

EE 238

Power Engineering - II

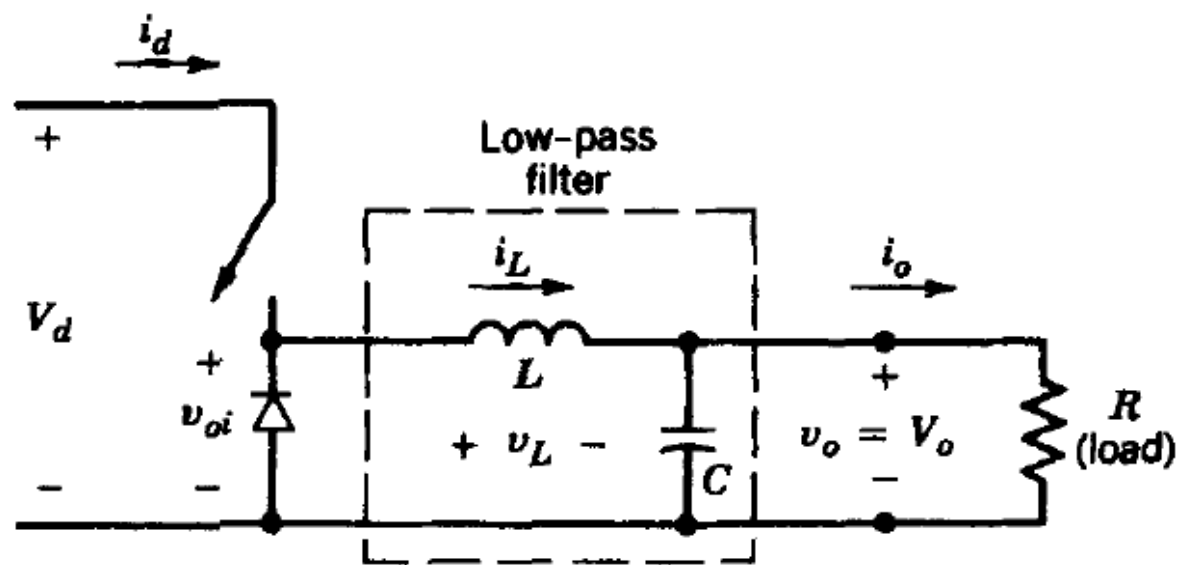
Power Electronics



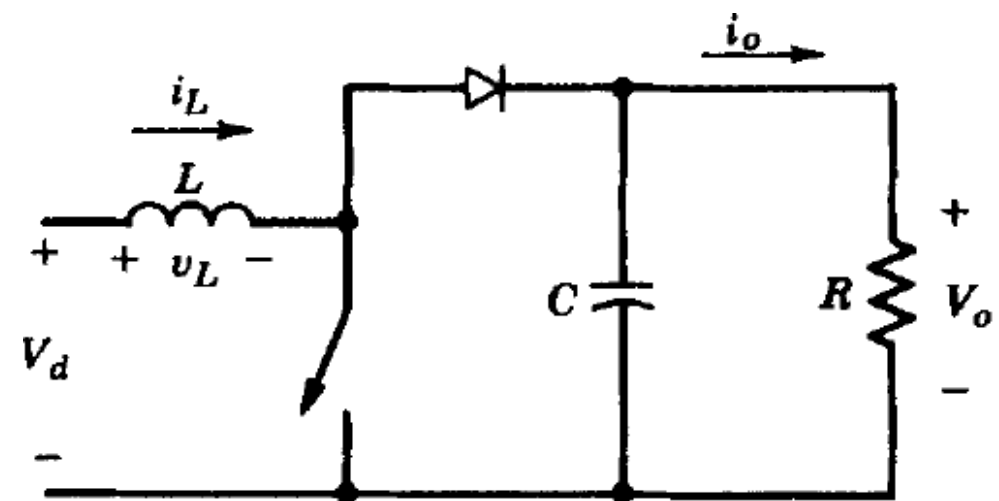
Lecture 11

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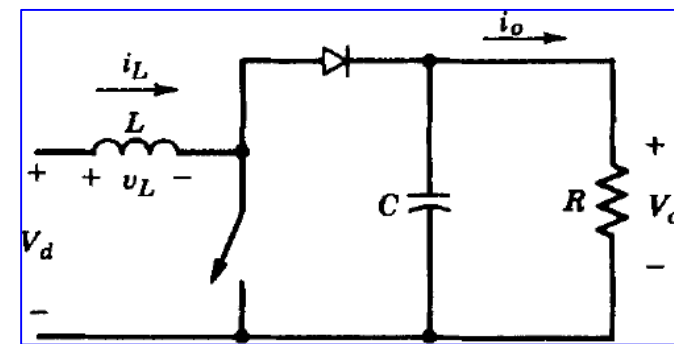
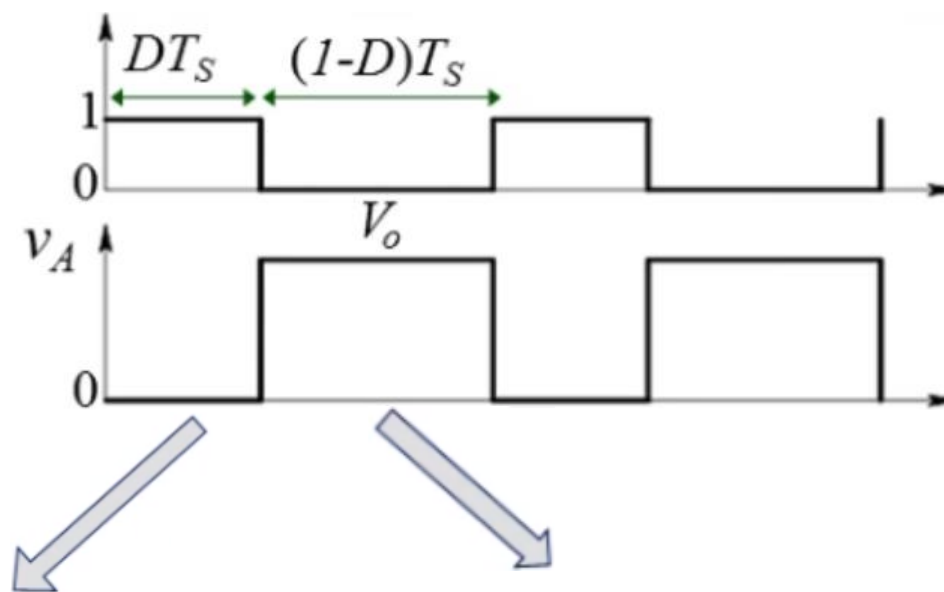
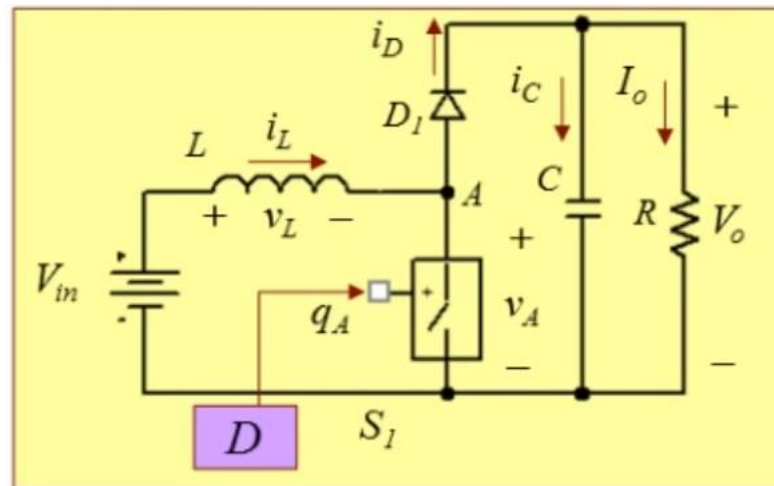


Buck

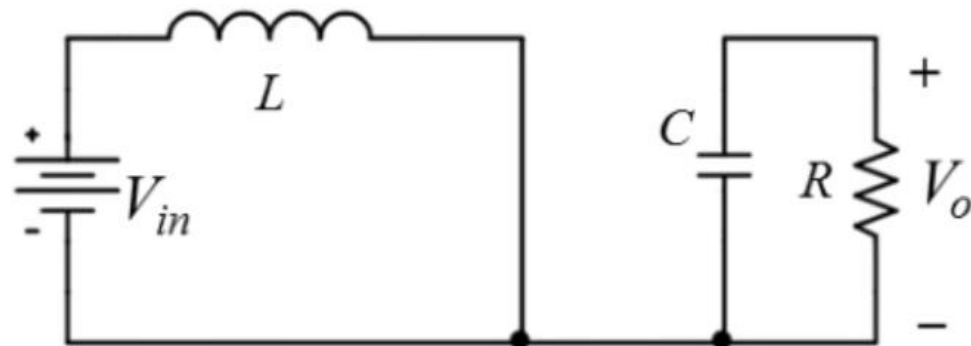


Boost

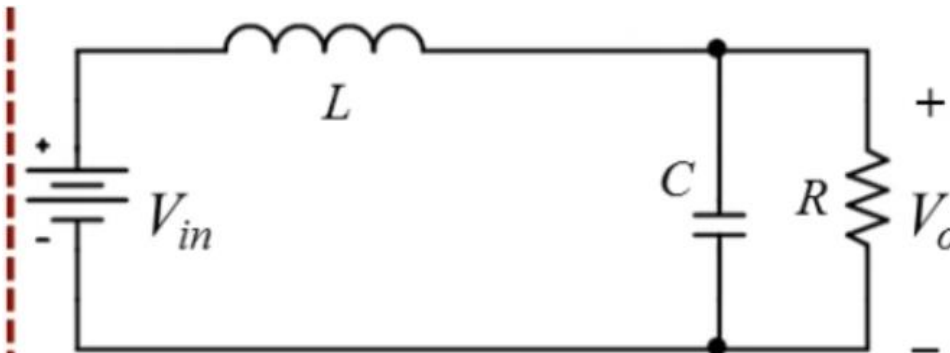
STEP-UP (BOOST) CONVERTER



DC Steady-state



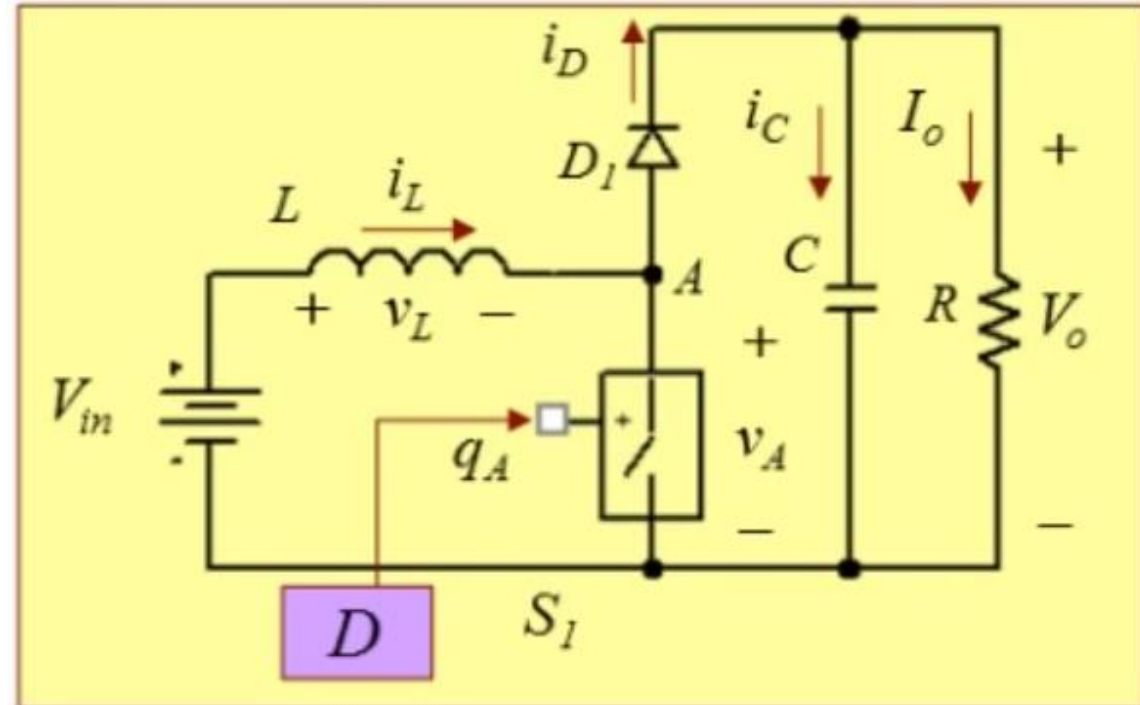
- $v_L = V_{in}$
- i_L and energy stored in L increase
- C supports load and discharges
- C large enough to maintain voltage almost constant (small ripple)



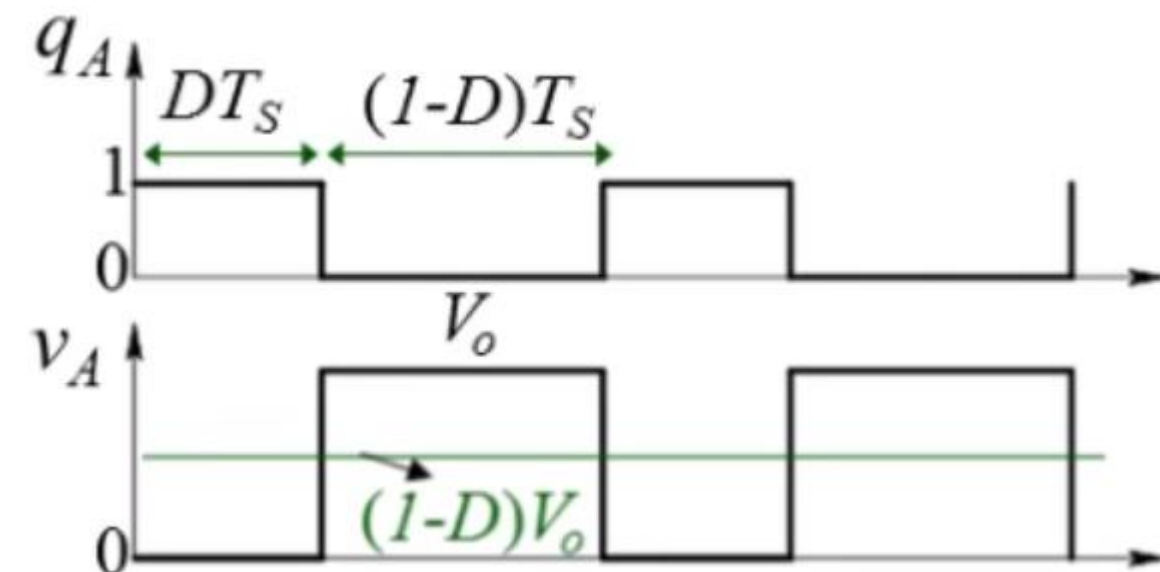
- $v_L = V_{in} - V_o$
- i_L and energy stored in L decrease, energy fed to C and R
- i_C positive and C charges up
- C large enough to maintain voltage almost constant (small ripple)

STEP-UP (BOOST) CONVERTER

Input-Output Voltage Relationship: SS operation



$$\bar{v}_A = V_{in} \text{ (since } \bar{v}_L = 0\text{)}$$



$$\bar{v}_A = (1-D)V_o \text{ (from waveform)}$$

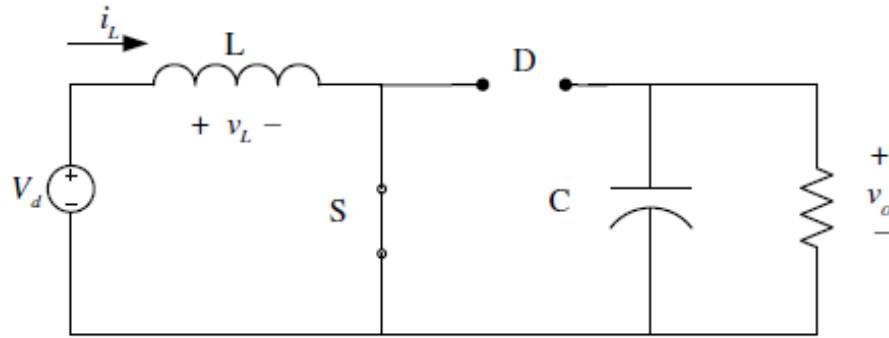
$$\frac{V_o}{V_{in}} = \frac{1}{1-D}$$

Input-output
relationship for
boost converter

STEP-UP (BOOST) CONVERTER

The Δi method

Input-Output Voltage Relationship: SS operation



$$v_L = V_d$$

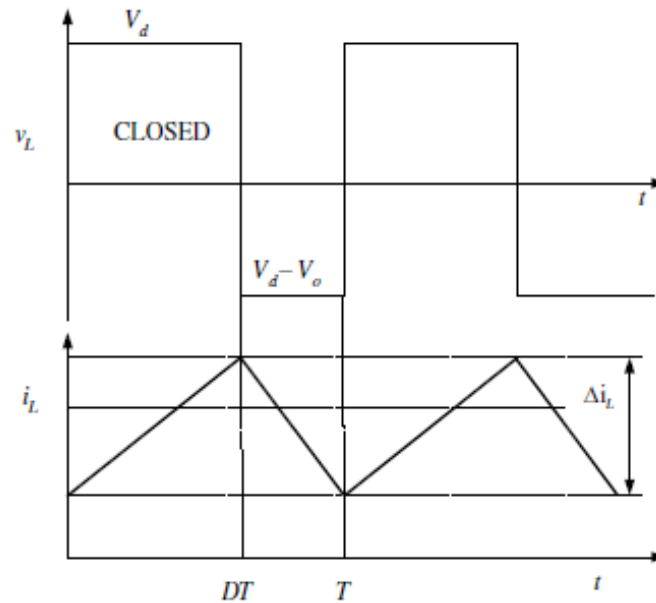
$$= L \frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$

$$(\Delta i_L)_{closed} = \frac{V_d DT}{L}$$

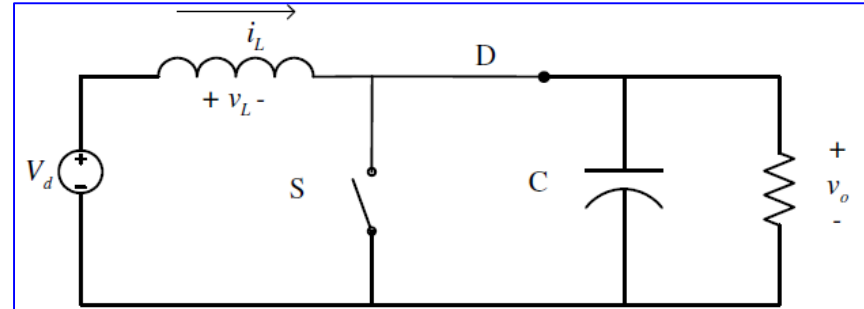


Boost converter produces output voltage that is greater or equal to the input voltage.

$$(\Delta i_L)_{closed} + (\Delta i_L)_{opened} = 0$$

$$\frac{V_d DT}{L} + \frac{(V_d - V_o)(1-D)T}{L} = 0$$

$$\Rightarrow V_o = \frac{V_d}{1-D}$$



$$v_L = V_d - V_o$$

$$= L \frac{di_L}{dt}$$

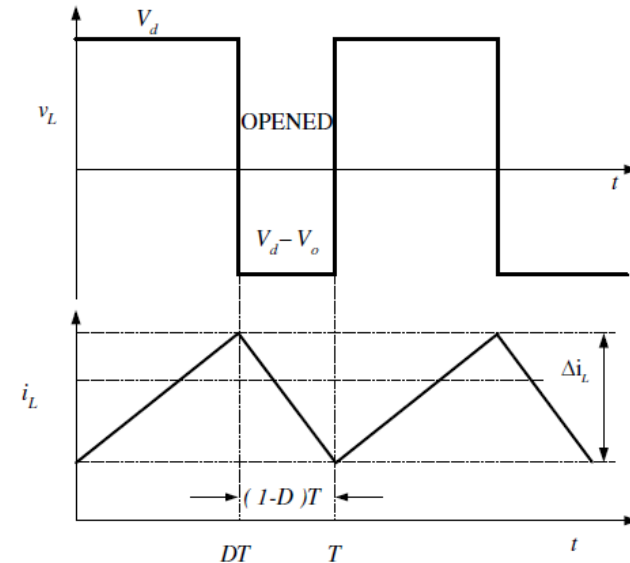
$$\Rightarrow \frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t}$$

$$= \frac{\Delta i_L}{(1-D)T}$$

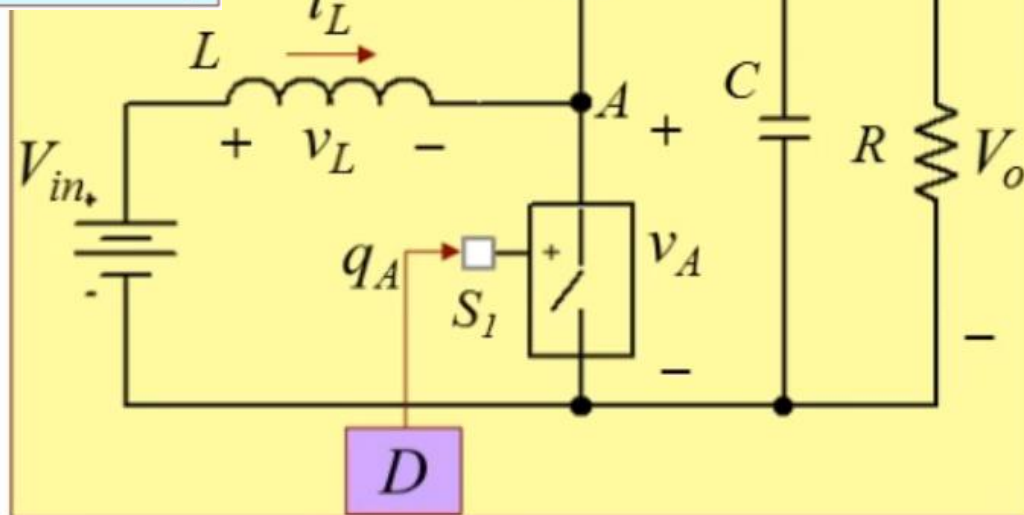
$$\Rightarrow \frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

$$\Rightarrow (\Delta i_L)_{opened} = \frac{(V_d - V_o)(1-D)T}{L}$$



Boost Converter Waveforms

$$\frac{V_o}{V_{in}} = \frac{1}{1-D} \Rightarrow D = 1 - \frac{V_{in}}{V_o}$$



Volt-sec balance for L

$$V_{in}DT_s + (V_{in} - V_o)(1-D)T_s = 0$$

$$V_{in}(D + 1 - D) = V_o(1-D)$$

$$\frac{V_o}{V_{in}} = \frac{1}{1-D}$$

For constant output voltage and variable input voltage applications

$$\left(1 - \frac{V_{in,max}}{V_o}\right) \leq D \leq \left(1 - \frac{V_{in,min}}{V_o}\right)$$

By power balance:

$$V_{in}I_{in} = V_oI_o$$

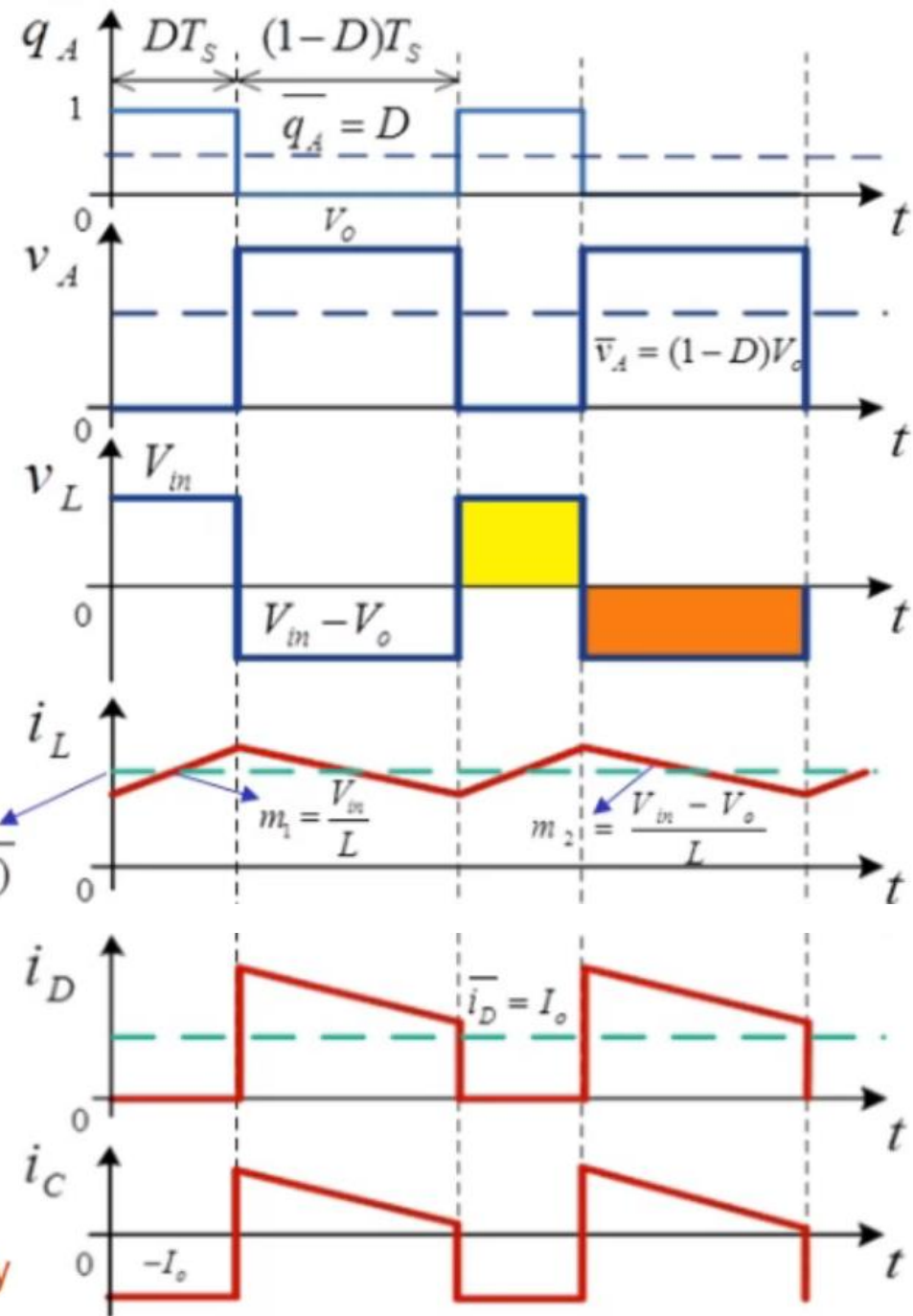
$$I_{in} = \frac{V_o}{V_{in}}I_o = \frac{I_o}{1-D}$$

Average KCL at node x

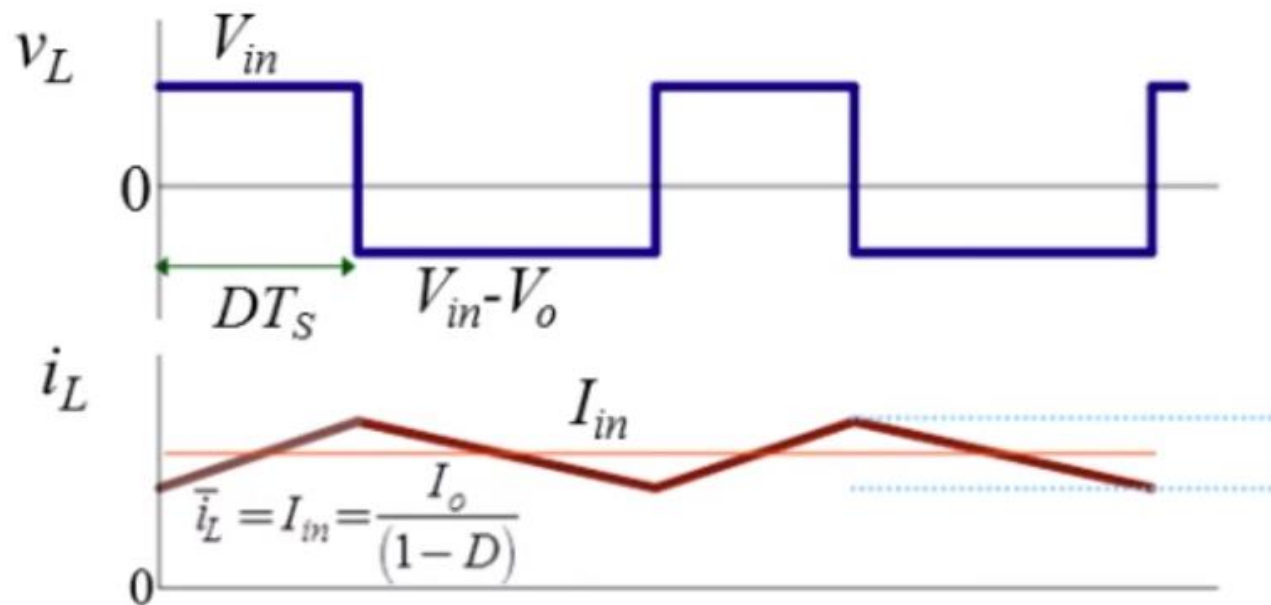
$$\bar{i}_D = \bar{i}_C + I_o = I_o$$

KCL at node x instantaneously

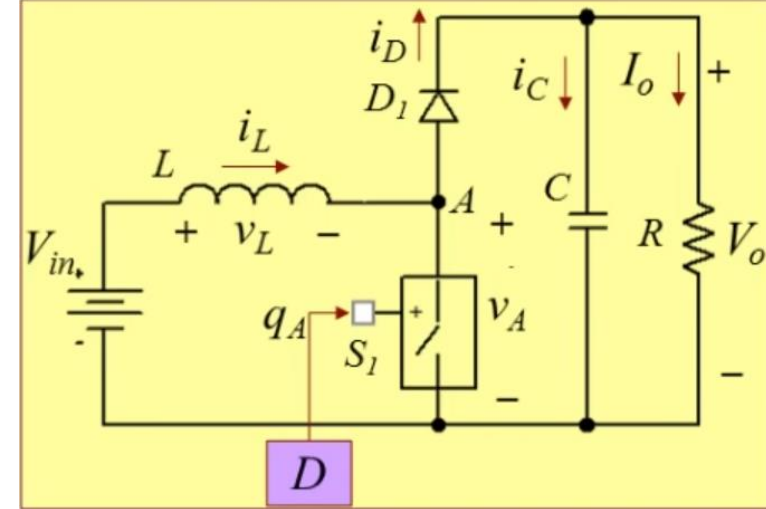
$$i_C = i_D - I_o$$



Selection of L



- L selected to limit peak-peak inductor current ripple to a chosen value
 - For example, 10-20% of max. I_{in}
 - Specifications on input current ripple
 - CCM considerations
- Choice of L does not significantly affect capacitor selection



ΔI_L Peak-peak ripple in inductor current

Consider the T_{ON} interval

$$L \frac{\Delta I_L}{DT_S} = V_{in} = V_o(1-D)$$

$$L = \frac{V_o D(1-D)T_S}{\Delta I_L}$$

Selection of L

- Design for **worst case condition**

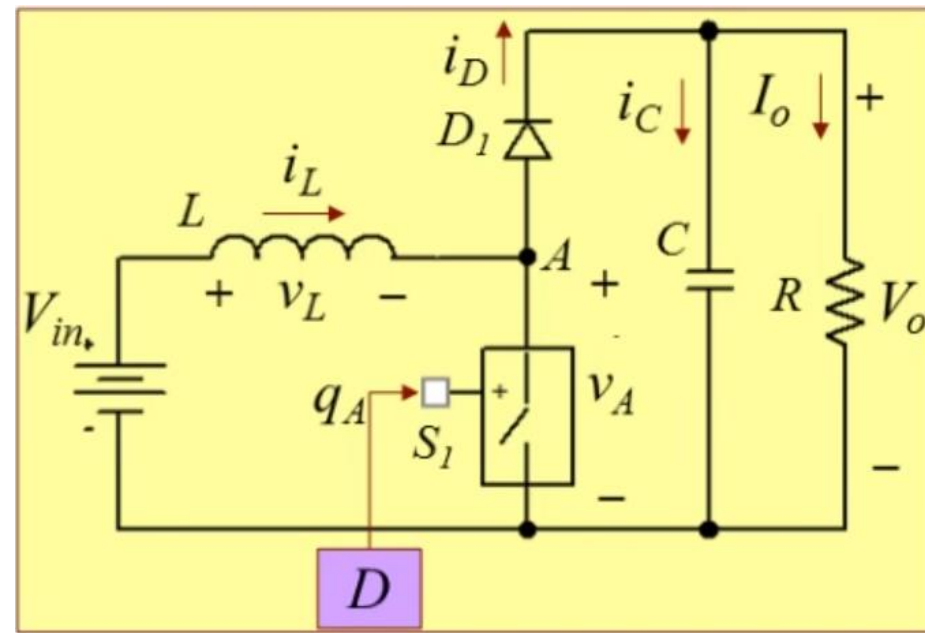
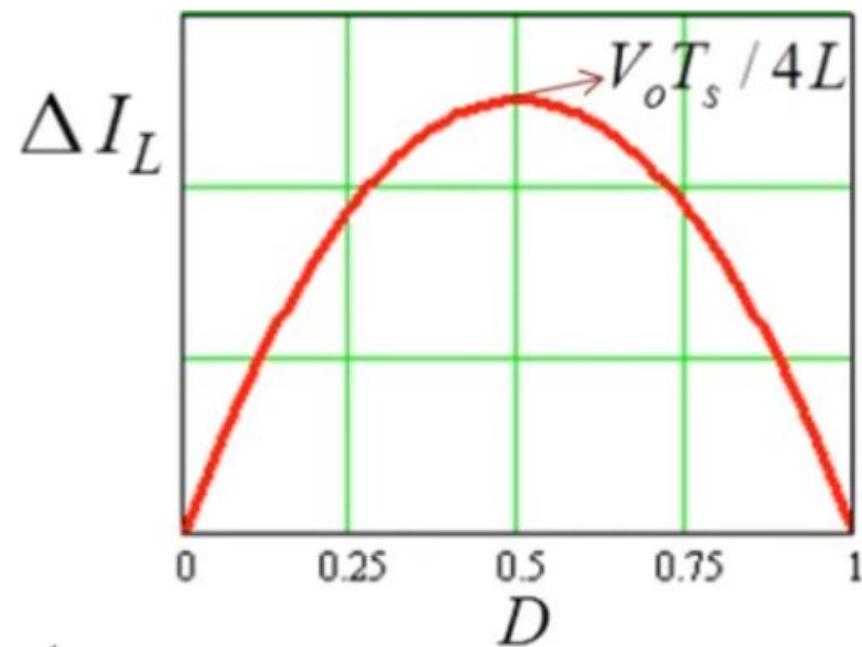
$$L = \frac{V_o D(1-D)T_s}{\Delta I_L}$$

Worst case condition for constant output voltage applications

$$\frac{dL(D)}{dD} = \frac{V_o T_s}{\Delta I_L} (1 - 2D) = 0$$

$$\Rightarrow D = 0.5 \quad (\text{or closest to 0.5 in the operating range of } D)$$

- L selected to limit peak-peak inductor current ripple to a chosen value
 - For example, 10-20% of max. I_{in}
 - Specifications on input current ripple
 - CCM considerations



BOOST CONVERTER ANALYSIS

Input power = Output power

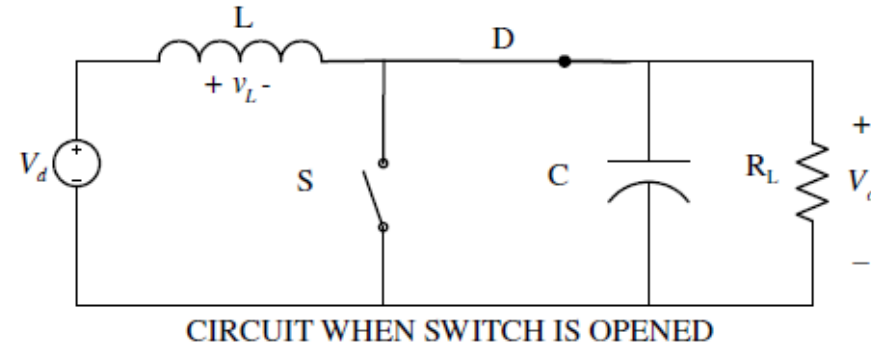
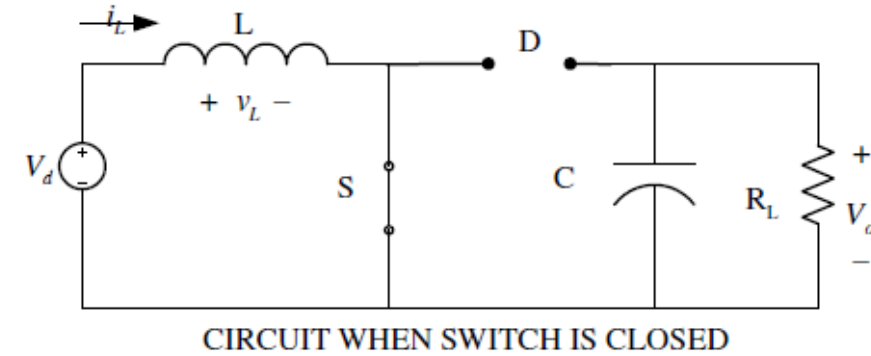
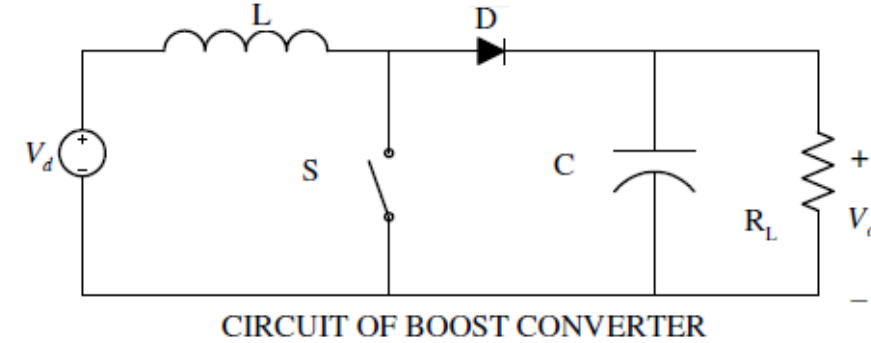
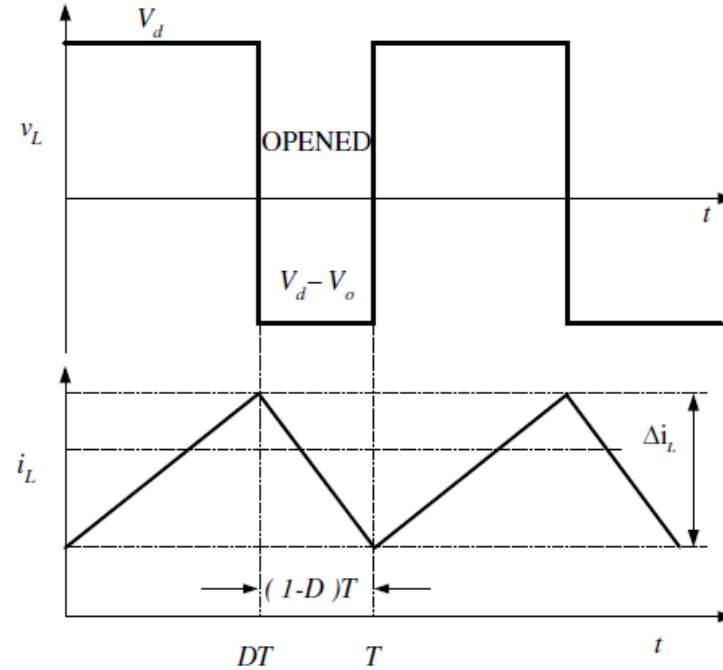
$$V_d I_d = \frac{V_o^2}{R}$$

$$V_d I_L = \frac{\left(\frac{V_d}{(1-D)}\right)^2}{R} = \frac{V_d^2}{(1-D)^2 R}$$

Average inductor current :

$$\Rightarrow I_L = \frac{V_d}{(1-D)^2 R}$$

Average, Maximum, Minimum Inductor Current



Maximum inductor current :

$$\Rightarrow I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} + \frac{V_d DT}{2L}$$

Minimum inductor current :

$$\Rightarrow I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} - \frac{V_d DT}{2L}$$

$$(\Delta i_L)_{\text{closed}} = \frac{V_d DT}{L}$$

$$(\Delta i_L)_{\text{opened}} = \frac{(V_d - V_o)(1-DT)}{L}$$

BOOST CONVERTER ANALYSIS

Maximum inductor current :

$$\Rightarrow I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} + \frac{V_d DT}{2L}$$

Minimum inductor current :

$$\Rightarrow I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} - \frac{V_d DT}{2L}$$

For CCM,

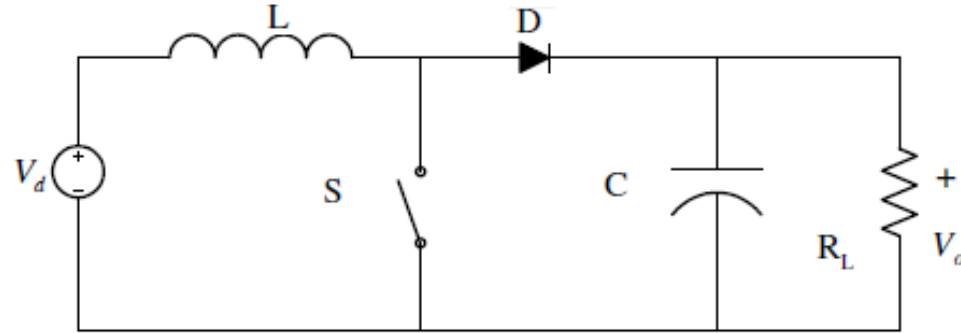
$$I_{\min} \geq 0$$

$$\frac{V_d}{(1-D)^2 R} - \frac{V_d DT}{2L} \geq 0$$

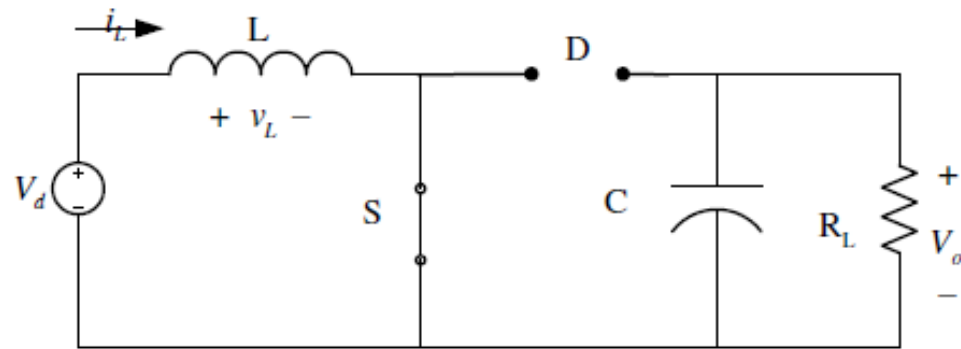
$$L_{\min} = \frac{D(1-D)^2 TR}{2}$$

$$= \frac{D(1-D)^2 R}{2f}$$

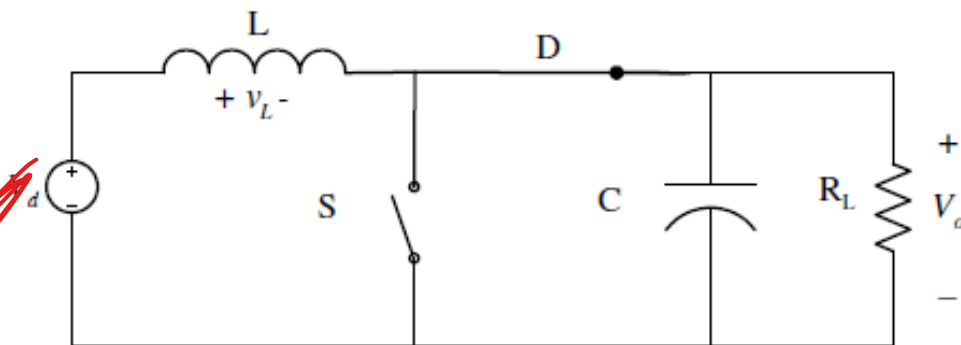
$$L = \frac{V_o D(1-D)T_s}{\Delta I_L}$$



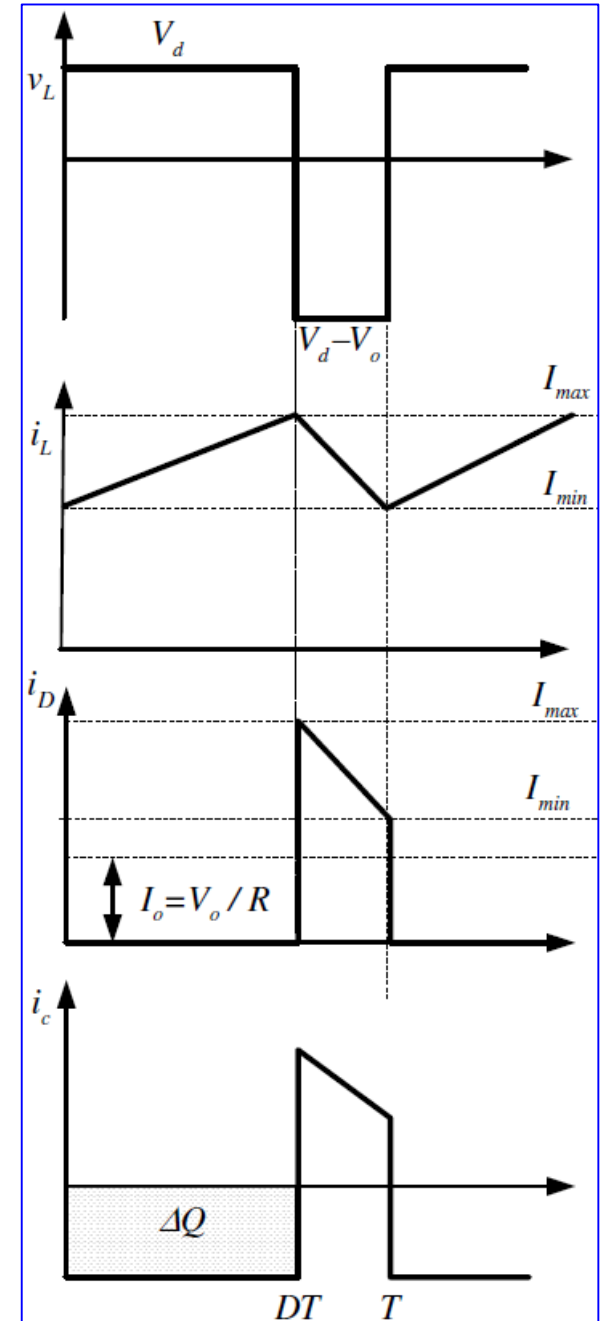
CIRCUIT OF BOOST CONVERTER



CIRCUIT WHEN SWITCH IS CLOSED



CIRCUIT WHEN SWITCH IS OPENED



BOOST CHOPPER

CONTINUOUS-CONDUCTION MODE

Since in steady state the time integral of the inductor voltage over one time period must be zero,

$$V_d t_{\text{on}} + (V_d - V_o) t_{\text{off}} = 0$$

Dividing both sides by T_s and rearranging terms yield

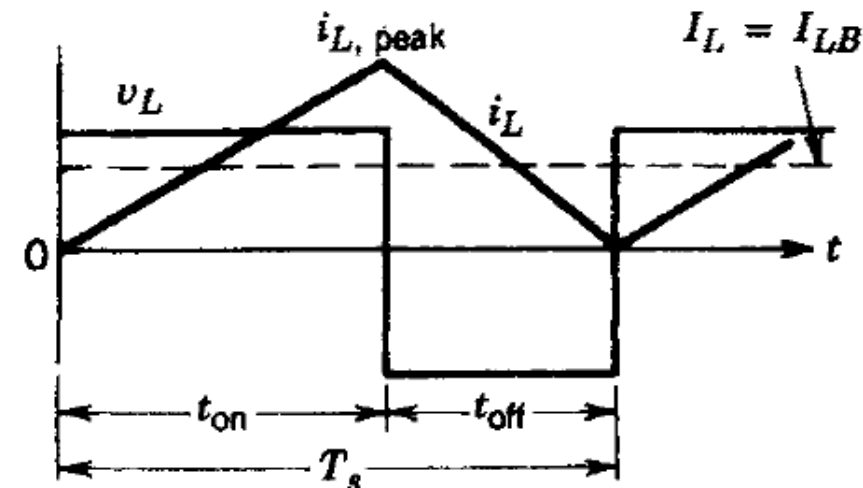
$$\frac{V_o}{V_d} = \frac{T_s}{t_{\text{off}}} = \frac{1}{1 - D}$$

Assuming a lossless circuit, $P_d = P_o$, $\therefore V_d I_d = V_o I_o$

$$\frac{I_o}{I_d} = (1 - D)$$

Ideally, the step up voltage ratio is continuously adjustable in the range from 1 ($D = 0$) to infinity ($D = 1$) by choice of D .

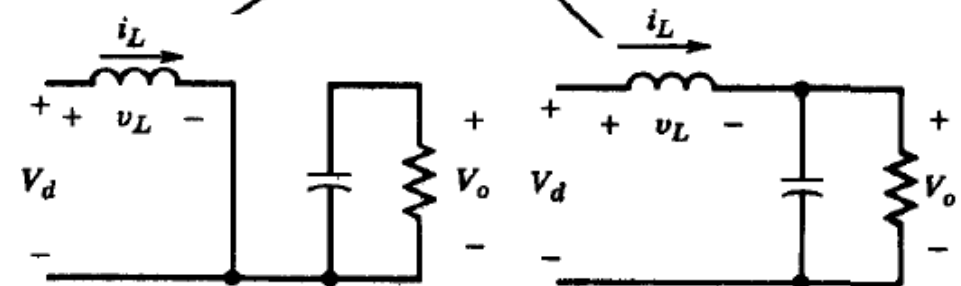
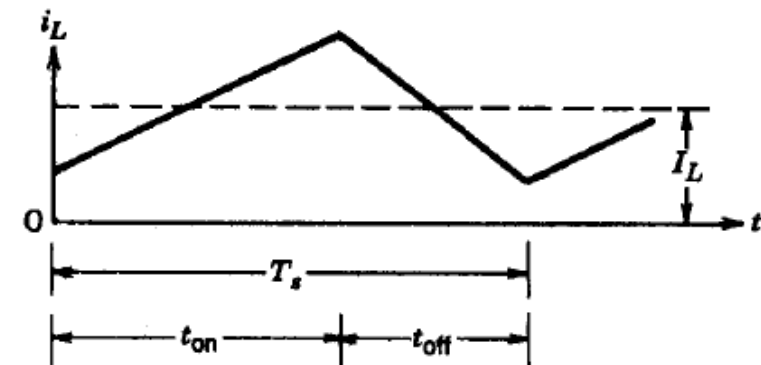
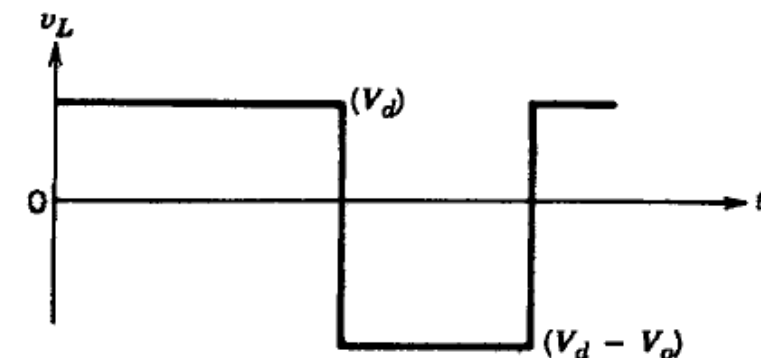
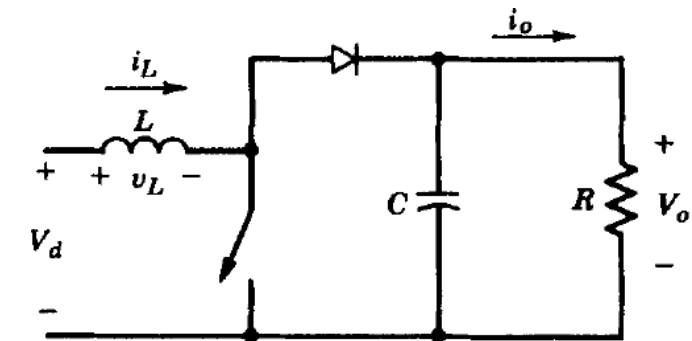
Boundary between Cont. and Discont. Modes



$$I_{LB} = \frac{1}{2} i_{L,\text{peak}}$$

$$= \frac{1}{2} \frac{V_d}{L} t_{\text{on}}$$

$$= \frac{T_s V_o}{2L} D(1 - D)$$



STEP-UP (BOOST) CONVERTER

Boundary between Cont. and Discont. Modes

$$I_{LB} = \frac{T_s V_o}{2L} D(1 - D) \frac{I_o}{I_d} = (1 - D) i_d = i_L$$

The average output current at the edge of cont. cond. is

$$I_{oB} = \frac{T_s V_o}{2L} D(1 - D)^2$$

In most Boost converter applications, V_o is kept constant.

I_{LB} reaches a maximum value at $D = 0.5$:

$$I_{LB, \max} = \frac{T_s V_o}{8L}$$

Also, I_{oB} has its maximum at $D = 0.333$

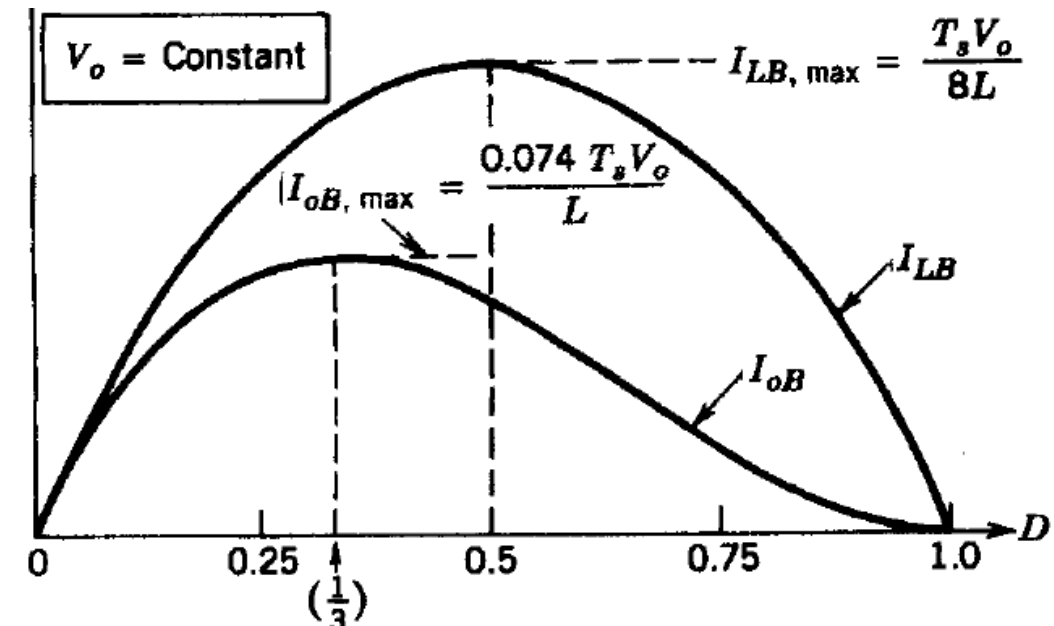
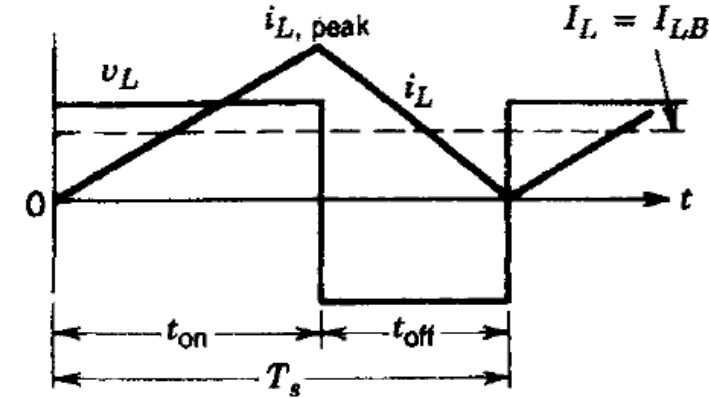
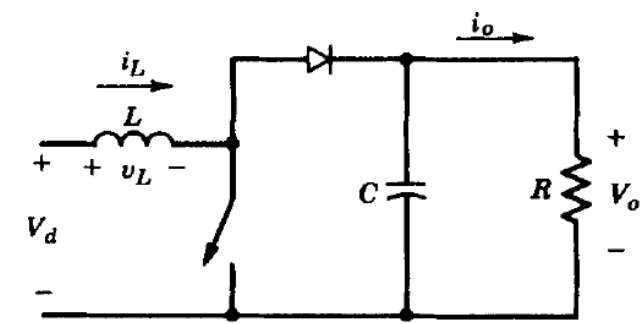
$$I_{oB, \max} = \frac{2}{27} \frac{T_s V_o}{L} = 0.074 \frac{T_s V_o}{L}$$

In terms of their maximum values,

$$I_{LB} = 4D(1 - D)I_{LB, \max}$$

$$I_{oB} = \frac{27}{4} D(1 - D)^2 I_{oB, \max}$$

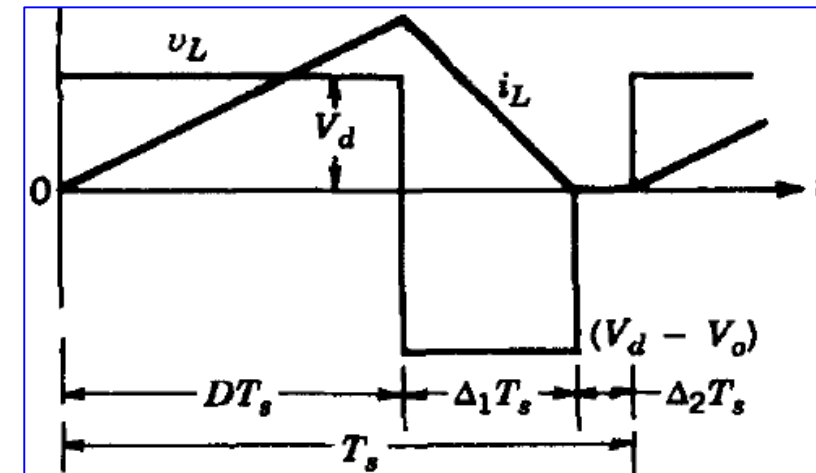
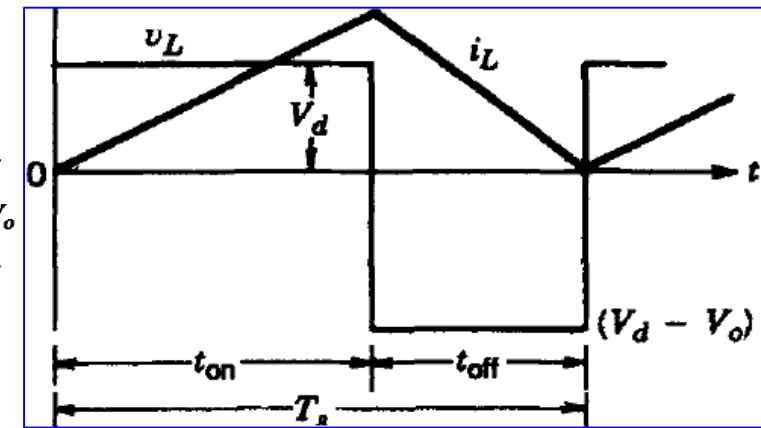
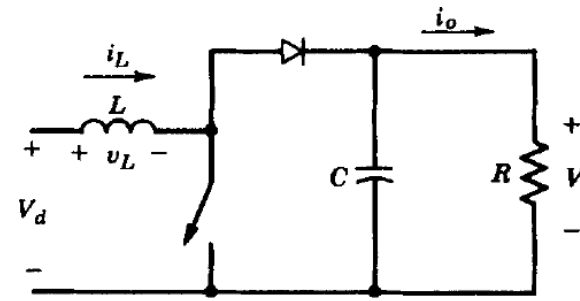
For a given D , with constant V_o , if the average load current drops below I_{oB} (and, hence, the average inductor current below I_{LB}), the current conduction will become discontinuous.



DISCONTINUOUS-CONDUCTION MODE

Assume that as the output load power decreases, V_d and D remain constant (even though, in practice, D would vary in order to keep V_o constant).

- The DCM occurs due to decreased $P_o (=P_d)$ and, hence, a lower $I_L (=I_d)$, since v_d is constant.
- Since $i_{L\text{peak}}$ is the same in both modes, a lower value of I_L (and, hence a discontinuous I_L) is possible only if V_o goes up.



If we equate the integral of the inductor voltage over one time period to zero,

$$V_d DT_s + (V_d - V_o) \Delta_1 T_s = 0$$

$$\therefore \frac{V_o}{V_d} = \frac{\Delta_1 + D}{\Delta_1}$$

$$\frac{I_o}{I_d} = \frac{\Delta_1}{\Delta_1 + D} \quad (\text{since } P_d = P_o)$$

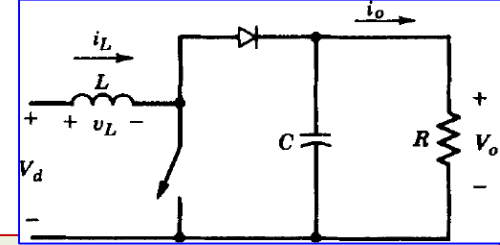
The average input current, which is also equal to the inductor current, is

$$I_d = \frac{V_d}{2L} DT_s (D + \Delta_1) \Rightarrow I_o = \left(\frac{T_s V_d}{2L} \right) D \Delta_1$$

DISCONTINUOUS-CONDUCTION MODE

$$I_o = \left(\frac{T_s V_d}{2L} \right) D \Delta_1 \therefore \frac{V_o}{V_d} = \frac{\Delta_1 + D}{\Delta_1}$$

$$I_{oB, \max} = \frac{2}{27} \frac{T_s V_o}{L} = 0.074 \frac{T_s V_o}{L}$$



In practice, since V_o is held constant and D varies in response to the variation in V_d , it is more useful to obtain the required duty ratio D as a function of load current for various values of V_o / V_d .

$$D = \left[\frac{4}{27} \frac{V_o}{V_d} \left(\frac{V_o}{V_d} - 1 \right) \frac{I_o}{I_{oB, \max}} \right]^{1/2}$$

D is plotted as a function of $I_o / I_{oB, \max}$ for various values of V_d / V_o .

