CMPS142 Homework 2

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1. Naive Bayes

2. Gaussian Discriminant Analysis

a.) Show that the set of points **x** such that $p(Y=0|x) = p(Y=1|x)$ is described by $\mathbf{w} \cdot \mathbf{x} = b$ for some **w** and b.

We can assume that the label probabilities for $Y=0$ and $Y=1$ are the same. Which is to say

$$p(Y=0) = p(Y=1) = \frac{1}{2} \tag{1}$$

Using Bayes Theorem, the conditional probabilities given **x** are as follows:

$$p(x|Y=0) = \frac{(Y=0|x)p(x)}{p(Y=0)} \; ; p(x|Y=1) = \frac{(Y=0|x)p(x)}{p(Y=1)} \tag{2}$$

Thus, we can deduce from (1) and (2) that:

$$p(Y=0) = \frac{(Y=0|x)p(x)}{p(x|Y=0)} = \frac{(Y=1|x)p(x)}{p(x|Y=1)} = p(Y=1) \tag{3}$$

We can deduce from this that:

$$p(Y=1|x) \cdot p(x|Y=0) = p(Y=0|x) \cdot p(x|Y=1) \tag{4}$$

We are attempting to classify values of x such that $p(Y=0|x) = p(Y=1|x)$, so we'll assume that this statement is true. Since these probabilities are equal, we have

$$p(Y=0|x) = 1 - p(Y=1|x) \tag{5}$$

Let $P = p(Y=0|x)$. Then from (5), it follows that $P = 1 - P = p(Y=1|x)$.
To sum up thus far, we have:

$$P \cdot p(x|Y=0) = P \cdot (x|Y=1) \tag{6}$$

Returning to our original models since, we can now see that

$$\frac{1}{(2\pi)^{\frac{n}{2}} |\sum|^{\frac{1}{2}}} \cdot \exp\left(-\frac{1}{2}((x-\mu_0)^T \Sigma^{-1}(x-\mu_0))\right) = \frac{1}{(2\pi)^{\frac{n}{2}} |\sum|^{\frac{1}{2}}} \cdot \exp\left(-\frac{1}{2}((x-\mu_1)^T \Sigma^{-1}(x-\mu_1))\right) \tag{7}$$

Which we can clearly cancel out the like terms on either side, and we have the equation

$$\exp -\frac{1}{2}((x-\mu_0)^T\Sigma^{-1}(x-\mu_0)) = \exp -\frac{1}{2}((x-\mu_1)^T\Sigma^{-1}(x-\mu_1)) \tag{8}$$

We can eliminate the $\exp$ and $-\frac{1}{2}$ by taking the log of both sides of the equation, then cancelling out $-\frac{1}{2}$. Now, the equation we're solving becomes:

$$(x-\mu_0)^T\Sigma^{-1}(x-\mu_0) = (x-\mu_1)^T\Sigma^{-1}(x-\mu_1) \tag{9}$$

By distributing the T,

$$(x^T-\mu_0^T)\Sigma^{-1}(x-\mu_0) = (x^T-\mu_1^T)\Sigma^{-1}(x-\mu_1) \tag{10}$$

Now, we can multiply $\Sigma^{-1}(x-\mu_0)$ by $(x^T-\mu_0^T)$

$$sd \tag{11}$$