Exploring the World of Science

Compound Machines

Science Olympiad

Cornell University Invitational

24 January 2015

Team: ________________________ School: ______________________________

ANSWER SHEET

Team: ________________________ School: ________________________

1. (i)

   (ii)

   (iii)

   (iv)

   (v)

   (vi)

2. (i)

   (ii)

3. (i)

   (ii)

   (iii)

4. (a) (i) Figure:

   (ii) $F_e^{\min} =$

   (iii) $F_e^{\min} =$

   (iv) $\text{IMA}_{plane} =$

   (v) $\text{IMA}_{plane} =$

   (b) $\text{IMA}_{wedge} =$

   (c) $\text{IMA}_{screw} =$

5. (i) Law:

   Significance:

   (ii) Law:

   Significance:

   (iii) Law:

   Significance:

6. Newton's laws:

7. (a)

   (b) eff =

8. (i) Helical:

   (ii) Planetary:

   (iii) Herringbone:

   (iv) Sector:

9. (a)

   (b)

   (c)

   (d)

10. (i)

   (ii)

   (iii)

   (iv)

   (v)

   (vi)

   (vii)

11. (a)

(b)

(c)

(d)

(e)

(f)

12. (i) IMA =

Class:

(ii) IMA =

Class:

(iii) IMA =

Class:

(iv) IMA =

Class:

(v) IMA =

Class:

13. (a) $\mu =$

(b) eff =

14. $r_{\mathrm{pt}} =$

15. (a) Figure:

(b) $M =$

(c) $\mathrm{IMA}_{plank} =$

16. $\mathrm{IMA}_{wedge} =$

eff =

17. (i) Example 1:

(ii) Example 2:

(iii) Other examples (optional):

18. The difference between *work* and *effort*:

19. Explanation:

20. (i) Machine 1:

(ii) Machine 2:

(iii) Machine 3:

1. State and classify the six types of simple machines.

2. Of the six types of simple machines, one specifically was noted by a famous ancient Greek mathematician and engineer through the following phrase: "Give me a place to stand, and I shall move the Earth with it." Identify **both** of the following:

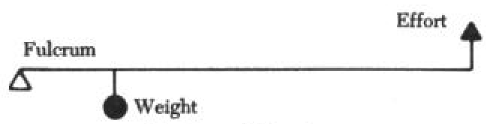
   (i) The mathematician described,

   (ii) The simple machine whose power is discussed by this mathematician.

3. Identify the class of each of the following levers:

   (i) 

   (ii) 

   (iii) 

4. Because complex machines are just conjunctions/compoundings (hence the name–"compound machines") of simple machines, we can often derive more difficult expressions from easier ones. As a result, you will be walked through the derivation of the mechanical advantage of a *simple* machine and then extend it to some other simple machines that are, in some sense, conjunctions of single machines. Follow the steps below carefully and show your work.

   (a) Consider an inclined plane of length L at an angle of θ with respect to the horizontal. It is positioned so that the top end of the plane is at a height h above the ground. Suppose additionally that a block of mass m is at rest on the (frictionless) surface of the plane.

      (i) Draw a figure representing the scenario, labeling all given quantities and any forces described/relevant to the situation.

      (ii) Consider a force F_e applied to the mass. What is the minimum value of F_e required to move the block up the plane in terms of m, g, and θ?

      (iii) Rewrite your answer in (ii) in terms of m, g, h and L.

      (iv) The ideal mechanical advantage (mechanical advantage without friction) is best described at how much relative force is required to *overcome* the resistance force (can be thought of as the *load*) in the mass (the inertial force, i.e. the force due to gravity). Let F_g represent this inertial force. In terms of F_e and F_g, find the ideal mechnanical advantage *IMA*.

      (v) Rewrite your answer in (iv) in terms of m, g, h and L based on values for F_e and F_g found above.

   (b) Repeat the process described in (a) for a wedge (an isoceles triangle) of length L (side length) and thickness t (base length). You only have to state the final answer as computed in step (v). *Hint: Treat this as two inclined planes put together.* This is a "compound machine" based on inclines.

   (c) A screw is a bit more complicated. If we consider it as essentially a "long" incline wrapped around a shaft, we can calculate ideal mechanical advantage *IMA* as usual. Suppose the screw has a *pitch* (the distance between adjacent threads of the screw) of P and a handle screwdriver of length L (which acts as a lever). This is also a "compound machine" based on levers and inclines. Again, repeat the process described in (a) for such a screw.

5. Clearly state Newton's three laws of motion. Explain the physical significance **and use** of each one.

6. (**Tiebreaker #4**) Mathematically describe each of Newton's laws written down in Problem 5. It is actually possible to write all of Newton's laws as consequences of Newton's second law. State boundary conditions under which the first and third laws can be derived from the first. Give two examples for each case.

7. Of course, as we see in Newton's second law from the previous two problems, ignoring the effects of friction allow us to ignore certain cases and assume a static equilibrium when this may not necessarily exist in a real situation. Thus, in not ignoring friction, we see that the *actual mechanical advantage* is less than or equal to the ideal mechanical advantage *IMA* (with equality if there is no friction) in any real situation. Let us define the *efficiency* of any system as always, adopting the idea of "actual over theoretical" as our ratio.

   (a) If a screw has a handle of length 31.6 cm and a pitch of length 31.4 mm and has 80% efficiency, calculate the *AMA* (actual mechanical advantage) of the system. Show all work.

   (b) Can you think of another equation for efficiency? Explain your thought process and show your derivation.

8. Draw, to the best of your ability, the external and internal structures for each of the following simple gears:

   (i) Helical gear

   (ii) Planetary gear

   (iii) Herringbone gear

   (iv) Sector gear

   Explain how these gears can connect together and form more complex structures through diagrams. Also, as a follow-up, how can one (mathematically) determine the number of teeth that the ring gear on a planetary gear has without counting?

9. In studying a subject, it is important to know both its theory as well as its history to better understand the context in which development occurred and can occur in the future. To this, please answer each of the following questions regarding developments of simple machines in history:

   (a) When was the waterwheel invented?

   (b) Where was the basic rudder invented?

   (c) Approximately how old is the standard hand axe tool (for woodchopping purposes)?

   (d) Who invented the water clock? How exactly does it work?

10. List the SI units (show derivations from fundamental units if necessary) for each of the following quantities:

   (i) Distance

   (ii) Force

   (iii) Momentum

   (iv) Torque

   (v) Energy

   (vi) Work

   (vii) Power

11. Classify each of the following theoretical machines according to a type of simple machine (as described in Problem 1):

   (a) A fork, used for eating

   (b) A scissor, used for cutting fabrics

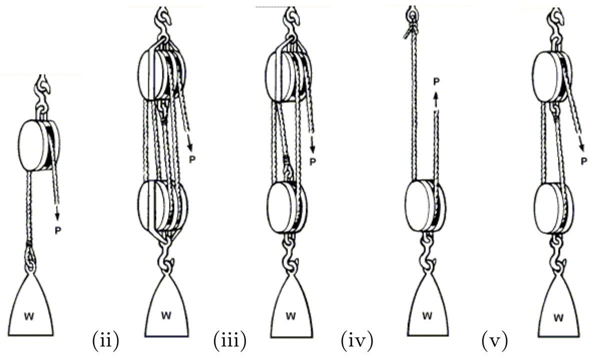
   (c) A wheelbarrow, used for holding dirt and mulch

   (d) A hand-drill with a turning-brace and a bit, used to drill

   (e) A hand yolk-beater, used in the kitchen

   (f) A block-and-tackle. What is such a machine used for?

12. Find the mechanical advantage of each of the following set-ups. (**Tiebreaker #3**) Classify each of the mass-pulley set-ups as well.



(i) (ii) (iii) (iv) (v)

13. A block of mass m is pushed horizontally d meters across a rough plank at constant speed with a force of F newtons.

    (a) Calculate the coefficient of friction μ in this set-up.

    (b) One end of the plank is lifted up onto the crossbar of a scaffold of height h. The same block is pushed d meters *up* the plane with a force of $5F/3$ newtons. What is the efficiency of the work on the plane?

14. (**Tiebreaker #1**) A solid material has a density of ρ. A uniformly distributed square bar made of this material has side lengths of ℓ cm and a beam length of L cm. A fulcrum is attached r cm from one end. If a mass m is placed $3r$ cm from the same end as the fulcrum, what volume of this material could be used to balance the beam? Where would the equilibrium balance point be (i.e. where would you have to put this solid material on the beam to bring it to balance)? Express all answers in terms of the given quantities.

15. (**Tiebreaker #2**) Consider the following theoretical description of a compound machine: A plank of length L has a fulcrum placed underneath the plank at the left end and a block of an unknown mass M fixed underneath the plank at the right end. Consider a second plank of length $3L/2$. This plank has a fulcrum placed at a distance of $L/4$ from the left end of the plank and a block of mass m fixed underneath the plank at the right end. The first plank is positioned horizontally in equilibrium and the second plank is placed on top of the first plank such that the left end of the second plank overlaps over $L/5$ of the right end of the first plank. Take the left end of the first plank to be the origin ($r = 0$).

    (a) Draw the scenario at hand and label all given quantities.

    (b) Calculate the unknown mass M in terms of the given quantities. *Hint: Use Newton's second law for rotational dynamics (net torque, etc.).*

    (c) What is the ideal mechanical advantage of the system?

16. What is the ideal mechanical advantage of a symmetrical wedge with a 21.4-degree angle of separation? What would its efficiency be if the actual mechanical advantage were 1.64? *Hint: Use the derivations in Problem 4.*

17. List at least two examples of first-class levers. (**Other Tiebreakers:** Listing more than two examples, in order of relevance/complexity...)

18. Draw the distinction between work and effort in your own words. What sets the two concepts apart (don't just state the definitions in a juxtaposition or similar fashion)?

19. Why is it impractical to measure the AMA of a wedge-shaped machine? State any practical limitations that justify your answer. Is the situation optimized or even worse for a $45 - 45 - 90$-degree triangular set-up? Why or why not?

20. In a household nail-clipper, what three simple machines does the compound machine comprise? If any of them is a lever, further classify the type of lever according to the system described in Problem 3.

SOME SPACE FOR SCRATCH WORK/SKETCHES