EE 238

Power Engineering - II

Power Electronics

Lecture 8

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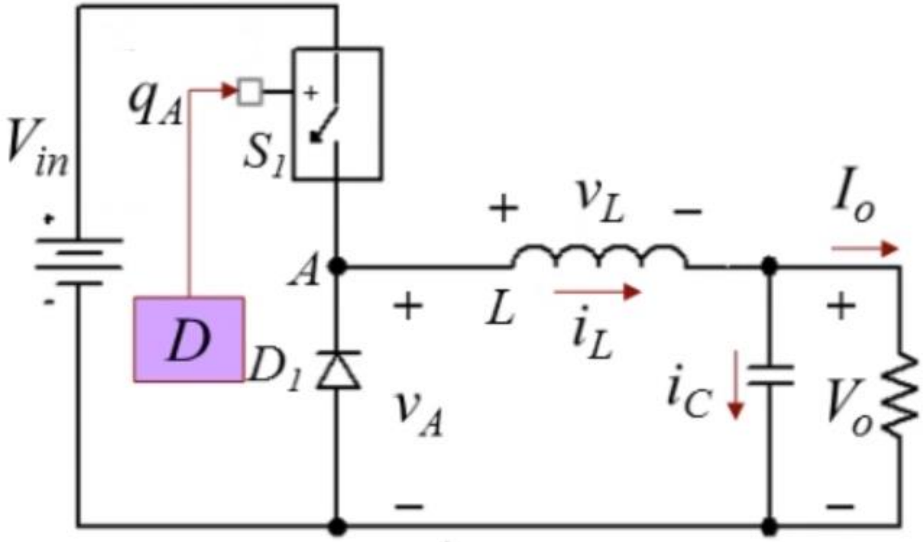
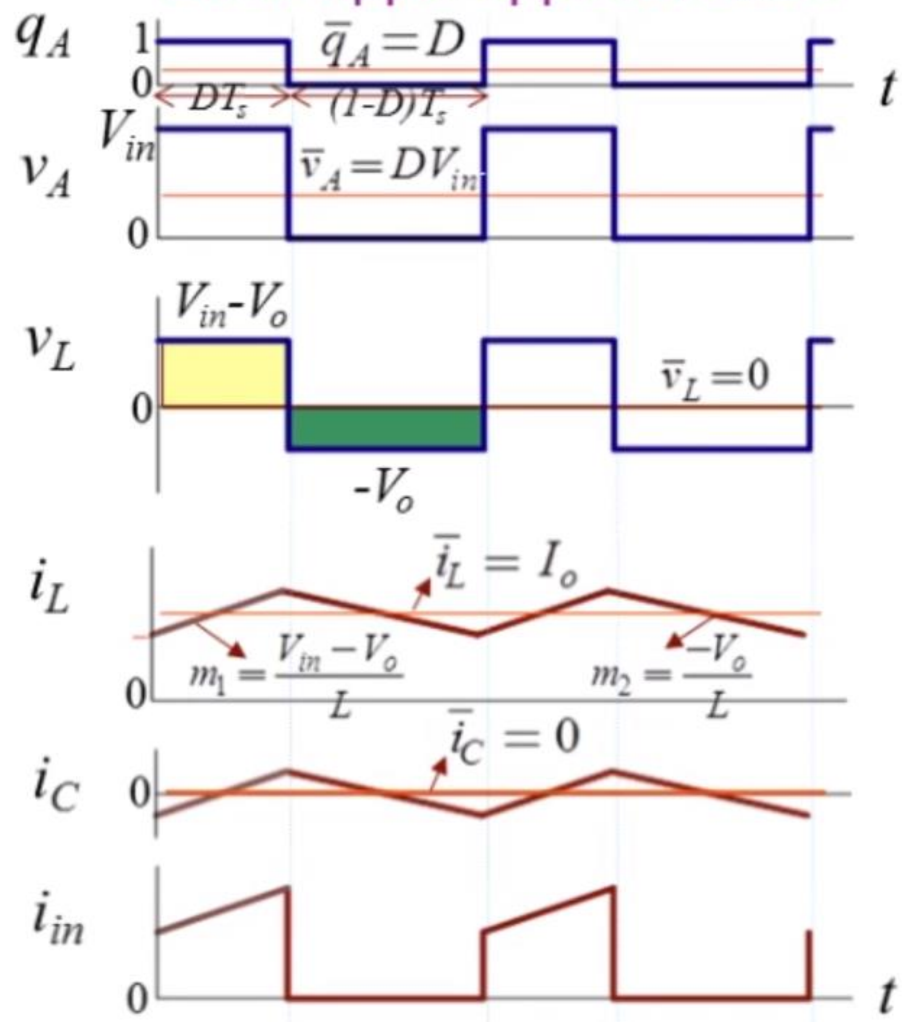
Buck Converter Waveforms

$$\frac{V_o}{V_{in}} = D$$



Small ripple approximation



$$i_L(t) = i_L(t_o) + \int_{t_o}^{t} v_L(\tau)\,d\tau$$

1. ON interval

$v_L(t) = V_{in} - V_o$ Slope of i_L : $m_1 = \frac{V_{in} - V_o}{L}$

2. OFF interval

$v_L(t) = -V_o$ Slope of i_L : $m_2 = \frac{-V_o}{L}$

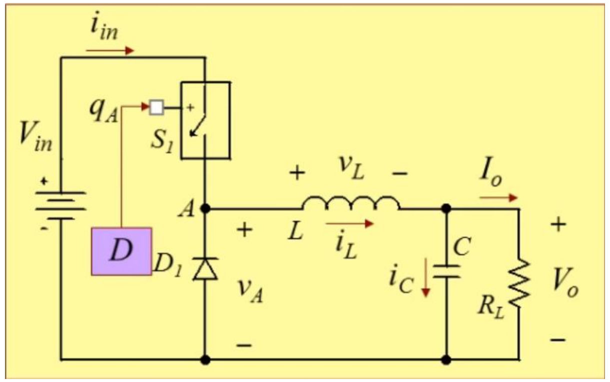
3. Average of i_L obtained by KCL at output node

$\bar{i}_L = I_o + \bar{i}_c = I_o$ (since $\bar{i}_c = 0$)

Instantaneous i_C obtained by KCL at output node

$i_c(t) = i_L(t) - I_o$

Selection of Output Filter Inductor, L



- L and C together determine output voltage ripple
- L selected to limit the inductor current ripple to a chosen value
  - e.g. 10-20% of its average current (I_o)
  - trade-off explained in next slide
  - CCM considerations
- **Worst case design**
  - minimum D for fixed output buck

For constant V_o and variable V_{in}

$$\boxed{\left(\frac{V_o}{V_{in,max}}\right) \le D \le \left(\frac{V_o}{V_{in,min}}\right)}$$

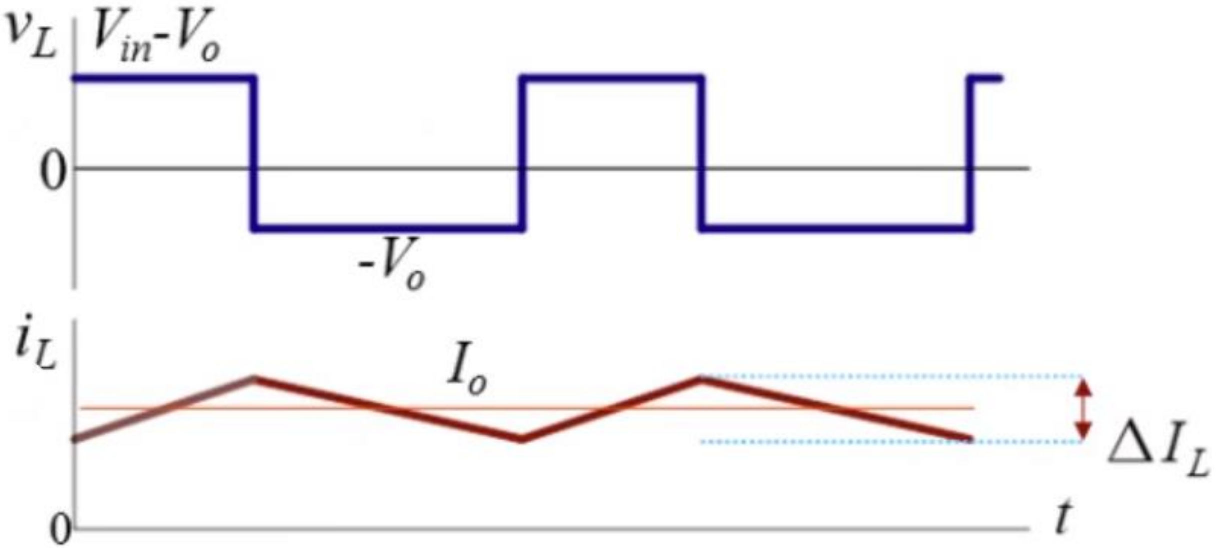
$$L\frac{di}{dt} = v_L$$

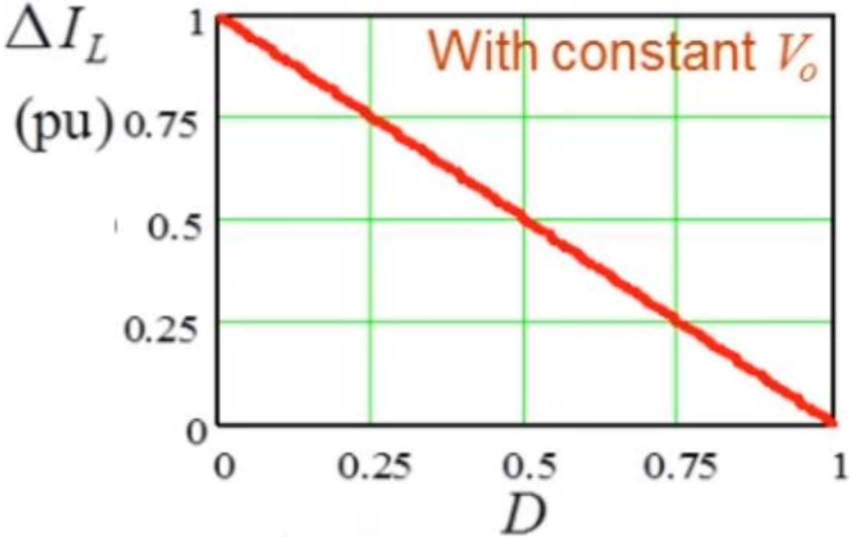
Consider the T_{OFF} interval

$$L\frac{\Delta I_L}{(1-D)T_S} = V_o$$

ΔI_L - Peak-peak ripple

$$\boxed{L = \frac{V_o(1-D)T_S}{\Delta I_L}}$$





Trade-off in Selection of Inductor Current Ripple

Larger L (small current ripple)

- Lower RMS current in switch, capacitor and inductor, hence lower conduction loss
- Smaller C is enough for same output voltage ripple

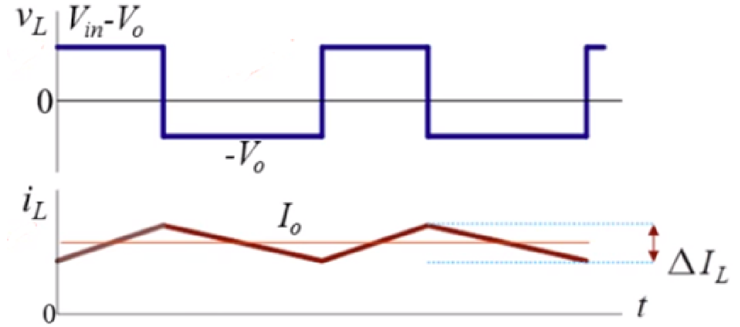
- Poor dynamic response to step loads (ramp rate of i_L)
- Bulky inductor
- Usually higher inductor resistance

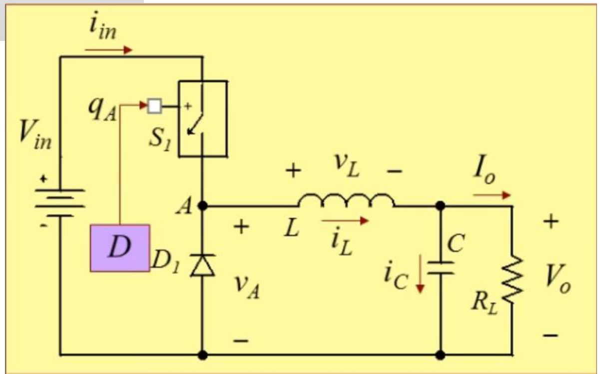
Smaller L (high current ripple)

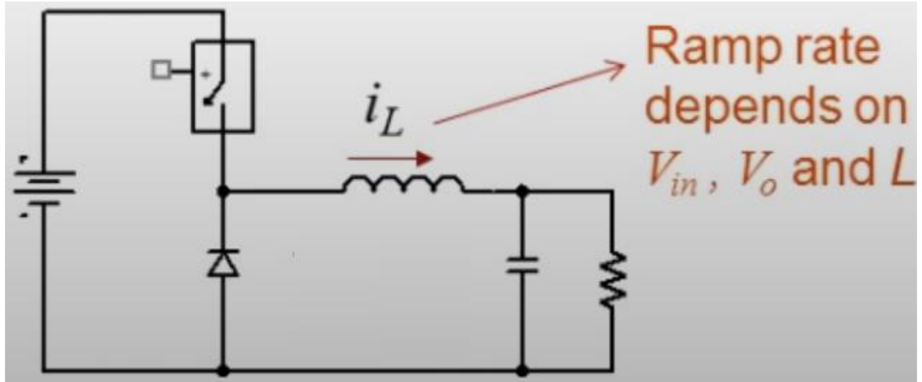
- Good dynamic response to step loads due to higher slew rate of inductor current
- Smaller size inductor, higher power density

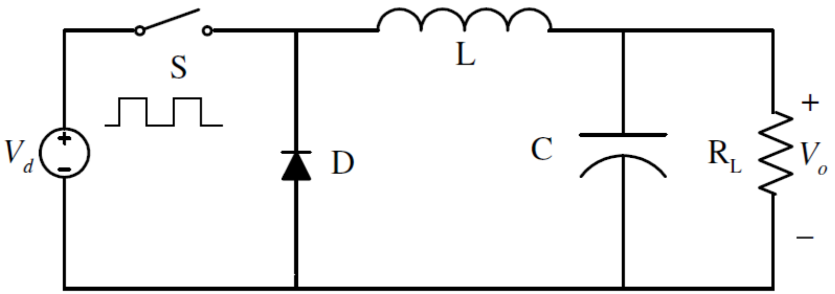
- Larger RMS current in switch, inductor and capacitor, hence higher conduction loss
- Larger flux swing at high frequency

CCM Considerations





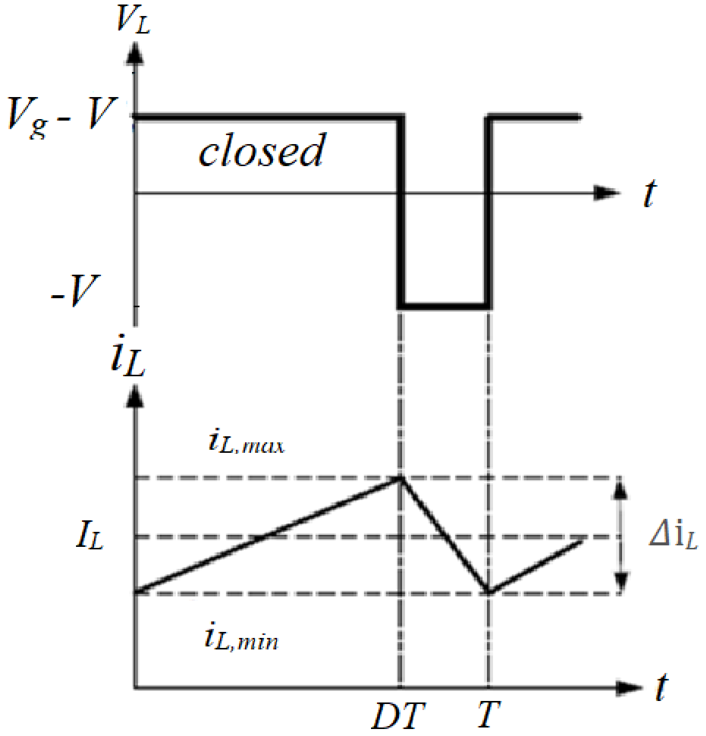




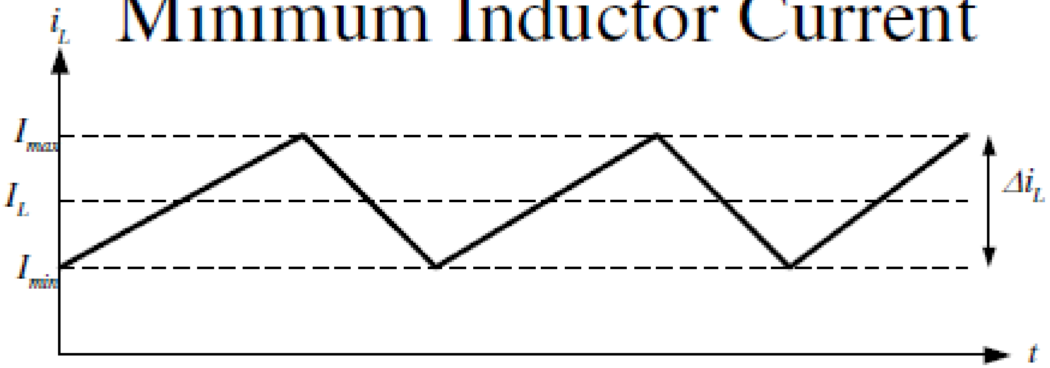
$$(\Delta i_L)_{closed} + (\Delta i_L)_{opened} = 0$$

$$\left(\frac{V_d - V_o}{L}\right) \cdot DT_s - \left(\frac{-V_o}{L}\right) \cdot (1-D)T_s = 0$$

$$\Rightarrow V_o = DV_d$$



Average, Maximum and Minimum Inductor Current



Average inductor current = Average current in R_L

$$\Rightarrow I_L = I_R = \frac{V_o}{R}$$

Maximum current :

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_o}{R} + \frac{1}{2}\left(\frac{V_o}{L}(1-D)T\right)$$

$$= V_o\left(\frac{1}{R} + \frac{(1-D)}{2Lf}\right)$$

Minimum current :

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = V_o\left(\frac{1}{R} - \frac{(1-D)}{2Lf}\right)$$

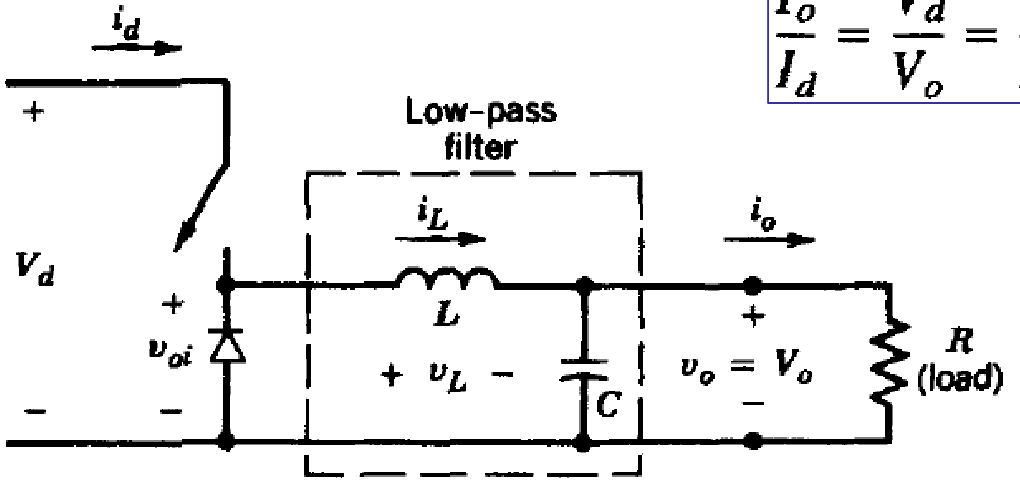
Inductor current ripple :

$$\Delta i_L = I_{\max} - I_{\min}$$

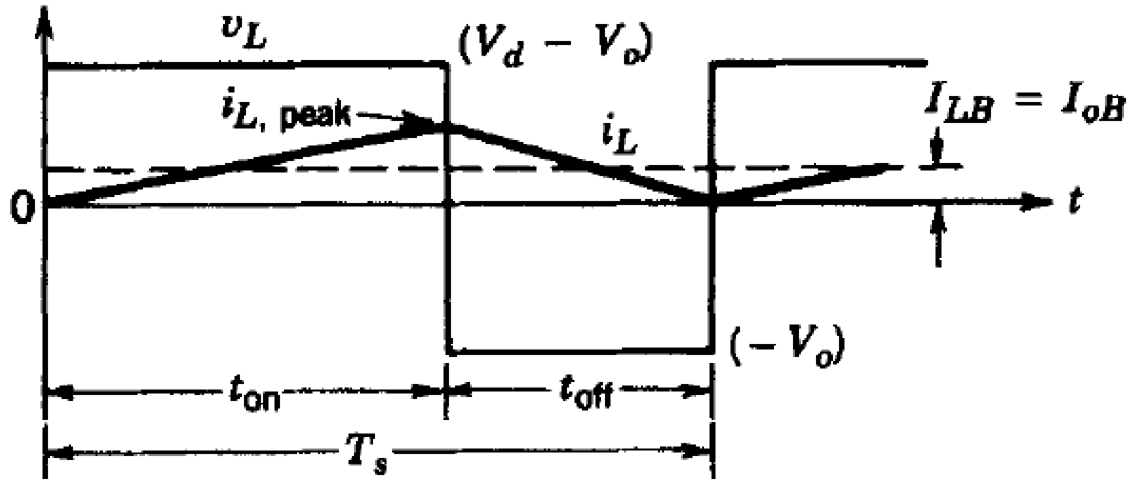
STEP-DOWN (BUCK) CONVERTER

BOUNDARY BETWEEN CONTINUOUS AND DISCONTINUOUS CONDUCTION

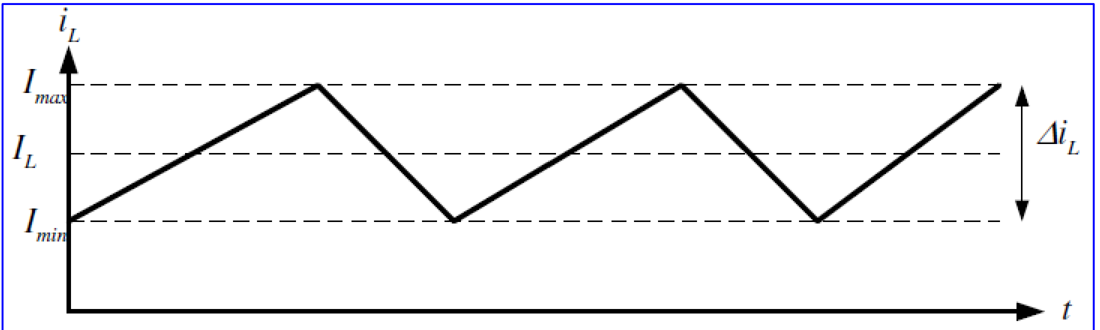
$$\frac{I_o}{I_d} = \frac{V_d}{V_o} = \frac{1}{D}$$



Equations that show the influence of various circuit parameters on the conduction mode of i_L .

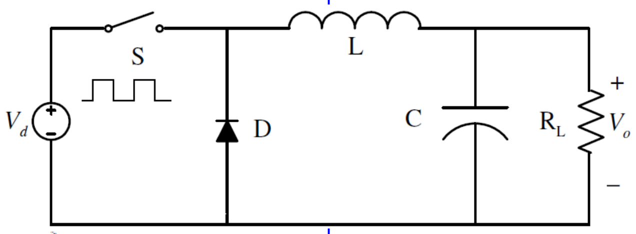


BOUNDARY BETWEEN CONTINUOUS AND DISCONTINUOUS CONDUCTION



$$\text{Average inductor current} = \text{Average current in } R_L$$

$$\Rightarrow I_L = I_R = \frac{V_O}{R}$$



Maximum current :

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_O}{R} + \frac{1}{2}\left(\frac{V_O}{L}(1-D)T\right)$$

$$= V_O\left(\frac{1}{R} + \frac{(1-D)}{2Lf}\right)$$

Minimum current :

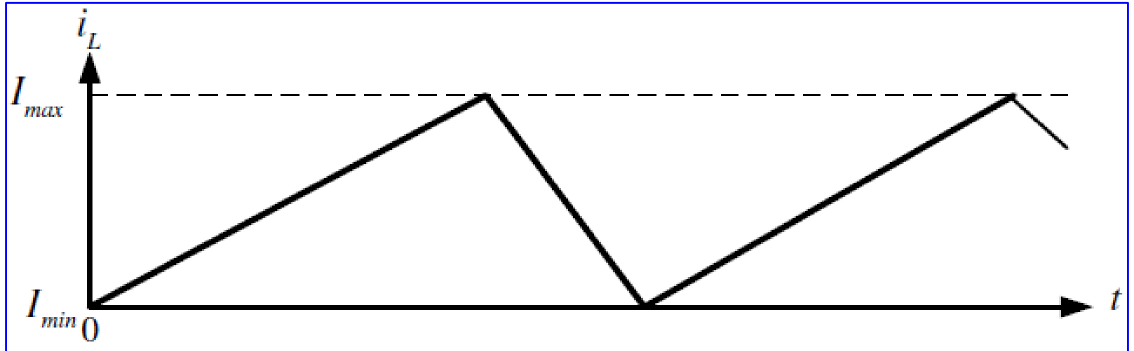
$$I_{\min} = I_L - \frac{\Delta i_L}{2} = V_O\left(\frac{1}{R} - \frac{(1-D)}{2Lf}\right)$$

Inductor current ripple :

$$\Delta i_L = I_{\max} - I_{\min}$$

For constant V_o and variable V_{in}

$$\boxed{\left(\frac{V_o}{V_{in,max}}\right) \le D \le \left(\frac{V_o}{V_{in,min}}\right)}$$



From previous analysis,

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = V_O\left(\frac{1}{R} - \frac{(1-D)}{2Lf}\right)$$

For continuous operation, $I_{\min} \ge 0$,

$$V_O\left(\frac{1}{R} - \frac{(1-D)}{2Lf}\right) \ge 0$$

$$\Rightarrow L \ge L_{\min} = \frac{(1-D)}{2f} \cdot R$$

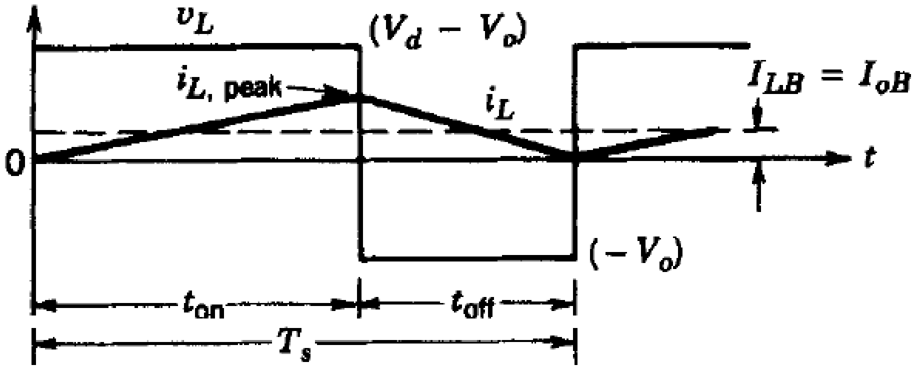
At the boundary,

***R**critical* **= ??**

This is the minimum inductor current to ensure continous mode of operation.

Normally L is chosen b be $>> L_{\min}$

BOUNDARY BETWEEN CONTINUOUS AND DISCONTINUOUS CONDUCTION



$$I_{\max} = I_L + \frac{\Delta i_L}{2}$$

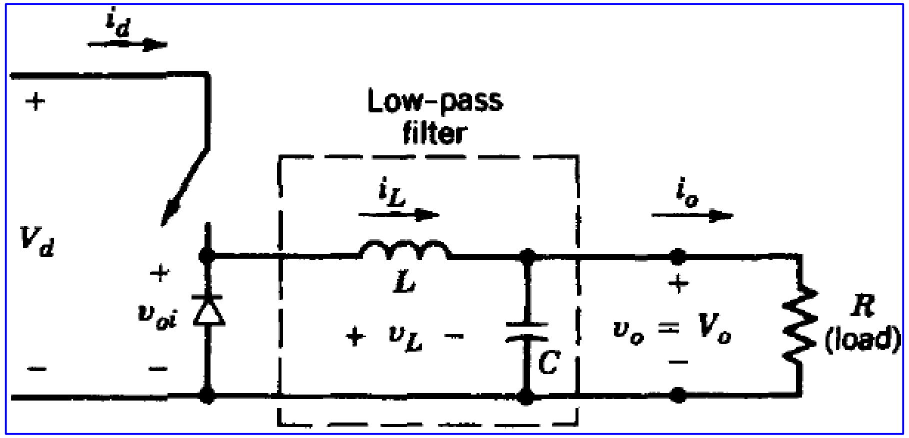
$$I_{\min} = I_L - \frac{\Delta i_L}{2}$$

$$(\Delta i_L)_{closed} = \left(\frac{V_d - V_o}{L}\right) \cdot DT$$

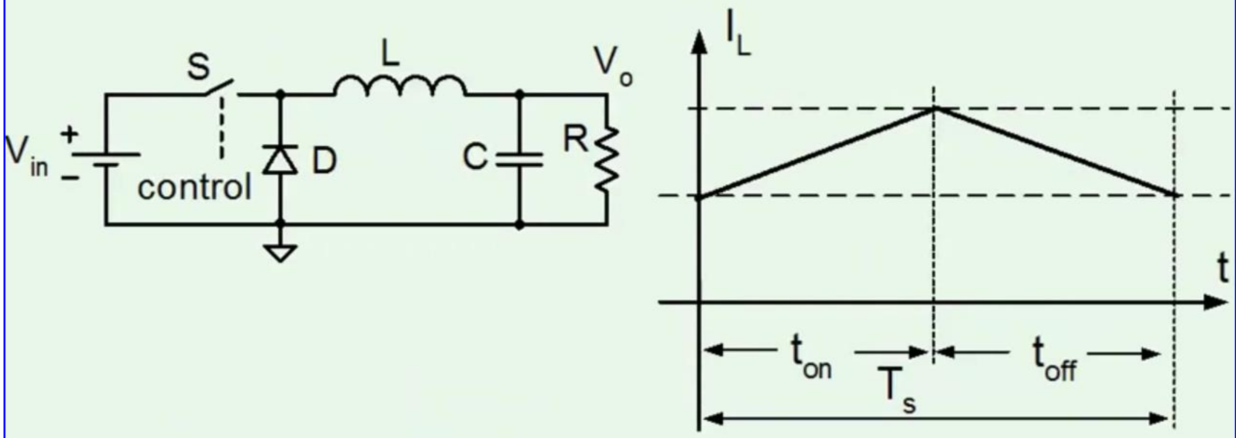
At the boundary, the average inductor current is

$$I_{LB} = \frac{1}{2} i_{L,\text{peak}} = \frac{t_{\text{on}}}{2L}(V_d - V_o) = \frac{DT_s}{2L}(V_d - V_o) = I_{oB}$$

Therefore, during an operating condition (with a given set of values for T_s, V_d , V_o, L, and D), if the average output current (and, hence, the average inductor current) becomes less than I_{LB}, then I_L will become discontinuous.

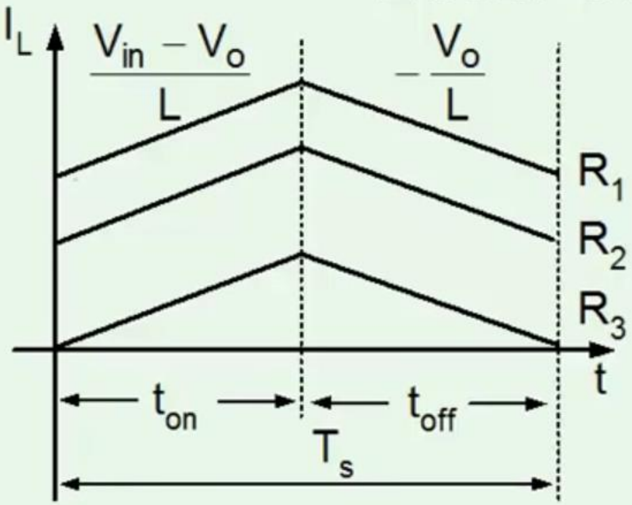


Load Change with Fixed D



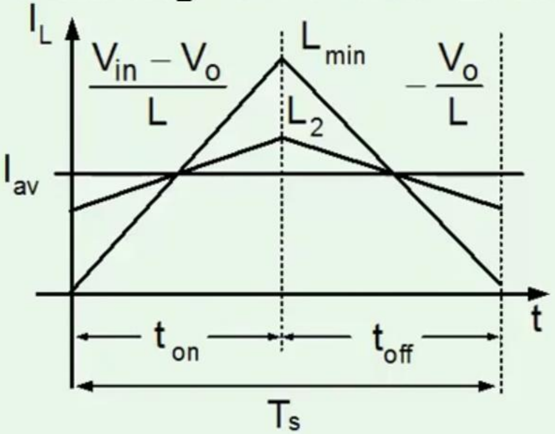
How will I_L change if R is getting smaller?

Load Change



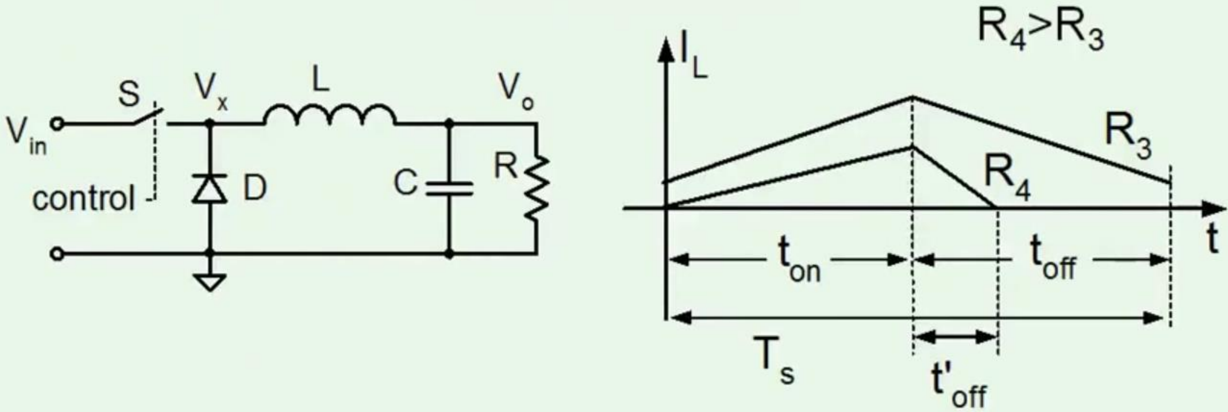
$R_1 < R_2 < R_3$

Boundary of CCM and DCM



- For CCM $\quad L > L_{min}$
- In Buck $\quad \frac{V_o}{L_{min}} t_{off} = I_{pk} = 2I_{av} \qquad L_{min} = \frac{V_o D_{off}}{2I_{av} f_s} = \frac{RD_{off}}{2f_s}$

Discontinuous Inductor Current Mode (DCM)



- Different voltage transfer ratio $\neq D_{on}$
- Higher ripple current

STEP-DOWN (BUCK) CONVERTER

Depending on application, either V_d or V_0 remains constant.

Discontinuous-Conduction Mode with Constant *Vd*

In many applications (e.g., dc motor speed control), V_d remains constant and V_0 is controlled by adjusting *D.*

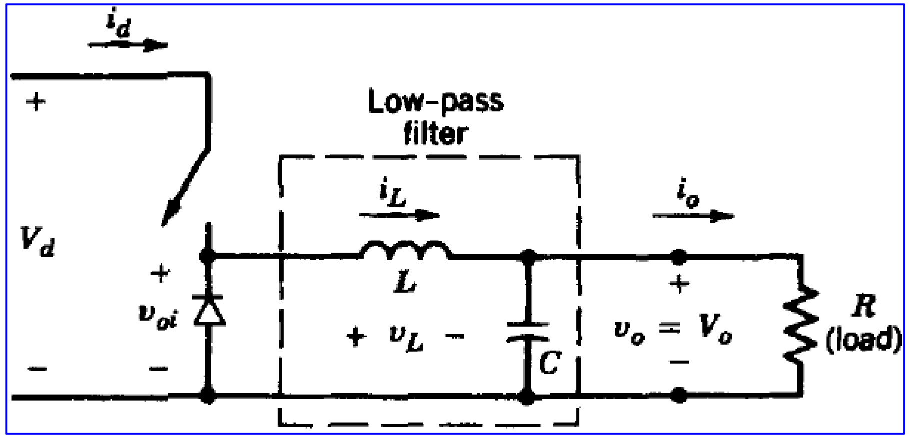
At the edge, $I_{LB} = \frac{DT_s}{2L}(V_d - V_o) = I_{oB}$ Since $V_0 = DV_d$, $I_{LB} = \frac{T_s V_d}{2L} D(1 - D)$

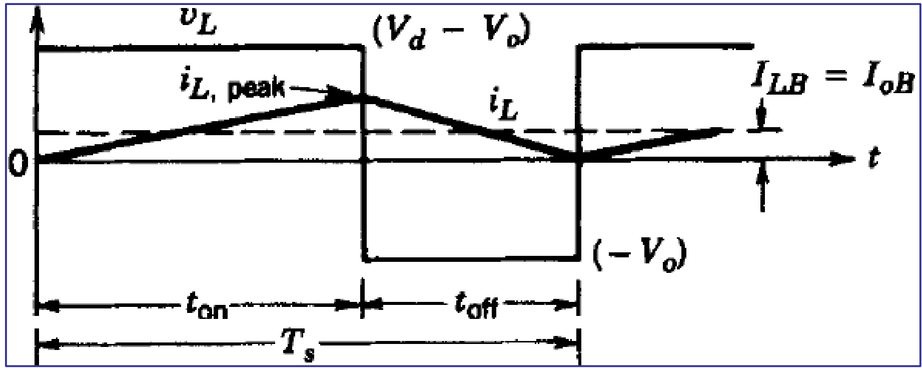
Output current required for CCM is maximum at D = 0.5:

$I_{LB,\max} = \frac{T_s V_d}{8L}$ => $I_{LB} = 4 I_{LB,\max} D(1 - D)$

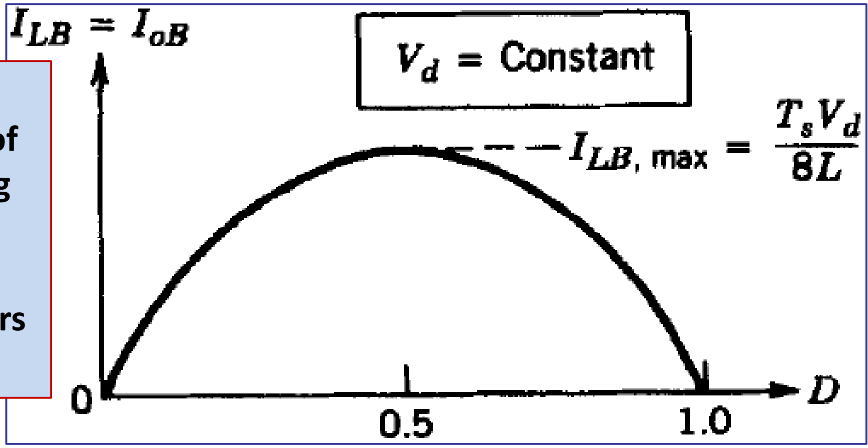
Assuming the converter is operating at the edge for given values of Ts L, *Vd* , and *D.* If these are kept constant and the output load power is decreased (i.e., the load resistance goes up), then the average inductor current will decrease.

Discontinuous-Conduction Mode





I_{LB} as a function of *D,* keeping V_d and all other parameters constant.

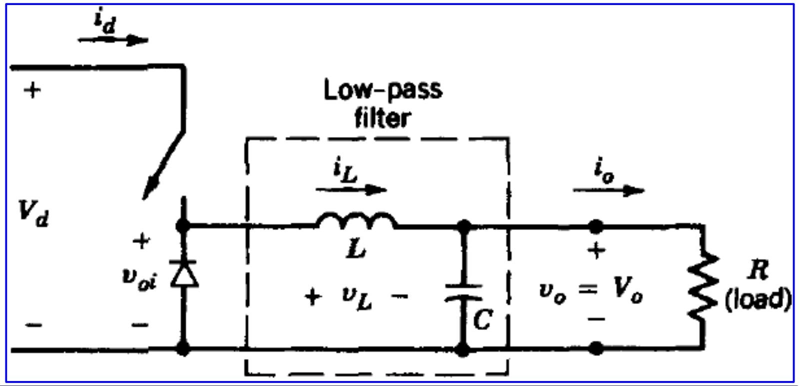


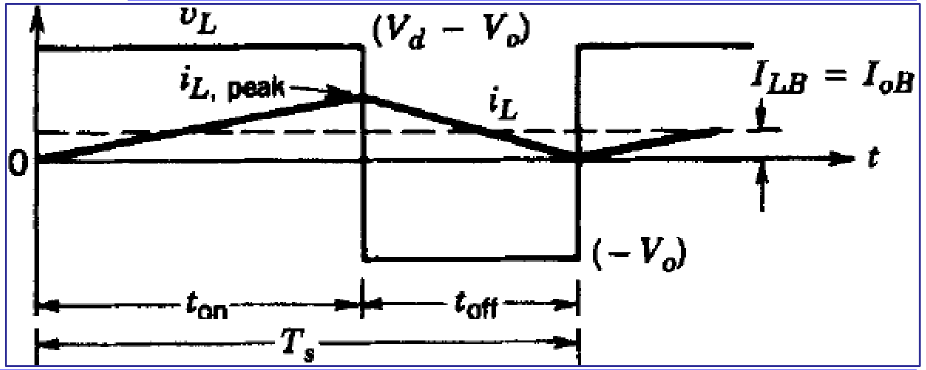
STEP-DOWN (BUCK) CONVERTER

Discontinuous-Conduction Mode

Assumption: converter is operating at the edge for given values of T, L, *Vd* , and *D,* which are kept constant
The output load power is decreased (i.e., the load resistance goes up), then the average inductor current will decrease.

During $\Delta_2 T_s$, power to R is supplied by C alone and *vL* is zero.





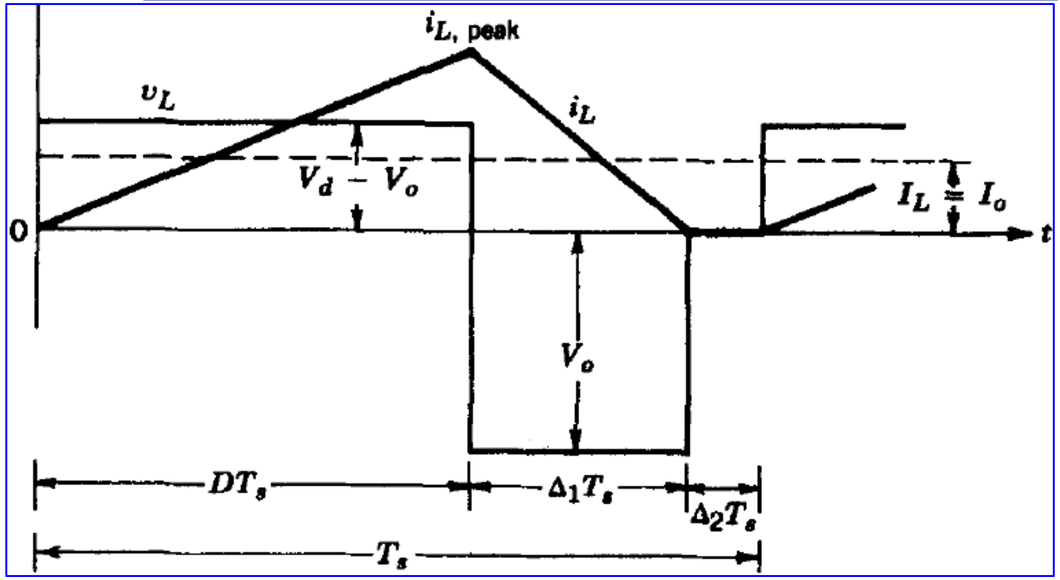
Again, equating the integral of *vL* over one time period to zero yields:

$$(V_d - V_o)\, DT_s + (-V_o)\Delta_1 T_s = 0$$

$$\therefore \frac{V_o}{V_d} = \frac{D}{D + \Delta_1}$$

where $D + \Delta_1 < 1.0$.

Hence, for same D, V0 in DCM > V0 in CCM.



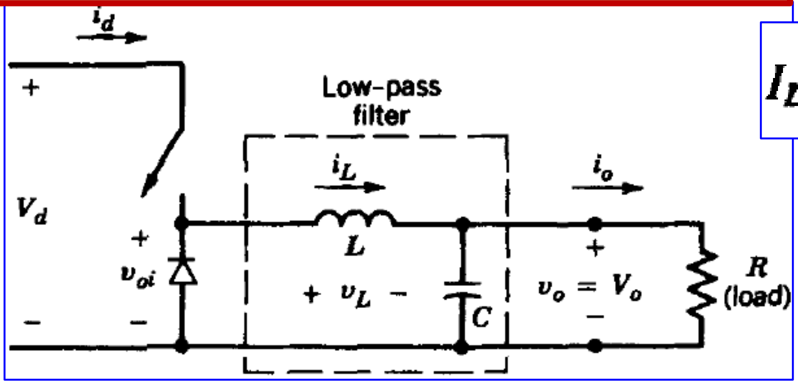
STEP-DOWN (BUCK) CONVERTER

Discontinuous-Conduction Mode

$$-v_0 = L\frac{di_L}{dt} => L\int_{i_{Lpeak}}^{0} di_L = -V_0\int_{0}^{\Delta_1 T_s} dt$$

$$=> i_{Lpeak} = \frac{V_0}{L}\Delta_1 T_s$$

$$\therefore \frac{V_o}{V_d} = \frac{D}{D+\Delta_1}$$



$$I_{LB,\max} = \frac{T_sV_d}{8L}$$

$$I_o = i_{L,\text{peak}}\frac{D+\Delta_1}{2} = \frac{V_oT_s}{2L}(D+\Delta_1)\,\Delta_1$$

$$= \frac{V_dT_s}{2L}D\Delta_1 = 4I_{LB,\max}D\Delta_1 \qquad \therefore \Delta_1 = \frac{I_o}{4I_{LB,\max}D}$$

$$\boxed{\therefore \frac{V_o}{V_d} = \frac{D}{D+\Delta_1} = \frac{D^2}{D^2 + \frac{1}{4}(I_o/I_{LB,\max})}}$$

The step-down converter characteristic in both modes of operation for a constant *Vd. V0/Vd* is plotted as a function of *I0/ILBmax* for various values of *D*. The boundary between CCM and DCM is shown by the dashed curve.