EE 238

Power Engineering - II

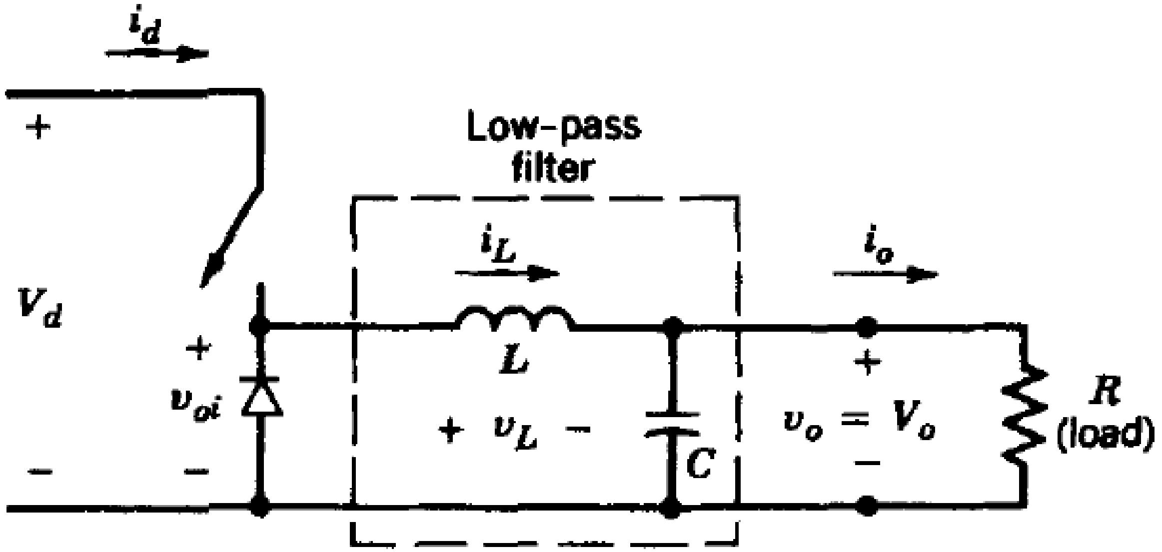
Power Electronics

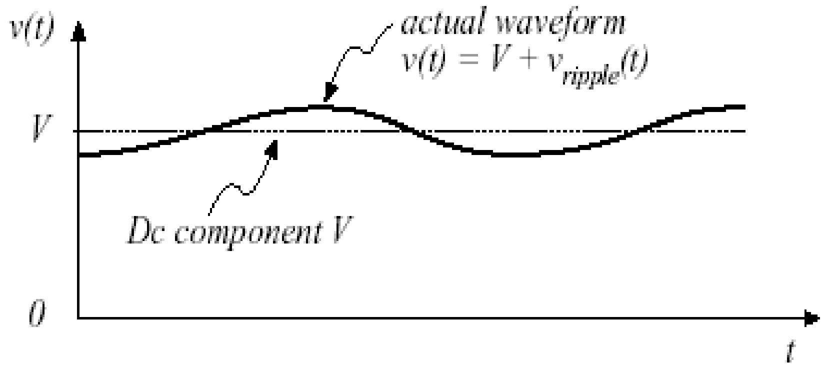
Lecture 6

Instructor: Prof. Anshuman Shukla
Email: ashukla@ee.iitb.ac.in

Thought process in analyzing basic DC/DC converters

- Basic operation principle (qualitative analysis)
  - How does current flow during different switching states
  - How is energy transferred during different switching states
- Verification of small ripple approximation
- Derivation of inductor voltage waveform during different switching states
- Quantitative analysis according to inductor volt-second balance or capacitor charge balance





$$\left| v_{ripple} \right| << V$$

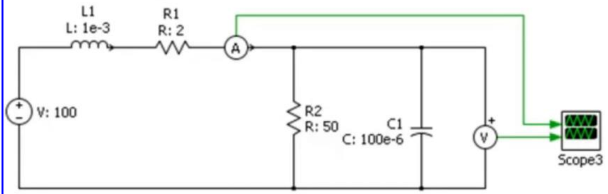
$$v(t) \approx V$$

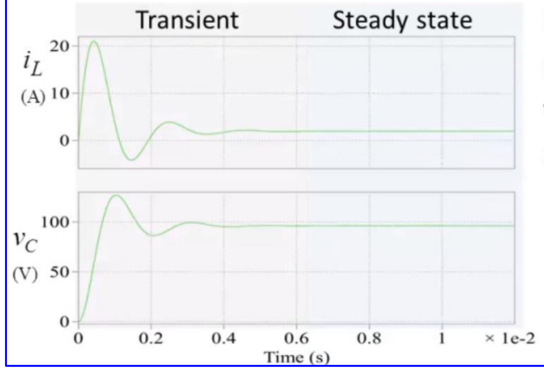
Steady-state Analysis

DC steady state in power converters

- DC steady state analysis is useful for
  - Thorough understanding of the operation, including various sub-intervals and modes of operation
  - Deriving input-output voltage and current relationships of various converter topologies
  - Design of various components such as inductors, capacitors, transformers
  - Selection of voltage and current ratings of semiconductor devices
  - Loss analysis

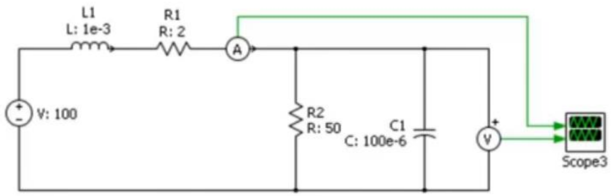
DC Steady state in non-switching circuits

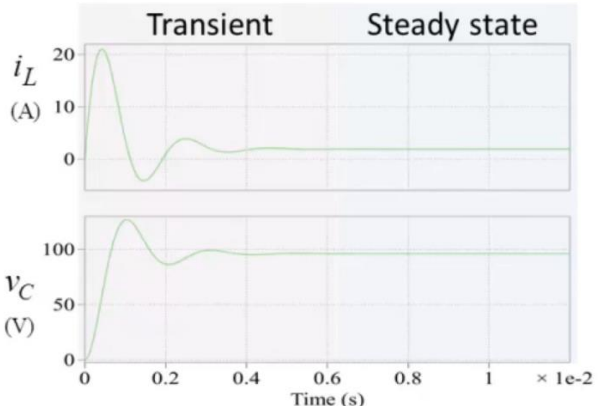




In non-switching circuits, DC steady state can be defined as a condition when all the variables (voltages, currents) are CONSTANT in time

DC Steady state in non-switching circuits





- Since,

  $v_L = L\dfrac{di_L}{dt}$ and i_L is constant, $v_L = 0$

  and inductor is considered as a short circuit in dc steady state for non-switching circuits

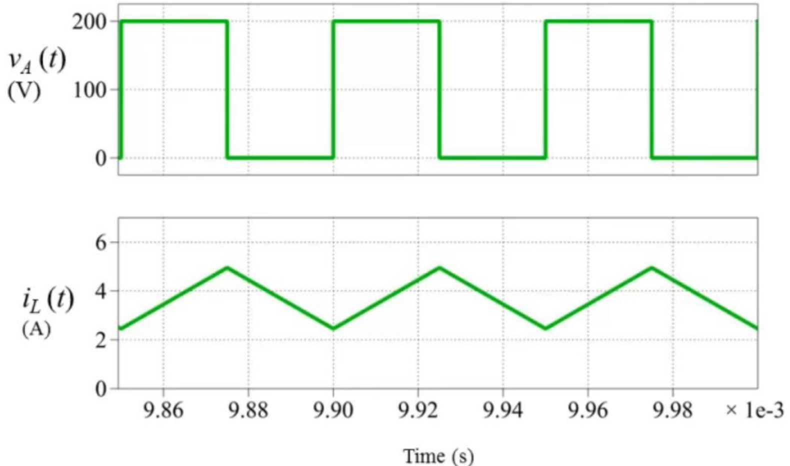
- Since,

  $i_C = C\dfrac{dv_C}{dt}$ and v_C is constant, $i_C = 0$

  and capacitor is considered as a open circuit in dc steady state

DC steady state in switching converters

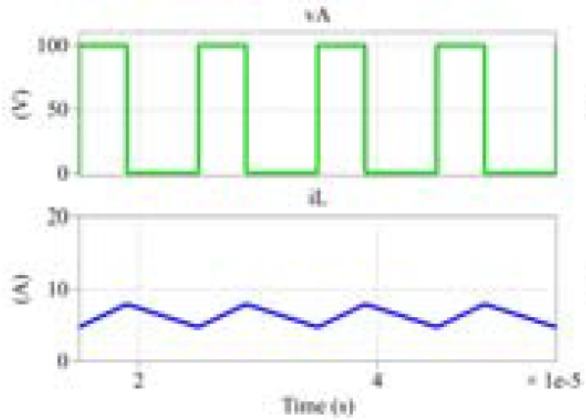
- In a switching converter, most of the voltages and currents are always switching or time varying
- Need for a different definition of DC steady state



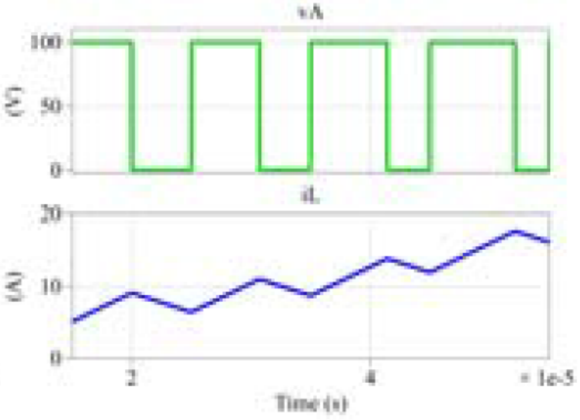
Concept of DC steady state in switching converters

A switching converter is in DC steady state, if

- ALL waveforms repeat exactly every switching period
  Example: $i_A(t) = i_A(t-T_S)$



Steady state

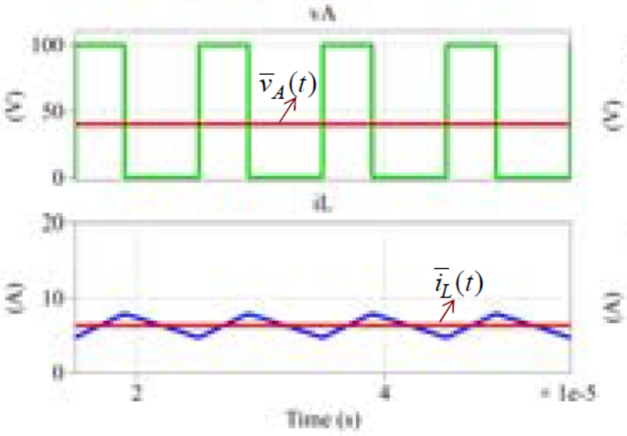


Transient

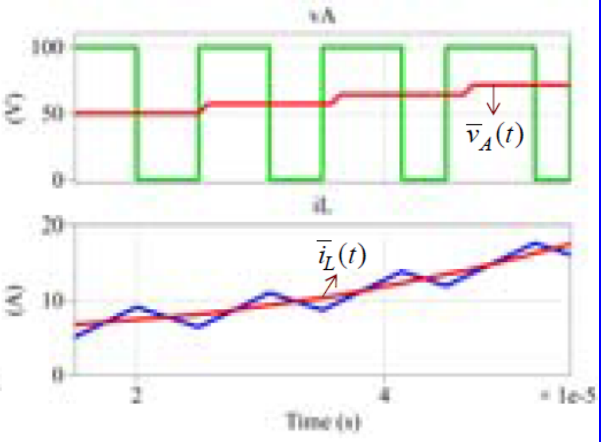
Concept of DC steady state in switching converters

A switching converter is in DC steady state, if

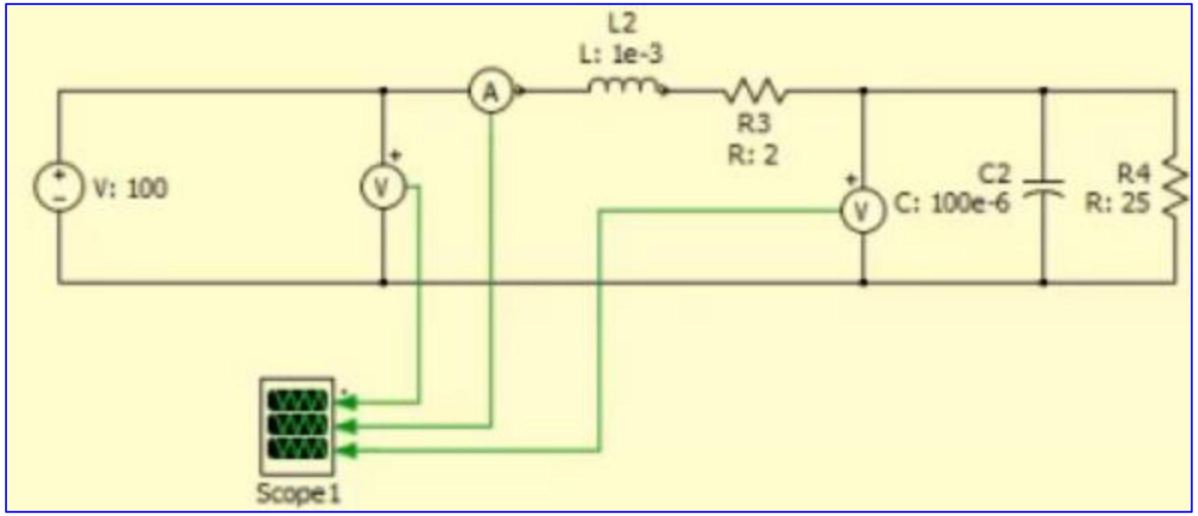
- CCA values of **ALL** variables remain constant

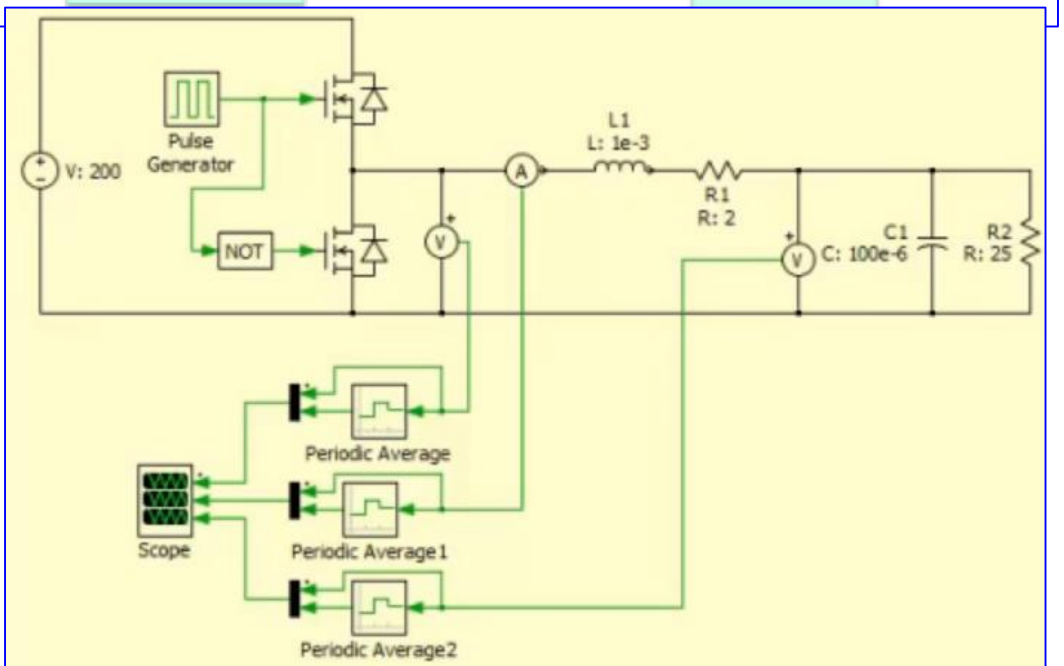


Steady state



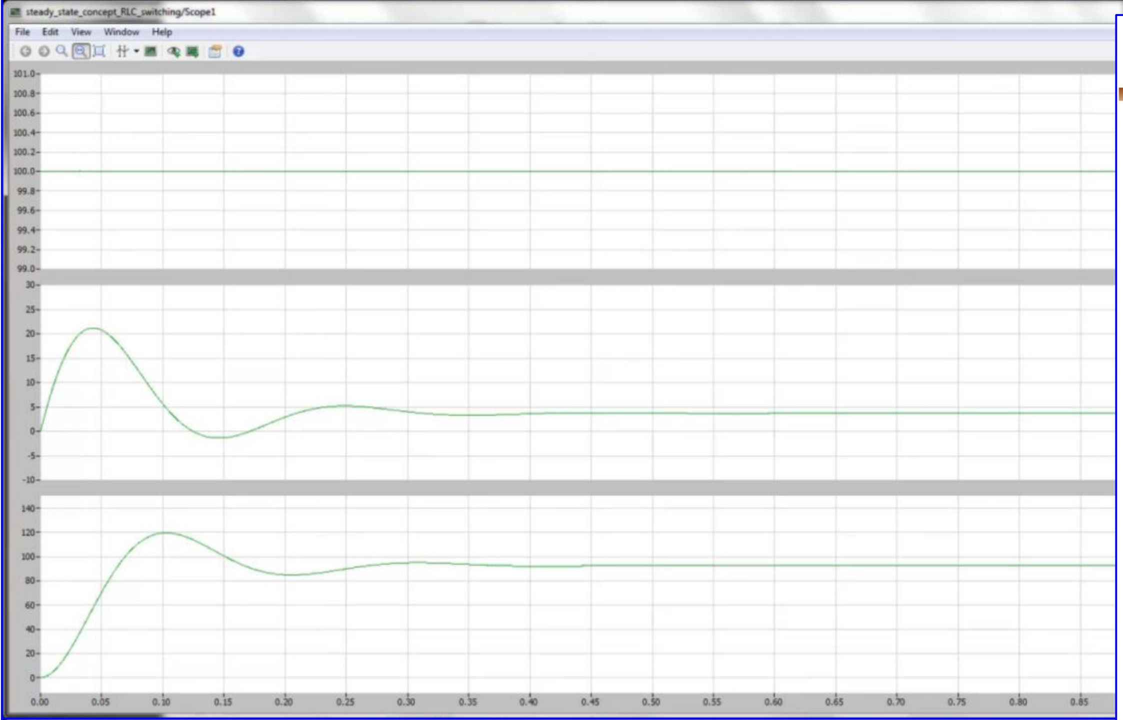
Transient



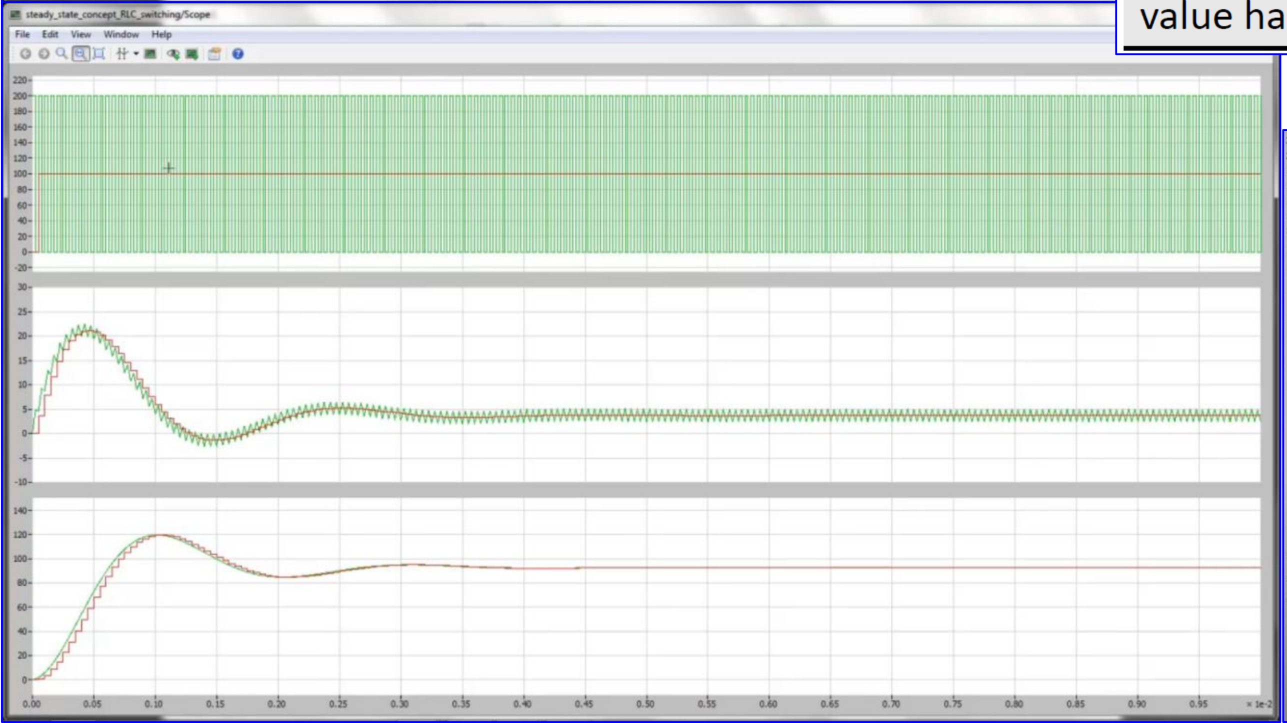


Simulation examples

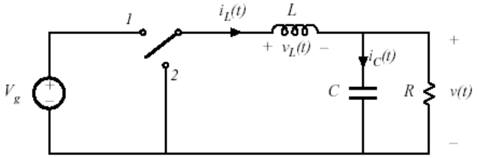
DC steady state characteristics



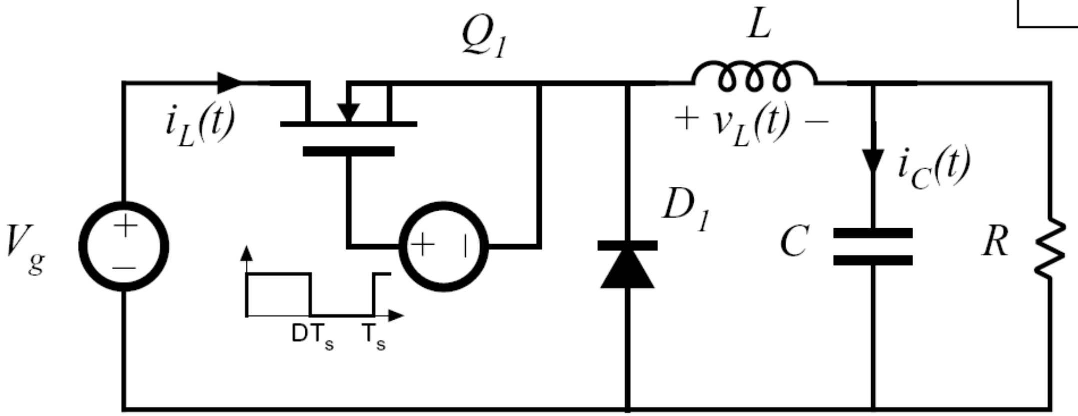
Non-switching circuits	Switching circuits
All instantaneous quantities (voltages, currents) are constant	All CCA (cycle-by-cycle) quantities (voltages, currents) are constant
Instantaneous voltage across an inductor is zero; inductor can be considered a short for analysis and L value has no impact in steady state	CCA voltage across inductor is zero (volt-sec balance principle); inductance determines the switching frequency current ripple in steady state
Instantaneous current through a capacitor is zero; capacitor can be considered open for analysis and C value has no impact in steady state	CCA current through a capacitor is zero (current-sec balance principle); capacitance determines the switching frequency voltage ripple in steady state



Buck converter analysis:

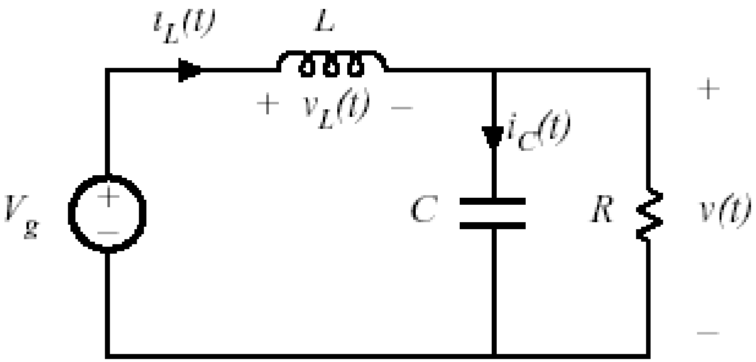


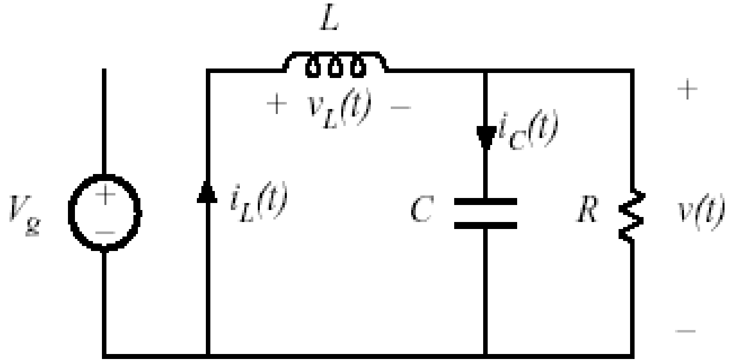
original converter



switch in position 1

switch in position 2

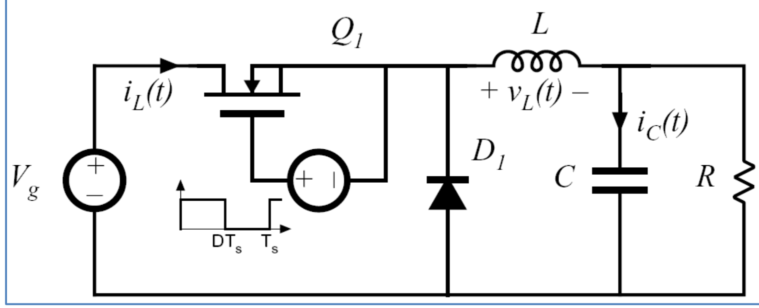




During the interval when the switch is on, the diode becomes reverse biased and the input provides energy to the load as well as to the inductor.

During the interval when the switch is off, the inductor current flows through the diode, transferring some of its stored energy to the load.

Inductor voltage and current
Subinterval 1: switch in position 1

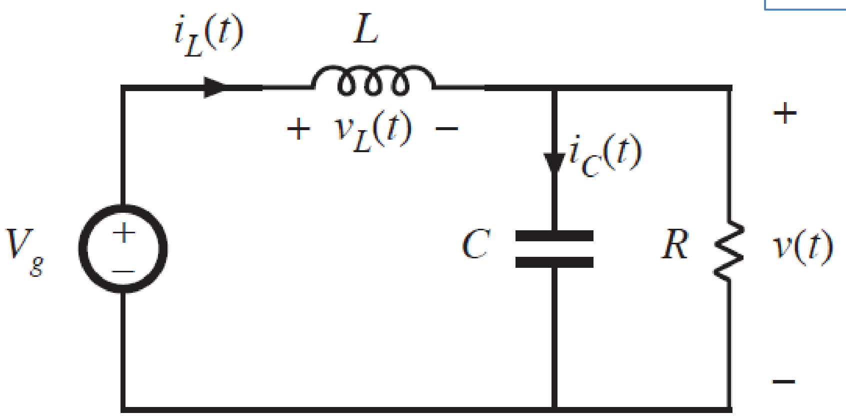


Inductor voltage

$$v_L = V_g - v(t)$$

Small ripple approximation:

$$v_L \approx V_g - V$$



Knowing the inductor voltage, we can now find the inductor current via

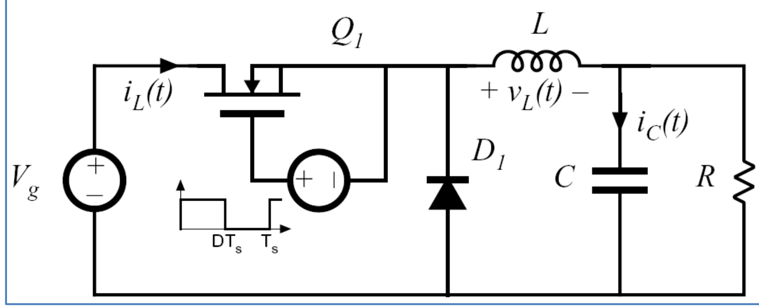
$$v_L(t) = L\frac{di_L(t)}{dt}$$

Solve for the slope:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \approx \frac{V_g - V}{L}$$

$\Rightarrow$ *The inductor current changes with an essentially constant slope*

Inductor voltage and current
Subinterval 2: switch in position 2

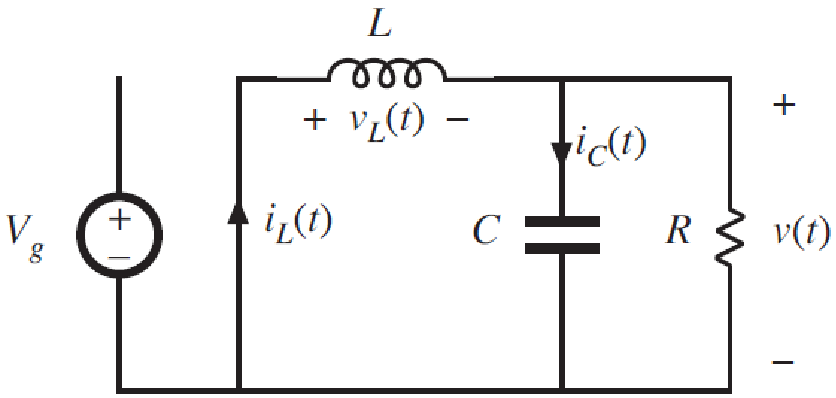


Inductor voltage

$$v_L(t) = -\,v(t)$$

Small ripple approximation:

$$v_L(t) \approx -\,V$$



Knowing the inductor voltage, we can again find the inductor current via

$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Solve for the slope:

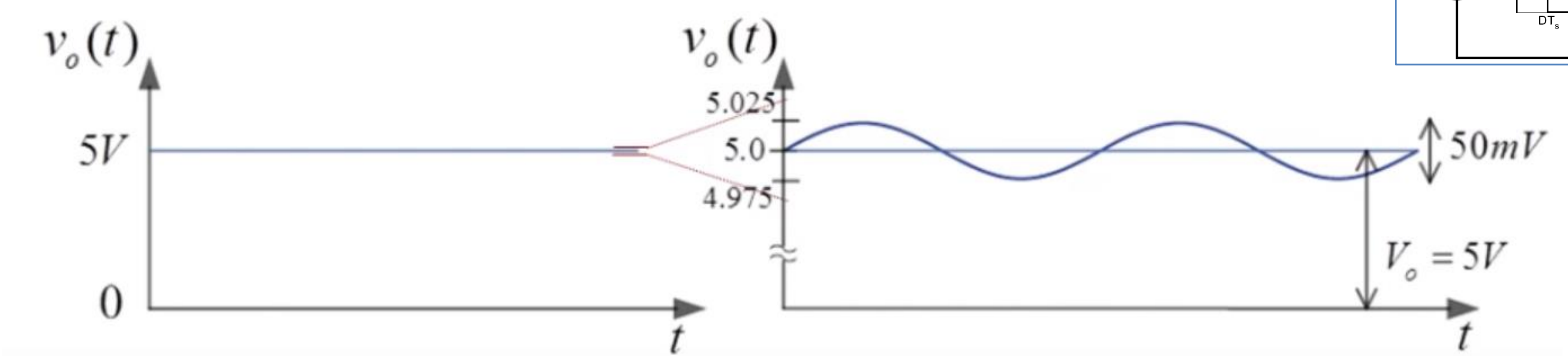
$$\frac{di_L(t)}{dt} \approx -\,\frac{V}{L}$$

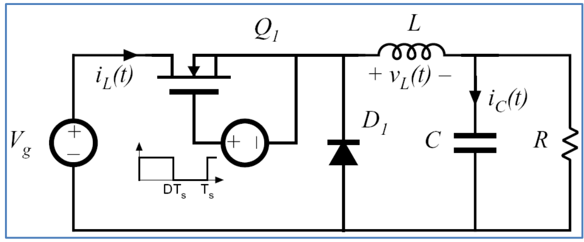
$\Rightarrow$ *The inductor current changes with an essentially constant slope*

Small Ripple Approximation

$$v_o(t) = V_o + v_{o,ripple}(t)$$ Peak-peak $v_{o,ripple}$ designed to be less than 1% of V_o

$$v_o(t) \approx V_o$$





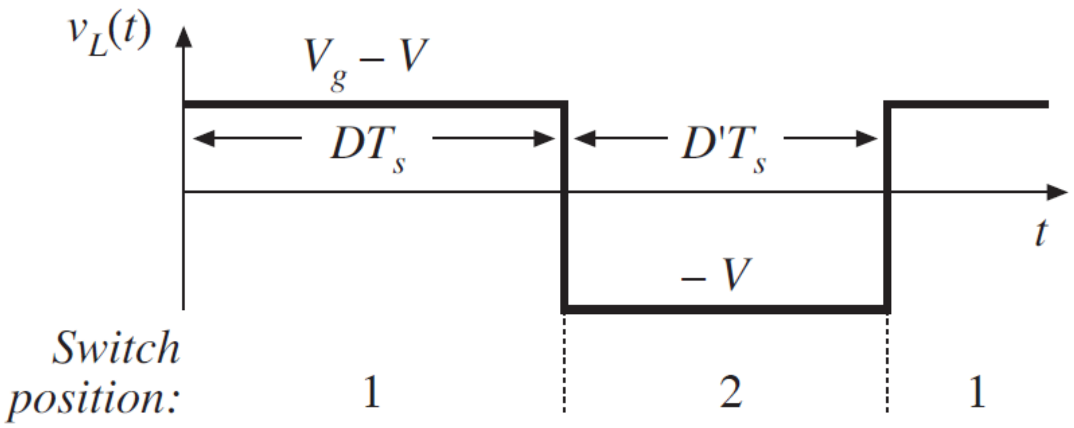
- For resistive loads as considered here initially,

$$i_o(t) \approx I_o$$

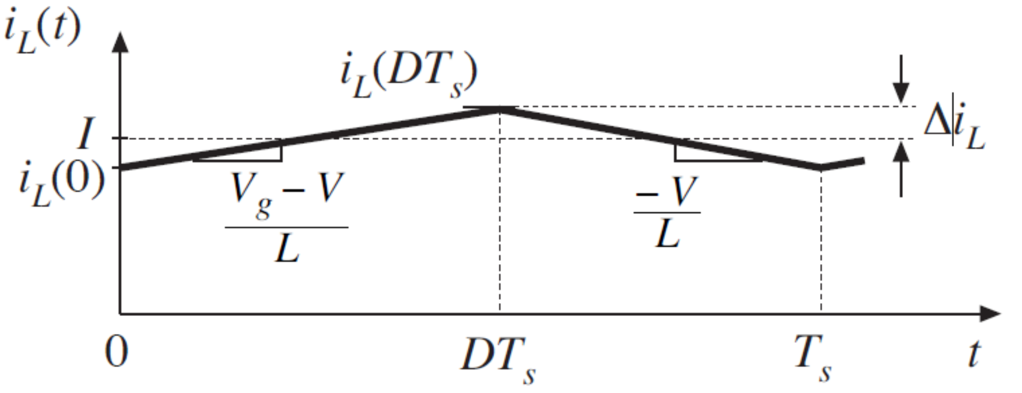
i.e., the high frequency component of inductor current flows only in the output capacitor, and negligible amount through the load

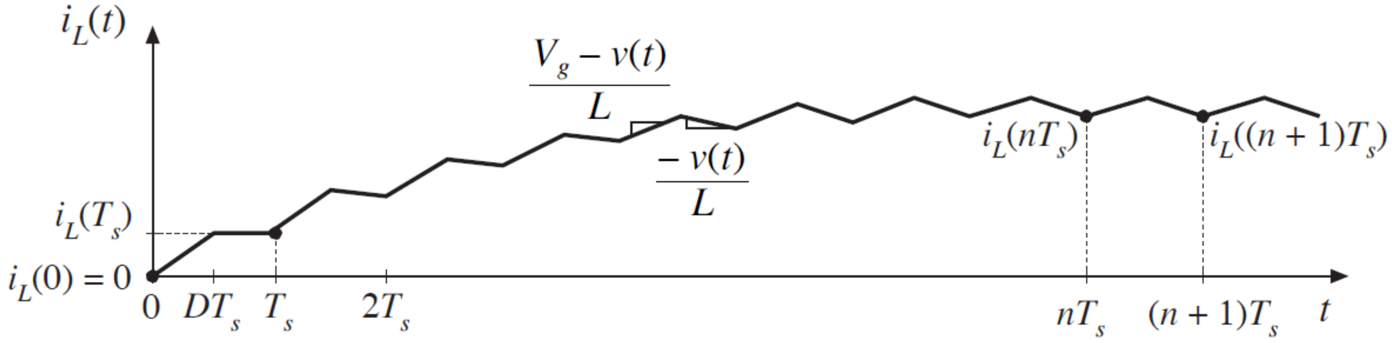
- Characteristics of practical loads such as processors can be quite different

Inductor voltage and current waveforms



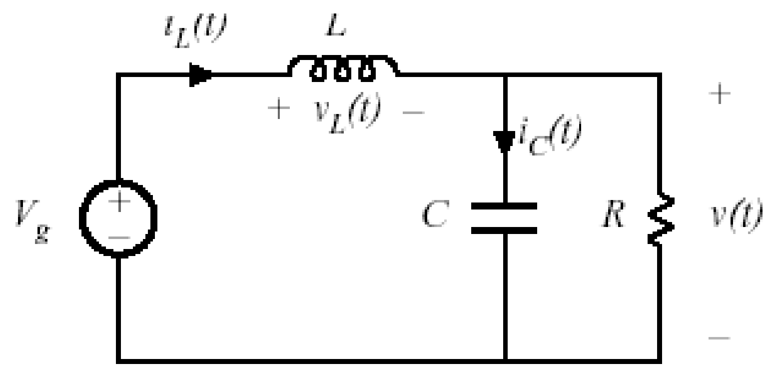
$$v_L(t) = L\,\frac{di_L(t)}{dt}$$



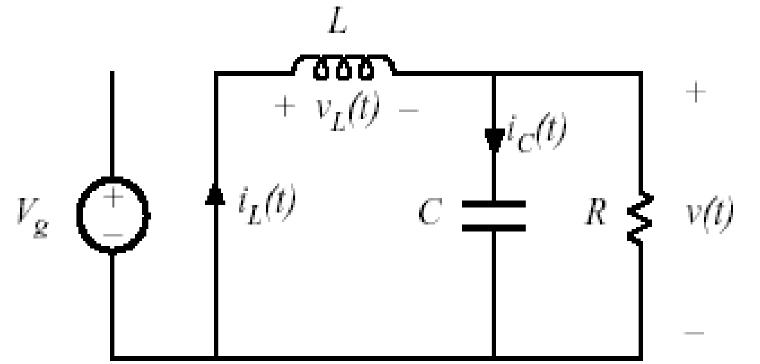


When the converter operates in equilibrium: $i_L((n+1)T_s) = i_L(nT_s)$

switch in position 1



switch in position 2



Buck converter Analysis

When the switch is closed (on) :

$$v_L = V_g - V = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_g - V}{L}$$

Derivative of i_L is a positive constant. Therefore i_L must increase linearly.

From Figure

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_g - V}{L}$$

$$(\Delta i_L)_{closed} = \left(\frac{V_g - V}{L}\right).DT$$

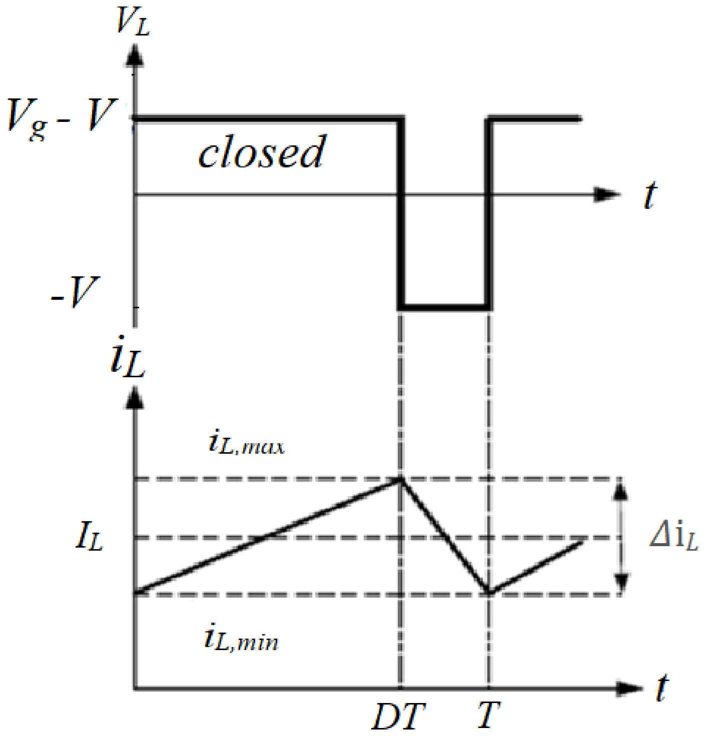
For switch opened,

$$v_L = -V = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{-V}{L}$$

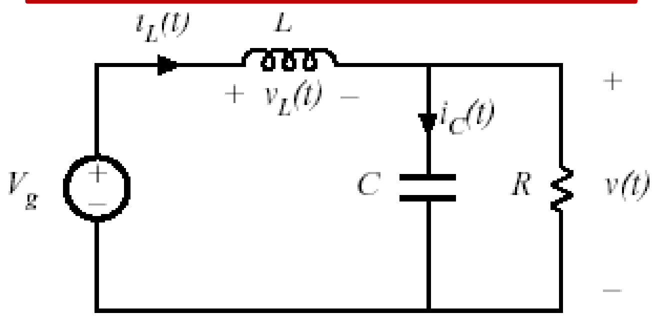
$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{-V}{L}$$

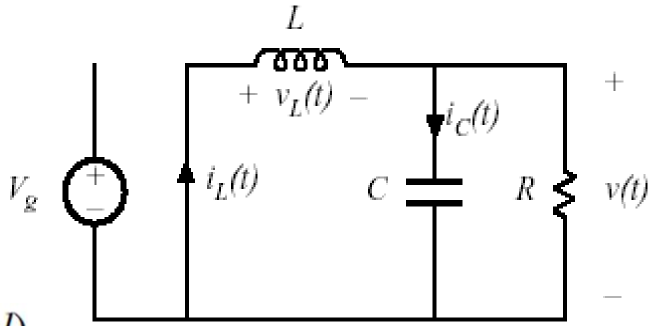
$$(\Delta i_L)_{opened} = \left(\frac{-V}{L}\right).\ (1 - D)T$$



$(change\ in\ i_L) = (slope)(length\ of\ subinterval)$

Determination of inductor current ripple magnitude



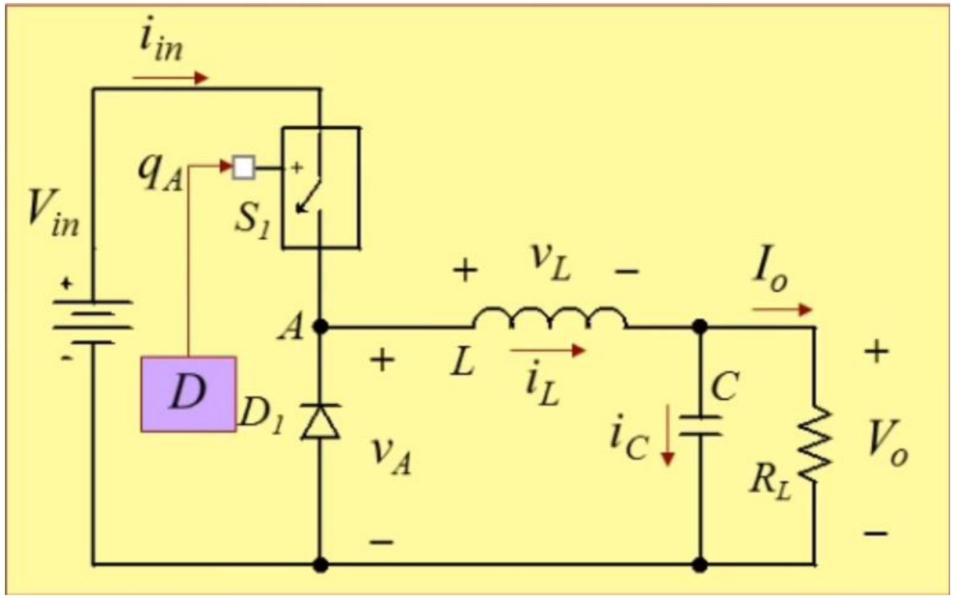


Steady- state operation requires that i_L at the end of switching cycle is the same at the beginning of the next cycle. That is the change of i_L over one period is zero i.e :

$$(\Delta i_L)_{closed} + (\Delta i_L)_{opened} = 0$$

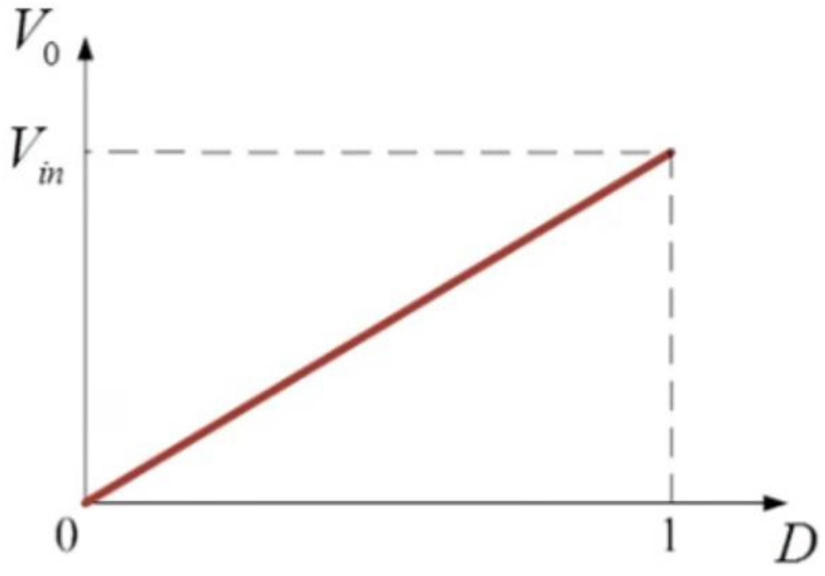
$$\left(\frac{Vg - V}{L}\right).\ DT_S - \left(\frac{-V}{L}\right).\ (1\text{-}D)T_S = 0$$

$$V = DV_g$$



$$\boxed{\frac{V_o}{V_{in}} = D}$$

Input-output relationship for Buck converter

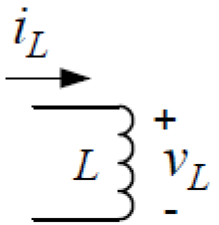


Neglecting power losses, the input power equals the output power.

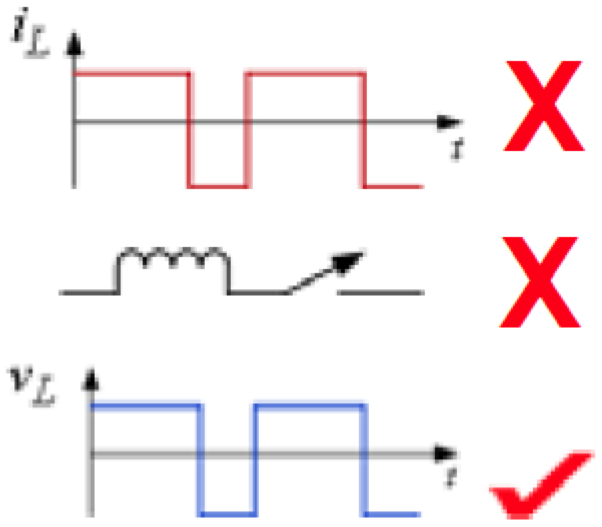
$$\boxed{P_d = P_o} \Rightarrow \boxed{V_d I_d = V_o I_o} \Rightarrow \boxed{\frac{I_o}{I_d} = \frac{V_d}{V_o} = \frac{1}{D}}$$

Characteristics of inductors and capacitors

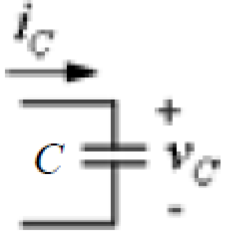
Inductor



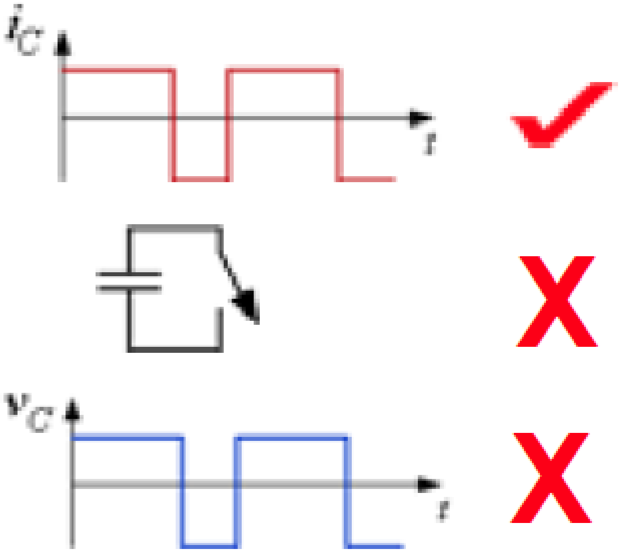
$$v_L(t) = L\frac{di_L(t)}{dt}$$



Capacitor



$$i_C(t) = C\frac{dv_C(t)}{dt}$$



Volt-sec balance in inductors

- The average (CCA) voltage across an inductor in DC steady-state is zero $\boxed{\overline{v}_L = 0}$

Volt-sec balance in Inductors

Instantaneous v-i relationship for inductor

$$v_L(t) = L\frac{di_L(t)}{dt}\ ; \quad i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(\tau)\,d\tau \ \ldots(1)$$

Substituting $t = t_0 + T_s$ in (1)

$$i_L(t_0 + T_s) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t_0+T_s} v_L(\tau)\,d\tau \ \ldots(2)$$

In steady state

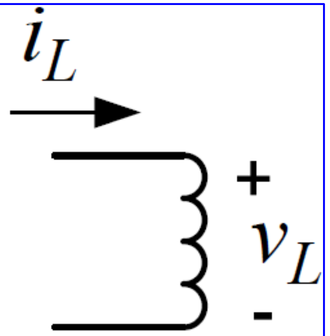
$$i_L(t_0 + T_s) = i_L(t_0) \ \ldots(3)$$

from (2) and (3)

$$\frac{1}{L}\int_{t_0}^{t_0+T_s} v_L(t)\,dt = \frac{T_s}{L}\overline{v}_L = 0$$

$$\overline{v}_L = 0$$

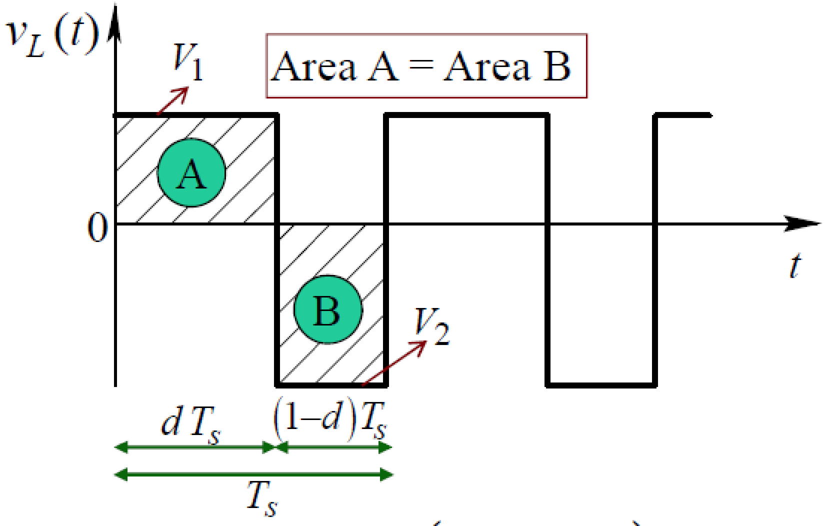
$$\overline{v}_L = L\frac{d\overline{i}_L}{dt} = 0$$



(since, $\overline{i}_L$ should be constant in steady state)

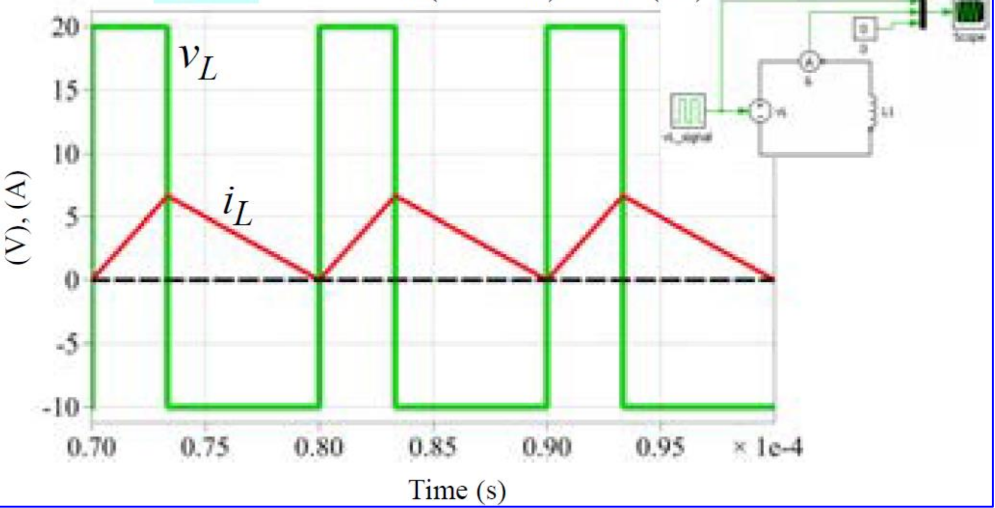
Volt-sec balance in Inductors

$\overline{v}_L = 0$ does not imply that the inductor voltage is zero instantaneously, only the average over a complete period is zero



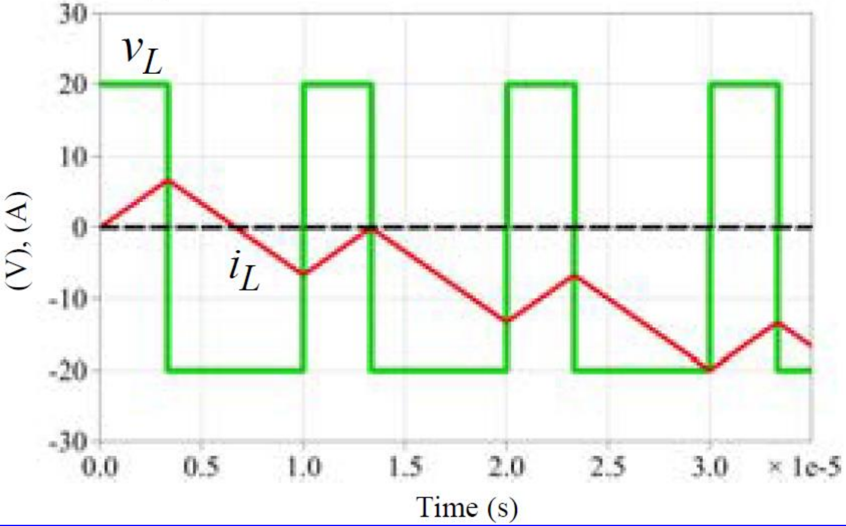
$$V_1 d + V_2 (1-d) = 0$$

$\overline{v}_L = 0$ $\quad i_L(t_0 + T_s) = i_L(t_0)$



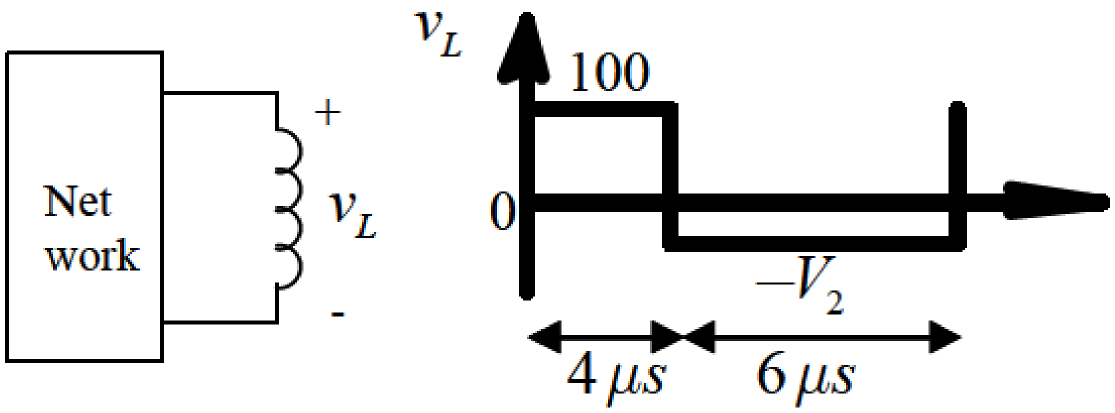
If volt-sec balance is violated

$\overline{v}_L < 0 \Rightarrow \overline{i}_L$ continuously decreases $\Rightarrow$ non-steady-state



Example

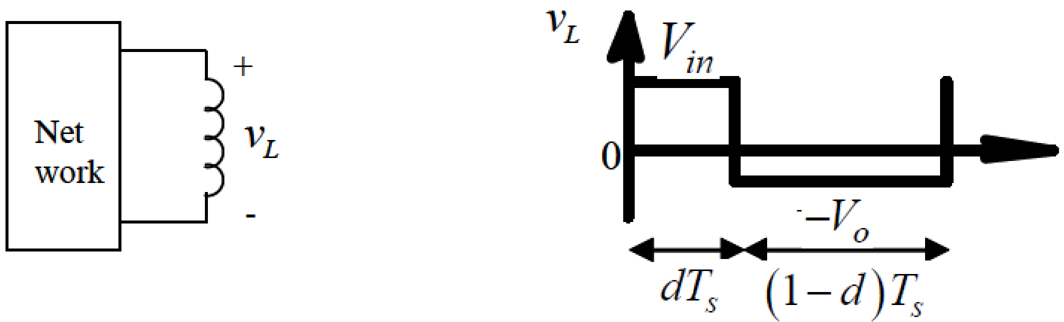
Given that the circuit is in DC steady state, calculate V_2



$V_2 = ?$

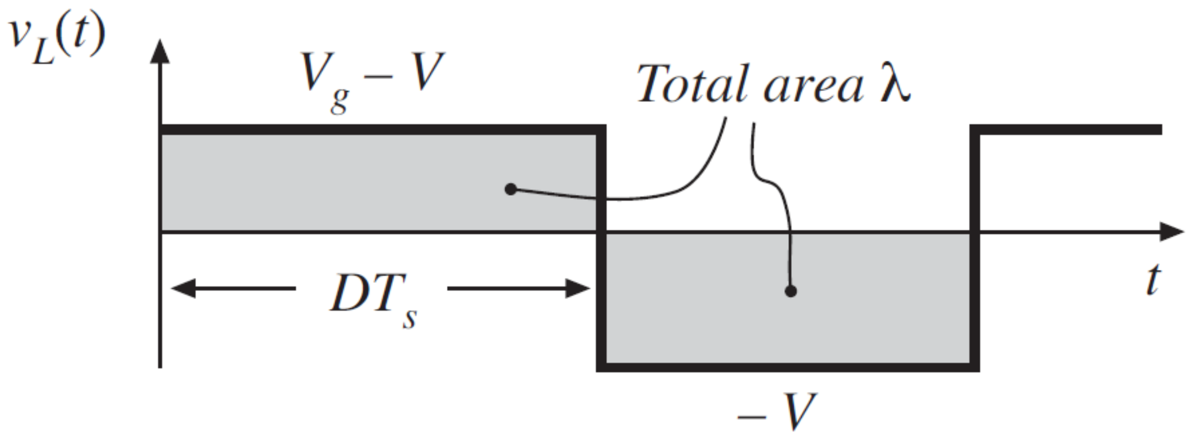
Example

Calculate the input-output relationship for a dc-dc converter, i.e, V_o / V_{in}, in terms of the duty ratio, d given the inductor voltage below



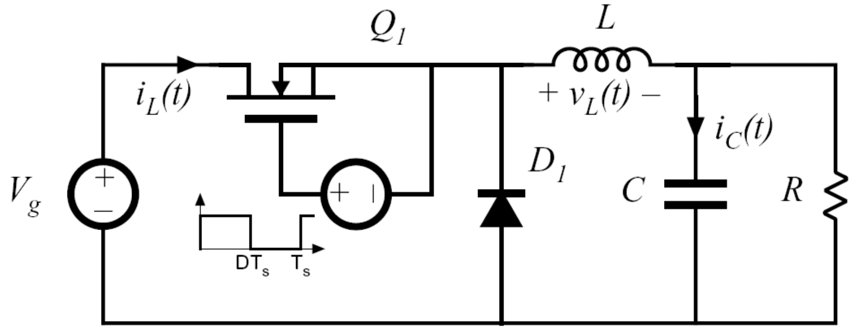
Inductor volt-second balance: Buck converter example

Inductor voltage waveform, previously derived:



Integral of voltage waveform is area of rectangles:

$$\lambda = \int_0^{T_s} v_L(t)\,dt = (V_g - V)(DT_s) + (-V)(D'T_s)$$



Average voltage is

$$\langle v_L \rangle = \frac{\lambda}{T_s} = D(V_g - V) + D'(-V)$$

Equate to zero and solve for *V:*

$$0 = DV_g - (D + D')V = DV_g - V \qquad \Rightarrow \qquad V = DV_g$$