DS203: Programming for Data Sciences

Assignment 1

Exercise 1. *A product is manufactured by two factories A and B. 80% of the product is manufactured in company A and rest in company B. 30% of the product manufactured by company A are defective while 10% of the product manufactured by company B are defective. A sample of the product is randomly selected from the market. Then,*

1. *What is the probability that the sample is defective.*
2. *What is the probability that a defective sample in the market is manufactured at company A.*

Exercise 2. *A particular webserver may be working or not working. If the webserver is not working, any attempt to access it fails. Even if the webserver is working, an attempt to access it can fail due to network congestion beyond the control of the webserver. Suppose that the a priori probability that the server is working is 0.8. Suppose that if the server is working, then each access attempt is successful with probability 0.9, independently of other access attempts. Find the following quantities.*

1. $P($ *first access attempt fails*$)$
2. $P($*server is working* $|$ *first access attempt fails* $)$
3. $P($*second access attempt fails* $|$ *first access attempt fails* $)$
4. $P($*server is working* $|$ *first and second access attempts fail* $)$.

Exercise 3. *Two dice are rolled. What is the probability that at least one is a six? If the two faces are different, what is the probability that at least one is a six?*

Exercise 4. *Suppose that 5 percent of men and 1 percent of women are color-blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females.*

Exercise 5. *(a) Suppose that an event E is independent of itself. Show that either $P(E) = 0$ or $P(E) = 1$.*
(b) Events A and B have probabilities $P(A) = 0.3$ and $P(B) = 0.4$. What is $P(A \cup B)$ if A and B are independent? What is $P(A \cup B)$ if A and B are mutually exclusive?
(c) Now suppose that $P(A) = 0.6$ and $P(B) = 0.8$. In this case, could the events A and B be independent? Could they be mutually exclusive?

Exercise 6. *Which of the following are valid CDF's? For each that is not valid, state at least one reason why. For each that is valid, find $P(X^2 > 5)$.*

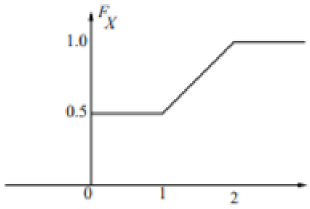
1. $$F(x) = \begin{cases} e^{-x^2}/4 & \text{if} \quad x < 0 \\ 1 - e^{-x^2}/4 & \text{if} \quad x \geq 0 \end{cases}$$

2. $$F(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ 0.5 + e^{-x} & \text{if} \quad 0 \leq x < 3 \\ 1 & \text{if} \quad x \geq 3 \end{cases}$$

3.

$$F(x) = \begin{cases} 0 & if \quad x < 0 \\ 0.5 + x/20 & if \quad 0 \le x \le 10 \\ 1 & if \quad x \ge 10 \end{cases}$$

Exercise 7. *Let X have the CDF shown.*



1. *Find* $P(X \le 0.8)$.
2. *Find* $E(X)$.
3. *Find* $Var(x)$.

Exercise 8. *If the density function of X equals*

$$f(x) = \begin{cases} ce^{-2x} & if\, 0 \le x < \infty \\ 0 & if\, x < 0 \end{cases}$$

find c. What is the value of $P\{X > 2\}$*?*

Exercise 9. *Suppose a coin having probability* 0.7 *of coming up heads is tossed three times. Let X denote the number of heads that appear in the three tosses. Determine the probability mass function of X.*

Ans Check

Exercise 10. *Let X is a random variable with probability density function*

$$f_X(x) = \begin{cases} 2x & if\ 0 \le x \le 1, \\ 0 & otherwise. \end{cases}$$

Find $P(X \ge 0.4 | X \le 0.8)$.

Exercise 11. *Let X is an exponentially distributed random variable with parameter* λ. *For any* $a, b > 0$, *find* $P(X > a + b | X > a)$.

Exercise 12. *Suppose five fair coins are tossed. Let E be the event that all coins land heads. Define a random variable* I_E

$$I_E = \begin{cases} 1 & if\ E\ occurs \\ 0 & if\ E^c\ occurs \end{cases}$$

For what outcomes in the original sample space does I_E *equals 1 ? what is* $P\{I_E = 1\}$

check ans & how to write

Exercise 13. *Suppose the distribution function of X is given by*

$$F(b) = \begin{cases} 0, & b < 0 \\ \frac{1}{2}, & 0 \leq b < 1 \\ 1, & 1 \leq b < \infty \end{cases}$$

What is the probability mass function of X?

Exercise 14. *A ball is drawn from an urn containing three white and three black balls. After the ball is drawn, it is then replaced and another ball is drawn. This goes on indefinitely. What is the probability that of the first four balls drawn, exactly two are white?*

Exercise 15. *A coin having probability p of coming up heads is successively flipped until the rth head appears. Argue that X, the number of flips required, will be n, $n \geq r$, with probability*

$$P(X = n) = \binom{n-1}{r-1} p^r (1-p)^{n-r}, n \geq r$$

Exercise 16. *Suppose that the number of typographical errors on a single page of this book has a Poisson distribution with parameter $\lambda = 1$. Calculate the probability that there is at least one error on this page.*