- 1. 1. Motion
- 2. 2. Introduction We use the word 'rest' very often. For example, when someone is doing no work or lying on the bed, we often say that the person is resting. This means that the person is not moving. Scientifically as well, the word 'rest' has a similar meaning. Scientifically, we say an object is at rest when the position of the object does not change with time, with respect to its surroundings. Similarly, motion is defined as the change of position of an object with time, with respect to its surroundings.
- 3. 3. What do we mean by with respect to the surroundings? We know that a moving train is in motion because its position changes with time. Now, consider a person sitting in the train. For someone standing on the platform, the person sitting in the train is in motion. But for the co-passengers, the person is at rest as the position of the person does not change with time. Therefore, we need to consider the surroundings or the point of observation while describing the state of motion of an object. The surroundings is called reference frame.
- 4. Types of Motion Motion can be broadly classified into three main categories: Translatory motion Rotational motion Periodic motion
- 5. Translatory motion Translatory motion is the motion of a particle in a straight line. A bus travelling on a straight road and an apple falling from a tree are examples of this kind of motion.
- 6. Rotational motion Rotational motion refers to the motion of a body around a fixed axis. A spinning top, a bead moving on a circular track and Earth's rotation are examples of this kind of motion.
- 7. Periodic motion Periodic motion refers to the motion that is repeated in a regular interval of time. An oscillating spring and the motion of a planet around the sun illustrate this type of motion.
- 8. Linear Motion The word 'linear' means 'straight' and the word 'motion' means 'change in position with respect to a frame of reference'. So, a body moving in a straight line with respect to a frame of reference is said to be in linear motion. An example of this is the motion of an ant on a straight wire. Points to remember regardinglinear motion: \*In linear motion, the object must move in a straight line. \*The motion of the object along the straight line may not be uniform.
- 9. Uniform motion If a body covers equal distances along a straight line in regular intervals of time, then the motion is said to be uniform. Examples: A ball pushed in free space will continue to move uniformly, covering equal distances in equal intervals of time along a straight path. If an ant covers equal distances in equal intervals of time along a straight wire, its motion is uniform.
- 10. 10. Non-uniform motion If a body covers unequal distances in regular intervals of time, then the motion is said to be non-uniform. Examples:- The ball takes a curved path when thrown. Its direction of motion changes with time. Also, it covers unequal distances in regular intervals of time. So, its motion is non-uniform. The ant is moving on a circular wire. It is travelling equal distances in equal intervals of time, but its direction of motion is not constant. So, its motion is non-uniform.
- 11. 11. Physical quantity A physical quantity is any physical property that can be expressed in numbers. For example, time is a physical quantity as it can be expressed in numbers, but anger is not as it cannot be expressed in numbers. Physical Quantities can be classified in into two types:- -Scalar Quantities -Vector Quantities
- 12. 12. Scalar Quantities If a physical quantity can be completely described only by its magnitude, then it is a scalar quantity. To measure the mass of an object, we only have to know how much matter is present in the object. Therefore, mass of an object is a physical quantity that only requires magnitude to be expressed. Therefore, we say that mass is a scalar quantity. Some more examples of scalar quantities are time, area, volume, and energy. We can add scalar quantities by simple arithmetic means. It is difficult to plot scalar quantities on a graph.

- 13. 13. Vector Quantities There are some physical quantities that cannot be completely described only by their magnitudes. These physical quantities require direction along with magnitude. For example, if we consider force, then along with the magnitude of the force, we also have to know the direction along which the force is applied. Therefore, to describe a force, we require both its magnitude and direction. This type of physical quantity is called a vector quantity. Some examples of vector quantities are velocity, force, weight, and displacement. Vector quantities cannot be added or subtracted by simple arithmetic means. Vector quantities can easily be plotted on a graph.
- 14. 14. Distance and Displacement Distance-Distance is the length of the path or the path length travelled by a body while moving from an initial position to a final position. It is a scalar quantity. Its SI unit is metre (m). Therefore, only magnitude is important, not the direction of movement. (Implies that path length can never be negative) Displacement-Displacement is the shortest distance between the initial and final positions of the body. It is a vector quantity. Its SI unit is also metre (m). In displacement, the direction of motion is always directed from the initial position toward the final position.
- 15. 15. Speed Speed is defined as the rate of distance covered by a body. Mathematically, speed is given as: speed = It is a scalar quantity; that means no direction is required. (Implies that speed cannot be negative) Average Speed A body travelling from one location to another might stop, slow down, speed up or move at a constant speed. The average speed of a body is defined as the total distance travelled divided by the total time taken. Mathematically, average speed is given as:-
- 16. 16. Velocity When we include the direction of motion with speed, we are talking of the physical quantity called velocity. Thus, velocity is speed with direction. Velocity is defined as the rate of change of displacement. Velocity = It is a vector quantity. Therefore, direction of movement is important. (Implies that velocity contains algebraic sign) Average Velocity- A body moving from one point to another may change its velocity a number of times, but it will have an average velocity of its journey. Average velocity of a body is defined as the net displacement divided by the total time of travel. It is a vector quantity. Its SI unit is m/s and it can be positive, negative or zero Average velocity=
- 17. 17. Acceleration Acceleration is defined as the rate of change of velocity. It is a vector quantity and its direction is given by the direction of the force causing the acceleration. Mathematically, acceleration is given as: Accelerations= Change in velocity Suppose the velocity of a car is u at time t1. Later, at time t2, its velocity becomes v. Change in velocity = (v u), time interval = t2 t1
- 18. 18. Uniform and Non Uniform Acceleration UniformAcceleration- If the rate of change of velocity remains constant, then the acceleration is uniform. Examples of uniform acceleration include a ball under free fall, a ball rolling on an inclined plane and a car accelerating on a straight, traffic-free road. Non-UniformAcceleration- If the rate of change of velocity changes with time, then the acceleration is non-uniform. An example of non- uniform acceleration is a car accelerating on a straight road with traffic.
- 19. 19. First Equation of Motion The first equation of motion is v=u + at. It gives the velocity acquired by a body in time t. Consider a body having initial velocity 'u'. Suppose it is subjected to a uniform acceleration 'a' so that after time 't' its final velocity becomes 'v'. Now, from the definition of acceleration we know that; —> Acceleration = Change in velocity —> Acceleration = Final velocity —Initial velocity —> a = v u —> at=v u —> v=u + at
- 20. 20. Second Equation of Motion The second equation of motion is s=ut + 1at. It gives the distance traveled by a body in time t. Consider a body having initial velocity 'u' and a uniform acceleration 'a' for time 't' so that its final velocity becomes 'v'. Let the distance traveled by the body in this time 's'. The distance travelled by a moving body in time 't' can be found out by considering its average velocity. Since the initial velocity of the body is 'u' and its final; velocity is 'v', the average velocity is given by: —> Average velocity=Initial velocity+ final velocity —> Average velocity=u + v

- 21. 21. Distance travelled= average velocity x Time  $\longrightarrow$  S=(u + v) x t ......(1) From the first equation of motion we have ,v=u + at. Putting the value of v in equation (1), we get  $\longrightarrow$  S=(u+ u+ at) x t  $\longrightarrow$  S=2ut+ at2  $\longrightarrow$  S= ut+1at2
- 22. 22. Third Equation of Motion The third equation of motion is v=u+2as. It gives the velocity acquired by a body in traveling a distance 's' Consider a body having initial velocity 'u' and a uniform acceleration 'a' for time 't' so that its final velocity becomes 'v'. Let the distance traveled by the body in this time 's'. The distance travelled by a moving body in time 't' can be found out by considering its average velocity. Since the initial velocity of the body is 'u' and its final; velocity is 'v', the average velocity is given by: The third equation of motion can be obtained by eliminating t between the first two equation of motion. From second equation of motion we have; S=ut+1at ......(1)
- 23. 23. And from the first equation of motion we have; —> v=u + at This can be rearranged and written as —> at=v u —> t=v u Putting the value of tin equation (1)we get; —> S=u (v-a) + 1a(v-u/a) —> S=2uv-2u+v+u-2uv —> V=u+2as
- 24. 24. Circular motion A body is said to be in circular motion when it rotates about a fix point. In circular motion, the velocity can never be constant, but the speed of the moving body can be constant. A body moving in a circular path at a constant speed is said to be in uniform circular motion. In Uniform Circular motion, the object in one revolution moves 2 r in T seconds

#### Introduction

In our daily life, we see lots of things moving around for example car passing through from one place to other, person riding on a bicycle and many more like this.

In scientific terms an object is said to be in motion, if it changes its position with the passage of time and if it does not change it position with the passage of time then it is said to be at rest

Both the motion and rest are relative terms for example mobile kept on the table is resting at its position but it is moving in the sense as earth is rotating on its axis. So for a person seeing mobile from earth it is at rest and for person on moon earth seems to change its position with time and somobile is moving.

Simplest case of motion is rectilinear motion which is the motion of the object in a straight line

In our description of object, we will treat the object as a point object.

Object under consideration can be treated as point object if the size of the object is much smaller than the distance traveled by it in reasonable time duration for example length of a motor car traveling a distance of 500km can be neglected w.r.t. distance traveled by it.

## Motion along a straight line

The simplest type of motion is the motion along a straight line.

Two different quantities Distance and Displacement are used to describe the overall motion of an object and to locate its final position with reference to its initial position at a given time.

Distance in physics, is the length of the path (the line or curve) described by an object moving through space. Distance is independent of direction. Thus, such physical quantities that do not require direction for their complete description are called scalars.

When a body moves from one position to another the shortest distance between the initial and final position of the body along with its direction is known as displacement. Displacement has both direction and magnitude for its complete description and hence such physical quantities are called avectors.

The distance travelled by a moving body cannot be zero but the final displacement of a moving body can be zero.

If a body covers equal distances in equal intervals of time then it is said to be having uniform motion.

If a body covers unequal distances in equal intervals or equal distances in unequal intervals then body is said be having non-uniform motion.

## **Speed**

Speed is defined as the total distance travelled by the object in the time interval during which the motion takes place. SI unit of speed is meter per second. So,

$$speed = \frac{distance \ travelled}{time \ taken} = \frac{s}{t}$$

where s is the distance travelled by the body and t is the time taken by the body to travel distance s.

Speed of a body gives us the idea how slow or fast that body is moving. The ratio of total distance to total time taken by the body gives its average speed. The speed of a body at a given instant is its instantaneous speed.

$$speed = \frac{total\ distance\ travelled}{total\ time\ taken}$$

A body is said to have constant or uniform speed if it travels equal distance in equal intervals of time.

## Velocity

The rate of change of displacement of a body with the passage of time is known as velocity of the body. Velocity of an object is measured in meter per second in SI

$$speed = \frac{total\ distance\ travelled}{total\ time\ taken}$$

Velocity is nothing but the speed of an object moving in a definite direction.

The velocity of an object can be uniform or variable. It can be changed by changing the object's speed, direction of motion or both.

So velocity of a body is a vector quantity involving both distance and displacement whereas speed of a body is a scalar quantity and it only has magnitude and does not have specific direction.

Thus a body is said to be moving with uniform velocity if it covers equal distances in equal intervals of time in a specified direction.

A body is said to be moving with non uniform velocity if it covers unequal distances in equal intervals of time and vice-versa in a specified direction or if it changes the direction of motion.

The velocity of a body can be changed in two ways first by changing the speed of the body and second by changing the direction of motion of the body by keeping the speed constant. Also both speed and direction of the body can be varied in order to change the velocity of the body.

When velocity of the object changes at a uniform rate, then average velocity is given by the arithmetic mean of initial velocity and final velocity for a given period of time. That is,

$$v = \frac{displacement}{time\ taken}$$

Where u is the initial velocity of the object and v is the final velocity of the object. **acceleration** 

Acceleration is a measure of the change in the velocity of an object per unit time and mathematically it is given as

$$Acceleration = \frac{change \ in \ velocity}{time \ taken}$$

If the velocity of an object changes from an initial value u to the final value v in time t, the acceleration a is given by,

$$a = \langle frac\{\{v - u\}\}\{t\}$$

and this kind of motion is called accelerated motion.

A body has uniform acceleration if it travels in a straight line and its velocity increases by equal amount in equal intervals of time for example freely falling bodies, motion of ball rolling down the inclined plane etc.

A body has non uniform acceleration if its velocity increases or decreases by unequal amount in equal intervals of time.

If acceleration is in the direction of the velocity then it is positive acceleration and if it is in the direction opposite to the direction of velocity then it is negative and the negative acceleration is termed retardation.

SI unit of acceleration is ms<sup>-2</sup>.

## **Equations of uniformly accelerated motion**

There are three equations of bodies moving with uniform acceleration which we can use to solve problems of motion

### First Equation of motion

The first equation of motion is v=u+at, where v is the final velocity and u is the initial velocity of the body.

First equation of motion gives velocity acquired by body at any time t.

Now we know that acceleration

$$a = \frac{\textit{change in velocity}}{\textit{time taken}} = \frac{\textit{Final velocity} - \textit{initial velocity}}{\textit{time taken}}$$

so, a=v-ut

and, at=v-u

rearranging above equation we get first equation of motion that is v=u+at

## **Second Equation of motion**

Second equation of motion is

s=ut+12at2

where u is initial velocity, a is uniform acceleration and s is the distance travelled by body in time t.

Second equation of motion gives distance travelled by a moving body in time t.

To obtain second equation of motion consider a body with

initial velocity u moving with acceleration a for time t its final velocity at this time be v. If body covered distance s in this time t, then averagevelocity of the body would be

$$average \ velocity = \frac{initial \ velocity + final \ velocity}{2} = \frac{u + v}{2}$$

Distance travelled by the body is

$$s = \frac{u+v}{2} \times t$$

From first equation of motion

v=u+at

So putting first equation of motion in above equation we get,

 $s=u+u+at2\times t=(2u+at)t/2=2ut+at^2/2$ 

Rearranging it we get

 $s=ut+1/2at^2$ 

## Third equation of motion

Third equation of motion is

 $v^2=u^2+2as$  where u is initial velocity, v is the final velocity, a is uniform acceleration and s is the distance travelled by the body.

This equation gives the velocity acquired by the body in travelling a distance s. Third equation of motion can be obtained by eliminating time t between first and second equations of motion.

So, first and second equations of motion respectively are

v=u+at and  $s=ut+1/2at^2$ 

Rearranging first equation of motion to find time t we get t=v-u/a

Putting this value of t in second equation of motion we get  $s=u(v-u)a+1/2a(v-ua)^2$ 

s=uv-u/2a+a(v2+u2-2uv)/2a

## Rearranging it we get

 $v^2=u^2+2as$ 

These three equations of motion are used to solve uniformly accelerated motion problems and following three important points should be remembered while solving problems

if a body starts moving from rest its initial velocity u=0

if a body comes to rest i.e., it stops then its final velocity would be v=0

If a body moves with uniform velocity then its acceleration would be zero.

## **Graphical representation of motion**

A graph is a pictorial representation of the relation between two sets of data of which one set is of dependent variables and the other set is of independent variables.

To describe the motion of an object, we can use line graphs. In this case, line graphs show dependence of one physical quantity, such as distance or velocity, on another quantity, such as time.

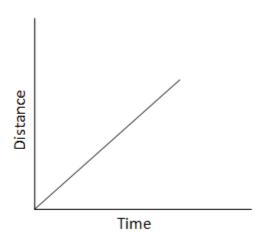
## **Distance Time Graphs**

The change in the position of an object with time can be represented on the distance-time graph.

In this graph, time is taken along the x-axis and distance is taken along the y-axis. Distance time graphs of a moving body can be used to calculate the speed of the body as they specifically represent velocity.

The distance time graph for a body moving at uniform speed is always a straight line as distance travelled by the body is directly proportional to time as shown below in the figure 1.

The distance time graph for a body moving with non uniform speed is a curve and is shown below in the figure 2.



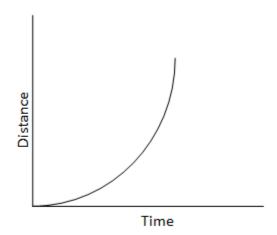
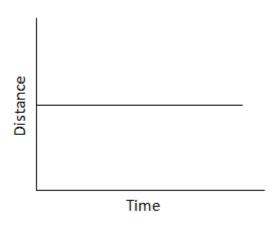
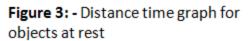


Figure 1: - Distance time graph for uniform speed

**Figure 2: -** Distance time graph for non uniform speed

The distance time graph is parallel to time axis when the object is at rest and is shown below in figure 3.





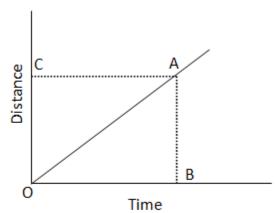


Figure 4: - Calculation of speed from distance time

To calculate speed of the body from distance time graph say at point A first draw a perpendicular AB on time axis and a perpendicular AC on distance axis so that AB represents the distance travelled by the body in time interval OB and since we know that

$$speed = \frac{distance\ travelled}{time\ taken} = \frac{AB}{OB}$$

### Velocity time graphs

The variation in velocity with time for an object moving in a straight line can be represented by a velocity-time graph.

In this graph, time is represented along the x-axis and the velocity is represented along the y-axis.

The product of velocity and time give displacement of an object moving with uniform velocity. The area enclosed by velocity-time graph and the time axis will be equal to the magnitude of the displacement.

If a body moves with a constant velocity then velocity time graph for this body would be straight line parallel to time axis as shown below in the figure 5

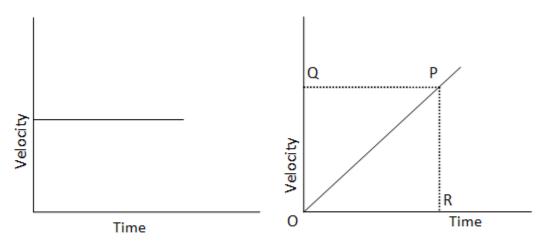


Figure 5: - Velocity time graph when speed remains constant (no acceleration)

**Figure 6: -** Velocity time graph showing uniform acceleration

The velocity time graph of uniformly changing velocity is shown in figure 6 and is a straight line. We can find out the value of acceleration using the velocity time graph.

For calculating acceleration at time corresponding to point R draw a perpendicular RP from point R as shown in figure 6 and we know that

$$a = \frac{change \ in \ velocity}{time \ taken}$$

Here change in velocity is represented by PR and time taken is equal to OR. So,

$$Acceleration = \frac{PR}{OR}$$

which is equal to the slope of velocity time graph. So we conclude that slope of velocity time graph of moving body gives its acceleration.

The distance travelled by moving body in a given time will be equal to area of triangle OPR as shown in figure 6

Distance travelled = Area of triangle 
$$OPR = \frac{1}{2}$$
 area of rectangle  $ORPQ$  so,

Distance travelled = 
$$\frac{1}{2} \times OQ \times OR$$

When the velocity of a body changes in an irregular manner then velocity time graph of the body is a curved line.

### Equations of motion by graphical method

We already know about equations of motion when an object moves along straight line with uniform acceleration. We already know how to derive them but these equations can also be derived by graphical method.

## **Equation for velocity time relation**

Consider the velocity-time graph of an object that moves under uniform acceleration as shown below in the figure 7.

From this graph, you can see that initial velocity of the object is u (at point A) and then it increases to v (at point B) in time t. The velocity changes at a uniform rate a.

Again from figure it is clear that time t is represented by OC, initial velocity u by OA and final velocity of object after time t by BC.

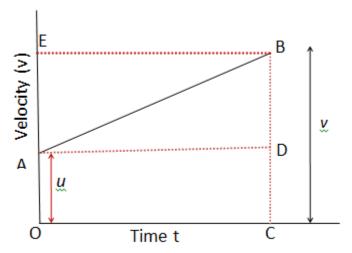


Figure 7:- v-t graph to derive equations of motion

From graph as given in figure 7 it is clear that BC=BD+DC=BD+OA.

So we have

$$v=BD+u$$
 (1)

We should now find out the value of BD. From the velocity-time graph (Fig. 7), the acceleration of the object is given by

$$a = \frac{change \ in \ velocity}{time \ taken} = \frac{BD}{AD} = \frac{BD}{OC} = \frac{BD}{t}$$

which gives, BD=at

putting this value of BD in equation 1 we get

v=u+at

which is the equation for velocity time relation.

## **Equation for position time relation**

Let us consider that the object has travelled a distance s in time t under uniform acceleration a. In Fig. 7, the distance travelled by the object is obtained by the area enclosed within OABC under the velocity-time graph AB.

Thus, the distance s travelled by the object is given by

s = area OABC (which is a trapezium)

s= area of the rectangle OADC + area of the triangle ABD So,

$$s=OA\times OC+(1/2)(AD\times BD)$$

Substituting OA=u, OC=AD=t and BD=at, we get

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s=(u\times t)+1/2\times(t\times at)
or,
s=ut+1/2at^2
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which is the equation of position time relation

## **Equation for position velocity relation**

Again consider graph in figure 7. We know that distance travelled s by a body in time t is given by the area under line AB which is area of trapezium OABC. So we have

 $distance \ travelled = s = Area \ of \ trapezium \ OABC$   $s = \frac{(sum \ of \ parallel \ sides) \times height}{2} = \frac{(OA + CB) \times OC}{2}$ 

Since OA+CB=u+v and OC=t, we thus have s=(u+v)t/2From velocity time relation t=v-ua putting this t in equation for s we get s=(u+v)2(v-ua) or we have  $v^2=u^2+2as$ 

which is equation for position velocity relation.

### Uniform circular motion

When an object moves in a circular path at a constant speed then motion of the object is called uniform circular motion.

In our everyday life, we came across many examples of circular motion for example cars going round the circular track and many more. Also earth and other planets revolve around the sun in a roughly circular orbits

If the speed of motion is constant for a particle moving in a circular motion still the particles accelerates because of constantly changing direction of the velocity. If an object moves in a circular path with uniform speed, its motion is called

If an object moves in a circular path with uniform speed, its motion is called uniform circular motion

Here in circular motion, we use angular velocity in place of velocity we used while studying linear motion.

Force which is needed to make body travel in a circular path is called centripetal force.

We know that the circumference of a circle of radius r is given by  $2\pi r$ . If the body takes t seconds to go once around the circular path of radius r, the velocity v is given by

 $v=2\pi rt$ 

One thing we must keep in mind is that uniform linear motion is not accelerated but uniform circular motion is accelerated motion.

## Examples of uniform circular motion are

- (a) Motion of artificial satellites around the earth
- (b) Moon, the natural satellite of earth, moves in uniform circular motion round the earth.
- (c) Cyclist moving on a circular track with a constant speed exhibits uniform circular motion.

An airplane accelerates down a runway at 3.20 m/s<sup>2</sup> for 32.8 s until is finally lifts off the ground. Determine the distance traveled before takeoff.

A car starts from rest and accelerates uniformly over a time of 5.21 seconds for a distance of 110 m. Determine the acceleration of the car.

Upton Chuck is riding the Giant Drop at Great America. If Upton free falls for 2.60 seconds, what will be his final velocity and how far will he fall?

A race car accelerates uniformly from 18.5 m/s to 46.1 m/s in 2.47 seconds. Determine the acceleration of the car and the distance travelled

A feather is dropped on the moon from a height of 1.40 meters. The acceleration of gravity on the moon is 1.67 m/s<sup>2</sup>. Determine the time for the feather to fall to the surface of the moon.

Rocket-powered sleds are used to test the human response to acceleration. If a rocket-powered sled is accelerated to a speed of 444 m/s in 1.83 seconds, then what is the acceleration and what is the distance that the sled travels?

A bike accelerates uniformly from rest to a speed of 7.10 m/s over a distance of 35.4 m. Determine the acceleration of the bike.

An engineer is designing the runway for an airport. Of the planes that will use the airport, the lowest acceleration rate is likely to be 3 m/s<sup>2</sup>. The takeoff speed for this plane will be 65 m/s. Assuming this minimum acceleration, what is the minimum allowed length for the runway?

A car traveling at 22.4 m/s skids to a stop in 2.55 s. Determine the skidding distance of the car (assume uniform acceleration).

A kangaroo is capable of jumping to a height of 2.62 m. Determine the takeoff speed of the kangaroo.

If Michael Jordan has a vertical leap of 1.29 m, then what is his takeoff speed and his hang time (total time to move upwards to the peak and then return to the ground)?

A bullet leaves a rifle with a muzzle velocity of 521 m/s. While accelerating through the barrel of the rifle, the bullet moves a distance of 0.840 m. Determine the acceleration of the bullet (assume a uniform acceleration).

A baseball is popped straight up into the air and has a hang-time of 6.25 s. Determine the height to which the ball rises before it reaches its peak. (Hint: the time to rise to the peak is one-half the total hang-time.)

The observation deck of tall skyscraper 370 m above the street. Determine the time required for a penny to free fall from the deck to the street below.

A bullet is moving at a speed of 367 m/s when it embeds into a lump of moist clay. The bullet penetrates for a distance of 0.0621 m. Determine the acceleration of the bullet while moving into the clay. (Assume a uniform acceleration.)

A stone is dropped into a deep well and is heard to hit the water 3.41 s after being dropped. Determine the depth of the well.

It was once recorded that a Jaguar left skid marks that were 290 m in length. Assuming that the Jaguar skidded to a stop with a constant acceleration of -3.90 m/s<sup>2</sup>, determine the speed of the Jaguar before it began to skid.

A plane has a takeoff speed of 88.3 m/s and requires 1365 m to reach that speed. Determine the acceleration of the plane and the time required to reach this speed.

A dragster accelerates to a speed of 112 m/s over a distance of 398 m. Determine the acceleration (assume uniform) of the dragster.

With what speed in miles/hr (1 m/s = 2.23 mi/hr) must an object be thrown to reach a height of 91.5 m (equivalent to one football field)? Assume negligible air resistance.

```
1
     Given:
                                                                                                                       Find:
     a = +3.2 \text{ m/s}^2
                                   t = 32.8 \text{ s}
                                                                           v_i = 0 \text{ m/s}
                                                                                                                       d = ??
d = v_i * t + 0.5 * a * t^2
d = (0 \text{ m/s})*(32.8 \text{ s}) + 0.5*(3.20 \text{ m/s}^2)*(32.8 \text{ s})^2
d = 1720 \text{ m}
2
     Given:
                                                                                                                       Find:
                                                                                                                       a = ??
     d = 110 \text{ m}
                                          t = 5.21 \text{ s}
                                                                             v_i = 0 \text{ m/s}
d = v_i * t + 0.5 * a * t^2
110 \text{ m} = (0 \text{ m/s})*(5.21 \text{ s}) + 0.5*(a)*(5.21 \text{ s})^2
110 \text{ m} = (13.57 \text{ s}^2)*a
a = (110 \text{ m})/(13.57 \text{ s}^2)
a = 8.10 \text{ m/ s}^2
3
     Given:
                                                                                                                       Find:
                                              t = 2.6 \text{ s} v_i = 0 \text{ m/s}
     a = -9.8 \text{ m}
                                                                                                                       d = ??
                                                                                                                       v_f = ??
d = v_i * t + 0.5 * a * t^2
d = (0 \text{ m/s})*(2.60 \text{ s}) + 0.5*(-9.8 \text{ m/s}^2)*(2.60 \text{ s})^2
d = -33.1 \text{ m} (- indicates direction)
v_f = v_i + a * t
v_f = 0 + (-9.8 \text{ m/s}^2)*(2.60 \text{ s})
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v_f = -25.5 m/s (- indicates direction)
4
      Given:
                                                                                                                           Find:
                                                v_f = 46.1 \text{ m/s}
                                                                                          t = 2.47 \text{ s}
                                                                                                                           d = ??
      v_i = 18.5 \text{ m/s}
                                                                                                                           a = ??
a = (Delta v)/t
a = (46.1 \text{ m/s} - 18.5 \text{ m/s})/(2.47 \text{ s})
a = 11.2 \text{ m/s}^2
d = v_i * t + 0.5 * a * t^2
d = (18.5 \text{ m/s})*(2.47 \text{ s}) + 0.5*(11.2 \text{ m/s}^2)*(2.47 \text{ s})^2
d = 45.7 \text{ m} + 34.1 \text{ m}
d = 79.8 \text{ m}
(Note: the d can also be calculated using the equation v_f^2 = v_i^2 + 2*a*d)
5
     Given:
                                                                                                                           Find:
                                                                         a = -1.67 \text{ m/s}^2
      v_i = 0 \text{ m/s}
                                      d = -1.40 \text{ m}
                                                                                                                           t = ??
d = v_i * t + 0.5 * a * t^2
-1.40 \text{ m} = (0 \text{ m/s})*(t) + 0.5*(-1.67 \text{ m/s}^2)*(t)^2
-1.40 \text{ m} = 0 + (-0.835 \text{ m/s}^2)*(t)^2
(-1.40 \text{ m})/(-0.835 \text{ m/s}^2) = t^2
1.68 \text{ s}^2 = t^2
t = 1.29 \text{ s}
6
     Given:
                                                                                                                           Find:
                                        v_f = 444 \text{ m/s}
                                                                                                                           a = ??
      v_i = 0 \text{ m/s}
                                                                                        t = 1.83 \text{ s}
                                                                                                                           d = ??
a = (Delta v)/t
a = (444 \text{ m/s} - 0 \text{ m/s})/(1.83 \text{ s})
a = 243 \text{ m/s}^2
d = v_i * t + 0.5 * a * t^2
d = (0 \text{ m/s})*(1.83 \text{ s}) + 0.5*(243 \text{ m/s}^2)*(1.83 \text{ s})^2
d = 0 \text{ m} + 406 \text{ m}
d = 406 \text{ m}
(Note: the d can also be calculated using the equation v_f^2 = v_i^2 + 2*a*d)
7
     Given:
                                                                                                                           Find:
      v_i = 0 \text{ m/s}
                                       v_f = 7.10 \text{ m/s}
                                                                                     d = 35.4 \text{ m}
                                                                                                                           a = ??
v_f^2 = v_i^2 + 2*a*d
(7.10 \text{ m/s})^2 = (0 \text{ m/s})^2 + 2*(a)*(35.4 \text{ m})
50.4 \text{ m}^2/\text{s}^2 = (0 \text{ m/s})^2 + (70.8 \text{ m})^*a
(50.4 \text{ m}^2/\text{s}^2)/(70.8 \text{ m}) = \text{a}
a = 0.712 \text{ m/s}^2
```

```
8
       Given:
                                                                                                                                             Find:
                                                                                                 a = 3 \text{ m/s}^2
       v_i = 0 \text{ m/s}
                                                 v_f = 65 \text{ m/s}
                                                                                                                                             d = ??
v_f^2 = v_i^2 + 2*a*d
(65 \text{ m/s})^2 = (0 \text{ m/s})^2 + 2*(3 \text{ m/s}^2)*d
4225 \text{ m}^2/\text{s}^2 = (0 \text{ m/s})^2 + (6 \text{ m/s}^2)*\text{d}
(4225 \text{ m}^2/\text{s}^2)/(6 \text{ m/s}^2) = d
d = 704 \text{ m}
9
       Given:
                                                                                                                                             Find:
       v_i = 22.4 \text{ m/s}
                                                            v_f = 0 \text{ m/s}
                                                                                                     t = 2.55 \text{ s}
                                                                                                                                             d = ??
d = (v_i + v_f)/2 *t
d = (22.4 \text{ m/s} + 0 \text{ m/s})/2 *2.55 \text{ s}
d = (11.2 \text{ m/s})*2.55 \text{ s}
d = 28.6 \text{ m}
10
      Given:
                                                                                                                                             Find:
      a = -9.8 \text{ m/s}^2
                                                                                                 d = 2.62 \text{ m}
                                                         v_f = 0 \text{ m/s}
                                                                                                                                             v_i = ??
v_f^2 = v_i^2 + 2*a*d
(0 \text{ m/s})^2 = v_i^2 + 2*(-9.8 \text{ m/s}^2)*(2.62 \text{ m})

0 \text{ m}^2/\text{s}^2 = v_i^2 - 51.35 \text{ m}^2/\text{s}^2
51.35 \text{ m}^2/\text{s}^2 = \text{v}_i^2
v_i = 7.17 \text{ m/s}
11
       Given:
                                                                                                                                             Find:
       a = -9.8 \text{ m/s}^2
                                                         v_f = 0 \text{ m/s}
                                                                                                 d = 1.29 \text{ m}
                                                                                                                                             v_i = ??
                                                                                                                                             t = ??
v_f^2 = v_i^2 + 2*a*d
(0 \text{ m/s})^2 = v_i^2 + 2*(-9.8 \text{ m/s}^2)*(1.29 \text{ m})
0 \text{ m}^2/\text{s}^2 = \text{v}_1^2 - 25.28 \text{ m}^2/\text{s}^2
25.28 \text{ m}^2/\text{s}^2 = \text{v}_i^2
v_i = 5.03 \text{ m/s}
To find hang time, find the time to the peak and then double it.
v_f = v_i + a * t
0 \text{ m/s} = 5.03 \text{ m/s} + (-9.8 \text{ m/s}^2)*t_{up}
-5.03 \text{ m/s} = (-9.8 \text{ m/s}^2)*t_{up}
(-5.03 \text{ m/s})/(-9.8 \text{ m/s}^2) = t_{up}
t_{up} = 0.513 \text{ s}
hang time = 1.03 \text{ s}
12
```

Given:

Find:

```
v_i = 0 \text{ m/s}
                                         v_f = 521 \text{ m/s}
                                                                                      d = 0.840 \text{ m}
                                                                                                                                  a = ??
v_f^2 = v_i^2 + 2*a*d
(521 \text{ m/s})^2 = (0 \text{ m/s})^2 + 2*(a)*(0.840 \text{ m})
271441 \text{ m}^2/\text{s}^2 = (0 \text{ m/s})^2 + (1.68 \text{ m})*\text{a}
(271441 \text{ m}^2/\text{s}^2)/(1.68 \text{ m}) = \text{a}
a = 1.62*10^5 \text{ m/s}^2
13
                                                                                                                                  Find:
      Given:
      a = -9.8 \text{ m/s}^2
                                                       v_f = 0 \text{ m/s}
                                                                                              t = 3.13 \text{ s}
                                                                                                                                  d = ??
(NOTE: the time required to move to the peak of the trajectory is one-half the total hang time -
3.125 s.)
First use: v_f = v_i + a * t
0 \text{ m/s} = v_i + (-9.8 \text{ m/s}^2)*(3.13 \text{ s})
0 \text{ m/s} = v_i - 30.7 \text{ m/s}
v_i = 30.7 \text{ m/s} (30.674 \text{ m/s})
Now use: v_f^2 = v_i^2 + 2*a*d
(0 \text{ m/s})^2 = (30.7 \text{ m/s})^2 + 2*(-9.8 \text{ m/s}^2)*(d)
0 \text{ m}^2/\text{s}^2 = (940 \text{ m}^2/\text{s}^2) + (-19.6 \text{ m/s}^2)*d
-940 \text{ m}^2/\text{s}^2 = (-19.6 \text{ m/s}^2)*\text{d}
(-940 \text{ m}^2/\text{s}^2)/(-19.6 \text{ m/s}^2) = d
d = 48.0 \text{ m}
14
                                                                                                                                  Find:
      Given:
                                                                                  a = -9.8 \text{ m/s}^2
                                          d = -370 \text{ m}
                                                                                                                                  t = ??
      v_i = 0 \text{ m/s}
d = v_i * t + 0.5 * a * t^2
-370 \text{ m} = (0 \text{ m/s})*(t) + 0.5*(-9.8 \text{ m/s}^2)*(t)^2
-370 \text{ m} = 0 + (-4.9 \text{ m/s}^2)*(t)^2
(-370 \text{ m})/(-4.9 \text{ m/s}^2) = t^2
75.5 \text{ s}^2 = \text{t}^2
t = 8.69 \text{ s}
15
      Given:
                                                                                                                                  Find:
      v_i = 367 \text{ m/s}
                                                v_f = 0 \text{ m/s}
                                                                                    d = 0.0621 \text{ m}
                                                                                                                                  a = ??
v_f^2 = v_i^2 + 2*a*d
(0 \text{ m/s})^2 = (367 \text{ m/s})^2 + 2*(a)*(0.0621 \text{ m})
0 \text{ m}^2/\text{s}^2 = (134689 \text{ m}^2/\text{s}^2) + (0.1242 \text{ m})*\text{a}
-134689 \text{ m}^2/\text{s}^2 = (0.1242 \text{ m})*\text{a}
(-134689 \text{ m}^2/\text{s}^2)/(0.1242 \text{ m}) = \text{a}
a = -1.08*10^6 \text{ m/s}^2
(The - sign indicates that the bullet slowed down.)
```

```
Given:
                                                                                                                                        Find:
      a = -9.8 \text{ m/s}^2
                                                          t = 3.41 \text{ s}
                                                                                                v_i = 0 \text{ m/s}
                                                                                                                                        d = ??
d = v_i * t + 0.5 * a * t^2
d = (0 \text{ m/s})*(3.41 \text{ s}) + 0.5*(-9.8 \text{ m/s}^2)*(3.41 \text{ s})^2
d = 0 \text{ m} + 0.5*(-9.8 \text{ m/s}^2)*(11.63 \text{ s}^2)
d = -57.0 \text{ m}
(NOTE: the - sign indicates direction)
17
      Given:
                                                                                                                                        Find:
      a = -3.90 \text{ m/s}^2
                                                           v_f = 0 \text{ m/s}
                                                                                                 d = 290 \text{ m}
                                                                                                                                        v_i = ??
v_f^2 = v_i^2 + 2*a*d
(0 \text{ m/s})^2 = v_i^2 + 2*(-3.90 \text{ m/s}^2)*(290 \text{ m})
0 \text{ m}^2/\text{s}^2 = v_1^2 - 2262 \text{ m}^2/\text{s}^2
2262 \text{ m}^2/\text{s}^2 = \text{v}_i^2
v_i = 47.6 \text{ m/s}
18
      Given:
                                                                                                                                        Find:
                                           v_f = 88.3 \text{ m/s}
                                                                                            d = 1365 \text{ m}
                                                                                                                                        a = ??
      v_i = 0 \text{ m/s}
                                                                                                                                        t = ??
v_f^2 = v_i^2 + 2*a*d
(88.3 \text{ m/s})^2 = (0 \text{ m/s})^2 + 2*(a)*(1365 \text{ m})
7797 \text{ m}^2/\text{s}^2 = (0 \text{ m}^2/\text{s}^2) + (2730 \text{ m})*\text{a}
7797 \text{ m}^2/\text{s}^2 = (2730 \text{ m})*\text{a}
(7797 \text{ m}^2/\text{s}^2)/(2730 \text{ m}) = \text{a}
a = 2.86 \text{ m/s}^2
v_f = v_i + a * t
88.3 \text{ m/s} = 0 \text{ m/s} + (2.86 \text{ m/s}^2)*t
(88.3 \text{ m/s})/(2.86 \text{ m/s}^2) = t
t = 30.8 s
19
      Given:
                                                                                                                                        Find:
      v_i = 0 \text{ m/s}
                                                                                               d = 398 \text{ m}
                                                                                                                                        a = ??
                                             v_f = 112 \text{ m/s}
v_f^2 = v_i^2 + 2*a*d
(112 \text{ m/s})^2 = (0 \text{ m/s})^2 + 2*(a)*(398 \text{ m})
12544 \text{ m}^2/\text{s}^2 = 0 \text{ m}^2/\text{s}^2 + (796 \text{ m})*\text{a}
12544 \text{ m}^2/\text{s}^2 = (796 \text{ m})*\text{a}
(12544 \text{ m}^2/\text{s}^2)/(796 \text{ m}) = \text{a}
a = 15.8 \text{ m/s}^2
20
                                                                                                                                        Find:
      Given:
```

$$a = -9.8 \text{ m/s}^2$$
  $v_f = 0 \text{ m/s}$   $d = 91.5 \text{ m}$   $v_i = ??$   $t = ??$ 

First, find speed in units of m/s:

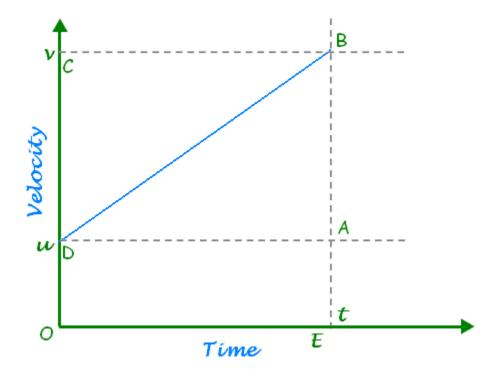
$$v_i^2 = v_i^2 + 2*a*d$$
  
 $(0 \text{ m/s})^2 = v_i^2 + 2*(-9.8 \text{ m/s}^2)*(91.5 \text{ m})$   
 $0 \text{ m}^2/s^2 = v_i^2 - 1793 \text{ m}^2/s^2$   
 $1793 \text{ m}^2/s^2 = v_i^2$   
 $v_i = 42.3 \text{ m/s}$   
Now convert from m/s to mi/hr:

 $v_i = 42.3 \text{ m/s} * (2.23 \text{ mi/hr})/(1 \text{ m/s})$ 

 $v_i = 94.4 \text{ mi/hr}$ 

## **Equation for Velocity – Time relation by graphical method – First** equation of Motion -

Let an object is moving with uniform acceleration.



Let the initial velocity of the object = u

Let the object is moving with uniform acceleration, a.

Let object reaches at point B after time, t and its final velocity becomes, v

Draw a line parallel to x-axis DA from point, D from where object starts moving.

Draw another line BA from point B parallel to y-axis which meets at E at y-axis.

Now, from the graph,

$$BE = AB + AE$$

$$\Rightarrow$$
 v = DC + OD (Since, AB = DC and AE = OD)

$$\Rightarrow$$
 v = DC + u (Since, OD = u)

$$\Rightarrow$$
 v = DC + u ----- (i)

$$Now, Acceleration (a) = \frac{Change in velocity}{Time \ taken}$$

$$\Rightarrow a = \frac{v - u}{t} = \frac{OC - OD}{t} = \frac{DC}{t}$$

$$\Rightarrow$$
 at = DC - - - - - - (ii)

By subtituting the value of DC from (ii)in (i)we get

$$v = at + u$$

$$\Rightarrow v = u + at$$

Above equation is the relation among initial velocity (u), final velocity (u), acceleration (a) and time (t). It is called first equation of motion.

## **Equation for distance –time relation:**

Distance covered by the object in the given time 't' is given by the area of the trapezium ABDOE

Let in the given time, t the distance covered by the moving object = s

The area of trapezium, ABDOE

= Distance (s) = Area of 
$$\triangle ABD + Area of ADOE$$

$$\Rightarrow s = \frac{1}{2} \times AB \times AD + (OD \times OE)$$

$$\Rightarrow s = \frac{1}{2} \times DC \times AD + (u \times t) \text{ [Since, AB = DC]}$$

$$\Rightarrow s = \frac{1}{2} \times at \times t + ut \quad [Since, DC = at from equation (ii)]$$

$$\Rightarrow s = \frac{1}{2}at^2 + ut$$

$$\Rightarrow s = ut + \frac{1}{2}at^2$$

The above expression gives the distance covered by the object moving with uniform acceleration. This expression is known as second equation of motion.

# **Equation for Distance Velocity Relation: Third equation of Motion:**

The distance covered by the object moving with uniform acceleration is given by the area of trapezium ABDO

Therefore,

Area of trapezium ABDOE

$$=\frac{1}{2}\times sum\ of\ parallel\ sides + distance\ between\ parallel\ sides$$
 
$$\Rightarrow Distance\ (s) = \frac{1}{2}(DO + BE)\ \times OE$$
 
$$\Rightarrow s = \frac{1}{2}(u+v)\times t - - - - - - - (iii)$$
 
$$Now, from\ equation\ (ii)\ a = \frac{v-u}{t}$$
 
$$Therefore, t = \frac{v-u}{a} - - - - - - (iv)$$
 
$$After\ substituting\ the\ value\ of\ t\ from\ equation\ (iv)in\ equation\ (iii)$$

$$\Rightarrow s = \frac{1}{2}(u+v) \times \frac{(v-u)}{a}$$

$$\Rightarrow s = \frac{1}{2a}(v+u)(v-u)$$

$$\Rightarrow 2as = (v+u)(v-u)$$

$$\Rightarrow 2as = v^2 - u^2$$

$$\Rightarrow 2as + u^2 = v^2$$

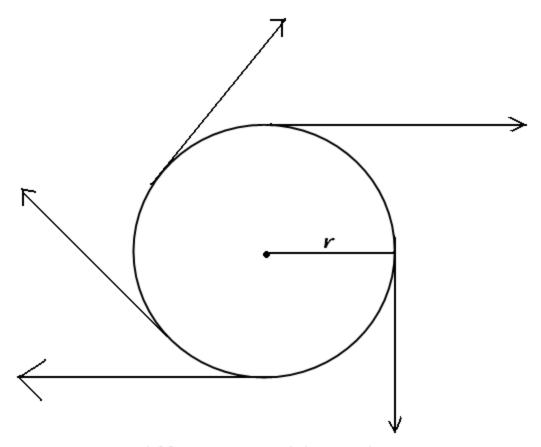
$$\Rightarrow v^2 = u^2 + 2as$$

The above expression gives the relation between position and velocity and is called the third equation of motion.

While solving the problems related to velocity, distance, time and acceleration following three points should be considered:

## Motion along a circular path:

Motion of an object along a circular path is called circular motion. Since, on a circular path the direction of the object is changing continuously to keep it on the path, the motion of the object is called accelerated motion.



Direction at different point while circular motion

## Velocity in the case of circular motion.

If the radius of circle is 'r'

Therefore, circumference =  $2\pi r$ 

Let time 't' is taken to complete one rotation over a circular path by any object

Therefore, velocity 
$$(v) = \frac{Distance}{time}$$

$$\Rightarrow v = \frac{Circumference}{t}$$

$$\Rightarrow v = \frac{2\pi r}{t}$$

Where, v = velocity, r = radius of circular path and t = time

Motion of earth around the sun, motion of moon around the earth, motion of a top, motion of blades of an electric fan, etc. are the examples of circular motion.

Question 1. A worker covers a distance of 40 km from his house to his place of work, and 10 km towards his house back. Then the displacement covered by the worker in the whole trip is

- (a) zero km
- (b) 10 km
- (c) 30 km
- (d) 50 km

Answer. (c) 30 km

### Question 2. Rate of change of displacement is called

- (a) Speed
- (b) Deceleration
- (c) Acceleration
- (d) Velocity

**Answer.** d) Velocity

### Question 3. Acceleration is a vector quantity, which indicates that its value

- (a) Is always negative
- (b) Is always positive
- (c) Is zero
- (d) Can be positive, negative or zero

**Answer.** (d) Can be positive, negative or zero

Question 4. A player moves along the boundary of a square ground of side 50 m in 200 sec. The magnitude of displacement of the farmer at the end of 11 minutes 40 seconds from his initial position is

- (a) 50 m
- (b) 150 m
- (c) 200 m
- (d)  $50\sqrt{2}$  m

**Answer.** (d)  $50\sqrt{2}$  m

Question 5. An object travels 40m in 5 sec and then another 80m in 5 sec. What is the average speed of the object?

- (a) 12 m/s
- (b) 6 m/s
- (c) 2 m/s
- (d) 0 m/s

Answer. (a) 12m/s

Question 6. The average velocity of a body is given by the expression :

(a) V = u + at

(b)  $2as = v^2 - u^2$ 

(c)  $V_{av} = (u + v)/2$ 

(d)  $S = ut + \frac{1}{2} a t^2$ 

**Answer.** (c)  $V_{av} = (u + v)/2$ 

### Question 7. SI Unit of measurement of acceleration is

(a) m/s

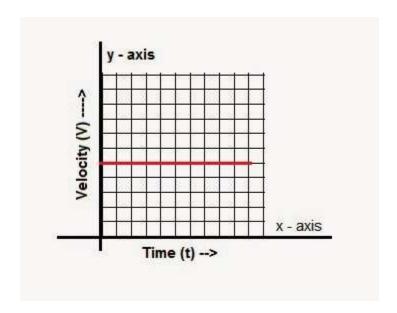
(b)  $m/s^2$ 

(c) m/hr

(d) M

**Answer.** (b) m/s<sup>2</sup>

### **Question 8. From the given Velocity-Time graph (figure)**



## it can be inferred that the object is moving with

- (a) Non uniform velocity
- (b) moving with uniform acceleration
- (c) At rest
- (d) Uniform velocity

Answer. (d) Uniform velocity

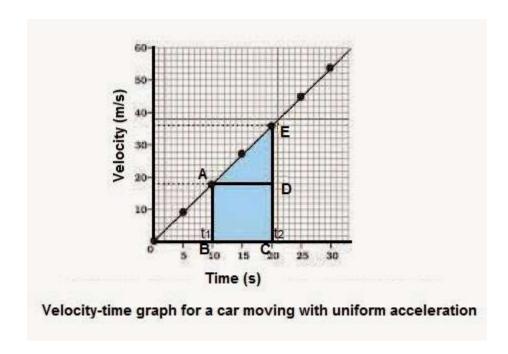
## Question 9. The acceleration of a body from a velocity –time graph is

- (a) Is denoted by a line parallel to the time axis at any point on the distance axis
- (b) Equal to the slope of the graph
- (c) Area under the graph
- (d) Is denoted by a line parallel to the distance axis at any point on the time axis **Answer.** (b) Equal to the slope of the graph

### Question 10. Distance covered by a body from velocity-time graph is

- (a) Is denoted by a line parallel to the distance axis at any point on the time axis
- (b) Is denoted by a line parallel to the time axis at any point on the distance axis
- (c) Equal to the slope of the graph
- (d) Area under the graph

**Answer.** (d) Area under the graph



From the above given Velocity-Time graph, answer the following questions (Q11 to Q15): -

### Question 11. From the above Velocity-Time graph, the body is moving with

- (a) Variable Acceleration
- (b) Zero Acceleration
- (c) Constant Acceleration
- (d) Zero velocity

Answer. (c) Constant Acceleration

# Question 12. Distance covered by the body during the interval from 10sec to 20 sec is

- (a) 180
- (b) 200
- (c) 240
- (d) 270

**Answer.** (d) 270

### Question 13. At the point A the body is at a distance of

- (a) 90m
- (b) 180m
- (c) 270m
- (d) 350m

Answer. 90m

### Question 14. The velocity of the body at the point 'B' is

- (a) 20m/s
- (b) 24m/s
- (c) 32m/s
- (d) 36m/s

Answer. (d) 36m/s

# Question 15. In the total journey, starting from the rest, the body has traveled up to a distance of

- (a) 270m
- (b) 360m
- (c) 450m
- (d) 540m

Answer. (b) 360m

### Question 16. What does the slope of distance - time graph give?

- (a) Acceleration
- (b) Uniform speed
- (c) Speed
- (d) both[b] and [c] depending upon the time of graph

Answer. (c) Speed

# Question 17. An example of a body moving with constant speed but still accelerating is

- (a) A body moving with constant speed on a straight road
- (b) A body moving in a helical path with constant speed
- (c) A body moving with constant speed in a circular path
- (d) A body moving with constant speed on a straight railway track

**Answer.** (c) A body moving with constant speed in a circular path