Lecture "Digital Signal Processing"

Prof. Dr. D. Klakow, summer term 2019

Tutorial 4

Submission deadline: 27.05.2019, 10:15

Submission Instructions:
You have one week to solve the tutorials.
The code should be well structured and documented. Do not use any Matlab-Toolbox or Python external libraries if not mentioned that you could use it.

- You are allowed to hand in your practical solutions in groups of two students.
- The theoretical part should be submitted before the lecture.
- For the practical tasks please submit files via the email address
  (Mon)Tutorial 1: dsp.tutorial1@gmail.com.
  (Thus)Tutorial 2: dsp.tutorial2@gmail.com.
- The subject of the letter should be [**DSP TUTORIAL 4**].
- Rename and pack the main directory:
  Ex04_matriculationnumber1_matriculationnumber2.zip.

The directory that you pack and submit should contain the following files:

- code files (contain *main.m* for matlab);
- file "answers.pdf" which contains answers to the questions appearing in the exercise sheet;
- file "README" that contains an information on all team members:
  name
  matriculation number
  email address.

1 (6P)Exercise

In this exercise we will discuss (two types of) cross-correlation, and how it can be used to determine the time delay between two sensors.

1.1 (2P) Subtask

Given a discrete, finite signal $x[n],\ n = 0, ...N-1$,the discrete Fourier transform is defined as

$$DFT\{x\}[k] = X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}, \quad k = 0, ...N-1 \tag{1}$$

and the inverse discrete Fourier transform, i.e. the original signal is

$$DFT^{-1}\{x\}[n] = x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{+j2\pi nk/N}, \quad n = 0, ...N-1 \tag{2}$$

The (circular) cross-correlation of two signals x, y is defined as

$$(x \star y)[n] = \sum_{m=0}^{N-1} x^{\star}[m]y[(m+n)\ mod\ N], \quad n = 0, ...N-1 \tag{3}$$

where $\ast$ denotes the comlex conjugate

What is the intuitive meaning of the cross-correlation as a function of n?
What is the asymptotic running time of the “naive” algorithm for calculating the cross-correlation vector?
Prove that

$$(x \star y)[n] = DFT^{-1}\{DFT^{*}\{x\} \cdot DFT\{y\}\}[n] \tag{4}$$

How can we use this fact to calculate the cross-correlation more quickly? What is the asymptotic running time of the improved algorithm?

1.2 (2P) Subtask

There are two microphone sensors recording the same source of audio.
Write a program that reads in the given audio file $Sensor1.wav$ and $Sensor2.wav$.
Use the fact shown in the subtask 1.1 to calculate the cross-correlation of the two signals.
Plot the cross-correlation against the time in seconds.
What does the maximum value of cross-correlation function mean?Corresponding value on X-axes.
What is the delay time between these two signals? which sensor did the audio arrive first?

Hint: you may use built-in functions for the transforms,but not for the cross-correlation(i.e. do not use xcorr())

1.3 (2P) Subtask

Another type of cross-correlation is GCC-PATH cross-correlation

$$\hat{G}_{PATH}(f) = \frac{X_i(f)X_j^*(f)}{|X_i(f)X_j^*(f)|} \tag{5}$$

where $X_i(f), X_j(f)$ are Fourier transforms of the two signals $x_i(n)$ and $x_j(n)$. $\ast$ denotes complex conjugate.

Please read GCC-PATH Note and write a summary of it.
Plot GCC-PATH for two signals from subtask 1.2. What do you observe? Compare it with the result of 1.2.
Include two plots (for both types of cross-correlation) into your report.

2 (4P)Exercise

In this part we will discuss linear microphone arrays.Please see *microphonearray.pdf* for information.

2.1 (2P) Subtask

Giving sampling rate $f_s = 45kHz$ and the distance between the microphones $\triangle x = 0.25m, c = 343\ m/s$.
Plot the angle of incidence φ (in degrees) for time delay $\triangle t = m \cdot T_s$ where $m = -24, ...24$ and T_s is period of sampling.
Change $\triangle x\ to\ 0.5m$, what do you observe? Include two plots into your report.
With the frequency of sound signal up to $900Hz$ and $\triangle x = 0.25m$,what is the maximum angle where the sound can be located ?

2.2 (2P) Subtask

Here is a simplified 2-D speaker-located system.Supposed there is only one speaker in an ideal environment (ignore the effect of environment such as temperature,winds and etc.) and no sound reflection. Read the audio files $Sensor1.wav$, $Sensor2.wav$ and $Sensor3.wav$ recorded by three different sensors.
Please use the function that you implement in 1.3 to locate the speaker and draw it in the graph ($c = 343m/s$).

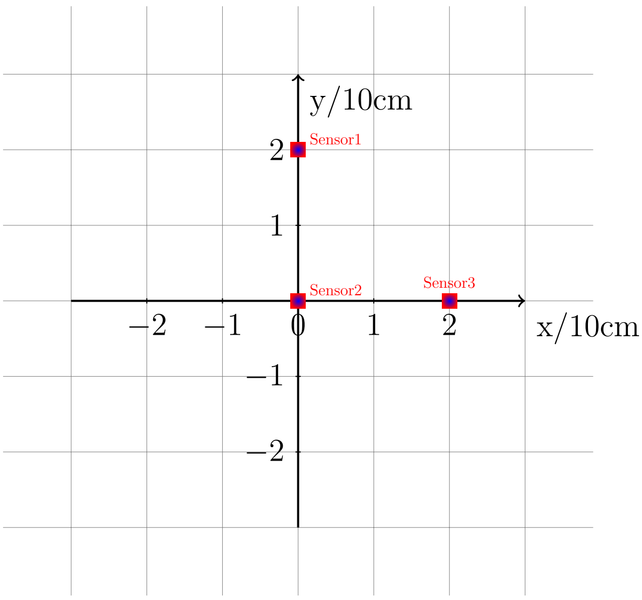

Figure 1

Is the speaker location in 3-D unique? Justify your answer.

2.3 (0P) Just thinking

The system will not be accurate since there is too much interference in the environment.Even a small error in φ could lead different location. Is there any method could improve the estimation of the location?