# Infrared thermography protocol for simple measurements of thermal diffusivity and conductivity

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| A R T I C L E   I N F O | A B S T R A C T |
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| | An accurate characterization of thermal properties requires the knowledge of both thermal diffusivity and conductivity. Most of the time, the thermal conductivity have to be measured with complicated setups. In this paper, we show that a combination of two experiments carried out with the same setup – stimulated infrared thermography – allows to estimate straightforwardly and quickly the absolute value of the thermal diffusivity and the thermal conductivity relatively to a reference material.<br> |

## 1. Introduction

In the process of thermal diffusion, thermal diffusivity $D$ is associated with the relation between time and space whereas thermal conductivity $k$ is associated with the relation between heat flux and temperature. These two quantities are however not independent since they are linked together by the quantity $\rho C$ as follows:

$$D = \frac{k}{\rho C} \tag{1}$$

where $\rho$ is the mass density and $C$ the specific heat. These non-independent physical parameters ($D$ and $k$) are not always easily accessible. To achieve thermal diffusivity measurements, contact transient or periodic methods are used for instance in the guarded hot plate, hot plate or hot plate transient schemes [1–5]. In these schemes, the size of the sample has to be calibrated and the detection devices are in contact with the sample surface. Therefore, these measurements are relatively delicate to achieve.

Photothermal thermography, which uses generally the flash method [6], appears as a versatile tool for these measurements with no contact detection [7–9]. However the comparison between experimental thermogram and a thermocinetic model to identify parameters allows only thermal diffusivity to deduced. In this paper, we measure with an infrared camera the surface temperature evolution of polymer samples under periodic thermal excitations in contact with the backside of the sample. Phase and amplitude images are analyzed to deduce the conductivity of the sample, after having measured its thermal diffusivity.

## 2. Conductive thermal transfer

Heat conduction, which is the spontaneous transfer of thermal energy through matter from a high temperature region to a low temperature region, tends to reduce temperature differences. The law of heat conduction, also known as Fourier's law, states that the rate of heat transfer through a material in a conductive process is proportional to the opposite of the temperature gradient. The heat flux density $\vec{\phi}$ (W/m²) is then proportional to the temperature gradient:

$$\vec{\phi} = -k\,\vec{\mathrm{grad}}\,(T) \tag{2}$$

where $k$ is the thermal conductivity (W/m/K). Taking into account the law of energy conservation, one obtains the heat diffusion equation:

$$\rho C \frac{\partial T}{\partial t} - \mathrm{div}\,(k\,\vec{\mathrm{grad}}(T)) = s \tag{3}$$

where $s$ (W/m³) is the heat source, $\rho$ (kg/m³) is the mass density and $C$ (J/K/kg) is the specific heat. When the considered material is uniform and heated over wide plane surface perpendicular to the depth axis $z$, the heat diffusion equation simplifies into:

$$\rho C \frac{\partial T(z,t)}{\partial t} - k \frac{\partial^2 T(z,t)}{\partial z^2} = s(z,t) \tag{4}$$

The solution of (4) for a uniform material of thickness $d$ when applying at $z = 0$ a temperature $T_m \exp(i\omega t)$ varying at angular frequency $\omega$ and assuming that heat flux is zero at $z = d$ is:

$$T(z,t) - T_0 = T_m \frac{\cos((i-1)(z-d)/\mu)}{\cos((i-1)d/\mu)} \exp(i\omega t) \tag{5}$$

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Here $T_0$ is the room temperature, $i = \sqrt{-1}$ and $\mu$ is the thermal length defined as

$$\mu = \sqrt{\frac{2D}{\omega}} \qquad (6)$$

Providing $d > \mu$, the phase difference $\Delta\varphi$ between the temperature variation at $z = d$ and at $z = 0$ is simply

$$\Delta\varphi = \arg(T(z = d)) - \arg(T(z = 0)) \approx -\frac{d}{\mu} \qquad (7)$$

That expression is correct at 13% when $d = \mu$, at 0.7% when $d = 2\mu$ and at 0.023% when $d = 3\mu$. Then the thicker the material the better the accuracy. Of course the temperature variation at $z = d$ reduces as the material becomes thicker but it remains reasonable for measurements since it is 77% the one at $z = 0$ when $d = \mu$, 27% when $d = 2\mu$ and still 10% when $d = 3\mu$. It is worth noting that if heat flux were not zero at $z = d$, expression (7) would still apply since it would affect only temperature amplitude but not its phase when $d > \mu$. The diffusivity $D$ can then be easily estimated by measuring $\Delta\varphi$ using (6) and (7) by

$$D = \frac{\omega}{2}\left(\frac{d}{\Delta\varphi}\right)^2 \qquad (8)$$

The relative estimation error $\delta D/D$ is

$$\frac{\delta D}{D} = \frac{\delta\omega}{\omega} + 2\frac{\delta d}{d} + 2\frac{\delta\Delta\varphi}{\Delta\varphi} \qquad (9)$$

so that the relative thickness error is the main limiting factor when $d \geqslant 2\mu$.

As to the thermal conductivity, $k$ cannot be estimated directly from (5). However, imposing a heat flux $\Phi_m \exp(i\omega t)$ at $z = 0$ and a zero heat flux at $z = d$ results in the temperature profile

$$T(z,t) - T_0 = \frac{\Phi_m \mu}{(1+i)k} \frac{\cos((i-1)(z-d)/\mu)}{\sin((i-1)d/\mu)} \exp(i\omega t) \qquad (10)$$

Therefore, knowing $\mu$, $k$ can be estimated by measuring the amplitude of the temperature variation at $z = d$. An accurate estimation of $\Phi_m$ is however difficult. The idea is to work in parallel with a known material (reference sample) subjected to the same heat flux. The ratio of the amplitude of the two temperature variation measurements leads to

$$\left|\frac{T(d,t) - T_0}{T_r(d,t) - T_0}\right| = \left|\frac{\mu}{\mu_r}\frac{k_r}{k}\frac{\sin((i-1)d_r/\mu_r)}{\sin((i-1)d/\mu)}\exp(i\omega t)\right| \\ = \frac{\mu}{\mu_r}\frac{k_r}{k}\exp\left(\frac{d_r}{\mu_r} - \frac{d}{\mu}\right) \qquad (11)$$

where $d$, $\mu$, $k$ are respectively the thickness, the diffusion length and the thermal conductivity and subscript $r$ denotes the reference sample. The coefficient $\mu$ and $\mu_r$ can be measured using (7) and thus knowing the thermal conductivity $k_r$ of the reference material, one deduces the unknown thermal conductivity $k$. It is worth noting that if heat flux were not zero at $z = d$, expression (11) would still apply if losses are similar for both samples. For that purpose, materials are black painted and thus present the same emissivity. When temperature variations are small, losses are proportional to these temperature variations and therefore disappear in the ration of (11). Obviously the thickness of painting must be sufficiently thin compared to thermal length.

The relative estimation error $\delta k/k$ is

$$\frac{\delta k}{k} = \frac{\delta d}{\mu} + \frac{\mu + d}{\mu^2}\delta\mu + \frac{\delta d_r}{\mu_r} + \frac{\mu_r + d_r}{\mu_r^2}\delta\mu_r + \frac{\delta k_r}{k_r} + \frac{\delta A}{A} + \frac{\delta A_r}{A_r} \qquad (12)$$

where $A = |T(d,t) - T_0|$ and $A_r = |T_r(d,t) - T_0|$. For $d = 2\mu$ and $\delta d/d = 0.5\%$, one has

$$\frac{\delta k}{k} \approx 5\% + \frac{\delta k_r}{k_r} + 2\frac{\delta A}{A} \qquad (13)$$

## 3. Measurements and results

### 3.1. Experimental set up

A Peltier module is used as the heat source. It is supplied by a sine current, creating a periodical heat source at frequency $f$. The temperature difference between each face of the Peltier module depends on its resistance and Seebeck coefficient. The back face of the Peltier module is thermatically coupled to a thermal tank. As a consequence, the front face of the Peltier module has an alternative temperature variation at frequency $f$.

The infrared radiations from the front face of the sample, of the Peltier module and of the reference sample are measured with a FLIR JADE MWIR InSb camera with $320 \times 256$ pixels (pixel size of 30 μm by 30 μm) operating in the 3–5.1 μm wavelength range. The thermal sensitivity of the camera is 20 mK. Each pixel is digitised on 14 bits.

The frequency $f$ of the Peltier module is locked with the infrared camera frame grabber. The infrared camera is coupled to a lock-in system developed by FLIR Corporation. The system collects a series of dated infrared images. Amplitude images and phase images are calculated with a real time Fast Fourier Transform process.

The frequency $f$ of the heating is chosen so that the thermal diffusion length is well smaller than the thickness of the sample as discussed in the previous section. All faces facing the infrared camera are black-painted in order to impose the same high emissivity of the measured faces as formerly discussed.

### 3.2. Measurement of diffusivity

A 0.7-A periodic current is injected into the Peltier module at a frequency $f$. One of its face is in dry contact with the sample while the other is in contact with thermal tank as shown in Fig. 1. Samples are sufficiently flat to insure a sufficiently good thermal contact between the front side of the Peltier module and the back side of the sample. The infrared camera is focused as to take images of the sample surface and of one part of the heating surface of the Peltier module (Fig. 2). The lock-in detection records images of the modulated infrared emission. Each measurement, which corresponds to 4 heating periods takes between 20 s and 80 s to be carried out according to the chosen measurement frequency $f$.

The temperature phase $\varphi_0$ of the studied sample (markers 3&4) and the temperature phase $\varphi_1$ of heating surface (markers 1&2) are
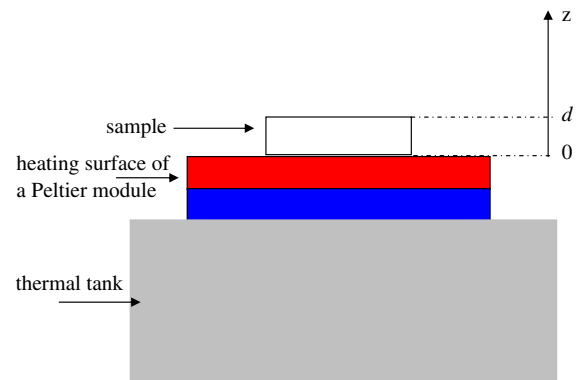


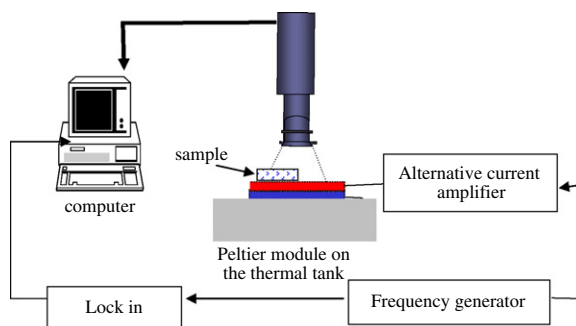**Fig. 1.** Schematic of the heating process in dry contact with the sample of thickness $d$.

**Fig. 2.** Experimental set-up for diffusivity measurements: the camera takes images of both sample and Peltier module surfaces.
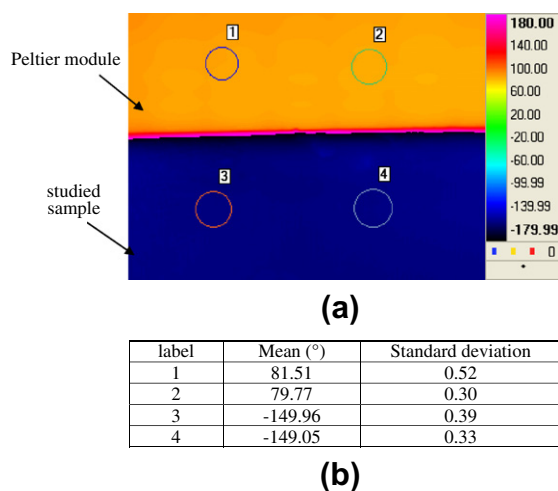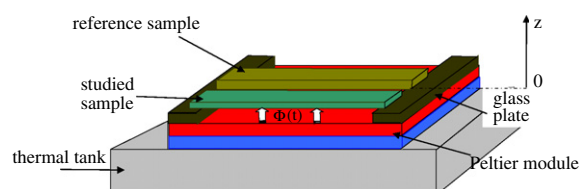


**Fig. 4.** Schematic of the heating process for conductivity measurement: in order to be in flow measuring conditions, an air gap is imposed between the Peltier module surface and the back sides of the studied and reference samples. Glass plate thickness = 1 mm.

The thermal diffusivity results are in good agreements with already reported results [11–14].

### 3.3. Measurement of conductivity

Peltier module is no longer in direct contact with the sample for conductivity estimation. An air gap is imposed at the back side of the sample in order to be as much as possible in flow conditions (Fig. 4). The infrared camera is focused in order to take images of the studied sample and the reference material. The lock-in detection records images of the modulated components of the infrared emission. The ratio of the temperature amplitude of the two sample surfaces ($A$ for the studied sample, $A_r$ for the reference sample) depends on the ratio of their conductivity according to (11).

Consequently, the measured conductivity of the studied sample can be estimated by:

$$k = \frac{\mu}{\mu_r} \frac{A_r}{A} \exp\left(\frac{-d_r}{\mu_r} - \frac{-d}{\mu}\right) k_r, \tag{14}$$

the error of measurement depending mainly on the accuracy of the value of the reference sample conductivity.

It is however necessary to ensure that both samples receive the same heat flux. That heat flux is imposed by the temperature difference across the air gap. That temperature difference can easily be calculated from the measurement of the front side temperature of the sample and from the estimated thermal diffusivity with (5). For the presented measurements, these two temperatures are similar, differences remaining well under the noise level. Sometimes however, the temperatures can be different at the back side of the sample and the reference material when their thermal properties are very different. In that case, (14) can be adjusted by taking into account that difference. Indeed, if $T_P$ refers to as the temperature of the Peltier module, $T_S$ as the temperature of the back side of the sample and $T_R$ as the temperature of the back side of the reference material (14) has to be multiplied by $\left|\frac{T_P - T_S}{T_P - T_R}\right|$ to give accurate results.

In Fig. 5a, the amplitude image shows 2 areas: the left markers 1&2 are on the reference sample, the right markers 3&4 are on the studied sample surface. The studied sample temperature amplitude $A$ and the reference sample temperature amplitude $A_r$ are deduced from the mean amplitude value inside the markers (Fig. 5b). The results obtained for a polypropylene sample using a PMMA as reference material are presented Table 2. The relative error on the



| label | Mean (°) | Standard deviation |
|-------|----------|--------------------|
| 1 | 81.51 | 0.52 |
| 2 | 79.77 | 0.30 |
| 3 | -149.96 | 0.39 |
| 4 | -149.05 | 0.33 |

**(b)**

**Fig. 3.** (a) Temperature phase image of the Peltier module surface (top) and of the studied sample (bottom). (image field: 4 cm × 3.2 cm). (b) Mean and standard deviation phase inside the markers.

deduced from the mean phase value (Fig. 3b) inside the markers (Fig. 3a). Table 1 shows the values of the relative phase shift $\Delta\varphi = \varphi_0 - \varphi_1$ of the surface temperature for three samples. The diffusivity is calculated from (6) taking into account the phase shift value. In each case the black painting thickness is small compared to the thermal diffusivity length and thus does not affect the results. Two polymers (PMMA and polypropylene samples) are studied here but the method can be extended to any types of materials by choosing the appropriate measurement frequency $f$. As a test, the thermal diffusivity of glass is also measured. We verify that the thermal length at the measurement frequency $f$ is well smaller than the sample thickness. We also verify the measuring condition by ensuring that temperature at the back side of the sample is similar to the one of the Peltier module. For that purpose the temperature at $z = 0$ is calculated from the measurements and from the estimated thermal length with (5). In the present case, the difference between the two temperatures is well under the measurement noise level.

**Table 1**
Experimental shift phase values (in degree) and deduced diffusivity for three materials of thickness $d$. $f$ is the heating process frequency.

|  | $f$ (Hz) | Thickness $d$ (mm) | Measured $\Delta\varphi$ (°) | Deduced $\mu$ (mm) | Measured diffusivity ($10^{-7}$ m²/s) |
|--|----------|--------------------|------------------------------|--------------------|----------------------------------------|
| Polypropylene | 0.05 | 1.95 | 139 ± 1 | 0.804 | 1.01 ± 0.02 |
| Pmma | 0.05 | 1.93 | 131 ± 1 | 0.850 | 1.13 ± 0.02 |
| Glass | 0.2 | 1 | 81 ± 1 | 0.707 | 3.14 ± 0.08 |

**(a)**

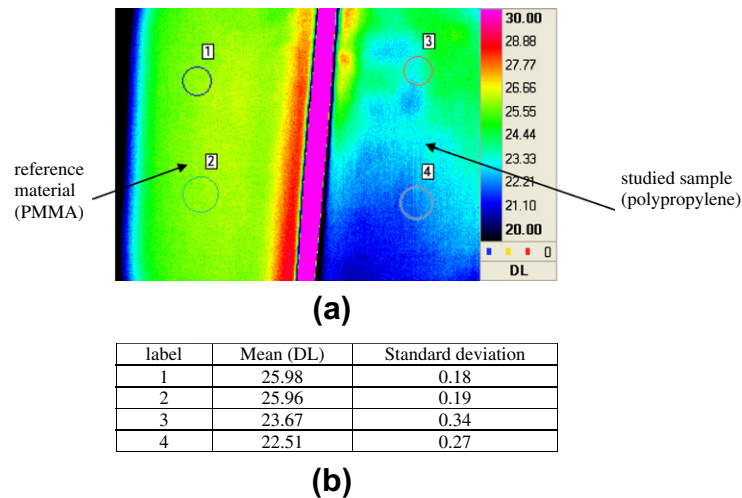| label | Mean (DL) | Standard deviation |
|-------|-----------|--------------------|
| 1 | 25.98 | 0.18 |
| 2 | 25.96 | 0.19 |
| 3 | 23.67 | 0.34 |
| 4 | 22.51 | 0.27 |

**(b)**

**Fig. 5.** (a) Amplitude image of the modulated component of the infrared emission. On the left: reference material; on the right: studied sample. (imaged field: 4 cm × 3.2 cm). (b) Mean and the standard deviation amplitude (in digital level) inside the markers.

**Table 2**
Experimental amplitude A (in digital level) measured by lock-in detection for two samples of thickness $d$. Conductivity is deduced from the values of A and the measured diffusivity. The reference material is PMMA and the studied sample is polypropylene.

| | Measured A (DL) | Thickness $d$ (mm) | Diffusivity $D$ ($10^{-7}$ m$^2$/s) (Table 1) | Deduced $\mu$ (mm) | Conductivity $k_0$ (W/m$^j$/K) | Literature conductivity (W/m/K) |
|---|---|---|---|---|---|---|
| Reference material: pmma | 22.01 ± 0.20 | 1.93 | 1.13 | 0.850 | 0.18 | 0.17–0.21[11,13] |
| Studied sample polypropylene | 19.76 ± 0.27 | 1.95 | 1.01 | 0.804 | 0.16 | 0.1–0.22[14] |

measured conductivity related to the components $A$ and $A_r$ is about 3%. The measured conductivity is in good agreement with already reported results [10–12].

## 4. Conclusion

In this paper, we show that infrared images obtained with a high sensitivity camera can provide absolute thermal diffusivity and relative thermal conductivity properties. The procedure is relatively rapid and requires little preparation, only plane sample surface blackening. The samples are positioned on a modulated heat source whose frequency must be selected according to the thickness of the sample and its thermal parameter. Thermal diffusivity is estimated from temperature phase measurements on the studied sample and the heat source surfaces. Thermal conduction is deduced from temperature amplitude measurements on the studied sample and the reference material surfaces in flow measuring conditions. The measuring condition can be checked during the measurements providing a good confidence on the result consistency. The development of this simple and accurate thermal measurement method should lead to more systematic studies: the differential measurement of conductivity is well adapted to performance comparison between materials with close thermal characteristics.

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