CMPS 142 — Spring Quarter 2017 – Homework 1

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Solution to Problem 1

2 - Linear Regression in Weka

a.) Root Mean Squared Error: 0.1897

b.) For $x = [3, 3, 5]$, our model $-0.1343(x_1) + 1.8477(x_2) + -0.8966(x_3) + 4.3608 = 5.018$

c.)

$$w = [1, 1, 1, 1] \ \eta = 0.1 \tag{1}$$

$$x^{(1)} = [1, 3, 9, 2] \ y^{(1)} = 19 \tag{2}$$

$$x^{(2)} = [1, 6, 9, 1] \ y^{(2)} = 19 \tag{3}$$

$$w_j = w_j + \eta(y^{(i)} - w^T x^{(i)})x_j^{(i)} \tag{4}$$

And using the update rule (4) for each feature j in instance i.

Step 1:

$$w_0 = 1 + 0.1(19 - 15)1 = 1.4$$
$$w_1 = 1 + 0.1(19 - 15)3 = 2.2$$
$$w_2 = 1 + 0.1(19 - 15)9 = 4.6$$
$$w_3 = 1 + 0.1(19 - 15)2 = 1.8$$
$$w = [1.4, 2.2, 4.6, 1.8]$$

Step 2:

$$w_0 = 1.4 + 0.1(19 - 57.8)1 = -2.48$$
$$w_1 = 2.2 + 0.1(19 - 57.8)6 = -21.08$$
$$w_2 = 4.6 + 0.1(19 - 57.8)9 = -30.32$$
$$w_3 = 1.8 + 0.1(19 - 59.6)1 = -2.08$$
$$w = [-2.48, -21.08, -30.32, -2.08]$$

d.)

$$\text{Closed Form: } w = (X^T X)^{-1} X^T y \tag{5}$$

$$X = \begin{bmatrix} 1,3,9,2 \\ 1,6,9,1 \\ 1,7,7,7 \\ 1,8,6,4 \\ 1,1,0,8 \end{bmatrix} \quad y = \begin{bmatrix} 19 \\ 19 \\ 10 \\ 11 \\ -3 \end{bmatrix} \quad X^T = \begin{bmatrix} 1,1,1,1,1 \\ 3,6,7,8,1 \\ 9,9,7,6,0 \\ 2,1,7,4,8 \end{bmatrix}$$

Now, we calculate $X^T X$ and $X^T y$

$$X^T X = \begin{bmatrix} 5, & 25, & 31, & 22 \\ 25, & 159, & 178, & 101 \\ 31, & 178, & 247, & 100 \\ 22, & 101, & 100, & 134 \end{bmatrix} \quad x^T y = \begin{bmatrix} 56 \\ 326 \\ 478 \\ 147 \end{bmatrix}$$

Putting it all together, we get:

$$w = \begin{bmatrix} 4.361 \\ -0.134 \\ 1.848 \\ -0.897 \end{bmatrix}$$

e.) //TODO