NLM
LECTURE - 5

AUROUS ACADEMY

Dynamics of Particles: Translation motion of accelerated bodies.

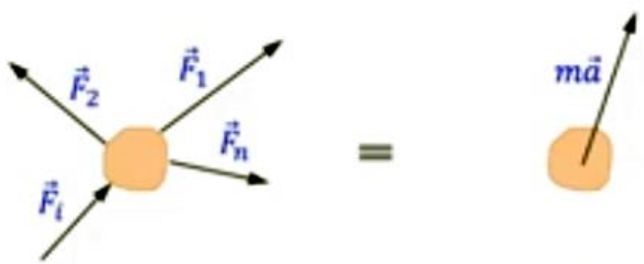
According to Newton's second law, forces acting on a body are considered as cause and rate of change in momentum or product of mass and acceleration as effect.

$$\Sigma\vec{F} = m\vec{a}$$

Strategy to use the law:

To write the equation of motion, it is recommended to draw the free body diagram, put a sign of equality and then draw the kinetic diagram.

Free Body Diagram (FBD) **Kinetic Diagram (KD)**



$$\Sigma\vec{F} \quad = \quad m\vec{a}$$

The equations obtained can either be solved by using triangle or polygon law of vector addition or by analytical method.

Triangle law may give very quick solutions in situations where only two forces are acting or some of the forces can be combined to reduce to a two force situation.

The analytical method is the most generalized one. In terms of Cartesian components, the analytical method yield the following equations.

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z$$

Ground or Laboratory Frame as Inertial Frame:

Newton's laws are valid only in inertial frames, so which frame we should prefer as a satisfactory inertial frame?

At present, we are interested in motion of terrestrial (earthly) bodies. For this purpose; a reference frame attached with the ground or a laboratory on the ground can be assumed a satisfactory inertial frame.

Free Space:

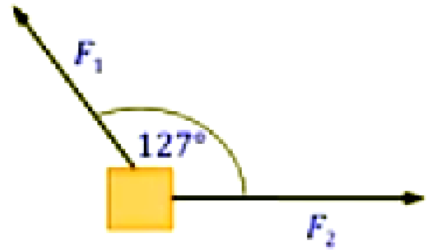
A region of space, where no force other then those we desire, can act on a body under consideration.

In mechanics, there is no interpretation of the term 'free space" other then the above mentioned one.

This term does not refer to vacuum or space.

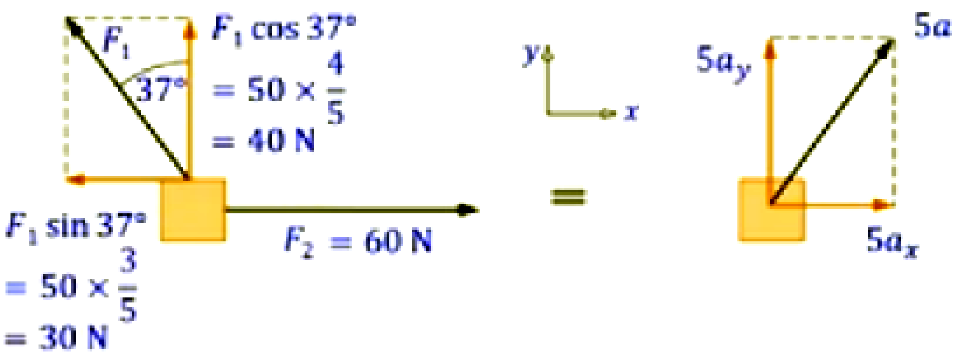
Example. 01

Two forces F_1 and F_2 of magnitudes 50 N and 60 N act simultaneously on a body of mass m = 5 kg as shown in the figure. Assuming the body in free space, find its acceleration vector?



Solution.

Free Body Diagram (FBD) **Kinetic Diagram (KD)**



$$\Sigma F_x = ma_x \qquad \Rightarrow 60 - 30 = 5a_x$$

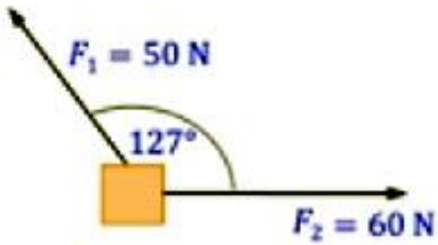
$$\Rightarrow a_x = 6 \text{ m/s}^2$$

$$\Sigma F_y = ma_y \qquad \Rightarrow 40 = 5a_y$$

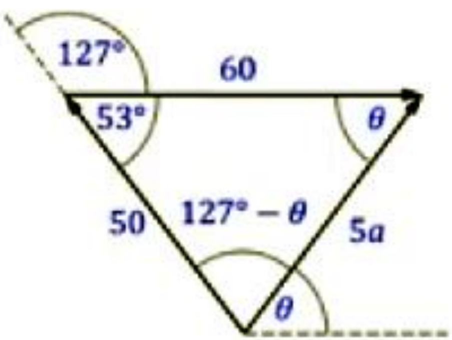
$$\Rightarrow a_y = 8 \text{ m/s}^2$$

$$\vec{a} = (6\hat{\imath} + 8\hat{\jmath}) \text{ m/s}^2$$

Graphical Method:



$$\vec{F}_1 + \vec{F}_2 = 5\vec{a}$$



Applying sin rule, we get:

$$\frac{5a}{\sin 53°} = \frac{50}{\sin\theta} = \frac{60}{\sin(127° - \theta)}$$

From the last two terms, we get:

$$5\sin\{90° + (37° - \theta)\} = 6\sin\theta$$

$$\Rightarrow 5\cos(37° - \theta) = 6\sin\theta$$

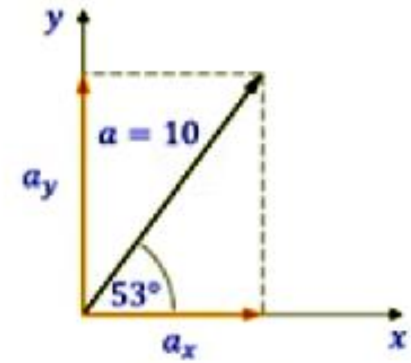
$$\Rightarrow 5\left(\frac{4}{5}\cos\theta + \frac{3}{5}\sin\theta\right) = 6\sin\theta$$

$$\Rightarrow 4\cos\theta = 3\sin\theta$$

$$\Rightarrow \tan\theta = \frac{4}{3} \quad \Rightarrow \theta = 53°$$

Now from the first two terms, we get:

$$\frac{5a}{\sin 53°} = \frac{50}{\sin 53°} \quad \Rightarrow a = 10$$



$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$$

$$= (10\cos 53°)\hat{\imath} + (10\sin 53°)\hat{\jmath}$$

$$= \left(10 \times \frac{3}{5}\right)\hat{\imath} + \left(10 \times \frac{4}{5}\right)\hat{\jmath}$$

$$= (6\hat{\imath} + 8\hat{\jmath})\ \mathrm{m/s^2}$$

Example 02.

A helium filled rigid spherical shell of mass m_1, when released in the air, starts ascending with an initial acceleration a. Another helium filled shell of the identical shape and size but of an unknown mass m_2, when released in the air, starts descending with an initial acceleration of the same magnitude as the previous one. Find mass of the second shell. Air drag is proportional to some unknown power of velocity.

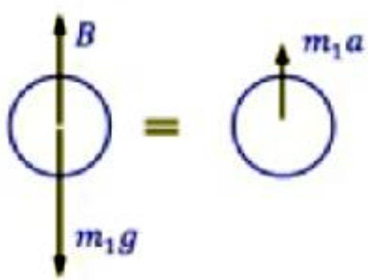
Solution.

Immediately after the release, velocity is zero so the air drag.

So, only two forces, the buoyant force and the weight, act on a shell immediately after they released.

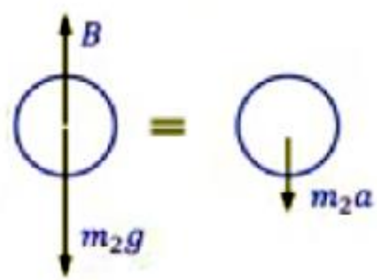
Buoyant force B on both the shells are equal as they displace the same amount of air due to equal volume.

For the first shell:



$$B - m_1 g = m_1 a \quad \text{..........} (1)$$

For the second shell:



$$m_2 g - B = m_2 a \quad \text{..........} (2)$$

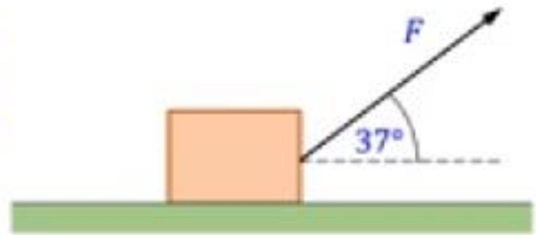
From the eq. (1) and (2), we get:

$$m_2 = m_1 \left(\frac{g + a}{g - a} \right)$$

Acceleration a cannot exceed g.

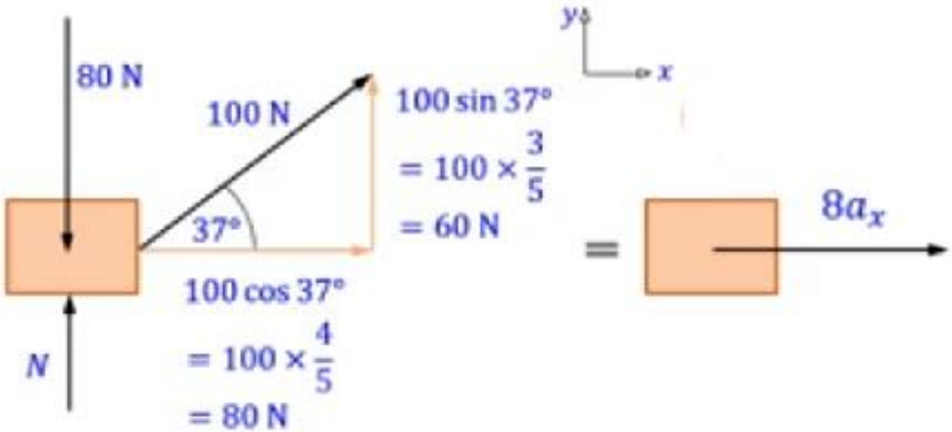
Example 03.

A block of mass 8 kg is being pulled by a constant force F of magnitude 100 N. If the horizontal floor is frictionless, find the acceleration of the block.



Solution.

Force acting on the block, the applied force F =100 N, the force of normal reaction from the ground N and weight 80 N are shown here:



The vertical component of the force F is smaller then the weight of the block, so the block will remain always in contact with the floor.

$$\Sigma F_x = ma_x \quad \Rightarrow 80 = 8a_x$$

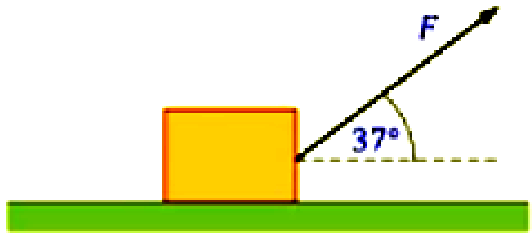
$$\Rightarrow a_x = 10\ \mathrm{m/s^2}$$

$$\Sigma F_y = 0 \quad \Rightarrow N + 60 - 80 = 0$$

$$\Rightarrow N = 20\ \mathrm{N}$$

Example 04.

A block of mass 2.4 kg is being pulled by a force of fixed direction but magnitude varying with time t as $F = 10t$ N. The horizontal floor is frictionless.

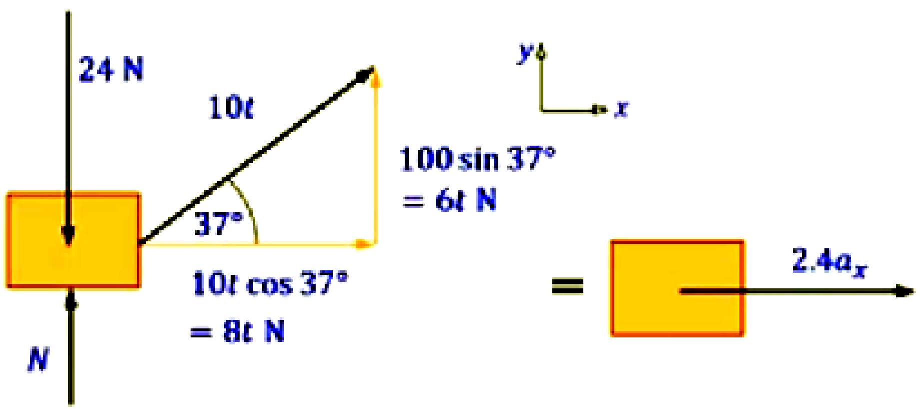


(a) When will the block leave the floor?

(b) Find speed of the block, when it is leaving the floor? *(c)* Find distance slid by the block on the floor.

Solution.

Force acting on the block, the applied force $F = 10t$ N, the force of normal reaction from the ground N and weight 24 N are shown here:



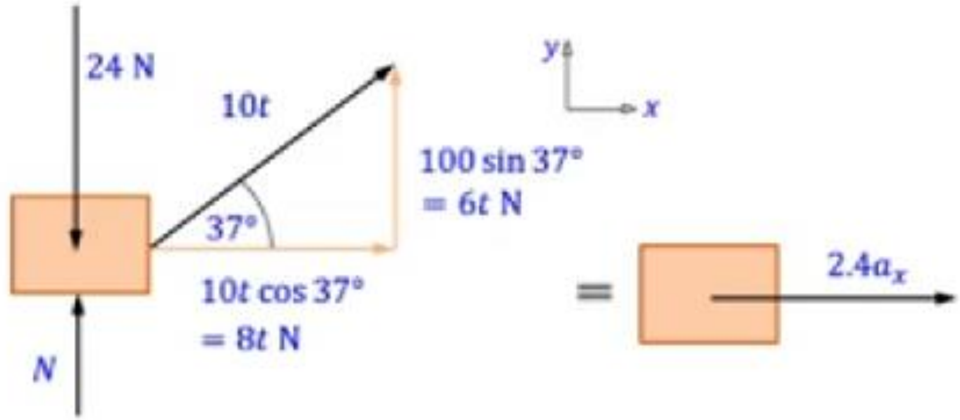
(a) When will the block leave the floor?

The moment, when the block leaves the floor, the force of normal reaction N will vanish and vertical component of acceleration be zero.

$$\Sigma F_y = 0 \qquad \Rightarrow 6t - 24 = 0$$

$$\Rightarrow t = 4 \text{ s}$$

(*b*) Find speed of the block, when it is leaving the floor?



$$\Sigma F_x = ma_x \quad \Rightarrow 8t = 2.4a_x$$

$$\Rightarrow a_x = \frac{10t}{3}\,\mathrm{m/s^2}$$

Acceleration is function of time, therefore:

$$a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t} \quad \Rightarrow \frac{\mathrm{d}v_x}{\mathrm{d}t} = \frac{10t}{3} \quad \Rightarrow \mathrm{d}v_x = \frac{10t}{3}\mathrm{d}t$$

$$\Rightarrow \int_0^{v_x} \mathrm{d}v_x = \frac{10}{3}\int_0^t t\mathrm{d}t$$

$$\Rightarrow v_x = \frac{5}{3}t^2 \quad \text{...............................}(1)$$

Substituting $t = 4$ s, we get the desired speed.

$$\Rightarrow v_x = \frac{5}{3} \times 4^2$$

$$\Rightarrow v_x = 26.67\ \mathrm{m/s}$$

(c) **Find distance slid by the block on the floor.**

Eq. (1) obtained in the previous part gives speed as function of time.

$$\Rightarrow v_x = \frac{5}{3}t^2 \quad \text{.............. ....(1)}$$

$$v_x = \frac{dx}{dt} \qquad \Rightarrow \frac{dx}{dt} = \frac{5}{3}t^2$$

$$\Rightarrow dx = \frac{5}{3}t^2 dt$$

$$\Rightarrow \int_0^x dx = \frac{5}{3}\int_0^4 t^2 dt$$

$$\Rightarrow x = \int_0^4 \frac{5}{3}t^2 dt$$

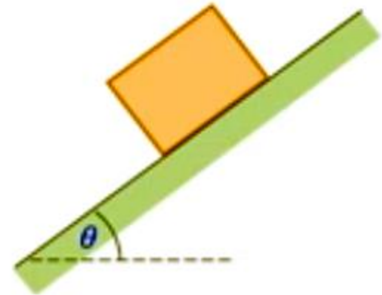
$$\Rightarrow x = 35.56 \text{ m}$$

Note:

In this problem, major part of the solution was from kinematics. Use of Newton's laws was limited to find time, when the block leaves the floor and horizontal component of acceleration.

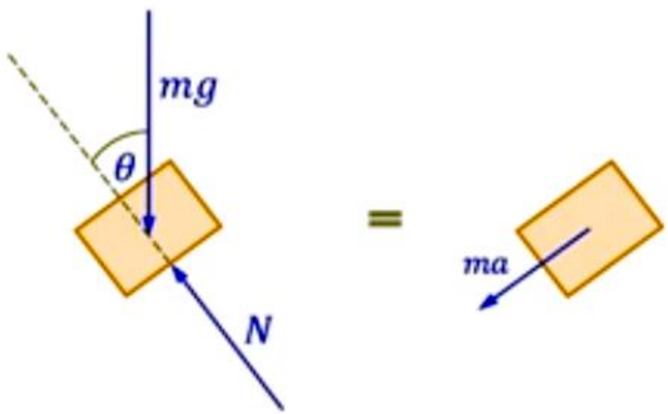
Example 05.

A block of mass m is sliding down a fixed frictionless inclined plane of inclination θ. Find acceleration of the block and the force of normal reaction.



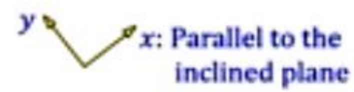
Solution.

Analytical Method.



How to orient the coordinate axes?

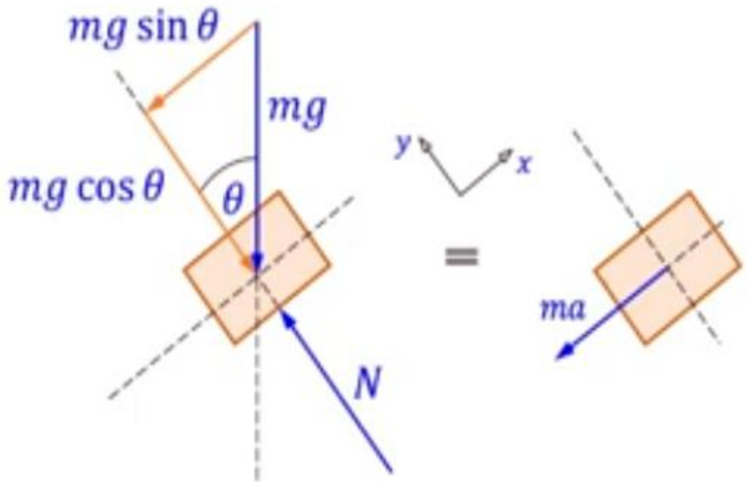
Orientation of the coordinate axes should be so chosen that maximum number of vectors in LHS as well RHS fall along the axes. In this way, minimum number of vectors has to be resolved into their components.



N and ma must be resolved into their components.

Only mg has to be resolved into its components.

Here in this problem, this choice of orientation is more appropriate then the first one.



$$\Sigma F_x = ma_x \quad \Rightarrow mg\sin\theta = ma$$

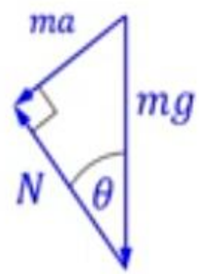
$$\Rightarrow a = g\sin\theta$$

$$\Sigma F_y = ma_y \quad \Rightarrow N - mg\cos\theta = 0$$

$$\Rightarrow N = mg\cos\theta$$

Graphical Method.

$$m\vec{g} + \vec{N} = m\vec{a}$$



$$\Rightarrow mg\sin\theta = ma$$

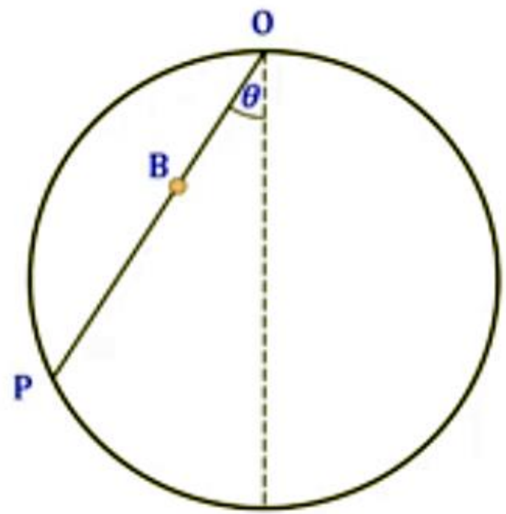
$$\Rightarrow a = g\sin\theta$$

$$\Rightarrow N = mg\cos\theta$$

Note:
Orientation of the coordinate axes should be so chosen that maximum number of vectors in LHS as well RHS fall along the axes. In this way, minimum number of vectors has to be resolve into their components.

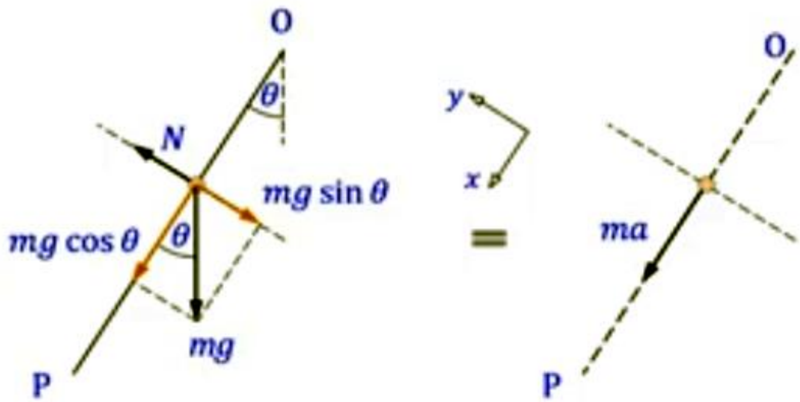
Example 06.

A circular wire frame of radius r is fixed in a vertical plane. From the top point O of the frame, a straight rigid wire OP is fixed in the frame along a chord as shown in the figure. A bead B can slide on the wire OP without friction. Find time of descend of the bead on the wire OP.



Solution.

Acceleration of the bead on the wire OP:



$$\Sigma F_x = ma_x \quad \Rightarrow mg\cos\theta = ma$$

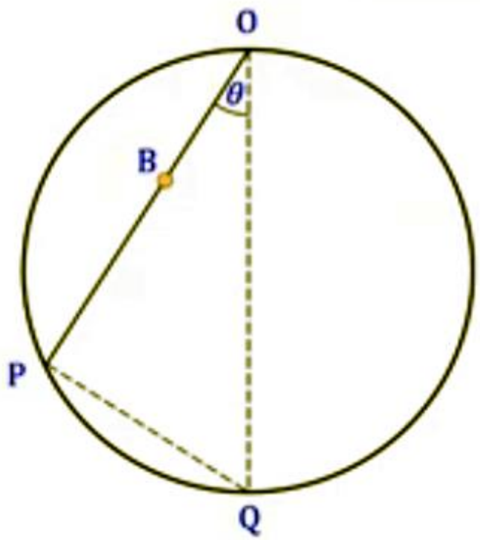
$$\Rightarrow a = g\cos\theta \quad \text{..........................} \quad (1)$$

$$\Sigma F_y = 0 \quad \Rightarrow N - mg\sin\theta = 0$$

$$\Rightarrow N = mg\sin\theta \quad \text{..................} \quad (2)$$

Time of descend of the bead on the wire OP:

Construct another chord PQ as shown.



ΔOPQ is a right angle triangle as angle subtended by a diameter on the circle is a right angle.

Denoting length OP by l:

$$l = 2r\cos\theta \quad \text{......................} \quad (3)$$

Eq. (1) reveals that the acceleration is a uniform one. Thus:

$$\Delta x = ut + \frac{1}{2}at^2 \quad \Rightarrow l = \frac{1}{2}at^2$$

Substituting expressions of a and l from eq. (1) and (3) respectively, we get:

$$\Rightarrow 2r\cos\theta = \tfrac{1}{2}(g\cos\theta)t^2$$

$$\Rightarrow t = 2\sqrt{\frac{r}{g}}$$

The time is independent of the angle θ.

System of Particles:

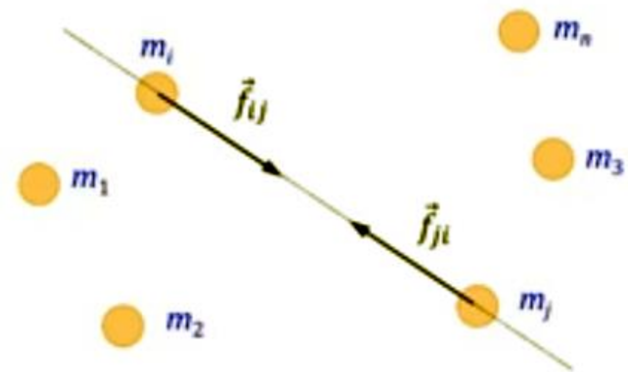
A system of particles is a well-defined collection of several or large number of particles, which may or may not apply forces of mutual interaction on each other.

By the term "particle", we mean either a material point or an extended body, only translational motion of which is to be considered.

As a schematic representation, consider a system of n particles of masses $m_1, m_2, \ldots\, m_i \ldots m_j \ldots\ldots\ldots$ and m_n respectively. They may be actual particles or rigid bodies in translation motion.

Some of them may interact with each other and some of them may not.

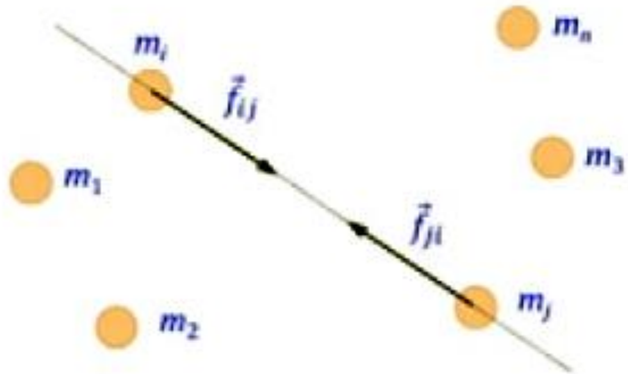
The particles, which interact with each other, apply forces on each other. The forces of interaction $\vec{f}_{ij}$ and $\vec{f}_{ji}$ between a pair of i^{th} and j^{th} particles are shown in the figure.



A System of Particles.

Internal and External Forces:

A System of Particles.



Internal Forces:

The forces of mutual interaction between the particles of a system are internal forces of the system.

The internal forces always exist in pairs of forces of equal magnitudes and opposite directions. In other words each pair is a Newton's third law pair.

Sum of all the internal forces of a system is a null vector.

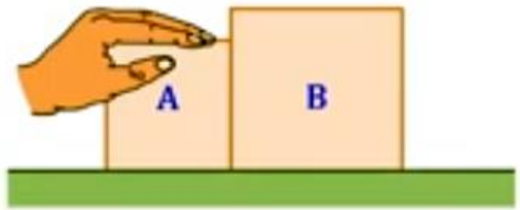
$$\Sigma\vec{f}_{ij} = \vec{0}$$

External Forces:

All those forces that are applied on a particle of a system by a body not included in the system.

Example. 07

Boxes A and B of masse m_A and m_B placed on a frictionless horizontal floor are being pushed horizontally by a force F as shown in the figure. Identify all the internal and external forces for the system of blocks A and B.



Solution.

All the forces acting on or within the system are listed here and shown in the adjacent figure.

$m_A g \stackrel{\text{def}}{=}$ Weight of the block A

$m_B g \stackrel{\text{def}}{=}$ Weight of the block B

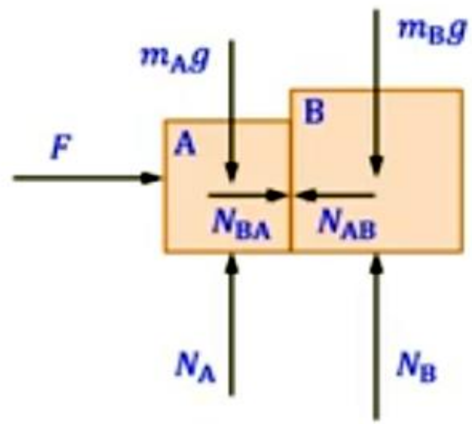
$N_A \stackrel{\text{def}}{=}$ Normal reaction of the floor on block A

$N_B \stackrel{\text{def}}{=}$ Normal reaction of the floor on block B

$N_{AB} \stackrel{\text{def}}{=}$ Normal reaction of the the block B on A

$N_{BA} \stackrel{\text{def}}{=}$ Normal reaction of the the block A on B

$F \stackrel{\text{def}}{=}$ Horizontal push of the hand



Internal Forces:

N_{AB} and N_{BA}

External Forces:

$m_A g$

$m_B g$

N_A

N_B

F