# EE 238

## Power Engineering - II

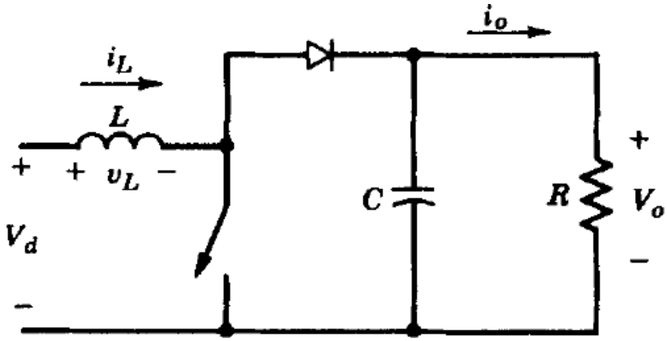
## Power Electronics

Lecture 12

Instructor: Prof. Anshuman Shukla
Email: ashukla@ee.iitb.ac.in

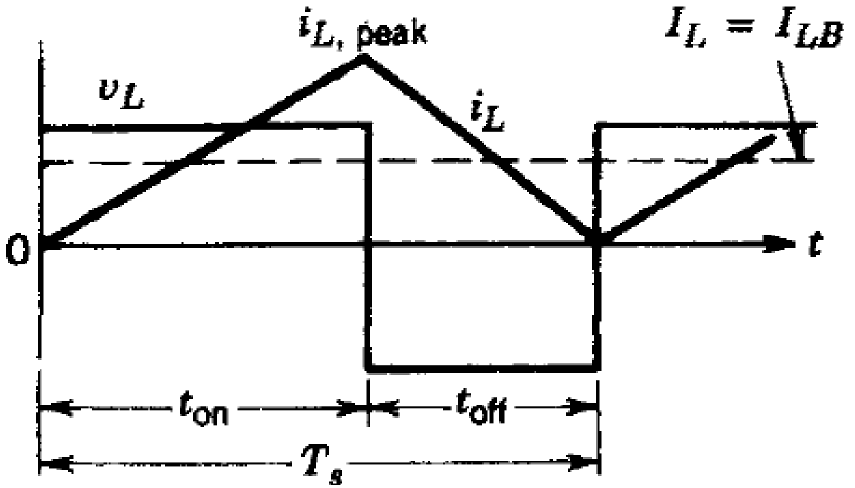
# BOOST CHOPPER

## Boundary between Cont. and Discont. Modes




$$\frac{V_o}{V_d} = \frac{T_s}{t_{\text{off}}} = \frac{1}{1 - D}$$

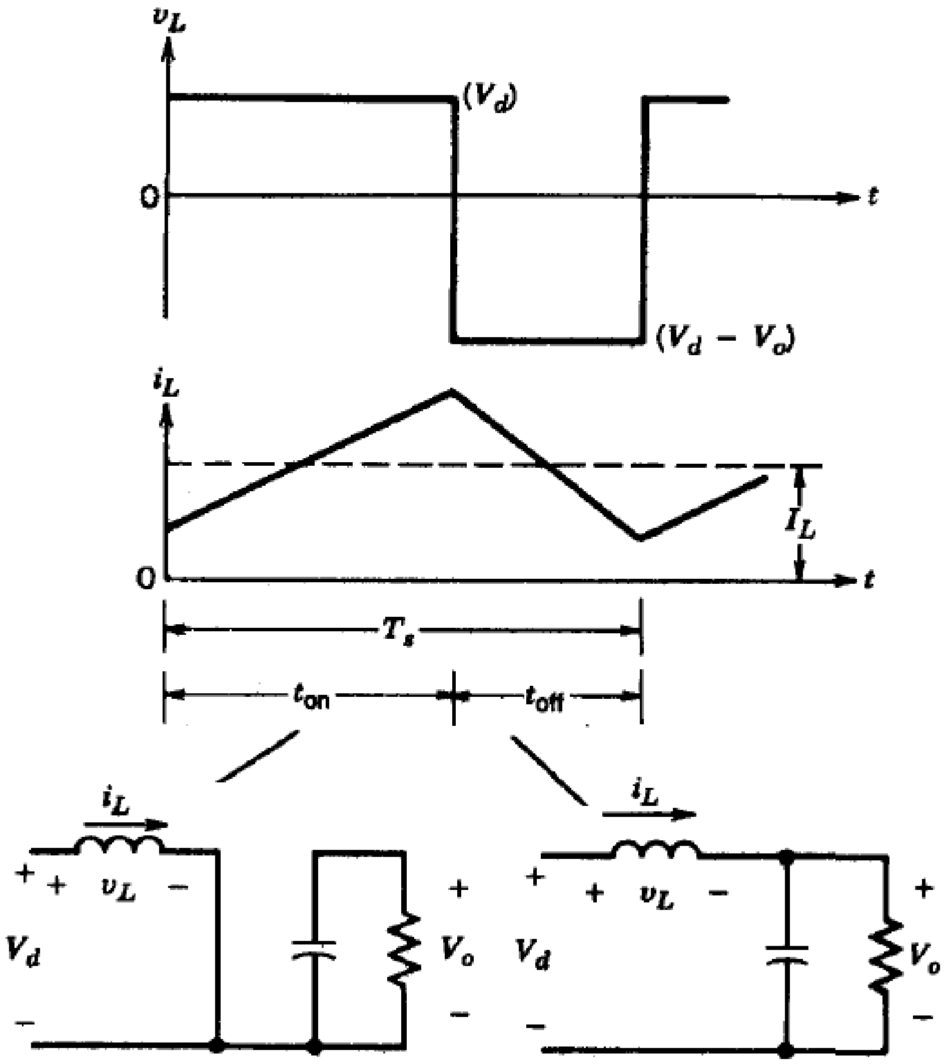
$$\frac{I_o}{I_d} = (1 - D)$$




$$I_{LB} = \frac{1}{2} i_{L,\text{peak}}$$

$$= \frac{1}{2} \frac{V_d}{L} t_{\text{on}}$$

$$= \frac{T_s V_o}{2L} D(1 - D)$$



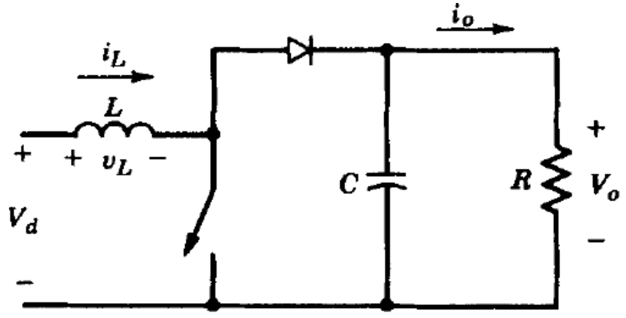

The average output current at the edge of cont. cond. is

$$I_{oB} = \frac{T_s V_o}{2L} D(1 - D)^2$$

# STEP-UP (BOOST) CONVERTER

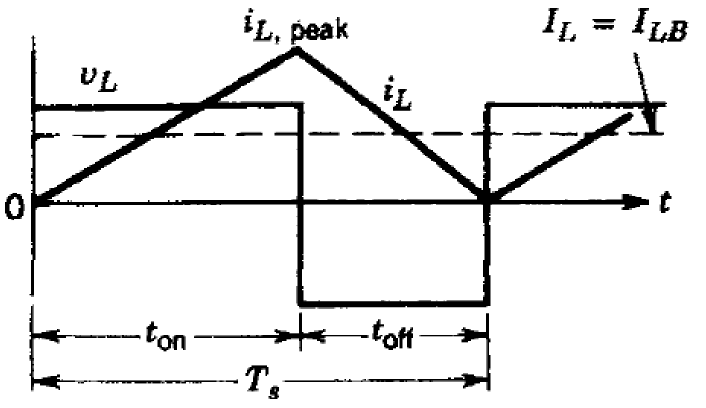
## Boundary between Cont. and Discont. Modes

$$I_{LB} = \frac{T_s V_o}{2L} D(1-D) \qquad \frac{I_o}{I_d} = (1-D) \qquad i_d = i_L$$




The average output current at the edge of cont. cond. is

$$I_{oB} = \frac{T_s V_o}{2L} D(1-D)^2$$




In most Boost converter applications, $V_0$ is kept constant.

$I_{LB}$ reaches a maximum value at D = *0.5*:

$$I_{LB,\max} = \frac{T_s V_o}{8L}$$

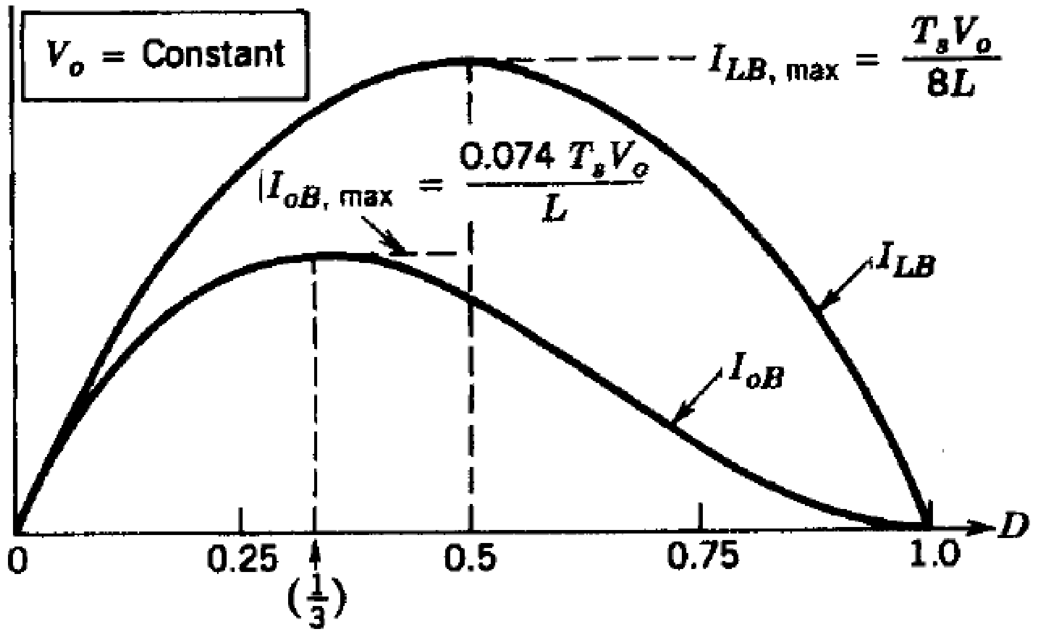
Also, $I_{0B}$ has its maximum at D = 0.333

$$I_{oB,\max} = \frac{2}{27}\frac{T_s V_o}{L} = 0.074\frac{T_s V_o}{L}$$

In terms of their maximum values,

$$I_{LB} = 4D(1-D)I_{LB,\max}$$

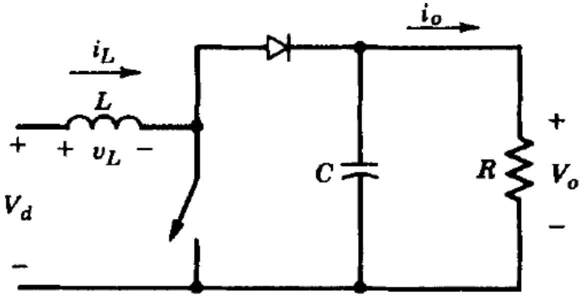
$$I_{oB} = \frac{27}{4}D(1-D)^2 I_{oB,\max}$$

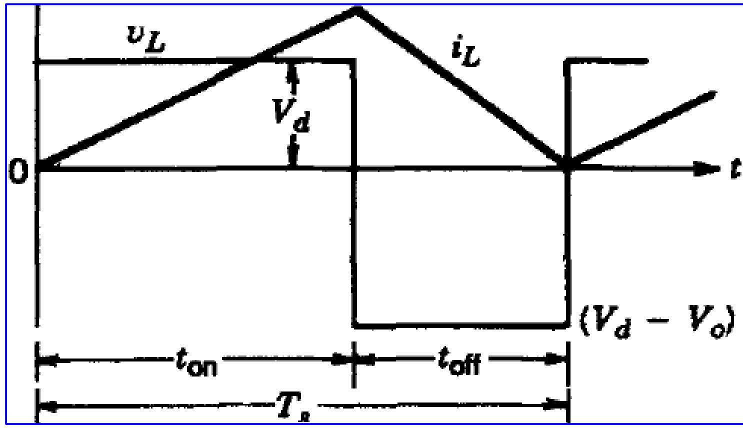



For a given *D,* with constant $V_0$, if the average load current drops below $I_{0B}$ (and, hence, the average inductor current below $I_{LB}$), the current conduction will become discontinuous.

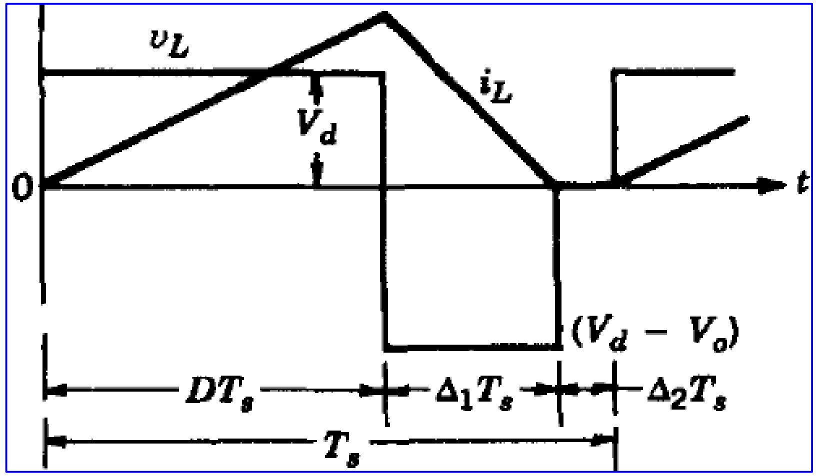
# DISCONTINUOUS-CONDUCTION MODE

Assume that as the output load power decreases, Vd and D remain constant (even though, in practice, D would vary in order to keep Vo constant).




- The DCM occurs due to decreased Po (=Pd) and, hence, a lower $I_L$ (= $I_d$), since $v_d$ is constant.
- Since $i_{Lpeak}$ is the same in both modes, a lower value of $I_L$ (and, hence a discontinuous $I_L$) is possible only if Vo goes up.




If we equate the integral of the inductor voltage over one time period to zero,

$$V_d D T_s + (V_d - V_o)\Delta_1 T_s = 0$$

$$\therefore \frac{V_o}{V_d} = \frac{\Delta_1 + D}{\Delta_1}$$

$$\frac{I_o}{I_d} = \frac{\Delta_1}{\Delta_1 + D} \quad (\text{since } P_d = P_o)$$

The average input current, which is also equal to the inductor current, is

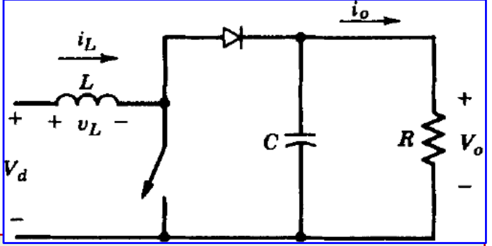
$$I_d = \frac{V_d}{2L} D T_s (D + \Delta_1)$$

$$\Rightarrow I_o = \left(\frac{T_s V_d}{2L}\right) D \Delta_1$$

# DISCONTINUOUS-CONDUCTION MODE

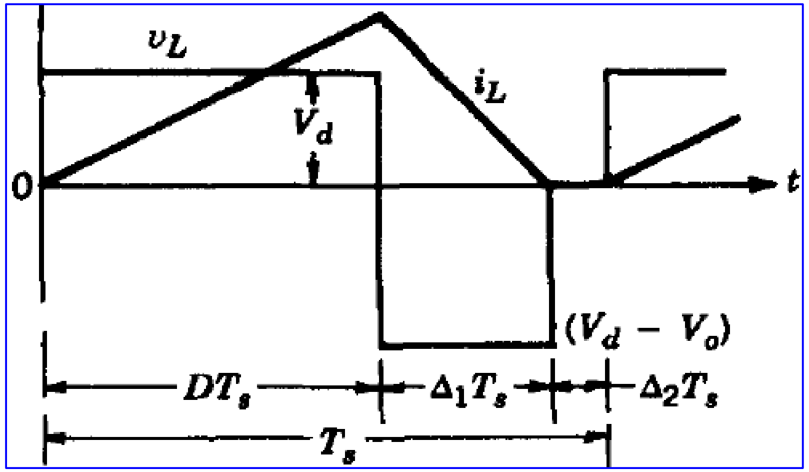
$$I_o = \left(\frac{T_s V_d}{2L}\right) D\Delta_1 \quad \therefore \frac{V_o}{V_d} = \frac{\Delta_1 + D}{\Delta_1}$$

$$I_{oB,\max} = \frac{2}{27}\frac{T_s V_o}{L} = 0.074\frac{T_s V_o}{L}$$

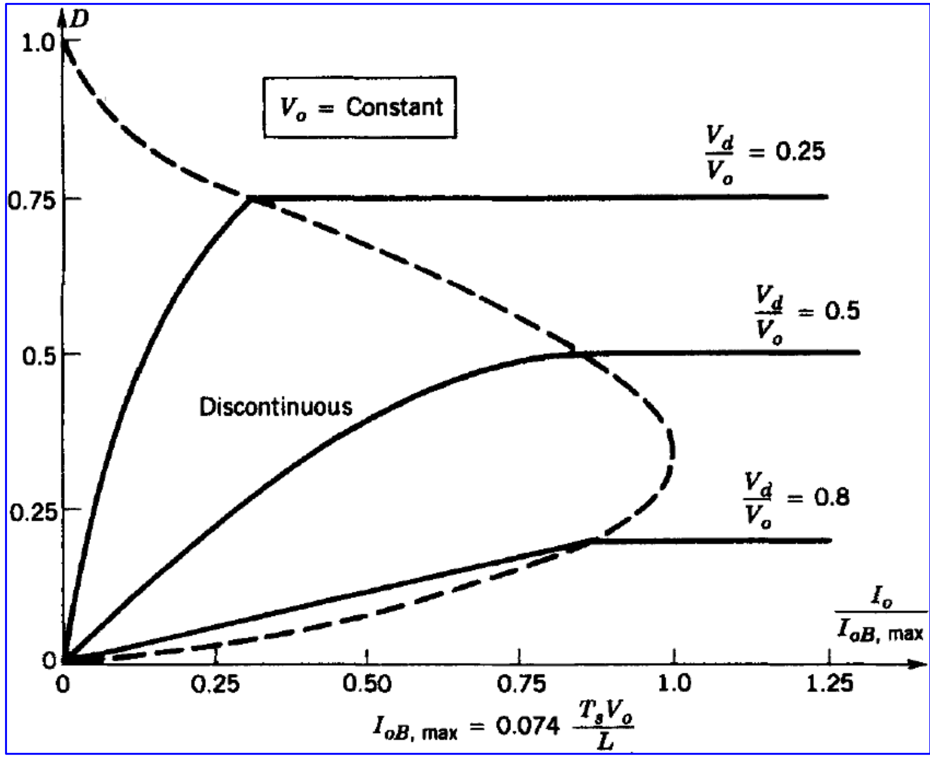



In practice, since $V_0$ is held constant and D varies in response to the variation in Vd, it is more useful to obtain the required duty ratio D as a function of load current for various values of $V_0$ / Vd.

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oB,\max}}\right]^{1/2}$$




D is plotted as a function of Io/IoBmax for various values of Vd/V0.




In the discontinuous mode, if $V_0$ is not controlled during each switching time period, at least

$$\frac{L}{2} i^2_{L,\text{peak}} = \frac{(V_d D T_s)^2}{2L} \quad \text{W-s}$$

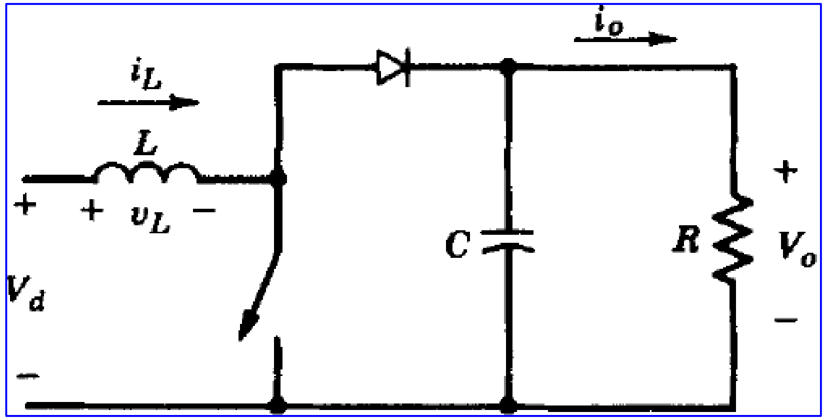
are transferred from the input to the output capacitor and to the load.

For constant output voltage and variable input voltage applications

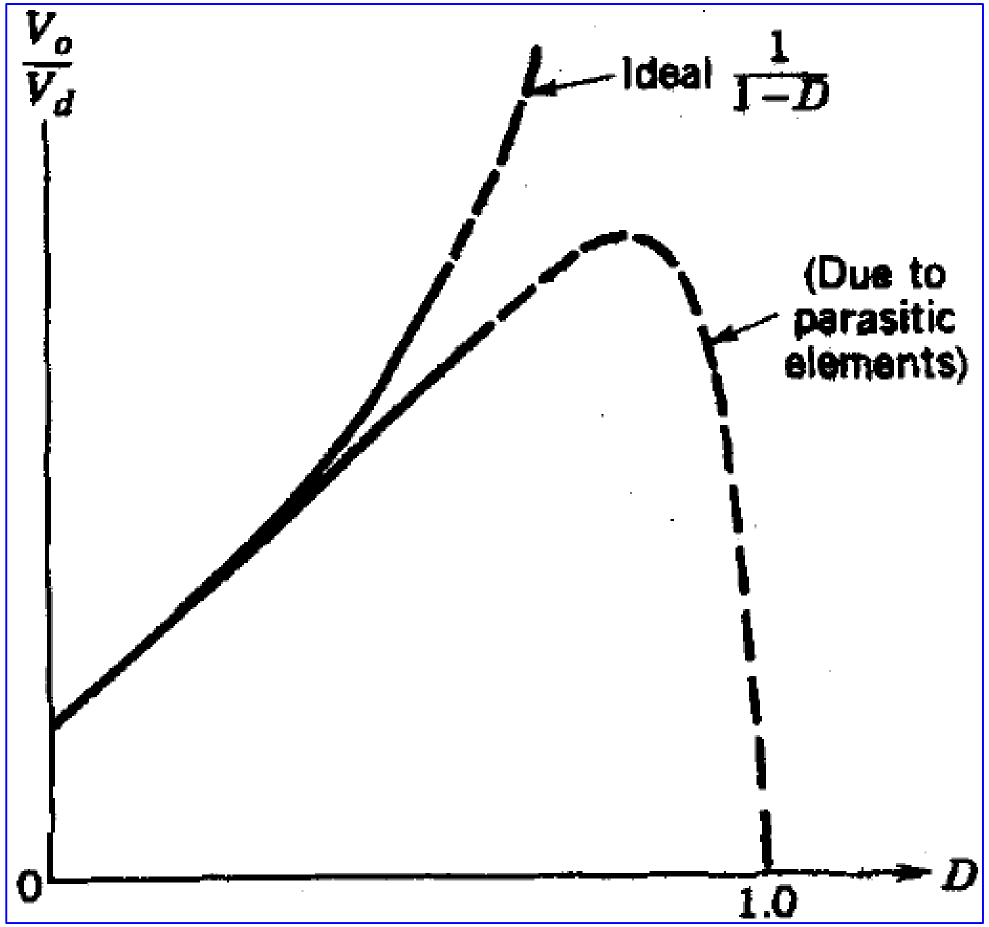
$$\left(1 - \frac{V_{in,max}}{V_o}\right) \le D \le \left(1 - \frac{V_{in,min}}{V_o}\right)$$

## STEP-UP (BOOST) CONVERTER | EFFECT OF PARASITIC ELEMENTS

The parasitic elements in a step-up converter are due to the losses associated with the inductor, the capacitor, the switch, and the diode.




Unlike the ideal characteristic, in practice, $V_0 / V_d$ declines as D approaches unity.

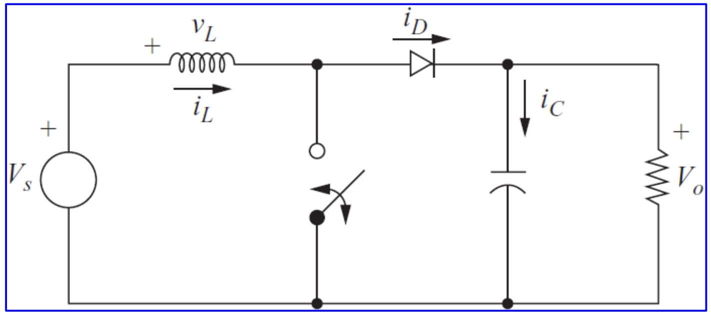



These parasitic elements can be incorporated into circuit simulation programs on computers for designing such converters.

# Inductor Resistance

- L should be designed to have small resistance to minimize power loss and maximize efficiency.
- The existence of a small inductor resistance does not substantially change the buck converter analysis.
- However, inductor resistance affects performance of the boost converter, especially at high duty ratios.

$$V_o = \frac{V_s}{1-D}$$




For analysis purpose, assume that $i_L$ is approximately constant.

Neglecting other losses. And applying power balance:

$$P_s = P_o + P_{r_L}$$
$$V_s I_L = V_o I_D + I_L^2 r_L$$

$r_L$ is the series resistance of $L$.

The diode current is equal to the inductor current when the switch is off and is zero when the switch is on. Therefore, the average diode current is $I_D = I_L(1-D)$

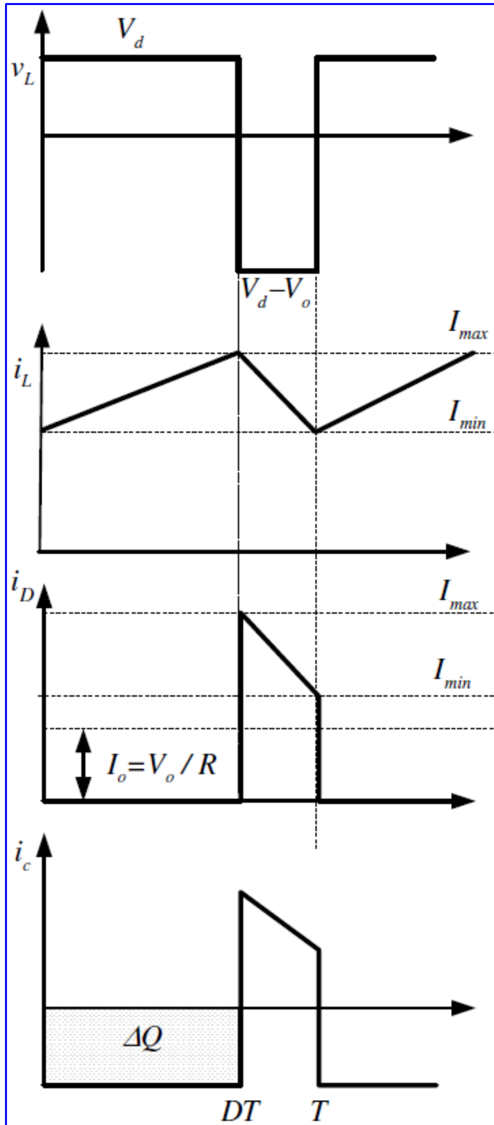
$$\Rightarrow I_L = \frac{I_D}{1-D} = \frac{V_o/R}{1-D}$$

From above, $V_s I_L = V_o I_L(1-D) + I_L^2 r_L$

$$\Rightarrow V_s = V_o(1-D) + I_L r_L$$

Also, $I_L = \frac{I_D}{1-D} = \frac{V_o/R}{1-D}$

$$\Rightarrow V_s = \frac{V_o r_L}{R(1-D)} + V_o(1-D)$$



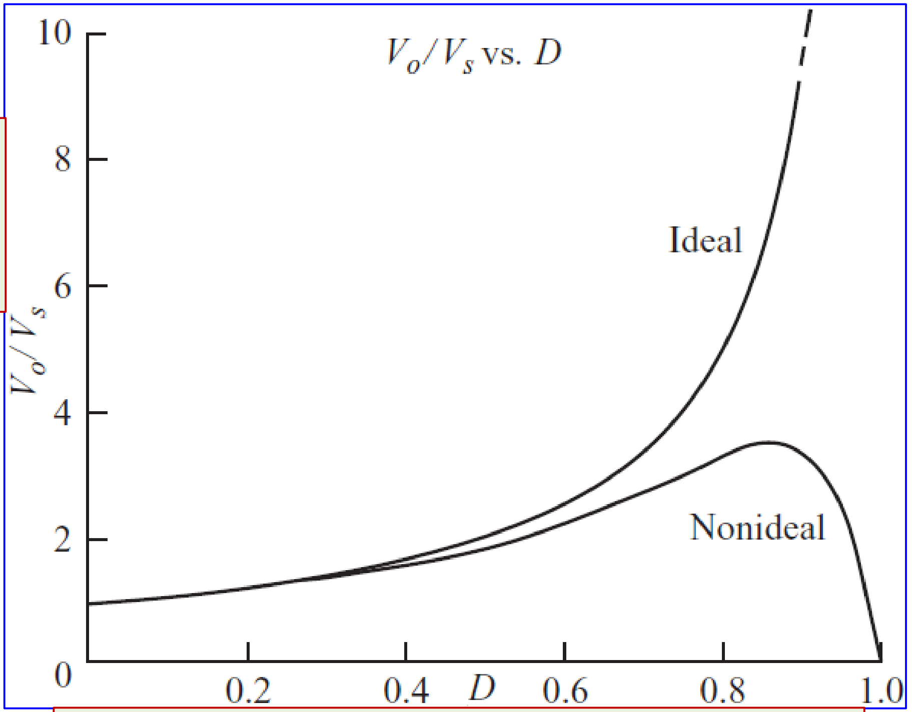
# Inductor Resistance

$$V_s = \frac{V_o r_L}{R(1-D)} + V_o(1-D)$$

Solving for $V_o$,

$$V_o = \left(\frac{V_s}{1-D}\right)\left(\frac{1}{1 + r_L/[R(1-D)^2]}\right)$$

This eqn. is similar to that for an ideal converter but includes a correction factor to account for rL.

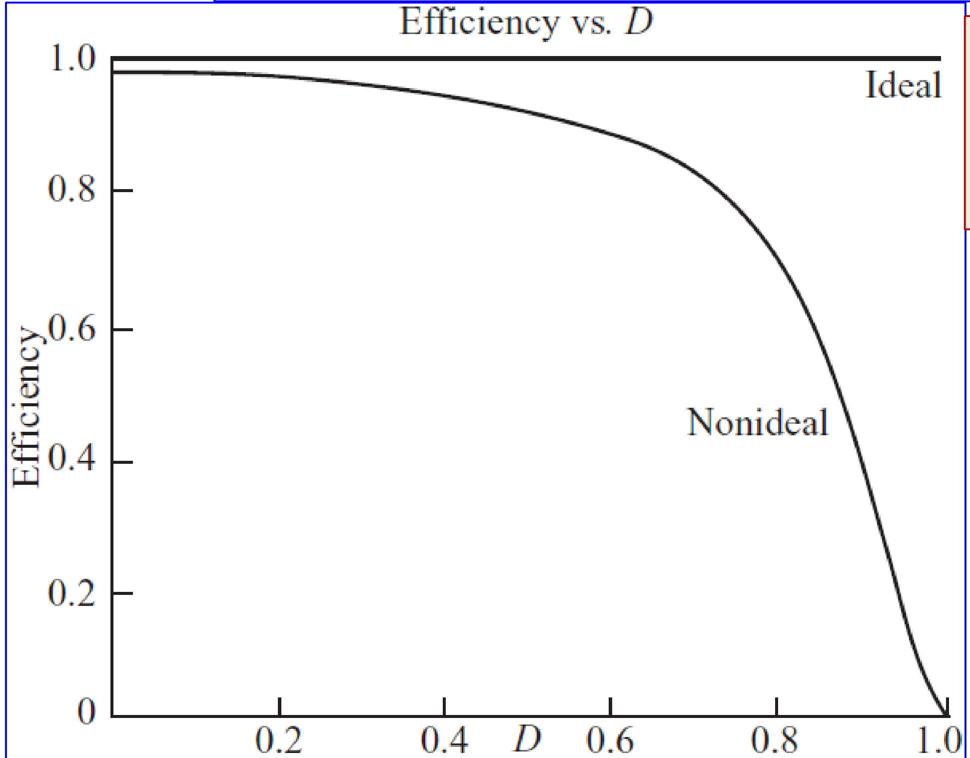



Output voltage of the boost converter with and without inductor resistance.

The inductor resistance also has an effect on the power efficiency of converters. Efficiency is the ratio of output power to output power plus losses. For the boost converter

$$\eta = \frac{P_o}{P_o + P_{loss}} = \frac{V_o^2/R}{V_o^2/R + I_L^2 r_L}$$

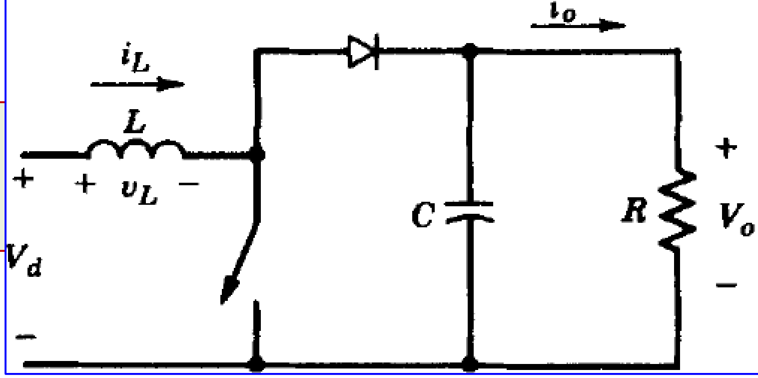
$$\eta = \frac{V_o^2/R}{V_o^2/R + (V_o/R)^2/(1-D)r_L} = \frac{1}{1 + r_L[R(1-D)^2]}$$




As D increases, the converter efficiency decreases.

- Effect is dominant at high D
- Difficult to achieve large conversion ratios ( > 10)
- No power transfer at D = 1

# OUTPUT VOLTAGE RIPPLE ($\Delta V_0$)

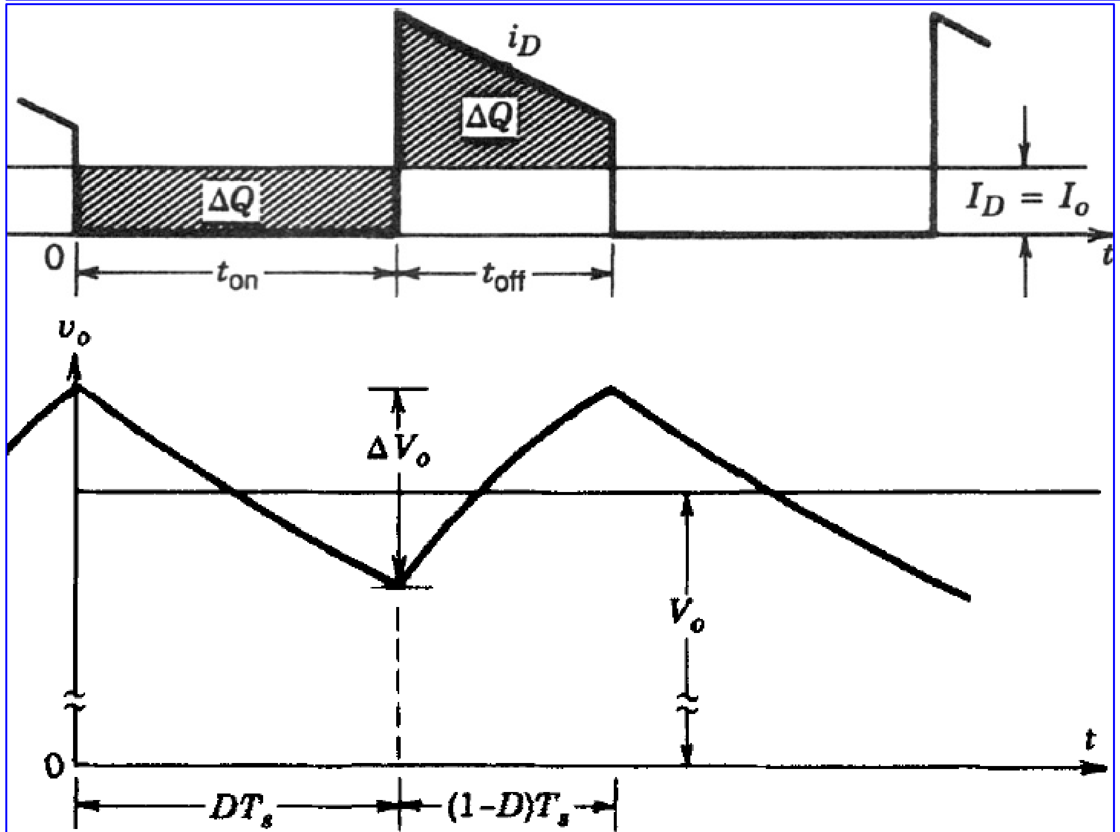


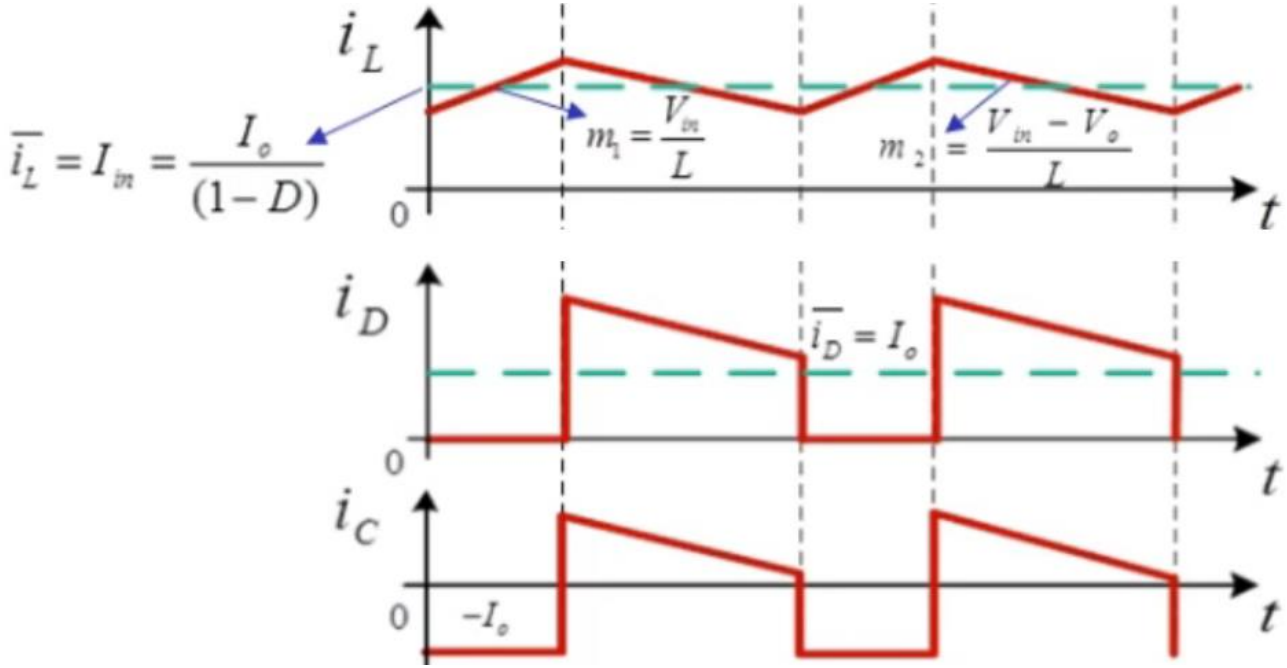

CCM: Assuming that all the ripple component $i_D$ flows through C and its average value flows through the load resistor, the shaded area represents charge ΔQ.

Current-sec. balance principle:

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{I_o D T_s}{C} = \frac{V_o}{R}\frac{D T_s}{C}$$

$$\therefore \frac{\Delta V_o}{V_o} = \frac{D T_s}{RC} = D\frac{T_s}{\tau} \quad (\text{where } \tau = RC \text{ time constant})$$

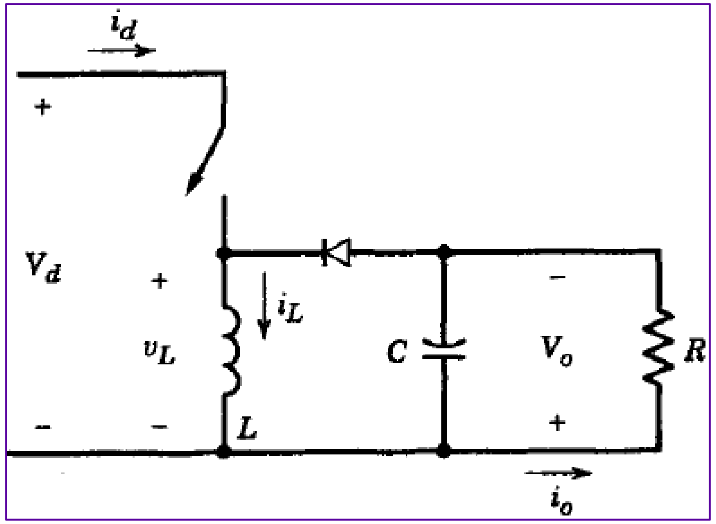



A similar analysis can be performed for the discontinuous mode of conduction.

# Buck-Boost Converter

The main application of a buck-boost converter is in regulated dc power supplies, where a **negative-polarity output** may be desired with respect to the common terminal of the input voltage, and the output voltage can be either higher or lower than the input voltage.

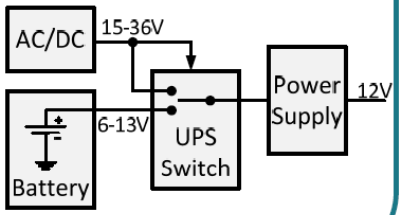



## Buck-boost applications

### Industrial PCs

**Application needs**

- 6 V-36 $V_{IN}$ from AC-powered supply or battery
- 12 V output, 60 W-200 W




### Automotive start/stop & DVRs


**Application needs**

- 9 V-16 $V_{IN}$, 3.5 V during start
- ~12 V output, 60 W-120 W

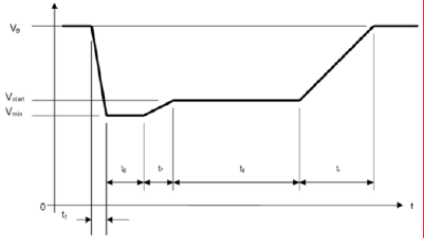


FIGURE 1: ENGINE CRANKING TEST PULSE

### USB power delivery


**Application needs**

- 12 V bus or battery, 9V–16 $V_{IN}$
- 5/12/20 $V_{OUT}$, 10 W–100 W

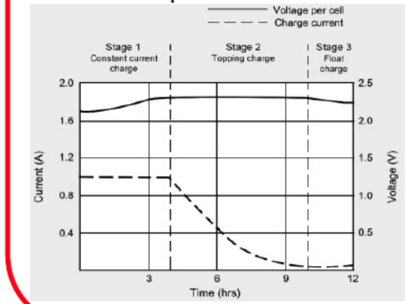
USB Power Delivery profiles

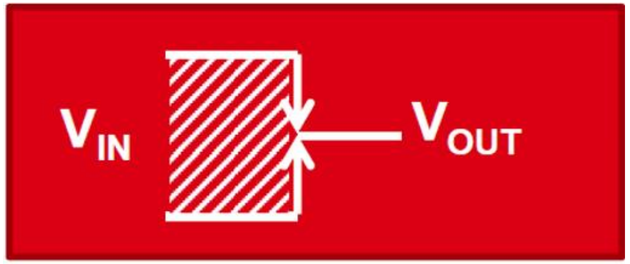
| Profile | +5 V | +12 V | +20 V |
|---|---|---|---|
| 1 | 2.0 A, 10 W | N/A | N/A |
| 2 | | 1.5 A, 18 W | N/A |
| 3 | | 3.0 A, 36 W | N/A |
| 4 | | | 3.0 A, 60 W |
| 5 | | 5.0 A, 60 W | 5.0 A, 100 W |

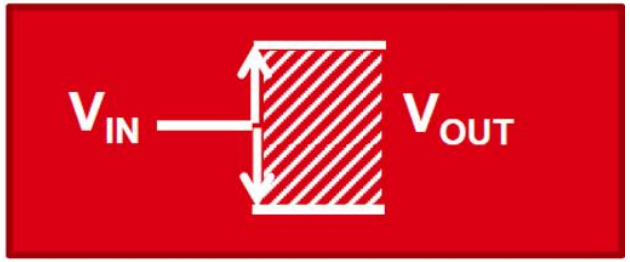
### Industrial & battery chargers

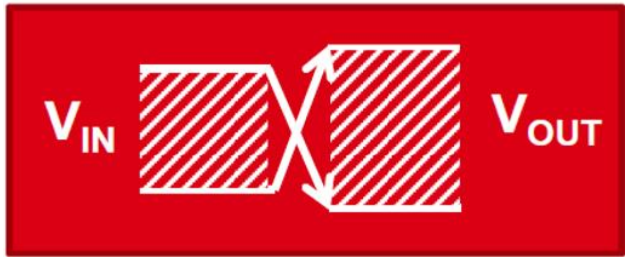
**Application needs**

- 12 V or 24 $V_{IN}$ or DC adapter
- CC/CV up to 200 W+



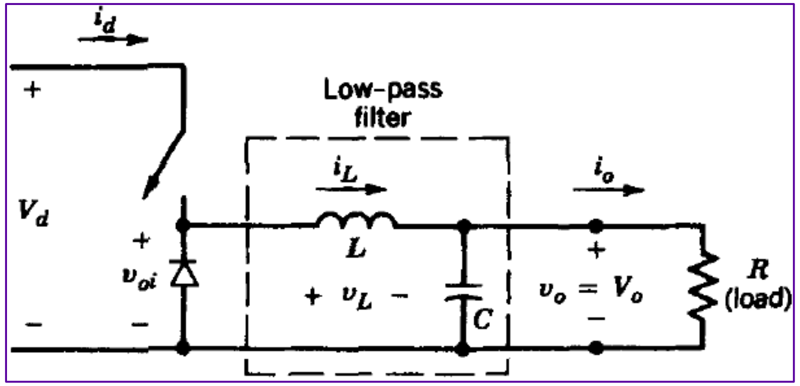



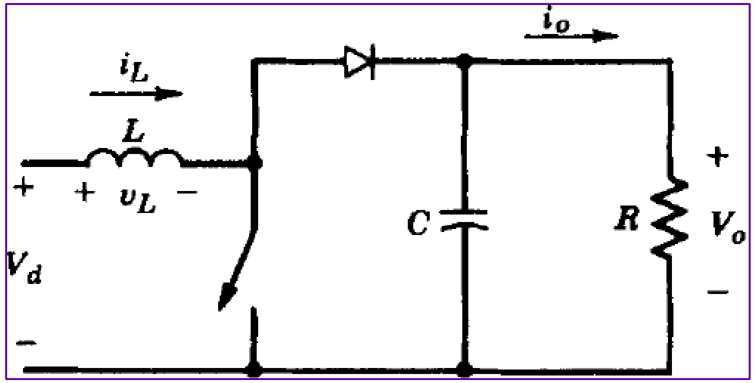
# BUCK-BOOST CONVERTER

A buck-boost converter can be obtained by the cascade connection of the two basic converters: the stepdown converter and the step-up converter.

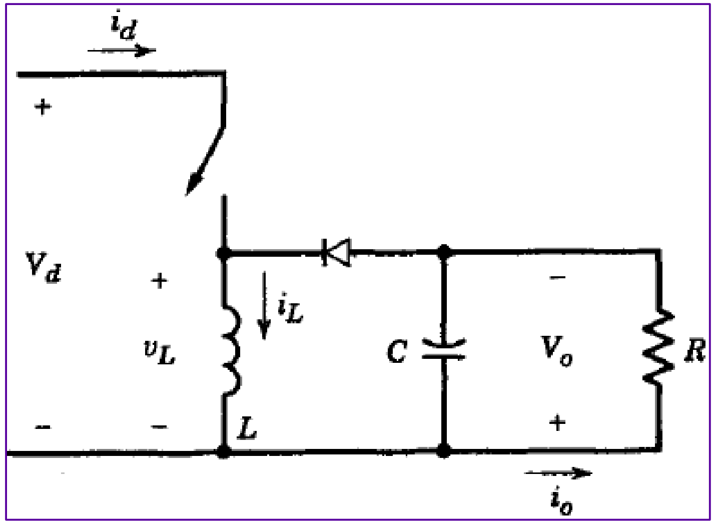
**Buck**




**Boost**




**Buck-Boost**

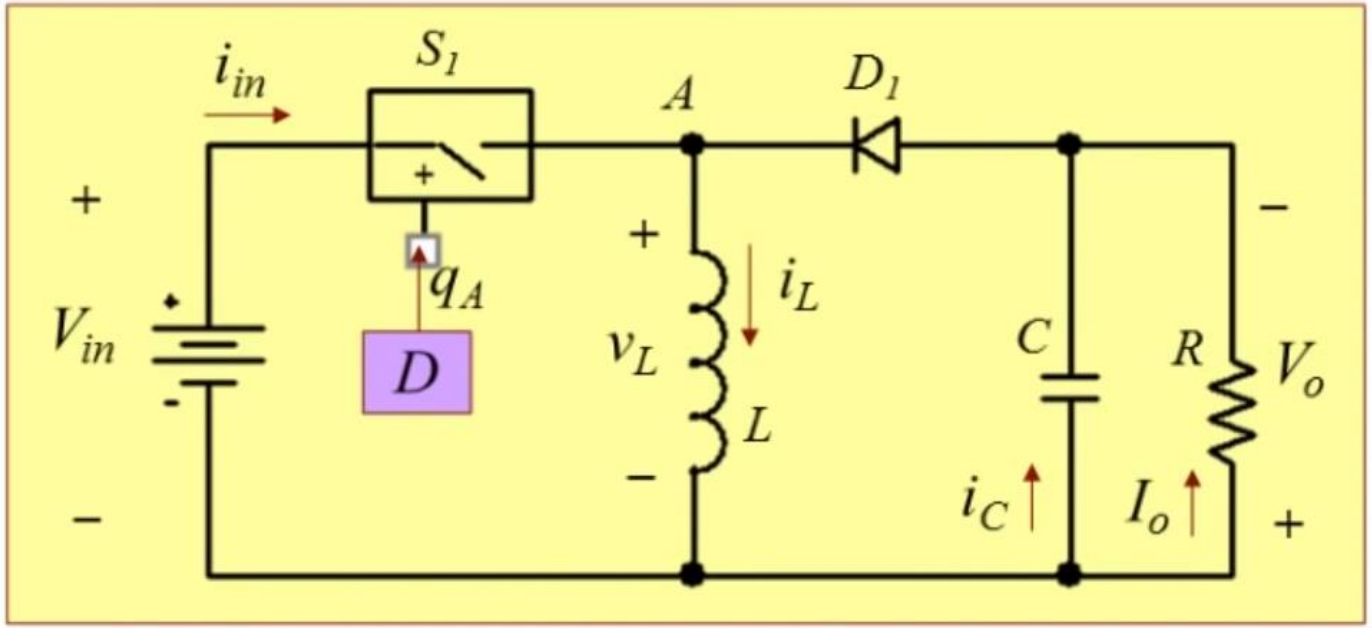


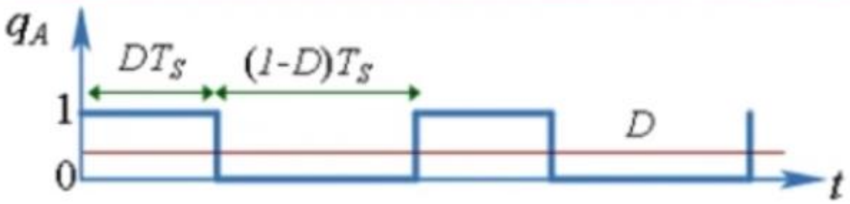

In steady state, the output-to-input voltage conversion ratio is the product of the conversion ratios of the two converters in cascade (assuming that switches in both converters have the same D):

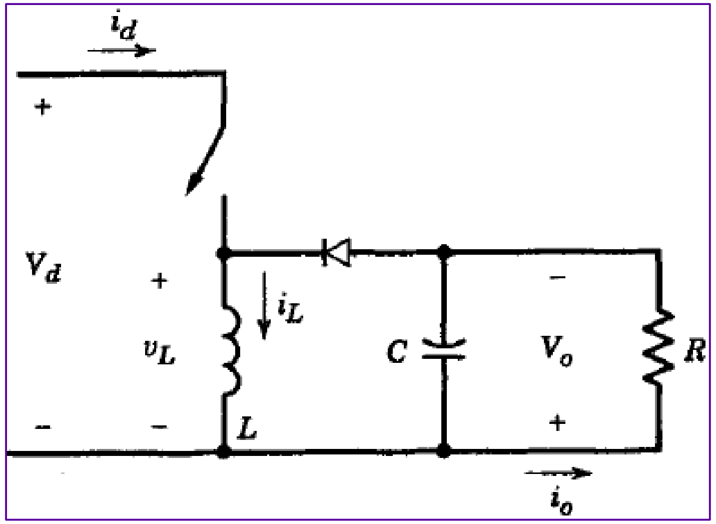
This allows the output voltage to be higher or lower than the input voltage, based on the duty ratio *D*.

$$\frac{V_o}{V_d} = D\,\frac{1}{1 - D}$$

# BUCK-BOOST CONVERTER



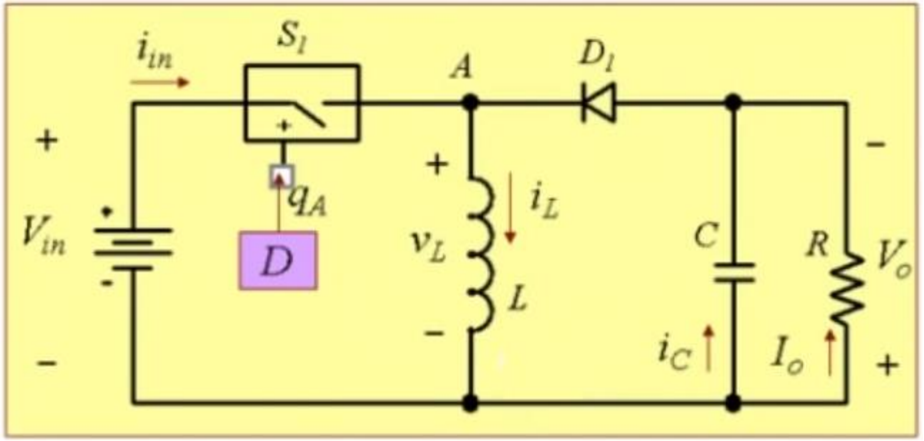



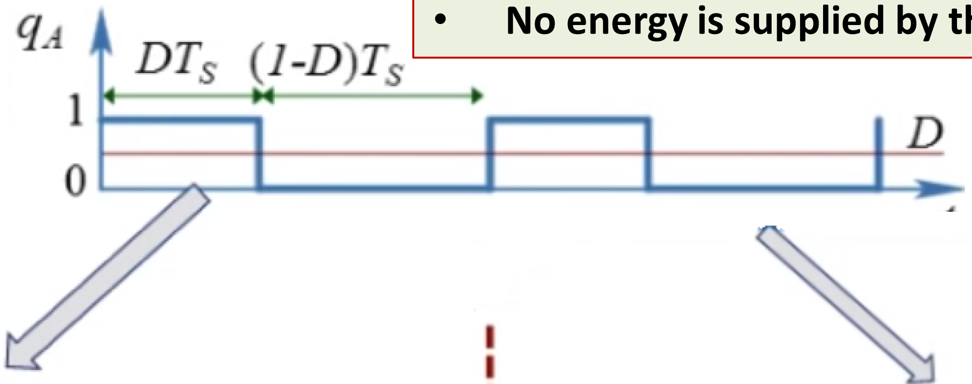



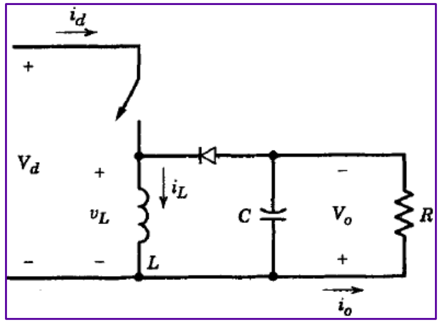

- Step up as well as step down depending on D
- Negative output (with respect to input ground)
- Isolated version – flyback converter quite popular at low (~100 W) power level

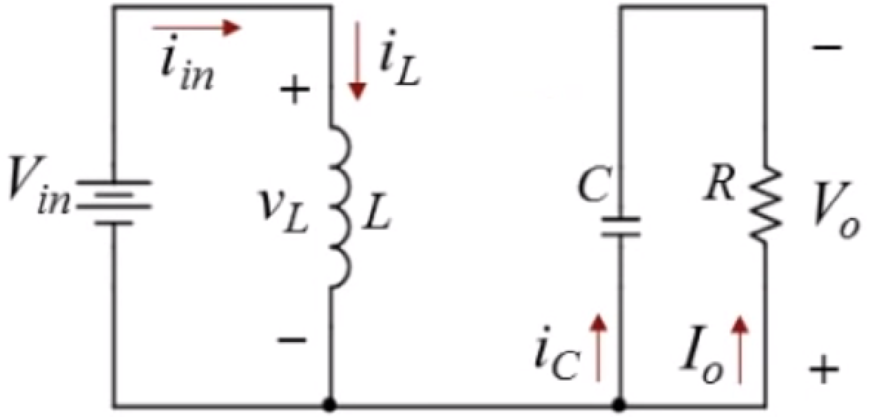
# BUCK-BOOST CONVERTER

- When the switch is closed, the input provides energy to the inductor and the diode is reverse biased.
- When the switch is open, the energy stored in the inductor is transferred to the output.
- No energy is supplied by the input during this interval.

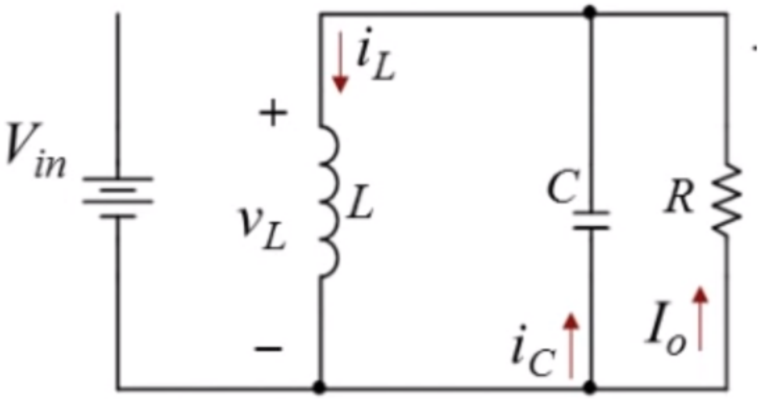


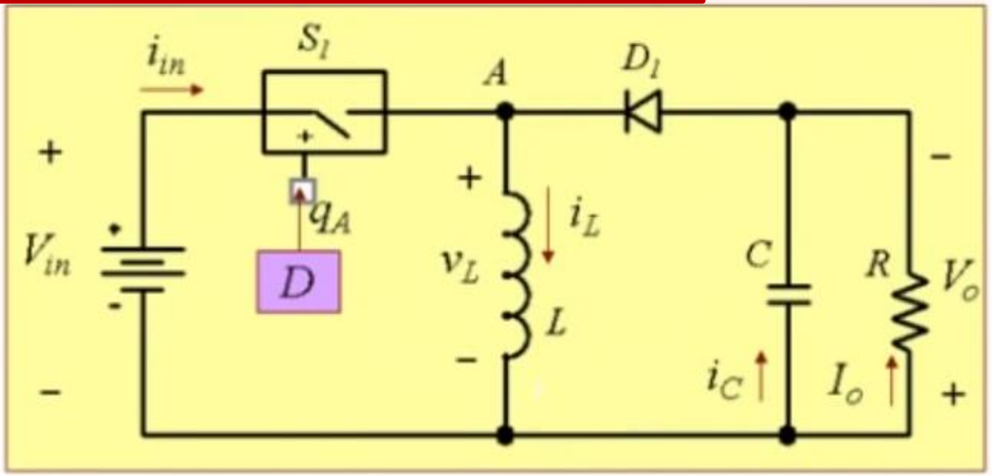




- $v_L = V_{in}$
- $i_L$ and energy stored in L increase
- C supports load and discharges
- C large enough to maintain voltage almost constant (small ripple)




- $v_L = -V_o$
- $i_L$ and energy stored in L decrease, energy fed to C and R
- $i_C$ positive and C charges up
- C large enough to maintain voltage almost constant (small ripple)

# BUCK-BOOST CONVERTER

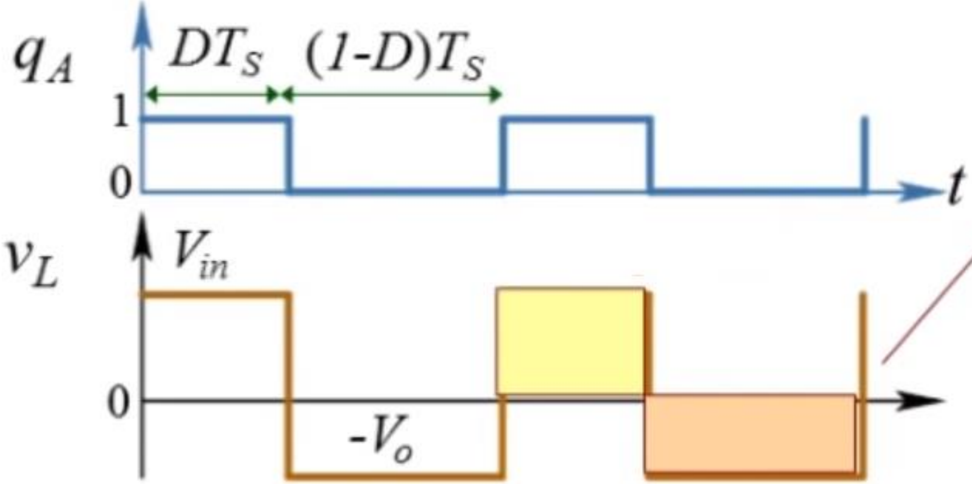



Buck operation for

$$D \le 0.5$$

Boost operation for

$$D \ge 0.5$$




$$V_{in} D T_s + (-V_o)(1-D) T_s = 0$$

$$V_{in} D = V_o (1-D)$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{D}{1-D}}$$

Input-output voltage relationship for buck-boost

$$\boxed{\frac{I_{in}}{I_o} = \frac{D}{1-D}}$$

Input-output current relationship using power balance

# Buck-boost analysis

## CONTINUOUS-CONDUCTION MODE

## The ΔI method

Switch closed

$$v_L = Vd = L\frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_d}{L}$$

$$\Rightarrow (\Delta i_L)_{closed} = \frac{V_d DT}{L}$$

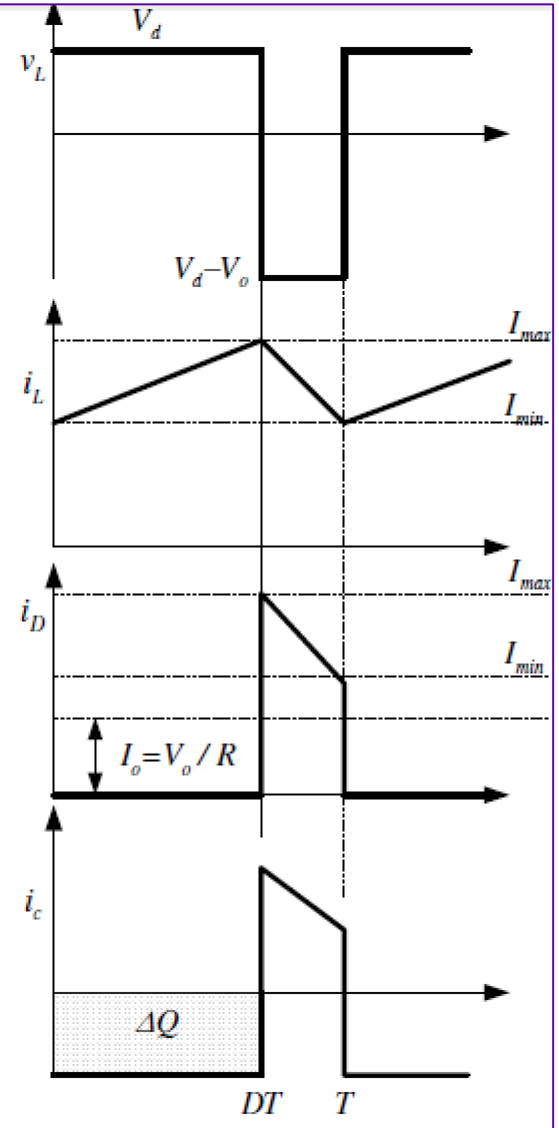
Switch opened

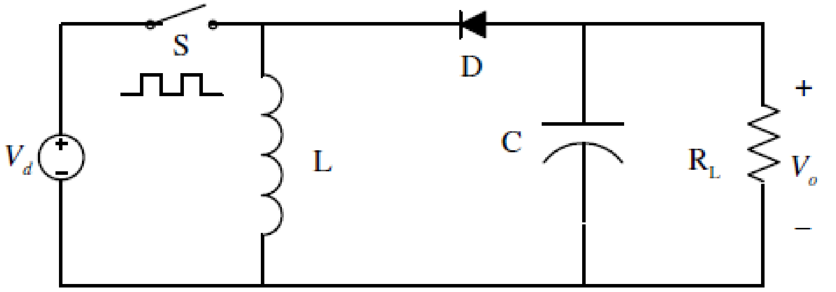
$$v_L = V_o = L\frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_o}{L}$$

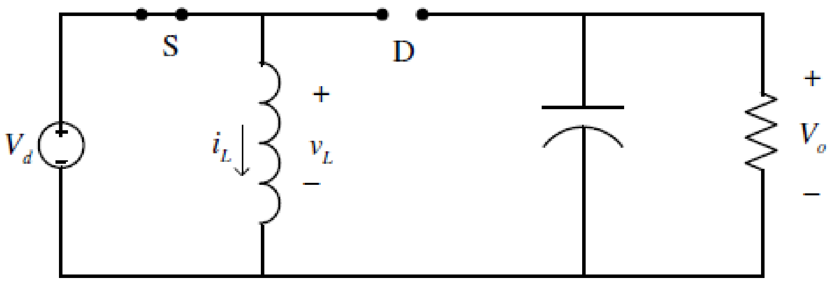
$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_o}{L}$$

$$\Rightarrow (\Delta i_L)_{opened} = \frac{V_o(1-D)T}{L}$$

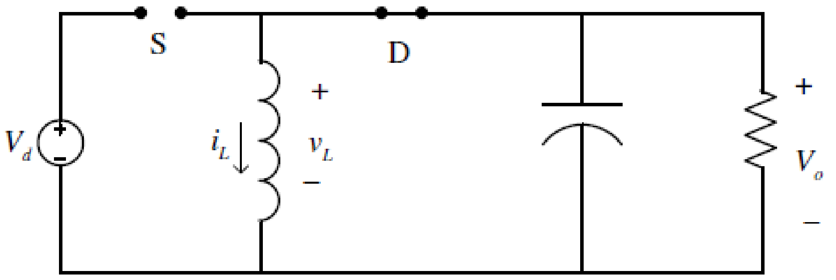



CIRCUIT OF BUCK-BOOST CONVERTER




CIRCUIT WHEN SWITCH IS CLOSED




CIRCUIT WHEN SWITCH IS OPENED

# Buck-boost analysis | CONTINUOUS-CONDUCTION MODE

**The ΔI method**

Steady state operation:

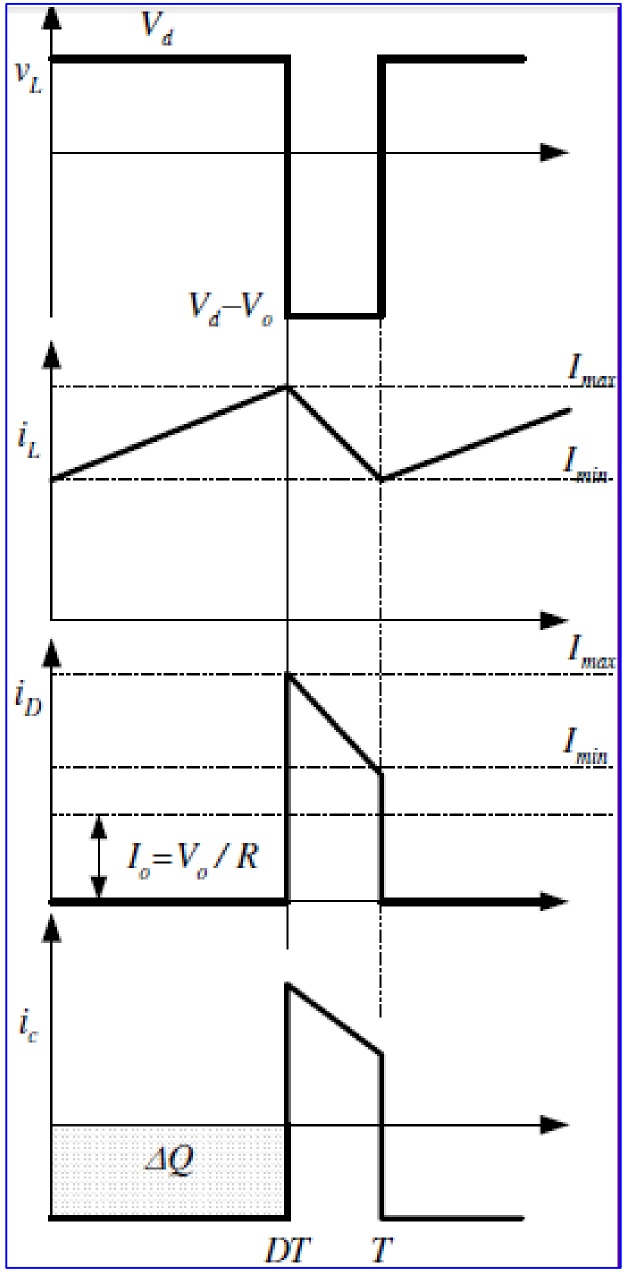
$$\Delta_{iL(closed)} + \Delta_{iL(opened)} = 0$$

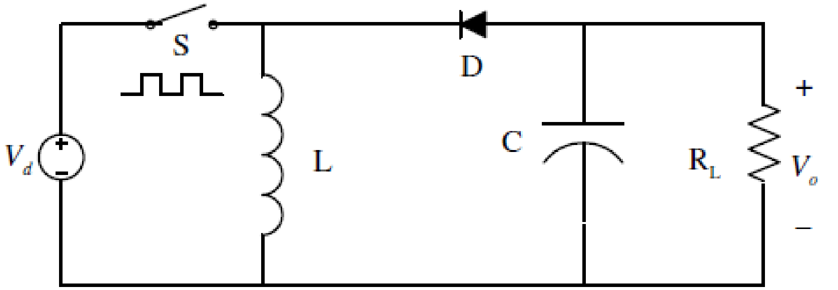
$$\Rightarrow \frac{V_d DT}{L} + \frac{V_o(1-D)T}{L} = 0$$

Output voltage:

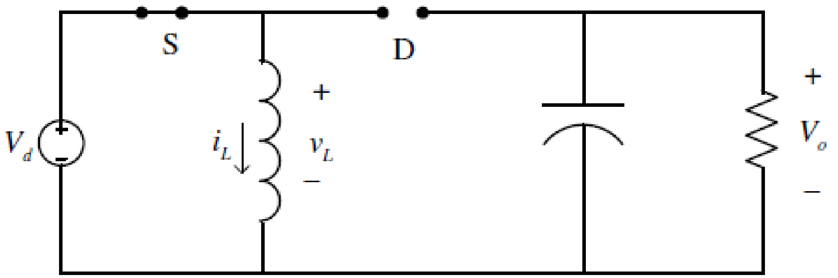
$$\Rightarrow V_o = -V_s\left(\frac{D}{1-D}\right)$$

- NOTE: Output of a buck-boost converter either be higher or lower than input.
  - If D>0.5, output is higher than input
  - If D<0.5, output is lower input
- Output voltage is always negative.
- Note that output is never directly connected to load.
- Energy is stored in inductor when switch is closed and transferred to load when switch is opened.

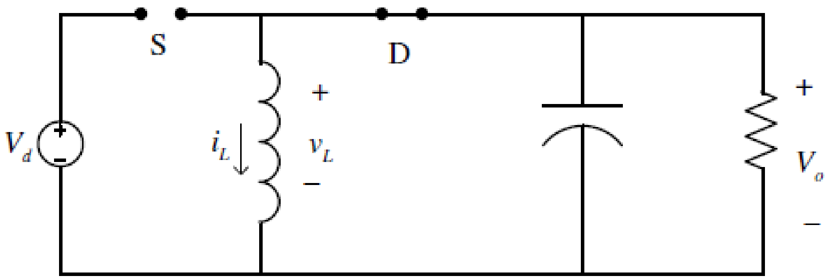


CIRCUIT OF BUCK-BOOST CONVERTER

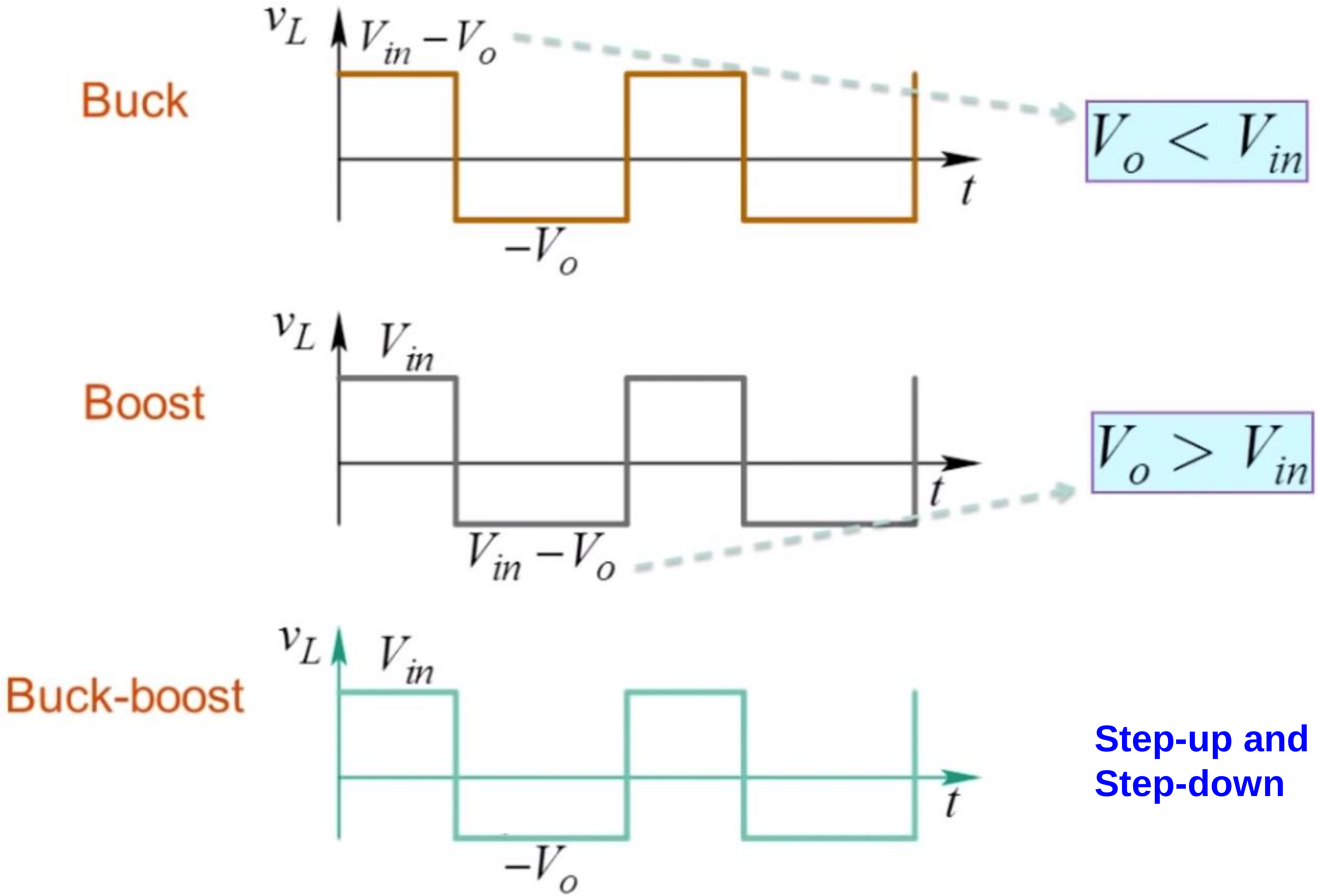


CIRCUIT WHEN SWITCH IS CLOSED



CIRCUIT WHEN SWITCH IS OPENED

# Comparison of $v_L$ in Buck, Boost and Buck-Boost




**Step-up and Step-down**