Chapter 2

Quadratic Equations

Objective Questions

SINGLE CORRECT

1. If m and n are are roots of this quadratic equation $x^2+2x-8=0$ and the roots of $x^2+10x-16p=0$ are $3m$ and $4n$, then the value of 'p' is :
   (A) -6 (B) 6 (C) 7 (D) 8

2. If α and β are the roots of the equation $x^2+x+k=0$ then $\left(\frac{\alpha-1}{2}\right)\left(\frac{\beta-1}{2}\right)$ is :
   (A) k (B) $\frac{k-1}{2}$ (C) $\frac{k+1}{2}$ (D) $\frac{k+2}{4}$

3. The roots of the quadratic equation $x^2-2015x+k=0$ are prime numbers, then k is equal to :
   (A) 4022 (B) 4026 (C) 2017 (D) 2016

4. Number of integral x satisfying the equation $(x^2-5x+5)^{x+5}=1$ is :
   (A) 1 (B) 2 (C) 3 (D) 4

5. If r,s,t are the roots of the equation $8x^3+1001x+2016=0$, then $(r+s)^3+(s+t)^3+(t+r)^3$ is equal to :
   (A) 751 (B) 752 (C) 753 (D) 756

6. Number of values of 'a' for which $Q(x)=(2-a)x^2-2ax-1$ is a perfect square of a linear expression with real coefficients :
   (A) 0 (B) 1 (C) 2 (D) 3

7. Let $P(x)=P(0)+P(1)x+P(2)x^2$ be a polynomial such that $P(-1)=1$, then $P(3)$ is :
   (A) 1 (B) 2 (C) 3 (D) 5

8. Let a,b,c,x are real number such that $(a+b)(b+c)(c+a)\neq 0$ and $\frac{a^2}{a+b}=\frac{a^2}{a+c}+20$, $\frac{b^2}{b+c}=\frac{b^2}{b+a}+14$ and $\frac{c^2}{c+a}=\frac{c^2}{c+b}+x$, then the value of x is :
   (A) -30 (B) -32 (C) -33 (D) -34

9. For $a, b \in Q - \{0\}$, if the quadratic equations $ax^2 - bx + 3 = 0$ and $x^2 - 4x + 1 = 0$ have a common root, then the area of the rectangle having sides as a and b is equal to :

(A) 18 (B) 24 (C) 36 (D) 72

10. If α, β are the roots of the equation $2x^2 + 3x - 4 = 0$, then the equation whose roots are $\frac{1}{\alpha - 2}$ and $\frac{1}{\beta - 2}$ is equal to :

(A) $10x^2 - 11x + 2 = 0$ (B) $10x^2 + 11x + 2 = 0$

(C) $11x^2 - 10x + 2 = 0$ (D) $x^2 + 11x - 10 = 0$

11. The roots of the cubic $x^3 + (\pi - 1)x^2 - \pi = 0$, are

(A) all three real and distinct.

(B) One real and two coincident.

(C) One real and two imaginary with product of the imaginary roots being π

(D) One real, two imaginary with sum of the imaginary root being π.

12. If α, β, γ are roots of the cubic $2011x^3 + 2x^2 + 1 = 0$, then which of the following relations is (are) correct?

(A) $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = -2011$ (B) $(\alpha\beta)^{-1} + (\beta\gamma)^{-1} = (\gamma\alpha)^{-1} = 2$

(C) $\alpha^2 + \beta^2 + \gamma^2 = \frac{4}{2011}$ (D) $\alpha^{-2} + \beta^{-2} + \gamma^2 = 4$

13. While applying the quadratic formula to a quadratic equation $f(x) = 0$, where $f(x) = ax^2 + bx + c (a, b, c \in R$ and $a \neq 0)$, it happens that $c = \frac{b^2}{4a}$, then the graph of $y = f(x)$ will certainly.

(A) have a maximum (B) have a minimum

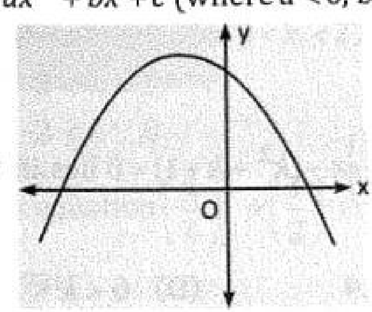
(C) be tangent to the x-axis (D) lie completely above x-axis

14. Number of integral values of k for which exactly one root of the equation $5x^2 + (k+1)x + k = 0$ lies in the interval $(1, 3)$ is equal to :

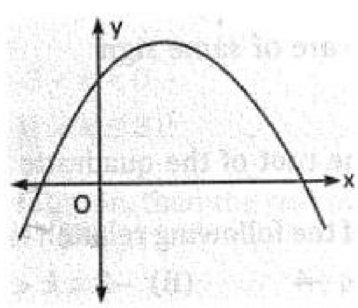
(A) 7 (B) 8 (C) 9 (D) 10

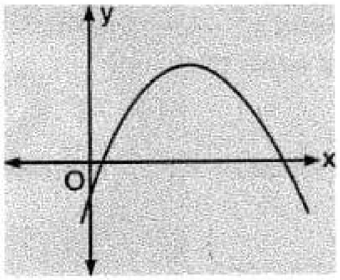
15. Which one of the following can best represent the graph of quadratic polynomial $y = ax^2 + bx + c$ (where $a < 0$, $b < 0$ and $c > 0$)?

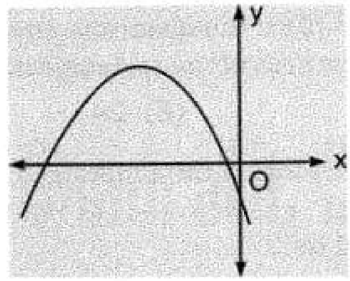
(A)



(B)



(C) 

(D) 

16. If the roots α, β of the equation $px^2 + qx + r = 0$ are real and of opposite sign (where p, q, r are real coefficient), then the roots of the equation $\alpha(x-\beta)^2 + \beta(x-\alpha)^2 = 0$ are

(A) positive (B) negative
(C) real and of opposite sign (D) imaginary

17. For $a, b, c \in R$ and $b^2 \geq 4ac$, if all the roots of the equation $ax^4 + bx^2 + c = 0$ are real, then :

(A) $b > 0, a < 0, c > 0$ (B) $b < 0, a > 0, c > 0$
(C) $b > 0, a > 0, c > 0$ (D) $b > 0, a < 0, c > 0$

18. The number of integral values of α for which the inequality $x^2 - 2(4\alpha - 1)x + 15\alpha^2 > 2\alpha + 7$ is true for every $x \in R$, is :

(A) 0 (B) 1 (C) 2 (D) 3

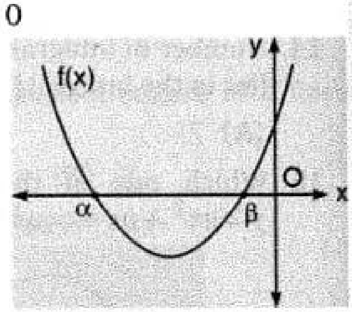
19. If two roots of the equation $(x-1)(2x^2 - 3x + 4) = 0$ coincide with roots of the equation $x^3 + (a+1)x^2 + (a+b)x + b = 0$ where $a, b \in R$, then $2(a+b)$ equals :

(A) 4 (B) 2 (C) 1 (D) 0

20. Let k be a real number such that $k \neq 0$. If α and β are non-zero complex numbers satisfying $\alpha + \beta = -2k$ and $\alpha^2 + \beta^2 = 4k^2 - 2k$, then a quadratic equation having $\frac{\alpha+\beta}{\alpha}$ and $\frac{\alpha+\beta}{\beta}$ as its roots is equal to :

(A) $4x^2 - 4kx + k = 0$ (B) $x^2 - 4kx + 4k = 0$
(C) $4kx^2 - 4x + k = 0$ (D) $4kx^2 - 4kx + 1 = 0$

21. The following figure shows the graph of $f(x) = ax^2 - bx + c$. Then which one of the following is correct ?



(A) $\frac{b}{c} > 0$

(B) a and c are of opposite sign

(C) a and b are of same sign

(D) None

22. If exactly one root of the quadratic equation $x^2 - \left(k + \frac{11}{3}\right)x - (k^2 + k + 1) = 0$ lies in $(0, 3)$ then which one of the following relation is correct?

(A) $-8 < k < -4$ (B) $-3 < k < -1$ (C) $1 < k < 4$ (D) $6 < k < 10$

23. Let a_1 and a_2 be two values of a for which the expression $f(x, y) = 2x^2 + 3xy + y^2 + ay + 3x + 1$ can be factorised into two linear factors then the product $(a_1 a_2)$ is equal to :

(A) 1 (B) 3 (C) 5 (D) 7

24. Let a, b and c be three distinct real roots of the cubic $x^3 + 2x^2 - 4x - 4 = 0$. If the equation $x^3 + qx^2 + rx + s = 0$ has roots $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$, then the value of $(q + r + s)$ is equal to :

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

25. Number of ordered pairs (x, y) of real numbers satisfying the equation $x^2 + y^2 - 24x - 26y + 313 = 0$ is equal to :

(A) infinite (B) finite but more than one
(C) exactly one (D) zero

26. If the roots of the quadratic equation $ax^2 + bx + c = 0$ are $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$ then $\left(\frac{a}{a+b+c}\right)^2$ equals :

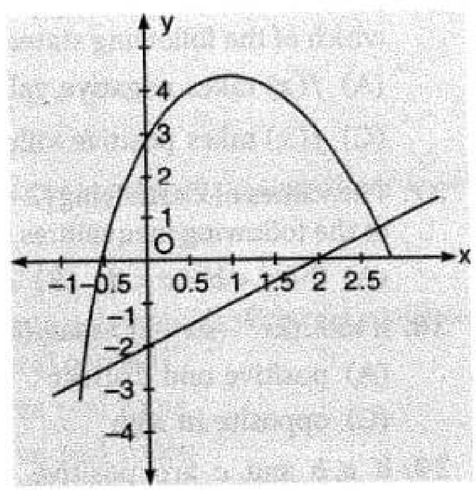
(A) k^2 (B) $(k+1)^2$ (C) $(k+2)^2$ (D) $k^2(k+1)^2$

27. If $c^2 = 4d$ and the two equations $x^2 - ax + b = 0$ and $x^2 - cx + d = 0$ have one common root, then the value of $2(b + d)$ is equal to :

(A) $\frac{a}{c}$ (B) ac (C) $2ac$ (D) $a + c$

28. The maximum vertical distance d between the parabola $y = -2x^2 + 4x + 3$ and the line $y = x - 2$ throughout the bounded region in the figure, is :

(A) $\frac{47}{8}$ (B) $\frac{49}{8}$
(C) $\frac{50}{8}$ (D) $\frac{48}{8}$



29. All values of k such that the quadratic equation $-2x^2 + kx + k^2 + 5 = 0$ has two distinct roots and only one of the roots satisfies $0 < x < 2$, is :

(A) $-3 < k < 1$ (B) $-3 < k < 0$
C) $-2 < k < 0$ (D) $-1 < k \leq 3$

30. If the equation $\left(\frac{1}{4}\right)^x + \left(\frac{1}{2}\right)^{x-1} + b = 0$ has a positive solution, then the real number b lies in the interval :

(A) $(-\infty, 0)$ (B) $(-\infty, -1)$ (C) $(-3, 1)$ (D) $(-3, 0)$

31. Range of the function $f(x)=\dfrac{x^2+3x+4}{x^2+3x+3}$ is equal to :

(A) $\left[1, \dfrac{7}{3}\right]$ (B) $[1, \infty)$ (C) $\left[1, \dfrac{7}{3}\right)$ (D) $\left(1, \dfrac{7}{3}\right]$

32. the solution of the inequality $\dfrac{x+5}{2x-1}>3$ is an interval. The length of the interval is :

(A) $\dfrac{7}{6}$ (B) $\dfrac{8}{7}$ (C) $\dfrac{9}{8}$ (D) $\dfrac{11}{10}$

33. Let x and y be real numbers satisfying the equation $4y^2+4xy+x+6=0$. the complete set of values for x, is :

(A) $\{x : -2 \le x \le 3\}$ (B) $\{x : x \le 2 \text{ or } x \ge 3\}$

(C) $\{x : -2 \le x \le 2\}$ (D) $\{x : x \le -2 \text{ or } x \ge 3\}$

34. If the equation $x^2+ax+b=0$ has distinct real roots and $x^2+a|x|+b=0$ has only one real root, then which one of the following is true?

(A) $b=0, a>0$ (B) $b=0, a<0$ (C) $b>0, a<0$ (D) $b<0, a>0$

35. One solution of the equation $x^3+5x^2-2x-4=0$ is $x=1$. Another solution of the given equation, is :

(A) $-3+\sqrt{5}$ (B) $-2+\sqrt{5}$ (C) $-5+\sqrt{3}$ (D) $-5+\sqrt{2}$

36. Consider the function $f(x)=ax^2+bx+c$ where $f(0)=6$ and $f(\sqrt{2})=f(\sqrt{3})=0$, $a, b, c \in R$, then which of the following statements is incorrect?

(A) $f(x)$ takes negative values in $(\sqrt{2}, \sqrt{3})$ (B) $f(x)$ takes positive values in $(\sqrt{3}, \infty)$

(C) $f(x)$ takes positive values in $(-\infty, \sqrt{2})$ (D) $a=6$

37. The values of x satisfying $|2-|2x-4||=1$ are distinct and can be ordered as $a<b<c<d$. Which of the following inequalities **hold good**?

(A) $0<a<b<c<d$ (B) $a<0<b<c<d$ (C) $a<b<0<c<d$ (D) $a<b<c<0<d$

38. If min. $(2x^2-ax+2)>$ max. $(b-1+2x-x^2)$ then roots of the equation $2x^2+ax+(2-b)=0$ are :

(A) positive and distinct (B) negative and distinct

(C) opposite in sign (D) imaginary

39. If a, b and c are positive real numbers such that $a>b>c$ and the quadratic equation $(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)=0$ has a root in the interval $(-1, 0)$ then the roots of the equation $x^2+(a+c)x+4b^2=0$ are :

(A) imaginary (B) real and unequal

(C) real and equal (D) less than -1

40. Let $f(x)=ax^2+bx+c$, where $a, b, c \in \boldsymbol{R}$ and $a \ne 0$. If $a^2+c^2-b^2+2ac<0$, then which one of the following statement is correct?

(A) $f(x)=0$ has imaginary roots. (B) $f(x)=0$ has equal roots.

(C) $f(x)=0$ has real and distinct roots. (D) Roots of $f(x)=0$ lies in the interval $(2, 3)$.

41. If α and β are the roots of equation $x^2 - a(x+1) - b = 0$ where $a, b \in R - \{0\}$ and $a + b \neq 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} - \frac{2}{a+b}$ is equal to :

(A) $\frac{4}{a+b}$ (B) $\frac{2}{a+b}$ (C) 0 (D) $\frac{1}{a+b}$

42. Let ABC be fixed triangle and P be variable point in the plane of triangle ABC. Suppose a, b, c are lengths of sides BC, CA, AB opposite to angles A, B, C respectively. If $a(PA)^2 + b(PB)^2 + c(PC)^2$ is minimum, then the point P with respect to ΔABC, is :

(A) centroid (B) circumcentre (C) orthocentre (D) incentre

43. The range of real values of 'p' for which the equation $2\log_3^2 x - |\log_3 x| + p = 0$ has four distinct solutions is :

(A) $\left(0, \frac{1}{8}\right)$ (B) $\left(0, \frac{1}{4}\right)$ (C) $\left(0, \frac{1}{3}\right)$ (D) $(0, 1)$

44. If $a, b, c, d \in R$ then the equation $(x^2 + ax + 3b)(-x^2 + cx + b)(-x^2 + dx - 2b) = 0$ has :

(A) 6 real roots (B) atleast two real roots

(C) 2 real and 4 imaginary roots (D) 4 real and 2 imaginary roots

45. Let $P(x) = x^2 + \frac{4x}{3} + \log_{10}(4.\overline{9})$, $A = \prod_{i=1}^{12} P(a_i)$ where $a_1, a_2, a_3, \ldots, a_{12}$ are positive reals and $B = \prod_{j=1}^{13} P(b_j)$ where $b_1, b_2, b_3, \ldots, b_{13}$ are non-positive reals, then which one of the following is always correct?

(A) $A > 0, B > 0$ (B) $A > 0, B < 0$

(C) $A < 0, B > 0$ (D) $A < 0, B < 0$

46. If the equation $\sin^2 x - a\sin x + b = 0$ has three real and distinct roots in $(0, \pi)$, then the true set of values of b is equal to :

(A) $(-1, 0)$ (B) $(0, 1)$ (C) $(-1, 1)$ (D) $(1, 2)$

47. Let $f(x) = x^2 + ax + b$ and $g(x) = x^2 + cx + d$ be two quadratic polynomials with real coefficients and satisfy $ac = 2(b+d)$. Then which of the following is (are) correct?

(A) Exactly one of either $f(x) = 0$ or $g(x) = 0$ must have real roots.

(B) Atleast one of either $f(x) = 0$ or $g(x) = 0$ must have real roots.

(C) Both $f(x) = 0$ and $g(x) = 0$ must have real roots.

(D) Both $f(x) = 0$ and $g(x) = 0$ must have imaginary roots.

48. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the roots of the equation $ax^2 - bx(x-1) + c(x-1)^2 = 0$ in terms of α and β is : (where $a \neq 0$ and $a - b + c \neq 0$)

(A) $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$ (B) $\frac{1-\alpha}{\alpha}, \frac{1-\beta}{\beta}$ (C) $\frac{1+\alpha}{\alpha}$ and $\frac{1+\beta}{\beta}$ (D) $\frac{\alpha}{1+\alpha}, \frac{\beta}{1+\beta}$

49. Let a,b,c be non-zero real numbers and p and q be the two arbitrary numbers, then the equation $\frac{a^2}{x-p}+\frac{b^2}{x-q}=c$ has :

(A) Both non-real complex roots (B) Both roots are irrational

(C) Both roots are real (D) Both roots are real and equal.

50. If $\sin x$ and $\sin y$ are roots of the quadratic equation $a\sin^2 z+b\sin z+c=0$ and $\sin x+2\sin y=1$, then the value of $a^2+2b^2+3ab+ac$ is equal to :

(A) 1 (B) 2 (C) 0 (D) -2

51. If x_1 and x_2 are roots of the equation $x^2-3x+7=0$, then $x_1^4+x_2^4$ is :

(A) -73 (B) -72 (C) -14 (D) -5

52. If x and y are two distinct real numbers such that $\{x\}=\{y\}$ and $\{x^3\}=\{y^3\}$ (wehre $\{\cdot\}$ denotes the fractional part function) then :

(A) $3nx^2+3n^2x+n^3-m=0 (n\neq 0)$ (B) $3x^2+3nx+n^2-m=0\ (n\neq 0)$

(C) $3n^2x^2+3nx+n^2-m=0 (n\neq 0)$ (D) $3n^2x^2+3nx+3n^2-m=0 (n\neq 0)$

[**Note :** n and m are integers.]

53. If the quadratic equation $ax^2-bx+12=0$ where a and b are positive integers not exceeding 10, has roots both greater than 2, then the number of possible ordered pair (a,b) is :

(A) 1 (B) 2 (C) 4 (D) 8

54. The number of values of k for which $(x^2-(k-2)x-2k)(x^2+kx+2k-4)$ is a perfect square is :

(A) 1 (B) 2 (C) 0 (D) More than 2

55. Let α and β be real roots of the equation $x^2-(k-2)x+(k^2+3k+5)=0$, then the maximum value of $(\alpha^2+\beta^2)$ is :

(A) 17 (B) 18 (C) 19 (D) 20

56. If $x^2-P_s x+s=0$ (where $s=1,2,3$) are three equations of which each pair has exactly one root common and no root is common to all the three equation, then number of triplets of (P_1,P_2,P_3) is:

(A) 1 (B) 2 (C) 3 (D) 4

57. Let $P(x)=x^5+x^2+1$ and the equation $P(x)=0$ has roots x_1,x_2,x_3,x_4 and x_5 and $g(x)=x^2-2$, then $g(x_1)g(x_2)g(x_3)g(x_4)g(x_5)$ is equal to :

(A) $\frac{1}{23}$ (B) 23 (C) -23 (D) None of these

58. If $k\cdot 3^{\tan x}+k\cdot 3^{-\tan x}-4=0$ has real solutions (where $x\in[0,\pi]-\{\pi/2\}$), then complete set of values of k is :

(A) $[-2,2]$ (B) $[-2,0]$ (C) $(0,2]$ (D) $(0,\infty)$

59. Complete set of values of a for which quadratic expression $ax^2+|2a-3|x-6$ is positive for exactly three integral values of x is :

(A) $\left(-\frac{3}{5},-\frac{1}{2}\right]$ (B) $\left[-\frac{3}{5},-\frac{1}{2}\right]$ (C) $\left[-\frac{3}{5},-\frac{1}{6}\right]$ (D) None of these

60. If $\frac{1}{2}$ lies between the roots of the quadratic equation $6x^2 + 3\cos\theta \cdot x - \sin^2\theta = 0$, then true set of values of θ in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to :

(A) $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$ (B) $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right)$ (C) $\left(\frac{4\pi}{3}, \frac{5\pi}{3}\right)$ (D) $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right) - \{\pi\}$

61. Let a, b, c are the roots of the equation $x^3 - 9x^2 + 11x - 1 = 0$ and $s = \sqrt{a} + \sqrt{b} + \sqrt{c}$, then $(s^4 - 18s^2 - 8s)$ is equal to :

(A) -27 (B) 27 (C) -37 (D) 37

62. If $T_n = (\sin\theta)^n + (\cos\theta)^n \ \forall n \in N$ and $5T_{n+2} - 2\sqrt{3}T_{n+1} + T_n = 0 \ \forall n \in N$, then the value of $\sin\theta + \cos\theta$ is :

(A) $\frac{\sqrt{3}}{5}$ (B) $2\sqrt{3}$ (C) $\frac{2\sqrt{3}}{5}$ (D) $\frac{2}{5}$

63. The area of region in xy-plane consisting of all points (a, b) such that quadratic equation $ax^2 + 2(a + b - 7)x + 2b = 0$ has fewer than 2 real solution is :

(A) 4π (B) 9π (C) 25π (D) 49π

64. Let $r_1, r_2, r_3, \ldots, r_n$ be the distinct real zeroes of the equation $x^8 - 14x^4 - 8x^3 - x^2 + 1 = 0$ then $r_1^2 + r_2^2 + \ldots + r_n^2$ is :

(A) 3 (B) 14 (C) 8 (D) 16

65. If α is a root of $x^3 - x - 1 = 0$ then the value of

$\alpha^{10} + 2\alpha^8 - \alpha^7 - 3\alpha^6 - 3\alpha^5 + 4\alpha^4 + 2\alpha^3 - 4\alpha^2 - 6\alpha + 2$ is :

(A) 2 (B) 3 (C) 4 (D) 5

66. Let $P(x) = x^2 + bx + c$ (where b and c are real numbers) . Suppose $P(P(1)) = P(P(2)) = 0$ and $P(1) \neq P(2)$, then $P(0)$ is :

(A) 1 (B) -1 (C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

67. The sum of all the real numbers x such that $5x^4 - 10x^3 + 10x^2 - 5x - 11 = 0$ is :

(A) 0 (B) 1 (C) 2 (D) None of these

68. Let a and b be real numbers and let r, s and t be the roots of $f(x) = x^3 + ax^2 + bx - 1$. Also $g(x) = x^3 + mx^2 + nx + p$ has roots r^2, s^2, t^2. If $g(-1) = -5$, then maximum possible value of 'b' is :

(A) $\sqrt{5}$ (B) $1 + \sqrt{5}$ (C) 0 (D) $\sqrt{5} - 1$

69. Let a, b, c, x, y and z be complex numbers such that

$$a = \frac{b+c}{x-2}, \ b = \frac{c+a}{y-2} \text{ and } c = \frac{a+b}{z-2}$$

If $xy + yz + zx = 2015$ and $x + y + z = 2016$ then the value of $|xyz|$ is :

(A) 2012 (B) 2013 (C) 2014 (D) 2015

70. $f(x)=\dfrac{(x+a)^2}{(a-b)(a-c)}+\dfrac{(x+b)^2}{(b-a)(b-c)}+\dfrac{(x+c)^2}{(c-a)(c-b)}$ where a,b,c are distinct real numbers. If 'P' denotes the number of natural numbers in the range of $f(x)$, then unit digit of $(P+8)^{2015}$ is :

(A) 1 (B) 3 (C) 7 (D) 9

71. If two distinct complex numbers x_1 and x_2 and a real number w satisfies the equation $x_1(x_1+1)=w, x_2(x_2+1)=w$ and $x_1^4+3x_1^3+5x_1=x_2^4+3x_2^3+5x_2$ then absolute value of w is :

(A) 1 (B) 5 (C) 7 (D) 9

72. If a,b,x and y are real numbers such that
$ax+by=3, ax^2+by^2=7, ax^3+by^3=16$ and $ax^4+by^4=42$, then ax^5+by^5 is :

(A) 10 (B) 20 (C) 23 (D) 58

73. Let $f(x)=x^3+x+1$. Suppose $g(x)$ is a cubic polynomial such that $g(0)=-1$ and roots of $g(x)=0$ are the squares of the roots of $f(x)$, then $g(4)$ is :

(A) 21 (B) 33 (C) 99 (D) 100

74. If 3 roots of $x^4+ax^2+bx+c=0$ are 1, 2 and 3, then the value of $(b-a-c)$ is :

(A) 121 (B) −61 (C) −60 (D) 25

75. If $x+y+z=0$, then the value of $\dfrac{(x^2+y^2+z^2)(x^5+y^5+z^5)}{(x^7+y^7+z^7)}$ is :

(A) 1 (B) $\dfrac{2}{5}$ (C) $\dfrac{5}{7}$ (D) $\dfrac{10}{7}$

76. If the equation $|\sin x|^2+|\sin x|+b=0$ has two distinct roots in $[0,\pi]$, then the number of integers in the range of b is equal to :

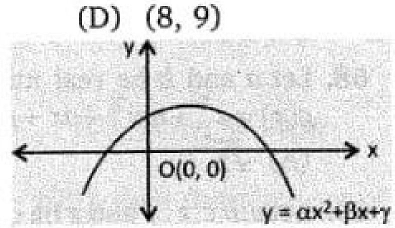
(A) 0 (B) 1 (C) 2 (D) 3

MULTIPLE CORRECT

1. For $x\in R$, the expression $\dfrac{x^2+34x-71}{x^2+2x-7}$ can not lie between.

(A) (5, 7) (B) (12, 19) (C) (1, 4) (D) (8, 9)

2. The following figure illustrate the graph of a quadratic trinomial $y=\alpha x^2+\beta x+\gamma$. Then which of the following is (are) correct ?



(A) $\alpha\beta>0$ (B) $\alpha^2+\beta\gamma>0$

(C) $\beta+\gamma-\alpha<0$ (D) $\alpha\beta\gamma<0$

3. For $m\in R$, let $f(x)=x^3+3mx^2-3x-3m+2$ has 3 real roots x_1, x_2 and x_3. Also k denotes the value of m for which $(x_1^2+x_2^2+x_3^2)$ is minimum. Which one of the following statement(s) is/are correct ?

(A) $f(x)$ has all 3 roots real and distinct corresponding to m for which $(x_1^2+x_2^2+x_3^2)$ is minimum.

(B) The value of k lies in $[-1,1]$.

(C) The minimum value of $\sum_{i=1}^{3} x_i^2$ equals 6.

(D) Number of roots of $\sin x = k$ in $[0, 2\pi]$ equals 3.

4. If the quadratic polynomial $P(x) = (p-3)x^2 - 2px + 3p - 6$ ranges from $[0, \infty)$ for every $x \in R$, then the value of p can be :

(A) 3/2 (B) 4 (C) 6 (D) 7

5. Let a, b and c be real numbers. Which of the following statement(s) about the equation $(x-a)(x-b) = c$ is/are incorrect ?

(A) If $c > 0$, then roots are always real
(B) If $c > 0$, then roots are always non-real
(C) If $c < 0$, then roots are always real
(D) If $c < 0$, then roots are always non-real

6. Let $P(x) = x^3 - 8x^2 + cx + d$ be a polynomial with real coefficients and with all its roots being distinct positive integers. The possible value of 'c' can be :

(A) 10 (B) 12 (C) 17 (D) 19

7. If quadratic equation $x^2 + 2(a+2b)x + (2a+b-1) = 0$ has unequal real roots for all $b \in R$ then the possible values of a can be equal to :

(A) 5 (B) −1 (C) −10 (D) 3

8. Given a, b, c are three distinct real numbers satisfying the inequality $a - 2b + 4c > 0$ and the equation $ax^2 + bx + c = 0$ has no real roots. Then the possible value of $\frac{4a+2b+c}{a+3b+9c}$ is/are :

(A) 2 (B) −1 (C) 3 (D) $\sqrt{2}$

9. If all values of x which satisfies the inequality $\log_{\frac{1}{3}}(x^2 + 2px + p^2 + 1) \geq 0$ also satisfy the inequality $kx^2 + kx - k^2 \leq 0$ for all real values of k, then all possible values of p lies in the interval:

(A) $[-1, 1]$ (B) $[0, 1]$ (C) $[0, 2]$ (D) $[-2, 0]$

10. If a, b, c are sides of ΔABC and $a > b > c$, then the equation

$a(x-b)(x+c) + b(x-a)(x+c) - c(x-a)(x-b) = 0$ has :

(A) real and unequal roots
(B) roots with opposite sign
(C) exactly one root in (b, a)
(D) imaginary roots

11. Let α and $\beta (\alpha > \beta)$ be roots of the quadratic equation $ax^2 + bx + c = 0$.

If $0 < a < c$ and $a + c = b$, then which of the following statement(s) is/are correct ?

(A) Larger root is 1
(B) Larger root is −1
(C) Smaller root is $\frac{c}{a}$
(D) b is greater than zero

12. If the vertex of the parabola $y = 3x^2 - 12x + 9$ is (a, b), then the parabola whose vertex is (b, a) is/are :

(A) $y = x^2 + 6x + 11$
(B) $y = x^2 - 7x + 3$
(C) $y = -2x^2 - 12x - 16$
(D) $y = -2x^2 + 16x - 13$

13. If one root of the quadratic equation $3x^2+px+3=0$ is square of another then the possible values of p can be :

(A) 6 (B) -6 (C) 3 (D) -3

14. Let a and b are non-zero real numbers then which of the following statements about the quadratic equation $ax^2+(a+b)x+ab=0$ is necessarily true ?

(A) It has at least one root negative
(B) It has at least one root positive
(C) Both its roots are real
(D) It has at least one root irrational.

15. Let $f(x)=ax^2+bx+c$ is a quadratic polynomial. If $3a+\sqrt{3}b+c=1$ and $4a+2b+c=0$ then which of the following is/are not possible ?

(A) $b^2-4ac>0$
(B) $b^2-4ac=0$
(C) $b^2-4ac<0$
(D) $c=4+2\sqrt{3}$

16. If a,b,c are rational numbers satisfying the equation $x^3+ax^2+bx+c=0$, then which of the following can be true ?

(A) $a+b^3+c^3=0$
(B) $a+b^2+c^3=5$
(C) $a+b^2+c^3=1$
(D) No such a,b,c exist

17. If α,β,γ are the roots of $x^2+2x^2-3x+1=0$, then the value of $\frac{\alpha\beta}{\alpha+\beta}+\frac{\alpha\gamma}{\alpha+\gamma}+\frac{\beta\gamma}{\beta+\gamma}$ is less than :

(A) 2 (B) 3 (C) 4 (D) 5

18. Consider the quadratic polynomial $f(x)=2x^2-2ax+a^2-a-6$, then :

(A) The equation $f(x)=0$ has real roots $\forall\ a\in[1-\sqrt{13},\ 1+\sqrt{13}]$
(B) The equation $f(x)=0$ has one positive and one negative root $\forall\ a\in(-2,3)$
(C) Number of integral values of a for which $f(x)=0$ has real roots is 8
(D) Number of integral values of a for which $f(x)=0$ has one root positive and one root negative is 4

19. If α and β are the roots of the equation $ax^2+bx+c=0$ and $\alpha\beta=3$, where a,b,c are in arithmetic progression, then $(\alpha^2+\beta^2)$ is :

(A) 2 (B) -2 (C) $\alpha^3+\beta^3$ (D) $\alpha+\beta$

20. Suppose a quadratic function $f(x)=ax^2+bx+c\,(a,b,c\in R$ and $a\neq 0)$ satisfies the following conditions :

(1) when $x\in R$, $f(x-4)=f(2-x)$ and $f(x)\geq x$

(2) when $x\in(0,2)$, $f(x)\leq\left(\frac{x+1}{2}\right)^2$

(3) The minimum value of $f(x)$ on R is 0.

If there exist $t\in R$ such that $f(x+t)\leq x\ \forall\ x\in[1,m]$, then possible values of $m(>1)$ can be :

(A) 2 (B) 3 (C) 9 (D) 10

21. Let x, y, z be real numbers such that $x+y+z=0$ and $xy+yz+zx=-3$, then the value of $x^3y+y^3z+z^3x$ is :

(A) -2 (B) -5 (C) -10 (D) -11

22. The real root of equation $8x^3-3x^2-3x-1=0$ can be written in the form $\frac{\sqrt[3]{m}+\sqrt[3]{n}+1}{p}$, where m,n and p are positive integer, then :

(A) $m+n+p=98$ (B) $m+n+p=97$ (C) $m+n=90$ (D) $n+p=17$

23. Let α, β are the roots of the equation $x^2+ax+b=0$ and γ, δ are the roots of $x^2-ax+b-2=0$. If $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}=\frac{5}{12}$ and $\alpha\beta\gamma\delta=24$, then:

(A) $a=5$ (B) $a=4$

(C) sum of values of b is 2 (D) Sum of values of b is 1

24. Suppose α, β, γ are complex numbers satisfying the following system of equations $\alpha+\beta+\gamma=6$, $\alpha^3+\beta^3+\gamma^3=87$ and $(\alpha+1)(\beta+1)(\gamma+1)=33$ then $\left[\frac{\alpha\beta\gamma}{\alpha\beta+\beta\gamma+\gamma\alpha}\right]$ (where $[\cdot]$ denotes greatest integer function) is greater than :

(A) -1 (B) 0 (C) 1 (D) 2

Comprehension Type Questions

Comprehension 1

Consider a quadratic polynomial 'C' and a line 'l' as $C : y=x^2-2x\cos\theta+\cos 2\theta+\cos\theta+\frac{1}{2}$ where $\theta \in [0,360°]$ and $l : y=x$.

1. If the vertex of C lies on the line l then the number of values of θ, is :

(A) 2 (B) 4 (C) 6 (D) 8

2. If C touches the line l then sum of all possible values of θ, is :

(A) 2π (B) 3π (C) 4π (D) 8π

3. Suppose that C has two intersection points with l and if maximum length of the line segment by which l cuts C is L, then $[L]$ equals :

(A) 2 (B) 3 (C) 4 (D) 6

Comprehension 2

Consider two quadratic trinomials

$$f(x)=x^2-2ax+a^2-1$$

and $$g(x)=(4b-b^2-5)x^2-(2b-1)x+3b, \text{ where } a,b \in R$$

1. The values of a for which both roots of the equation $f(x)=0$ are greater than -2 but less than 4, lie in the interval :
   (A) $-\infty < a < -3$ (B) $-2 < a < 0$ (C) $-1 < a < 3$ (D) $5 < a < \infty$
2. If roots of the quadratic equation $g(x)=0$ lie on either side of unity, then number of integral values of b is equal to :
   (A) 1 (B) 2 (C) 3 (D) 4
3. If $f(x) < 0 \ \forall \ x \in [0,1]$, then a lies in the interval :
   (A) $-1 < a < 1$ (B) $0 < a < 2$ (C) $0 < a < 1$ (D) $a > 3$

Comprehension 3

Let m,n be positive integers and the quadratic equation $4x^2 + mx + n = 0$ has two distinct real roots p and $q (p < q)$. Also the quadratic equations $x^2 - px + 2q = 0$ and $x^2 - qx + 2p = 0$ have a common root say α.

1. The value of α lies in :
   (A) (0, 1) (B) (1, 2) (C) $(-1, 0)$ (D) $(-3, -1)$
2. Number of possible ordered pairs (m,n) is equal to :
   (A) 1 (B) 2 (C) 3 (D) 4
3. If p and q are rational, then uncommon root of the equation $x^2 - px + 2q = 0$ is equal to :
   (A) 1 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{-1}{2}$

Comprehension 4

Consider a rational function $f(x) = \frac{x^2 - 3x - 4}{x^2 - 3x + 4}$ and a quadratic function $g(x) = x^2 - (b+1)x + b - 1$, where b is a parameter.

1. The sum of integers in the range of $f(x)$, is :
   (A) -5 (B) -6 (C) -9 (D) -10
2. If both roots of the equation $g(x)=0$ are greater than -1, then b lies in the interval :
   (A) $(-\infty, -2)$ (B) $\left(-\infty, \frac{-1}{4}\right)$ (C) $(-2, \infty)$ (D) $\left(\frac{-1}{2}, \infty\right)$
3. The largest natural number b satisfying $g(x) > -2 \ \forall \ x \in R$, is :
   (A) 1 (B) 2 (C) 3 (D) 4

Comprehension 5

Let α, β be real roots of the equation $(x^2+2)^2 + 8x^2 = 6x(x^2+2)$ and $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of the quadratic equation $f(x)=0$.

1. If $f(0) = -3$, then greatest value of $y = f(x)$, is :
(A) -3
(B) 1
(C) 3
(D) not possible to determine

2. The equation $\sin x + \sqrt{2}\cos x = \frac{\alpha}{2}(\alpha < \beta)$ is satisfied for :
(A) $\frac{\pi}{4}$
(B) $\frac{3\pi}{4}$
(C) $\frac{2\pi}{3}$
(D) $\frac{7\pi}{4}$

3. If α^2 and β^2 are the roots of the quadratic equation $g(x) = 0$ and vertex of $y = g(x)$ is $(p, -64)$, then the value of $g(1)$, is :
(A) -7
(B) -14
(C) 7
(D) 10

Comprehension 6

For $a, b \in R - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right) \forall x \in R$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

1. The value of $(a + b)$ is equal to :
(A) 4
(B) 5
(C) 6
(D) 7

2. The minimum value of $f(x)$ in $\left[0, \frac{3}{2}\right]$ is equal to :
(A) $\frac{-33}{8}$
(B) 0
(C) 4
(D) -2

Comprehension 7

Consider a quadratic expression $f(x) = tx^2 - (2t - 1)x + (5t - 1)$.

1. If $f(x)$ can take both positive and negative values than t must lie in the interval :
(A) $\left(\frac{-1}{4}, \frac{1}{4}\right)$
(B) $\left(-\infty, \frac{-1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$
(C) $\left(\frac{-1}{4}, \frac{1}{4}\right) - \{0\}$
(D) $(-4, 4)$

2. If $f(x)$ is non-negative $\forall x \geq 0$ then t lies in the interval :
(A) $\left[\frac{1}{5}, \frac{1}{4}\right]$
(B) $\left[\frac{1}{4}, \infty\right)$
(C) $\left[\frac{1}{4}, \frac{1}{4}\right]$
(D) $\left[\frac{1}{5}, \infty\right)$

Comprehension 8

If α and β are the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ and γ and δ are the roots of the equation $x^2 - 2(a + 1)x + a(a - 1) = 0$.

1. If $\alpha, \beta \in (\gamma, \delta)$, then the number of integral values of a is/are :

(A) 1 (B) 2 (C) 3 (D) 4

2. If α and β both roots of the given equation are positive then complete set of values of a is :

(A) $(1, 3)$ (B) $(-3, -1)$ (C) $(-1, 1)$ (D) $(0, 1)$

3. If exactly one root of the given equations is common then the number of values of a is :

(A) 1 (B) 2 (C) 3 (D) 4

Match the Columns

1. Match the following for the equation $x^2 + b|x| + 1 = 0$ where b is a real parameter.

Column-I		Column-II	
(A)	No real root	(P)	$b < -2$
(B)	Two distinct real roots	(Q)	$b = -2$
(C)	Three distinct real roots	(R)	$b \in \phi$
(D)	Four distinct real roots	(S)	$b \geq 0$
		(T)	$b > 2$

2. The expression $y = ax^2 + bx + c$ $(a, b, c \in R$ and $a \neq 0)$ represents a parabola which cuts the x-axis at the points which are roots of the equation $ax^2 + bx + c = 0$. Column-II contains values which correspond to the nature of roots mentioned in column-I.

Column-I		Column-II	
(A)	For $a = 1, c = 4$, if both roots are greater than 2 then b can be equal to	(P)	4
(B)	For $a = -1, b = 5$, if roots lie on either side of -1 then c can be equal to	(Q)	8
(C)	For $b = 6, c = 1$, if one root is less than -1 and the other root greater than $-\frac{1}{2}$ then a can be equal to	(R)	10
		(S)	no real value

3. If x, y and $z \in R$ and are connected by $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$, then :

Column-I		Column-II	
(A)	Minimum value of x is	(P)	2
(B)	Maximum value of x is	(Q)	5/3
(C)	When x is minimum, y is	(R)	1
(D)	When x is maximum, z is	(S)	2/3

4. Given the equation $x^4 + x^2(1-2k) + k^2 - 1 = 0$, then :

	Column-I		Column-II
(A)	Complete values of k for which given equation has no solution is	(P)	1
(B)	Complete values of k for which the given equation has 1 solution is	(Q)	-1
(C)	Complete values of k for which the given equation has 2 solutions is	(R)	$(-\infty,-1)\cup\left(\frac{5}{4},\infty\right)$
(D)	Complete values of k for which the given equation has 3 solutions is	(S)	$(-1,1)\cup\left\{\frac{5}{4}\right\}$

5. Let $f(x) = 4(a-3)x$ and $g(x) = 4x^2 + 4ax + a^2$, then :

	Column-I		Column-II
(A)	If $g(x)$ has minimum value 2 for $x \in [0,1]$, then the number of integral value(s) of a is /are	(P)	0
(B)	Number of integral values of a for which the both the points of intersection of $f(x)$ and $g(x)$ are real and distinct and less than 2	(Q)	2
(C)	Number of integral values of a for which exactly one point of intersection of $f(x)$ and $g(x)$ lies in $(-2, 0)$	(R)	4
(D)	If the point of intersection of $f(x)$ and $g(x)$ lies on either side of -1 *i.e.*, one is less than -1 and other is greater than -1, the sum of integral values of a is	(S)	5

Integer Type Questions

1. Let x_1 and x_2 be real solutions of the equation $x^2 + bx + c = 0 (b, c \in R)$. If $x_1 - x_2 = 4$ and $x_1^2 + x_2^2 = 40$, then find the value of b^2.

2. Let M be the minimum value of $f(\theta) = (3\cos^2\theta + \sin^2\theta)(\sec^2\theta + 3\,\text{cosec}^2\theta)$, for permissible real values of θ and P denotes the product of all real solutions of the equation $\frac{(x-1)(50-10x)}{x^2-5x} = x^2 - 8x + 7$. Find (P M).

3. Find the smallest positive integral value of a for which the greater root of the equation $x^2 - (a^2 + a + 1)x + a(a^2 + 1) = 0$ lies between the roots of the equation $x^2 - a^2x - 2(a^2 - 2) = 0$.

4. If all the solutions of the inequality $x^2 - 6ax + 5a^2 \le 0$ are also the solutions of inequality $x^2 - 14x + 40 \le 0$ then find the number of possible integral values of a.

5. If the quadratic equation $x^2 + (2 - \tan\theta)x - (1 + \tan\theta) = 0$ has two integral roots, then sum of all possible values of θ in interval $(0, 2\pi)$ is $k\pi$. Find the value of k.

6. Let a,b,c be real numbers such that $a+b+c=6$ and $ab+bc+ca=9$. If exactly one root of the equation $x^2-(m+2)x+5m=0$ lies between minimum and maximum values of c, then find the number of integral values of m.

7. If the equation $2\sin\left(\frac{\pi}{3}-\theta\right)x^2+(\sqrt{3}-\tan\theta)x+\cos\theta=0$ has exactly one root at infinity, then find the number of values of θ in the interval $(0,\pi)-\left\{\frac{\pi}{2}\right\}$.

8. If sum of maximum and minimum value of $y=\log_2(x^4+x^2+1)-\log_2(x^4+x^3+2x^2+x+1)$ can be expressed in form $((\log_2 m)-n)$, where m and 2 are coprime then compute $(m+n)$.

9. If $1-\log_x 2+\log_{x^2} 9+\log_{x^3} 64<0$, then range of x is (a,b). Find the minimum value of $(a+9b)$.

10. Let A denotes the set of values of x for which $\frac{x+2}{x-4}\leq 0$ and B denotes the set of values of x for which $x^2-ax-4\leq 0$. If B is the subset of A, then find the number of possible integral values of a.

11. Find the number of non-zero integers which can be the value of $f(x)=\frac{\sec x}{\sec 3x}$.

12. Let α,β be real roots of the quadratic equation $x^2-kx+k^2+k-5=0$. If m and M are respectively the minimum and maximum value of $\alpha^2+\beta^2$, then find $(m+M)$.

13. Let x_1,x_2 be the roots of the quadratic equation $x^2+ax+b=0$ and x_3,x_4 be the roots of the quadratic equation $x^2-ax+b-2=0$. If $\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}=\frac{5}{6}$ and $x_1x_2x_3x_4=24$, then find the value of a.

14. Number of values of θ in $[0,2\pi]$ for which the expression

$$y=\frac{\tan(x-\theta)+\tan x+\tan(x+\theta)}{\tan(x-\theta)\tan x\tan(x+\theta)} \text{ is independent of } x.$$

$\left(\text{You may use the fact that, the ratio } \frac{ax^2+bx+c}{px^2+qx+r} \text{ is independent of } x \text{ if } \frac{a}{p}=\frac{b}{q}=\frac{c}{r}\right)$

15. Find the largest integral value of x satisfying the inequality $\log_{0.09}(x^2+2x)\geq\log_{0.3}\sqrt{x+2}$.

16. If α,β are roots of the equation $2x^2+6x+b=0$ where $b<0$, then find the least integral value of $\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)$.

17. Find sum of all integral values of a in $[1,100]$ for which the equation

$x^2-(a-5)x+\left(a-\frac{15}{4}\right)=0$ has atleast one root greater than zero.

18. Find the smallest integral value of a such that $|x+a-3|+|x-2a|=|2x-a-3|$ is true $\forall x\in R$.

19. Let p be the product of the non-real roots of the equation

$$x^4 - 4x^3 + 6x^2 - 4x = 2008$$

where $[*]$ denotes the greatest integer function, then find $[p]$.

20. Let a,b,c,d be four distinct real numbers in A.P. Find the smallest positive value of k satisfying $2(a-b)+k(b-c)^2+(c-a)^3 = 2(a-d)+(b-d)^2+(c-d)^3$.

21. Let x_1, x_2, x_3 be 3 roots of the cubic $x^3 - x - 1 = 0$.

Then the expression $x_1(x_2-x_3)^2 + x_2(x_3-x_1)^2 + x_3(x_1-x_2)^2$ equals a rational number. Find the absolute value of the number.

22. Let x_1, x_2, x_3, x_4, x_5 are roots of the equation $x^5 - 4x^2 + x - 13 = 0$. Compute $\sum_{r=1}^{5} \frac{1}{2-x_r}$.

23. If the range of parameter t in the interval $(0, 2\pi)$, satisfying $\frac{(-2x^2+5x-10)}{(\sin t)x^2 + 2(1+\sin t)x + 9\sin t + 4} > 0$ for all real values of x is (a,b), then $(a+b) = k\pi$. Find the value of k.

24. If exactly one root of the equation $x^2 - 2kx + k^2 - 1 = 0$ satisfies the inequality $\log_{\sqrt{3}}(2-x) \le 0$, then find sum of all possible integral values of k.

25. Find the number of solution of the equation $\sqrt[3]{2\tan\theta - 1} + \sqrt[3]{\tan\theta - 1} = 1$ in interval $(0, 2\pi)$.

26. Find number of solutions of the equation $(\log_2 \cos\theta)^2 + \log_{\frac{4}{\cos\theta}}(16\cos\theta) = 2$, in interval $[0, 2\pi)$.

27. $a\alpha^2 + b\alpha + c = 3\alpha^2 - 4\alpha + 1$, $a\beta^2 + b\beta + c = 3\beta^2 - 4\beta + 1$ and $a\gamma^2 + b\gamma + c = 3\gamma^2 - 4\gamma + 1$ where α, β and γ are distinct real numbers, then find sum of roots of the quadratic equation $3ax^2 + 9bx + 7c = 0$.

28. The number of integers n such that the equation $nx^2 + (n+1)x + (n+2) = 0$ has rational roots only is :

29. If the minimum value of $(x-1)(x-2)(x-3)(x-4)$ is M then $(M+4)$ is :

30. If the real numbers x, y, z satisfy the equations $\sqrt{x} + \sqrt{y} + \sqrt{z} = 5$ and $\sqrt{xy} + \sqrt{yz} + \sqrt{zx} = 8$ then $(x+y+z)$ equals :

31. If the quadratic polynomial $p(x) \ge 0$ for all real numbers and $p(3) = 0$, $p(2) = 4$, then the value of $p(5)$ is :

32. Number of integral values of n for which $n^2 + 3n + 5$ is divisible by 121 is :

33. If $(x^2+1)(y^2+1) + 9 = 6(x+y)$ $(x, y \in R)$, then the value of (x^2+y^2) is :

34. Let the polynomial $f(x) = ax^2 - bx + c$ (where a,b,c are positive integers). If $f(p) = f(q) = 0$, where $0 < p < q < 1$, then minimum possible value of 'a' is :

35. Three pairs of real numbers $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) satisfy both equations $x^3 - 3xy^2 = 2005$ and $y^3 - 3x^2y = 2004$, then the value of $\frac{y_1 y_2 y_3}{2(y_1-x_1)(y_2-x_2)(y_3-x_3)}$ is :

Previous Year Questions

JEE MAINS

Single Correct

1. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is: **[2002]**

(A) $3x^2 - 19x + 3 = 0$
(B) $3x^2 + 19x - 3 = 0$
(C) $3x^2 - 19x - 3 = 0$
(D) $x^2 - 5x + 3 = 0$

2. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then: **[2002]**

(A) $a + b + 4 = 0$ (B) $a + b - 4 = 0$ (C) $a - b - 4 = 0$ (D) $a - b + 4 = 0$

3. If p and q are the roots of the equation $x^2 + px + q = 0$, then: **[2002]**

(A) $p = 1, q = -2$ (B) $p = 0, q = 1$ (C) $p = -2, q = 0$ (D) $p = -2, q = 1$

4. If a, b, c are distinct $+ve$ real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is: **[2002]**

(A) less than 1 (B) equal to 1 (C) greater than 1 (D) any real number

5. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{c}$ and $\frac{c}{b}$ are in: **[2003]**

(A) arithmetic-geometric progression
(B) arithmetic progression
(C) geometric progression
(D) harmonic progression

6. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is: **[2003]**

(A) $-\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{2}{3}$ (D) $\frac{1}{3}$

7. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is:

(A) 3 (B) 2 (C) 4 (D) 1

8. The real number x when added to its inverse gives the minimum value of the sum at x equal to: **[2003]**

(A) -2 (B) 2 (C) 1 (D) -1

9. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation: **[2004]**

(A) $x^2 - 18x - 16 = 0$
(B) $x^2 - 18x + 16 = 0$
(C) $x^2 + 18x - 16 = 0$
(D) $x^2 + 18x + 16 = 0$

10. If $(1-p)$ is a root of quadratic equation $x^2 + px + (1-p) = 0$ then its roots are: **[2004]**

(A) $-1, 2$ (B) $-1, 1$ (C) $0, -1$ (D) $0, 1$

11. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is: **[2004]**

(A) 4 (B) 12 (C) 3 (D) $\frac{49}{4}$

12. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are roots of $ax^2 + bx + c = 0, a \neq 0$ then: **[2005]**

(A) $a = b + c$ (B) $c = a + b$ (C) $b = c$ (D) $b = a + c$

13. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval: **[2005]**

(A) $(5, 6]$ (B) $(6, \infty)$ (C) $(-\infty, 4)$ (D) $[4, 5]$

14. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$, respectively, then the value of $2 + q - p$ is: **[2006]**

(A) 2 (B) 3 (C) 0 (D) 1

15. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4, lie in the interval. **[2006]**

(A) $-2 < m < 0$ (B) $m > 3$ (C) $-1 < m < 3$ (D) $1 < m < 4$

16. If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is: **[2006]**

(A) $\frac{1}{4}$ (B) 41 (C) 1 (D) $\frac{17}{7}$

17. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is: **[2007]**

(A) $(3, \infty)$ (B) $(-\infty, -3)$ (C) $(-3, 3)$ (D) $(-3, \infty)$

18. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is: **[2009]**

(A) 1 (B) 4 (C) 3 (D) 2

19. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is: **[2009]**

(A) less than $4ab$ (B) greater than $-4ab$

(C) less than $-4ab$ (D) greater than $4ab$

20. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[2010]**

(A) -1 (B) 1 (C) 2 (D) -2

21. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3,2). The correct roots of equation are: **[2011 RS]**

(A) 6, 1 (B) 4, 3 (C) $-6, -1$ (D) $-4, -3$

22. Let for $a \neq a_1 \neq 0$,

$f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$

and $p(x) = f(x) - g(x)$.

If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value $p(2)$ is : **[2011 RS]**

(A) 3 (B) 9 (C) 6 (D) 18

23. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has: **[2012]**

(A) infinite number of real roots (B) no real roots

(C) exactly one real root (D) exactly four real roots

24. The number of values of k, for which the system of equations : **[JEE M 2013]**

$$(k+1)x + 8y = 4k$$
$$kx + (k+3)y = 3k - 1$$

has no solution is:

(A) infinite (B) 1 (C) 2 (D) 3

25. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$ have a common root, then $a:b:c$ is: **[JEE M 2013]**

(A) 1:2:3 (B) 3:2:1 (C) 1:3:2 (D) 3:1:2

26. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is : **[JEE M 2014]**

(A) $\frac{2\sqrt{13}}{9}$ (B) $\frac{\sqrt{61}}{9}$ (C) $\frac{2\sqrt{17}}{9}$ (D) $\frac{\sqrt{34}}{9}$

27. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to : **[JEE M 2015]**

(A) -3 (B) 6 (C) -6 (D) 3

JEE ADVANCED

Single Correct

1. If α and $\beta (\alpha < \beta)$ are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then: **[2000 S]**

(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$ (C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

2. If $b > a$, then the equation $(x-a)(x-b) - 1 = 0$ has : **[2000 S]**

(A) both roots in (a, b)

(B) both roots in $(-\infty, a)$

(C) both roots in $(b, +\infty)$

(D) one root in $(-\infty, a)$ and the other in $(b, +\infty)$

3. For the equation $3x^2 + px + 3 = 0, p > 0$, if one of the root is square of the other, then p is equal to: **[2000 S]**

(A) 1/3 (B) 1 (C) 3 (D) 2/3

4. The set of all real numbers x for which $x^2 - |x+2| + x > 0$, is: **[2002 S]**

(A) $(-\infty, -2) \cup (2, \infty)$
(B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
(C) $(-\infty, -1) \cup (1, \infty)$
(D) $(\sqrt{2}, \infty)$

5. For all 'x', $x^2 + 2ax + 10 - 3a > 0$, then the interval in which 'a' lies is: **[2004 S]**

(A) $a < -5$ (B) $-5 < a < 2$ (C) $a > 5$ (D) $2 < a < 5$

6. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is: **[2004 S]**

(A) $p^3 - q(3p-1) + q^2 = 0$
(B) $p^3 - q(3p+1) + q^2 = 0$
(C) $p^3 + q(3p-1) + q^2 = 0$
(D) $p^3 + q(3p+1) + q^2 = 0$

7. Let a, b, c be the sides of a triangle where $a \neq b \neq c$ and $\lambda \in R$. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ are real, then: **[2006 - 3M, -1M]**

(A) $\lambda < \frac{4}{3}$ (B) $\lambda > \frac{5}{3}$ (C) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

8. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is: **[2007-3 Marks]**

(A) $\frac{2}{9}(p-q)(2q-p)$
(B) $\frac{2}{9}(q-p)(2p-q)$
(C) $\frac{2}{9}(q-2p)(2q-p)$
(D) $\frac{2}{9}(2p-q)(2q-p)$

9. Let a, b, c, p, q be real number. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are roots of the equation $ax^2 + 2bx + x = 0$, where $\beta^2 \notin \{1, 0, 1\}$. **[JEE 2008]**

Statement-1 $(p^2 - q)(b^2 - ac) \geq 0$

and

Statement-2 $b \neq pa$ or $c \neq qa$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is not correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

10. Let p and q be real numbers such that $p \neq 0, p^3 \neq q$ and $p^3 \neq -q$. If α and β are non-zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is : **[JEE 2010]**

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

11. A value of b for which the equations

$$x^2 + bx - 1 = 0$$
$$x^2 + x + b = 0$$

have one root in common is: **[JEE 2011]**

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

12. Let α and β be the roots of $x^2 - 6x - 2 = 0$ with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{a_9}$ is : **[JEE 2011]**

(A) 1 (B) 2 (C) 3 (D) 4

Multiple Correct

1. Let S be the set of all non-zero numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is (are) a subset (s) of S? **[JEE 2015, 4M, –0M]**

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$ (C) $\left(0, \frac{1}{\sqrt{5}}\right)$ (D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Subjective Type Questions

1. If α, β are the roots of $ax^2 + bx + c = 0, (a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant δ, then prove that $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$. **[2000- 4 Marks]**

2. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. **[2001- 4 Marks]**

3. If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$ then find the values of a for which equation has unequal real roots for all values of b. **[2003-4 Marks]**

4. Let a and b be the roots of the equation $x^2 - 10cx - 11d = 0$ and those of $x^2 - 10ax - 11b = 0$ are c, d then the value of $a + b + c + d$, when $a \neq b \neq c \neq d$ is. **[2006 - 6M]**

5. The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is. **[JEE 2009]**

Answer

SINGLE CORRECT

1. B	2. D	3. B	4. D	5. D	6. A	7. D	8. D
9. C	10. B	11. C	12. B	13. C	14. B	15. A	16. C
17. B	18. B	19. C	20. B	21. D	22. B	23. C	24. C
25. C	26. D	27. B	28. B	29. A	30. D	31. D	32. D
33. D	34. A	35. A	36. D	37. A	38. D	39. A	40. C
41. C	42. D	43. A	44. B	45. A	46. B	47. B	48. D
49. C	50. C	51. A	52. A	53. A	54. A	55. B	56. B
57. C	58. C	59. A	60. D	61. C	62. C	63. D	64. C
65. C	66. C	67. B	68. B	69. C	70. D	71. C	72. B
73. C	74. A	75. D	76. C				

MULTIPLE CORRECT

1. A,D	2. B,D	3. B,C,D	4. C	5. B,C,D	6. C,D	7. B,C	8. A,C,D
9. A,B,C	10. A,B,C	11. B,D	12. A,C	13. B,C	14. A,C	15. C,D	16. A,B,C
17. A,B,C,D	18. A,B,D	19. B,D	20. A,B,C	21. C,D	22. A,C	23. A,C	24. A,B

COMPREHENSION TYPE

Comprehension —1	1. B	2. C	3. A
Comprehension —2	1. C	2. B	3. C
Comprehension —3	1. D	2. C	3. C
Comprehension —4	1. B	2. D	3. B
Comprehension —5	1. C	2. D	3. B
Comprehension —6	1. B	2. D	
Comprehension —7	1. C	2. D	
Comprehension —8	1. A	2. C	3. C

MATCH THE COLUMN

1. (A) S, T; (B) Q; (C) R; (D) P
2. (A) S; (B) Q, R: (C) P
3. (A) S; (B) P; (C) Q; (D) R
4. (A) R; (B) Q; (C) S; (D) P
5. (A) Q; (B) S; (C) R; (D) P

INTEGER TYPE

1. 64	2. 24	3. 3	4. 0	5. 4	6. 8	7. 0	8. 5
9. 25	10. 3	11. 3	12. 13	13. 10	14. 4	15. 1	16. 10

17. 5011	**18.** 1	**19.** 45	**20.** 16	**21.** 9	**22.** 13	**23.** 3	**24.** 2
25. 2	**26.** 3	**27.** 4	**28.** 3	**29.** 3	**30.** 9	**31.** 16	**32.** 0
33. 7	**34.** 5	**35.** 501					

PREVIOUS YEARS

JEE MAINS

Single Correct

1. A	**2.** A	**3.** A	**4.** A	**5.** D	**6.** B	**7.** C	**8.** C
9. B	**10.** C	**11.** D	**12.** B	**13.** C	**14.** B	**15.** C	**16.** B
17. C	**18.** D	**19.** B	**20.** B	**21.** A	**22.** D	**23.** B	**24.** B
25. A	**26.** A	**27.** D					

JEE ADVANCED

Single Correct

1. B	**2.** D	**3.** C	**4.** B	**5.** B	**6.** A	**7.** A	**8.** D
9. B	**10.** B	**11.** B	**12.** C				

Multiple Correct

1. A,D

Subjective Type

2. $\alpha^2\beta, \alpha\beta^2$	**3.** $a > 1$	**4.** 1210	**5.** 2