DS - 203 Assign 1

① E = manufactured by A or B

F = defective or not

$P(A) = 0.8$ $P(B) = 0.2$

$P(D|A) = 0.3$ $P(D|B) = 0.1$

(a) $P(D) = P(D|A) \cdot P(A) + P(D|B) \cdot P(B) = (0.8)(0.3) + (0.2)(0.1)$

$\Rightarrow \boxed{P(D) = 0.26}$

(b) $P(A|D) = \dfrac{P(D|A) \cdot P(A)}{P(D|A) \cdot P(A) + P(D|B) \cdot P(B)} = \dfrac{0.24}{0.26} = \boxed{\dfrac{12}{13}}$

11.00 ② (a) $P(FA|NW) = 1 \quad P(FA|W) = 0.1$

12.00 $P(W) = 0.8 \quad P(NW) = 0.2$

01.00 $P(FA) = P(FA|NW) \cdot P(NW) + P(FA|W) \cdot P(W)$

$= (1)(0.2) + (0.8)(0.1)$

02.00 $= \boxed{0.28}$

03.00 (b) $P(W|FA) = \dfrac{P(FA|W) \cdot P(W)}{P(FA|W) \cdot P(W) + P(FA|NW) \cdot P(NW)}$

$= \dfrac{(0.8)(0.1)}{0.28} = \boxed{\dfrac{2}{7}}$

(c) $P(2^{nd}\ FA \mid 1^{st}\ FA) = \dfrac{P(1^{st}\ \&\ 2^{nd}\ FA)}{P(1^{st}\ FA)}$

$P(\text{both fail}) \Rightarrow$ Server $\rightarrow$ W $\rightarrow$ 2 FA $\Rightarrow p = (0.1)(0.1)$

$\qquad\qquad\qquad\qquad\quad \downarrow$ NW $\rightarrow$ -||- $\Rightarrow p = (1)(1)$

$\Rightarrow P(\text{both fail}) = (0.1)(0.1)(0.8) + (1)(1)(0.2)$

$= 0.008 + 0.2 = 0.208$

$\Rightarrow P(2^{nd}\ FA \mid 1^{st}\ FA) = \dfrac{0.208}{0.28} = \boxed{0.743}$

(d) $P(W \mid \text{both fail}) = \dfrac{P(\text{both fail} \mid W) \cdot P(W)}{P(\text{both fail} \mid W) \cdot P(W) + P(\text{both fail} \mid NW) \cdot P(NW)}$

$= \dfrac{(0.01)(0.8)}{(0.01)(0.8) + (0.2)(1)} = \dfrac{0.008}{0.208}$

$= \boxed{0.038}$

③ 36 outcomes possible.

If identical dice → 11 have at least 1 '6'.

different — 11 — → 12 — 11

$\Rightarrow \boxed{(a)\ \frac{11}{36} \qquad (b)\ \frac{1}{3}}$

(4) $P(CB|M) = 0.05 \quad P(CB|W) = 0.01 \quad P(M) = P(W) = 0.5$

$$P(M|CB) = \frac{P(CB|M) \cdot P(M)}{P(CB|M) \cdot P(M) + P(CB|W) \cdot P(W)} = \frac{(0.05)(0.5)}{(0.05)(0.5) + (0.01)(0.5)}$$

$$= \frac{0.05}{0.06} = \boxed{5/6}.$$

5 (a) Statement $\Rightarrow P(E \cap E) = P(E) \cdot P(E) = [P(E)]^2 = P(E)$

$\Rightarrow P(E) = [P(E)]^2$

$\Rightarrow \boxed{P(E) = 0, 1}$ Hence Proved.

(b) (i) A, B independent $\Rightarrow P(A \cap B) = P(A) \cdot P(B)$

$= 0.12$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 - 0.12 = \boxed{0.58}$

(ii) A, B mut. exc. $\Rightarrow P(A \cap B) = 0$

$\Rightarrow \boxed{P(A \cup B) = 0.7}$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1.4 - P(A \cap B)$

≤ 1

$\Rightarrow P(A \cap B) \geq 0.4$

$P(A) \cdot P(B) = 0.48$

$\Rightarrow$ A, B can be independent but not mut exc.

(6) (a) $F(x) = \begin{cases} \frac{e^{-x^2}}{4} & x < 0 \\ 1 - \frac{e^{-x^2}}{4} & x \geq 0 \end{cases}$ $\frac{d}{dx}(F(x)) = \begin{cases} \frac{(e^{-x^2})(-2x)}{4} & x < 0 \\ 1 - \frac{e^{-x^2}(-2x)}{4} & x \geq 0 \end{cases}$

$\Rightarrow \frac{d}{dx}(F(x)) = \begin{cases} -\frac{x}{2} e^{-x^2} & x < 0 \\ 1 + \frac{x}{2} e^{-x^2} & x \geq 0 \end{cases}$ $\Rightarrow \frac{d}{dx}(F(x)) \geq 0 \Rightarrow$ non-dcing.

$x \to \infty \Rightarrow F(x) \to 1$ $x \to -\infty \Rightarrow F(x) \to 0$ $\Rightarrow F(x)$ is CDF.

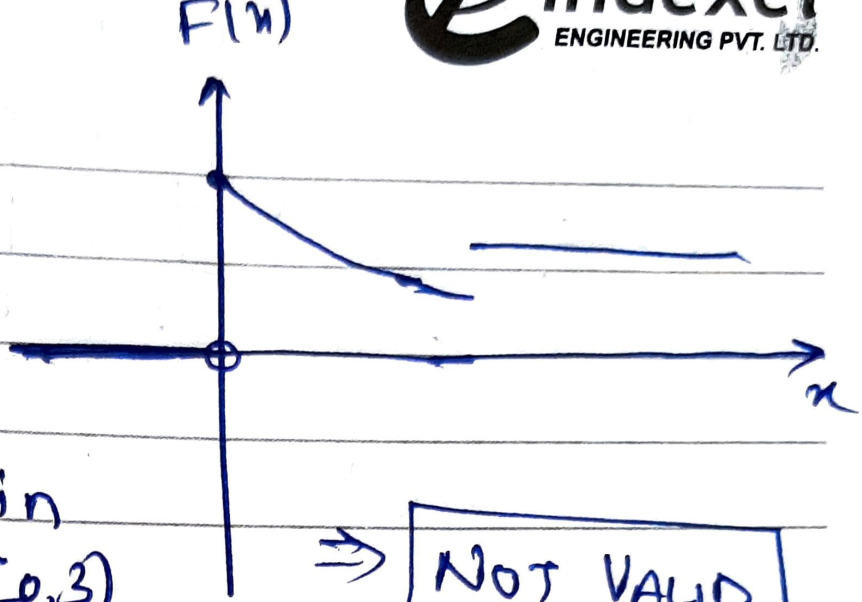
Any person receiving the said information agrees to

$P(X^2>5) \Rightarrow P(X<-\sqrt{5}) + P(X>+\sqrt{5}) = \lim_{x\to h^+} F(-\sqrt{5}-h) + \lim_{x\to h^+} F(\sqrt{5}+h)$

$1-F(\sqrt{5})$

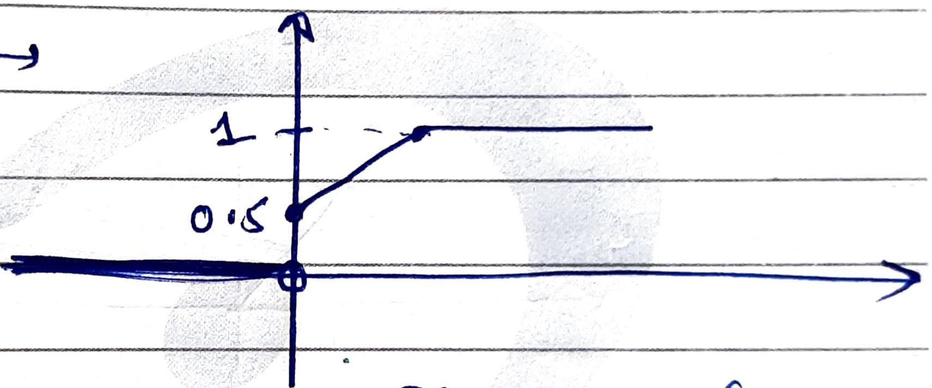
$= \frac{e^{-5}}{4} + 1-\left(1-\frac{e^{-5}}{4}\right) = \boxed{\frac{e^{-5}}{2}}.$

6 (b) $F(x) \rightarrow$



Not non-dec. in $x \in [0,3)$ $\Rightarrow$ $\boxed{\text{NOT VALID}}$

6 (c) $F(x) \rightarrow$



Non-dec ✓

$x \to -\infty \Rightarrow F(x) = 0$ ✓

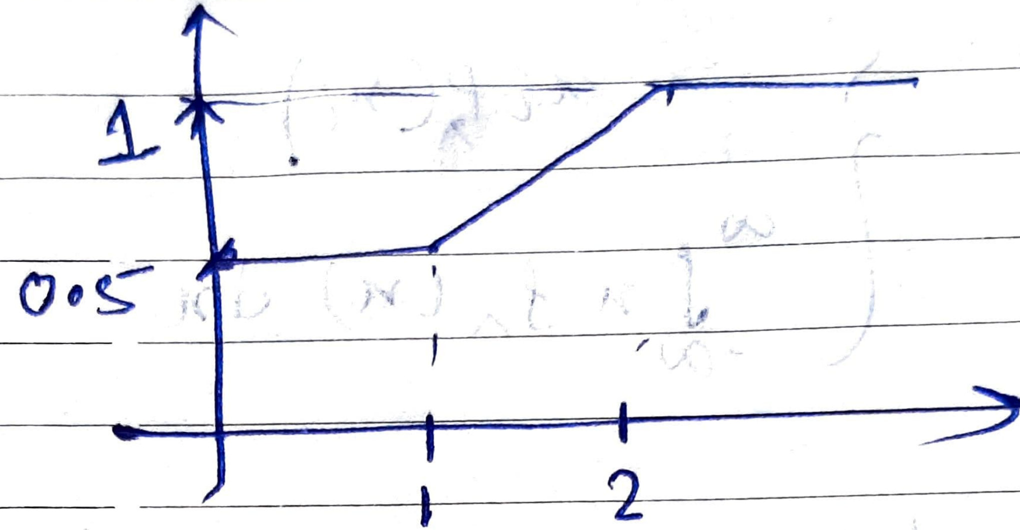
$x \to +\infty \Rightarrow F(x) = 1$ ✓

$\Rightarrow$ $\boxed{\text{VALID}}$

$$P(X^2 > 5) = \lim_{h \to 0^+} F(-\sqrt{5}-h) + 1 - F(\sqrt{5})$$

$$= 0 + 1 - 0.5 - \frac{\sqrt{5}}{20} = 0.5 - \frac{\sqrt{5}}{20} = \boxed{0.388}$$

7) CDF:



08.00

09.00

10.00

PMF = $\begin{cases} 0 & x < 0 \\ 1/2 & x \geq 0 \\ 0 & 0 < x < 1 \\ x/2 & 1 \leq x \leq 2 \\ 0 & x > 2 \end{cases}$

11.00

12.00

01.00

(a) $P(X \leq 0.8) = 0.5$

02.00

(b) $E(X) = \begin{cases} \sum_{i=1}^{\infty} x_i P_X(x_i) \\ \int_{-\infty}^{\infty} x f_X(x)\, dx \end{cases}$

$\cancel{E[X]}$ for cont. RV $\rightarrow$ $F_X(x) = \frac{x}{2}$

$f_X(x) = \frac{d}{dx} F_X(x) = 1/2$

$\Rightarrow E(X) = \left(\frac{1}{2}\right)(0) + \int_1^2 x f_X(x)\, dx = \int_1^2 \frac{x}{2}\, dx$

$= \left[\frac{x^2}{4}\right]_1^2 = \frac{3}{4} = \boxed{0.75}$

(c) $Var(x) = \begin{cases} \sum_{i=1}^{\infty} (x_i - E(x))^2 P_X(x_i) \\ \int_{-\infty}^{\infty} (x - E(x))^2 f_X(x)\, dx \end{cases}$

$\Rightarrow Var(x) = (-0.75)^2 \left(\frac{1}{2}\right) + \int_1^2 (x - 0.75)^2 \frac{1}{2}\, dx$

$= \frac{9}{32} + \left[\frac{(x - 3/4)^3}{6}\right]_1^2 =$

$= \frac{9}{32} + \frac{\frac{125}{64} - \frac{1}{64}}{6}$

$= \boxed{\frac{29}{48}}$

(8) $f(x) = \begin{cases} c e^{-2x} & x \in [0, \infty) \\ 0 & x < 0 \end{cases}$

$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$

$\Rightarrow 0 + \int_{0}^{\infty} c e^{-2x}\,dx = 1 \Rightarrow c\left[\frac{e^{-2x}}{(-2)}\right]_0^{\infty} = 1$

$\Rightarrow \left(-\frac{c}{2}\right)(-1) = 1 \Rightarrow \boxed{c = 2}$

$P\{X > 2\} = \int_{2}^{\infty} f(x)\,dx = \int_{2}^{\infty} 2(e^{-2x})\,dx$

$= 2\frac{\left[e^{-2x}\right]_2^{\infty}}{(-2)} = \boxed{e^{-4}}$

(9) $P(H) = 0.7 \qquad X = \{0, 1, 2, 3\}$

$P(X=0) = (0.3)^3 = 0.027$

$P(X=1) = {}^3C_1 (0.7)(0.3)^2 = 0.189$

$P(X=2) = {}^3C_2 (0.7)^2 (0.3) = 0.441$

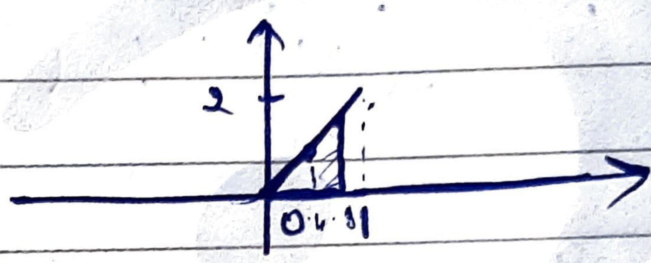
$P(X=3) = {}^3C_3 (0.7)^3 = 0.343$

9

(10) $P(X \geq 0.4 \mid x \leq 0.8) = \dfrac{P((X \geq 0.4) \cap (X \leq 0.8))}{\cancel{P(X \geq 0.4)}\, P(X \leq 0.8)}$

$$= \frac{\int_{0.4}^{0.8} f_X(x)\,dx}{\cancel{\left(\int_{0.4}^{\infty} f_X(x)\,dx\right)} \left(\int_{-\infty}^{0.8} f_X(x)\,dx\right)}$$

$f_X(x) =$



$$\Rightarrow p = \frac{(0.8)^2 - (0.4)^2}{\cancel{(1-(0.4)^2)}\left((0.8)^2\right)} = \frac{\cancel{0.48}}{\cancel{(0.84)(0.64)}}$$

6 3 8 0.42 0.14

$\Rightarrow \boxed{p = 0.75}$

(11) $f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

$a, b > 0$

$$P(X > (a+b) \mid X > a) = \frac{P((X > (a+b)) \cap (X > a))}{P(X > a)}$$

$$\Rightarrow p = \frac{\int_{a+b}^{\infty} \cancel{\lambda} e^{-\lambda x}\, dx}{\int_{a}^{\infty} \cancel{\lambda} e^{-\lambda x}\, dx} = \frac{\left[\dfrac{e^{-\lambda x}}{-\lambda}\right]_{a+b}^{\infty}}{\left[\dfrac{e^{-\lambda x}}{-\lambda}\right]_{a}^{\infty}}$$

$$\Rightarrow p = \frac{e^{-\lambda(a+b)}}{e^{-\lambda(a)}} = \boxed{e^{-\lambda(b)}}$$

12) (a) $\{HHHHH\}$ (b) $P(I_5 = 1) = (0.5)^5$

$$= \boxed{0.025}$$

$$= \boxed{1/32}$$

(13) $PDF = \frac{d}{db} F(b) \Rightarrow PDF = \frac{1}{2}\delta(x) + \frac{1}{2}\delta(x-1)$

Dirac Delta fxn.

$$\Rightarrow PMF_X = \begin{cases} 1/2 & x = 0 \\ 1/2 & x = 1 \end{cases}$$

(4). $P(W) = 0.5$

$${}^4C_2 \cdot (0.5)^4 = \frac{4!}{2!\,2!} \cdot \frac{1}{2^4} = \frac{6}{2^4}$$

$$= \boxed{3/8}$$

(15) $P(H) = p$

$\underbrace{\text{- - - - - - - - -}}_{n-1 \text{ have } r-1\ H}\ H$

$$\boxed{\binom{n-1}{r-1} p^{r} \cdot (1-p)^{n-r}}$$

Hence Proved

(16) $PMF(X = i) = \dfrac{e^{-\lambda} \lambda^{i}}{i!} = \dfrac{1}{e(i!)}$

$1 - P(0 \text{ error}) = \boxed{1 - \dfrac{1}{e}}$.