Forage RRT - An Efficient Approach To Task-Space Goal Planning for Redundant Manipulators

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***Abstract*—Achieving efficient end-effector planning for manipulators in real world workspaces is challenging due to the fact that planning is performed in manipulator joint space, while the planning goal is given in end-effector or tool space. For manipulator planning, the problem becomes a joint path planning and inverse kinematics problem to be resolved efficiently, in spite of the potentially infinite number of inverse solutions to the end-effector goal state and the nonlinear relationship between joint configurations and world obstacles. The Forage-RRT algorithm described in this paper attempts to efficiently and quickly resolve the end-effector or tool planning problem. Using ideas from foraging theory, Forage-RRT implements a diffusion-based search strategy with two rates of diffusion, one high and one low, which simulate both long jumps (coarse search) and focused small area exploration (fine search) in the joint space, respectively. During coarse search, it is important to keep track of past fine searches, therefore the traditional RRT algorithm is augmented with a goal heap that keeps track of potential focused search regions and discards them when they result in failure. By mixing between two search distributions with different spreads, the search space is rapidly covered and potentially fruitful avenues are finely explored. The search coverage advantages of foraging identified in the biological research literature are demonstrated here for end-effector based, manipulator path planning.**

I. Introduction

Task-based, manipulator motion planning algorithms seek to automatically and quickly find a collision-free path in a general environment between any given initial joint configuration (usually modeled by R^m where m is the degrees of freedom of all the joints) and a goal point specified in end-effector or task space (a subspace of $SE(3)$). Here, the goal is specified in end-effector space because it is almost always the end-effector (or tool) which interacts with the object to be manipulated. The specific configuration of the final joint configuration is usually not important so long as it is not in collision with workspace obstacles. Examples of end effector tools which interact with objects include hands, magnets, suction cups, paint sprayers, and welders. Any practical manipulator planner has to convert the given initial information into a trajectory fully residing in joint space and meeting existing collision constraints.

The literature consists of two main approaches to solving the task-space planning problem: either first solve end-effector goal directly using inverse kinematics, or incorporate the search for the end-effector goal into planning. Solving for a feasible inverse kinematics is difficult due to the need to identify a goal joint configuration with collision-free connectivity to the initial joint configuration. To date, we are not aware of an inverse kinematics algorithm for general n-link manipulators which is complete, fast, and returns a configuration guaranteed to be reachable from the start configuration. Were it to exist, then the task-space planning problem would be converted into a joint-space planning problem with task-space obstacle constraints. An efficient solution would then be the Bidirectional RRT **[CITE]**, which is in the class of Rapidly-Exploring Random Tree (RRT) algorithms **[CITE]**, or any other solution known to be fast (see §II).

a) Problem Statement: This paper tackles the second approach to task-space planning. The initial state is given in joint space. The goal of the plan is expected to be specified as an end-effector configuration, either position only or position and orientation. The only assumptions are that the model of the manipulator is known (at least enough to calculate a Jacobian) and that the world can be queried to see if a given manipulator configuration would produce a collision or not. The task-space motion planner for (redundant) manipulators should be complete, single-query, and reliably fast for all reasonable problems posed.

b) Contribution: The proposed solution, Forage-RRT, is an RRT algorithm with multiple diffusion rates and a greedy, goal-directed phase as part of planning. The main idea behind Forage-RRT is to initially explore the manipulator space quickly with a large step-size RRT (high diffusion rate) and then attempt to connect to the goal using a small step-size RRT (low diffusion rate) from promising nodes in the large step-size RRT. The algorithm alternates between the high and low diffusion rate modes. To enable sensible greedy, goal-directed searches, the RRT is augmented with a task-space goal heap (RRT+GH), which orders the RRT joint space configurations via a heuristic score. The ordering determines which node will be used to initiate the greedy search. Promising coarse step-size tree nodes are selected and removed from the goal heap, then used to initialize a small step-size RRT search. Forage RRT lends itself to parallel (or multi-core) implementation by running several low-diffusion rate searches simultaneously. We show that Forage-RRT provides competitive average planning times that are further improved through parallelization. The result is a reliable planner which is fast and consistent at solving a potentially wide range of manipulation problems in environments with obstacles.

c) Organization: The paper presents previous work we build upon as well as other approaches to the same problem

(§II), an analysis of the shortcomings of the RRT in bug-trap problems (§III), the implementation of the Forage-RRT algorithm (§IV), experiments and results compared to other planners having the same problem statement (§V), and finally a discussion of possible improvements to Forage-RRT (§VI).

II. Related Work

Achieving tractable, complete motion planning for high dimensional systems such as (redundant) manipulators in general work environments present several challenges. Algorithms to resolve the high-dimensional aspect have been the first to arise. One of the most famous and most cited approaches is the Artificial Potential Field Method originally presented in [1]. Although this method is fast enough to be used in a single query planner, it depends on obstacles being of a simple geometry and suffers from several fundamental issues such as getting stuck in local minima, no passage between closely spaced obstacles, and oscillations in certain conditions [2]. Attempts have been made to solve these issues by the formulation of new potential functions [3], [4], however, these functions either take prohibitively long to compute or do not resolve all of the aforementioned pitfalls.

Ariadne's Clew Algorithm [5] approximates the feasible joint space via sampling. It uses an explore-search approach to achieve resolution completeness and was shown to be respectably fast for most problems. Algorithms based on probabilistic roadmaps [6] use sampling to generate a roadmap for multi-query problems which allows subsequent path planning problems to be solved efficiently. Because the roadmap preprocessing step has a baseline overhead, it is not optimal for our stated goal of a single query planner for general environments. Within the category of efficient resolution-complete sampling based algorithms, the Rapidly-Exploring Random Tree (RRT) family is popular and effective at solving many high dimensional planning problems [7], [8] For manipulation, standard RRTs depend on start and goal states given in joint space. This bidirectional version [8], which grows two trees (sometimes randomly, sometimes toward each other), is effective in solving bug-trap problems. Since goals are most often not specified in joint space for manipulation problems, a method for finding inverse kinematics algorithms to transform the goal state into joint space is first needed [9], [10], [11], [12]. None of these inversion algorithms are complete and guaranteed to be reachable from the start configuration. An inverse kinematics approach that was guaranteed to return a configuration reachable from start was presented in [13].

When the planning problem incorporates also the inverse kinematics problem as part of it, then the inverse kinematics solution will be resolution complete and guaranteed reachable from the start. In [14], the RRT search is biased to be around the existing nodes which were closest in end-effector space to goal. Significant speed improvements to the biased search were achieved by using the Jacobian transpose or Jacobian pseudo-inverse to take steps in the direction of the goal from existing nodes [15], [16]. Including such a bias is susceptible to getting stuck when the goal is occluded by an obstacle (similar to a bug-trap). Other RRT modifications include growing an additional end-effector space tree which is then used to guide the growth of the joint space tree [17], [18]. The former, [17], grows the end-effector space tree from the goal in a bidirectional-type approach whereas the latter, [18], creates end-effector paths from start to goal. The latter approach struggles to find solutions quickly in bug-trap problems.

III. RRT and the Bug-Trap Problem

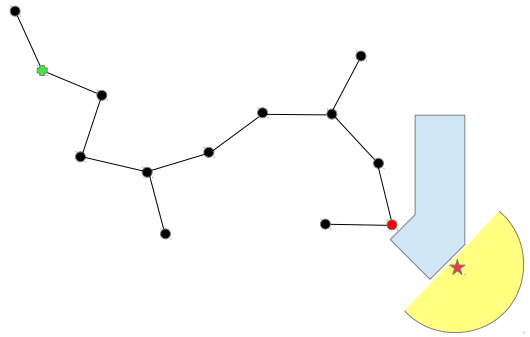


Fig. 1. An RRT attempting a bug-trap problem.

Figure 1 shows an example of a uni-directional RRT attempting to solve a problem where the goal is occluded by an object. The state of the RRT is shown after 12 iterations of the extend operation (with the probability of taking a step to goal was 35%). At this state of the tree, we can see that two problems emerge in our quest to connect the tree to the goal node:

1) Future attempts to step to goal will be taken from the red node since it is closest. All these attempts will fail because there is an obstacle in the way, and are a waste of time.
2) To get to the goal, the RRT will have to find its way into the yellow semicircle around the goal by virtue of only random steps. Any nodes outside of this area will be either further than the red node or obstructed by the obstacle. The probability of this happening quickly is very low. Essentially, it would require that random configurations to extend toward constantly be in the bottom right corner of the workspace. For this simple example, the probability of getting such random configurations looks to be around 1/15. In a different scenario with a larger workspace and smaller step size, the probability would be much smaller.

The important takeaway is that fast RRTs tend to approach the goal in a very directed fashion, but will be blocked by bug-trap type situations. Traditional RRTs rely on diffusion through random walks to go around obstacles, but this process is slow for low probability situations.

IV. Forage RRT Implementation

The Forage-RRT algorithm, described in this section, utilizes ideas from the biological foraging literature where it is shown that a multi-diffusion rate process lowers the mean time to passage for foraging problems **[CITE]**. The foraging problem is the problem of finding a reward that has low probability of occurence in a large space (with a penalty imposed on movement and searching). The foraging process model is a search process that has two diffusion rates (high and low), that result in two modes of search, or at least two distinct movement types[1] (far and near, respectively). By alternating between the two, search for an unknown goal state in a large search space is faster than traditional diffusion-based search, and covers a larger region versus greedy searches. Each movement mode is also characterized as having a specific cost: low for the high diffusion rate (far mode) and high for low diffusion rate (near mode). These costs are typically associated to the attention or energy given to search versus movement.

Forage-RRT replicates this idea within the context of our problem statement. The high diffusion rate mode utilizes a coarse step-size RRT algorithm, with low probability of seeking the goal, and is akin to an exploration phase. The low diffusion rate mode attempts to connect a node from the coarse tree to the goal via a fine step-size RRT algorithm with a high probability of greedily seeking the goal. The two RRTs have parameters chosen to replicate the low/high attention characteristics found in foraging. The RRT in the fine-step/high-attention is more guided to the goal and will also more often use the manipulator Jacobian to move. The effect of this approach for bug-trap type problems is to solve the problem presented in Section III by using the coarse step-size tree to cover joint space in search of promising approach directions to the goal, then using the fine step-size tree to avoid any small occlusions that remain on the path to the goal.

A. Goal Heap

An inefficiency in the basic RRT algorithm is that unsuccessful steps to goal may continuously be taken from the same node because that node is the closest to goal, as illustrated in §III. Our solution to the problem is to implement a goal heap. When new nodes are added to the RRT, they are also added to the goal heap along with their value (some heuristic that evaluates the fitness of the joint state with regards to proximity to goal). In our case, the value of a node was simply the inverse of its Euclidean distance to goal position. The heap data structure automatically orders the nodes by value so that the top of the heap is the highest value node. Once a step to goal is attempted from a node, that node is removed from the goal heap because using it again for a step to goal would be a fruitless endeavor. While the goal heap can apply to any RRT implementation, it is essential for the Forage RRT because the heap provides a node in the coarse tree for use when attempting a fine step-size J+RRT to connect to goal.

[1]Some researchers prefer to use a single, heavy-tailed distribution to generate the two movement types **[CITE]**

B. RRT Extend Implementations

Both coarse and fine RRT implementations use the J+RRT algorithm [15] with the proposed goal heap augmentation (J+RRT+GH). Random extensions of the search tree use the standard extension procedure of picking a random point in joint space and taking a step towards it from the nearest neighbor already in the tree. Goal-directed extensions use the Moore-Penrose pseudo-inverse Jacobian to take a step in the proper direction. The overall extend implementation is shown in Algorithm 1.

d) Coarse versus fine search.: The coarse search should focus more on rapidly exploring the space rather than seeking the goal. This is done by lowering the probability of stepping to goal in the coarse J+RRT+GH implementation, which also lowers the exploration cost. Since the coarse J+RRT+GH takes large step sizes, for each node extension, the link between the new node and its parent must be checked for validity by iterating through its length at an acceptable resolution and making sure each smaller step is valid. In contrst, the fine search should focus on exploring a small region with a greater intent to find the final goal state. The fine J+RRT implementation has a higher probability of stepping to goal. For the fine step-size J+RRT+GH, we assume that if the new node is valid, then the path connecting the parent node to the new node is also valid.

C. Explore/Search

The main Forage RRT algorithm is shown in Algorithm 2. The first while loop initially explores the space and builds up several promising nodes to initialize the fine searches with. The second while loop attempts to connect the promising nodes from the coarse step-size tree to the goal with a fine step-size tree. Once a fine step-size RRT experiences a certain number of collisions, we give up on it and try to start another one from the next most promising node. After a given number of fine step-size searches fail, we grow the coarse step-size tree to replenish the heap with promising start nodes. The finite search cost and restarts from alternative promising nodes allows us to make the fine step-size RRT greedy, making it fast in easy situations but also robust to more challenging situations. A full list of parameters we used is given in Section V.

Once the fine step-size RRT has reached the goal, the found path is obtained by tracing to the root of the fine step-size tree, which is a node in the coar step-size tree, then tracing to the root of the coarse step-size tree. Due to the mixed step-size RRTs, the raw final plan will not be attractive, especially near the start. For this purpose, path smoothing is required to make the path acceptable. This is done in a quick manner by taking pairs of nodes, attempting to connect them with a straight line, and deleting all nodes in between if successful. Empirically, we have found that picking a node from the coarse step-size plan and trying to connect it to the fine step-size plan works well. The second step is to go through remaining coarse steps and subdivide them into the desired step size to match the rest of the path. We have found that 15-20 successful smoothing

Data: $RRT, goal, StepSize$
Result: $UpdatedRRT, StepResult$
$\rho \leftarrow random([0, 100])$;
if $\rho < randomExtendProbability$ **then**
  $x_{rand} \leftarrow RANDOM\_STATE()$;
  $x_{near} \leftarrow NEAREST\_NODE(x_{rand}, RRT)$;
  $x_{new} \leftarrow TAKE\_STEP(x_{rand}, x_{near}, StepSize)$;
  if $isLegalPath(x_{near}, x_{new})$ **then**
    $RRT \leftarrow addVertex(x_{new})$;
    $RRT \leftarrow addEdge(x_{near}, x_{new}, StepSize)$;
    $NodeValue \leftarrow value(x_{new})$;
    $GoalHeap \leftarrow insert(x_{new}, NodeValue)$;
    if $x_{new} = x_{randomConfig}$ **then**
      return $STEP\_REACHED$;
    else
      return $STEP\_PROGRESS$;
    end
  else
    return $STEP\_COLLISION$;
  end
else
  $x_{near} \leftarrow GoalHeap \rightarrow top$;
  $x_{new} \leftarrow JAC\_STEP(x_{near}, goal, StepSize)$;
  if $isLegalPath(x_{near}, x_{new})$ **then**
    $RRT \leftarrow addVertex(x_{new})$;
    $RRT \leftarrow addEdge(x_{near}, x_{new}, StepSize)$;
    if $x_{new} = goal$ **then**
      return $GOAL\_REACHED$;
    else
      $NodeValue \leftarrow value(x_{new})$;
      $GoalHeap \leftarrow insert(x_{new}, NodeValue)$;
      $GoalHeap \leftarrow remove(x_{near})$;
      return $STEP\_PROGRESS$;
    end
  else
    $GoalHeap \leftarrow remove(x_{near})$;
    return $STEP\_COLLISION$;
  end
end

Algorithm 1: J+RRT+GH Extend Operation

steps result in a smooth path. The computational cost for path smoothing is low compared to the path planning procedure.

D. Parallel Implementation

The two search strategies in Forage-RRT are parellelizable. Using a Master-Worker parallel implementation, the main steps are:

- **Initial Step:** Grow coarse step-size tree to an initial size greater than the number of threads to be used.
- **Worker Threads:** Grow a fine step-size tree to goal with a seeded start configuration from the coarse step-size tree. Terminate upon success or when a certain number of collisions occur.

Data: start, goal
Result: path from start to goal
$CoarseRRT \leftarrow INIT(largeStepSize, start, goal)$;
while $CoarseRRT \rightarrow size < initialSize$ **do**
  $CoarseRRT \rightarrow EXTEND()$;
end
while $result \neq GOAL\_REACHED$ **do**
  $numFailures \leftarrow 0$;
  $FineRRT \leftarrow$
  $INIT(fineStepSize, CoarseRRT \rightarrow$
  $GoalHeap \rightarrow top)$;
  while $numCollisions < maxNumCollisions$ **do**
    $FineRRT \rightarrow EXTEND()$;
  end
  increment $numFailures$;
  if *numFailures = maxNumFailures* **then**
    for *1:percentIncrease*initialSize* **do**
      $CoarseRRT \rightarrow EXTEND()$;
    end
  end
end

Algorithm 2: Forage-RRT

- **Master Thread:** 1) Grow coarse step-size RRT; 2) Maintain worker threads by seeding them with the best available node from coarse step-size RRT when they terminate without success.

V. Experiments and Evaluation

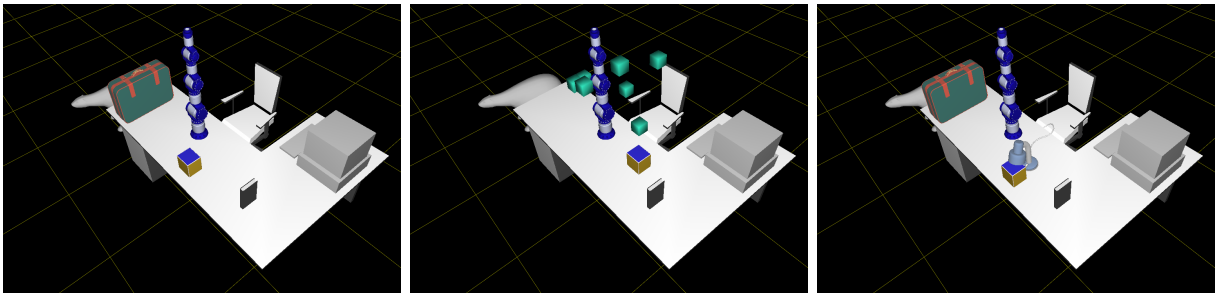
A. Experiments

1) Sequential Implementation: Experiments were conducted on a Schunk 7 DOF arm with the DART/GRIP simulator (developed by the GOLEMS lab at Georgia Tech) on a 2.27GHz, 8 core machine. We compared the following algorithms to the path-smoothed Forage-RRT:

1) RRT-JT [16]
2) J+RRT with goal directed steps [15]
3) BiSpace RRT [17]
4) Kinematic Roadmap [13]. The paths produced from this algorithm would be too long to be used for most applications. Rather only the inverse kinematics solution, which is guaranteed to be reachable from start, would be used.

The smoothing time was included in Forage RRT results but not for the other results because the paths produced did not strictly require it. Moreover, any RRT reaching 10,000 nodes was restarted to improve the average planning time of all planners (empirically it seems that an RRT that grows too large will have trouble connecting to the goal, so it is better to restart). The Forage-RRT parameters used for testing are given in Table I (with common parameters for the other algorithms equaling those from the table).

Each algorithm was tested on an easy, medium, and hard case. Figure (2) depicts the cases. The goal is the point where the three shown faces of the gold and blue cube meet.



(a) Easy case for testing: no obstacles. (b) Medium case for testing: obstacles throughout space. (c) Hard case for testing: goal directly under obstacle, near edge of workspace

Fig. 2. Depiction of different test cases.

TABLE I
FORAGE RRT PARAMETERS.

initialSize	50
randomExtendProbability - coarse	90
randomExtendProbability - fine	65
largeStepSize	1.3
fineStepSize	.02
maxNumCollisions	5
maxNumFailures	10
percentIncrease	.25

50 random collision-free start configurations were computed for each case (easy, medium, and hard) and the same 50 start configurations were used for each algorithm. Each start configuration was the seed for 40 runs, thus totaling 2000 runs per algorithm per case. In the interest of time, a run was considered a failure if the RRT had to be restarted 25 times and no solution was found (such a run would have taken 400-600 seconds so it should be penalized).

2) Parallel Implementation: To test the parallel algorithm, experiments were conducted on the same cases as the sequential Forage-RRT. The parallel implementations were tested for 10 precomputed start configurations per case, at 40 runs per start configuration. Thus, each version was tested 400 times. The implementation was tested with one to six worker threads.

B. Evaluation

1) Sequential Implementation: The results of the sequential implementation simulation are shown in Table II. The reported times are averaged over completed cases only. For all cases, the Forage-RRT provided the lowest average completion time and also maintained a 100% completion rate for the tasks.

2) Parallel Implementation: The results of the parallel implementation for the tested number worker threads are shown in Figure 3. The decline in search time is sub-linear. Because the solution is often found in the first few attempts to connect to goal from the coarse RRT, we hypothesize that additional threads beyond a certain amount will initiate a fine search that will not be needed. For our experiments, the results imply that improvements after four working threads would be minimal. The best average planning times achieved with the parallel implementation are summarized in Table III and apply to the case when four worker threads were implemented.

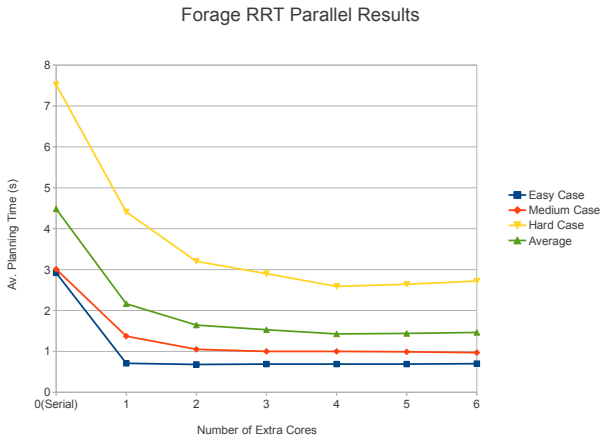


Fig. 3. Average planning times per number of extra cores used.

TABLE III
PARALLEL (4 CORE) PLANNING RESULTS. ALL TIMES IN SECONDS.

Case:	easy	Medium	Hard	Average
Avg. Time:	.69	1.00	2.59	**1.43**

VI. DISCUSSION

The benefits of the Forage-RRT arise from the two different diffusion rates associated to the two RRT implementations. For the purpose of visualization, the end-effector positions of a sample, initial coarse RRT are shown in Figure 4. The green node is the start and the red node is the goal. The closest nodes to the goal come from many different directions, making the fine RRTs more likely to be effective.

A. Completeness

From [7], any RRT is resolution complete because its coverage of the workspace converges to the sampling distribution. Because the coarse RRT in the Forage-RRT algorithm searches indefinitely, the Forage-RRT is also resolution complete. Needless to say, it is highly unlikely in practice that the coarse step-size tree will be the one to come upon the goal.

TABLE II
SEQUENTIAL EXPERIMENT RESULTS. ALL TIMES IN SECONDS.

Algorithm	**Easy Avg**	**% Comp**	**Med Avg**	**% Comp**	**Hard Avg**	**% Comp**
Forage RRT	**2.92**	**100**	**3.01**	**100**	**7.52**	**100**
RRT-JT	12.12	93.7	30.51	82.7	62.62	29.0
Jinv-RRT	4.45	96.9	21.42	92.0	65.92	53.6
BiSpace RRT	4.57	100	21.94	99.96	36.87	80.6
Kin. Roadmap	5.22	100	4.72	100	22.21	100

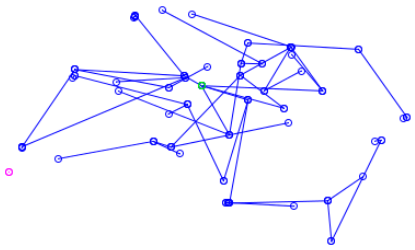


Fig. 4. End-effector positions for an initial coarse RRT.

B. Parameters

The Forage RRT parameters used to achieve the results in Table II are given in Table I. These parameters were arrived at empirically by running a few test cases, and are not guaranteed to be ideal ones. It may be of interest to investigate ideal values for these parameters, for instance when to switch from fine searching to coarse searching. Perhaps there should be a heuristic function rather than a hard constraint on failures. Moreover, parameters such as initial coarse tree size and the coarse step-size may be good candidates for learning over time. We conjecture that the ideal values for these parameters depend on specific environment factors such as the obstacle density and obstacle geometry.

VII. CONCLUSION

We have presented the Forage-RRT algorithm for motion planning. The premise behind the algorithm is that a foraging-based search algorithm provides an excellent compromise between diffusion-based searching (via random samples) and greedy search (goal-directed movements) in single-query, task-based, manipulator planning algorithms. The effectiveness of the method has been shown through randomized trials on scenarios with variable difficulty.

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