Lecture “Digital Signal Processing”

Prof. Dr. D. Klakow, summer term 2019

Tutorial 6

Submission deadline: 10.06.2019, 10:15

Submission Instructions:
You have one week to solve the tutorials.
The code should be well structured and documented. Do not use any Matlab-Toolbox or Python external libraries if not mentioned that you could use it.

- You are allowed to hand in your practical solutions in groups of two students.
- The theoretical part should be submitted before the lecture.
- For the practical tasks please submit files via the email address
  (Mon)Tutorial 1: dsp.tutorial1@gmail.com.
  (Thus)Tutorial 2: dsp.tutorial2@gmail.com.
- The subject of the letter should be [**DSP TUTORIAL 6**].
- Rename and pack the main directory:
  Ex06_matriculationnumber1_matriculationnumber2.zip.

The directory that you pack and submit should contain the following files:

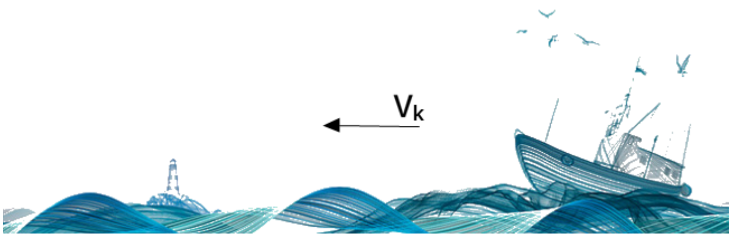
- code files (contain *main.m* for matlab);
- file “answers.pdf” which contains answers to the questions appearing in the exercise sheet;
- file “README” that contains an information on all team members:
  name
  matriculation number
  email address.

1 (7P)Exercise

In this exercise, we will implement a Kalman filter for the sailing boat. This reference may help you.

1.1 (1P) Subtask

Imagine, we are sailing a boat over the sea to an island. We could only control the boat either forward or backward towards the island with speed v_k.



Then the coordinate of the boat varies according to the following equation:

$$x_{k+1} = x_k + v_k \cdot \triangle t$$

We could also accelerate sailing with α:

$$v_{k+1} = v_k + \alpha \cdot \triangle t$$

Combine the equations we get the ideal coordinate of the boat:

$$x_{k+1} = x_k + v_k \cdot \triangle t + \frac{1}{2}\alpha \triangle t^2$$

In real life, we need to take into account small perturbations that affect the boat (wind, wave resistance, etc.), then the real position of the boat will differ from the calculated one. In this case, the random variable ζ_k should be added to the equation:

$$x_{k+1} = x_k + v_k \cdot \triangle t + \frac{1}{2}\alpha \triangle t^2 + \zeta_k$$

Where $\zeta \sim \mathcal{N}(0, \sigma_\zeta^2)$.

Assume this is the ground truth position of the boat. Implement a function for it and output all the x_k and v_k with inputs $x_0, v_0, \alpha, \triangle t, \sigma_\zeta^2, N$. Where N is the total number of time steps.

1.2 (1P) Subtask

On the boat, we have GPS for position measurement and IMU(Inertial Measurement Unit) for speed measurement. However both of them are not very accurate, e.g measurements are with Gaussian noise. For position: $\xi \sim \mathcal{N}(0, \sigma_\xi^2)$, for speed: $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$

$$z_k = x_k + \xi_k$$
$$V_k = v_k + \eta_k$$

Write a function of adding some gaussian noise to the output of subtask 1.1 and use the output as measurements.

1.3 (4P) Subtask

Now, we get the equations for the real coordinate and the sensor readings of the boat, respectively, which are:

$$x_{k+1} = x_k + v_k \cdot \triangle t + \frac{1}{2}\alpha\triangle t^2 + \zeta_k$$
$$z_k = x_k + \xi_k$$

Recall that our predict model is:

$$x_{k+1} = x_k + v_k \cdot \triangle t + \frac{1}{2}\alpha\triangle t^2$$

Implement Kalman filter to estimate the optimal coordination of the boat with the following conditions:

1. $x_0 = 1, v_0 = 0, \alpha = 0.15\ m/s^2$
2. $\triangle t = 0.2s, N = 100$
3. $\sigma_\xi^2 = 4,\ \sigma_\eta^2 = 4,$
4. $\sigma_\zeta^2 = 0.01$
5. The noise ξ, η and error ζ are independent to each other.

Plot the ground truth x_k, measurementz_k and optimal estimation x_k^{opt}(output of Kalman filter) in the same graph.

1.4 (1P) Subtask

Vary parameters σ_ξ^2, σ_η^2 and error σ_ζ^2, describe and explain the results.(For convenience, you could set $\sigma_\xi^2 = \sigma_\eta^2$).

2 (3P)Exercise

There are many good properties if a matrix is positive semi-definite, e.g Cholesky decomposition. Either in LPC or Wiener filter, we need to calculate the correlation matrix of signal x[n]. One can prove that the correlation matrix defined in the lecture is positive semi-definite.

2.1 (3P) Subtask

Prove that the $(n+1) \times (n+1)$-matrix R

$$R_{ij} = \frac{1}{n+1}\sum_{k=0}^{n} x[k+i]x[k+j]$$

is positive semidefinite[1]. A matrix is positive semi-definite, if for each vector $\mathbf{l} = [l(0)\ l(1)\ \cdots\ l(n)]$ the equation

$$\mathbf{l}R\mathbf{l}^{\mathbf{T}} \geq 0.$$

holds true. Additionally, prove that the matrix has only real and non-negative Eigenvalues.

[1] check `https://en.wikipedia.org/wiki/Positive-definite_matrix`