# TME260 Fatigue & Fracture
# Assignment 2

## Fatigue crack growth – digital twin

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- Supervisor: MICHELE MAGLIO

Time to finish and hand over to supervisor **2018-09-30**

You must include **all code** in your report if you are to pass the assignment.

A flow chart for crack growth prediction is included as an appendix. Further, code skeletons are available at the course home page. Note that the code skeletons are (mainly) for the last task. To follow/use these is optional, but you will not get *programming support* if they are not followed.

## Background

This assignment deals with crack growth in a rotating axle. The crack growth is promoted by rail bending due to passing wheels. The axles are designed and tested for existing defects (cracks). During operation the axles are subjected to inspections that can improve the prediction of the remaining fatigue crack growth life.

The first task in the assignment is to predict the fatigue life for the axle. In the second task the fatigue life prediction is updated and subsequent inspection intervals established based on (lack of) crack geometry data obtained from inspections. This second model constitutes a (very small and limited) digital twin where numerical models of the operational components are supporting maintenance and inspection decisions.

# Axle geometry and loads

The geometry of the axle and the crack is given in figure 1.

The geometric and material data of the axle are:

- Axle radius $r = 80$ [mm]
- Elasticity modulus $E = 210$ [GPa]
- Fracture toughness $K_{\mathrm{Ic}} = 60\,[\mathrm{MPa}\sqrt{\mathrm{m}}]$
- Material parameters in Paris law $C = 5 \cdot 10^{-9}$ and $m = 3$ for $\mathrm{d}a/\mathrm{d}N$ in mm/cycle and $\Delta K_{\mathrm{I}}$ in $\mathrm{MPa}\sqrt{\mathrm{m}}$.
- Walker parameters $\gamma = 1$ for $R \geq 0$ and $\gamma = 0$ for $R < 0$.
- Semi-elliptical crack according to figure 1with $b/s = 0.4$.
- The reference load consist of a rotating bending moment $M = \pm 60$ [kNm]

# Stress intensity factors

For the studied configuration, the stress intensity factor can be expressed as

$$K_{\mathrm{I}} = S \cdot F\sqrt{\pi s}, \;\; S = \frac{4M}{\pi r^3} \tag{1}$$

Here $M$ is the bending moment, $r$ the radius of the axle, and $s$ the arc length according to figure 1.

The magnitude of $F$ (and thereby $K$) will generally depend on the position of the crack front, and also on the orientation of the crack in relation to the neutral axis of the axle.

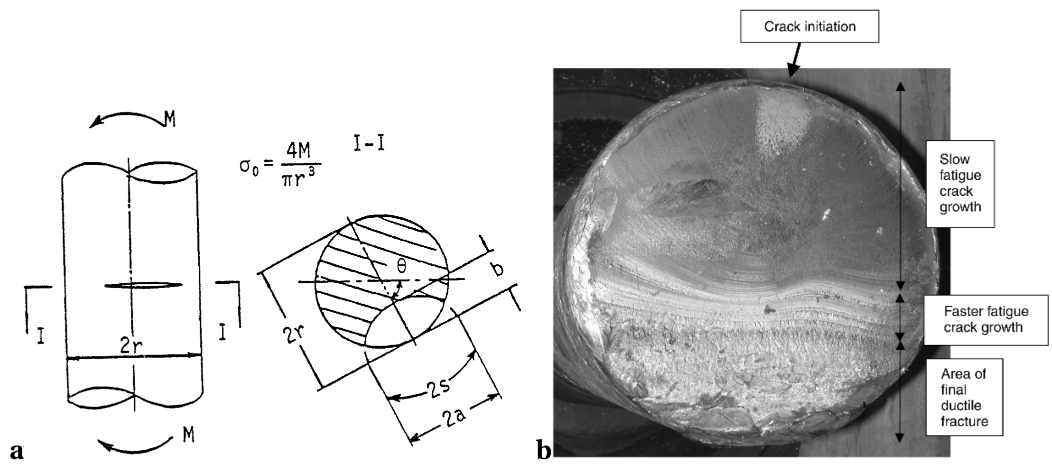


Figure 1: a) Geometry for axle containing a crack (right). b) Fracture surface of a train wagon axle (from D S Hoddinott, Railway axle failure investigations and fatigue crack growth monitoring of an axle, IMechE Journal of Rail and Rapid Transit, vol. 218, pp 283–292, 2004).

The highest loading of the crack is obtained for $\theta = 90°$. An approximation of the geometry factor $F$ for this case is tabulated in table 1. The geometry factor relates to the stress intensity at the deepest point of the crack edge.

| $b/s$ \ $b/r$ | $\approx 0$ | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|---|---|
| 0.4 | 0.595 | 0.558 | 0.554 | 0.564 | 0.608 | 0.695 | 0.858 |

Table 1: Geometry factor $F$.

Note that we have presumed that the ratio between the axes of the semi-elliptical crack is constant, $b/s = 0.4$. In practice this is not the case. To evaluate the actual evolution of $b/s$, the stress intensities both at the deepest point of the crack edge and at the intersection with the axle surface can be evaluated. Crack propagation is then evaluated separately for these two points.

# Tasks

## 1. Estimate the fatigue life of the axle

**a** Evaluate analytically the critical crack depth corresponding to fracture at the reference load.

**b** Evaluate the number of cycles to fracture given an initial crack depth of $b = 1$ mm.

- Plot the evolution of the crack size with the number of cycles
- Plot the evolution of the stress intensity factor as function of the number of cycles
- Plot the evolution of the stress intensity factor as a function of the crack depth

**c.** Consider the case of variable amplitude loading with three load magnitudes:

- $M_- = \pm 40$ [kNm] corresponding to 25% of the load cycles
- $M_0 = \pm 60$ [kNm] corresponding to 50% of the load cycles
- $M_+ = \pm 75$ [kNm] corresponding to 25% of the load cycles

Evaluate the number of cycles to fracture.[1] Do this in two ways:

1. By introducing an equivalent stress range magnitude (see section 11.7.2 in [1]). Note that the critical crack depth for this approach will depend on the highest load magnitude!

[1] The definition of "fracture" requires some considerations. Use the conservative approach of equalling fracture with propagation to a crack depth that $b_c$ that corresponds to $K_I = K_{Ic}$ for the largest load magnitude.

2. By a cycle-by-cycle evaluation of the crack depth.

Also vary the load magnitude in two ways:

1. By successively changing load magnitude: $M = 40, 60, 60, 75, 40, 60, \ldots$ etc
2. By taking the loading in blocks of 100 000 cycles, i.e. 100 000 cycles at 40 kNm followed by 200 000 cycles at 60 kNm etc.

Plot the evolution of crack depth as function of the number of cycles for the two cases!

**d** Explain the (lack of) differences in predicted number of cycles to fracture between numerical integration and cycle-by-cycle evaluation for the two load sequences in in **c**. Identify and comment on the design flaw regarding equivalent stress evaluation in the code skeleton.

**e** Transform the given material parameters $C$ and $m$ so that they correspond to regular SI-units: $\mathrm{d}a/\mathrm{d}N$ in m/cycle and $\Delta K_\mathrm{I}$ in $\mathrm{N/m}^{(3/2)}$!
(Similar transformations can be used e.g. to transform crack growth data in inches per cycle.)

## 2. Enhance inspection intervals – digital twin

The machine is now considered to be in operation featuring the reference (constant amplitude) load. Non-destructive tests[2] (NDT) for occurring cracks are carried out. Much can be discussed on the reliability of test results and their interpretation in terms of crack depth.[3] Here we will avoid this topic and just assume that the theoretical crack growth curve is followed, but that the actual crack length, $b_\mathrm{test}$ has a distortion following a normal distribution with $\mu = b_\mathrm{int}, \sigma = b_\mathrm{int}/10$ where $b_\mathrm{int}$ is the crack depth predicted by numerical integration.

**f** Prescribe the next inspection interval so that there are always three inspection intervals before failure (using the evaluated critical crack length). Plot the inspection interval as a function of the number of cycles.

**g** Use the inspection data (crack sizes and pertinent number of cycles) to estimate material parameters $m$ and $C$! Note that you need to translate the inspection data to crack growth rates ($\mathrm{d}b/\mathrm{d}N$) between the inspection intervals. These crack growth rates need to be related to the pertinent average stress intensity rates in the inspection interval. See example 11.1 in [1] and in particular the Discussion part of that example.

[2] Regarding cracks, two common methods are eddy current testing (ECT) [2] and ultrasonic testing (UT) [3]. Also methods such as magnetic particle inspection (MPI) [4] and dye penetrant (DP) inspection [5] can be used, but they are more limited to surface features of the fatigue crack.

[3] For example, if no crack is detected, any defect (i.e. crack) cannot be smaller than the detection limit, but it is unknown how large it is.

# References

[1] DOWLING N E (2013). **Mechanical behavior of materials**, 4th edition. (Pearson Education) *Prentice Hall*, Upper Saddle River, NJ, USA, 936 pp.

[2] https://en.wikipedia.org/wiki/Eddy-current_testing

[3] https://en.wikipedia.org/wiki/Ultrasonic_testing

[4] https://en.wikipedia.org/wiki/Magnetic_particle_inspection

[5] https://en.wikipedia.org/wiki/Dye_penetrant_inspection