Wavelets

Applications in Remote Sensing

DFT Vs Wavelet

Fourier basis functions: Complex exponentials (most "*continuous*" functions !)

It fails to provide localization property in both the domains

Fourier analysis doesn't work well on discontinuous, "bursty" data

music, video, power, earthquakes,…

DFT versus Wavelets

DFT

Loses time/space (location) coordinate completely

Analyses the ***whole*** signal

Short pieces lose “frequency” meaning

Wavelets

Localized time-frequency (space-scale) analysis

Short signal pieces also have significance

Scale = Frequency band

Multiresolution Analysis

MRA provides a coarse-to-fine and scale-invariant decomposition for interpreting the image information

Different scales give different details of an image

lower scale ➔ spectral information

higher scale➔ spatial details

Lower resolution provides a global view, while the higher resolution provides the details of the scene

Need of MRA

Difficult to analyze information content just from the pixel value

The local changes of the intensity of an image are more important than the gray level intensity of that image

Different resolution levels are suitable for different sizes of objects (Buildings, shopping malls, small houses)

The size of an object sets the resolution of reference; difficult to define a single optimal resolution for image analysis.

MRA methods

Quadtree

Pyramids

Regular, Irregular, Laplacian, Gaussian

Wavelets

Wavelet Based MRA

Wavelet based MRA

Mathematical algorithm used to describe images in multiple resolutions

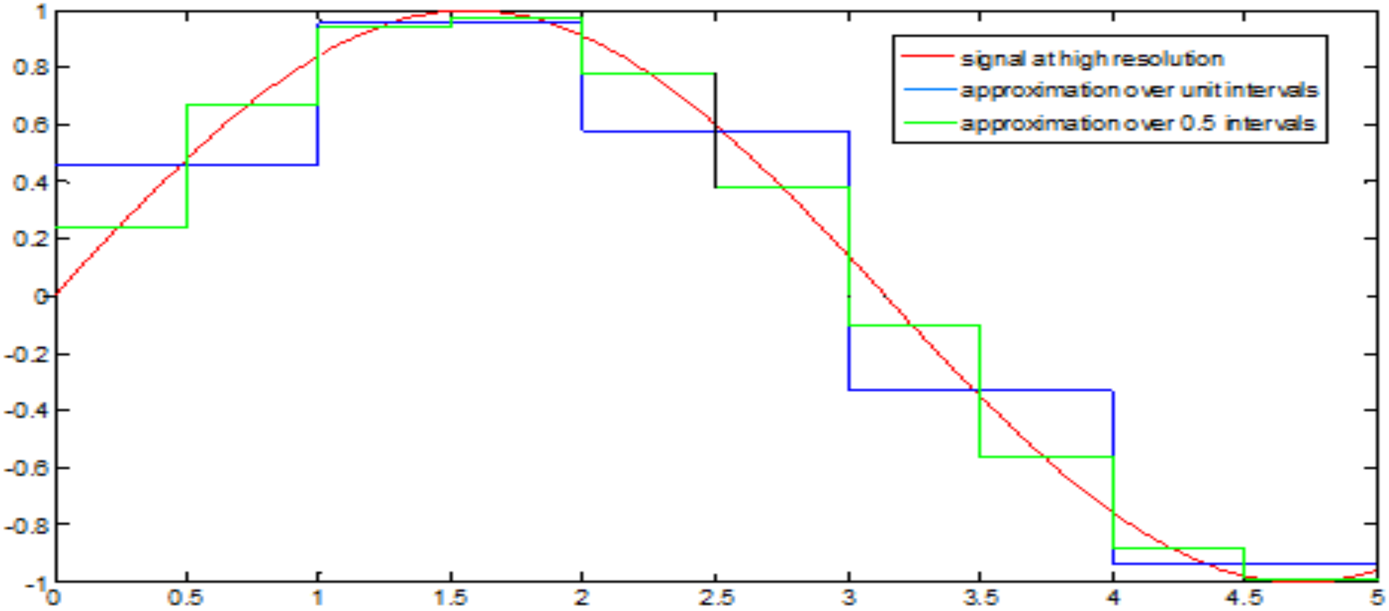
Allows a perfect reconstruction of the original image

Reduces noise (Lowpass filtering is involved)

Compared to Pyramid approach more flexible because it allows to choose a wavelet appropriate to particular segmentation

Ability to perform local analysis; Optimal for compression

Signal Representation



Basis Functions

$$\varphi(t) = \sum_n h_n \varphi(2^m t - n)$$

$$\psi(t) = \sum_n g_n \varphi(2^m t - n)$$

Scaling function ➔ Provides information at a resolution

hn ➔ lowpass filter coefficients

m controls dilation (between scale parameter)

n controls translation (for shifting the function in different intervals

Wavelet function ➔ Provides incremental information OR information BETWEEN two resolutions

gn ➔ highpass filter coefficients

Haar scaling function

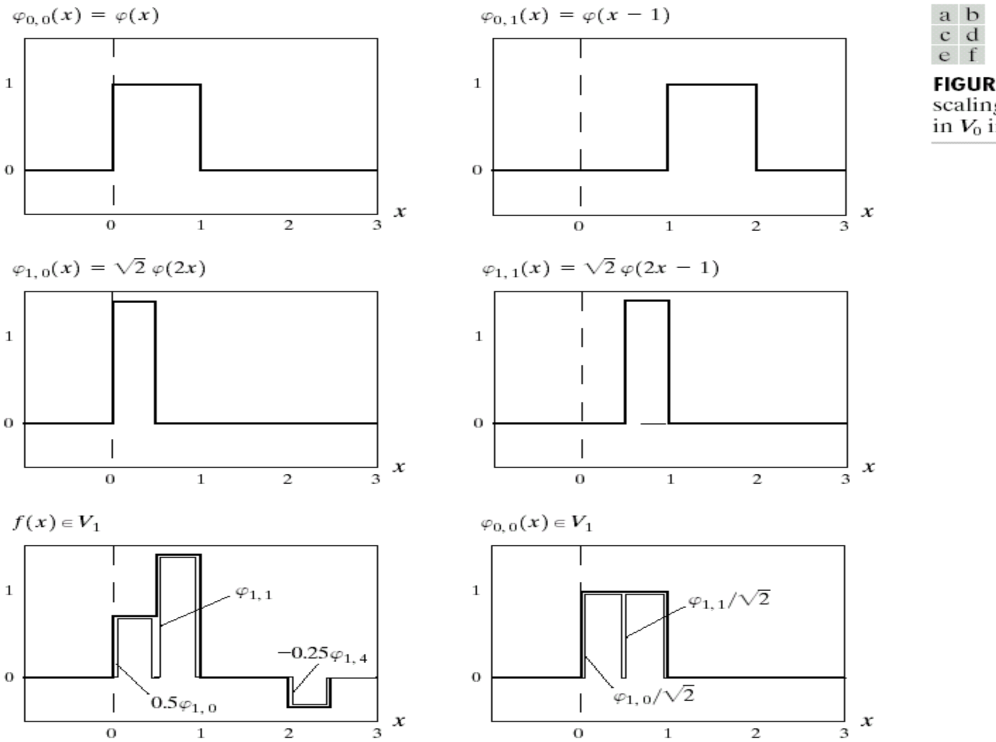
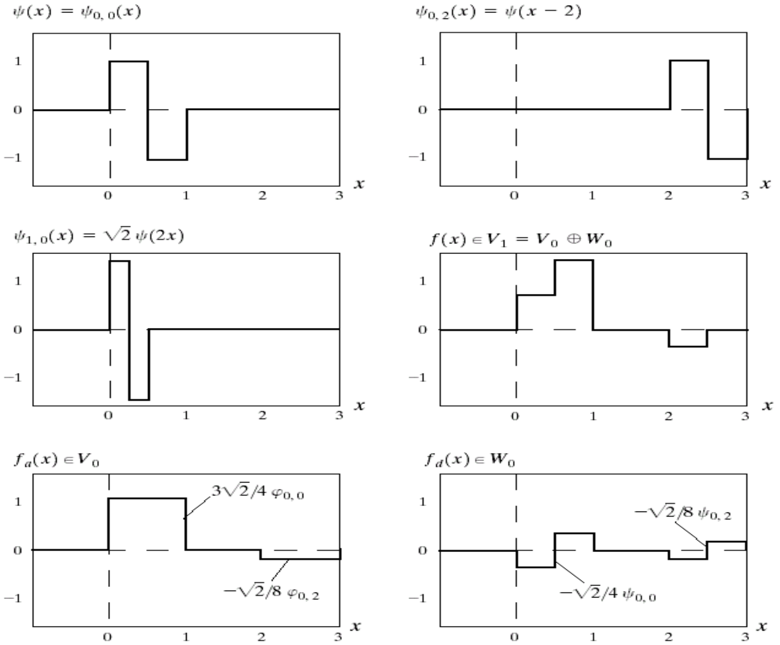


FIGURE 7.9 Haar scaling functions in V_0 in V_1.

Illustration of Scaling function

(Courtesy, *Digital Image Processing*, Rafael Gonzalez, 3rd Edition)

Wavelet Functions



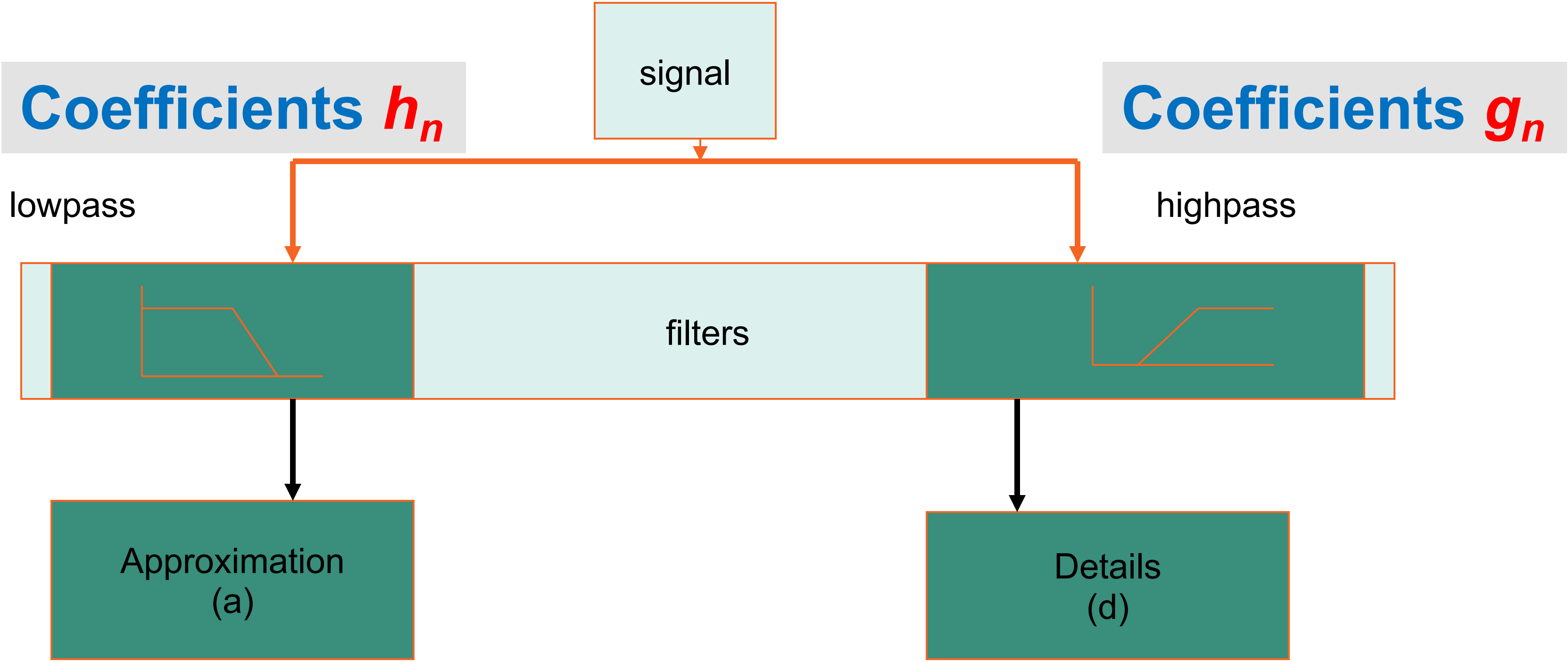
a b
c d
e f

FIGURE 7.12 Haar wavelet functions in W_0 and W_1.

Illustration of Wavelet function

(Courtesy, *Digital Image Processing*, Rafael Gonzalez, 3rd Edition)

Discrete Wavelet transform



Results of wavelet transform: approximation and details

Low frequency:

approximation (a)

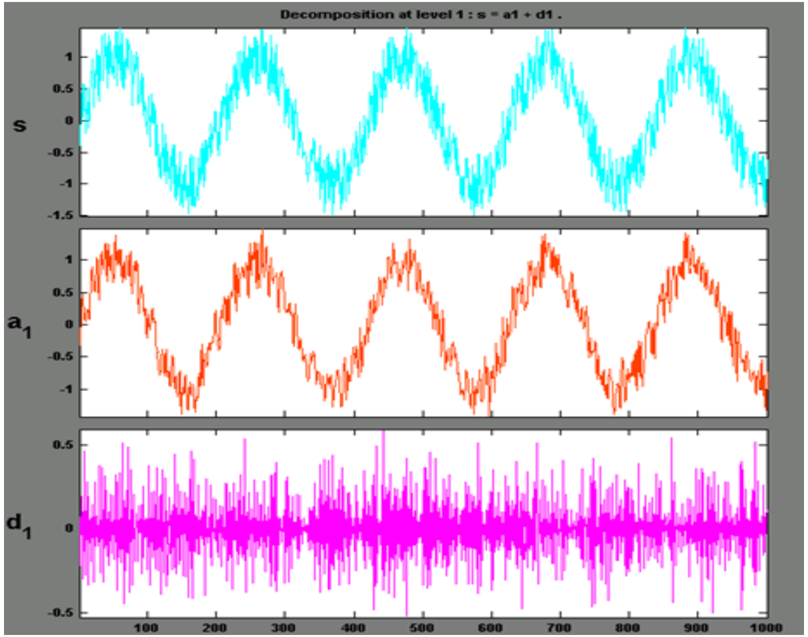
High frequency

Details (d)

“Decomposition”

can be performed

iteratively



A few applications

Noise Filtering

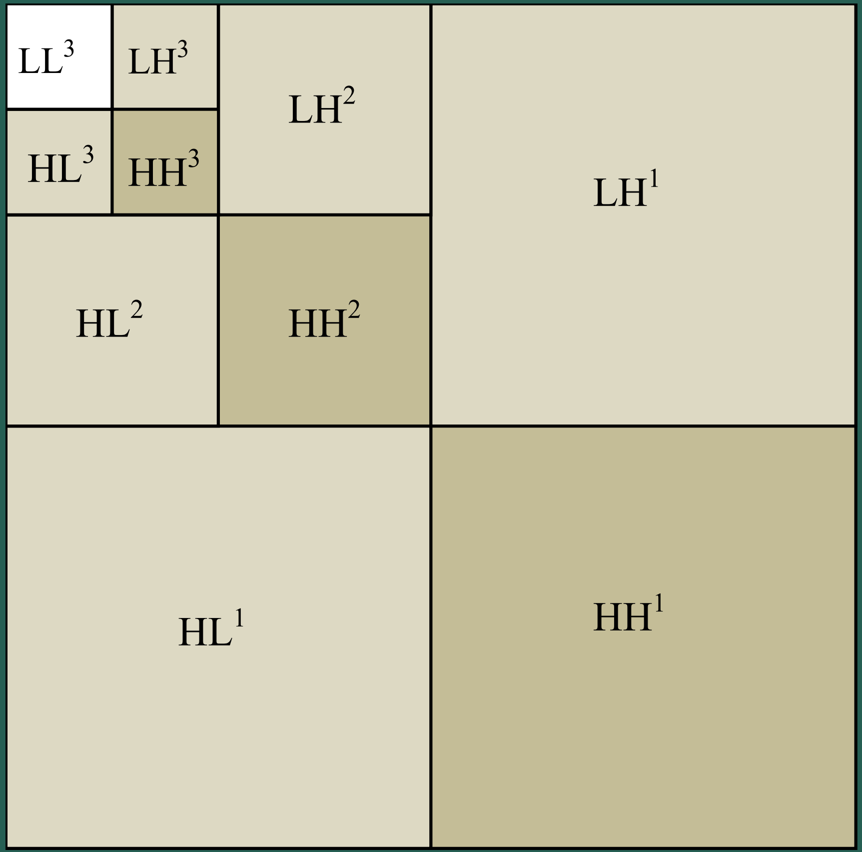
Edge Detection

Texture Segmentation

Compression

Structure of three level wavelet decomposition

Wavelets in 2D



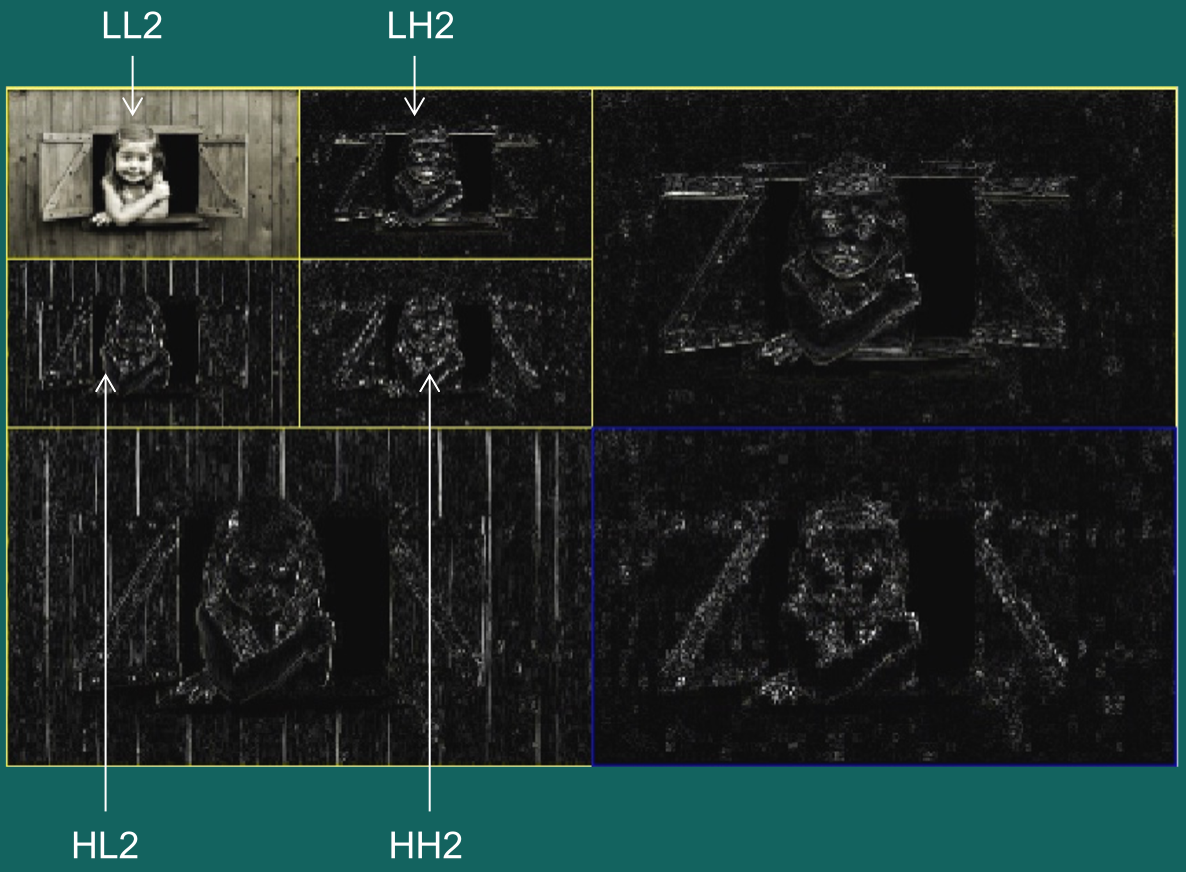
An example

One level wavelet decomposition

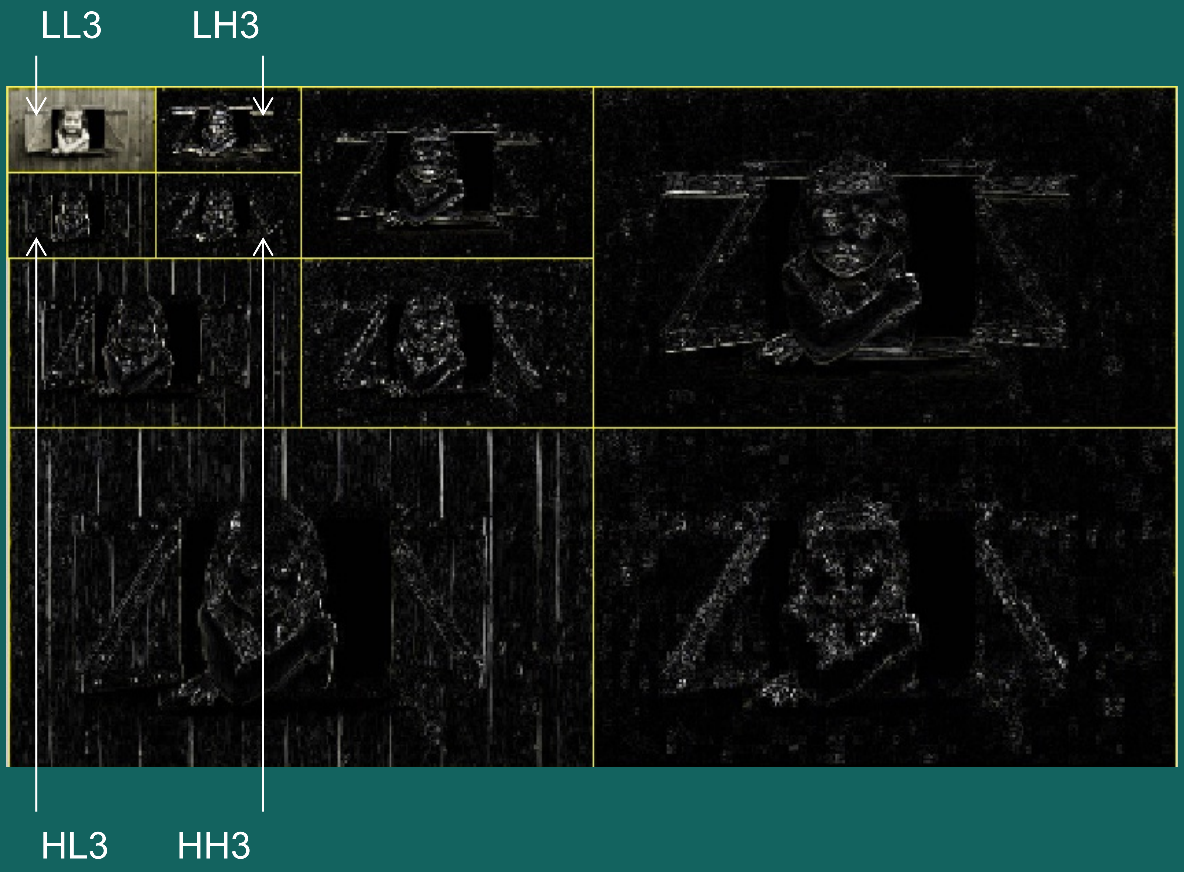
LL1 LH1

HL1 HH1

Two level wavelet decomposition



Three level wavelet decomposition



Noise Filtering

Thresholding on MRA coefficients

In most of the cases, noise tends to be represented by MRA coefficients at finer scales

Discarding these coefficients filters out the noise naturally

Inverse transform of the thresholded coefficients is noise filtered image

Thresholding

Hard Threshold

Select the threshold = $3\sigma_{jl}$ for all bands except the finest scale where it is set at $4\sigma_{jl}$;   (Candes & Donoho 2000)

σ_{jl} :the noise level of a coefficient at scale *j* and angle *l*

$$\bar{C} = \begin{cases} C; & |C| \geq T \\ 0; & |C| < T \end{cases}$$

Thresholding

Soft Threshold

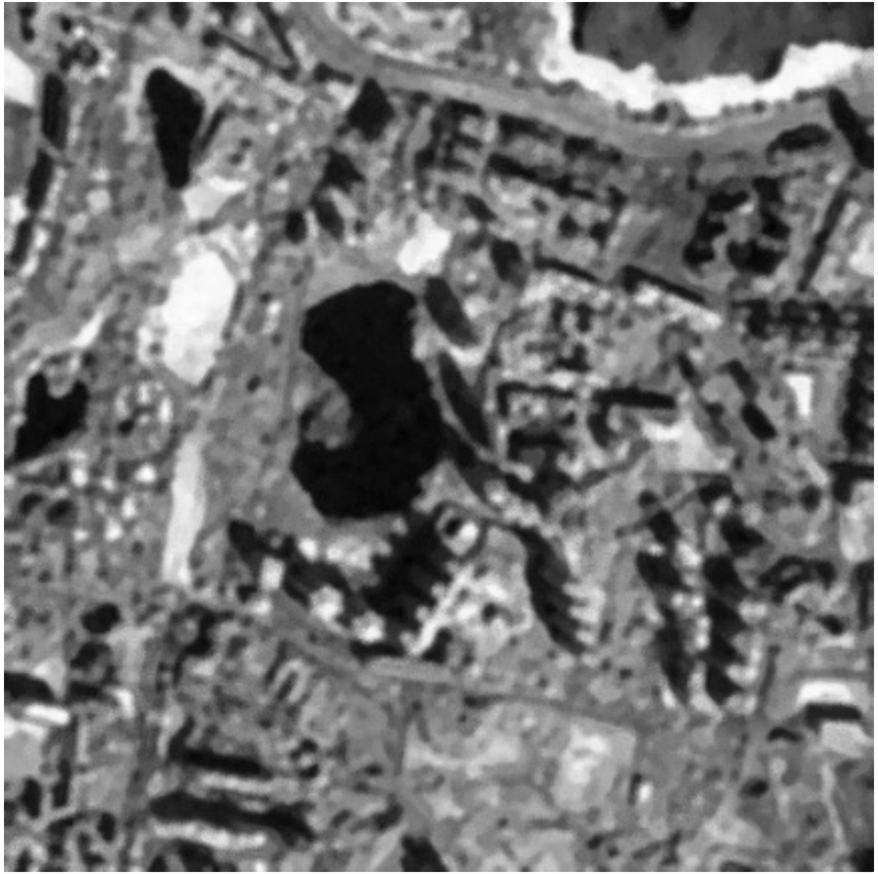
$$\bar{C} = \begin{cases} sign(C)\left(|C| - T\right) & ;\ |C| \geq T \\ 0 & ;\ |C| < T \end{cases}$$

Original Worldview2 image

Noisy Image (σ=20)


Wavelet based


Directional wavelet based



Bayesian based

Anisotropic diffusion based

Original Quickbird image

Noisy Image (σ=25)

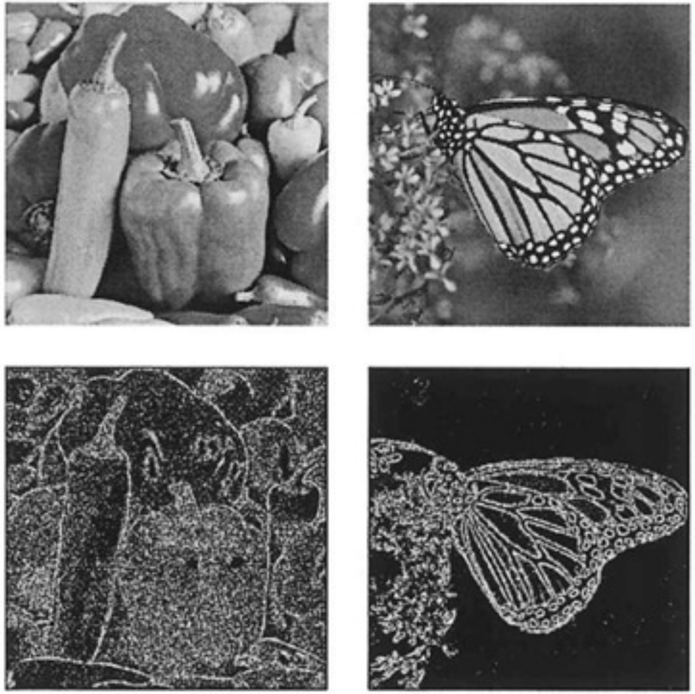
Wavelet based

Directional wavelet based

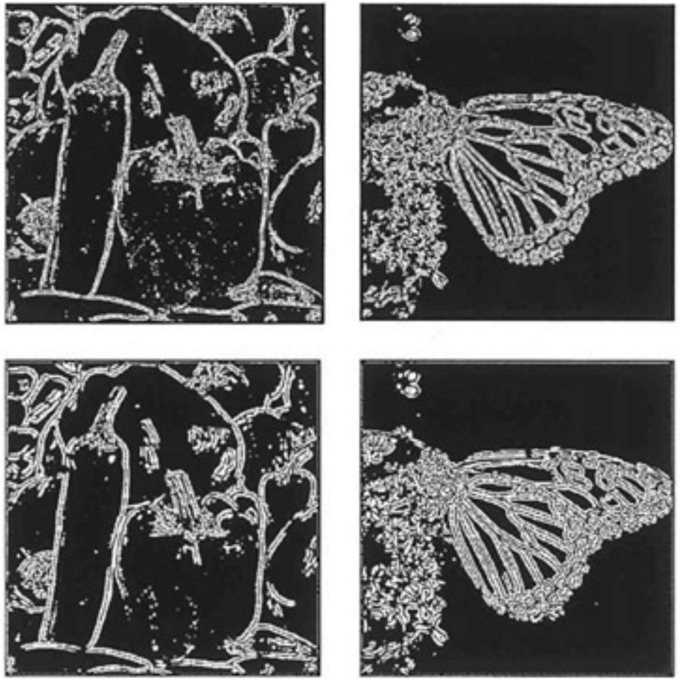
Bayesian based

Anisotropic based

Edge detection



levels = 1



levels = 3

levels = 6

(a) (b)