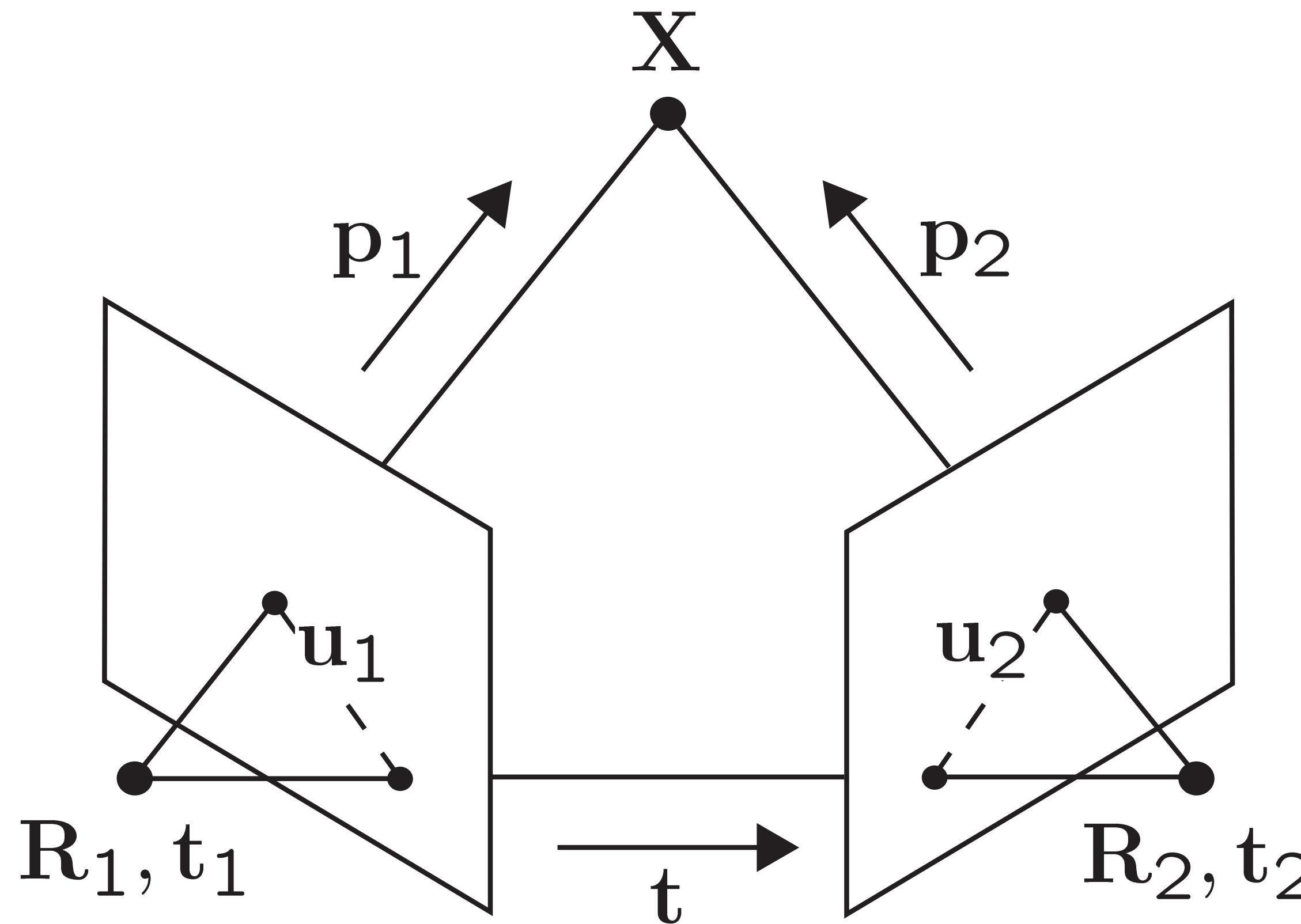


# Epipolar Lines



# Epipolar Geometry

- For rays to intersect at a point ( $X$ ), the two rays and the camera translation must lie in the same plane



14.1

# Epipolar Geometry

- Example: 2-view matching in 3D



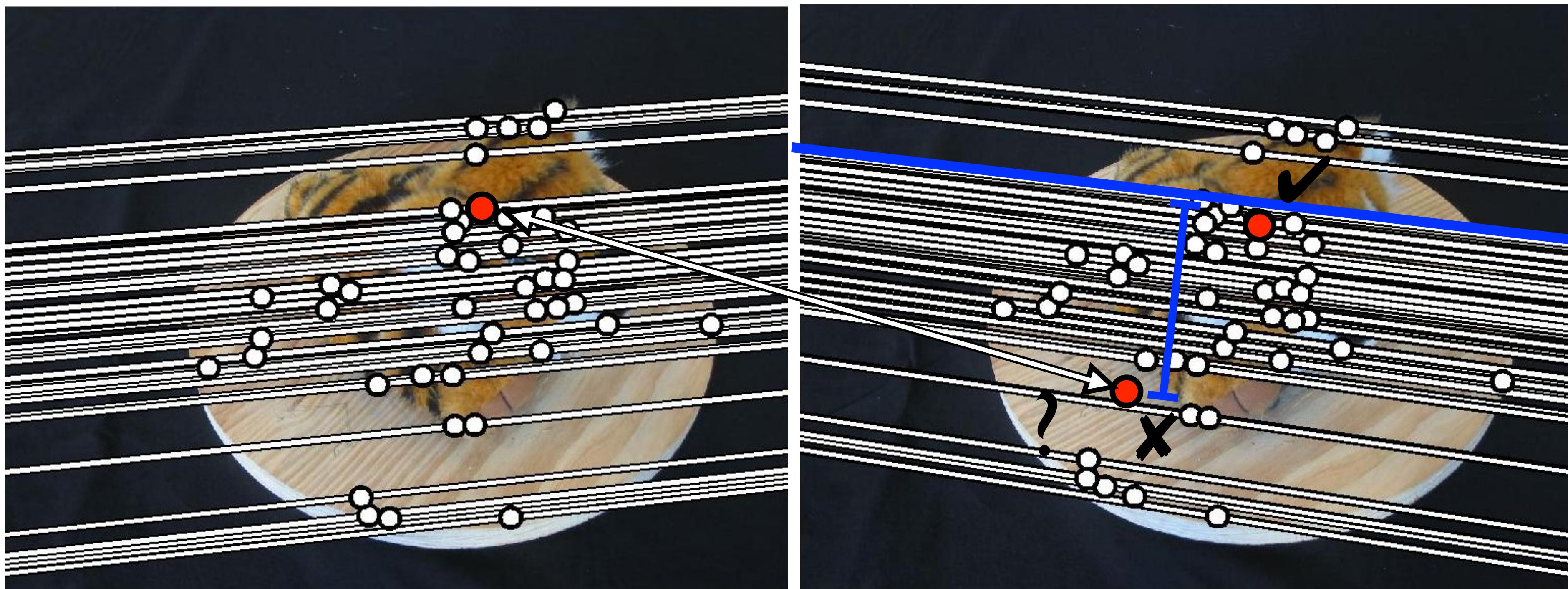
# Epipolar Geometry

- Raw SIFT matches



# Epipolar Geometry

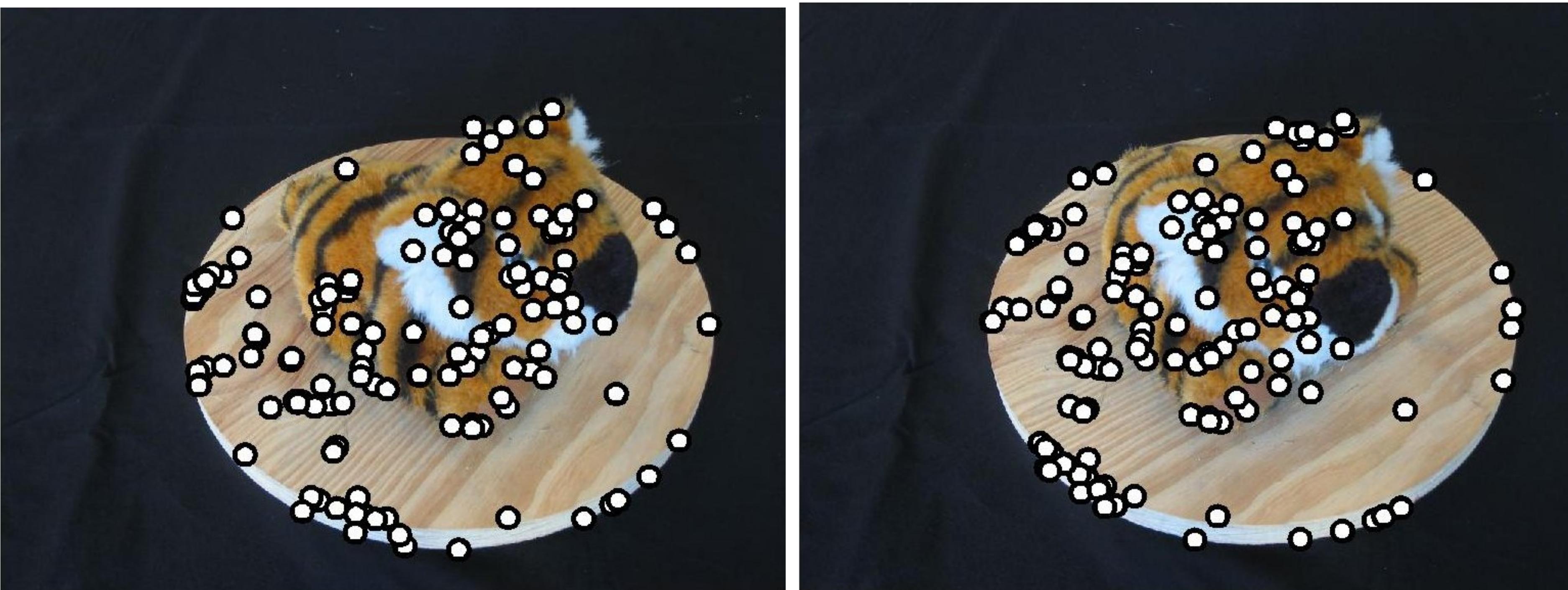
- Epipolar lines



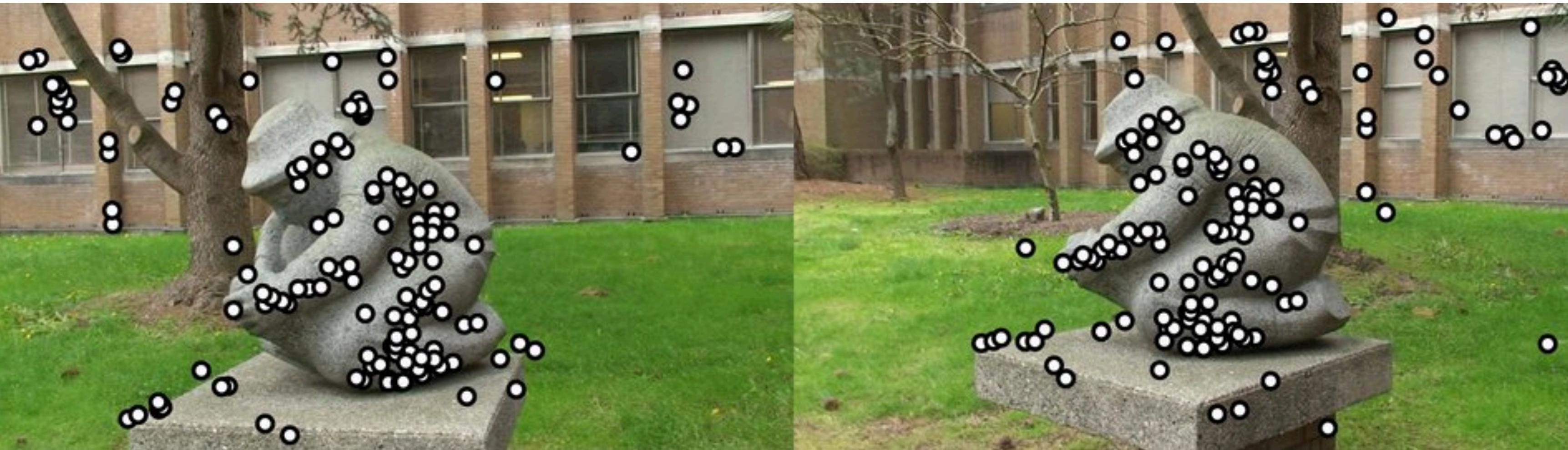
Can use RANSAC to find inliers with  
small distance from epipolar line

# Epipolar Geometry

- Consistent matches



# RANSAC for Epipolar Geometry



Raw feature matches (after ratio test filtering)

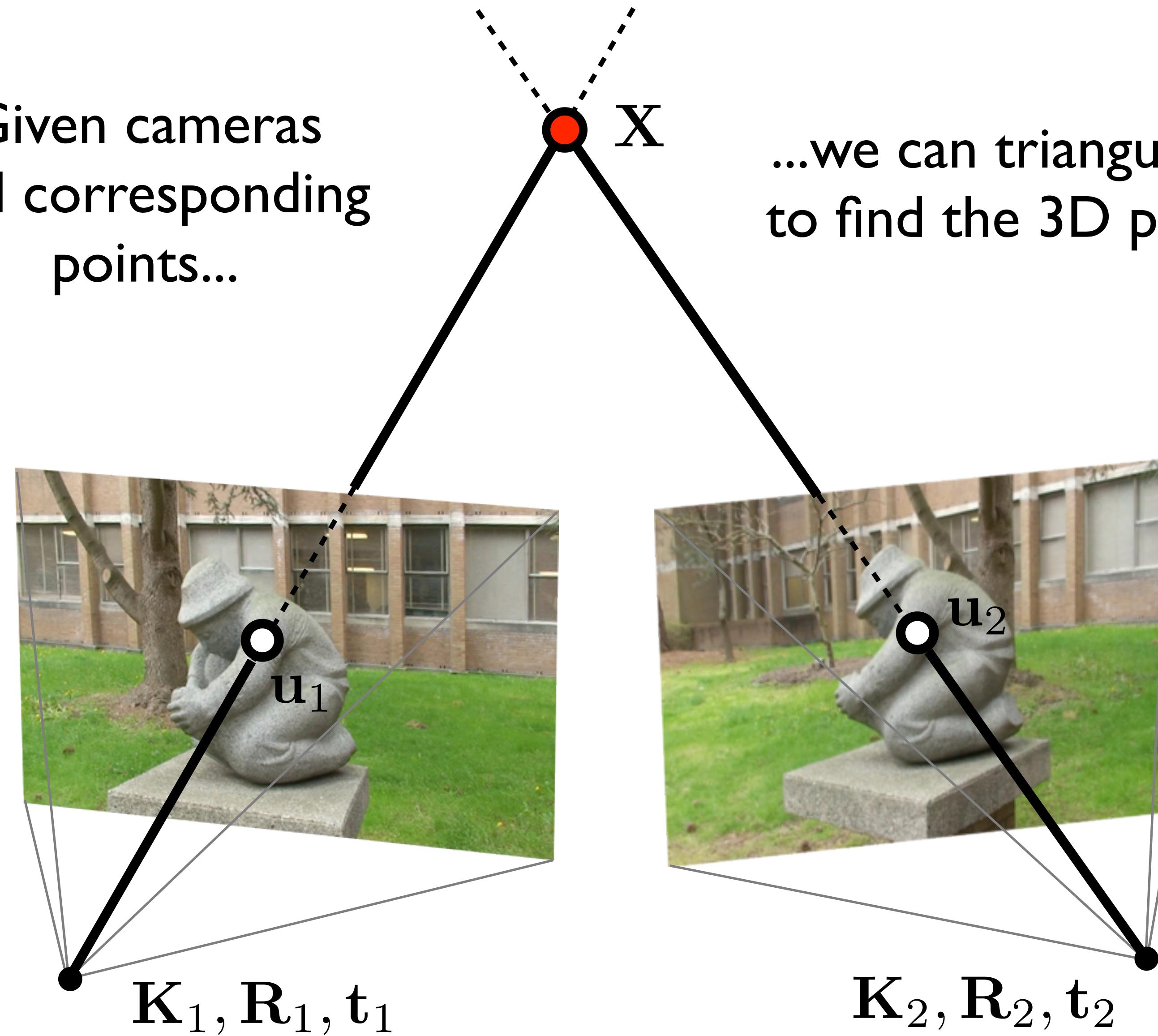


Solve for camera geometry and RANSAC inliers

# Triangulation

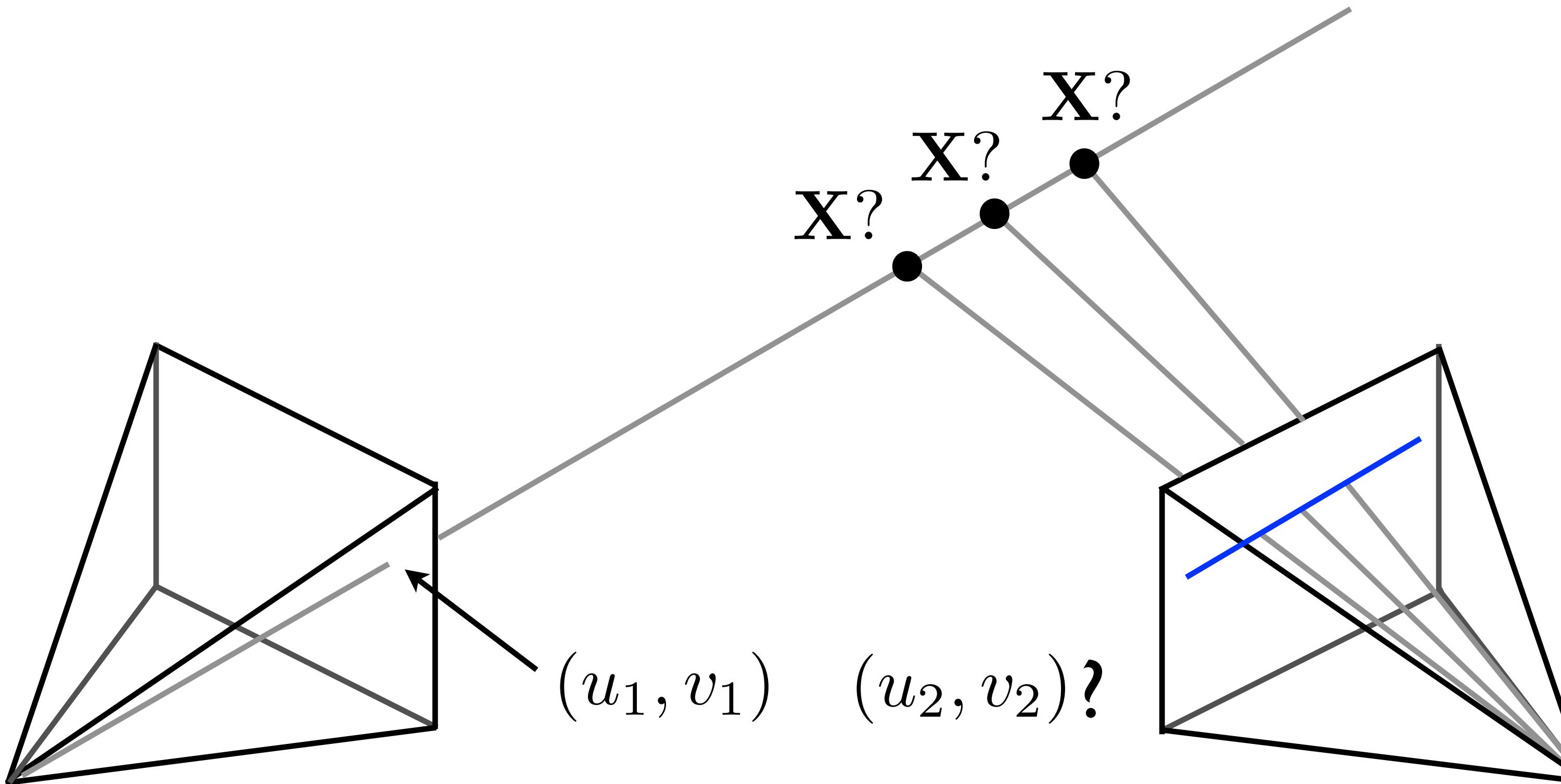
Given cameras  
and corresponding  
points...

...we can triangulate  
to find the 3D point



# Epipolar Geometry

- A point in one view may lie on a line in the 2nd



Position in image 2 depends on the **depth** of the 3D point

# 2-view Stereo

- Camera motion only, points constrained to epipolar lines



ID Search

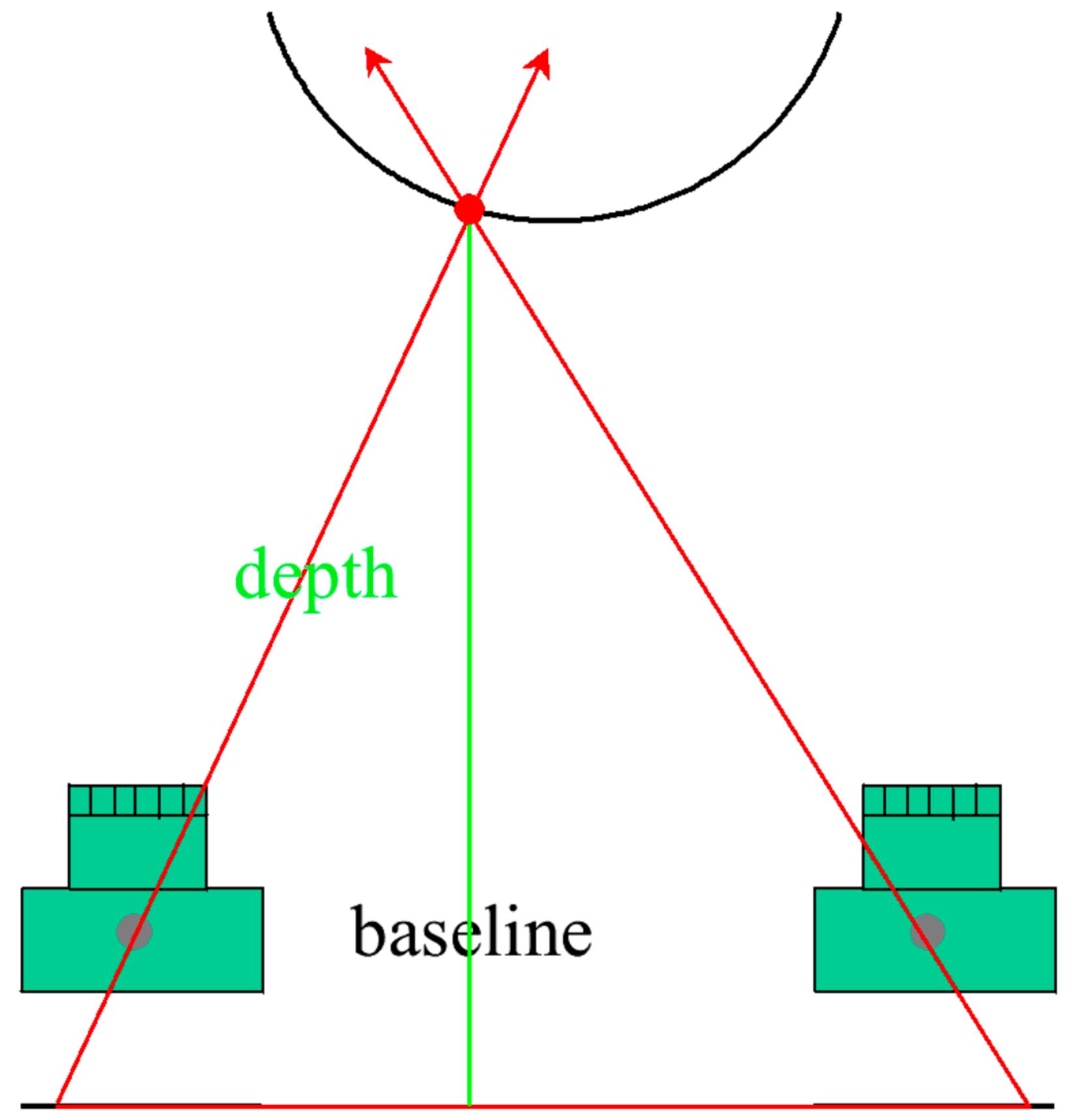
# Stereo Camera Configuration

- Humans and many stereo cameras have parallel optical axes



# Axis Aligned Stereo

- A common stereo configuration has camera optical axes aligned, with cameras related by a translation in the x direction



# Stereo Matching

- In a standard stereo setup, where cameras are related by translation in the x direction, epipolar lines are horizontal



- Stereo algorithms search along scanlines for matches
- Distance along the scanline (difference in x coordinate) for a corresponding feature is called **disparity**

# Disparity and Depth: R



# Disparity and Depth: L



# Disparity and Depth: R+L



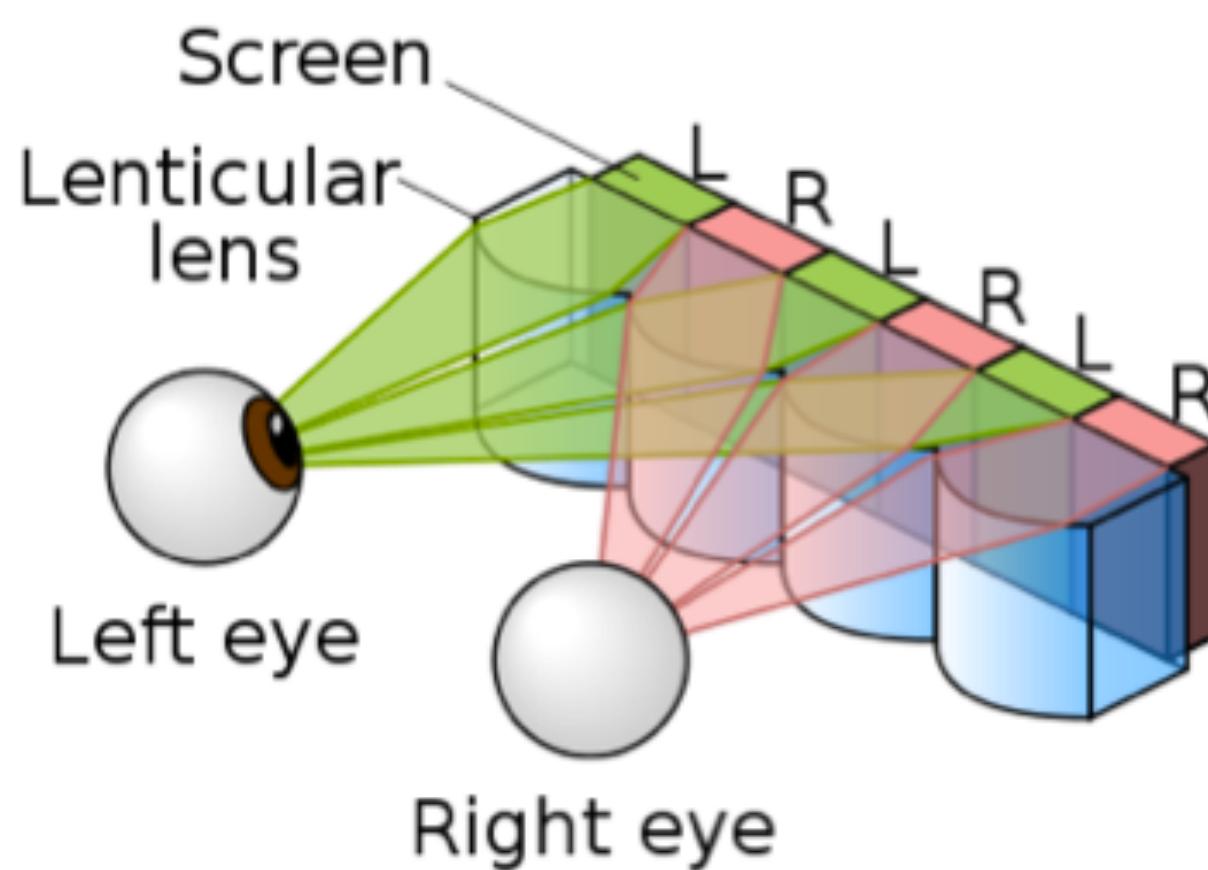
# Anaglyph

- Stereo pair with images encoded in different color channels



# Stereo Displays

- Field sequential (shutter) glasses transmit alternate left/right image at 120Hz



**Lenticular lenses** send different images directly to each eye, without the need for glasses

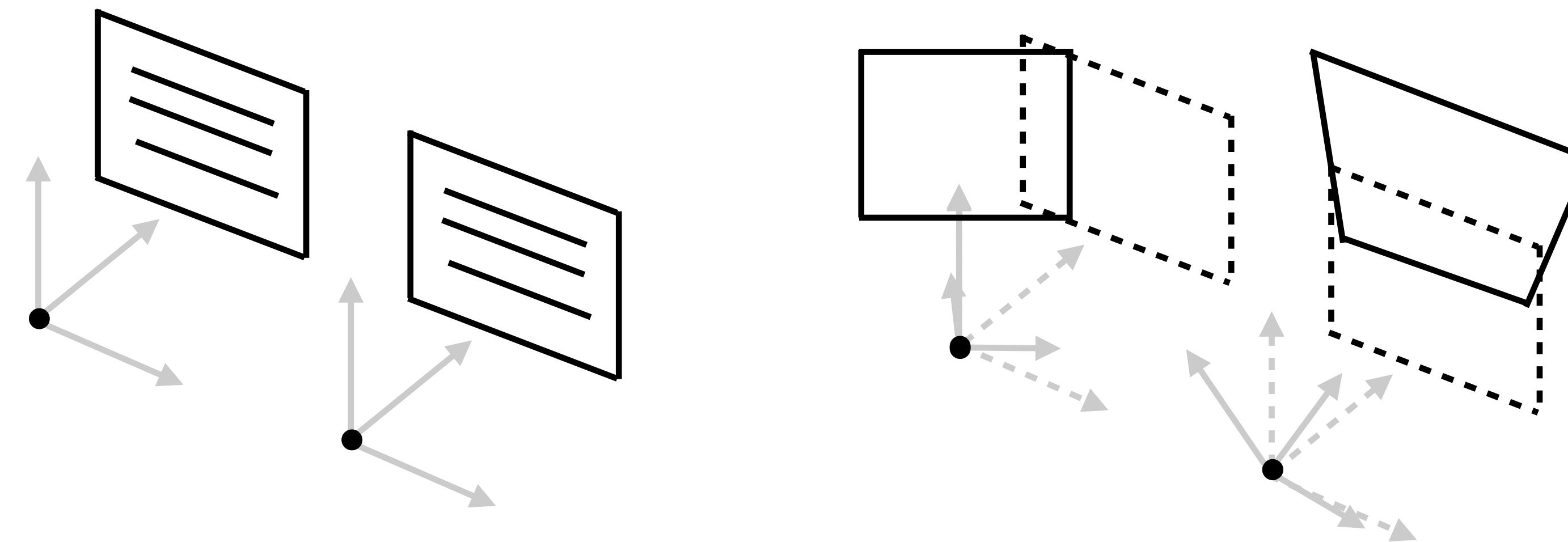
# Stereo Displays

- VR headsets send L/R images directly to each eye



# Stereo Rectification

- If the optical axes are not aligned, we can rotate the images (homography) until they are perpendicular to the baseline

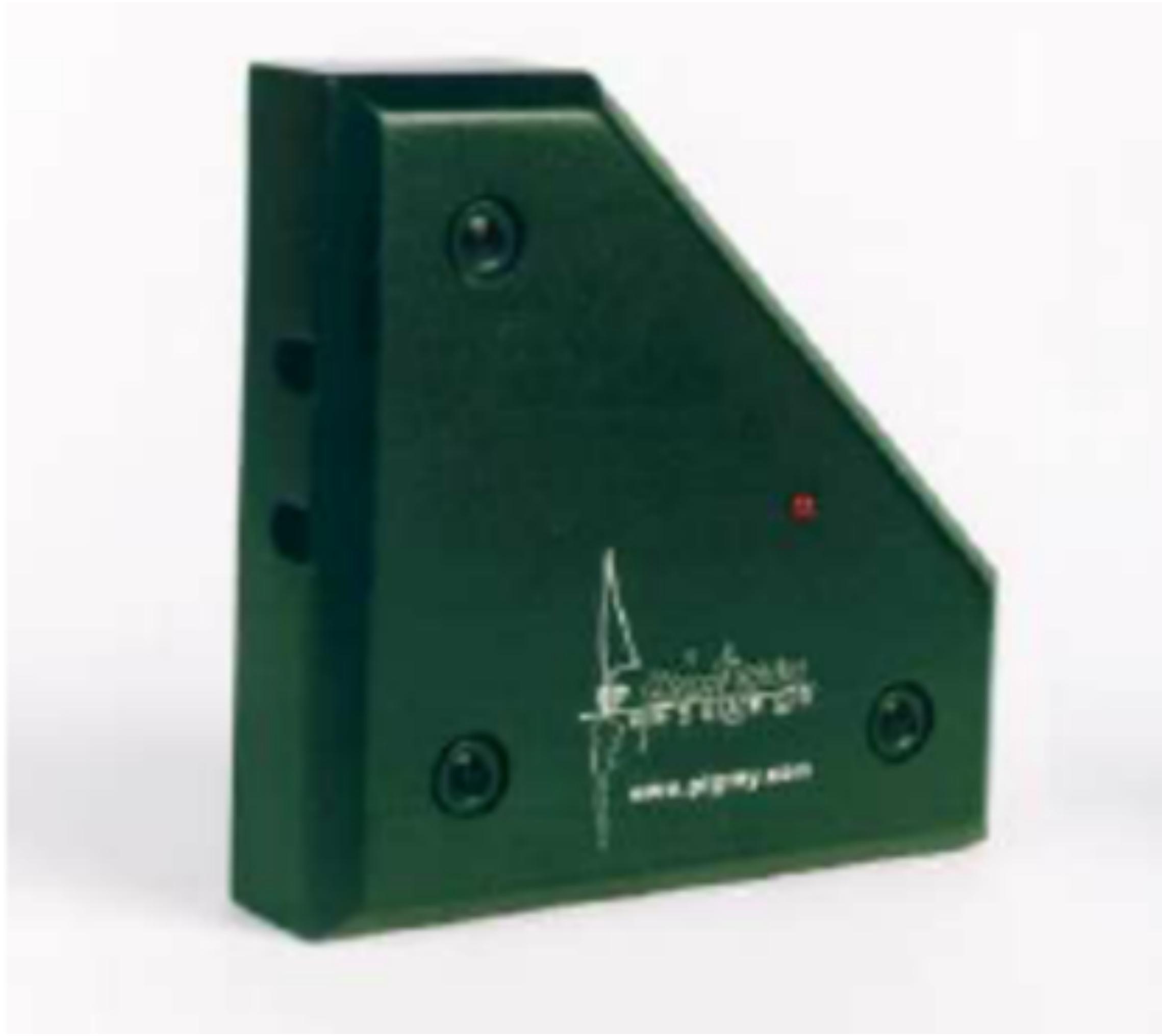


# Stereo Rectification

- Transform (rotate) images so that epipolar lines are horizontal



# Point Grey Research **Digiclops**



**Image credit:** Point Grey Research

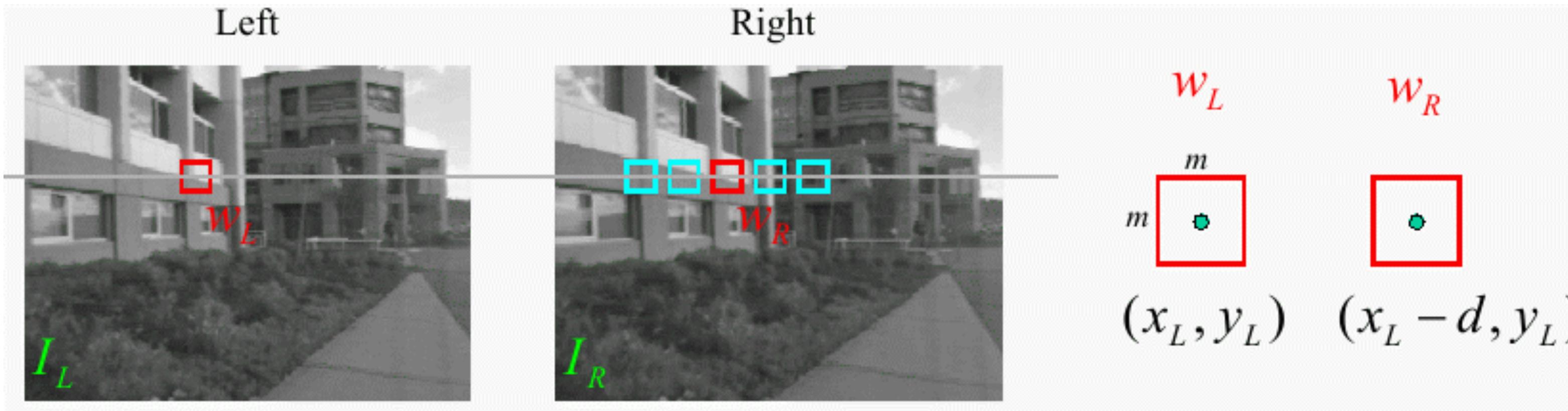
# Stereo Matching

- In a standard stereo setup, where cameras are related by translation in the x direction, epipolar lines are horizontal



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- Distance along the scanline (difference in x coordinate) for a corresponding feature is called **disparity**

# Matching along a Scanline



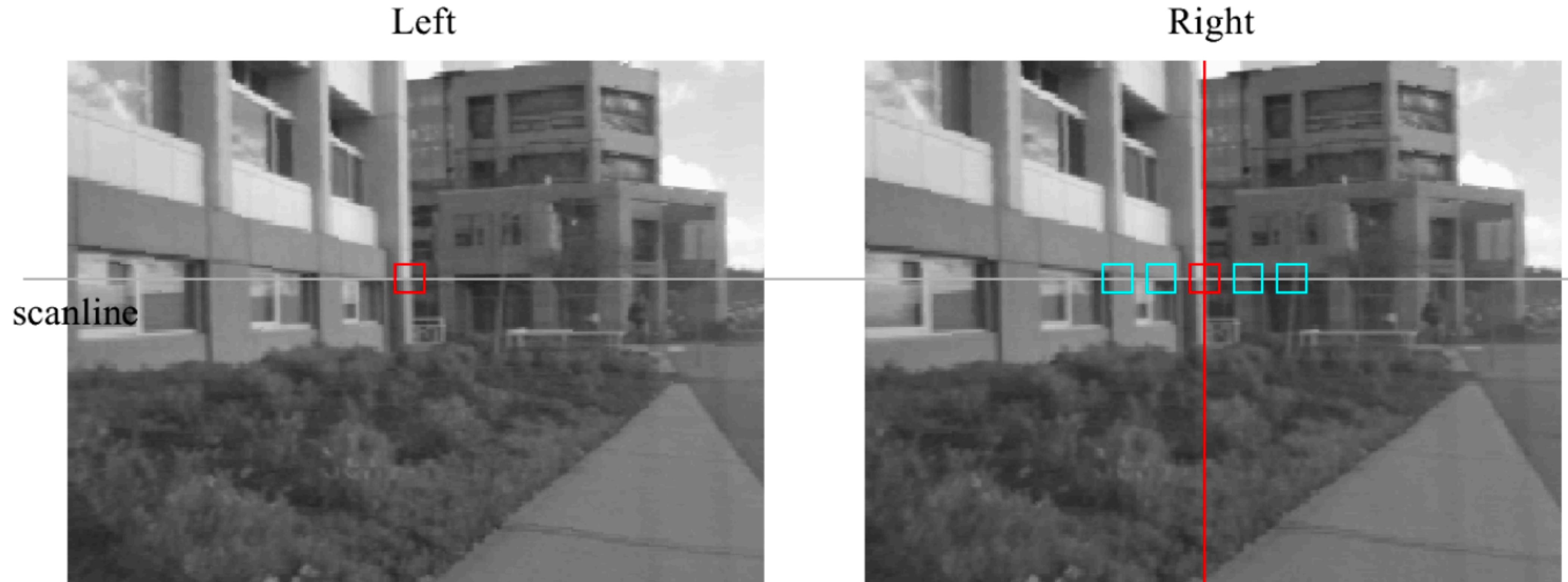
$w_L$  and  $w_R$  are corresponding  $m \times m$  windows of pixels

Define a distance function between image patches, e.g.,

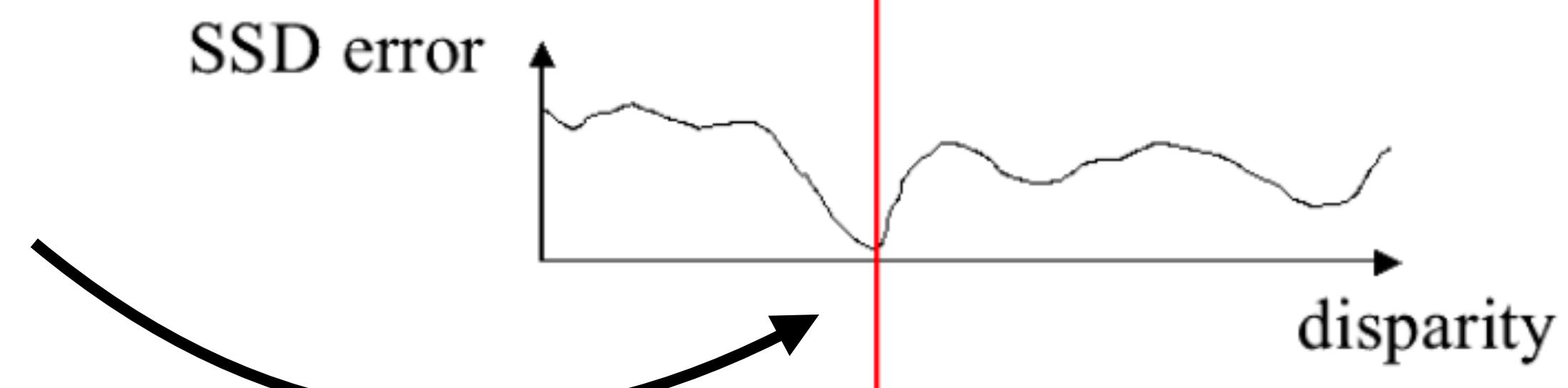
$$\text{SSD} = \|\mathbf{w}_L - \mathbf{w}_R(d)\|^2$$

$$\text{correlation} = \mathbf{w}_L \cdot \mathbf{w}_R(d) = \cos \theta$$

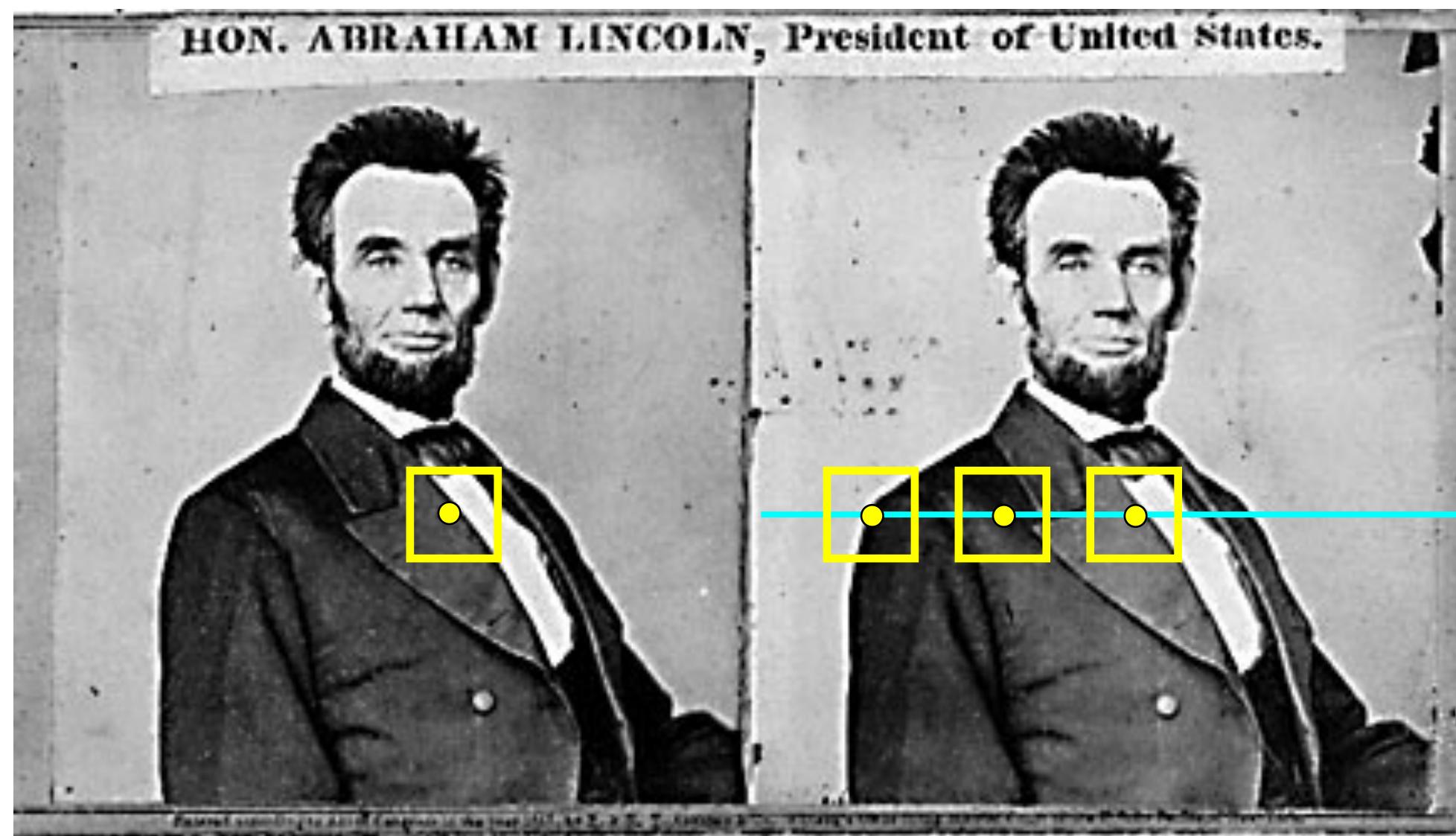
# Matching along a Scanline



Best match is at minimum of  
SSD function along a scanline



# (simple) Stereo Algorithm



1. Rectify images  
(make epipolar lines horizontal)
2. For each pixel in image 1
  - a. Search along epipolar line in image 2
  - b. Find best match and record offset = disparity
  - c. Compute depth from disparity

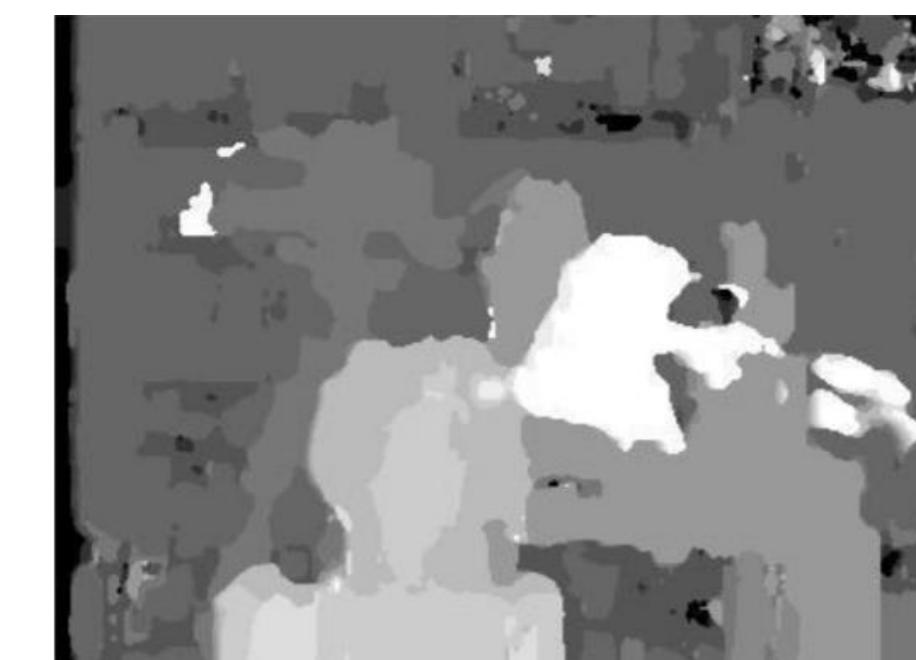
$$Z = f \frac{\Delta X}{disp}$$

# Effect of Window Size

- Larger windows → smoothed result



$W=3$



$W=11$



$W=25$

## Smaller window

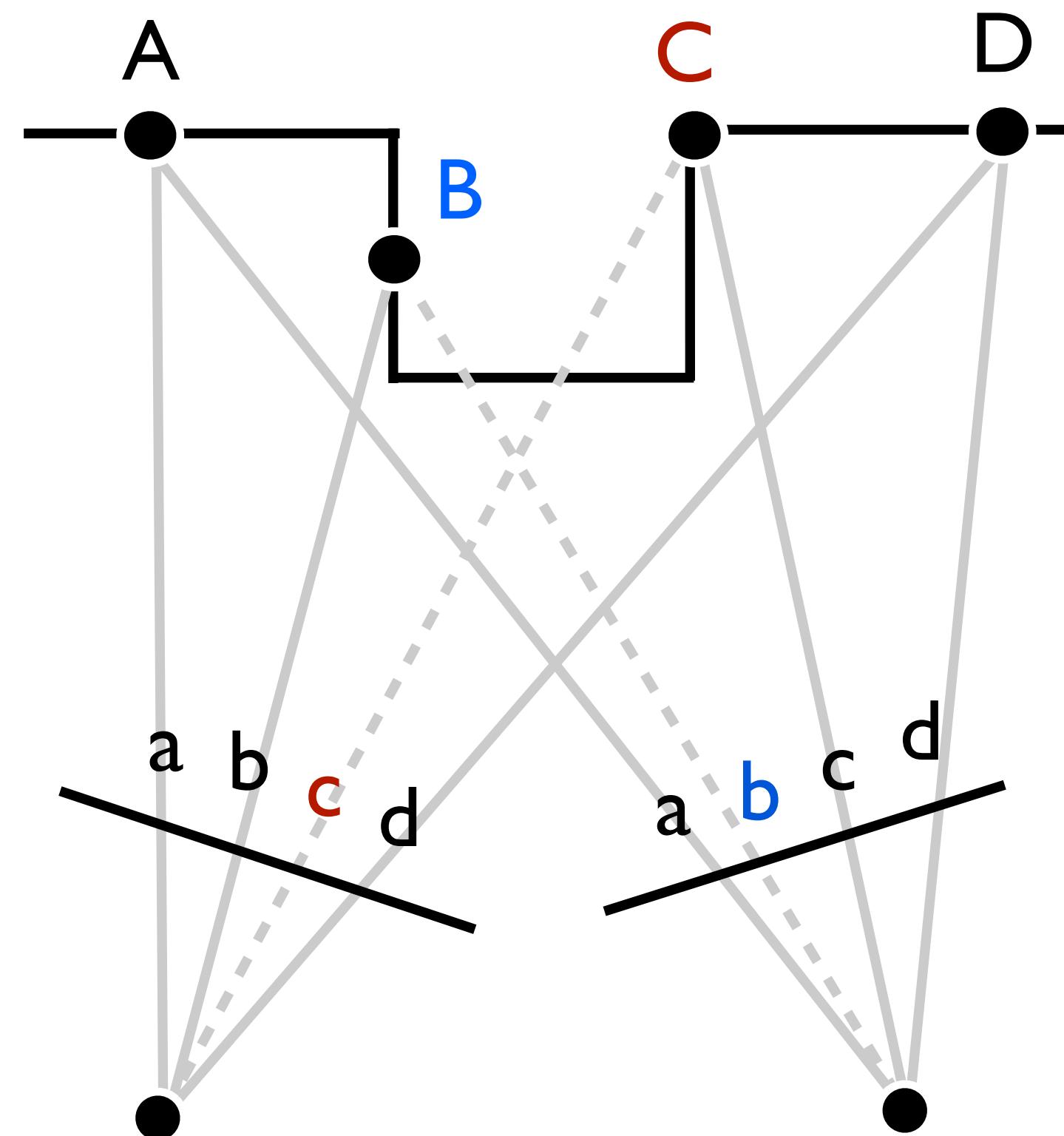
- + More detail
- More noise

## Larger window

- + Smoother disparity maps
- Less detail
- Fails near boundaries

# Occlusions

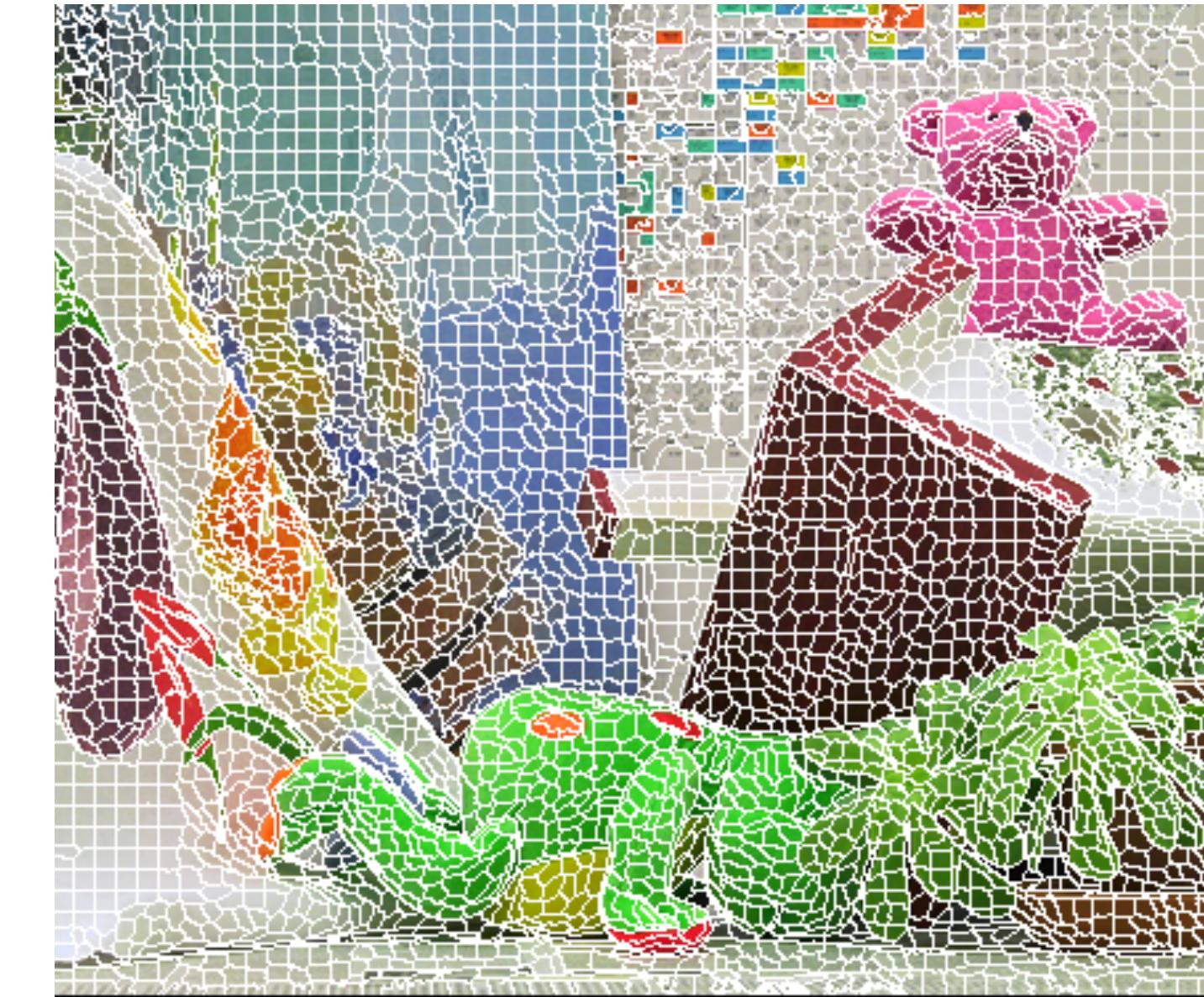
- Sometimes a point in image 1 does not appear in image 2, or vice-versa (this is called an **occlusion**)



- Occlusions cause gaps in the stereo reconstruction
- + Matching is difficult nearby as aggregation windows often overlap the occluded region

# Edge Aware Stereo

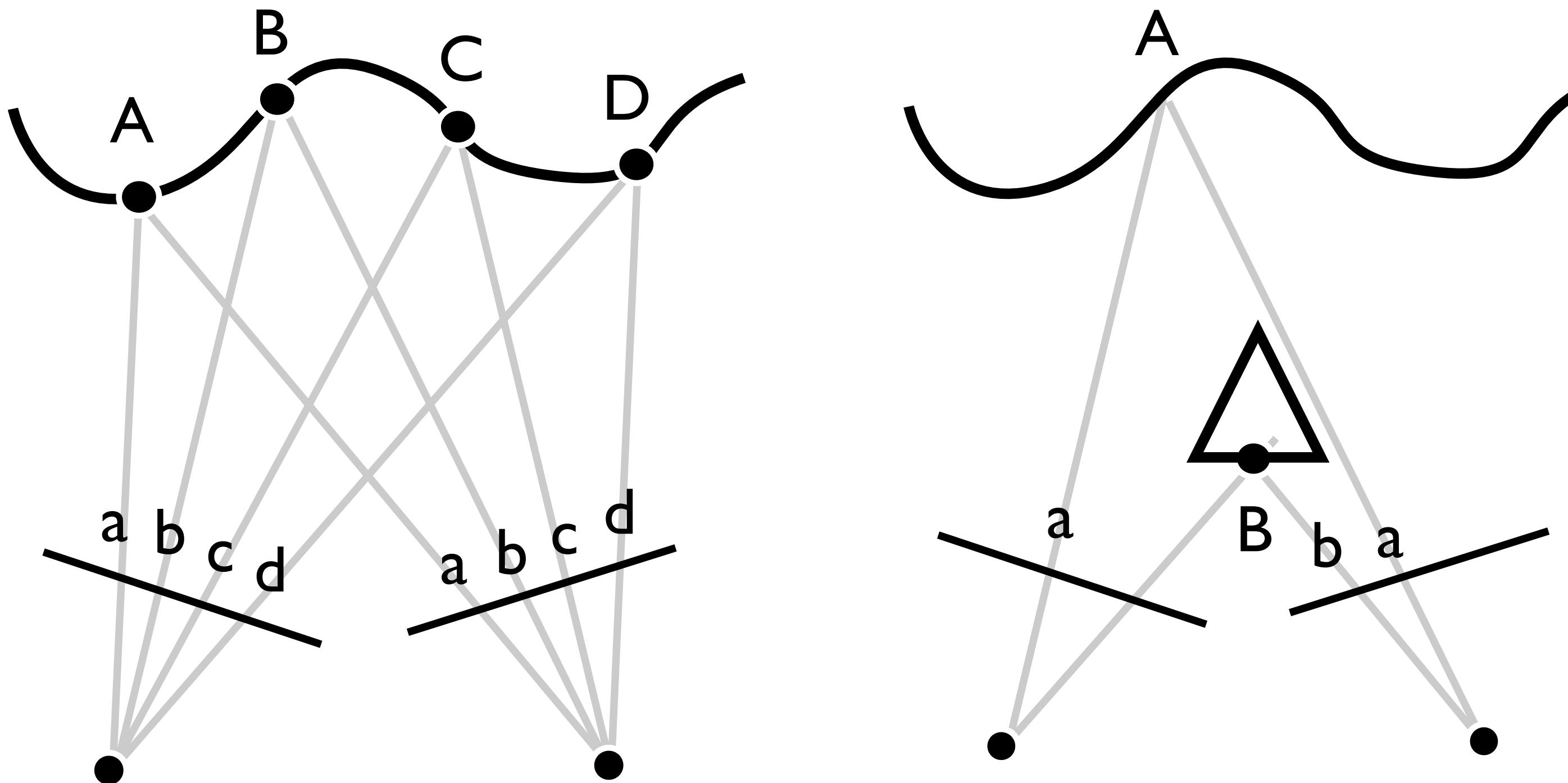
- Occlusions and depth discontinuities cause problems for stereo matching, as aggregation windows overlap multiple depths



- Segmentation-based stereo approaches aim to solve this by trying to guess the depth edges (e.g., joint segmentation and depth estimation [ Taguchi et al 2008 ])

# Ordering Constraint

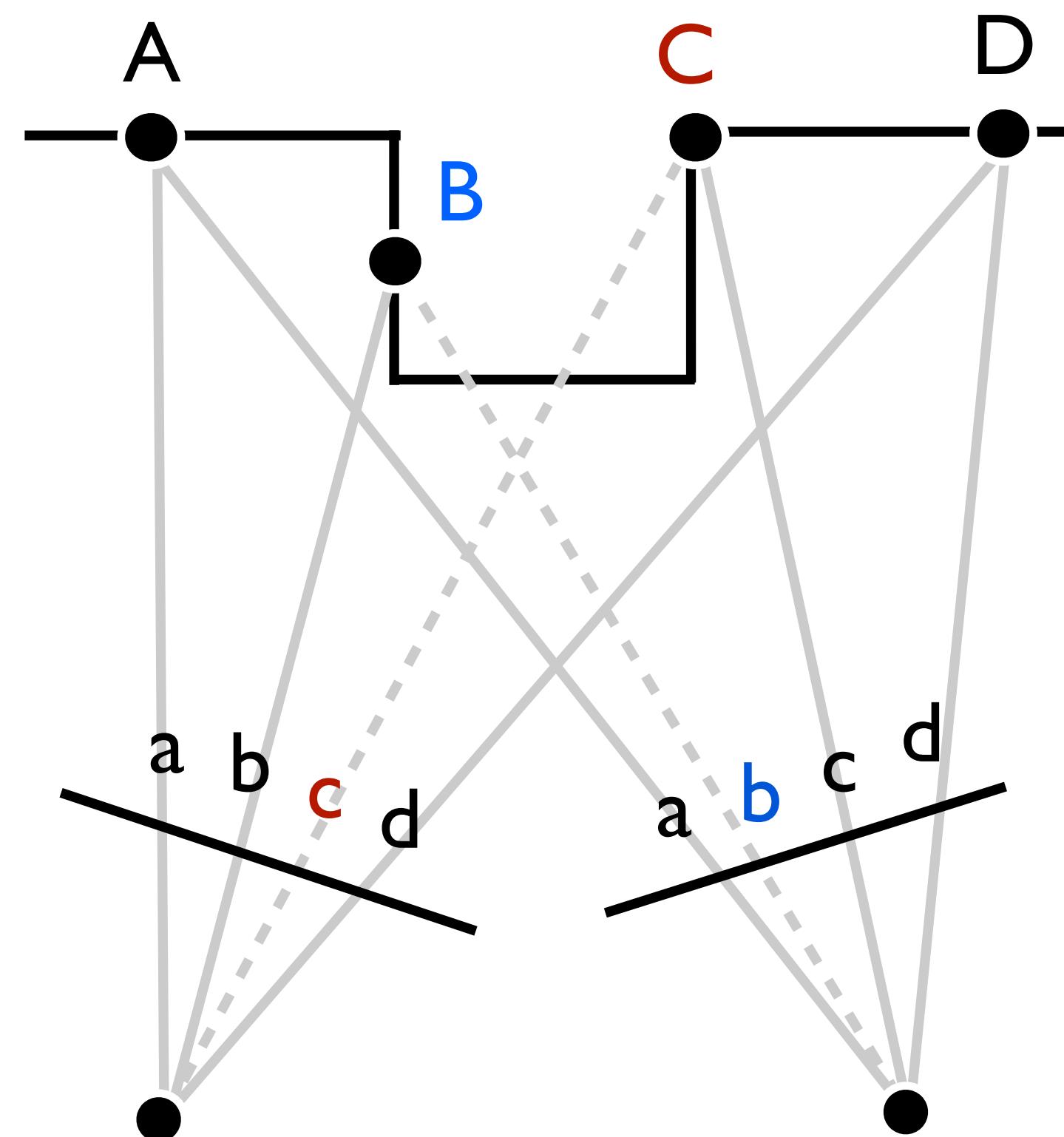
- If point B is to the right of point A in image 1, the same is *usually* true in image 2



Not always, e.g., if an object  
is wholly within the ray triangle  
generated by A

# Occlusions + Ordering

- Note that the ordering constraint is still maintained in the presence of occlusions (unless there is an object off surface as in the previous slide)



# Stereo Cost Functions

- Energy function for stereo matching based on disparity  $d(x,y)$
- Sum of data and smoothness terms

$$E(d) = E_d(d) + \lambda E_s(d)$$

- Data term is cost of pixel  $x,y$  allocated disparity  $d$  (e.g., SSD)

$$E_d(d) = \sum_{(x,y)} C(x, y, d(x, y))$$

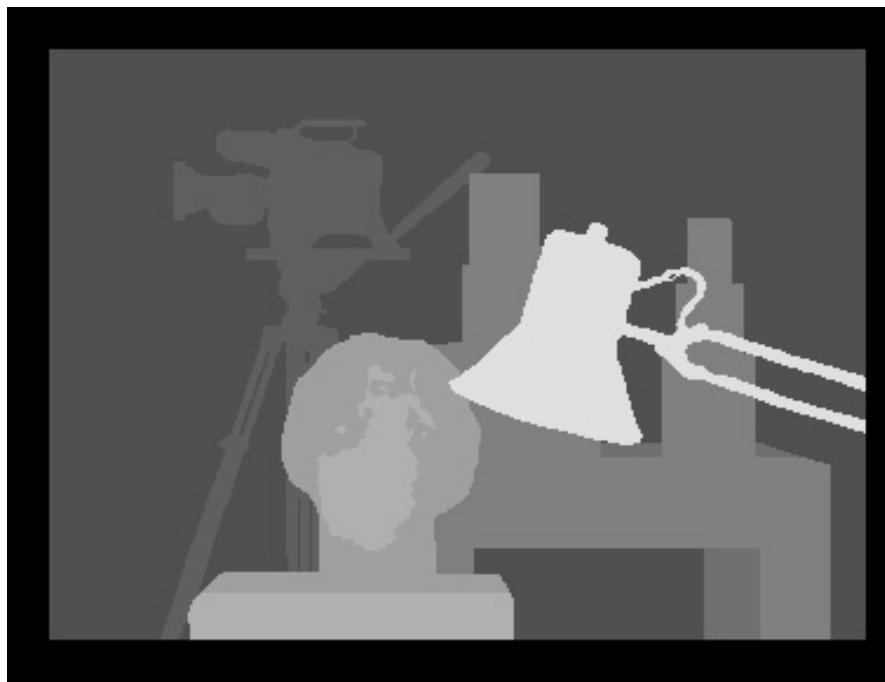
- Smoothness cost penalises disparity changes with robust  $\rho(\cdot)$

$$E_s(d) = \sum_{(x,y)} \rho(d(x, y) - d(x + 1, y)) + \rho(d(x, y) - d(x, y + 1))$$

- This is a Markov Random Field (MRF), which can be solved using techniques such as Graph Cuts

# Stereo Comparison

- Global vs Scanline vs Local optimization



Ground  
truth



Graph Cuts  
[ Kolmogorov  
Zabih 2001 ]

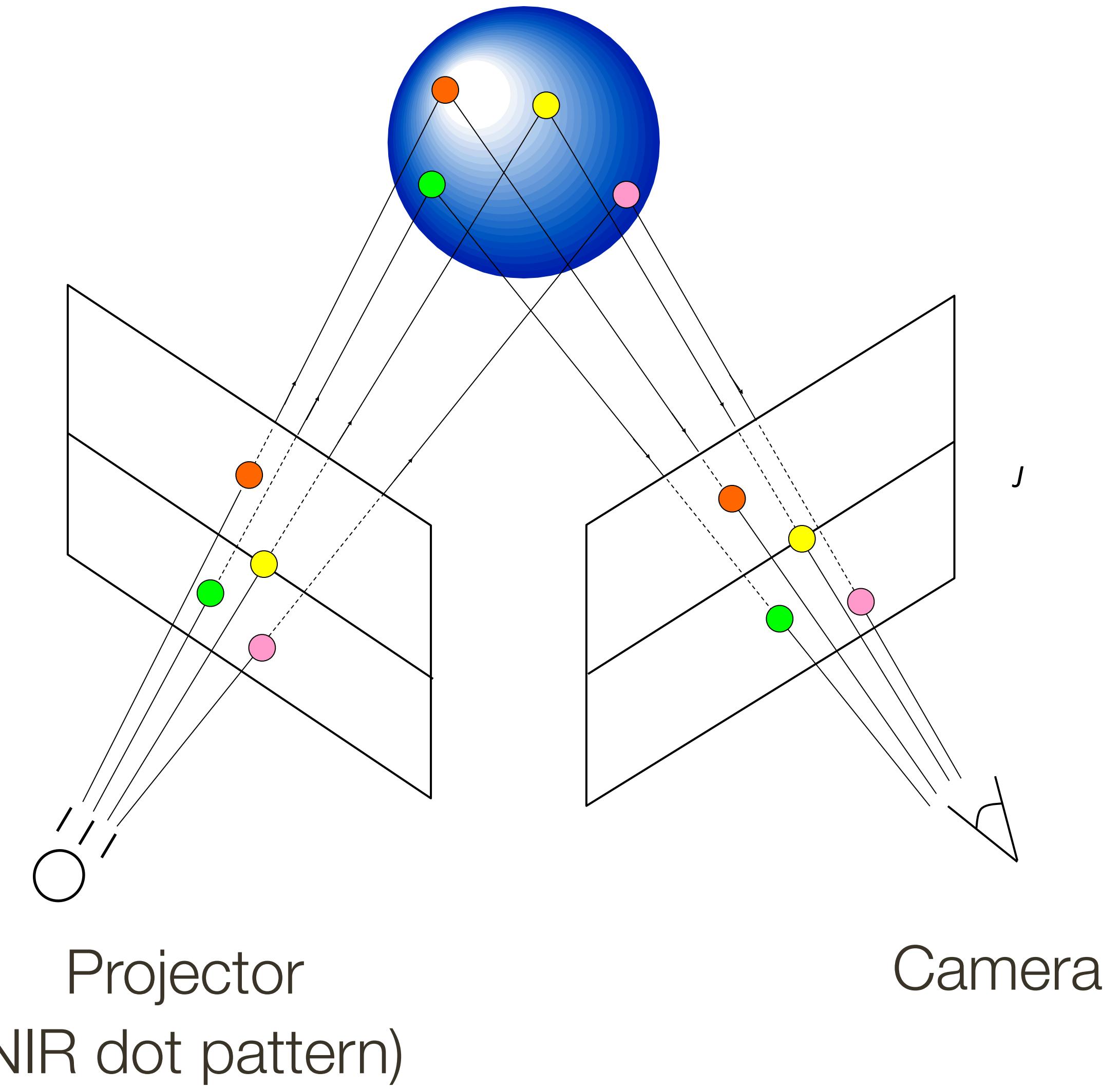
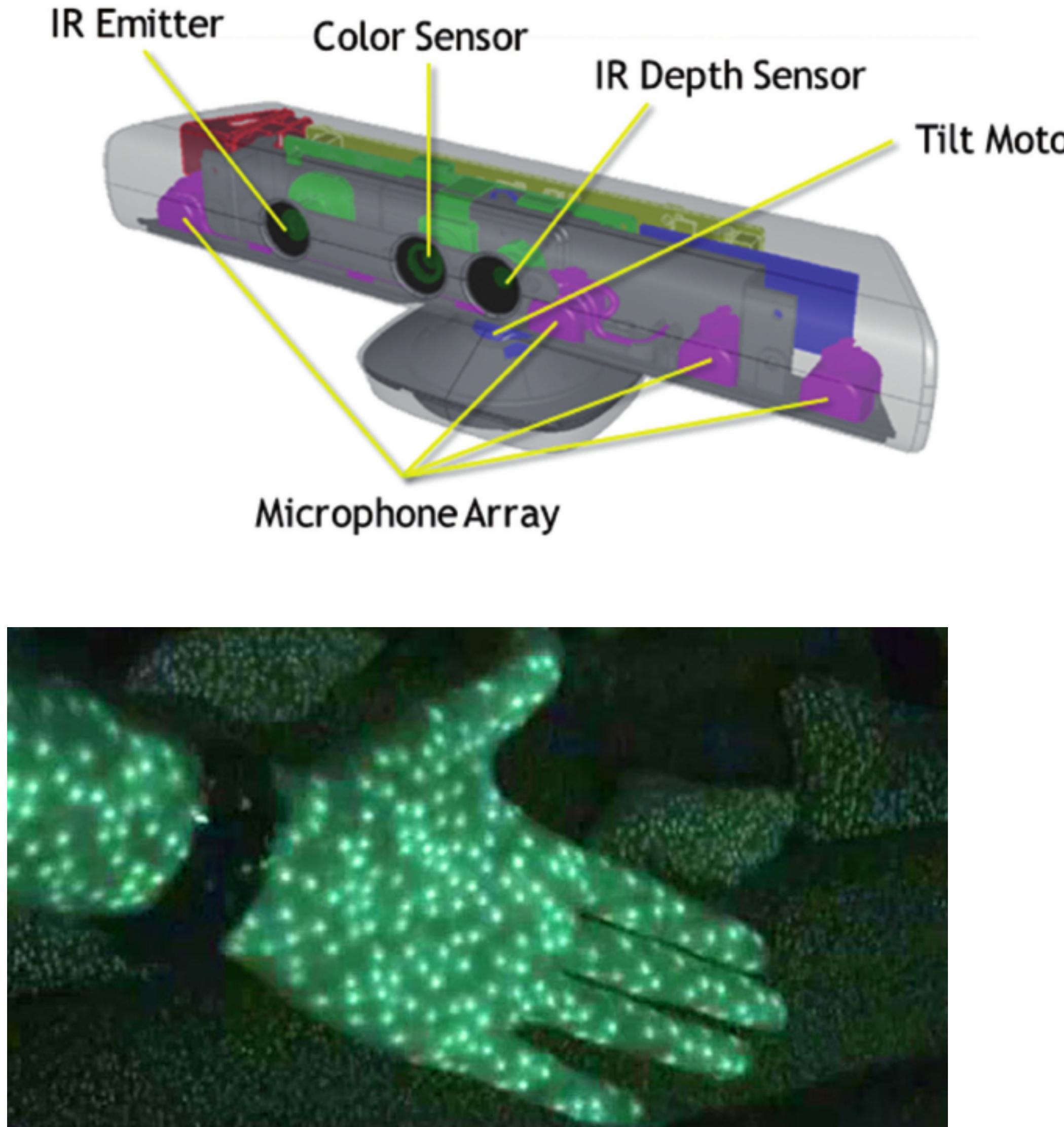


Dynamic  
Programming



SSD 21px  
aggregation

# Application: Microsoft Kinect



# Stereo Vision Summary

With two eyes, we acquire images of the world from slightly different viewpoints

We perceive **depth** based on **differences in the relative position of points** in the left image and in the right image

**Stereo algorithms** work by finding **matches** between points along corresponding lines in a second image, known as epipolar lines.

**A point** in one image projects to an **epipolar line** in a second image

In an axis-aligned / rectified stereo setup, matches are found along horizontal scanlines

# Menu for Today

## Topics:

- 3D Correspondence, **Epipolar** Geometry
- **Stereo** Vision
- **Quiz 4**

## Readings:

- **Today's** Lecture: Szeliski 12.1, 12.3-12.4, 9.3

## Reminders:

- **Assignment 4:** RANSAC and Panoramas due **November 9th**