TME260 Fatigue & Fracture
Assignment 4

Multiaxial Fatigue

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- Supervisor: MICHELE MAGLIO

Time to finish and hand over to supervisor **2019-10-28**.

You must include **all code** in your report if you are to pass the assignment.

A flow chart for multiaxial fatigue assessment is included as an appendix. Further, code skeletons are available at the course home page. To follow/use these is optional, but you will not get *programming support* if they are not followed.

The assignment deals with multiaxial fatigue analysed using the Dang Van and Crossland equivalent stress criteria. In the first part an analytical assessment is made of errors introduced by quantifying fatigue loading by the largest principal stress. The second part considers out-of-phase loading in combined bending and torsion. This is a common load case in automotive application, e.g. in axles and shafts. This part is to be solved numerically, e.g., by using MATLAB.

Tasks

Analytical assignment using the Dang Van criterion

Assume a material that has a material parameter in the Dang Van criterion of $c_{\mathrm{DV}} = 1/3$.

1. Derive the Dang Van equivalent stress when the material is loaded in uniaxial alternating loading with a stress amplitude of σ_{a}, see figure 1a!

2. Derive the Dang Van stress when the material is subjected to a biaxial in-phase loading with principal stress amplitudes $\sigma_{1,\mathrm{a}} = \sigma_{\mathrm{a}}$, $\sigma_{2,\mathrm{a}} = \sigma_{\mathrm{a}}$, $\sigma_{3,\mathrm{a}} = 0$, see figure 1b!
Evaluate the error if fatigue had been characterised by $\sigma_{1,\mathrm{a}}$ (i.e. under the implicit presumption that $\sigma_{2,\mathrm{a}} = \sigma_{3,\mathrm{a}} = 0$)!
The error is defined as $\left(\sigma_{\mathrm{eq,dv}}/\sigma_{\mathrm{eqdev,appr}}\right) - 1$, where $\sigma_{\mathrm{eq,dv}}$ is the correct and $\sigma_{\mathrm{eqdev,appr}}$ the approximate Dang Van stress.

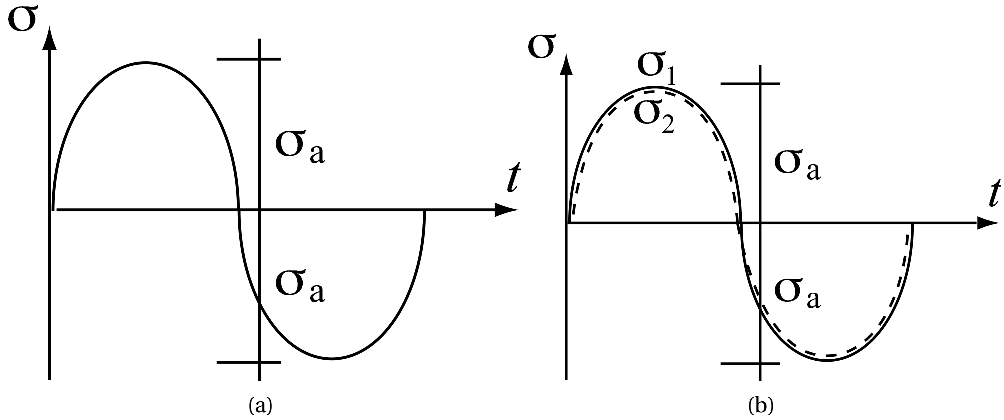

Figure 1: Alternating (a) uniaxial and (b) biaxial stress cycles.

3. Construct an approximative Wöhler curve under the assumption that a fatigue life of 1 000 cycles corresponds to a Dang Van equivalent stress pertinent to the uniaxial stress amplitude of $\sigma_{\rm a} = 0.9\sigma_{\rm u}$, where $\sigma_{\rm u} = 2\sigma_{\rm fl}$ is the ultimate tensile stress. Further the Dang Van stress corresponding to the uniaxial stress amplitude $\sigma_{\rm fl}$ taken equal to the stress magnitude evaluated in task 1, corresponds to the fatigue limit which is assumed to be $2 \cdot 10^6$ cycles.
Use this approximate Wöhler curve to estimate the error (in number of cycles) in fatigue life prediction corresponding to the erroneously evaluated stress magnitude from task 2!

4. Consider an 180° out-of-phase loading with $\sigma_1 = \sigma_2 = 0 \to \sigma_{\rm a} \to 0 \to -\sigma_{\rm a} \to 0$, $\sigma_3 = 0 \to -\sigma_{\rm a} \to 0 \to \sigma_{\rm a} \to 0$, see Figure 2.
Evaluate the Dang Van equivalent stress!
Evaluate the error if the fatigue evaluation had been based on the magnitude of $\sigma_{1,\rm a}$ (i.e. under the implicit presumption that $\sigma_{2,\rm a} = \sigma_{3,\rm a} = 0$)!

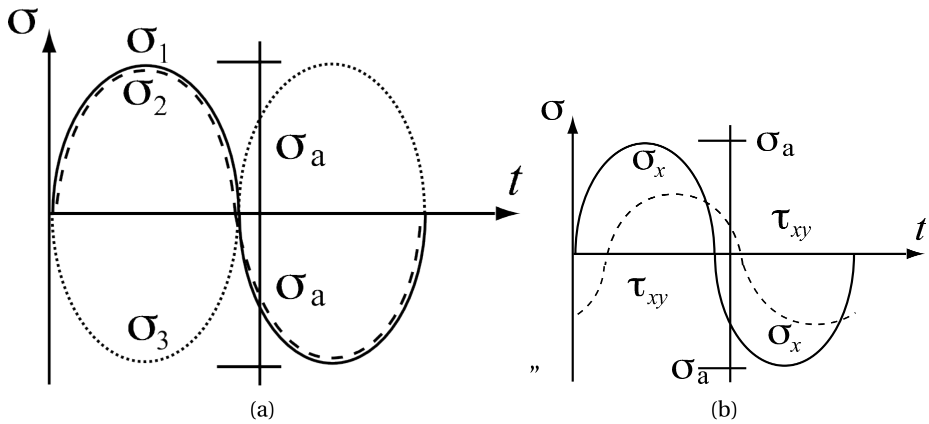

Figure 2: a) An alternating multiaxial stress cycle with one of three normal stress components 180° out of phase.
b) An alternating multiaxial stress cycle with a shear stress component 45° out of phase.

Programming assignment using the Crossland criterion

Based on the theory in the course material in multiaxial fatigue, implement a code that evaluates the time evolution of the Crossland equivalent stress in a material point that is subjected to a general loading. The input to the program should be:

- Stress history in terms of the stress tensor
- The material parameter c_c

The output of the program should be:

- The maximum Crossland equivalent stress during a stress cycle
- A plot (or table) of the Crossland equivalent stress as a function of time.

An outline for the design of the code is given in Appendix and in the code skeleton that you can download from the course homepage. The program shall be verified against tasks 1, 2 and 4 above. Note that the default code skeleton uses subroutines to enforce $\sigma^{\mathrm{d}}_{ij,\mathrm{mid}}$ to be deviatoric. You can evaluate what the difference in results will be by excluding these subroutines.

6. A driving axle in a machine is made of a material with a (reduced) fatigue limit in alternating bending of $\sigma_{\mathrm{FLB}} = 260$ MPa, and a (reduced) fatigue limit in pulsating bending $\sigma_{\mathrm{FLPB}} = 220 \pm 220$ MPa. Derive the material parameters c_c and σ_{ec} in the Crossland criterion.

7. The driving axle has a circular solid cross section with the diameters $d_2 = 60$ mm and $d_1 = 50$ mm. The axle is subjected to a driving torque that can act in both directions. The maximum value of the torque is 1.2 kNm. It is further presumed that the axle can be subjected to a bending moment of at most the same magnitude (i.e. 1.2 kNm). These two stress components are uncorrelated, i.e. they can be described by the equations $\sigma_x = \sigma_{\max} \cdot \sin(\omega_1 t)$ MPa, $\tau_{xy} = \tau_{\max} \cdot \sin(\omega_2 t + \phi)$ where ϕ is unknown and $\sigma_{\max}$ and $\tau_{\max}$ are obtained from the given loading and geometry, see figure 2b
The axle contains a notch with $r/d = 0.1$ according to figure 3.

1. Determine (by numerically varying ϕ) the most dangerous phase difference. Presume that the angular frequency is the same for the two stress components, i.e. $\omega_1 = \omega_2$. Evaluate and tabulate the Crossland equivalent stress for phase differences $\phi = n\pi/8$ with $n = 0$ to 7.
2. Derive for this "worst case scenario" the Crossland equivalent stress and corresponding safety factor. Include the stress concentration caused by the notch.

Hint 1: You may presume that, for both bending and torsion, the stress is given as $k_t (= K_t)$ times the nominal stress (i.e. not using k_f). The conservative error of this simplification depends on the shape of the notch and can for bending be quantified using the notch sensitivity factor q. For torsion, the simplification is smaller than what q would imply due to the lower stress gradients.

Hint 2: Probably the easiest way (or at least the most general way) of accounting for stress concentrations in multiaxial loading is to increase the magnitudes of each stress component before deriving the equivalent stress (in contrast to the approach taken by Dowling in reducing the *S*–*N*-curve).

8. By inspection it is found that a tensile residual stress of $\sigma_{x,\mathrm{res}} = 100$ MPa exists in the notch due to manufacturing. Derive the Crossland equivalent stress when accounting for this residual stress.

APPENDIX

Derivation of the Crossland equivalent stress

Following the procedure outlined in EKBERG, "Multiaxial fatigue".

1. Read the material parameter c_c and the stress history for the stress cycle and material point considered.

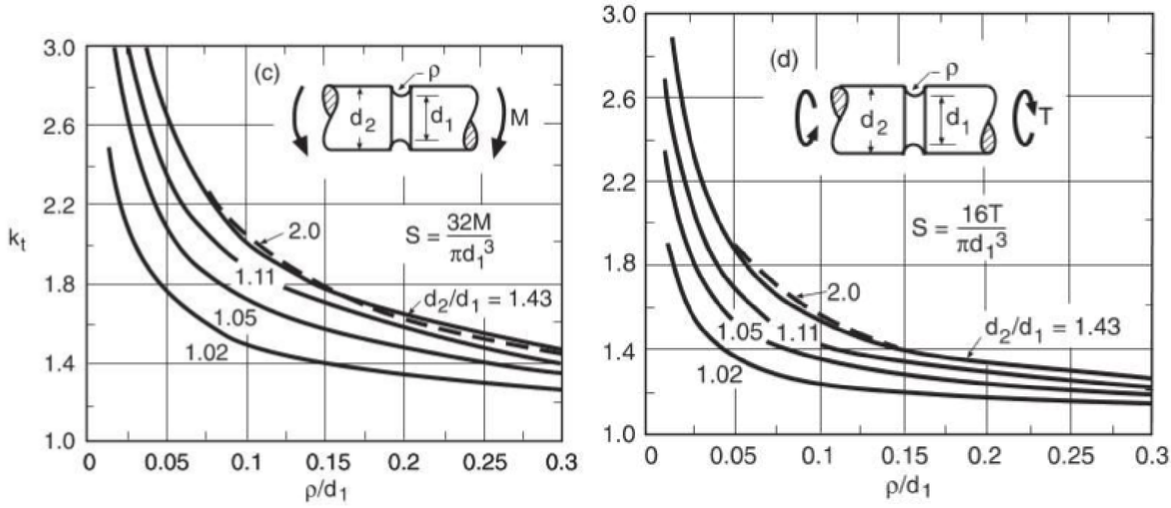

Figure 3: Stress concentration factors for a notched axle subjected to bending and torsion. (from Dowling)

2. Evaluate the hydrostatic stress $\sigma_{\mathrm{h}}(t)$ for all instants in time.

3. Evalute the deviatoric stress tensor for all time increments as $\sigma^{\mathrm{d}}_{ij}(t) = \sigma_{ij}(t) - \sigma_{\mathrm{h}}(t) \cdot \delta_{ij}$, where δ_{ij} is the Kronecker delta function.

4. Create a function that derives the von Mises stress $J_2 \equiv \sigma_{\mathrm{vM}} = \sqrt{\frac{3}{2}\sigma^{\mathrm{d}}_{ij}\sigma^{\mathrm{d}}_{ij}}$ given a deviatoric stress tensor.

5. Use the function `fminsearch` from MATLAB's Optimization Toolbox and the created function to derive the mid value of the deviatoric stress tensor, $\sigma^{\mathrm{d}}_{ij,\mathrm{mid}}$ for the current stress cycle and material point as the solution to the min–max problem $\min\{\max_{t\in T} J_2(\sigma^{\mathrm{d}}_{ij}(t) - \sigma^{\mathrm{d}}_{ij,\mathrm{mid}})\}$. The code should assure that `smid` is deviatoric. A possible call could be
```
smid = fminsearch(@(smid) J2(smid,sdev),(max(sdev)+min(sdev))/2)
```
Here `smid` is the mid value you search for and the third input parameter your initial guess. Further, `sdev` contains the deviatoric stress history in a 6 by N matrix, with N being the number of time increments studied and 6 the number of stress components. Note that the function J2 (that you must provide) should return the maximum (over time) value of the J_2-stress. This and more is described in the MATLAB documentation for `fminsearch`, which is recommended reading. Finally, make sure that your input includes a sufficiently long time series to capture the extreme Crossland stress.

6. Evaluate the deviatoric stress "amplitude" (i.e. the deviation of the deviatoric stress tensor from its mid-value) as $\sigma^{\mathrm{d}}_{ij,\mathrm{a}}(t) = \sigma^{\mathrm{d}}_{ij}(t) - \sigma^{\mathrm{d}}_{ij,\mathrm{mid}}$.

7. Evaluate the maximum hydrostatic stress $\sigma_{\mathrm{h.max}}$.

8. Evaluate the Crossland equivalent stress as $\sigma_{\mathrm{eq,c}}(t) = \sqrt{\frac{3}{2}\sigma^{\mathrm{d}}_{ij,\mathrm{a}}(t)\sigma^{\mathrm{d}}_{ij,\mathrm{a}}(t)} + c_{\mathrm{c}} \cdot \sigma_{\mathrm{h,max}}$.

9. Plot or tabulate $\sigma_{\mathrm{eq,c}}(t)$ and evaluate $\max_t(\sigma_{\mathrm{eq,c}}(t))$.