

# Chapter

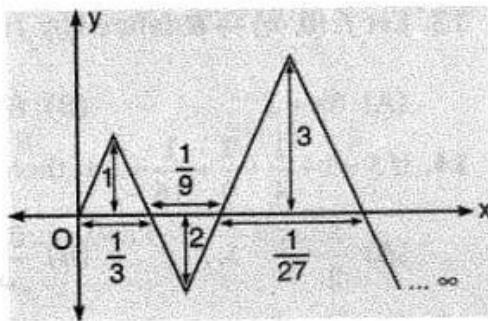
# 3

# Sequence and Progression

## Objective Questions

### SINGLE CORRECT

1. Let  $a_1, a_2, a_3, a_4, \dots, a_{11}$  be a geometric sequence. If  $\prod_{k=1}^{11} a_k = 6$ , then the value of  $(a_5 a_6 a_7)$  is equal to :
- (A)  $(6)^{\frac{5}{11}}$       (B)  $(25)^{\frac{1}{11}}$       (C)  $(216)^{\frac{1}{11}}$       (D)  $(343)^{\frac{1}{11}}$
2. If  $a, b$  and  $c$  are in A.P. and  $p, p'$  are the A.M. and G.M. between  $a$  and  $b$  respectively, while  $q, q'$  are the A.M. and G.M. between  $b$  and  $c$  respectively, then :  
(A)  $p^2 + q^2 = p'^2 + q'^2$       (B)  $pq = p'q'$   
(C)  $p^2 - q^2 = p'^2 - q'^2$       (D)  $pq' = p'q$
3. For  $x > 0$ , the sum of the series  $\frac{1}{1+x} - \frac{(1-x)}{(1+x)^2} + \frac{(1-x)^2}{(1+x)^3} - \dots \infty$  is equal to :
- (A)  $\frac{1}{4}$       (B)  $\frac{1}{2}$       (C)  $\frac{3}{4}$       (D) 1
4. The limiting sum of the infinite series  $\frac{1}{10} + \frac{2}{10^2} + \frac{3}{10^3} + \dots$  whose  $n^{\text{th}}$  term is  $\frac{n}{10^n}$  is equal to :  
(A)  $\frac{1}{9}$       (B)  $\frac{10}{81}$       (C)  $\frac{1}{8}$       (D)  $\frac{17}{72}$
5. Infinite number of triangles are formed as shown in figure. If total area of these triangles is  $A$  then  $8A$  is equal to :  
(A) 3  
(B) 4  
(C) 1  
(D) 2



6. Let  $\{a_n\}$  be an arithmetic sequence whose first term is 1 and  $\{b_n\}$  be a geometric sequence whose first term is 2. If the common ratio of geometric sequence is half the common difference of arithmetic sequence, then the minimum value of  $(a_4 b_1 + a_3 b_2 + 2a_1 b_3)$  is equal to :

(A)  $\frac{25}{12}$       (B)  $\frac{-3}{2}$       (C)  $\frac{3}{2}$       (D)  $\frac{-25}{12}$

7. If roots of the cubic  $64x^3 - 144x^2 + 92x - 15 = 0$  are in arithmetic progression, then the difference between the largest and smallest root is equal to :

(A) 2      (B) 1      (C) 1/2      (D) 3/8

8. If the equation  $2^x + 2^{-2} = 2k$  has exactly one real solution, then sum of all integral values of  $k$  in  $[-100, 100]$  is equal to :

(A) 5050      (B) 10100      (C) 0      (D) -5050

9. If  $H_1, H_2, H_3, \dots, H_{101}$  are in H.P., then  $\sum_{i=1}^{100} (-1)^i \left( \frac{H_i + H_{i+1}}{H_i - H_{i+1}} \right)$  is equal to :

(A) 99      (B) 101      (C) 100      (D) 1

10. If  $\frac{1}{a} + \frac{1}{a-2b} + \frac{1}{c} + \frac{1}{c-2b} = 0$  and  $a, b, c$  are not in A.P. then :

(A)  $a, b, c$  are in G.P.      (B)  $a, \frac{b}{2}, c$  are in A.P.      (C)  $a, \frac{b}{2}, c$  are in H.P.      (D)  $a, 2b, c$  are in H.P.

11. If  $S_n = n^2 a + \frac{n}{4}(n-1)d$  is the sum of first  $n$  terms of an A.P., then common difference is equal to :

(A)  $a + 2d$       (B)  $2a + d$       (C)  $\frac{a+d}{2}$       (D)  $2a + \frac{d}{2}$

12. A line segment initially 1 unit long grows according to the law

$$1 + \frac{\sqrt{2}}{4} + \frac{1}{4} + \frac{\sqrt{2}}{16} + \frac{1}{16} + \frac{\sqrt{2}}{64} + \frac{1}{64} + \dots \infty$$

If the growth process continues indefinitely, then length of the line is equal to :

(A)  $\frac{4}{3}$       (B)  $\frac{8}{3}$       (C)  $\frac{4+\sqrt{2}}{3}$       (D)  $\frac{2}{3}(4+\sqrt{2})$

13. Let  $f: (0, \infty) \rightarrow \mathbb{R}$  defined by  $f(x) = x^2 + \frac{16\pi^4}{x^2} - \cos 3x$ , then the minimum value of  $f(x)$  is :

(A)  $8\pi + 1$       (B)  $8\pi - 1$       (C)  $8\pi^2 + 1$       (D)  $8\pi^2 - 1$

14. If  $S = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots \infty$ , then  $1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \dots \infty$  is equal to :

(A)  $\frac{S}{2}$       (B)  $\frac{3S}{4}$       (C)  $S - \frac{1}{4}$       (D)  $S - \frac{1}{2}$

15. The quadratic equation whose roots are A.M. and H.M. of the roots of the equation  $ax^2 + bx + c = 0$  ( $a, b, c \in \mathbb{R}, a \neq 0$ ) is :

- (A)  $2ax^2 + (b^2 + 4ac)x + 2bc = 0$       (B)  $2abx^2 + (b^2 + 4ac)x + 2bc = 0$   
 (C)  $abx^2 + (b^2 + 4ac)x + bc = 0$       (D)  $ax^2 + (b^2 + 4ac)x + bc = 0$

16. The value of  $\frac{1 \cdot 2}{3!} + \frac{2 \cdot 2^2}{4!} + \frac{3 \cdot 2^3}{5!} + \dots + \frac{15 \cdot 2^{15}}{17!}$  equals :

- (A)  $2 - \frac{16 \cdot 2^{17}}{17!}$       (B)  $2 - \frac{2^{17}}{17!}$       (C)  $1 - \frac{16 \cdot 2^{17}}{17!}$       (D)  $1 - \frac{2^{16}}{17!}$

17. Given distinct straight lines  $OA$  and  $OB$ . From a point in  $OA$ , a perpendicular is drawn to  $OB$ , from the foot of this perpendicular a line is drawn perpendicular to  $OA$ . From the foot of this second perpendicular, a line is drawn perpendicular to  $OB$  and so on indefinitely. The lengths of the first and second perpendiculars are ' $a$ ' and ' $b$ ', respectively. Then the sum of the lengths of the perpendiculars approaches a limit as the number of perpendiculars grow beyond all bound. This limit is equal to :

- (A)  $\frac{b}{a-b}$       (B)  $\frac{a}{a-b}$       (C)  $\frac{ab}{a-b}$       (D)  $\frac{a^2}{a-b}$

18. In a convex polygon, the degree measures of the interior angles form an arithmetic progression. If the smallest angle is  $159^\circ$  and the largest angle is  $177^\circ$ , then the number of sides in the polygon is :

- (A) 21      (B) 27      (C) 30      (D) 31

19. Given a sequence of ten numbers, if the first number is 2 and each other number is the square of the preceding number, then the tenth number is :

- (A) between  $10$  and  $10^5$       (B) between  $10^5$  and  $10^{10}$   
 (C) between  $10^{50}$  and  $10^{100}$       (D) more than  $10^{100}$

20. The value of  $\sum_{n=3}^{\infty} \frac{1}{n^5 - 5n^3 + 4n}$  is equal to :

- (A)  $\frac{1}{120}$       (B)  $\frac{1}{96}$       (C)  $\frac{1}{24}$       (D)  $\frac{1}{144}$

21. A ball is dropped onto a floor from a height of 1 meter. Each time that the ball hits the floor, it rebounds to half its previous height. (After falling one meter it rebounds to a height of  $\frac{1}{2}$  meter.)

The next time it hits the floor, it rebounds to a height of  $\frac{1}{4}$  meter, etc.) How far has the ball travelled when it hits the floor for the 40<sup>th</sup> time?

- (A)  $T = 2 + \frac{(2^{38} - 1)}{2^{38}}$       (B)  $T = 1 + \frac{(2^{28} - 1)}{2^{39}}$       (C)  $T = 8$       (D)  $T > 3$

22. For  $x \in (1, \infty)$ , if  $P = x^3 - \frac{1}{x^3}$  and  $Q = x - \frac{1}{x}$  then the minimum value of  $\frac{P}{Q^2}$  equals :

- (A)  $2\sqrt{3}$       (B)  $4\sqrt{3}$       (C)  $\sqrt{3}$       (D)  $3\sqrt{3}$       (E)  $\frac{\sqrt{3}}{2}$

23. Given that  $a$  and  $b$  are positive real numbers with  $a + b = 4$ , the minimum value of  $\left(1 + \frac{1}{a}\right)\left(1 + \frac{1}{b}\right)$  is :  
 (A) 2 (B) 8/3 (C) 9/4 (D) 3

24. The value of the sum  
 $S = \frac{1}{2} + \frac{1}{6}(1^2 + 2^2) + \frac{1}{12}(1^2 + 2^2 + 3^2) + \frac{1}{20}(1^2 + 2^2 + 3^2 + 4^2) + \dots + \frac{1}{3660}(1^2 + 2^2 + 3^2 + \dots + 60^2)$   
 is :  
 (A) 620 (B) 720 (C) 520 (D) 420

25. For a given arithmetic series the sum of the first 50 terms is 200 and the sum of the next 50 terms is 2700. The first term of the series is :  
 (A) -12221 (B) -21.5 (C) -20.5 (D) 3

26. The 3<sup>rd</sup> term of an arithmetic progression is 7 and its 7<sup>th</sup> term is 2 more than thrice of its 3<sup>rd</sup> term. The sum of its first 20 terms equals :  
 (A) 470 (B) 740 (C) 704 (D) 770

27. If  $a, b, c$  are in H.P., then  $\log_e(a+c) + \log_e(a-2b+c)$  equals :  
 (A)  $2\log_e(a+c)$  (B)  $\log_e(a^2 + b^2)$  (C)  $2\log_e|a-c|$  (D)  $\log_e|a^2 - b^2|$

28. The sum of the series,  $1 + 2 \cdot \left(1 + \frac{1}{n}\right) + 3 \cdot \left(1 + \frac{1}{n}\right)^2 + \dots$  is :  
 (A)  $n^2$  (B)  $n(n+1)$  (C)  $n\left(1 + \frac{1}{n}\right)^2$  (D)  $(n+1)(n+2)$

29. A sequence is such that the sum of its any number of terms, beginning from the first, is four times as large as the square of the number of terms. If the  $n^{\text{th}}$  term of such a sequence is 996, then value of  $n$  is equal to :  
 (A) 100 (B) 112 (C) 125 (D) 132

30. The value of  $\frac{\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \infty\right)}{\left(1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots \infty\right)}$  is :  
 (A) 1 (B) 2 (C) 3/2 (D) 5/2

31. The least integral value of  $\frac{(x-1)^7 + 3(x-1)^6 + (x-1)^5 + 1}{(x-1)^5}$   $\forall x > 1$  is :  
 (A) 2 (B) 4 (C) 6 (D) 8

32. The value of  $\sum_{k=1}^{\infty} \frac{6^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})}$  is :  
 (A) 1 (B) 2 (C) 3 (D) 4

23. Given that  $a$  and  $b$  are positive real numbers with  $a+b=4$ , the minimum value of  $\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)$  is :  
 (A) 2 (B) 8/3 (C) 9/4 (D) 3

24. The value of the sum  
 $S = \frac{1}{2} + \frac{1}{6}(1^2 + 2^2) + \frac{1}{12}(1^2 + 2^2 + 3^2) + \frac{1}{20}(1^2 + 2^2 + 3^2 + 4^2) + \dots + \frac{1}{3660}(1^2 + 2^2 + 3^2 + \dots + 60^2)$   
 is :  
 (A) 620 (B) 720 (C) 520 (D) 420

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26. The 3<sup>rd</sup> term of an arithmetic progression is 7 and its 7<sup>th</sup> term is 2 more than thrice of its 3<sup>rd</sup> term. The sum of its first 20 terms equals :  
 (A) 470 (B) 740 (C) 704 (D) 770

27. If  $a, b, c$  are in H.P., then  $\log_e(a+c) + \log_e(a-2b+c)$  equals :  
 (A)  $2\log_e(a+c)$  (B)  $\log_e(a^2 + b^2)$  (C)  $2\log_e|a-c|$  (D)  $\log_e|a^2 - b^2|$

28. The sum of the series,  $1+2\cdot\left(1+\frac{1}{n}\right)+3\cdot\left(1+\frac{1}{n}\right)^2+\dots$  is :  
 (A)  $n^2$  (B)  $n(n+1)$  (C)  $n\left(1+\frac{1}{n}\right)^2$  (D)  $(n+1)(n+2)$

29. A sequence is such that the sum of its any number of terms, beginning from the first, is four times as large as the square of the number of terms. If the  $n^{\text{th}}$  term of such a sequence is 996, then value of  $n$  is equal to :  
 (A) 100 (B) 112 (C) 125 (D) 132

30. The value of  $\frac{\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\infty\right)}{\left(1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\dots\infty\right)}$  is :  
 (A) 1 (B) 2 (C) 3/2 (D) 5/2

31. The least integral value of  $\frac{(x-1)^7 + 3(x-1)^6 + (x-1)^5 + 1}{(x-1)^5}$   $\forall x > 1$  is :  
 (A) 2 (B) 4 (C) 6 (D) 8

32. The value of  $\sum_{k=1}^{\infty} \frac{6^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})}$  is :  
 (A) 1 (B) 2 (C) 3 (D) 4

#### **Excellence and Progression |**

33. If  $a_1, a_2, a_3, \dots$  and  $b_1, b_2, b_3, \dots$  are two geometric progressions with  $a_1 = 2\sqrt{3}$  and  $b_1 = \frac{52}{9}\sqrt{3}$ . If  $3a_{99}b_{99} = 104$ , then  $\sum_{i=1}^{102} a_i b_i$  is :

(A) 102      (B) 3536      (C) 2040      (D) None of these

34. If  $a+b+c+d+e = 12$ , where  $a, b, c, d$  and  $e$  are positive numbers and  $a^4 b^3 c^3 d e = 4 \times 6^6$ , then the value of  $(4a+3b+3c+d+e)$  can be :

(A) 32      (B) 36      (C) 38      (D) 40

35. Let  $a, b, c, d$  and  $e$  are positive integers such that  $abcde = a+b+c+d+e$ , then the maximum possible value of 'e' is :

(A) 2      (B) 5      (C) 6      (D) 7

36. The value of  $S = \frac{1}{3^2 + 1} + \frac{1}{4^2 + 2} + \frac{1}{5^2 + 3} + \dots \infty$  is :

(A)  $\frac{1}{3}$       (B)  $\frac{13}{36}$       (C)  $\frac{7}{18}$       (D)  $\frac{5}{12}$

37. Let  $F_0 = 0, F_1 = 1$  and  $F_n = F_{n-1} + F_{n-2}$  ( $n \geq 2$ ), then the value of the sum  $\sum_{n=1}^{\infty} \frac{F_n}{3^n}$  is :

(A)  $\frac{3}{5}$       (B)  $\frac{1}{3}$       (C)  $\frac{2}{3}$       (D)  $\frac{7}{9}$

38. Let two sequences of rational numbers are defined as  $a_{n+1} = \frac{a_n^2}{b_n}$  and  $b_{n+1} = \frac{b_n^2}{a_n}$  (where  $a_0 = 2$  and  $b_0 = 3$  and  $n \geq 0$ ). If  $b_8 = \frac{3^m}{2^n}$ , then  $(m-n)$  (where  $m$  and  $n \in N$ ) is equal to :

(A) 1      (B) 2      (C) 3      (D) 4

39. The maximum value attained by  $\frac{x^4 - x^2}{x^6 + 2x^3 - 1}$  over real numbers  $x > 1$  is :

(A)  $\frac{1}{3}$       (B)  $\frac{1+\sqrt{5}}{2}$       (C)  $\frac{1}{6}$       (D)  $\frac{1}{8}$

40. If  $a, b, c$  are in A.P. and  $a, b, d$  are in G.P., then  $a, a-b, d-c$  are in :

(A) A.P.      (B) G.P.      (C) H.P.      (D) None of these

41. If  $P$  is the product of geometric progression,  $S_1$  be their sum and  $S_2$  be the sum of their reciprocals, then  $P$  in terms of  $S_1, S_2$  and  $n$  is :

(A)  $(S_1 S_2)^{n/2}$       (B)  $\left(\frac{S_1}{S_2}\right)^{n/2}$       (C)  $(S_1 S_2)^{n-2}$       (D)  $\left(\frac{S_1}{S_2}\right)^n$

42. The sum of infinite terms of the series

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots \infty$$
 is :

(A)  $\frac{1}{2}$

(B)  $\frac{1}{3}$

(C) 1

(D)  $\frac{1}{4}$

43. Let  $n$  be a positive integer such that  $\langle a_n \rangle$  is a sequence satisfying  $a_0 = 1$  and  $a_n = \frac{1}{2}a_{n-1}(na_n + 2)$ , then  $a_n$  is :

(A)  $\frac{4}{4-n^2-n}$

(B)  $\frac{4-n^2-n}{4}$

(C)  $4-n^2-n$

(D)  $\frac{6}{n^2+n-24}$

44. If the distinct non-zero numbers  $x(y-z)$ ,  $y(z-x)$ ,  $z(x-y)$  form a geometric progression with common ratio  $r$ , then  $r$  satisfies the equation :

(A)  $r^2 + r + 1 = 0$     (B)  $r^2 - r + 1 = 0$     (C)  $r^4 + r^2 - 1 = 0$     (D)  $(r-1)^4 + r = 0$

45. If roots of the equation  $x^3 + ax^2 + bx + c = 0$  are in arithmetic progression then  $2a^3 - 9ab + 27c$  is (where  $a, b, c$  are non-zero) :

(A) 0

(B) 9

(C) 18

(D) 27

46. The value of  $\sum_{n=2}^{\infty} \frac{n^4 + 3n^2 + 10n + 10}{2^n(n^4 + 4)}$  is :

(A) 1

(B)  $\frac{11}{10}$

(C)  $\frac{3}{2}$

(D) 2

47. Let  $a_0 = \frac{6}{7}$  and  $a_{n+1} = \begin{cases} 2a_n & \text{if } a_n < \frac{1}{2} \\ 2a_n - 1 & \text{if } a_n \geq \frac{1}{2} \end{cases}$ , then  $a_{2015}$  is :

(A)  $\frac{3}{7}$

(B)  $\frac{4}{7}$

(C)  $\frac{5}{7}$

(D)  $\frac{6}{7}$

48. The value of  $11^2 - 1^2 + 12^2 - 2^2 + 13^2 - 3^2 + \dots + 20^2 - 10^2$  is :

(A) 2015

(B) 2050

(C) 2100

(D) 2200

49. Let  $a = \frac{1^2}{1} + \frac{2^2}{3} + \frac{3^2}{5} + \dots + \frac{1001^2}{2001}$  and  $b = \frac{1^2}{3} + \frac{2^2}{5} + \frac{3^2}{7} + \dots + \frac{1001^2}{2003}$ , then the integer which is closest to  $a-b$  is :

(A) 500

(B) 501

(C) 1000

(D) 1001

50. The minimum value of the expression  $E = 3^{x+y}(3^{x-1} + 3^{y-1} - 1)$  is :

(A) -1

(B) -2

(C)  $-\frac{1}{3}$

(D)  $-\frac{2}{3}$

51. The value of  $\sum_{n=1}^{9999} \frac{1}{(\sqrt[n]{n} + \sqrt[n+1]{n})(\sqrt[4]{n} + \sqrt[4]{n+1})}$  is :

(A) 9

(B) 99

(C) 999

(D) 9999

52. Let  $\{a_n\}_{n \geq 1}$  be an arithmetic sequence and  $\{g_n\}_{n \geq 1}$  be a geometric sequence such that first four terms of  $\{a_n + g_n\}$  are 0, 0, 1 and 0 in that order then the tenth term of  $\{a_n + g_n\}$  is :

(A) 54

(B) -54

(C) 27

(D) -27

## Sequence and Progression

53. Let  $a_0, a_1, a_2, \dots$  denote the sequence of real numbers such that  $a_0 = 2$  and  $a_{n+1} = \frac{a_n}{1+a_n}$  for  $n \geq 0$ ,

then  $a_{2015}$  is :

- (A)  $\frac{2}{4025}$       (B)  $\frac{2}{4027}$       (C)  $\frac{2}{4029}$       (D)  $\frac{2}{4031}$

54. The value of the sum  $1 \cdot 2^2 + 2 \cdot 3^2 + 3 \cdot 4^2 + \dots + 19 \cdot 20^2$  is :

- (A) 44100      (B) 44000      (C) 41230      (D) 40000

55. The value of  $\sum_{n=1}^{\infty} \sum_{k=1}^{n-1} \frac{k}{2^{n+k}}$  is :

- (A)  $\frac{14}{9}$       (B)  $\frac{5}{9}$       (C)  $\frac{4}{9}$       (D)  $\frac{10}{9}$

56. The number of ordered pair  $(a, b)$  for which the real numbers  $10, a, b, ab$  is in arithmetic progression :

- (A) 1      (B) 2      (C) 3      (D) 4

57. Let  $ABC$  be a triangle with  $AB = 5$ ,  $BC = 4$  and  $AC = 3$ . Let  $P$  and  $Q$  be squares inside  $ABC$  with disjoint interiors such that they both have one side lying on  $AB$ . Also, the two squares each have an edge lying on a common line perpendicular to  $AB$  and  $P$  has one vertex on  $AC$  and  $Q$  has one vertex on  $BC$ . The minimum value of sum of the areas of two square is :

- (A)  $\frac{12}{7}$       (B)  $\frac{144}{49}$       (C)  $\frac{13}{7}$       (D)  $\frac{25}{49}$

58. The value of  $\sum_{n=1}^{\infty} \frac{7n+32}{n(n+2)} \cdot \frac{3^n}{4^n}$  is :

- (A)  $\frac{3}{2}$       (B)  $\frac{15}{2}$       (C)  $\frac{33}{2}$       (D)  $\frac{17}{2}$

59. The value of  $S = \sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \dots + \sqrt{1 + \frac{1}{(2014)^2} + \frac{1}{(2015)^2}}$  is :

- (A) 2015      (B)  $2015 - \frac{1}{2015}$       (C)  $2016 - \frac{1}{2016}$       (D)  $2014 - \frac{1}{2014}$

60. Let  $S = x_1 + x_2 + x_3 + \dots + x_n$ , where  $x_i > 0 \quad \forall i = 1, 2, 3, \dots, n$  then the minimum value of  $\frac{S}{S-x_1} + \frac{S}{S-x_2} + \dots + \frac{S}{S-x_n}$  is :

- (A)  $\frac{n}{n-1}$       (B)  $\frac{n}{n^2-1}$       (C)  $\frac{n^2}{n^2-1}$       (D)  $\frac{n^2}{n-1}$

61. Let  $xyz = 1$ , then the sum of minimum and maximum values of

$S = \frac{1}{1+x+xy} + \frac{1}{1+y+yz} + \frac{1}{1+z+zx}$  is :

- (A) 2      (B) 3      (C) 5      (D) Does not exist

62. Let  $a, b, c$  be positive real numbers then least value of  $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$  is :
- (A)  $\frac{1}{2}$       (B) 1      (C)  $\frac{3}{2}$       (D) 2
63. For  $a, b > 0$ , let  $g_1, g_2, g_3$  and  $g_4$  be geometric means between  $a$  and  $b$ , then the roots of the equation  $(g_2 g_3)x^2 - \left(\frac{g_2}{g_1 + g_3}\right)x - g_1 g_4 = 0$  are :
- (A) both negative      (B) both positive  
 (C) one negative and one positive      (D) imaginary
64. Which of the following statement(s) is(are) correct?
- (A) Sum of the reciprocal of all the  $n$  harmonic means inserted between  $a$  and  $b$  is equal to  $n$  times the harmonic mean between two given numbers  $a$  and  $b$ .  
 (B) Sum of the cubes of first  $n$  natural number is equal to square of the sum of the first  $n$  natural numbers.  
 (C) If  $a, A_1, A_2, A_3, \dots, A_{2n}, b$  are in A.P., then  $\sum_{i=1}^{2n} A_i = (n+1)(a+b)$ .  
 (D) If the first term of the geometric progression  $g_1, g_2, g_3, \dots, \infty$  is unity, then the value of the common ratio of the progression such that  $(4g_2 + 5g_3)$  is minimum equals  $\frac{2}{5}$ .

### MULTIPLE CORRECT

1. If  $1, x, y$  be a geometric sequence and  $x, y, 3$  be an arithmetic sequence, then possible values of  $(x+y)$  is /are :
- (A) 0      (B) 1      (C)  $\frac{15}{4}$       (D) 4
2. If  $x \in R$ , the numbers  $(5^{1+x} + 5^{1-x}), a/2, (25^x + 25^{-x})$  are in A.P., then 'a' can be :
- (A) 5      (B) 10      (C) 12      (D) 14
3. If the sum of first 100 terms of an arithmetic progression is  $-1$  and the sum of the even terms lying in first 100 terms is 1, then which of the following statement(s) is (are) correct ?
- (A) Common difference of the arithmetic progression is  $-\frac{3}{50}$ .  
 (B) First term of the arithmetic progression is  $\frac{149}{50}$ .  
 (C)  $100^{\text{th}}$  term of the arithmetic progression is  $\frac{74}{25}$ .  
 (D) Sum of an infinite geometric progression whose first term is  $\frac{47}{25}$  and common ratio is common difference of arithmetic sequence, equals 2 .

4. If  $\sin \theta, \sin^2 \theta, 1$  are the first 3 terms of an A.P., where  $\theta \in (-\pi, \pi)$  then the sum of the first 3 terms of an A.P. can be equal to :
- (A) 3      (B)  $\frac{3}{4}$       (C)  $\frac{15}{4}$       (D)  $\frac{17}{4}$
5. Which of the following statement(s) is (are) correct ?
- (A) If the discriminant of the quadratic trinomial  $P(x) = ax^2 + bx + c$  ( $a, b, c \in R$ ) vanishes then the range of  $P(x)$  is  $[0, \infty)$ .
- (B) If  $\sin^2 x + \sec^2 y = 1$  then number of ordered pairs  $(x, y)$  of real numbers where  $x, y \in [0, 2\pi]$  is equal to 9.
- (C) If triangle ABC, let  $a, b$  and  $c$  denote the lengths of the sides opposite to vertices  $A, B$  and  $C$  respectively. If  $(a+c-b)(a-c+b) = bc$ , then angle  $A$  is equal to  $\frac{\pi}{3}$ .
- (D) If  $a, b > 0$ , then minimum value of  $\left( \frac{8a}{b} + \frac{b}{2a} \right)$  is equal to 4.
6. An arithmetic progression has the following property: For an even number of terms, the ratio of the sum of first half of the terms to the sum of second half is always equal to a constant 'k'. Let the first term of arithmetic progression is 1. Then which of the following statement(s) is(are) correct ?
- (A) Absolute difference of all possible values of  $k$  is  $\frac{2}{3}$ .
- (B) The sum of all possible values of  $k$  is  $\frac{4}{3}$ .
- (C) If the number of terms of arithmetic progression is 20, then the sum of all terms of all possible arithmetic progression is 420.
- (D) The number of possible non-zero values of common difference of arithmetic progression is 1.
7. The first term of an infinite geometric series is 21. The second term and the sum of the series are both positive integers. The possible value(s) of the second term can be :
- (A) 12      (B) 14      (C) 18      (D) 20
8. The first term of an A.P. and a G.P. of positive terms, are same and equal to the distance travelled by the body moving rectilinearly with the speed  $v(t) = 3t^2 - 4t + 1$  m/s in first two seconds. Let the third term of both the sequences are also equal and the non-negative difference of their second terms is 4. If  $S_n$  denotes the sum to  $n$  terms of A.P. and  $\sigma_n$  denotes the corresponding for G.P., then which of the following is(are) correct ?
- (A) First term of the G.P. is 1      (B) Common difference of the A.P. is 8  
 (C)  $S_5 > \sigma_4$       (D)  $n^{\text{th}}$  term of the G.P. is  $(3^n - 1)$ .
9. In an A.P. whose first term and common difference is non-zero, the ratio  $R$  of the sum of the first  $n$  terms to the sum of  $n$  terms succeeding them does not depend on  $n$ . Then :
- (A) ratio of common difference of A.P. to its first term is 2.



- (C) remaining angle of the polygon is  $130^\circ$   
 (D) remaining angle of the polygon is  $144^\circ$

16. Consider a sequence  $\{a_n\}$  with  $a_1 = 2$  and  $a_n = \frac{a_{n-1}^2}{a_{n-2}}$  for all  $n \geq 3$ , terms of the sequence being distinct. Given that  $a_2$  and  $a_5$  are positive integers and  $a_5 \leq 162$  then the possible value(s) of  $a_5$  can be :

(A) 2

(B) 32

(C) 64

(D) 162

17. The sum of the first three numbers in an A.P. is 24 and their product is 384. Then which of the following hold(s) good ?

(A) Sum to  $n$  terms can be  $2n(n + 1)$ .

(B) Sum to  $n$  terms can be  $2n(n + 3)$

(C) Sum to  $n$  terms can be  $14n - 2n^2$

(D) Sum of the squares of the 3 numbers is 224.

18. If  $5x - y, 2x + y, x + 2y$  are in A.P. and  $(x - 1)^2, (xy + 1), (y + 1)^2$  are in G.P.,  $x \neq 0$ , then  $(x + y)$  equals :

(A)  $\frac{3}{4}$

(B) 3

(C) -5

(D) -6

19. For the A.P. given by  $a_1, a_2, \dots, a_n, \dots$  the equations satisfied are :

(A)  $a_1 + 2a_2 + a_3 = 0$

(B)  $a_1 - 2a_2 + a_3 = 0$

(C)  $a_1 + 3a_2 - 3a_3 - a_4 = 0$

(D)  $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

20. An increasing sequence is formed so that the difference between consecutive terms is a constant. If the first four terms of this sequence are  $x, y, 3x + y$  and  $x + 2y + 2$ , then which of the following statements hold good ?

(A)  $y - x$  is equal to 6.

(B)  $y = 3x$ .

(C)  $n^{\text{th}}$  term of the sequence is  $2(3n - 2)$ .

(D) sum of the first 43 terms of the series is 5050.

21.  $a, b, c$  are the first three terms of geometric series. If the H.M. of  $a$  and  $b$  is 12 and that of  $b$  and  $c$  is 36 then which of the following hold(s) good ?

(A) Sum of the first term and common ratio of the G.P. is 11.

(B) Sum of the first five terms of the G.P. is 948.

(C) If the value of the first term and common ratio of the given G.P. is taken as the first term and common difference of an A.P. then its 8<sup>th</sup> term is 29.

(D) The number 648 is one of the term of the G.P.

22. The real numbers  $x_1, x_2, x_3$  satisfying the equation  $x^3 - x^2 + \alpha x + \beta = 0$  are in A.P., then select the correct alternative(s).

(A)  $\alpha \leq \frac{1}{3}$

(B)  $\beta \geq -\frac{1}{27}$

(C)  $\alpha \geq \frac{1}{3}$

(D)  $\beta \leq -\frac{1}{27}$

23. Let  $a, b, c$  be unequal real numbers. If  $a, b, c$  are in G.P. and  $a + b + c = bx$ , then 'x' cannot be equal to:

(A) -1

(B) 0

(C) 2

(D) 3

24. Let  $f_n(\theta) = \sum_{n=0}^n \frac{1}{4^n} \sin^4(2^n \theta)$ . Then which of the following alternative(s) is/are correct?

(A)  $f_2\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

(B)  $f_3\left(\frac{\pi}{8}\right) = \frac{2+2\sqrt{2}}{4}$

(C)  $f_4\left(\frac{3\pi}{2}\right) = 1$

(D)  $f_5(\pi) = 0$

25. For any odd integer  $n \geq 1$ , if the value of the sum :

$n^3 - (n-1)^3 + (n-2)^3 + \dots + (-1)^{n-1}(1)^3$  equals 208, then 'n' cannot be :

(A) 5

(B) 7

(C) 9

(D) 11

26. Let  $S_n$  denotes the sum of  $n$  terms of an arithmetic progression whose first term is -4 and common difference is 1. If  $V_n = 2S_{n+2} - 2S_{n+1} + S_n$  ( $n \in N$ ), then :

(A)  $V_n = -\frac{9n^2 + 5n - 12}{2}, n \in N$

(B)  $V_n = \frac{n^2 - 5n - 12}{2}, n \in N$

(C) minimum value of  $V_n$  is -9(D) minimum value of  $V_n$  is  $-\frac{73}{8}$ 

27. The  $p^{\text{th}}$  term  $T_p$  of H.P. is  $q(p+q)$  and  $q^{\text{th}}$  term  $T_q$  is  $p(p+q)$  when  $p > 2, q > 2$ , then :

(A)  $T_{p+q} = pq$

(B)  $T_{pq} = p + q$

(C)  $T_{p+q} > T_{pq}$

(D)  $T_{p+q} < T_{pq}$

28. If  $\sum_{i=1}^4 a_i^2 x^2 - 2 \sum_{i=1}^4 a_i a_{i+1} x + \sum_{i=1}^4 a_{i+1}^2 \leq 0 \forall x > 0$  where  $a_i > 0$  and all are distinct then :

(A)  $a_1 + a_5 > 2a_3$

(B)  $\sqrt{a_1 a_5} = a_3$

(C)  $\frac{2}{\sqrt{a_1 a_4}} > \frac{1}{a_1} + \frac{1}{a_4}$

(D)  $\prod_{i=1}^5 a_i = a_3^5$

## COMPREHENSION TYPE QUESTIONS

### Comprehension

1

Let  $a_1, a_2, a_3, \dots$  be an arithmetic progression and  $b_1, b_2, b_3, \dots$  be a geometric progression. The sequence  $c_1, c_2, c_3, \dots$  is such that  $c_n = a_n + b_n \forall n \in N$ . Suppose  $c_1 = 1, c_2 = 4, c_3 = 15$  and  $c_4 = 2$ .

1. The common ratio of geometric progression is equal to :

(A) -2

(B) -3

(C) 2

(D) 3

2. The value of the sum  $\sum_{i=1}^{20} a_i$  is equal to :

(A) 480

(B) 770

(C) 960

(D) 1040

## Comprehension 2

Let  $\alpha, \beta, \gamma$  be three numbers such that  $\alpha + \beta + \gamma = 2$ ,  $\alpha^2 + \beta^2 + \gamma^2 = 6$  and  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{1}{2}$ .

1. The value of  $(\alpha^3 + \beta^3 + \gamma^3)$  is equal to :  
 (A) -2      (B) -1      (C) 6      (D) 8
2. If roots of the quadratic equation  $x^2 = 4ax + 12$  lies on either side of  $(\alpha\beta + \beta\gamma + \gamma\alpha)$ , then the largest integral value of  $a$  equals :  
 (A) 2      (B) 3      (C) 4      (D) 5
3. The sum of first 10 terms of a geometric progression whose first term and common ratio are  $\alpha^2 + \beta^2 + \gamma^2$  and  $\alpha\beta\gamma$  respectively, is equal to :  
 (A) -1023      (B) -2025      (C) -2046      (D) -4045

## Comprehension 3

The first four terms of a sequence are given by  $T_1 = 0, T_2 = 1, T_3 = 1, T_4 = 2$ .

The general term is given by  $T_n = A\alpha^{n-1} + B\beta^{n-1}$  where  $A, B, \alpha, \beta$  are independent of  $n$  and  $A$  is positive.

1. The value of  $(\alpha^2 + \beta^2 + \alpha\beta)$  is equal to :  
 (A) 1      (B) 2      (C) 5      (D) 4
2. The value of  $5(A^2 + B^2)$  is equal to :  
 (A) 2      (B) 4      (C) 6      (D) 8
3. The quadratic equation whose roots are  $\alpha$  and  $\beta$  is given by :  
 (A)  $x^2 - 2x - 1 = 0$       (B)  $x^2 - 2x - 2 = 0$       (C)  $x^2 - x - 1 = 0$       (D) None of these

## Comprehension 4

Let  $\langle a_n \rangle$  and  $\langle b_n \rangle$  be the arithmetic sequences each with common difference 2 such that  $a_1 < b_1$  and let  $c_n = \sum_{k=1}^n a_k, d_n = \sum_{k=1}^n b_k$ . Suppose that the points  $A_n(a_n, c_n), B_n(b_n, d_n)$  are all lying on the parabola :  $C: y = px^2 + qx + r$  where  $p, q, r$  are constants.

1. The value of  $p$  equals :  
 (A)  $\frac{1}{4}$       (B)  $\frac{1}{3}$       (C)  $\frac{1}{2}$       (D) 2
2. The value of  $q$  equals :  
 (A)  $\frac{1}{4}$       (B)  $\frac{1}{3}$       (C)  $\frac{1}{2}$       (D) 2
3. If  $r = 0$ , then the value of  $a_1$  and  $b_1$  are :  
 (A)  $\frac{1}{2}$  and 1      (B) 1 and  $\frac{3}{2}$       (C) 0 and 2      (D)  $\frac{1}{2}$  and 2

**Comprehension 5**

Consider  $\triangle XYZ$  whose sides  $x, y$  and  $z$  opposite to angular points  $X, Y$  and  $Z$  are in geometric progression.

1. If  $r$  be the common ratio of G.P., then :

(A)  $\frac{\sqrt{5}-1}{2} < r < \frac{\sqrt{5}+1}{2}$

(C)  $\frac{\sqrt{5}-1}{3} < r < \frac{\sqrt{5}+1}{3}$

(B)  $\frac{\sqrt{5}-2}{2} < r < \frac{\sqrt{5}+2}{2}$

(D)  $\frac{\sqrt{5}-2}{3} < r < \frac{\sqrt{5}+2}{3}$

2. The integral values of  $\frac{\sin Y}{\sin X}$  is :

(A) prime only      (B) even

(C) composite      (D) odd

3. The maximum value of  $\frac{\sin Z}{\sin Y}$  is :

(A) irrational number

(B) rational number but not integer

(C) integer

(D) not defined

**Comprehension 6**

Given  $a, b, c, d, e$  are such that  $a + b + c + d + e = 8$  and  $a^2 + b^2 + c^2 + d^2 + e^2 = 16$ , then

1. Greatest value of  $e$  is :

(A)  $\frac{3}{5}$

(B)  $\frac{6}{5}$

(C)  $\frac{8}{5}$

(D)  $\frac{16}{5}$

2. The value of 'a' when  $e$  attains its greatest value is :

(A)  $\frac{3}{5}$

(B)  $\frac{6}{5}$

(C)  $\frac{8}{5}$

(D)  $\frac{16}{5}$

**Match the Columns**

1.

**Column-I**

- (A) Number of positive integral ordered pairs of  $(a, b)$  such that  $6, a, b$  are in harmonical progression, is

- (B) If  $x, y$  and  $z$  are positive real numbers and  $x = \frac{12 - yz}{y + z}$ .  
The maximum value of  $(xyz)$  equals

- (C) If the roots of the equation  $3x^3 - 26x^2 + 52x - 24 = 0$  are in G.P. then the largest root, is

**Column-II**

6

(P)

7

(Q)

8

(R)

9

(S)

2.

**Column-I**

- (A) The value of  $3 \left( \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \dots + \infty \text{ terms} \right)$

- (B) If  $1 + \cos x + \cos^2 x + \cos^3 x + \dots \infty = 4 + 2\sqrt{3}$ , then number of solutions in  $[0, 4\pi]$  will be

- (C) If the roots of the equation  $x^3 - 18x^2 + px - q = 0$  are in A.P., then maximum value of  $(q - 2p + 7)$  is

- (D) If roots of the quadratic equation  $4x^2 - 2x - 1 = 0$  are  $\alpha$  and  $\beta$ , then the value of  $\left( \sum_{r=0}^{\infty} (\alpha^r + \beta^r) \right)$  is

**Column-II**

- (P) 6

- (Q) 7

- (R) 8

- (S) 9

3. If  $\sum_{r=1}^{\infty} \frac{8}{(2r+3)\sqrt{(2r+7)(2r+9)} + (2r+7)\sqrt{(2r+3)(2r+5)}} = \sqrt{\frac{a}{5}} + \sqrt{\frac{b}{7}} - \sqrt{c}$

where  $a, b, c$  are natural numbers, then :

**Column-I**

- (A)  $a$  is equal to  
 (B)  $b$  is equal to  
 (C)  $c$  is equal to  
 (D)  $\left( \frac{a}{7} + \frac{b}{3} + c \right)$  is equal to

**Column-II**

- (P) 4

- (Q) 7

- (R) 8

- (S) 9

4. If  $x^6 - 12x^5 + ax^4 - bx^3 + cx^2 - dx + 64 = 0$  has all positive real roots, then :

**Column-I**

- (A)  $a$  is equal to  
 (B)  $b$  is equal to  
 (C)  $c$  is equal to  
 (D)  $d$  is equal to

**Column-II**

- (P) 240

- (Q) 192

- (R) 160

- (S) 60

5. If the integers  $p, q, r$  and  $s$  are in increasing arithmetic progression such that  $s = p^2 + q^2 + r^2$  then :

**Column-I**

- (A)  $(p + q + r + s)$  is equal to  
 (B)  $(p - q - r + 2)$  is equal to  
 (C)  $(p^2 + q^2 + r^2 + s^2)$  is equal to  
 (D)  $(p^3 + q^3 + r^3 + s^3)$  is equal to

**Column-II**

- (P) 0  
 (Q) 2  
 (R) 6  
 (S) 8

## Integer Type Questions

1. Suppose that  $x, x^2 - 24$  and  $x + x^2$  are consecutive terms in arithmetic progression, then sum of all possible values of the common difference of sequence is :
2. If the roots of  $x^5 - 40x^4 + Px^3 + Qx^2 + Rx + S = 0$  are in geometric progression and the sum of their reciprocals is 10, then the value of  $|S|$  is :
3. If  $a_1, a_2, a_3, \dots$  is in arithmetic progression with common difference 1 and  $a_1 + a_2 + a_3 + \dots + a_{98} = 137$ , then the value of  $a_2 + a_4 + a_6 + \dots + a_{98}$  is :
4. Let  $a$  and  $b$  be positive integers. The value of  $xyz$  is 55 or  $\frac{343}{55}$  according as  $a, x, y, z, b$  are in arithmetic progression or harmonic progression. Find the value of  $(a^2 + b^2)$ .
5. Find the sum of all integral values of  $x$  in interval  $[-4, 100]$  satisfying  $\left| 2x - \sqrt{(2x-1)^2} \right| = 1$ .
6. A geometric sequence  $\{a_n\}$  has  $a_1 = \sin x, a_2 = \cos x$  and  $a_3 = \tan x$  for some real number  $x$ . If  $a_n = (1 + \cos x)$ , then find the value of  $n$ .
7. If the sum of the series  $\sigma = 0.9 \left( 99 + 999 + 9999 + \dots + \underbrace{999 \dots 9}_{\substack{\text{string of 101} \\ \text{digits all equal to 9}}} \right)$  can be expressed in the form of  $(10^a - b)$  where  $a, b \in N$ , then find  $(a + b)$ .
8. There are two sets  $M_1$  and  $M_2$  each of which consists of three numbers in arithmetic sequence whose sum is 15. Let  $d_1$  and  $d_2$  be the common differences such that  $d_1 = 1 + d_2$  and  $8p_1 = 7p_2$  where  $p_1$  and  $p_2$  are the product of the numbers respectively in  $M_1$  and  $M_2$ . If  $d_2 > 0$  then find the value of  $\left( \frac{p_2 - p_1}{d_1 + d_2} \right)$ .
9. Let the first term of a certain sequence is  $\sin \frac{\pi}{3}$  and each of its successive term is also a sine term whose arguments bear the same common difference equal to  $\frac{\pi}{3}$  with the arguments of the immediately preceding sine term. If the sum of the first 5050 terms can be expressed in the form as  $\sqrt{\frac{p}{q}}$  where  $p$  and  $q$  are relatively prime. Find  $(p + q)$ .
10. Let  $S_k, k = 1, 2, 3, \dots$  denote the sum of infinite geometric series whose first term is  $(k^2 - 1)$  and the common ratio is  $\frac{1}{k}$ . Find the value of  $\sum_{k=1}^{\infty} \frac{S_k}{2^{k-1}}$ .
11. Let  $3\cos \theta + 5\cos^2 \theta + 7\cos^3 \theta + \dots + \infty = \frac{-1}{2}$ , where  $|\cos \theta| < 1$ . Find the value of  $\left( 1 + 2\sin^2 \frac{\theta}{2} \right)^2$ .

12. For  $a, b, c \in R - \{0\}$ , let  $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$  are in A.P. If  $\alpha, \beta$  are the roots of the quadratic equation

$$2acx^2 + 2abc x + (a+c) = 0, \text{ then find the value of } (1+\alpha)(1+\beta).$$

13. Let  $T_n$  denotes the  $n^{\text{th}}$  term of a G.P. with common ratio 2 and  $(\log_2(\log_3(\log_{512} T_{100}))) = 1$ . If three sides of a triangle  $ABC$  are the values of  $(T_1 + T_2), T_2$  and  $T_3$  then area of the triangle is  $\frac{\sqrt{2160}}{N}$ , where  $N$  is positive integer. Find the remainder when  $N$  is divided by  $2^{10}$ .

14. If the sum of first  $n$  terms of an A.P. (having positive terms) is given by  $S_n = (1+2T_n)(1-T_n)$

$$\text{where } T_n \text{ is the } n^{\text{th}} \text{ term of series then } T_2^2 = \frac{\sqrt{a}-\sqrt{b}}{4} (a, b \in N). \text{ Find the value of } (a+b).$$

15. Let  $\{a_n\}$  be an arithmetic sequence with first term 77 and common difference -3. Find the maximum value of  $\sum_{k=1}^n a_k$ .

16. Three non-zero real numbers  $a, b, c$  are said to be in harmonic progression if  $\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$ . Number of harmonic progression  $a, b, c$  of strictly increasing positive integers in which  $a = 20$  and  $b$  divides  $c$ .

17. The number of values of  $x$  such that the three numbers  $2^x, 2^{x^2}$  and  $2^{x^3}$  form a non-constant arithmetic progression in same order.

18. If  $a_n = \sqrt{1 + \left(1 - \frac{1}{n}\right)^2} + \sqrt{1 + \left(1 + \frac{1}{n}\right)^2}$  ( $n \geq 1$ ), then the value of  $\sum_{i=1}^{20} \frac{1}{a_i}$  is :

19. Let  $a, b$  and  $c$  be positive real numbers. If the least value of  $\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1+c}{2a}$  can be expressed as  $\frac{p}{q}$  (where  $p$  and  $q$  are relatively prime) then  $(p+q)$  is:

20. If  $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (n \cdot 3^m + m \cdot 3^n)} = \frac{p}{q}$  (where  $p$  and  $q$  are relatively prime), then  $(p+q)$  is :

21. Let  $a_0, a_1, a_2, \dots, a_n$  be a sequence of numbers satisfying  $(3-a_{n+1})(6+a_n) = 18$  and  $a_0 = 3$ . If  $\sum_{i=0}^n \frac{1}{a_i} = \frac{1}{k}(2^{n+2} - n - 3)$ , then  $k$  is.

22. The number of ordered pairs  $(x, y)$  where  $x \in \left[0, \frac{\pi}{2}\right]$  such that  $\frac{(\sin x)^{2y}}{(\cos x)^{y^2/2}} + \frac{(\cos x)^{2y}}{(\sin x)^{y^2/2}} = \sin 2x$  is:

23. A sequence of integers  $a_1, a_2, a_3, \dots$  is chosen such that  $a_n = a_{n-1} - a_{n-2}$  for each  $n \geq 3$ . What is the sum of first 2019 terms of this sequence if the sum of first 1492 term is 1985 and sum of first 1985 terms is 1492.

24. For any sequence of real numbers of  $A = (a_1, a_2, a_3, \dots)$ , let us define a sequence denoted by  $\Delta A$  as  $(a_2 - a_1, a_3 - a_2, \dots)$ . Suppose all terms of  $\Delta(\Delta A)$  are 1 and  $a_{19} = a_{92} = 0$ , then  $a_1$  is :
25. Given that  $A_k = \frac{k(k-1)}{2} \cos\left(\frac{k(k-1)\pi}{2}\right)$ , then  $|A_{19} + A_{20} + \dots + A_{98}|$  is:
26. Given that  $x_1 = 211, x_2 = 375, x_3 = 420, x_4 = 523, x_5 = 267$  and  $x_n = x_{n-1} - x_{n-2} + x_{n-3} - x_{n-4}$  (when  $n \geq 5$ ) then the value of  $|x_{2015} + x_{2016} + x_{2017}|$  is :
27. A sequence of positive integers with  $a_1 = 1$  and  $a_9 + a_{10} = 646$  is formed so that the first three terms are in geometric progression, the second, the third and the fourth terms are in arithmetic progression and in general for  $n \geq 1$ , the terms,  $a_{2n-1}, a_{2n}, a_{2n+1}$  are in G.P. and the terms  $a_{2n}, a_{2n+1}, a_{2n+2}$  are in A.P. Let  $a_n$  be the greatest term of this sequence that is less than 1000 then  $(n+a_n)$  is :
28. The sequence  $(a_n)$  satisfies  $a_1 = 1$  and  $5^{(a_{n+1}-a_n)} - 1 = \frac{3}{3n+2}$  for  $n \geq 1$ . Let  $k$  be the least integer greater than 1 for which  $a_k$  is an integer then  $k^2$  is :
29. Let  $x_1, x_2, \dots, x_6$  be non-negative real numbers such that  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1$  and  $x_1x_3x_5 + x_2x_4x_6 \geq \frac{1}{540}$ . If the maximum possible value of  $x_1x_2x_3 + x_2x_3x_4 + x_3x_4x_5 + x_4x_5x_6 + x_5x_6x_1 + x_6x_1x_2$  can be expressed as  $\left(\frac{p}{q}\right)$  (where  $p$  and  $q$  are coprime, then  $(p+q)$  is:
30. If  $\sum_{k=1}^{360} \frac{1}{k\sqrt{k+1} + (k+1)\sqrt{k}} = \frac{p}{q}$  (where  $p$  and  $q$  are coprime, then  $(p+q)$  is :
31. For any positive integer  $n$ ,  $\langle n \rangle$  denotes the closest integer to  $\sqrt{n}$  then  $\sum_{n=1}^{\infty} \frac{2^{\langle n \rangle} + 2^{-\langle n \rangle}}{2^n}$  is :
32. Let  $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$  and  $T_n = \frac{1}{(n+1)H_n H_{n+1}}$ , then the value of  $(T_1 + T_2 + T_3 + \dots + \infty)$  is :
33. The value of  $\sum_{k=1}^{511} \frac{1}{\sqrt[3]{k^2} + \sqrt[3]{k(k+1)} + \sqrt[3]{(k+1)^2}}$  is .
34. Let  $\langle a_n \rangle$  be an arithmetic sequence. If  $\sum_{r=1}^{10^{99}} a_{2r} = 10^{100}$  and  $\sum_{r=1}^{10^{99}} a_{2r-1} = 10^{99}$ , then find the common difference of arithmetic sequence.
35. If three non-zero distinct real numbers form an arithmetic progression and the squares of these numbers taken in the same order constitute a geometric progression, then find the sum of all possible common ratios of the geometric progression.
36. Find sum of integral values of  $b$  in the interval  $[1, 20]$  for which the equation  $4^x + (1-b)2^{x+1} + b = 0$  has roots of opposite sign.

## Sequence and Progression

37. A cricketer has to score 4500 runs. Let  $a_n$  denotes the number of runs he scores in the  $n^{\text{th}}$  match. If  $a_1 = a_2 = \dots = a_{10} = 150$  and  $a_{10}, a_{11}, a_{12}, \dots$  are in A.P. with common difference -2, then find the total number of matches played by him to score 4500 runs.
38. If the value of  $\sum_{n=1}^{20} \frac{n^2 - \frac{1}{2}}{n^4 + \frac{1}{4}} = \frac{p}{q}$ , where  $p, q \in N$  and are relatively prime to each other, then find the value of  $(p+q)$ .
39. If  $a_1, a_2, a_3, \dots, a_{4001}$  are terms of an A.P. such that  $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \dots + \frac{1}{a_{4000} a_{4001}} = 10$  and  $a_2 + a_{4000} = 50$ , then find the value of  $|a_1 - a_{4001}|$ .
40. Let 'a' denotes the number of non-negative values of  $p$  for which the equation  $p \cdot 2^x + 2^{-x} = 5$  possess a unique solution. If  $a, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_{20}, 6$  are in H.P. and  $a, \beta_1, \beta_2, \beta_3, \dots, \beta_{20}, 6$  are in A.P., then find the value of  $\alpha_{18} \beta_3$ .

## Previous Year Questions

### JEE MAINS

#### ■ Single Correct

1. If  $1, \log_9(3^{1-x} + 2), \log_3(4 \cdot 3^x - 1)$  are in A.P. then  $x$  equals: [2002]  
 (A)  $\log_3 4$       (B)  $1 - \log_3 4$       (C)  $1 - \log_4 3$       (D)  $\log_4 3$
2.  $l, m, n$  are the  $p^{\text{th}}$ ,  $q^{\text{th}}$  and  $r^{\text{th}}$  terms of a G.P. all positive, then  $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$  equals: [2002]  
 (A) -1      (B) 2      (C) 1      (D) 0
3. The value of  $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \dots \infty$  is: [2002]  
 (A) 1      (B) 2      (C) 3/2      (D) 4
4. Fifth term of a G.P. is 2, then the product of its 9 terms is: [2002]  
 (A) 256      (B) 512      (C) 1024      (D) none of these
5. Sum of infinite number of terms of G.P. is 20 and sum of their square is 100. The common ratio of G.P. is: [2002]  
 (A) 5      (B) 3/5      (C) 8/5      (D) 1/5
6.  $1^3 - 2^3 + 3^3 - 4^3 + \dots + 9^3 =$  [2002]  
 (A) 425      (B) -425      (C) 475      (D) -475
7. If  $S_n = \sum_{r=0}^n \frac{1}{n C_r}$  and  $t_n = \sum_{r=0}^n \frac{r}{n C_r}$ , then  $\frac{t_n}{S_n}$  is equal to: [2004]  
 (A)  $\frac{2n-1}{2}$       (B)  $\frac{1}{2}n-1$       (C)  $n-1$       (D)  $\frac{1}{2}n$

8. Let  $T_r$  be the  $r^{\text{th}}$  term of an A.P. whose first term is  $a$  and common difference is  $d$ . If for some positive integers  $m, n, m \neq n$ ,  $T_m = \frac{1}{n}$  and  $T_n = \frac{1}{m}$ , then  $a - d$  equals: [2004]
- (A)  $\frac{1}{m} + \frac{1}{n}$       (B) 1      (C)  $\frac{1}{mn}$       (D) 0
9. The sum of the first  $n$  terms of the series  $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \dots$  is  $\frac{n(n+1)^2}{2}$  when  $n$  is even. When  $n$  is odd, the sum is: [2004]
- (A)  $\left[\frac{n(n+1)}{2}\right]^2$       (B)  $\frac{n^2(n+1)}{2}$       (C)  $\frac{n(n+1)^2}{4}$       (D)  $\frac{3n(n+1)}{2}$
10. If the coefficients of  $r^{\text{th}}, (r+1)^{\text{th}}$  and  $(r+2)^{\text{th}}$  terms in the binomial expansion of  $(1+y)^m$  are in A.P., then  $m$  and  $r$  satisfy the equation: [2005]
- (A)  $m^2 - m(4r-1) + 4r^2 - 2 = 0$   
 (B)  $m^2 - m(4r+1) + 4r^2 + 2 = 0$   
 (C)  $m^2 - m(4r+1) + 4r^2 - 2 = 0$   
 (D)  $m^2 - m(4r-1) + 4r^2 + 2 = 0$
11. If  $x = \sum_{n=0}^{\infty} a^n$ ,  $y = \sum_{n=0}^{\infty} b^n$ ,  $z = \sum_{n=0}^{\infty} c^n$  where  $a, b, c$  are in A.P. and  $|a| < 1, |b| < 1, |c| < 1$  then  $x, y, z$  are in: [2005]
- (A) G.P      (B) A.P  
 (C) arithmetic-geometric progression      (D) H.P
12. Let  $a_1, a_2, a_3, \dots$  be terms of an A.P. If  $\frac{a_1 + a_2 + \dots + a_p}{a_1 + a_2 + \dots + a_q} = \frac{p^2}{q^2}, p \neq q$ , then  $\frac{a_6}{a_{21}}$  equals: [2006]
- (A)  $\frac{41}{11}$       (B)  $\frac{7}{2}$       (C)  $\frac{2}{7}$       (D)  $\frac{11}{41}$
13. If  $a_1, a_2, \dots, a_n$  are in H.P., then the expression  $a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n$  is equal to: [2006]
- (A)  $n(a_1 - a_n)$       (B)  $(n-1)(a_1 - a_n)$       (C)  $n a_1 a_n$       (D)  $(n-1)a_1 a_n$
14. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is: [2007]
- (A)  $\sqrt{5}$       (B)  $\frac{1}{2}(\sqrt{5}-1)$       (C)  $\frac{1}{2}(1-\sqrt{5})$       (D)  $\frac{1}{2}\sqrt{5}$
15. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is: [2008]
- (A) -4      (B) -12      (C) 12      (D) 4
16. The sum to infinite term of the series  $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots$  is: [2009]
- (A) 3      (B) 4      (C) 6      (D) 2

17. A person is to count 4500 currency notes. Let  $a_n$  denotes the number of notes he counts in the  $n^{\text{th}}$  minute. If  $a_1 = a_2 = \dots = a_{10} = 150$  and  $a_{10}, a_{11}, \dots$  are in an A.P. with common difference -2, then the time taken by him to count all notes is: [2010]

(A) 34 minutes      (B) 125 minutes      (C) 135 minutes      (D) 24 minutes

18. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after: [2011]

(A) 19 months      (B) 20 months      (C) 21 months      (D) 18 months

19. Let  $a_n$  be the  $n^{\text{th}}$  term of an A.P. If  $\sum_{r=1}^{100} a_{2r} = \alpha$  and  $\sum_{r=1}^{100} a_{2r-1} = \beta$ , then the common difference of the A.P. is: [2011 RS]

(A)  $\alpha - \beta$       (B)  $\frac{\alpha - \beta}{100}$       (C)  $\beta - \alpha$       (D)  $\frac{\alpha - \beta}{200}$

20. Statement- 1: The sum of the series  $1 + (1+2+4) + (4+6+9) + (9+12+16) + \dots + (361+380+400)$  is 8000.

Statement- 2:  $\sum_{k=1}^n (k^3 - (k-1)^3) = n^3$ , for any natural number  $n$ . [2012]

- (A) Statement-1 is false, Statement-2 is true.  
 (B) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.  
 (C) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.  
 (D) Statement-1 is true, Statement-2 is false.

21. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is: [JEE M 2013]

(A)  $\frac{7}{81}(179 - 10^{-20})$       (B)  $\frac{7}{9}(99 - 10^{-20})$       (C)  $\frac{7}{81}(179 + 10^{-20})$       (D)  $\frac{7}{9}(99 + 10^{-20})$

22. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is: [JEE M 2014]

(A)  $2 + \sqrt{3}$       (B)  $\sqrt{2} + \sqrt{3}$       (C)  $3 + \sqrt{2}$       (D)  $2 - \sqrt{3}$

23. If  $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + \dots + 10(11)^9 = k(10)^9$ , then  $k$  is equal to: [JEE M 2014]

(A) 110      (B)  $\frac{121}{10}$       (C)  $\frac{441}{100}$       (D) 100

24. The sum of first 9 terms of the series

$$\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \dots \text{ is:} [JEE M 2015]$$

(A) 192      (B) 71      (C) 96      (D) 142

25. If  $m$  is the A.M. of two distinct real numbers  $l$  and  $n$  ( $l, n > 1$ ) and  $G_1, G_2$  and  $G_3$  are three geometric means between  $l$  and  $n$ , then  $G_1^4 + 2G_2^4 + G_3^4$  equals: [JEE M 2015]

(A)  $4l^2m^2n^2$       (B)  $4l^2mn$       (C)  $4lm^2n$       (D)  $4lmn^2$

## JEE ADVANCED

## ■ Single Correct

1. Consider an infinite geometric series with first term  $a$  and common ratio  $r$ . If its sum is 4 and the second term is  $3/4$ , then: [2000S]

(A)  $a = \frac{4}{7}, r = \frac{3}{7}$       (B)  $a = 2, r = \frac{3}{8}$       (C)  $a = \frac{3}{2}, r = \frac{1}{2}$       (D)  $a = 3, r = \frac{1}{4}$

2. If  $a, b, c, d$  are positive real numbers such that  $a + b + c + d = 2$ , then  $M = (a + b)(c + d)$  satisfies the relation: [2000S]

(A)  $0 \leq M \leq 1$       (B)  $1 \leq M \leq 2$       (C)  $2 \leq M \leq 3$       (D)  $3 \leq M \leq 4$

3. Let  $\alpha, \beta$  be the roots of  $x^2 - x + p = 0$  and  $\gamma, \delta$  be the roots of  $x^2 - 4x + q = 0$ . If  $\alpha, \beta, \gamma, \delta$  are in G.P., then the integral values of  $p$  and  $q$  respectively are: [2001S]

(A)  $-2, -32$       (B)  $-2, 3$       (C)  $-6, 3$       (D)  $-6, -32$

4. Let the positive numbers  $a, b, c, d$  be in A.P. Then  $abc, abd, acd, bcd$  are: [2001S]

(A) NOT in A.P. / G.P. / H.P.      (B) in A.P.  
(C) in G.P.      (D) in H.P.

5. If the sum of the first  $2n$  terms of the A.P.  $2, 5, 8, \dots$ , is equal to the sum of the first  $n$  terms of the A.P.  $57, 59, 61, \dots$ , then  $n$  equals: [2001S]

(A) 10      (B) 12      (C) 11      (D) 13

6. Suppose  $a, b, c$  are in A.P. and  $a^2, b^2, c^2$  are in G.P. if  $a < b < c$  and  $a + b + c = \frac{3}{2}$ , then the value of  $a$  is: [2002S]

(A)  $\frac{1}{2\sqrt{2}}$       (B)  $\frac{1}{2\sqrt{3}}$       (C)  $\frac{1}{2} - \frac{1}{\sqrt{3}}$       (D)  $\frac{1}{2} - \frac{1}{\sqrt{2}}$

7. If  $a_1, a_2, \dots, a_n$  are positive real numbers whose product is a fixed number  $c$ , then the minimum value of  $a_1 + a_2 + \dots + a_{n-1} + 2a_n$  is: [2002S]

(A)  $n(2c)^{1/n}$       (B)  $(n+1)c^{1/n}$       (C)  $2nc^{1/n}$       (D)  $(n+1)(2c)^{1/n}$

8. If  $\alpha \in \left(0, \frac{\pi}{2}\right)$  then  $\sqrt{x^2 + x} + \frac{\tan^2 \alpha}{\sqrt{x^2 + x}}$  is always greater than or equal to: [2003S]

(A)  $2 \tan \alpha$       (B) 1      (C) 2      (D)  $\sec^2 \alpha$

9. An infinite G.P. has first term ' $x$ ' and sum '5', then  $x$  belongs to: [2004S]

(A)  $x < -10$       (B)  $-10 < x < 0$       (C)  $0 < x < 10$       (D)  $x > 10$

10. In the quadratic equation  $ax^2 + bx + c = 0$ ,  $\Delta = b^2 - 4ac$  and  $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$ , are in G.P., where  $\alpha, \beta$  are the roots of  $ax^2 + bx + c = 0$ , then: [2005S]

(A)  $\Delta \neq 0$       (B)  $b\Delta = 0$       (C)  $c\Delta = 0$       (D)  $\Delta = 0$

11. If the sum of first  $n$  terms of an A.P. is  $cn^2$ . Then the sum of squares of these  $n$  terms is: [2009]

(A)  $\frac{n(4n^2 - 1)c^2}{6}$       (B)  $\frac{n(4n^2 + 1)c^2}{3}$       (C)  $\frac{n(4n^2 - 1)c^2}{3}$       (D)  $\frac{n(4n^2 - 1)c^2}{6}$

12. Let  $a_1, a_2, a_3, \dots$  be in harmonic progression with  $a_1 = 5$  and  $a_{20} = 25$ . The least positive integer  $n$  for which  $a_n < 0$  is: [JEE 2012, 3M, -1 M]

**Multiple Correct**

1. A straight line through the vertex  $P$  of a triangle  $PQR$  intersects the side  $QR$  at the point  $S$  and the circumcircle of the triangle  $PQR$  at the point  $T$ . If  $S$  is not the centre of the circumcircle then:

[JEE 2008]

- (A)  $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$

(B)  $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$

(C)  $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{OR}$

(D)  $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

2. Let  $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$ . Then  $S_n$  can take value(s): [JEE Advanced 2013, 4, (-1)]

## ■ Comprehension Type Questions

## Comprehension

1

Let  $V_r$  denotes the sum of first  $r$  terms of an arithmetic progression (A.P.) whose first term is  $r$  and the common difference is  $(2r - 1)$ .

Let  $T_r = V_{r+1} - V_r - 2$  and  $Q_r = T_{r+1} - T_r$  for  $r = 1, 2, \dots$

1. The sum  $V_1 + V_2 + \dots + V_n$  is: [2007-4 Marks]

  - (A)  $\frac{1}{12}n(n+1)(3n^2 - n + 1)$
  - (B)  $\frac{1}{12}n(n+1)(3n^2 + n + 2)$
  - (C)  $\frac{1}{2}n(2n^2 - n + 1)$
  - (D)  $\frac{1}{3}(2n^3 - 2n + 3)$

2.  $T_r$  is always : [2007-4 Marks]

  - (A) an odd number
  - (B) an even number
  - (C) a prime number
  - (D) a composite number

3. Which one of the following is a correct statement? [2007-4 Marks]

  - (A)  $Q_1, Q_2, Q_3, \dots$  are in A.P with common difference 5.
  - (B)  $Q_1, Q_2, Q_3, \dots$  are in A.P with common difference 6.
  - (C)  $Q_1, Q_2, Q_3, \dots$  are in A.P with common difference 11.
  - (D)  $Q_1 = Q_2 = Q_3 = \dots$

## Comprehension

2

Let  $A_1, G_1, H_1$  denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers for  $n \geq 2$ . Let  $A_{n-1}$  and  $H_{n-1}$  have arithmetic, geometric and harmonic means as  $A_n, G_n, H_n$  respectively.

1. Which one of the following statement is correct?

[2007-4 Marks]

- (A)  $G_1 > G_2 > G_3 > \dots$   
 (B)  $G_1 < G_2 < G_3 < \dots$   
 (C)  $G_1 = G_2 = G_3 = \dots$   
 (D)  $G_1 < G_3 < G_5 < \dots$  and  $G_2 > G_4 > G_6 > \dots$

2. Which one of the following statement is correct?

[2007-4 Marks]

- (A)  $A_1 > A_2 > A_3 > \dots$   
 (B)  $A_1 < A_2 < A_3 < \dots$   
 (C)  $A_1 > A_3 > A_5 > \dots$  and  $A_2 < A_4 < A_6 < \dots$   
 (D)  $A_1 < A_3 < A_5 < \dots$  and  $A_2 > A_4 > A_6 > \dots$

3. Which one of the following statement is correct?

[2007-4 Marks]

- (A)  $H_1 > H_2 > H_3 > \dots$   
 (B)  $H_1 < H_2 < H_3 < \dots$   
 (C)  $H_1 > H_3 > H_5 > \dots$  and  $H_2 < H_4 < H_6 < \dots$   
 (D)  $H_1 < H_3 < H_5 < \dots$  and  $H_2 > H_4 > H_6 > \dots$

**■ Subjective Type Questions**

1. The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer.

[2004- 4 Marks]

2. Let  $a_1, a_2, \dots, a_n$  be positive real numbers in geometric progression. For each  $n$ , let  $A_n, G_n, H_n$  be respectively, the arithmetic mean, geometric mean, and harmonic mean of  $a_1, a_2, \dots, a_n$ . Find an expression for the geometric mean of  $G_1, G_2, \dots, G_n$  in terms of  $A_1, A_2, \dots, A_n, H_1, H_2, \dots, H_n$ .

[2001 - 5 Marks]

3. Let  $a, b$  be positive real numbers. If  $a, A_1, A_2, b$  are in arithmetic progression,  $a, G_1, G_2, b$  are in geometric progression and  $a, H_1, H_2, b$  are in harmonic progression.

$$\text{Show that } \frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a+b)(a+2b)}{9ab}$$

[2002 - 5 Marks]

4. If  $a, b, c$  are in A.P.,  $a^2, b^2, c^2$  are in H.P., then prove that either  $a = b = c$  or  $a, b, -c/2$  form a G.P.

[2003- 4 Marks]

5. If  $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \dots (-1)^{n-1} \left(\frac{3}{4}\right)^n$  and  $b_n = 1 - a_n$ , then find the least natural number  $n_0$  such that  $b_n > a_n \forall n \geq n_0$ .

[2006 - 6 M]

6. Let  $a_1, a_2, a_3, \dots, a_{11}$  be real numbers satisfying

$$a_1 = 15, 27 - 2a_2 > 0 \text{ and } a_k = 2a_{k-1} - a_{k-2} \text{ for } k = 3, 4, \dots, 11.$$

If  $\frac{a_1^2 + a_2^2 + \dots + a_{11}^2}{11} = 90$ , then the value of  $\frac{a_1 + a_2 + \dots + a_{11}}{11}$  is equal to : [JEE 2010]

7. The minimum value of the sum of real numbers  $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$  and  $a^{10}$  with  $a > 0$  is :

[JEE 2011]

8. Let  $a_1, a_2, a_3, \dots, a_{100}$  be an arithmetic progression with  $a_1 = 3$  and  $S_p = \sum_{i=1}^p a_i, 1 \leq p \leq 100$ . For any integer  $n$  with  $1 \leq n \leq 20$ , let  $m = 5n$ . If  $\frac{S_m}{S_n}$  does not depend on  $n$ , then  $a_2$  is : [JEE 2011]

9. A pack contains  $n$  cards numbered from 1 to  $n$ . Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is  $k$ , then  $k - 20 =$  [JEE Advanced 2013, 4, (-1)]

10. Let  $a, b, c$  be positive integers such that  $\frac{b}{a}$  is an integer. If  $a, b, c$  are in geometric progression and the arithmetic mean of  $a, b, c$  is  $b+2$ , then the value of  $\frac{a^2 + a - 14}{a+1}$  is : [JEE 2014]

11. Suppose that all terms of an A.P. are natural numbers. If the ratio of the sum of first seven terms to the sum of first eleven terms is  $6 : 11$  and the seventh term lies in between 130 and 140, then the common difference of A.P is : [JEE 2015]

**Answer****SINGLE CORRECT**

1. C	2. C	3. B	4. B	5. A	6. D	7. B	8. A
9. C	10. D	11. D	12. C	13. D	14. B	15. B	16. D
17. D	18. C	19. D	20. B	21. A	22. A	23. C	24. A
25. C	26. B	27. C	28. A	29. C	30. B	31. C	32. B
33. B	34. B	35. B	36. B	37. A	38. A	39. C	40. B
41. B	42. A	43. A	44. A	45. A	46. B	47. A	48. C
49. B	50. C	51. A	52. B	53. D	54. C	55. C	56. B
57. B	58. C	59. B	60. D	61. A	62. C	63. C	64. B

**MULTIPLE CORRECT**

1. A,C	2. C,D	3. C,D	4. A,B	5. B,C,D	6. A,B,C,D	7. A,B,C,D	8. B,C
9. A,B	10. A,B,C,D	11. A,B,D	12. A,C,D	13. B,C,D	14. C,D	15. B,C	16. B,D
17. A,C,D	18. A,D	19. B,D	20. A,C	21. A,C,D	22. A,B	23. A,B,C,D	24. C,D
25. A,C,D	26. B,C	27. A,B,C	28. A,B,D				

**COMPREHENSION TYPE**

Comprehension —1	1. B	2. C					
Comprehension —2	1. D	2. A	3. C				
Comprehension —3	1. B	2. A	3. C				
Comprehension —4	1. A	2. C	3. C				
Comprehension —5	1. A	2. D	3. D				
Comprehension —6	1. D	2. B					

**MATCH THE COLUMN**

- |           |        |        |       |
|-----------|--------|--------|-------|
| 1. (A) Q; | (B) R; | (C) P  |       |
| 2. (A) S; | (B) R; | (C) Q; | (D) P |
| 3. (A) Q; | (B) S; | (C) P; | (D) R |
| 4. (A) S; | (B) R; | (C) P; | (D) Q |
| 5. (A) Q; | (B) P; | (C) R; | (D) S |

**INTEGER TYPE**

1. 50	2. 32	3. 93	4. 50	5. 5050	6. 8	7. 201	8. 5
9. 7	10. 14	11. 5	12. 1	13. 0	14. 6	15. 1027	16. 5
17. 0	18. 7	19. 9	20. 41	21. 3	22. 1	23. 986	24. 819
25. 40	26. 319	27. 973	28. 1681	29. 559	30. 37	31. 3	32. 1
33. 7	34. 9	35. 6	36. 204	37. 34	38. 1641	39. 30	40. 12

## PREVIOUS YEARS

## JEE MAINS

## ■ Single Correct

1. B	2. D	3. B	4. B	5. B	6. A	7. D	8. D
9. B	10. C	11. D	12. D	13. D	14. B	15. B	16. A
17. A	18. C	19. B	20. B	21. C	22. A	23. D	24. C
25. C							

## JEE ADVANCED

## ■ Single Correct

1. D	2. A	3. A	4. D	5. A	6. D	7. A	8. A
9. C	10. C	11. C	12. D				

## ■ Multiple Correct

1. B,D	2. A,D						
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## ■ Comprehension Type Questions

Comprehension — 1	1. B	2. D	3. B				
Comprehension — 2	1. C	2. A	3. B				

## ■ Subjective Type

2. $G = \{A_1 A_2 \dots A_n, H_1 H_2 \dots H_n\}^{1/2n}$			5. 6	6. 0	7. 8	8. 9 or 3	
9. 5	10. 4	11. 9					