To my family and friends
for giving me the most valuable gift,
genuine moral values.

Financial Data Analysis of Italian Stocks

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Abstract

This dissertation focuses on the implementation of financial models using Python. Starting from the methodology, it gradually covers topics such as the statistical analysis of financial data, the Capital Asset Pricing Model, the Portfolio Selection Model and option pricing via Black-Scholes or Monte Carlo simulations. Despite the aforementioned concepts have already been widely discussed, the goal is to assess what can be learned by applying these models to different contexts, in this case, the Italian stock market.

Contents

1 Introduction

1.1 Preamble

This work is the result of a fervent interest in the world of quantitative finance. The themes and approach have been strongly inspired by a series of lectures held by Mauricio Labadie Martinez, former quant trader of Credit Suisse, at the National Autonomous University of Mexico.

For each topic I start with an explanation of the underlying theory and then I procede by commenting on the main results and intuitions that can be gained by analyzing those results. As John Maynard Keynes said: “the master-economist must possess a rare combination of gifts. He must reach a high standard in several different directions and must combine talents not often found together. He must be mathematician, historian, statesman, philosopher, in some degree.” Initially, I would have liked to provide a more thorough treatment on the actual implementation of the code since there are many interesting aspects behind it that shade additional light on the subjects of interest, but given the lenght constraint and the fact that it would have turned more into a discussion concerning computer science, I decided to omit that part. For further deepening, the source code and data set are available at: https://github.com/chiqui-quant/QuantPadawan.

Despite the rise of sophisticated mathematical approaches, in finance, differently from mathematics, not always a solution exists, and if it does, it is definitely not unique. The challenge, thus, lies in combining a vast set of methods and tools to a given problem in order to solve specific applications. Just as economics is the study of the allocation of scarce resources which have alternative uses and of the decisions that lead to such allocations, so do the fields of statistics and finance present us with tradeoffs. In the first case, between bias and variance, and in the second, between risk and return.

1.2 Data

Fortunately, in finance, the data wrangling process is much less tedius compared with statitics. The main reasons inherent to the price system, due to the evolution of technology and the concerns about informational asymmetries. (elaborate better)

The dataset that has been used consists in 5 year historical daily italian stock prices (from June 2nd 2017 to June 2nd 2022) downloaded from yahoo finance with the addition of an ETF of the FTSE MIB index and some European ETFs. The purpose is to compare the fexibility of the models being used by applying them in different markets. (be more informative)

As of today, there are 40 components in the FTSE MIB Index (https://www.borsaitaliana.it/borsa/azioni/ftse-mib/lista.html). However, for the seek of this dissertation, I focused mainly to a few randomly selected sample of stocks to carry on my analysis, the reason is that it allows to highlight the different views that one can have on a specific problem. Moreover, apart from specific metrics, what I try to convey is an explanation such that the same observations and rationales can be applied to other cases.

2 Are stock returns normally distributed?

2.1 Jarque-Bera normality test

There have been a lot of (concerns/polemics) ... about the normality of stock returns ... The reason is due to the fact that the most commonly used models relied on that assumption. Rather than relying on intuition, how can we assess wether a distribution is normal or not? One possible approach is to use a Jarque-Bera normality test. The JB test statistic is defined as:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right)$$

where n is the number of observations, S is the sample skewness and K is the sample kurtosis. Intuitively, when skewness is zero and kurtosis is 3, as in the case of a standard normal distribution, then the JB statistic would be close to zero, otherwise deviations would lead to an increase in the JB statistic. If data comes from a normal distribution, the JB statistic is asymptotically distributed as a chi-squared distribution with two degrees of freedom. This can be reduced at testing the joint null hypothesis that skewness and excess kurtosis are both zero, which in practice allows to assess wether a distribution is normal or not.

As a rule of thumb, any time a new algorithm or method is implemented it is useful to see how it works in a simpler and more controlled setting in which mistakes or unexpected results are easier to detect. Therefore I start by performing the test in a setting of simulated random variables.

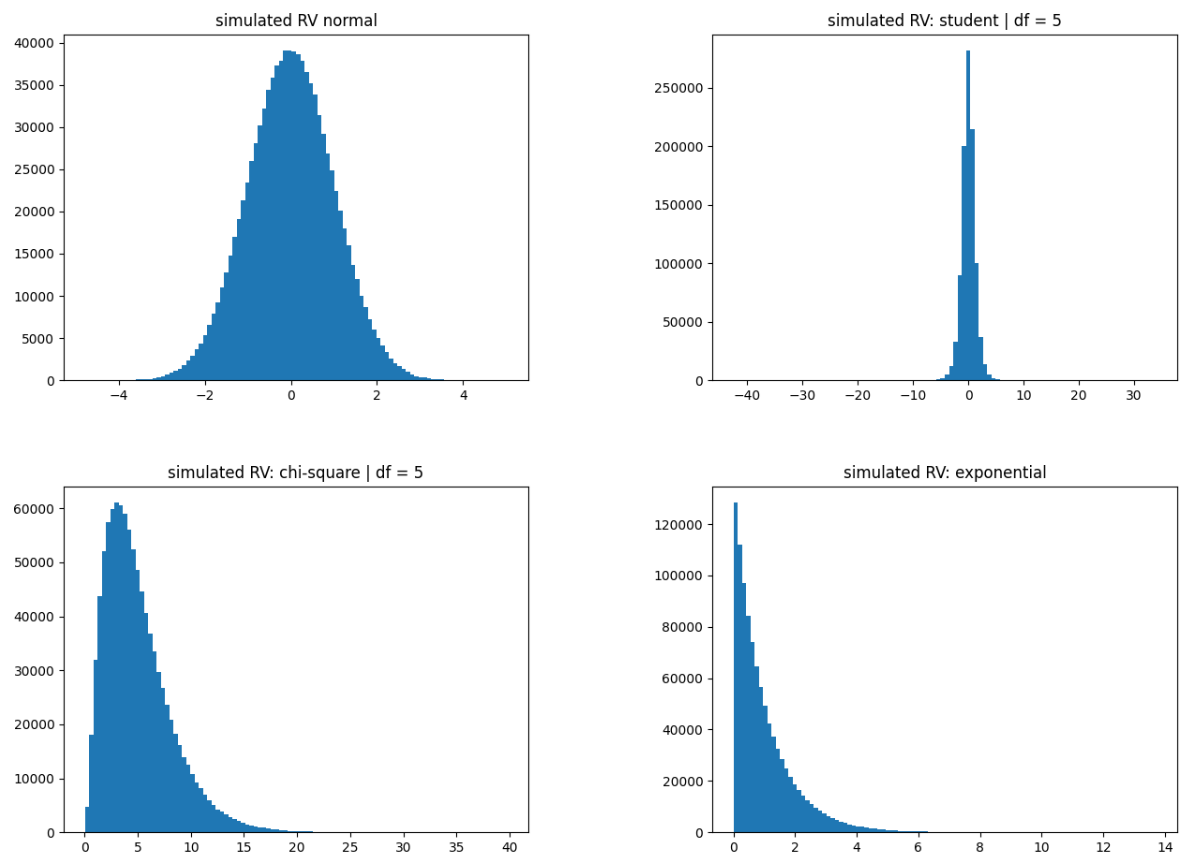

Figure 1: Different simulated distributions with 10^6 observations each.

One might reasonably ask, why are the degrees of freedom equal to 5 if the sample size is 1 million? The answer is that they have been chosen arbitrarily and the reson is that the simulation in Python allows to draw observations as if directly drawn from a given "population" distribution with pre-specified degrees of freedom. As we shall see, this will turn to be particularly useful for the question of interest in this section. One additional and relevant observation is that the samples are actually pseudo-randomly generated. Unless one is working with quantum computers that rely on true random properties of physics. In the case of a common computers, numbers are determined by an initial value called 'seed' and therefore not truly random.

Once the random simulations have been generated it is possible to compute a series of useful statistical metrics and use the Jarque-Bera formula to test for normality. The critical value for a chi-squared distribution with 2 degrees of freedom at the 5% significance level is 5.99, therefore we expect to reject the null hypothesis that the distribution is normal when the JB statistic is higher than this value. The associated p-value is the area on the right side of the critical value and we can compute it as 1 minus the cumulative density function of the chi-squared distribution up to the value we computed.

The output from the console (in the case of the normal distribution) is the following:

```
simulated RV: normal
mean is -0.0006352440319614184
standard deviation is 1.0001160793114432
skewness is -0.002692937851235039
excess kurtosis is -0.003512381715584656
JB statistic is 1.7226867666012902
p-value is 0.4225939951077672
is normal True
```

Note: the values are not exact because... talk a bit about simulation algorithms and the libraries... if we run the code multiple times we get slightly different results...

As expected, the computed statistical metrics are close to the ones of a normally distributed normal variable (mean 0, standard deviation 1, excess kurtosis 0, skweness 0) leading to a JB statistic lower than the critical value of 5.99. The p-value is 0.42 which means there is not enough empirical evidence for rejecting the null hypothesis that the distribution is normal. Having defined a 5% significance level, that means that 5% of the time the test will reject the null when it is actually true. For instance, by repeately simulating a normal random variable, one of the outputs was the following:

```
simulated RV: normal
mean is -0.0016239200778600778
standard deviation is 0.999860759582705
skewness is -0.007109991897447154
excess kurtosis is 0.002045368334968156
JB statistic is 8.599644614697796
p-value is 0.013570970259647597
is normal False
```

Here, despite having generated a normal distribution, the computed JB statistic is higher than the critical value of 5.99 and the corresponding p-value is less than 5% leading to a rejection of the null hypothesis of the distribution being normal (even though it actually was). This serves to highlight a relevant question concerning any application, that is, that knowledge about the assumptions and implications, along with judgement are required when conducting a statistical analysis.

2.2 Real market data

By applying a few adjustments, it is possible to check wether real market data are normally distributed or not.

Let's consider for example the stock of A2A.MI...

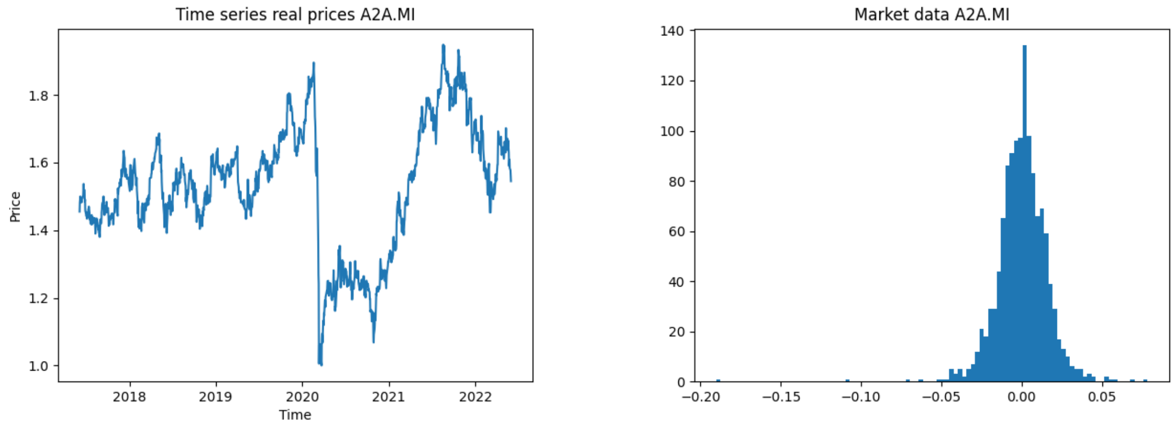

Figure 2: Timeseries and distribution of returns of A2A

```
---Real market data---
Ric is A2A.MI
mean is 0.00016557498162846543
standard deviation is 0.015978161935158598
skewness is -1.513417176329895
excess kurtosis is 18.42946380505845
JB statistic is 18428.629226984514
p-value is 0.0
is normal False
```

Here the story is quite different: mean and standard deviation are close to those of a normal distribution, however the negative skewness and remarkable kurtosis lead to a large JB statistic. Since running the test multiple times for different stocks yields similar results, empirical evidence suggests with a high degree of confidence that the returns of Italian stocks are not normally distributed.

For a more general view ... Theoretically, due to the fact that it represents the aggregate result (see weighting of the FTSE) ... it should be closer to a normal distribution...

3 Implementing the CAPM

The Capital Asset Pricing Model is used to define the expected rate of return of an asset by considering its sensitivity to non-diversifiable risk, that is, systematic or market risk and is formally defined as:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

where $E(R_i)$ is the expected return of the individual asset, R_f is the risk free rate of interest, $E(R_m)$ is the expected return of the market, $E(R_m) - R_f$ is known as market premium and $E(R_i) - R_f$ is known as risk premium.

From the equation above it is possible to derive the beta coefficient that corresponds to the sensitivity of the asset with respect to the market:

$$\beta_i = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)} = \rho_{i,m}\frac{\sigma_i}{\sigma_m}$$

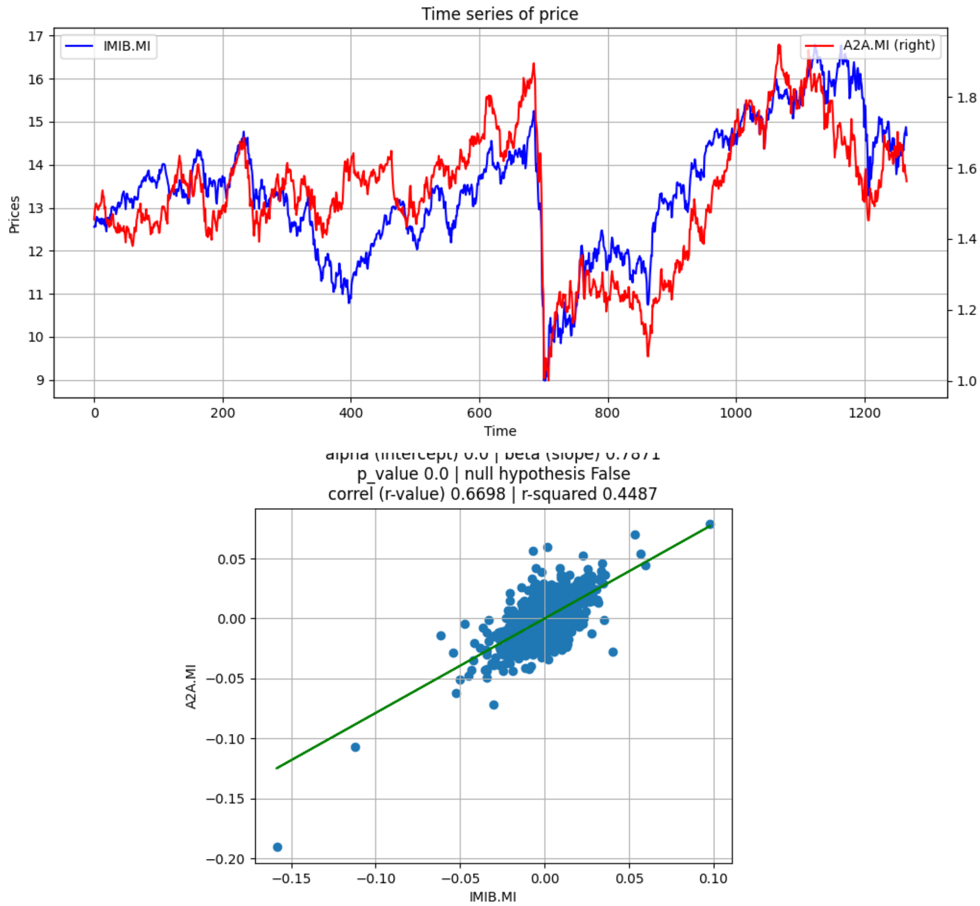

Figure 3: normalized time series and linear regression of A2A.MI with respect to the market IMIB.MI

```
Linear regression | security A2A.MI | benchmark IMIB.MI
alpha (intercept) 0.0 | beta (slope) 0.7871
p_value 0.0 | null hypothesis False
correl (r-value) 0.6698 | r-squared 0.4487
```

The estimated beta coefficient for A2A.MI is 0.7871, which means that the asset is less volatile with respect to the market. The p-value computed above is used to test the hypothesis that the regression has explanatory power...

Now, we know the volatility of a specific stock with respect to the market. But how can we make use of this knowledge?

3.1 Delta and beta hedging

One aspect that has not been mentioned in the previous section is the fact that the CAPM assumes that the rate of return of a security beyond risk (or absolute return) commonly referred to as α, that is, the fraction of the total return of the asset that cannot be explained by the market, is equal to zero. However, taking a closer look at the figure on the linear regression of the

Important: regression of stock with respect to market vs regression of market with respect to stock.

By taking into account only α and β it is possible to define 4 different kinds of investment strategies:

- Index tracker: whose goal is to replicate the performance of a benchmark ($\beta = 1, \alpha = 0$).
- Long-only asset management: whose goal is to outperform the market and achieve extra, uncorrelated return ($\beta = 1, \alpha > 0$).
- Smart beta: which aims to outperform the market by dynammically adjusting the weights of a portfolio ($\beta > 1$ when the market is up, $\beta < 1$ when the market is down, $\alpha = 0$).
- Hedge fund: that focuses on delivering absolute returns uncorrelated with the market ($\beta = 0, \alpha > 0$).

Given this non-exhaustive list of approaches, I procede by discussing about the last one. As a first example, which will be extended and generalized in the next section, suppose the current holdings in a portfolio consist only in one stock. Is it possible to hedge the exposure of such an asset by buying or selling another security, or even better, a number of different securities?

Although the concept is most commonly associated with options, on a general level, delta neutrality refers to a portfolio whose value remains unchanged, whereas beta neutrality defines a portfolio whose return is uncorrelated with the market. But why would an individual (wether individual investor or institution) employ such a strategy in the first place? More specifically, why would an investor choose to hold a portfolio whose value stays constant over time instead of increasing? These are perfectly reasonable questions that shine light to an important aspect about financial markets, that is, the fact that there is a variety of players with different goals.

For instance, a player that might be interested in the beta-delta neutrality is a market maker, whose function is to enhance market liquidity and efficiency by matching orders. To perform such a function, a market maker holds an inventory of securities and profits from the difference between bid and ask prices.

Mathematically, a hedging strategy with one security can be defined as follows. Let S_0 be the value of the initial holding in a portfolio (in euros):

- Delta neutral: find S_1 such that $S_0 + S_1 = 0$ (the value of the portfolio does not change).
- Beta neutral: find S_1 such that $\beta_0 S_0 + \beta_1 S_1 = 0$ (the sensitivity of the portfolio to the market is null).

More in general, a hedging strategy with N securities is:

- Delta neutral: find $S_1, ..., S_n$ such that $S_0 + \sum_{i=1}^{N} S_i = 0$.
- Beta neutral: find $S_1, ..., S_n$ such that $\beta_0 S_0 + \sum_{i=1}^{N} \beta_i S_i = 0$.

For example, if $N = 2$ the problem consists in finding $x = \begin{bmatrix} S_1 & S_2 \end{bmatrix}$ (the allocation of the notional of the 2 hedging stocks) such that:

$$\begin{bmatrix} 1 & 1 \\ \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = - \begin{bmatrix} S_0 \\ \beta_0 S_0 \end{bmatrix}$$

If $\beta_1 \neq \beta_2$ then the solution exits, is unique and is given by:

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = - \begin{bmatrix} 1 & 1 \\ \beta_1 & \beta_2 \end{bmatrix}^{-1} \begin{bmatrix} S_0 \\ \beta_0 S_0 \end{bmatrix}$$

As a practical example, suppose the current portfolio holds 10 million euro worth of 'A2A.MI' (see sector) shares and we want to hedge by combining 'pinco' and 'pallino' whose sectors are unrelated to ...

```
------
Input portfolio:
Delta mlnEUR for A2A.MI is 10
Beta for A2A.MI vs IMIB.MI is 0.7871
Beta mlnEUR for A2A.MI vs IMIB.MI is 7.871
------
Input hedges:
Beta for hedge[0] = AMP.MI vs IMIB.MI is 0.7315
Beta for hedge[1] = ATL.MI vs IMIB.MI is 0.9617
------
Optimization result - exact solution using linear algebra
Delta portfolio: 10
Beta portfolio EUR: 7.871
Delta hedge: -9.999999999999996
Beta hedge EUR: -7.870999999999998
------
Optimal hedge:
[-7.58470895 -2.41529105]
```

As shown above, the inputs display the delta to hedge, that is the starting 10 million euros of the starting porfolio, the beta of A2A.MI computed from the market index and the beta to hedge in terms of euros, which is a fictitious (???) yet helpful measure as it allows to ...

The second step entails the computation of the betas for the two stocks we want to hedge with. Finally, by using the theoretical approach discussed previously, the optimal hedge results in shorting around 7.58 million euros of 'AMP.MI' and 2.42 million of 'ATL.MI'.

3.2 Introducing numerical solutions

There is one limitation about the analytical approach. If we want to hedge the initial holded security by using more than 2 assets, that is $N > 2$, the system is undefined and there are infinitely many solutions and Python is not able to find an exact solution using linear algebra, yielding an error message. In such a case, it would be ideal to choose the one with smallest weights. This leads to finding the weights that lead to the smallest norm $||x||$ (either l_1 or l_2). Why? Recall that the unitary l_1 and l_2 norms are given by:
Therefore one could try to minimize the norm under the constraints of beta and delta neutrality. However this does not solve the issue of large weights..., moreover a one-sided hedge would be more desirable...
In order to solve this it is possible to add a new parameter that penalizes for having a large norm. Mathematically, let $x = \begin{bmatrix} S_1 & \cdots & S_n \end{bmatrix}$, $\beta = \begin{bmatrix} \beta_1 & \cdots & \beta_n \end{bmatrix}$ and $e = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$, the function to minimize is:

$$f(x, \varepsilon) = f_{\text{delta}}(x) + f_{\text{beta}}(x) + f_{\text{penalty}}(x, \varepsilon)$$

where:

$$f_{\text{delta}}(x) = (e^{\mathrm{T}} x + S_0)^2$$
$$f_{\text{beta}}(x) = (\beta^{\mathrm{T}} x + \beta_0 S_0)^2$$
$$f_{\text{penalty}}(x, \varepsilon) = \varepsilon ||x||^2$$

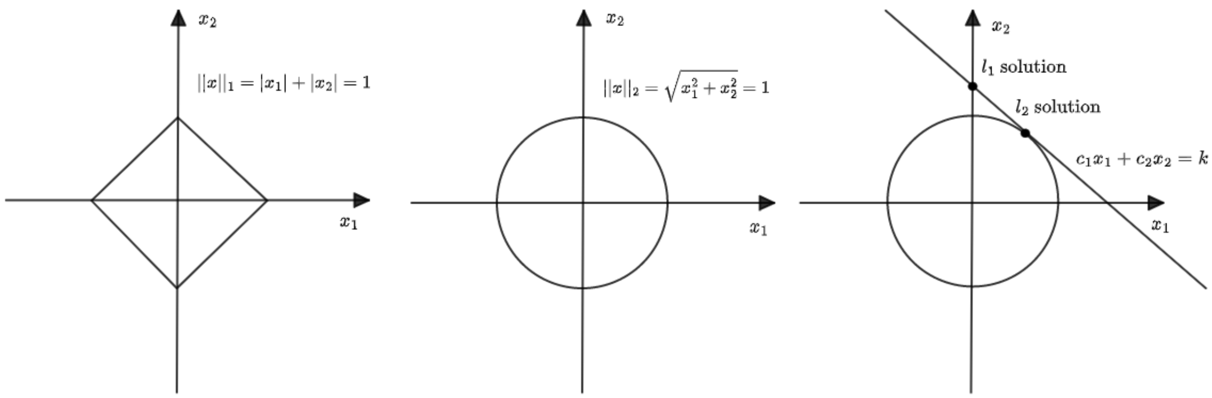

Figure 4: example of l_1 and l_2 norms in R^2, the line in the right panel in the set of infinite solutions, given that set we would like to find the solutions that minimize either the l_1 or l_2 norm.

The parameter $\varepsilon \geq 0$ makes the penalty larger as the norm of x grows. Although the solution will not be perfectly beta and delta neutral as before, the solution is easier to implement in a real world setting.
Practical example (same as before but with 4 securities and regularization)

4 Portfolio selection in practice

4.1 Computing the efficient frontier

The portfolio optimization problem consists in computing the efficient frontier, that is, computing the set of portfolios with the maximum rate of return for every given level of risk (or alternatively the minimum risk for every level of return). In the absence of a risk-free security the problem can be mathematically stated as:

$$\text{min. } x^{\mathrm{T}} V x$$

$$\text{sub. } x^{\mathrm{T}} \mu = m, \;\; x^{\mathrm{T}} e = 1$$

where: e is a vector of ones (to make notation consistent), m is a fixed value of mean return, x is the vector of weights for a unitary portfolio such that $\sum_{i=1}^{n} x_i = 1$, whose mean return and variance are respectively:

$$\mathrm{E(P)} = m = x^{\mathrm{T}} \mu = \mu^{\mathrm{T}} x \qquad \mathrm{Var(P)} = \sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{i,j} = x^{\mathrm{T}} V x$$

The variance-covariance matrix V has some important properties: it is symmetric and semidefinite positive. Hence, the first order conditions are sufficient for finding the minimum. Finally, the Lagrangian function that has been implemented in the code is:

$$L(x, \lambda_1, \lambda_2) = x^{\mathrm{T}} V x + \lambda_1 (m - x^{\mathrm{T}} \mu) + \lambda_2 (1 - x^{\mathrm{T}} e)$$

The actual implementation using Python is straightfoward since there are methods which allow for optimization by simply declaring the objective function and the underlying contraints. Back in 1952, when the portfolio selection model of Markowitz was first published, the efficient frontier was expensive in terms of computation, with the advance of technology that is not an issue anymore.

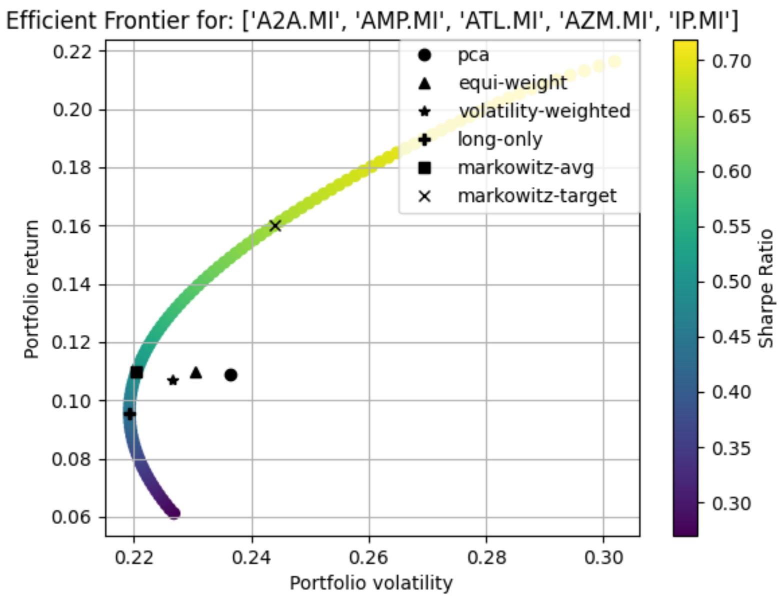

Figure 5: Markowitz's efficient frontier of 5 Italian stocks

Since reporting the computed results of the entire frontier of the top 40 Italian stocks might be a tedius, I will focus on a simplified case whose principles can easily be extended.
As it can be seen in the figure, each portfolio in the efficient frontier is associated to a given Sharpe ratio. The Sharpe ratio defines how many units of risk premium are present in the portfolio for every unit of standard deviation (or risk):

$$S = \frac{R_p - R_f}{\sigma_p}$$

where R_p and R_f are respectively the returns of the portfolio and of the risk free security and σ_p is the standard deviation of the portfolio.
Although I omitted the results about the frontier taking into account all the stocks of the FTSE MIB index, there is an important observation that is worth mentioning. As the number of stocks taken into account grow, the efficient shifts to the left, yielding portfolios with higher returns for a given level of risk. This can be proved by simply looking at the Markovitz target return portfolio (in which the target return has been arbitrarily defined as 0.16 in both figures), in the first case the volatility of the portfolio is is more than 0.24, whereas in the second case it is 0.18. Another important observation, related with the previous one, is that in the second frontier it is possible to obtain portfolios that yield a higher Sharpe ratio, hence a better mix of risk and return. Finally, the preference about which kind of portfolio to choose according to the mean-variance principle may change as the universe of securities expands.

4.2 Computing different portfolios

In the previous section I mentioned the 'Markowitz target portfolio', here I explain the rationale of the different portfolios that I reported in the figures of the efficient frontier. As it has already previously highlighted, different individuals might have different investment prefereces. In order to satisfy such preferences it is possible to construct different kinds of portfolios:

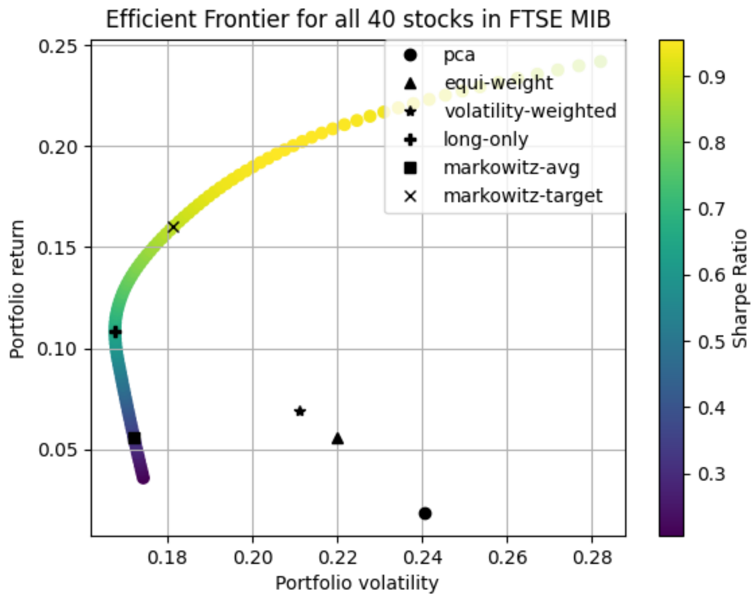

Figure 6: Markowitz's efficient frontier of the main 40 Italian stocks

- Min-variance using eigenvectors: computed by diagonalizing the variance-covariance matrix and picking the eigenvector associated to the smallest eigenvalue.
- PCA or max-variance using eigenvectors: which instead picks the eigenvector associated with the larger eigenvalue. Why would one construct a portfolio with the highest variance? The main reason is for replication purposes. (explain better)
- Equi-weighted: in which the notional is equally splitted between securities.
- Volatility-weighted: which weights more the securities with lower volatility.
- Long-only: the minimum variance portfolio that only admits long positions.
- Markowitz average: the classical Markowitz portfolio whose return is the average return of the stocks.
- Markowitz target: minimum variance portfolio given a target return.
- Make Max Sharpe Portfolio: ...

Remark static view of volatility and returns. The computed portfolios taking into account the 5 stocks considered for the first efficient frontier are the following:

```
------
Portfolio type: min-variance
Rics:
['A2A.MI', 'AMP.MI', 'ATL.MI', 'AZM.MI', 'BAMI.MI']
Weights:
[ 5.97808895 -1.03199789 -0.74840773 -1.98060661 -0.26089882]
Notional (mln EUR): 10.000000000000002
Delta (mln EUR): 1.9561779000472335
Variance explained: 0.06299448174082709
Profit and loss annual (mln EUR): -0.22415920417722016
Volatility annual (mln EUR): 1.2661412982994997
Return annual: -0.022415920417722016
Volatility annual: 0.12661412982994993
Sharpe ratio annual: -0.1770412231859737
```

(Using eigenvectors) The first one is the optimal portfolio if one considers the lowest overall volatility, however the historical return of this portfolio has been negative and since common sense suggests that any investor prefers a positive return, it is not worth spending further words.

```
------
Portfolio type: min-variance-l2
Rics:
['A2A.MI', 'AMP.MI', 'ATL.MI', 'AZM.MI', 'BAMI.MI']
Weights:
[ 5.97771194 -1.03267415 -0.74823322 -1.98225261 -0.25912808]
Notional (mln EUR): 10.0
Delta (mln EUR): 1.9554238830079715
Profit and loss annual (mln EUR): -0.22426706668044422
Volatility annual (mln EUR): 1.2661755283537062
Return annual: -0.022426706668044423
Volatility annual: 0.1266175528353706
Sharpe ratio annual: -0.1771216246550259
```

The portfolio minimizing the norm of the weights, which as previously seen is only a numerical (not exact) solution yields very similar results as before and therefore presents the same issue of negative returns. (Cost function????)

```
------
Portfolio type: min-variance-l1
Rics:
['A2A.MI', 'AMP.MI', 'ATL.MI', 'AZM.MI', 'BAMI.MI']
Weights:
[5.22882917 2.01821135 1.08884199 1.25504747 0.40907001]
Notional (mln EUR): 10.0
Delta (mln EUR): 10.0
Profit and loss annual (mln EUR): 0.8955487473140936
Volatility annual (mln EUR): 2.265556811289524
Return annual: 0.08955487473140937
Volatility annual: 0.22655568112895247
Sharpe ratio annual: 0.39528858550422286
```

On the other hand, minimizing using the l_1 norm, which is another numerical solution that only approximates the true solution, yields remarkably different results. This time the portfolio is profitable and despite volatility being doubled it yields a positive annualized return of almost 9%.

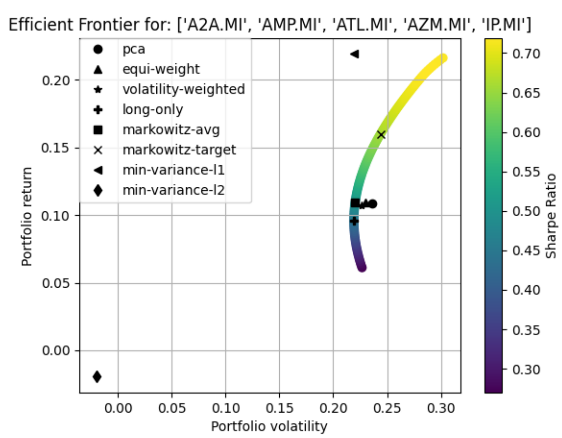

Figure 7: Efficient frontier with the addition of min-variance-l1 and min-variance-l2 portfolios.

CHANGE PLOT The plot above emphasizes the importance of analyzing a problem from multiple perspectives.

```
Portfolio type: long-only
Rics:
['A2A.MI', 'AMP.MI', 'ATL.MI', 'AZM.MI', 'BAMI.MI']
Weights:
[5.22882915 2.01821137 1.08884201 1.25504748 0.40907   ]
Notional (mln EUR): 9.999999999999996
Delta (mln EUR): 9.999999999999996
Profit and loss annual (mln EUR): 0.8955487516919051
Volatility annual (mln EUR): 2.2655568112877
Return annual: 0.08955487516919051
Volatility annual: 0.22655568112877003
Sharpe ratio annual: 0.39528858743687467
```

The ... indeed they both lie in the efficient frontier.

```
Portfolio type: pca
Rics:
['A2A.MI', 'AMP.MI', 'ATL.MI', 'AZM.MI', 'BAMI.MI']
Weights:
[1.36473264 1.53229535 2.17626302 2.1300496  2.79665939]
Notional (mln EUR): 10.0
Delta (mln EUR): 10.0
Variance explained: 0.5085770084985362
Profit and loss annual (mln EUR): 1.010186705743057
Volatility annual (mln EUR): 2.582219964385345
Return annual: 0.10101867057430569
Volatility annual: 0.2582219964385345
Sharpe ratio annual: 0.39120861881474733

Portfolio type: equi-weight
Rics:
['A2A.MI', 'AMP.MI', 'ATL.MI', 'AZM.MI', 'BAMI.MI']
Weights:
[2. 2. 2. 2. 2.]
Notional (mln EUR): 10.0
Delta (mln EUR): 10.0
Profit and loss annual (mln EUR): 1.03894
Volatility annual (mln EUR): 2.445286077333284
Return annual: 0.10389400000000001
Volatility annual: 0.24452860773332843
Sharpe ratio annual: 0.4248746229042534

Portfolio type: volatility-weighted
Rics:
['A2A.MI', 'AMP.MI', 'ATL.MI', 'AZM.MI', 'BAMI.MI']
Weights:
[2.65034562 1.96814335 1.74165354 2.03792703 1.60193046]
Notional (mln EUR): 10.0
Delta (mln EUR): 10.0
Profit and loss annual (mln EUR): 1.005354995199421
Volatility annual (mln EUR): 2.3801366899371157
Return annual: 0.10053549951994209
Volatility annual: 0.23801366899371154
Sharpe ratio annual: 0.42239380597338

Portfolio type: markowitz
Rics:
['A2A.MI', 'AMP.MI', 'ATL.MI', 'AZM.MI', 'BAMI.MI']
Weights:
```

```
[6.94030805e+00 5.41001647e-02 1.46143017e+00 1.54416161e+00
 2.16840434e-18]
Notional (mln EUR): 10.0
Delta (mln EUR): 10.0
Profit and loss annual (mln EUR): 0.5000000000237974
Volatility annual (mln EUR): 2.3602489458427427
Target return: 0.05
Return annual: 0.05000000000237974
Volatility annual: 0.23602489458427425
Sharpe ratio annual: 0.21184206051843785
```

Finally, combining the visual interpretation of the efficient frontier with the analytical insights...

5 An informal introduction to option pricing

5.1 Black-Scholes

In this section I necessarily have to focus in discussing the main interpretations given by the outputs, as the derivation of the Black-Scholes formula requires an extense discussion on stochastic calculus. An option is ...

Why is it useful determine the theoretical value of an option? Just as the map is not the territory, so a model is not reality, yet it provides a guide for understanding it better. In our case, an option pricing model provides a starting point for determining an appropriate strategy and more importantly for quantifying the risks that the strategy entails.

What are some important considerations to take into account? (Intuitions from option volatility and pricing)

The assumptions are: ...

The Black-Scholes formula for determining the value of an european call option for a non-paying stock is:

$$C(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)}$$

where $d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + (r + \frac{\sigma^2}{2})(T-t)\right]$ and $d_2 = d_1 - \sigma\sqrt{T-t}$

Therefore there are five characteristics that are needed for the computation, namely: strike price, time to expiration, price of the underlying, interest rate and volatility. Remark static ...
Example for Italian stock

5.2 Pricing options through Monte Carlo simulation

In essence, a Monte Carlo Simulation (or Monte Carlo method) is a way of estimating the possible outcome of an uncertain event by using random numbers. One simple example that helps to clarify how this approach works is the following: assume we want to determine the value of π, to achieve that it is possible to start with a unit circle inscribed in a square and generate random points in the space. Since we know the ratio of the area of the circle with respect to the square we can estimate the value of π by counting how many points fall inside the circle compared to the total number of points. The answer will approximate the true value.

In our context, the goal is to estimate the value of the theoretical price of a given option by simulating random variables. Assuming that the behavior of stock prices follows a Geometric Brownian Motion, that is:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where W_t is a standard Brownian motion (or Wiener process), μ the drift (return) and σ the volatility, then we can determine the price of a stock at time t (that is, S_t) given an arbitrary initial value of the stock S_0 at time $t = 0$. The solution is given by:

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right)$$

This way we can simulate a given number of future stock prices and then use them to find the discounted expected option payoffs.

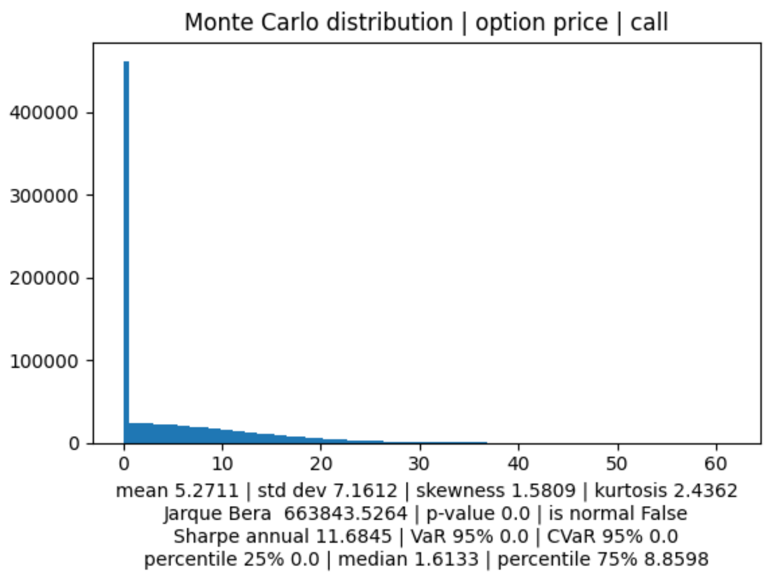

Figure 8: distribution of option prices given by the Monte Carlo simulation. The large frequency of value 0 is due to the fact that the price of the option is worthless at expiration (out of the money).

```
Price using Black-Scholes formula | call
5.280950048178042
------
Monte Carlo simulation for option pricing | call
number of simulations 1000000
confidence radius 0.014043764152302083
confidence interval [5.26803327 5.2961208]
price 5.282077037478256
probability of exercise 0.563328
probability of profit 0.365157
```

The average price of the option using the same predicted by the Black Scholes formula. Since the computed value is a mean, by the law of large numbers, as the sample size increases, the sample mean converges to the real mean, moreover, by the central limit theorem, the mean converges in distribution to $N(\mu, \frac{\sigma^2}{n})$. Therefore (as reported in the output above) it is possible to construct a 95% confidence interval for the average option price as:

$$S_n \pm 1.96SE = S_n \pm 1.96 \times \frac{\sigma}{\sqrt{n}}$$

Along with this information, by using this approach it is possible to estimate the probability of exercising the option, that is, when ... and the probability of profit...

5.3 An intuitive approach to greeks

To conclude the discussion, I will comment on which additional information can be gained by simply looking at a figure comparing the price of an option and the price of the underlying changes. In finance, the Greeks represent the sensitivity of the price of a derivative with respect to several parameters. For instance, ...

Putting the mathematics aside, the basic interpretation of the most common Greeks is the following:

- Delta: sensitivity of the option price to a change in the price of the underlying.
- Gamma: sensitivity of the option delta to a change in the price of the underlying.
- Vega: sensitivity of the option price to a change in implied volatility.
- Theta: expected change in the option price as time passes.
- Rho: sensitivity of the option price to interest rates or dividend payout.

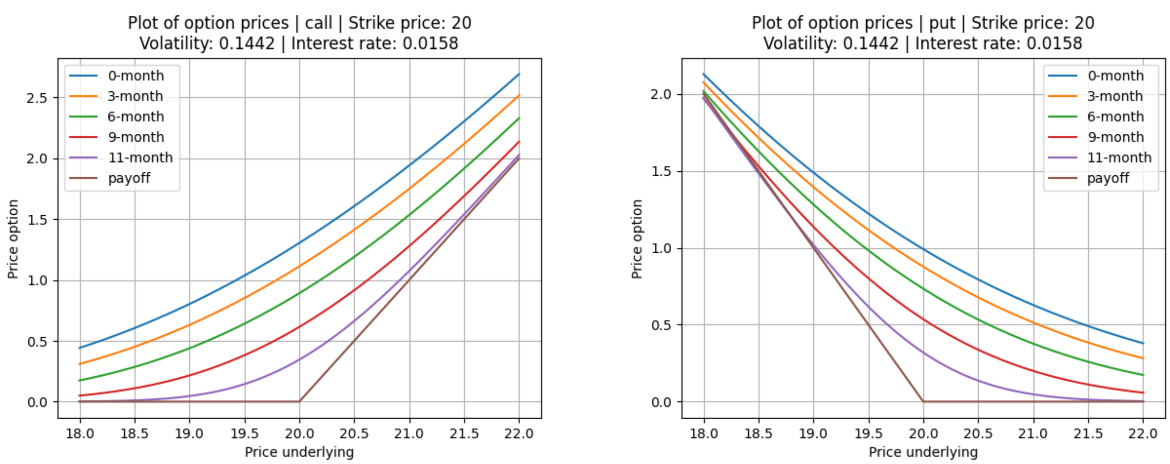

Figure 9: plot of call and put option prices with same input parameters