EE 238

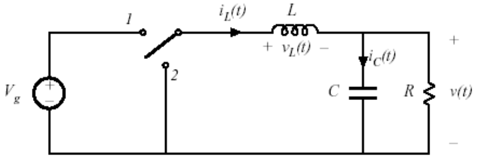
Power Engineering - II

Power Electronics

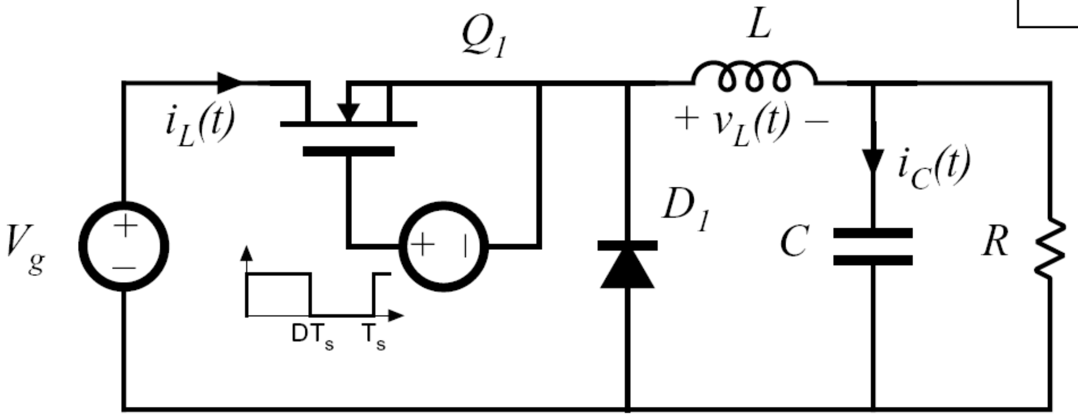
Lecture 7

Instructor: Prof. Anshuman Shukla
Email: ashukla@ee.iitb.ac.in

Buck converter analysis:

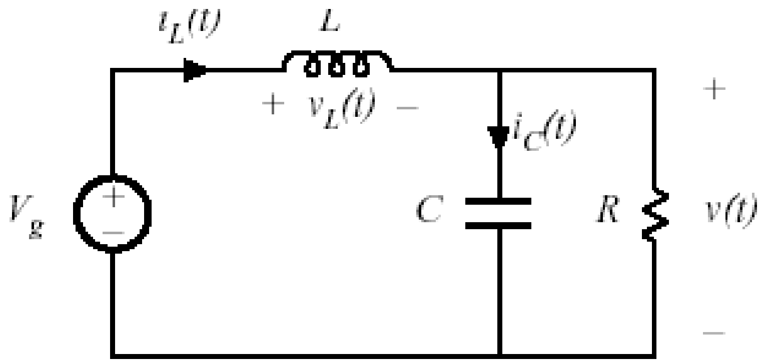


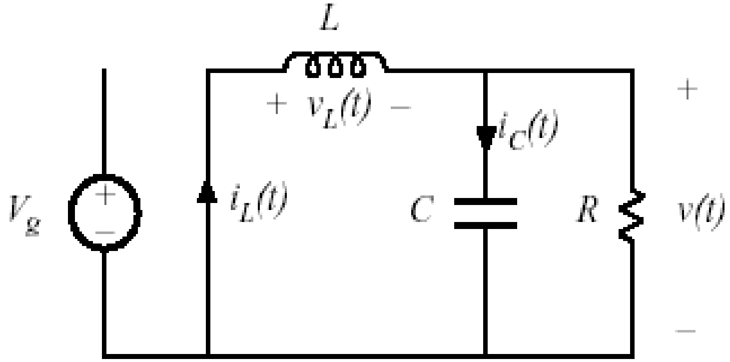
original converter



switch in position 1

switch in position 2





During the interval when the switch is on, the diode becomes reverse biased and the input provides energy to the load as well as to the inductor.

During the interval when the switch is off, the inductor current flows through the diode, transferring some of its stored energy to the load.

Buck converter analysis:

Switch is turned on (closed)

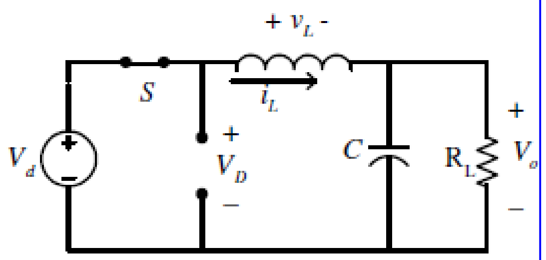
- Diode is reversed biased.
- Switch conducts inductor current
- This results in positive inductor voltage, i.e:

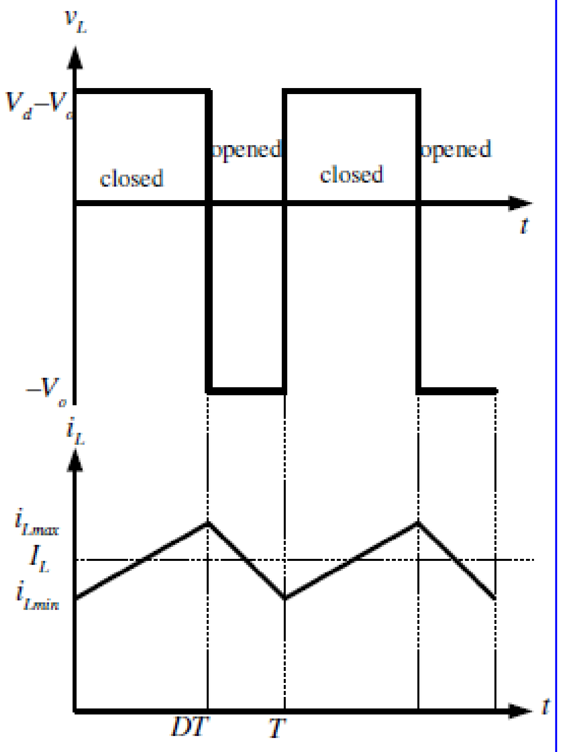
$$v_L = V_d - V_o$$

- It causes linear increase in the inductor current

$$v_L = L\frac{di_L}{dt}$$

$$\Rightarrow i_L = \frac{1}{L}\int v_L dt$$

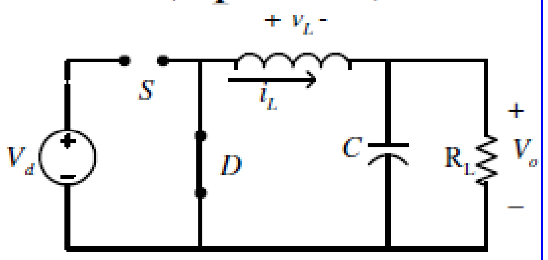


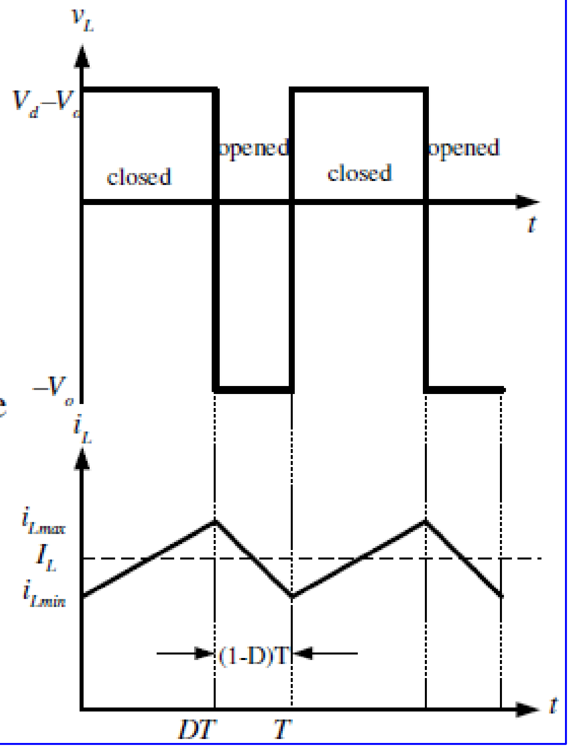


Switch turned off (opened)

- Because of inductive energy storage, i_L continues to flow.
- Diode is forward biased
- Current now flows (freewheeling) through the diode.
- The inductor voltage can be derived as:

$$v_L = -V_o$$





Buck converter Analysis

When the switch is closed (on) :

$$v_L = V_g - V = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_g - V}{L}$$

Derivative of i_L is a positive constant. Therefore i_L must increase linearly.

From Figure

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_g - V}{L}$$

$$(\Delta i_L)_{closed} = \left(\frac{V_g - V}{L}\right).DT$$

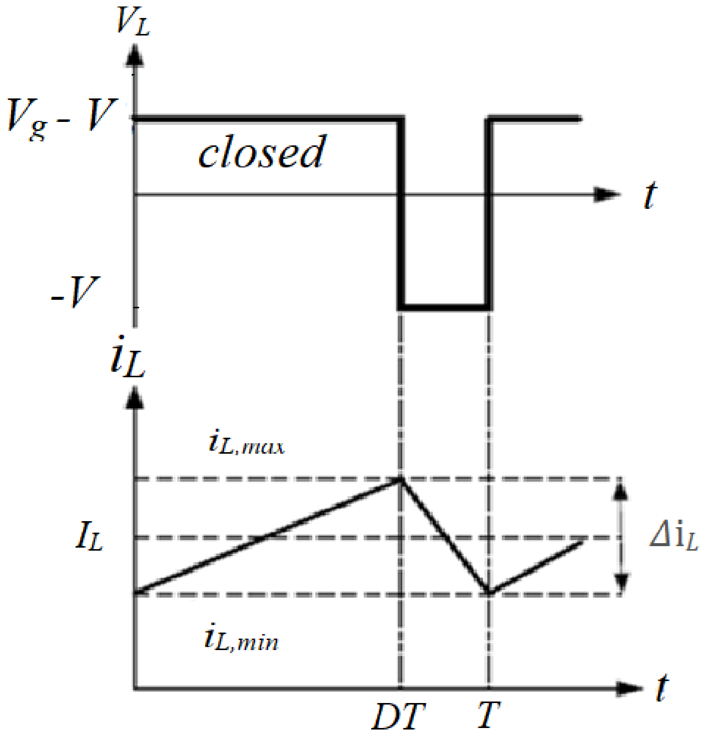
For switch opened,

$$v_L = -V = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{-V}{L}$$

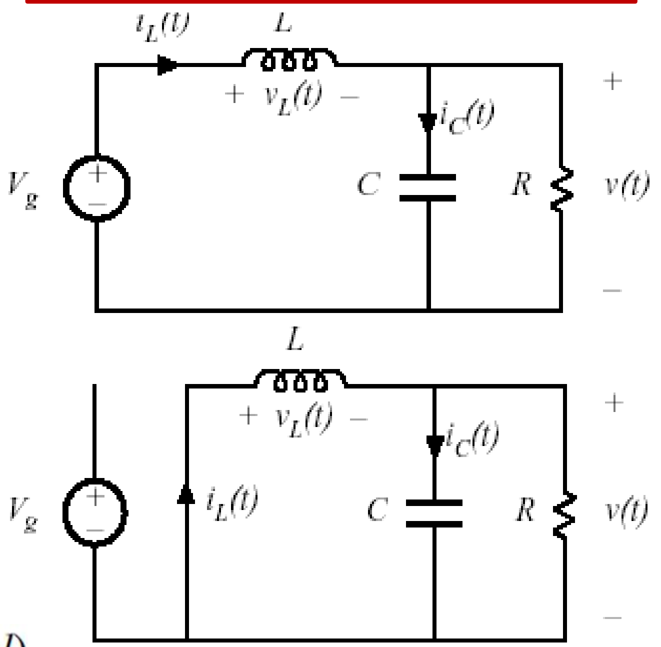
$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{-V}{L}$$

$$(\Delta i_L)_{opened} = \left(\frac{-V}{L}\right).\ (1 - D)T$$



$(\text{change in } i_L) = (\text{slope})(\text{length of subinterval})$

Determination of inductor current ripple magnitude

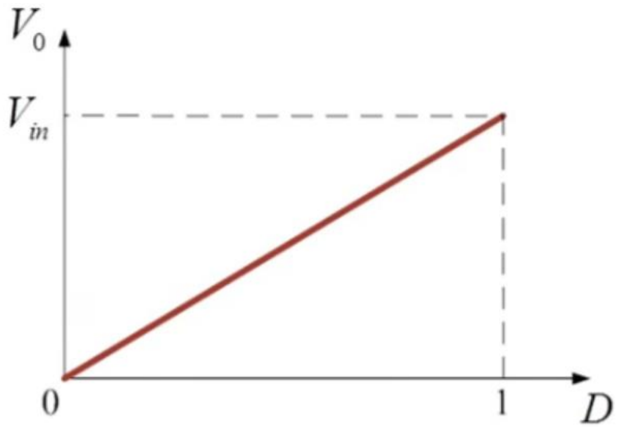


Steady- state operation requires that i_L at the end of switching cycle is the same at the beginning of the next cycle. That is the change of i_L over one period is zero i.e :

$$(\Delta i_L)_{closed} + (\Delta i_L)_{opened} = 0$$

$$\left(\frac{Vg - V}{L}\right).\ DT_S - \left(\frac{-V}{L}\right).\ (1\text{-}D)T_S = 0$$

$$V = DV_g$$



Volt-sec balance in Inductors

Instantaneous v-i relationship for inductor

$$v_L(t) = L\frac{di_L(t)}{dt} \;;\quad i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(\tau)\,d\tau \quad \ldots(1)$$

Substituting $t = t_0 + T_s$ in (1)

$$i_L(t_0+T_s) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t_0+T_s} v_L(\tau)\,d\tau \ldots(2)$$

In steady state

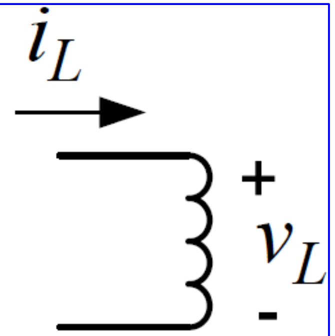
$$i_L(t_0+T_s) = i_L(t_0) \quad \ldots(3)$$

from (2) and (3)

$$\frac{1}{L}\int_{t_0}^{t_0+T_s} v_L(t)\,dt = \frac{T_s}{L}\,\overline{v}_L = 0$$

$$\boxed{\overline{v}_L = 0}$$

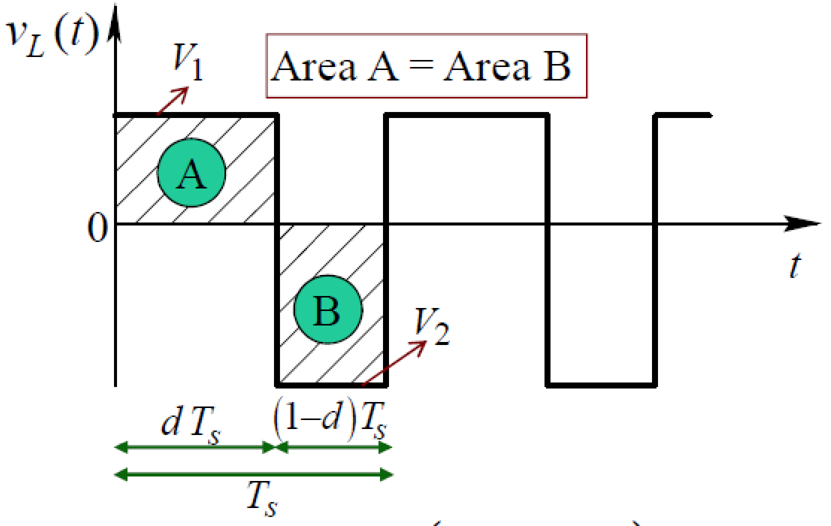
$$\overline{v}_L = L\frac{d\,\overline{i}_L}{dt} = 0$$



(since, $\overline{i}_L$ should be constant in steady state)

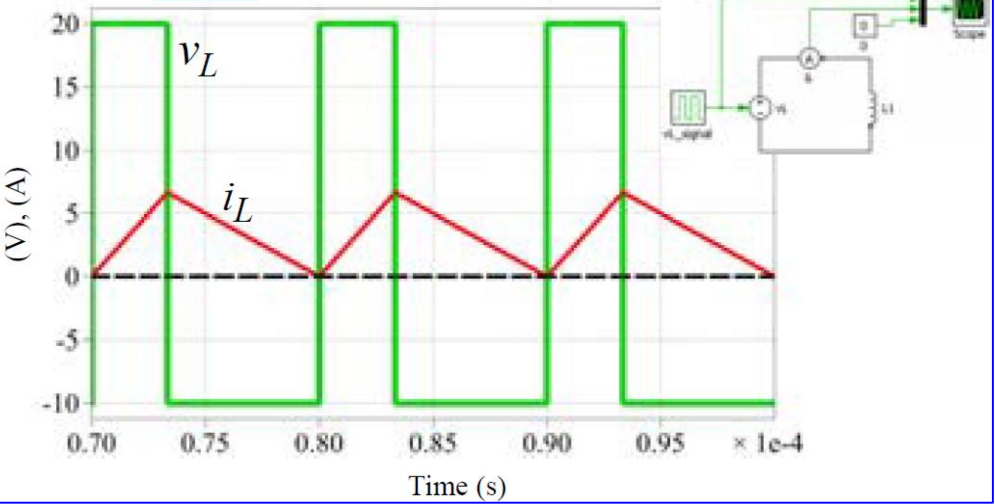
Volt-sec balance in Inductors

$\overline{v}_L = 0$ does not imply that the inductor voltage is zero instantaneously, only the average over a complete period is zero



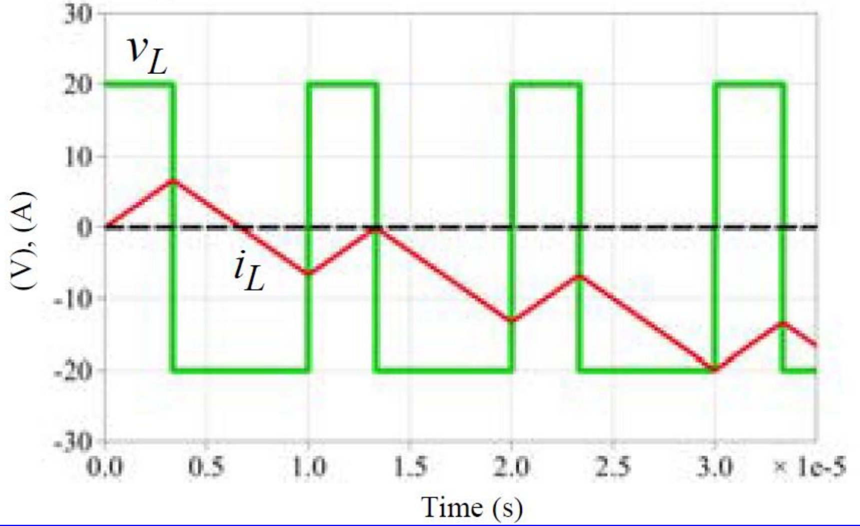
$$V_1 d + V_2 (1 - d) = 0$$

$\overline{v}_L = 0$ $\quad i_L(t_0 + T_s) = i_L(t_0)$

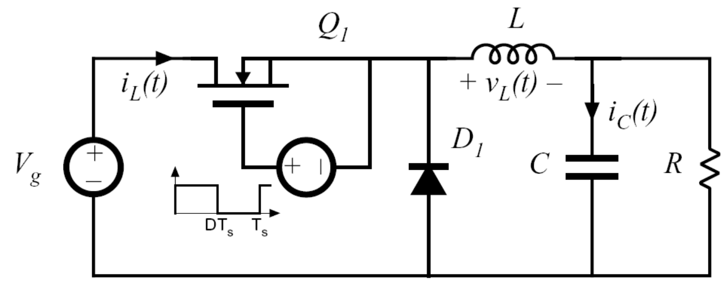


If volt-sec balance is violated

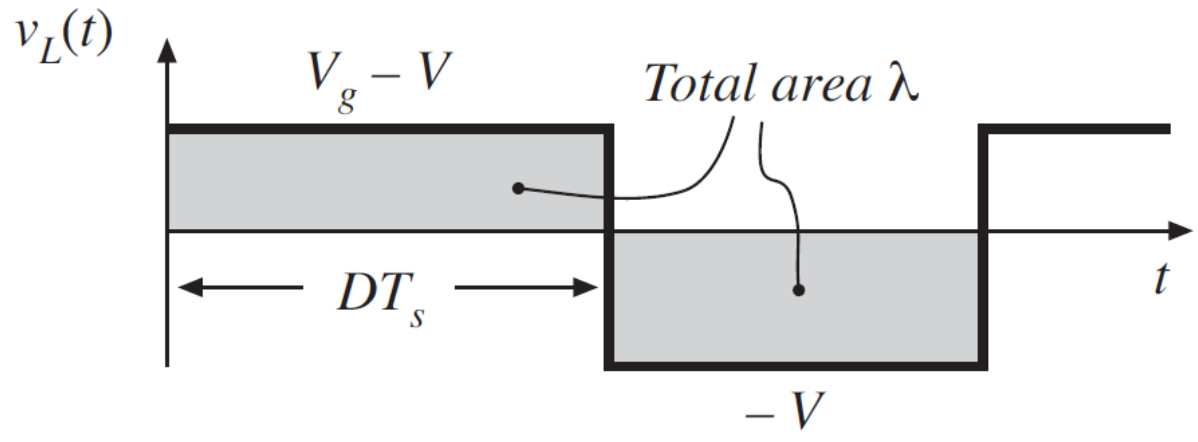
$\overline{v}_L < 0 \Rightarrow \overline{i}_L$ continuously decreases $\Rightarrow$ non-steady-state



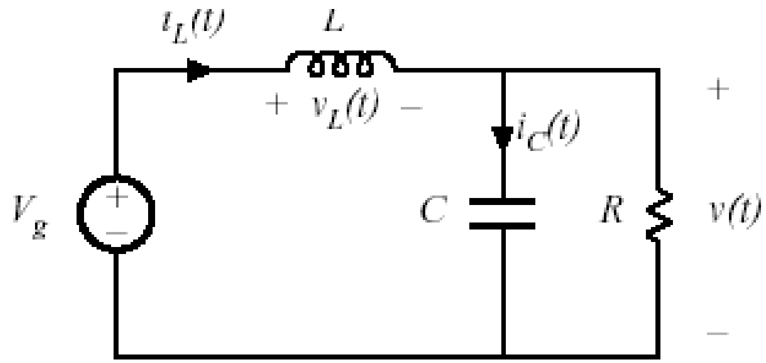
Inductor volt-second balance: Buck converter example



Inductor voltage waveform, previously derived:



switch in position 1



Integral of voltage waveform is area of rectangles:

$$\lambda = \int_0^{T_s} v_L(t)\, dt = (V_g - V)(DT_s) + (-V)(D'T_s)$$

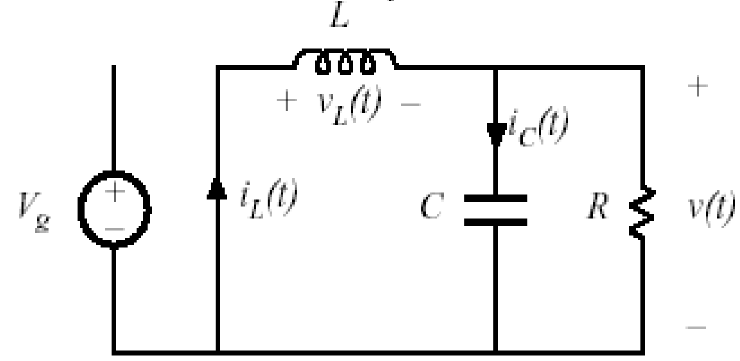
Average voltage is

$$\langle v_L \rangle = \frac{\lambda}{T_s} = D(V_g - V) + D'(-V)$$

Equate to zero and solve for *V*:

$$0 = DV_g - (D + D')V = DV_g - V \quad \Rightarrow \quad \boxed{V = DV_g}$$

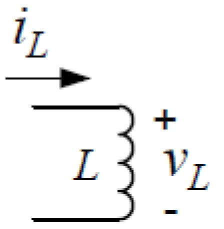
switch in position 2



Current-sec balance in Capacitors

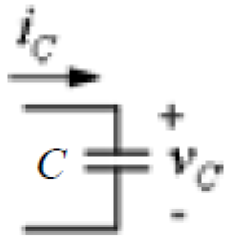
Characteristics of inductors and capacitors

Inductor

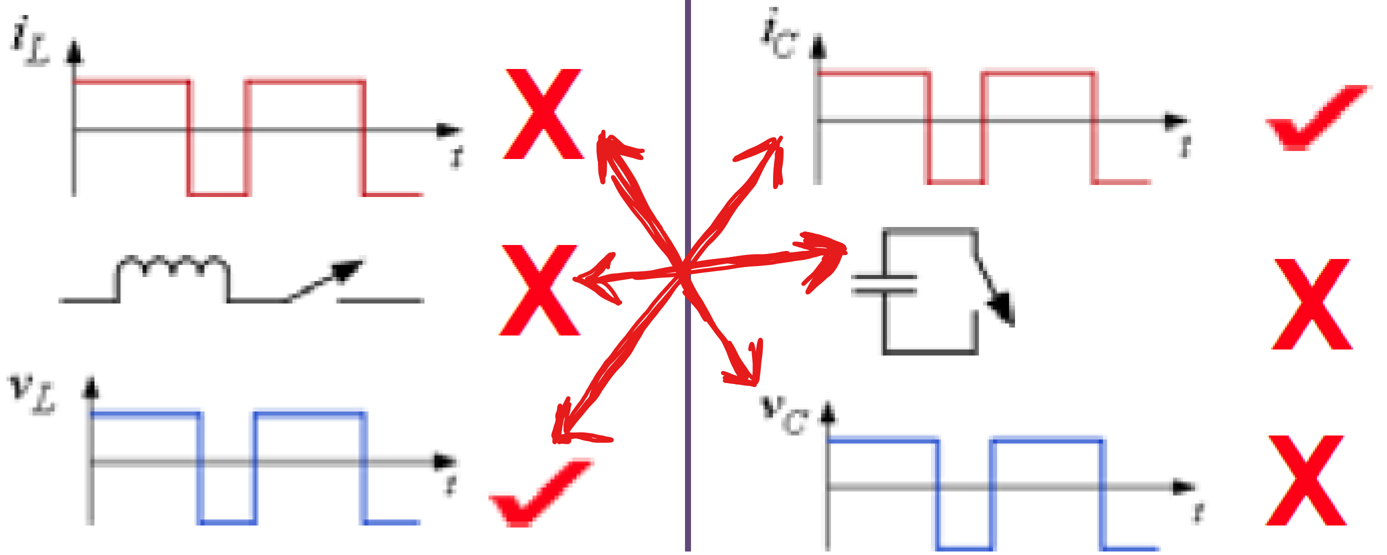


$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Capacitor



$$i_C(t) = C\,\frac{dv_C(t)}{dt}$$



Current-sec balance in capacitors

- The average (CCA) current through a capacitor in steady-state is zero

$$\bar{i}_C = 0$$

Current-sec balance: Derivation

Instantaneous v-i relationship for a capacitor

$$i_C(t) = C\frac{d\,v_C(t)}{dt}\;;\quad v_C(t) = v_C(t_0) + \frac{1}{C}\int_{t_0}^{t} i_C(\tau)\,d\tau \;\ldots(1)$$

Substituting $t = t_0 + T_s$ in (1)

$$v_C(t_0+T_s) = v_C(t_0) + \frac{1}{C}\int_{t_0}^{t_0+T_s} i_C(\tau)\,d\tau \;\ldots(2)$$

In steady state

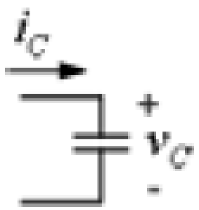
$$v_C(t_0+T_s) = v_C(t_0) \;\ldots(3)$$

from (2) and (3)

$$\frac{1}{C}\int_{t_0}^{t_0+T_s} i_C(\tau)\,d\tau = \frac{T_s}{C}\,\overline{i}_C = 0$$

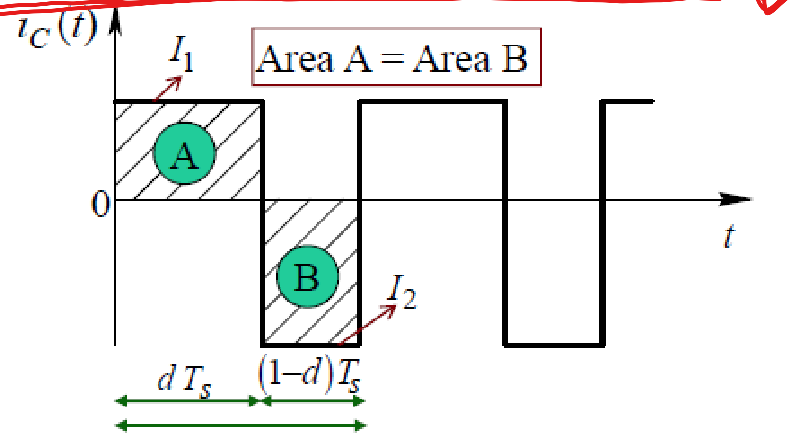
$$\overline{i}_C = 0$$

$$\overline{i}_C = C\frac{d\,\overline{v}_C}{dt} = 0$$



(since, $\overline{v}_C$ should be constant in steady state)

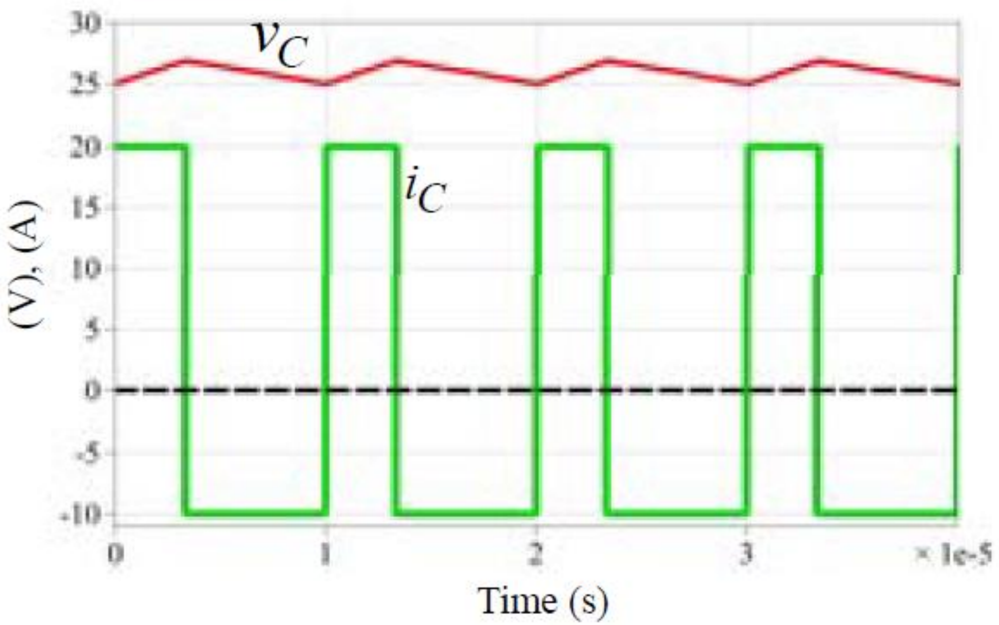
$\overline{i}_C = 0$ does not imply that the capacitor current is zero instantaneously, only the average over a complete period is zero



$$I_1 d + I_2(1-d) = 0$$

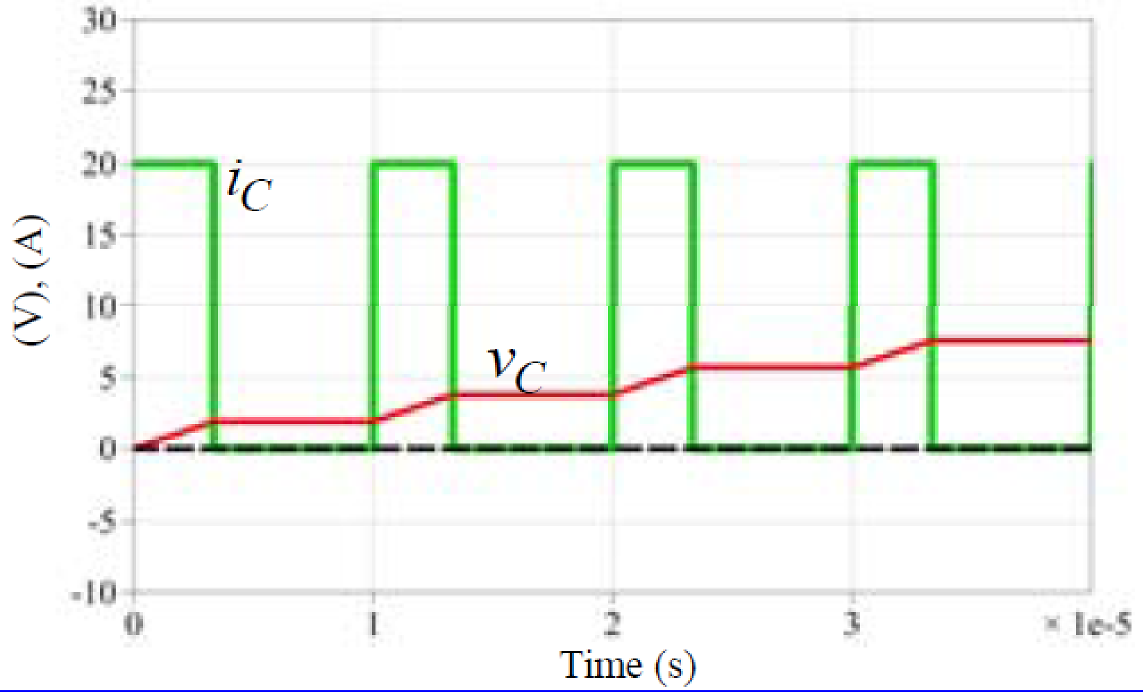
Current-sec balance: Derivation

$\overline{i_C} = 0 \quad \Rightarrow \overline{v}_C$ remains constant



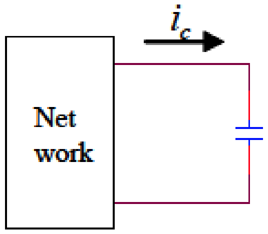
If current-sec balance is violated

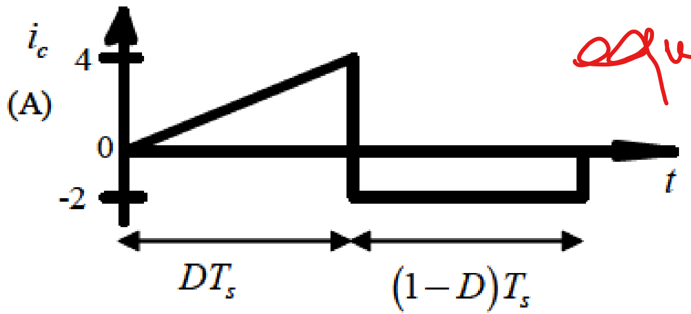
$\overline{i_C} > 0 \;\Rightarrow\; \overline{v}_C$ continuously increases $\Rightarrow$ non-steady-state



Example

Given that the circuit is in DC steady state, calculate *D*





equate areas

$\boxed{D = ?}$

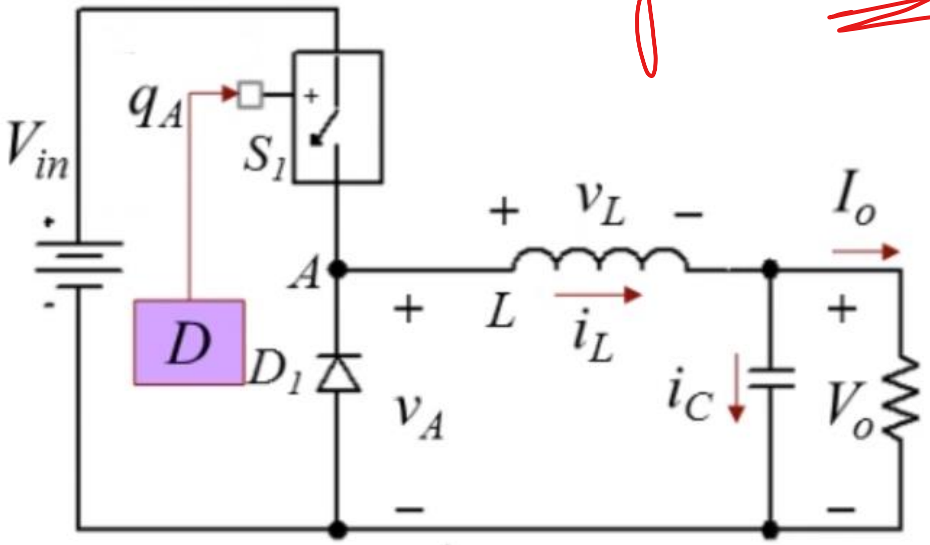
Buck Converter Waveforms

Waveforms IMP

$$\boxed{\frac{V_o}{V_{in}} = D}$$

$P_o = P_d$

same as P_{in}



$$i_L(t) = i_L(t_o) + \int_{t_o}^{t} v_L(\tau)\,d\tau$$

1. ON interval

$v_L(t) = V_{in} - V_o$ Slope of i_L : $m_1 = \dfrac{V_{in} - V_o}{L}$

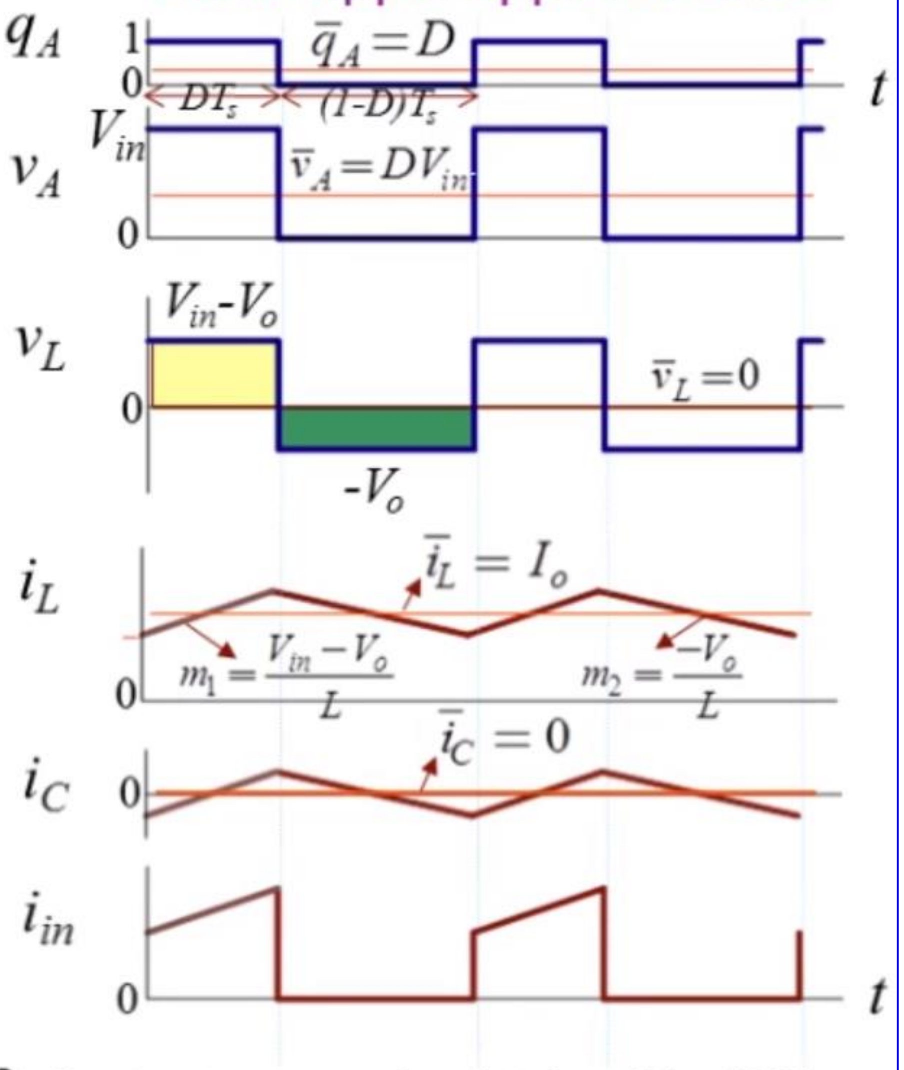
2. OFF interval

$v_L(t) = -V_o$ Slope of i_L : $m_2 = \dfrac{-V_o}{L}$

3. Average of i_L obtained by KCL at output node

$\bar{i}_L = I_o + \bar{i}_c = I_o$ (since $\bar{i}_c = 0$)

Small ripple approximation



Instantaneous i_C obtained by KCL at output node

$i_c(t) = i_L(t) - I_o$

Selection of Output Filter Inductor, L

$f_s >>> f_c = \frac{1}{2\pi\sqrt{LC}}$

- L and C together determine output voltage ripple
- L selected to limit the inductor current ripple to a chosen value
  - e.g.10-20% of its average current (I_o)
  - CCM considerations

$$L\frac{di}{dt} = v_L$$

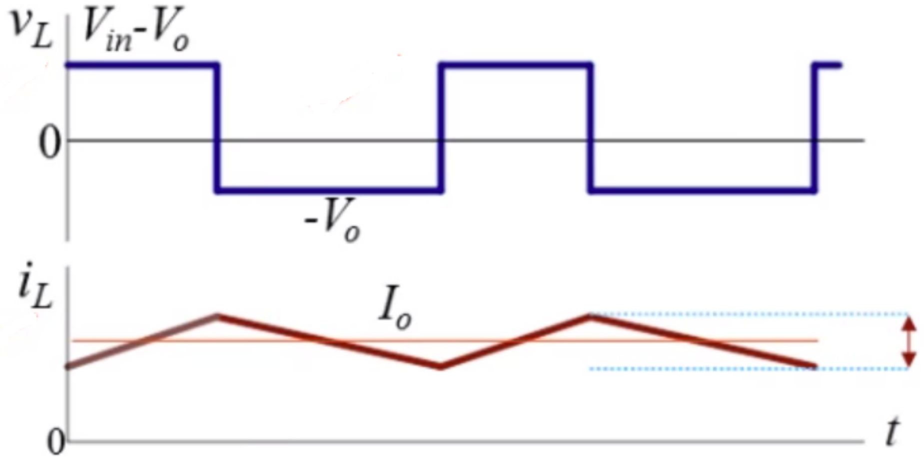
Consider the T_{OFF} interval

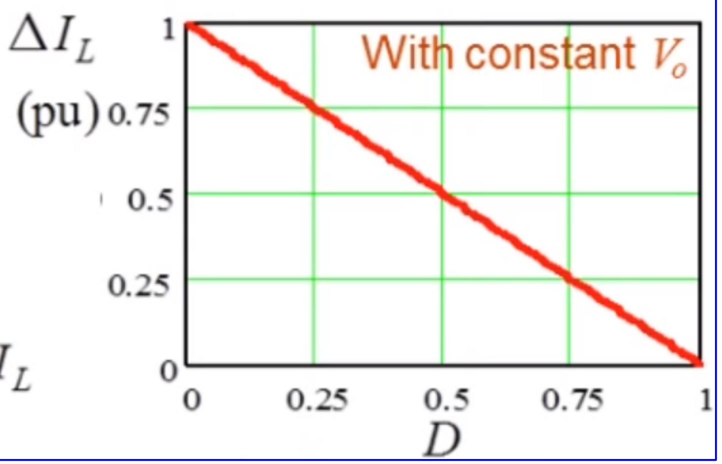
$$L\frac{\Delta I_L}{(1-D)T_S} = V_o$$

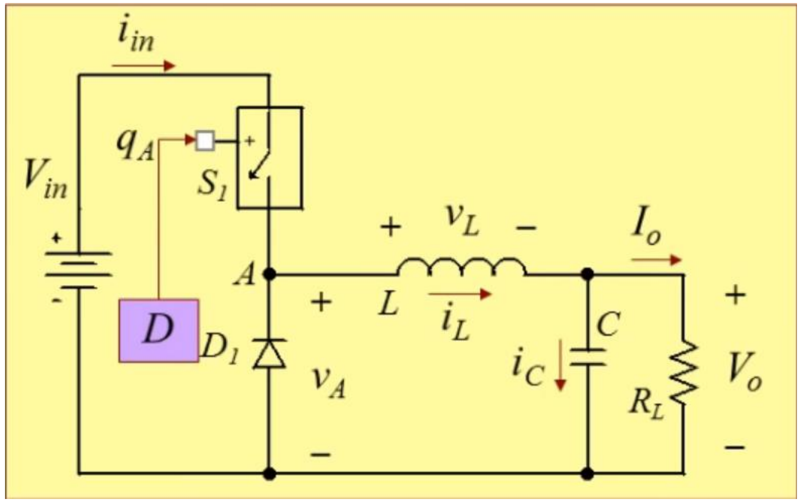
ΔI_L - Peak-peak ripple

$$L = \frac{V_o(1-D)T_S}{\Delta I_L}$$

- **Worst case design**
  - minimum D for fixed output buck







For constant V_o and variable V_{in}

$$\left(\frac{V_o}{V_{in,max}}\right) \leq D \leq \left(\frac{V_o}{V_{in,min}}\right)$$

Trade-off in Selection of Inductor Current Ripple

Larger L
(small current ripple)

- Lower RMS current in switch, capacitor and inductor, hence lower conduction loss
- Smaller C is enough for same output voltage ripple

- Poor dynamic response to step loads (ramp rate of i_L)
- Bulky inductor
- Usually higher inductor resistance

Smaller L
(high current ripple)

- Good dynamic response to step loads due to higher slew rate of inductor current
- Smaller size inductor, higher power density

- Larger RMS current in switch, inductor and capacitor, hence higher conduction loss
- Larger flux swing at high frequency

CCM Considerations ⇨