EE 238

Power Engineering - II

Power Electronics

Lecture 11

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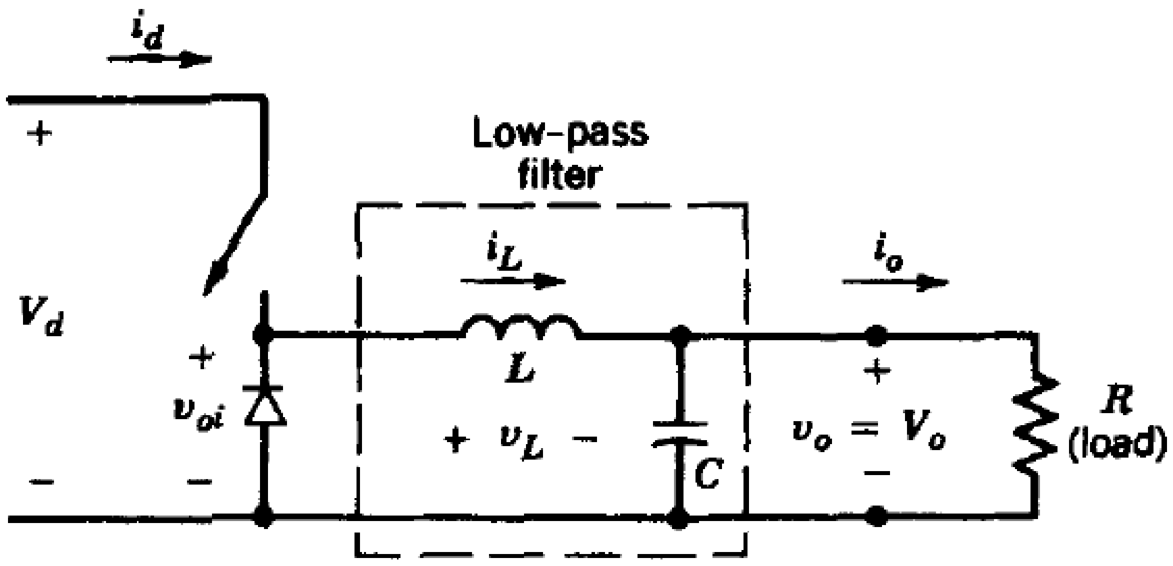
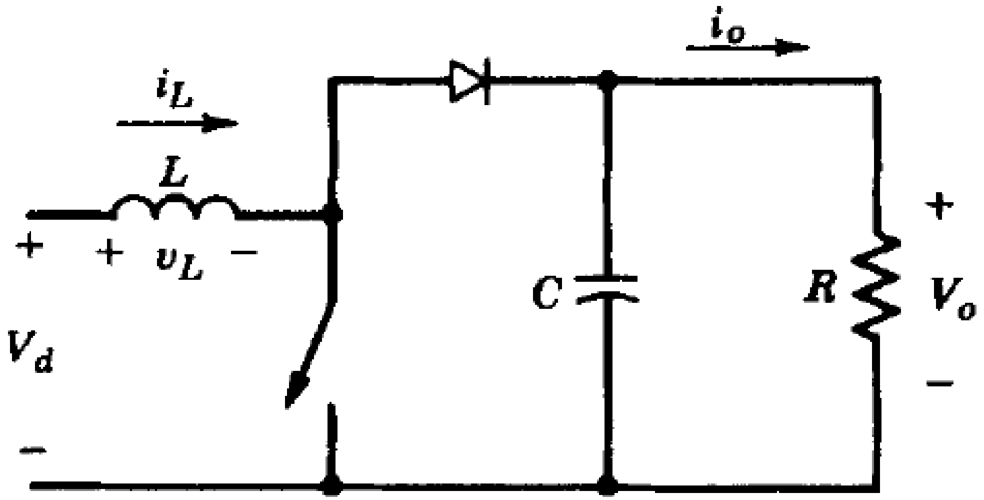


i_d
+
V_d
−
+
v_{oi}
−
Low-pass
filter
i_L
L
+ v_L −
C
i_o
+
$v_o = V_o$
−
R
(load)

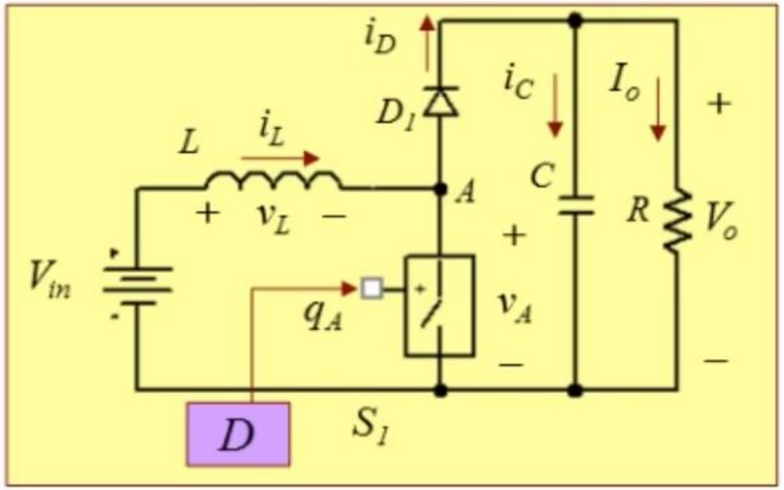
Buck

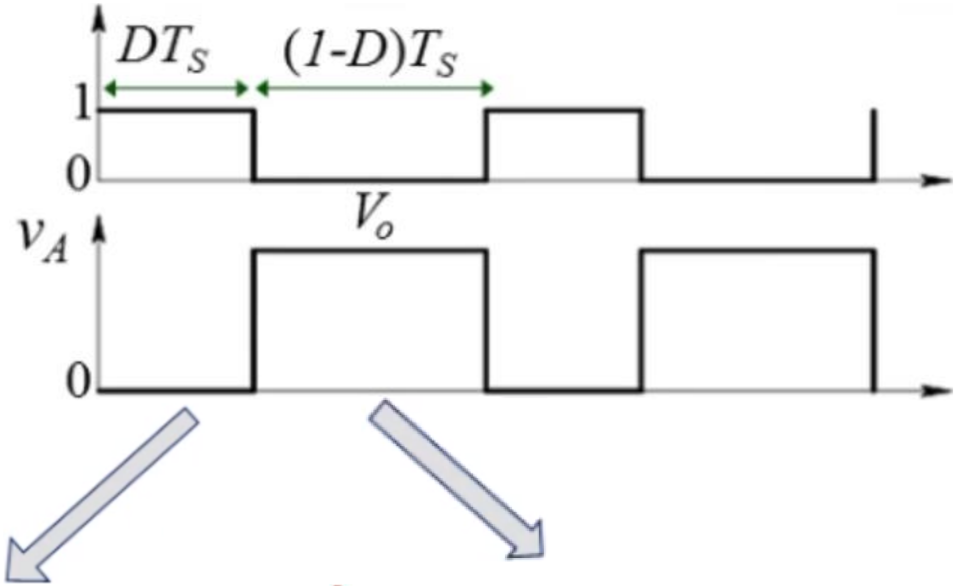


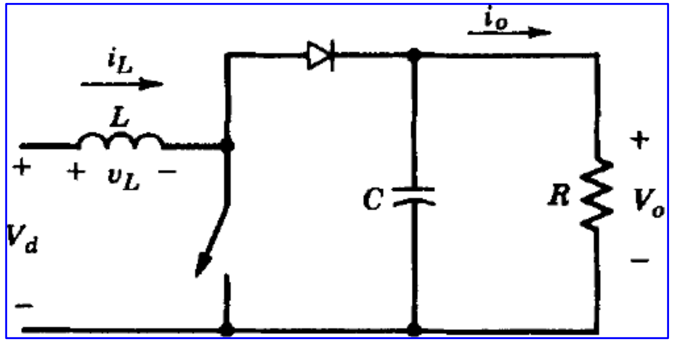
i_L
L
+
+ v_L −
V_d
−
i_o
C
R
+
V_o
−

Boost

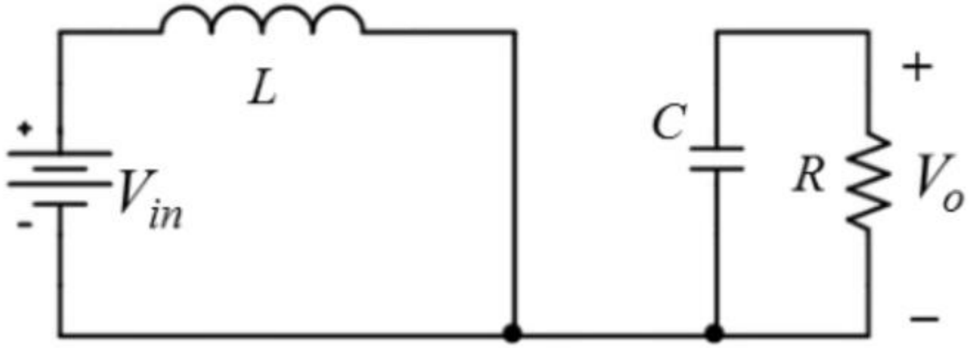
STEP-UP (BOOST) CONVERTER



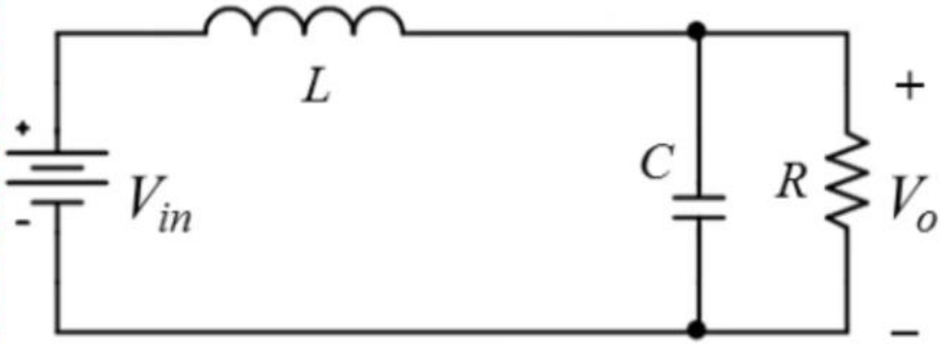




DC Steady-state



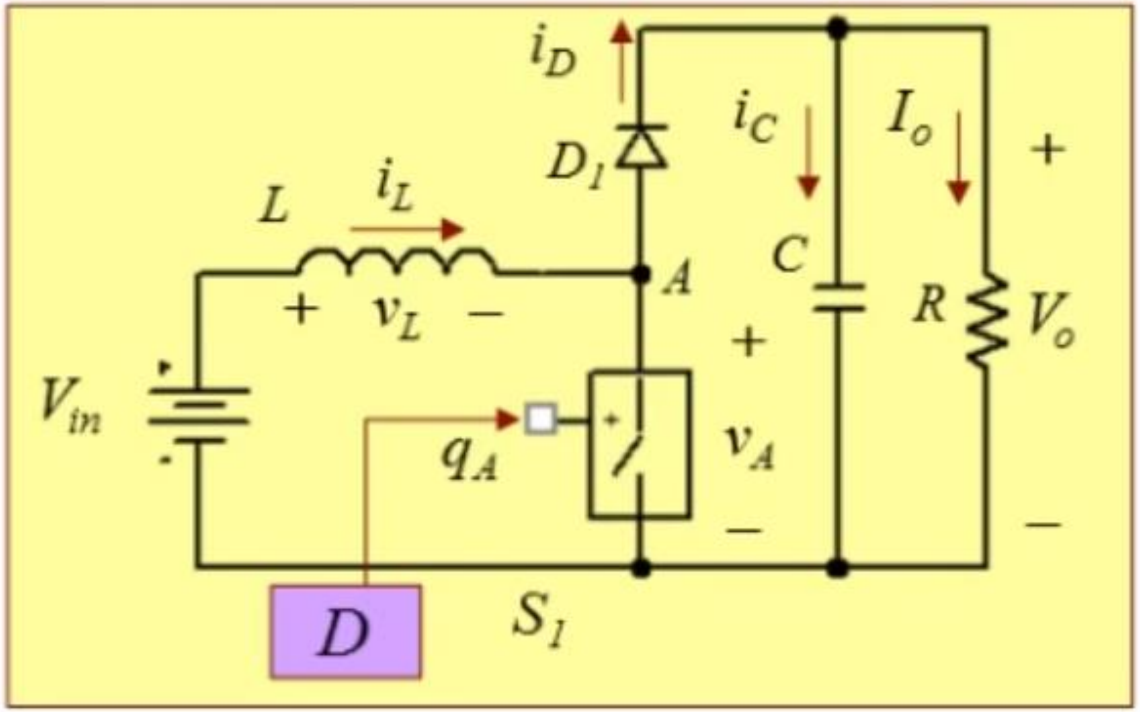
- $v_L = V_{in}$
- i_L and energy stored in L increase
- C supports load and discharges
- C large enough to maintain voltage almost constant (small ripple)



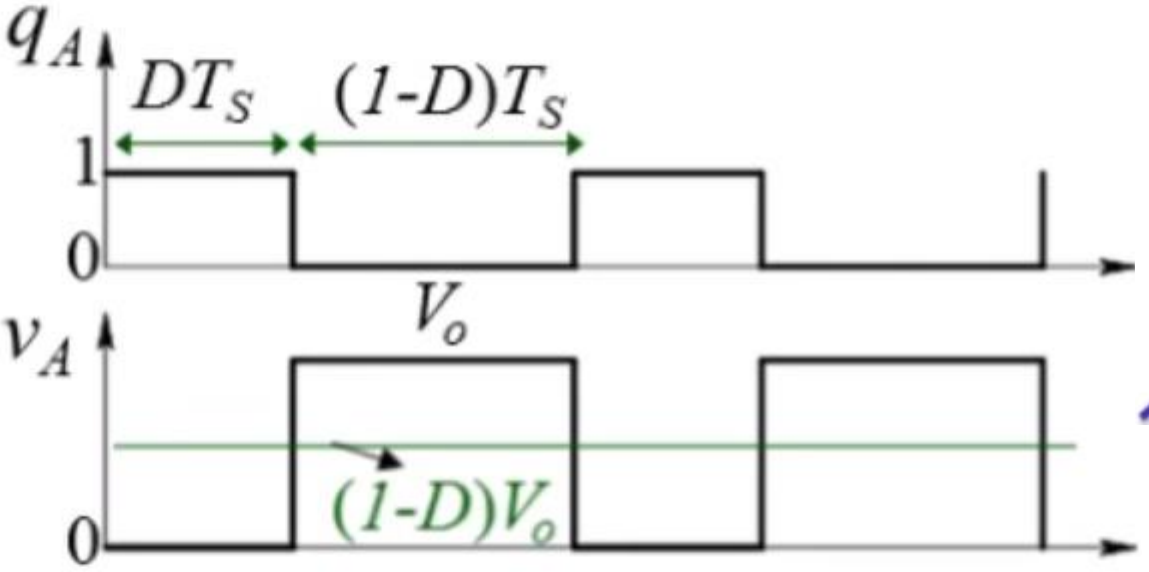
- $v_L = V_{in} - V_o$
- i_L and energy stored in L decrease, energy fed to C and R
- i_C positive and C charges up
- C large enough to maintain voltage almost constant (small ripple)

STEP-UP (BOOST) CONVERTER

Input-Output Voltage Relationship: SS operation



$$\bar{v}_A = V_{in} \; (\text{since } \bar{v}_L = 0)$$



$$\bar{v}_A = (1-D)V_o \; (\text{from waveform})$$

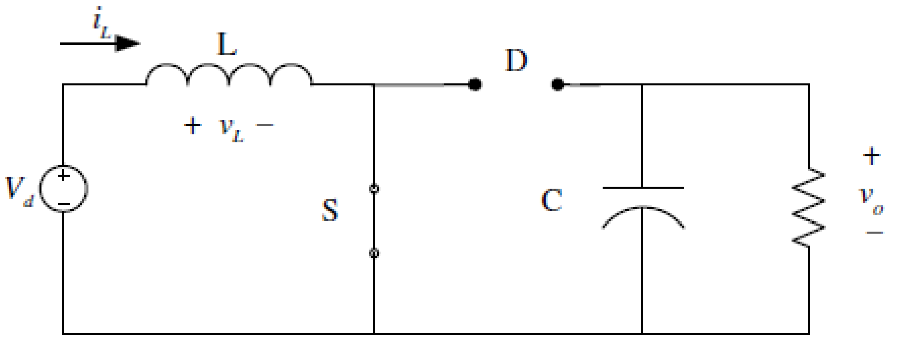
$$\boxed{\frac{V_o}{V_{in}} = \frac{1}{1-D}}$$

Input-output relationship for boost converter

STEP-UP (BOOST) CONVERTER

Input-Output Voltage Relationship: SS operation

The ΔI method



$$v_L = V_d$$

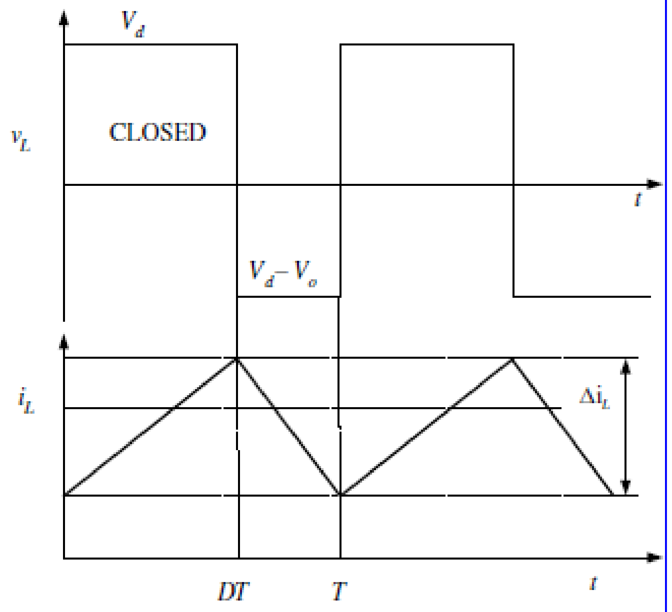
$$= L\frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$

$$(\Delta i_L)_{closed} = \frac{V_d DT}{L}$$

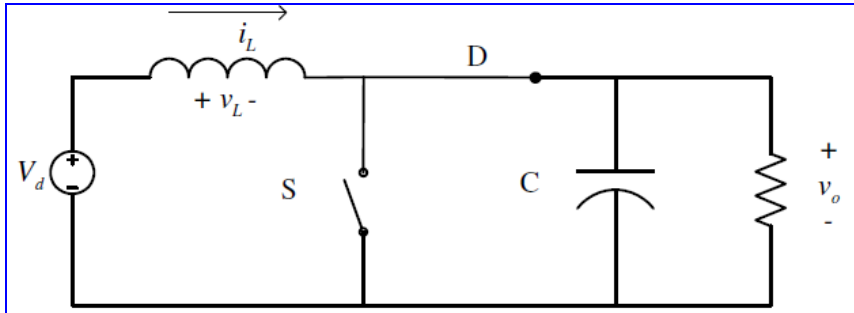


Boost converter produces output voltage that is greater or equal to the input voltage.

$$(\Delta i_L)_{closed} + (\Delta i_L)_{opened} = 0$$

$$\frac{V_d DT}{L} + \frac{(V_d - V_o)(1-D)T}{L} = 0$$

$$\Rightarrow V_o = \frac{V_d}{1-D}$$



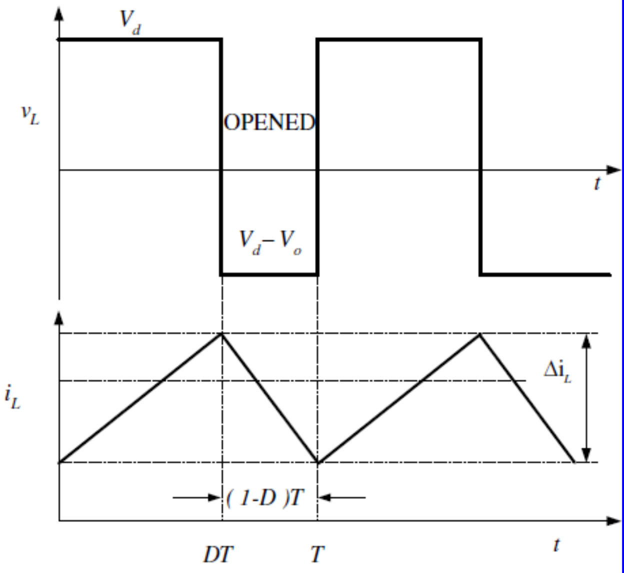
$$v_L = V_d - V_o$$

$$= L\frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t}$$

$$= \frac{\Delta i_L}{(1-D)T}$$

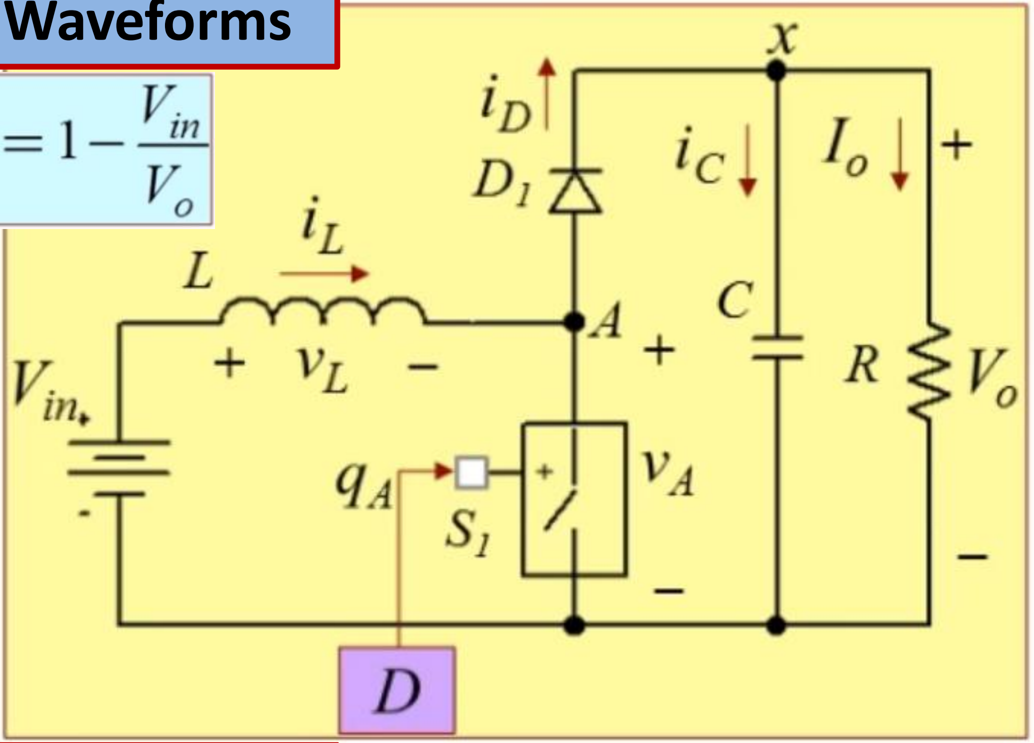


$$\Rightarrow \frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

$$\Rightarrow (\Delta i_L)_{opened} = \frac{(V_d - V_o)(1-DT)}{L}$$

Boost Converter Waveforms

$$\frac{V_o}{V_{in}} = \frac{1}{1-D} \Rightarrow D = 1 - \frac{V_{in}}{V_o}$$



Volt-sec balance for L

$$V_{in} DT_s + (V_{in} - V_o)(1-D)T_s = 0$$

$$V_{in}(D+1-D) = V_o(1-D)$$

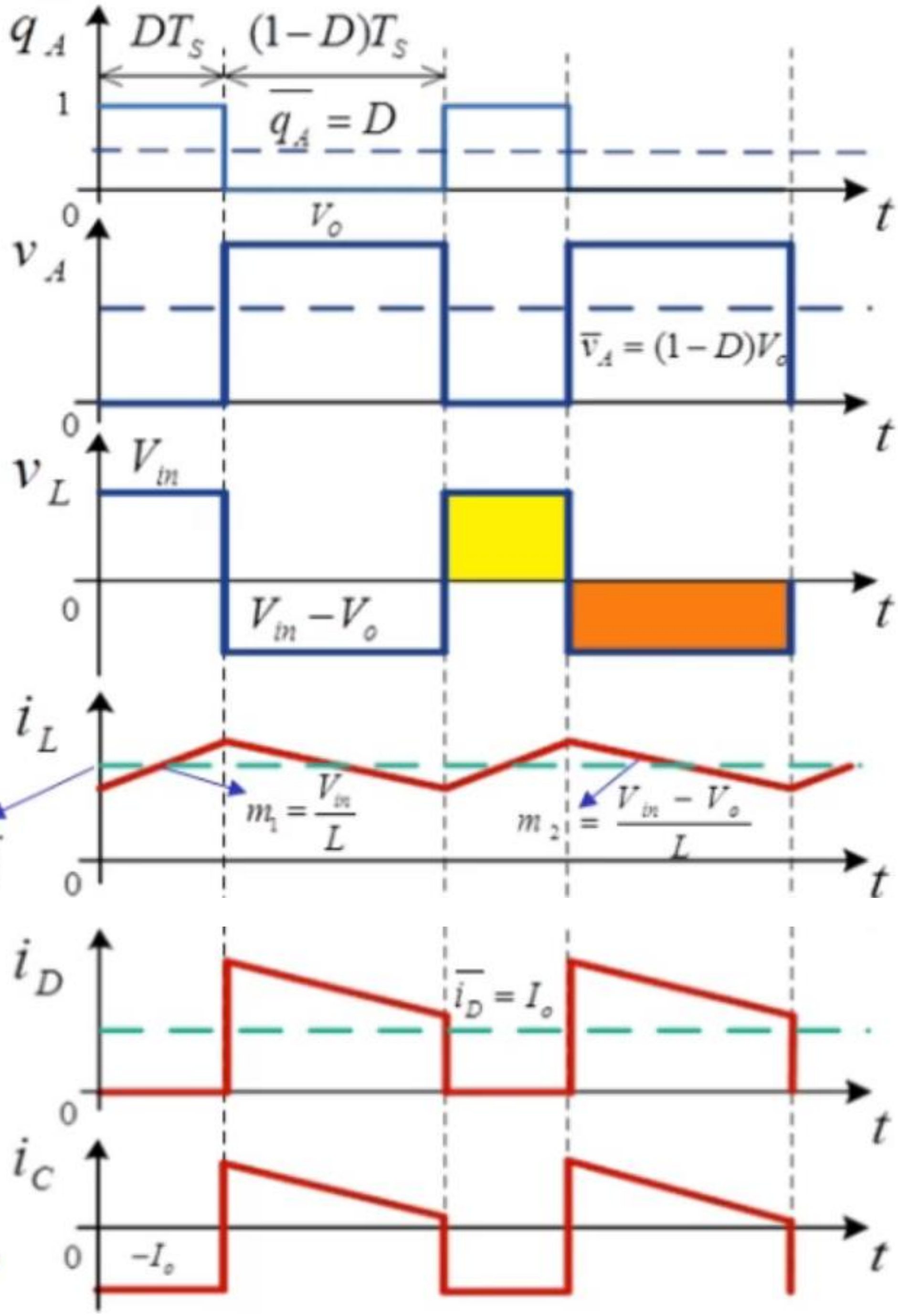
$$\frac{V_o}{V_{in}} = \frac{1}{1-D}$$

By power balance:

$$V_{in} I_{in} = V_o I_o$$

$$I_{in} = \frac{V_o}{V_{in}} I_o = \frac{I_o}{1-D}$$

$$\bar{i}_L = I_{in} = \frac{I_o}{(1-D)}$$



For constant output voltage and variable input voltage applications

$$\left(1 - \frac{V_{in,max}}{V_o}\right) \le D \le \left(1 - \frac{V_{in,min}}{V_o}\right)$$

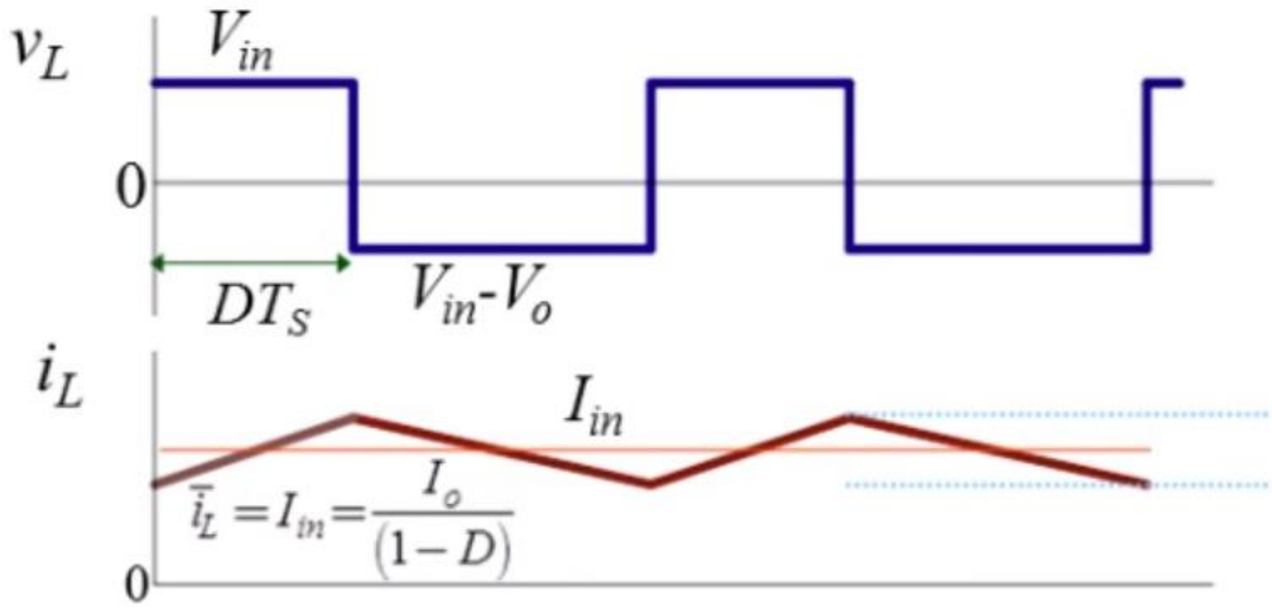
Average KCL at node x

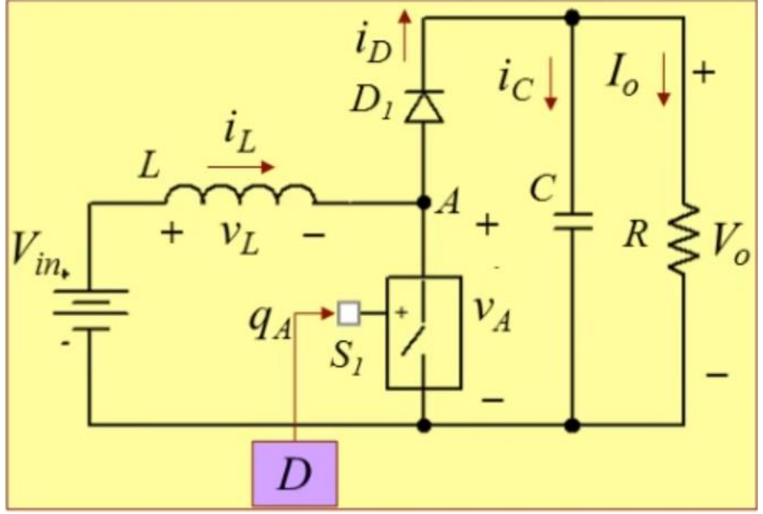
$$\bar{i}_D = \bar{i}_C + I_o = I_o$$

KCL at node x instantaneously

$$i_C = i_D - I_o$$

Selection of L





ΔI_L Peak-peak ripple in inductor current

- L selected to limit peak-peak inductor current ripple to a chosen value
  - For example, 10-20% of max. I_{in}
  - Specifications on input current ripple
  - CCM considerations
- Choice of L does not significantly affect capacitor selection

Consider the T_{ON} interval

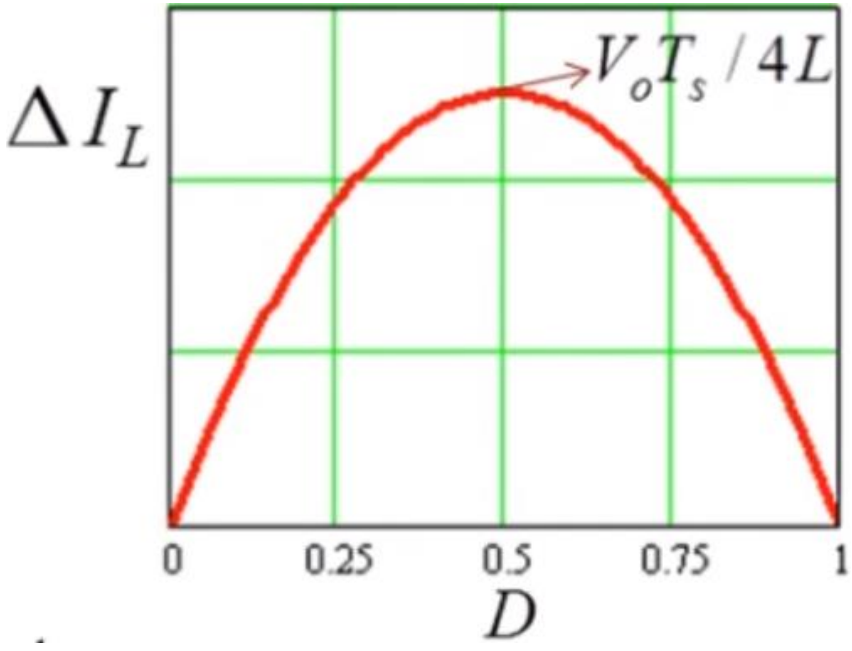
$$L\frac{\Delta I_L}{DT_S} = V_{in} = V_o(1-D)$$

$$\boxed{L = \frac{V_o D(1-D)T_S}{\Delta I_L}}$$

Selection of L

- Design for **worst case condition**

$$\boxed{L = \frac{V_o D(1-D)T_S}{\Delta I_L}}$$

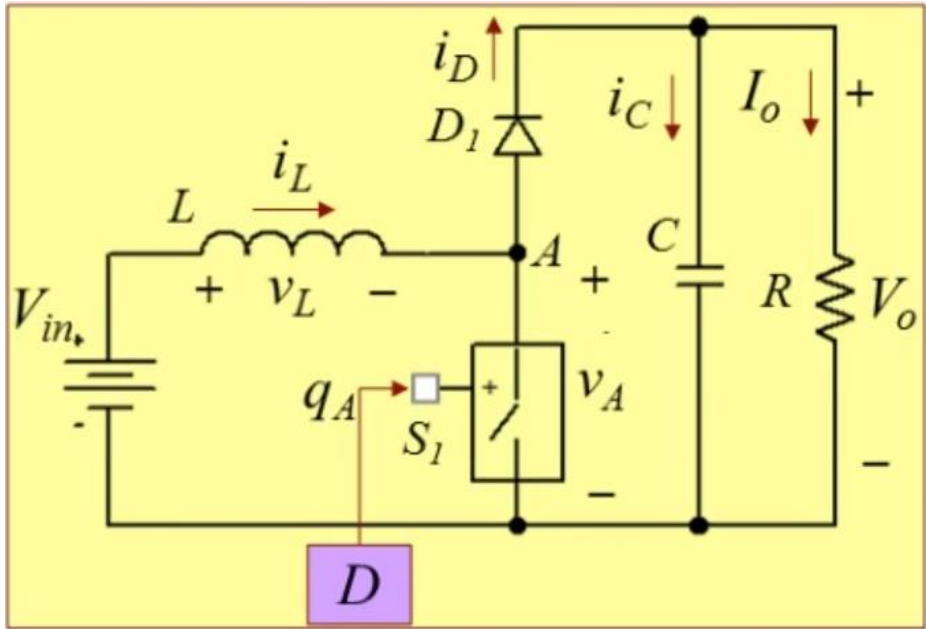


Worst case condition for constant output voltage applications

$$\frac{dL(D)}{dD} = \frac{V_o T_S}{\Delta I_L}(1-2D) = 0$$

$$\Rightarrow D = 0.5$$ (or closest to 0.5 in the operating range of D)

- L selected to limit peak-peak inductor current ripple to a chosen value
  - For example, 10-20% of max. I_{in}
  - Specifications on input current ripple
  - CCM considerations



BOOST CONVERTER ANALYSIS

Average, Maximum, Minimum Inductor Current

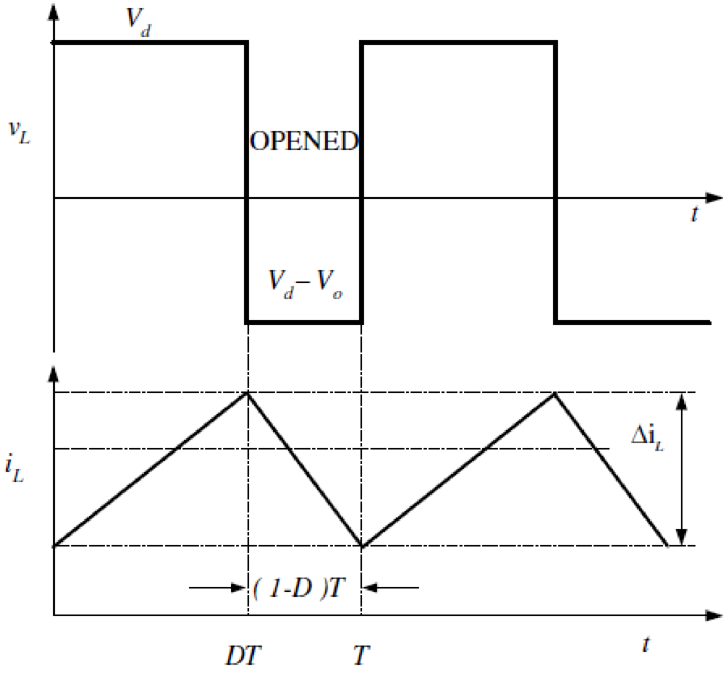
Input power = Output power

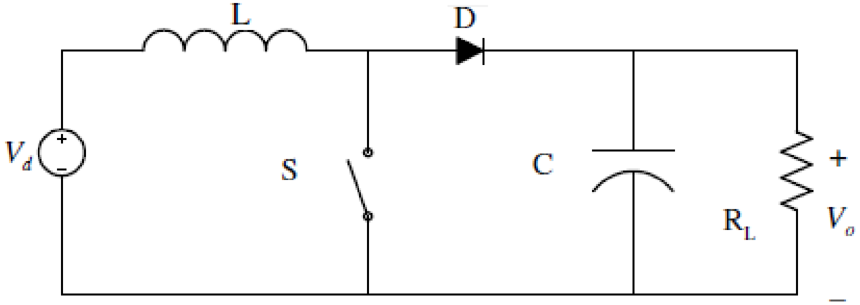
$$V_d I_d = \frac{{V_o}^2}{R}$$

$$V_d I_L = \frac{\left(\frac{V_d}{(1-D)}\right)^2}{R} = \frac{{V_d}^2}{(1-D)^2 R}$$

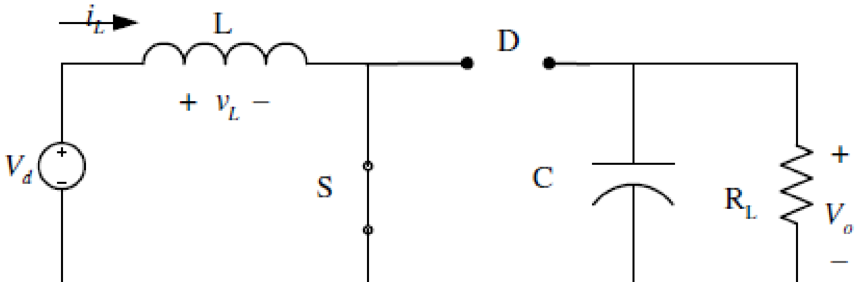
Average inductor current :

$$\Rightarrow I_L = \frac{V_d}{(1-D)^2 R}$$

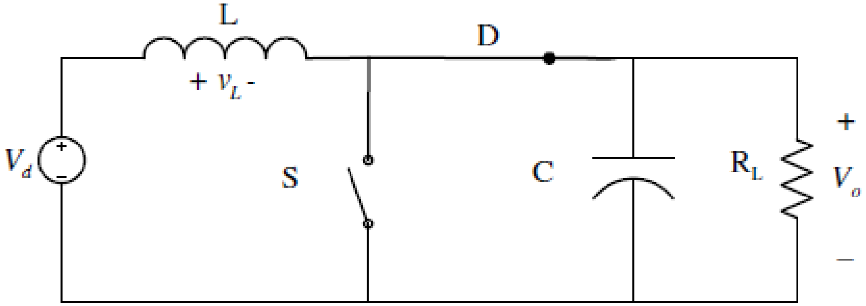




CIRCUIT OF BOOST CONVERTER



CIRCUIT WHEN SWITCH IS CLOSED



CIRCUIT WHEN SWITCH IS OPENED

$$(\Delta i_L)_{closed} = \frac{V_d DT}{L}$$

Maximum inductor current :

$$\Rightarrow I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} + \frac{V_d DT}{2L}$$

$$(\Delta i_L)_{opened} = \frac{(V_d - V_o)(1-DT)}{L}$$

Minimum inductor current :

$$\Rightarrow I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} - \frac{V_d DT}{2L}$$

BOOST CONVERTER ANALYSIS

Maximum inductor current :

$$\Rightarrow I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} + \frac{V_d DT}{2L}$$

Minimum inductor current :

$$\Rightarrow I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} - \frac{V_d DT}{2L}$$

For CCM,

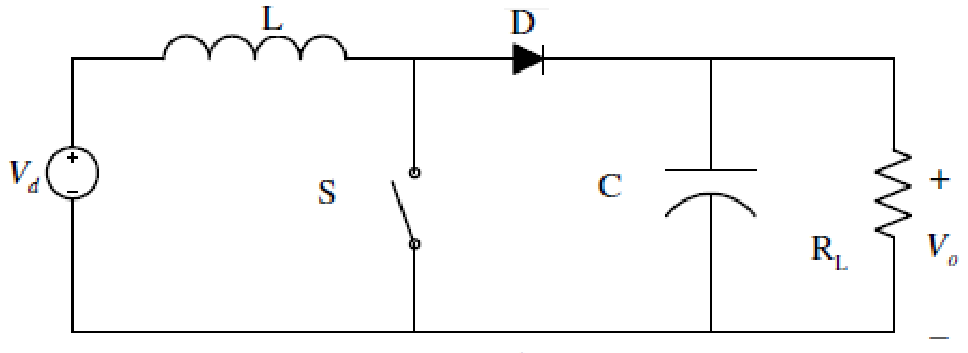
$$I_{\min} \geq 0$$

$$\frac{V_d}{(1-D)^2 R} - \frac{V_d DT}{2L} \geq 0$$

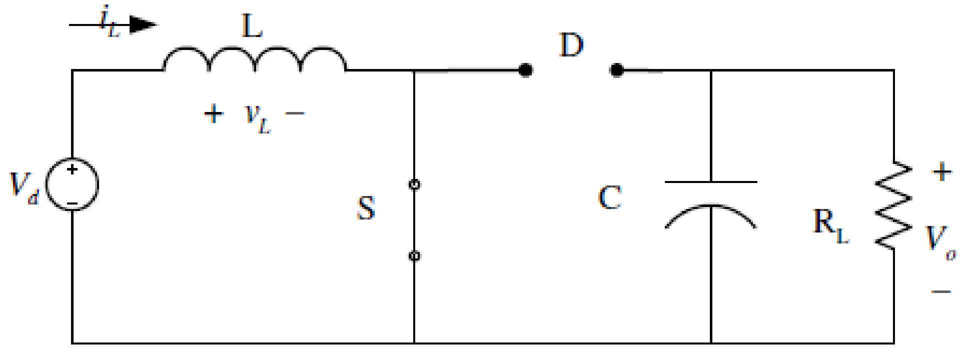
$$L_{\min} = \frac{D(1-D)^2 TR}{2}$$

$$= \frac{D(1-D)^2 R}{2f}$$

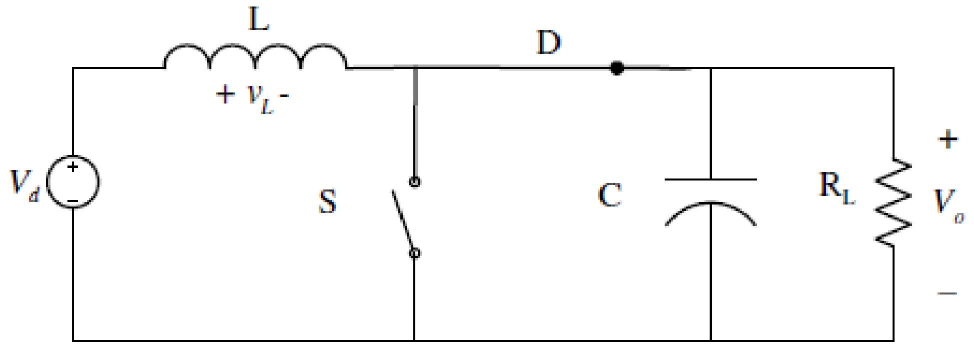
$$L = \frac{V_o D(1-D) T_S}{\Delta I_L}$$



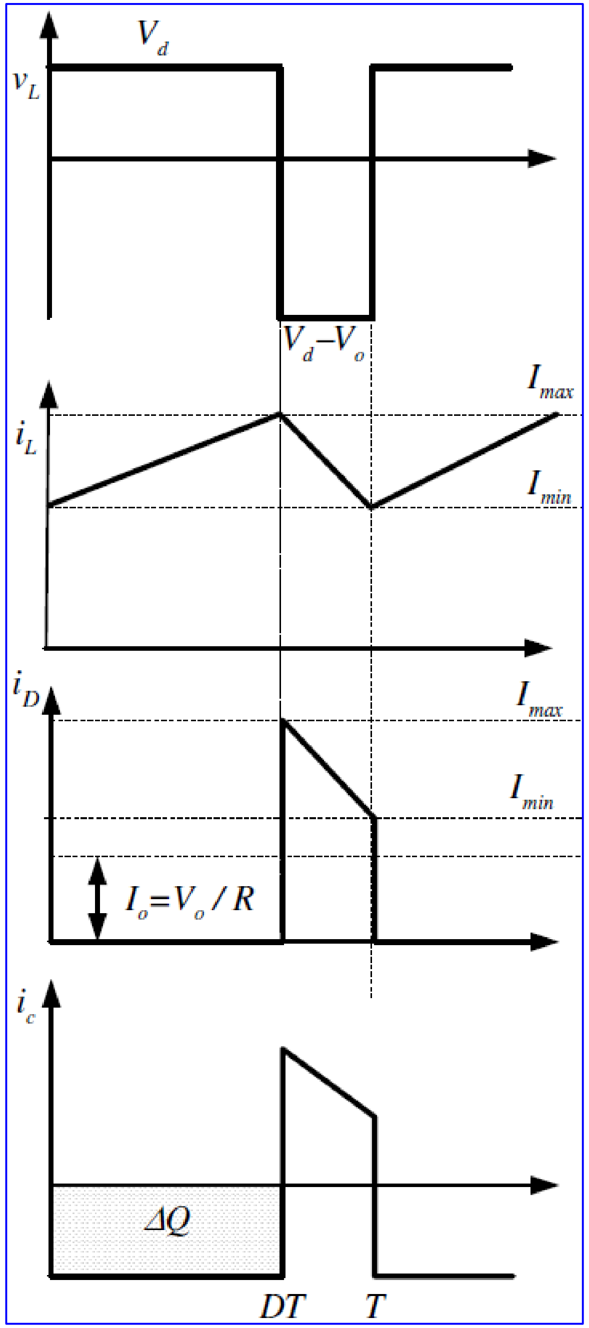
CIRCUIT OF BOOST CONVERTER



CIRCUIT WHEN SWITCH IS CLOSED

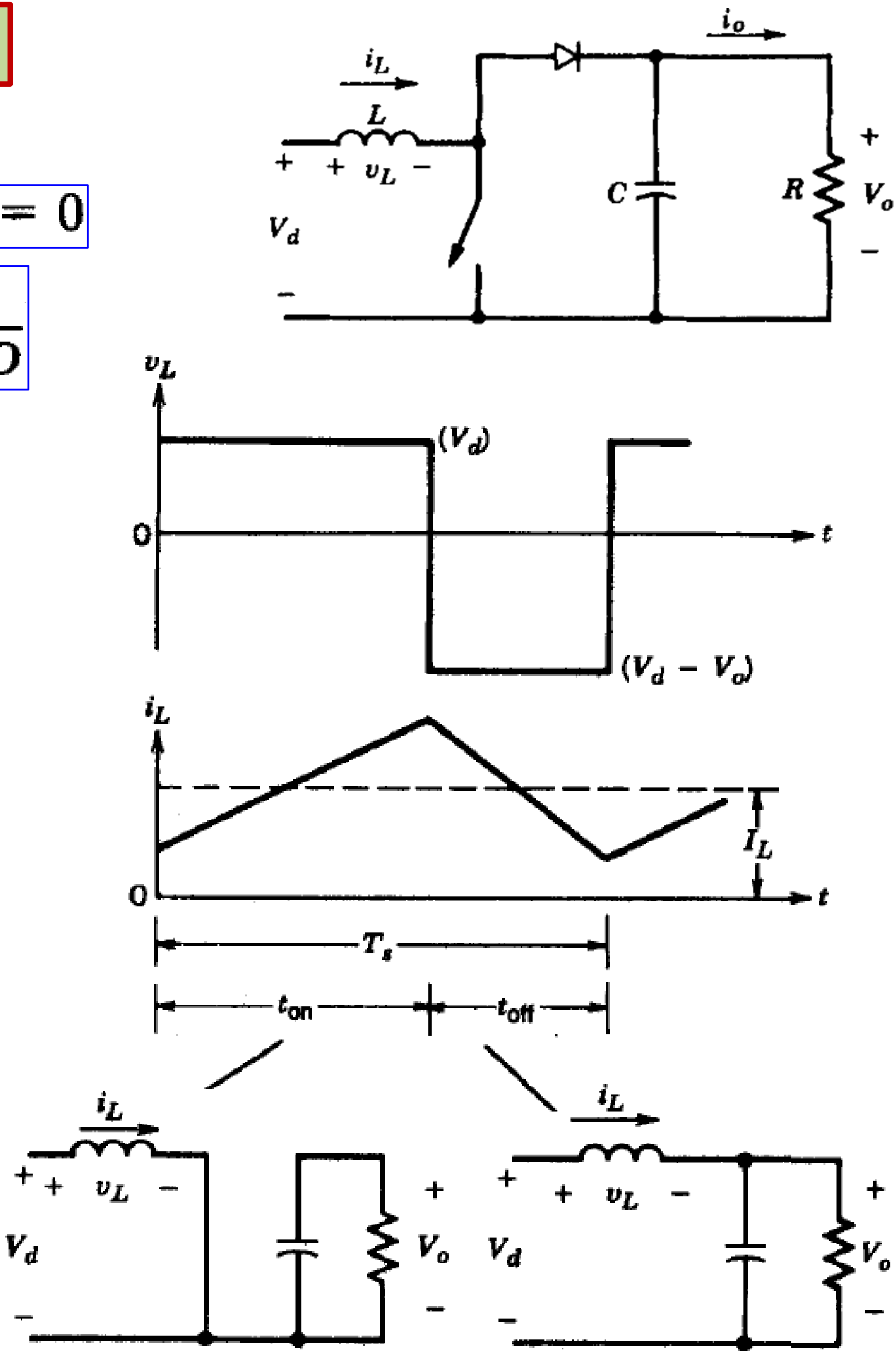


CIRCUIT WHEN SWITCH IS OPENED



BOOST CHOPPER

CONTINUOUS-CONDUCTION MODE



Since in steady state the time integral of the inductor voltage over one time period must be zero,

$$V_d t_{on} + (V_d - V_o)t_{off} = 0$$

Dividing both sides by *Ts* and rearranging terms yield

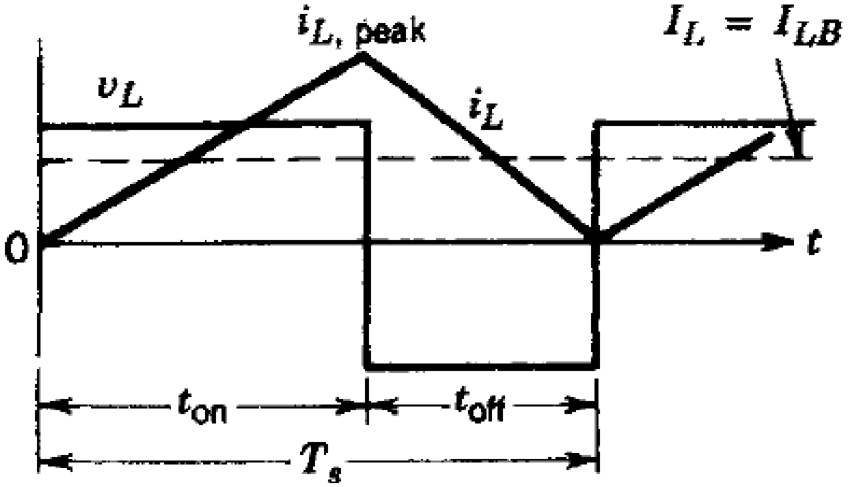
$$\frac{V_o}{V_d} = \frac{T_s}{t_{off}} = \frac{1}{1-D}$$

Assuming a lossless circuit, $P_d = P_o$, $\therefore V_d I_d = V_o I_o$

$$\frac{I_o}{I_d} = (1-D)$$

Ideally, the step up voltage ratio is continuously adjustable in the range from 1 (D = 0) to infinity (D = 1) by choice of D.

Boundary between Cont. and Discont. Modes



$$I_{LB} = \frac{1}{2} i_{L,\text{peak}}$$

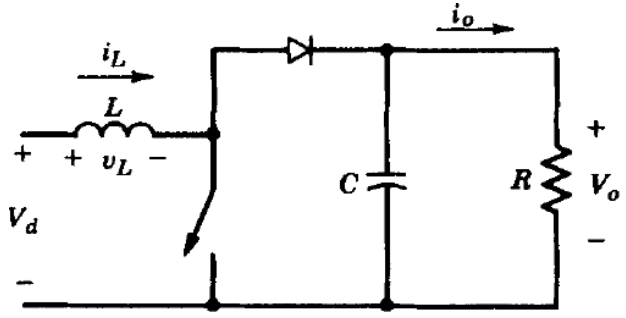
$$= \frac{1}{2}\frac{V_d}{L} t_{on}$$

$$= \frac{T_s V_o}{2L} D(1-D)$$

STEP-UP (BOOST) CONVERTER

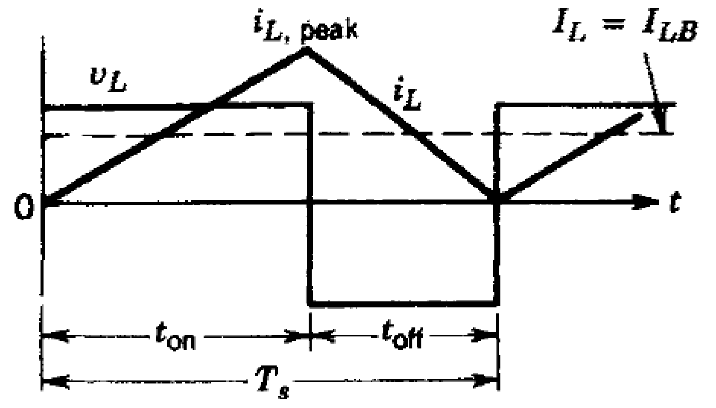
Boundary between Cont. and Discont. Modes

$$I_{LB} = \frac{T_s V_o}{2L} D(1-D) \qquad \frac{I_o}{I_d} = (1-D) \qquad i_d = i_L$$



The average output current at the edge of cont. cond. is

$$I_{oB} = \frac{T_s V_o}{2L} D(1-D)^2$$



In most Boost converter applications, V_0 is kept constant.

I_{LB} reaches a maximum value at D = *0.5*: $I_{LB,\max} = \frac{T_s V_o}{8L}$

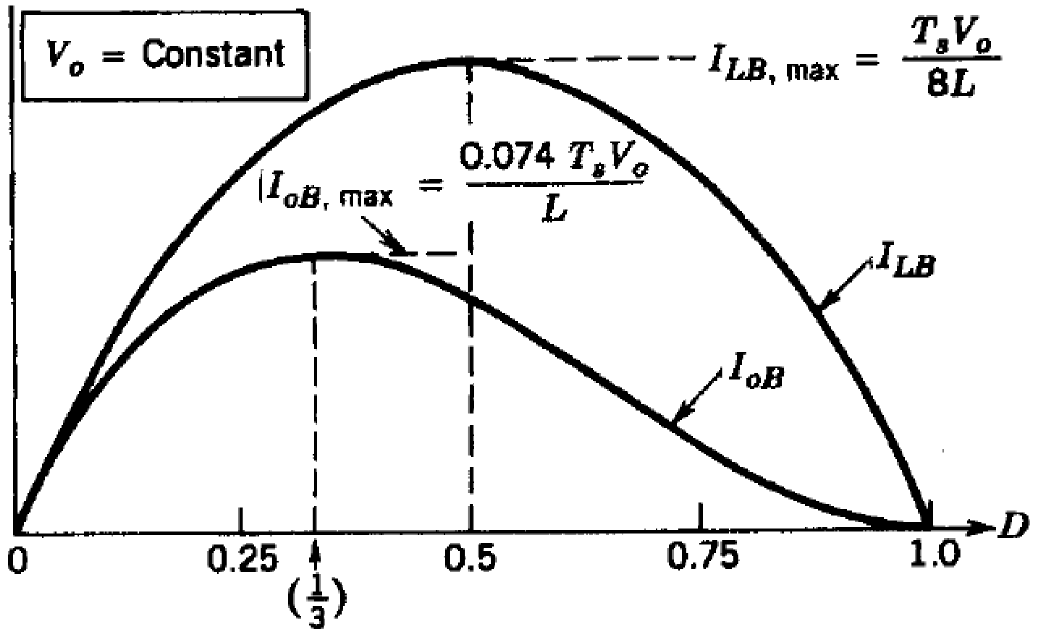
Also, I_{0B} has its maximum at D = 0.333 $I_{oB,\max} = \frac{2}{27}\frac{T_s V_o}{L} = 0.074\frac{T_s V_o}{L}$

In terms of their maximum values,

$$I_{LB} = 4D(1-D)I_{LB,\max}$$

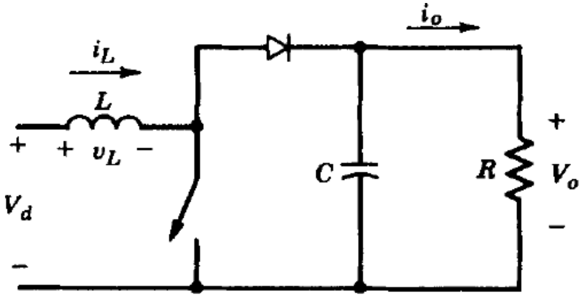
$$I_{oB} = \frac{27}{4} D(1-D)^2 I_{oB,\max}$$

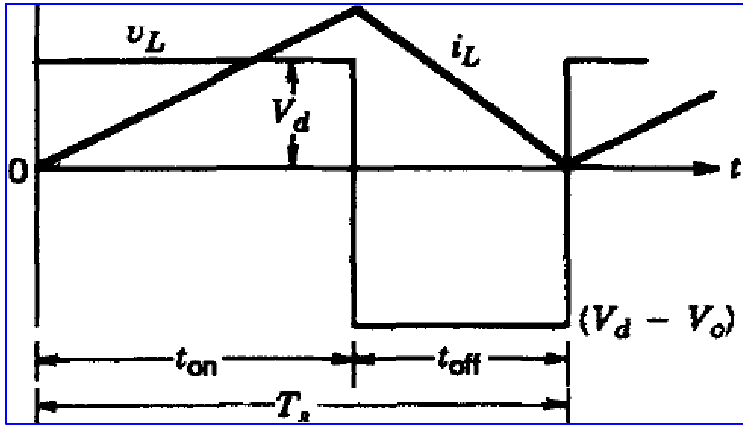
For a given *D,* with constant V_0, if the average load current drops below I_{0B} (and, hence, the average inductor current below I_{LB}), the current conduction will become discontinuous.



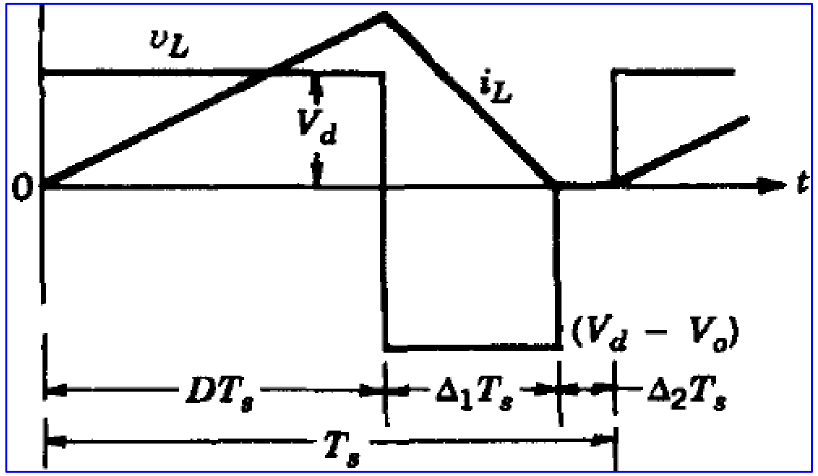
DISCONTINUOUS-CONDUCTION MODE

Assume that as the output load power decreases, Vd and D remain constant (even though, in practice, D would vary in order to keep Vo constant).





- The DCM occurs due to decreased Po (=Pd) and, hence, a lower I_L (= I_d), since v_d is constant.
- Since i_{Lpeak} is the same in both modes, a lower value of I_L (and, hence a discontinuous I_L) is possible only if Vo goes up.



If we equate the integral of the inductor voltage over one time period to zero,

$$V_d DT_s + (V_d - V_o)\Delta_1 T_s = 0$$

$$\therefore \frac{V_o}{V_d} = \frac{\Delta_1 + D}{\Delta_1}$$

$$\frac{I_o}{I_d} = \frac{\Delta_1}{\Delta_1 + D} \quad (\text{since } P_d = P_o)$$

The average input current, which is also equal to the inductor current, is

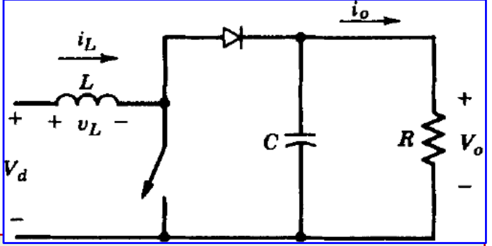
$$I_d = \frac{V_d}{2L} DT_s(D + \Delta_1)$$

$$\Rightarrow I_o = \left(\frac{T_s V_d}{2L}\right) D\Delta_1$$

DISCONTINUOUS-CONDUCTION MODE

$$I_o = \left(\frac{T_s V_d}{2L}\right) D\Delta_1 \quad \therefore \frac{V_o}{V_d} = \frac{\Delta_1 + D}{\Delta_1}$$

$$I_{oB,\max} = \frac{2}{27}\frac{T_s V_o}{L} = 0.074\frac{T_s V_o}{L}$$



In practice, since V_0 is held constant and D varies in response to the variation in Vd, it is more useful to obtain the required duty ratio D as a function of load current for various values of V_0 / Vd.

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oB,\max}}\right]^{1/2}$$

D is plotted as a function of Io/IoBmax for various values of Vd/V0.