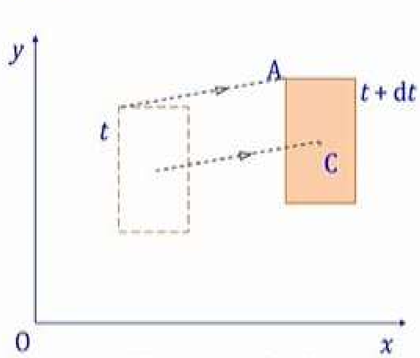
NLM
Lecture - 1

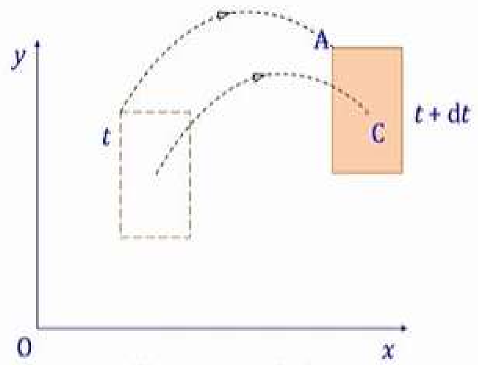
Translational Motion:

A body in translational motion, changes its location without rotation.

In translational motion, a body can move on a rectilinear or on a curvilinear path.



Rectilinear Translation



Curvilinear Translation

Characteristics of translational motion:

All the particles of the body and the mass center move on identical parallel trajectories.

All the particles and mass center of the body cover identical segments of their trajectories in a given time interval. Therefore, at any instant of time all of them have equal velocities and accelerations.

Particle:

Translational motion of a body can be represented by motion of any of its particle.

This is why; we can consider a body in translational motion as a particle, irrespective of its size.

In mechanics, a particle is either a material point or an extended body in translational motion.

Quantity of Motion:

If two bodies of unequal masses are moving with equal velocities, more effort is required to stop the body having greater mass.

If two bodies of equal masses are moving with different velocities, more effort is required to stop the body moving with greater velocity.

If any one, either mass or velocity vanishes, no effort is required to stop the body.

Thus, quantity of motion in a body depends on its velocity as well as its mass.

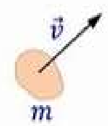
The simplest relation satisfying all the above criteria to describe the quantity of motion is the product of mass and velocity.

Linear Momentum:

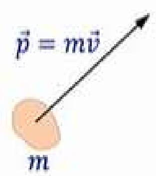
The product of mass and velocity of a body, known as linear momentum is equal to quantity of motion in the body.

$$\vec{p} = m\vec{v}$$

Linear momentum or simply momentum of a body is the quantity of motion in the body.



A body of mass m moving with velocity $\vec{v}$.



Quantity of motion in a body is equal to the momentum.

Inertia:

The tendency of an object to preserve its state of uniform motion or of rest is known as inertia of the object. It was first conceived by Galileo.

In other word:

The tendency of an object to preserve its quantity of motion is known as inertia of the object.

Due to its inertia, an object opposes any change in its momentum.

Mass: Quantity of Inertia.

In common usage, the term mass of a body refers to quantity of matter in that body.

But in mechanics, we are not concerned at all with quantity of matter in a material body but with other aspect that is related to change in its quantity of motion.

If two bodies of unequal masses are moving with the equal velocities, more effort is required to stop the body having greater mass. Therefore, mass serves as a suitable measure of inertia.

In fact in physical sciences, mass has no interpretation other then inertia.

Mass is measured in kilogram (kg), gram (g), and pound (lb) in SI, CGS, and FPS systems of units respectively.

$$1 \text{ kg} = 1000 \text{ g} = 2.205 \text{ lb}$$

Force:

A physical force or simply a force arises due to mutual interaction between two objects.

The concept of force explains mutual interaction between two objects as the action of one body on another in form of push or pull, which brings out or tries to bring out a change in the state of motion of the two bodies.

Action and Reaction:

A mutual interaction between two bodies, which creates force on one body, also creates force on the other body.

Force on the body under investigation is known as **action** and that on the other body as **reaction**.

Contact and Field Forces:

Forces between two bodies created by interactions due to direct contact are known as *contact forces* and forces created without any physical contact between the interacting bodies are known as *field forces.*

Tensile force of string, force applied by a spring, force of normal reaction and friction are the typical examples of contact forces; whereas, gravitational pull of the earth, electric force and magnetic force are examples of field forces.

Study of mechanics deals with how forces between two bodies affect their states of motion. It does not include study of nature of the mutual interactions. Therefore in mechanics, it becomes immaterial to know the nature of interaction i.e. electromagnetic, gravitational or nuclear or their combination or of some other kind.

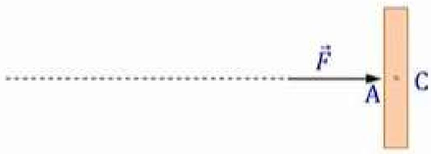
Fundamental characteristics of a force:

1. Magnitude. **2. Direction.** **3. Line of Action.**

To predict effects of a force on the motion of a body, magnitude, direction and point of application of the force must be known.

Line of action of a force is decided by its point of application and direction.

Consider a uniform bar placed on a frictionless horizontal floor. A force $\vec{F}$ is applied on the bar.



If the line of action of the force passes through the mass center C, the bar undergoes translational motion.

If line of action of the force does not pass through the mass center C, the bar undergoes translational and rotational motion both.

The magnitude and direction of a force decide its effect on translation motion.

The magnitude, direction and the line of action a of a force decide its effect on rotational motion.

Laws of Motion:

These laws are of fundamental nature and consider forces acting on a body as the only cause of alteration in the state of motion of the body.

The first law of motion: Law of Inertia.

Every material body preserves its state of rest or of uniform motion in a straight line, unless it is compelled to change that state by external forces impressed on it

In other word, force is the only cause of acceleration or change in momentum of a body.

A force is an agent of change in the quantity of motion; it does not sustain motion.

If there is no force acting on a body, momentum of the body will remain unchanged.

Necessity of a force to change momentum of a body reveals that every material body has a natural tendency to preserve its state of motion and force applied is necessary to overcome this tendency. This tendency of a body to preserve its state of motion is the inertia. This is why, this law is also known as the **law of inertia.**

Inertial Frame of Reference:

The first law requires a frame of reference in which for any acceleration produced in a body; only the forces acting on the body can be responsible and not the acceleration of the frame of reference. These frames of reference are known as inertial frames.

Obviously, an inertial frame of reference must not be an accelerated reference frame.

The second law of motion:

The first law suggests that a force is necessary to change state of motion. The second law provides necessary quantitative description and relates the force applied with corresponding change in momentum.

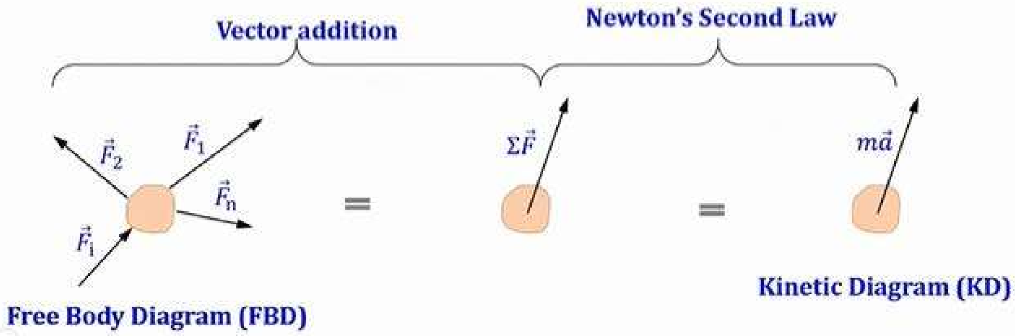
The law: The rate of change in momentum of a body is equal to and occurs in the direction of the net force applied.

A body of mass m in translational motion with velocity $\vec{v}$, if acted upon by a net external force $\Sigma\vec{F}$, the second law suggests:

$$\Sigma\vec{F} = \frac{\mathrm{d}}{\mathrm{d}t}(m\vec{v})$$

If mass of the body is a constant, the above equation relates acceleration $\vec{a}$ of the body with the net force $\Sigma\vec{F}$ acting on it.

$$\Sigma\vec{F} = m\vec{a}$$



The third law of motion:

The first law describe qualitatively and the second law quantitively, what happens to a body, if a net force acts on it. Both of these laws do not reveal anything about what happens to the other bodies participating in the interactions responsible for the net force.

The third law accounts for this aspect of the force.

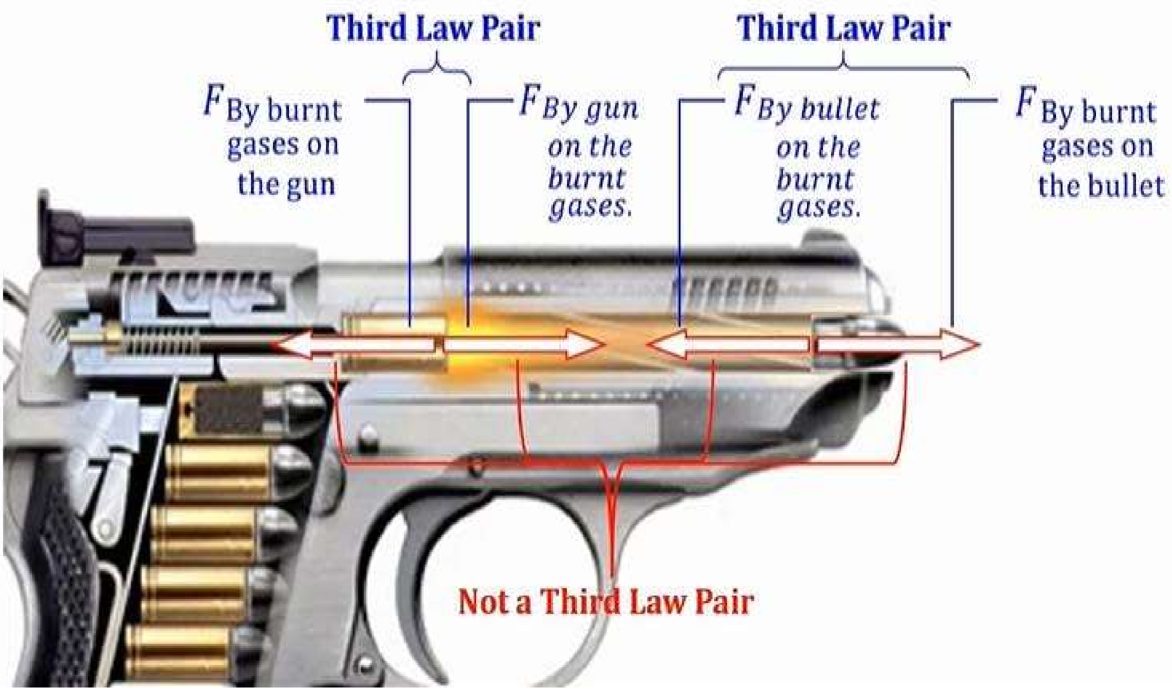
The action and reaction originating from a mutual interaction between two bodies are equal in magnitude and opposite in direction.

Fundamental nature of the three laws of motion:

These laws are fundamental in nature. The first law tells us under what conditions there is no net external force, the second law shows how to measure a force when it exists and the third law reminds us that a force is interaction between two bodies.

The third law pair: Action-reaction pair:

Recoil of a Gun:



Contact Forces.

All the contact forces are the reaction forces arising due to unnoticeable or noticeable deformations produced in the bodies in contact.

Most common contact forces are tensile force of a string, force of normal reaction, friction, spring force etc.

Tensile Force of Strings:

A string or similar flexible connecting link such as a thread or a chain etc. are used to transmit a force from one end to the other.

Due to flexibility, a string can only pull a body connected to it by applying a force always along the string.

This pulling force of a string is known as the tensile force of the string or mor commonly tension in the string. It is developed due almost negligible (unnoticeable) extension produced in the string.

In the figure, various forces between a string and a body connected to it are shown.

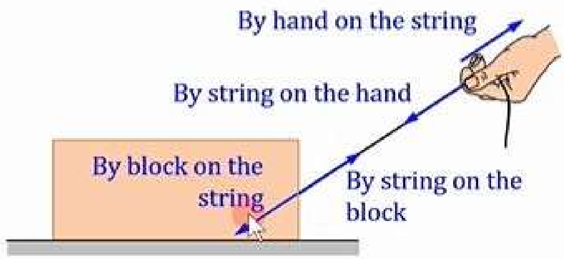
Question 01.

Identify the action-reaction pairs.

Ans.

Force by hand on the string and the force by string on the hand.

Force by string on the block and the force by block on the string.



Ideal String:

A string is used to pull an object connected at one end by transmitting the applied force from the other end.

An ideal string must have the following characteristics.

Inextensible: The pulled object must shift by the same amount as the end being pulled.

For this, length of the string must be a constant irrespective of the forces applied i.e. the string must be inextensible.

A string cannot be perfectly inextensible. In actual practice, extensions produced in the string must be negligible as compared to the length of the string.

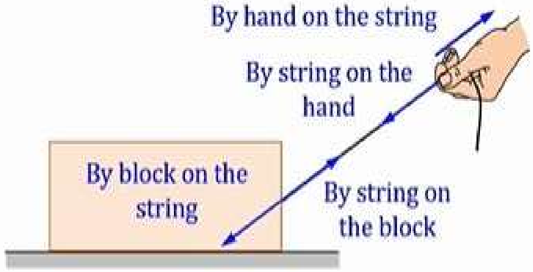
Massless: The applied force must be transmitted undiminished to the other end.

For this the string must be massless.

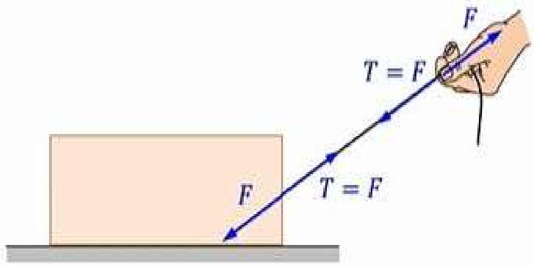
A material object cannot be massless so a string. In actual practice, mass of the string must be negligible as compared to the mass of the object being pulled.

Question 02.

If the force applied by the hand is F, show magnitude of all the forces considered in the following setup.



Ans.



Note:

A string must be assumed ideal, until it is not specified to be a non-ideal one i.e. having mass or being extensible or both.

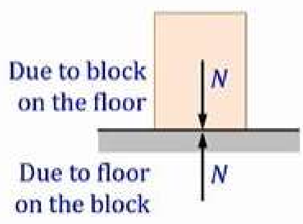
Force of Normal Reaction:

Two bodies in contact, when press each other, apply equal and opposite forces on each other. These forces constitute an action-reaction pair.

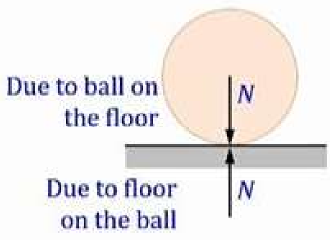
If surfaces of the bodies in contact are frictionless, the contact force acts along normal to the surfaces at the point of contact and known as the force of normal reaction or simply normal reaction.

If surfaces of the bodies are not frictionless, in addition the normal reactions, frictional forces also act.

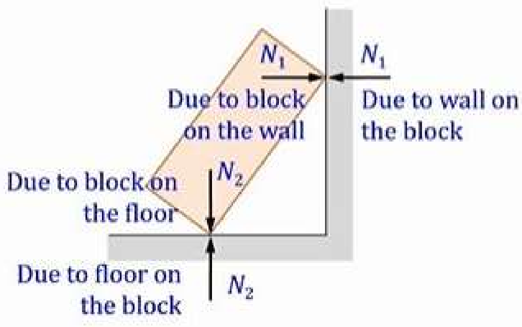
In the following figures, forces of normal reactions in some common situations are shown.



Block on a horizontal floor



Ball on a horizontal floor



Block on a corner

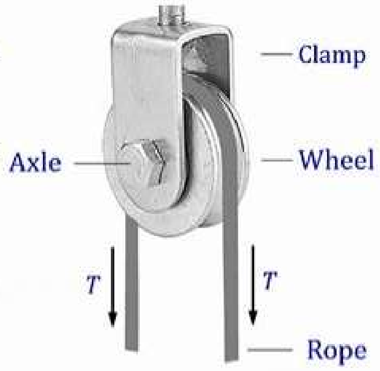
Pulley:

A pulley consists of a wheel free to rotate about an axle that is attached to a clamp.

The wheel rotates about an axle that may be a part of the clamp or of the wheel.

A pulley is used to change direction on force.

An ideal pulley must change direction of force undiminished.

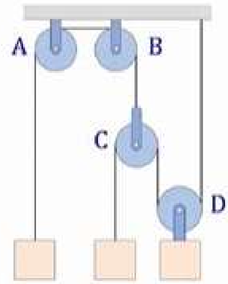


An ideal pulley must have the following characteristics.

Massless: For fixed pulleys, the wheel must be massless.

In case of a movable pulley, the entire pulley must be massless. Actually its mas must be negligible as compared the load on the pulley.

Frictionless: There must no friction between the axle and wheel, but between the rope and wheel, there must be sufficient friction to prevent the rope from slipping on the wheel.



In case of fixed pulleys as A and B, only the wheel must be massless.

In case of movable pulleys as C and D.

Necessity of Free Body Diagram (FBD):

A force is a two-body interaction. Therefore, in every situation, where forces are involved, there must be two or more bodies.

For quantitative description of motion of a body, we have to use Newton's second law.

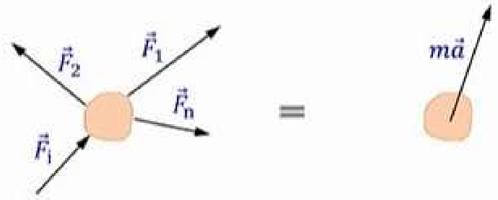
Therefore, to analyze a given problem, we have to consider each of the bodies separately one by one.

This idea lads us to the concept of free body diagram.

Free Body Diagram (FBD):

A free body diagram is a pictorial representation in which the body under study is assumed free from rest of the system, then it is drawn in its actual shape and orientation showing all the forces acting it.

$$\Sigma\vec{F} = m\vec{a}$$



Free Body Diagram (FBD) **Kinetic Diagram (KD)**

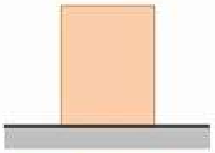
I. Separate the body under consideration from the rest of the system and draw it separately in actual shape and orientation.

II. Show all the forces whether known or unknown acting on the body at their respective points of application.

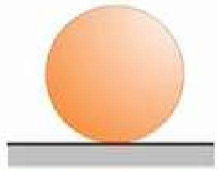
For the purpose, count every contact where we separate the body under study from other bodies. At every such contact point, there may be a contact force. After showing, all the contact forces show all the field forces.

Example 01.

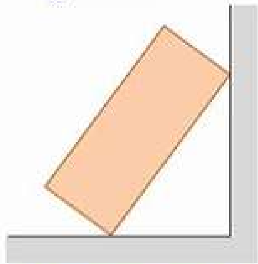
Denoting weight of the body by W, draw FBD in the following cases.



Block on a horizontal floor

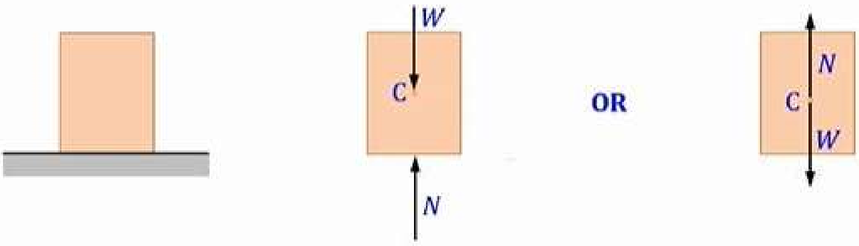


Ball on a horizontal floor



Block on a corner

Solution. **Block on a horizontal floor**

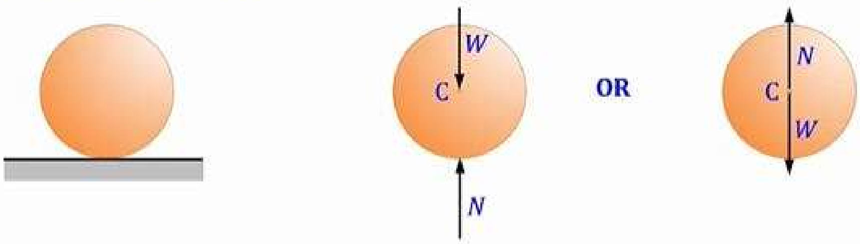


Note:

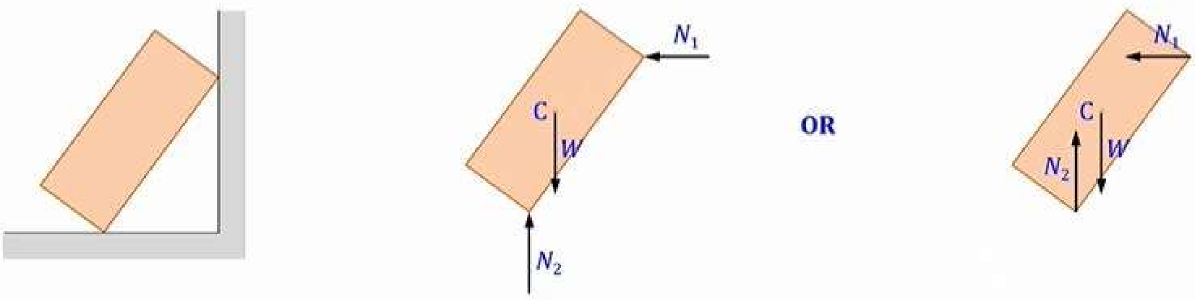
Weight is assumed to act at the mass center of the body.

In analyzing motion, a force can be shifted anywhere on its line of action.

Ball on a horizontal floor

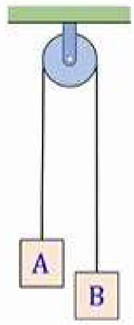


Block on a corner



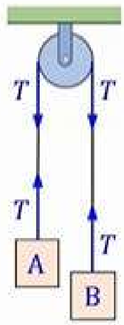
Example 02.

Draw FBD of the blocks and the pulley. Weight of the blocks B (W_B) is more then that (W_A) of A.

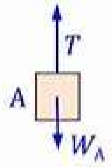


Solution.

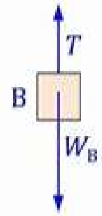
First consider how tensile force is transmitted in the string.



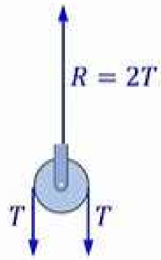
Here forces by the string are shown.



FBD of block A



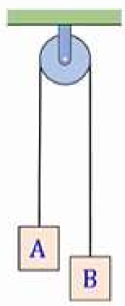
FBD of block B



FBD of the pulley

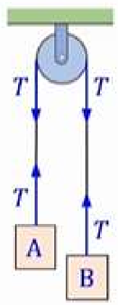
Example 03.

Draw FBD of the pulley, if weight of the pulley is W_P.

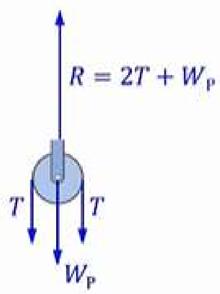


Solution.

First consider how tensile force is transmitted in the string.



Here forces by the string are shown.



FBD of the pulley

Note:

A pulley and a string must be assumed ideal, until it is not specified to tread any one of them nonideal.

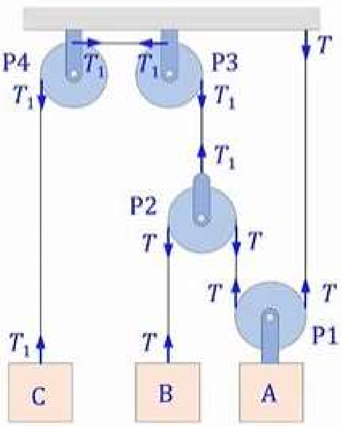
Thus, be the term string we mean an ideal string and by the term pulley we mean an ideal pulley .

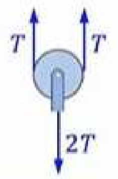
Example 04.

Draw FBD of the blocks and the pulleys. Weight of the blocks A, B and C are W_A, W_B and W_C respectively.

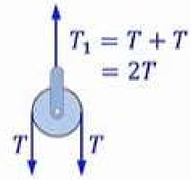
Solution.

Tensile forces in the strings.

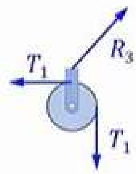




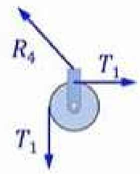
FBD of P1



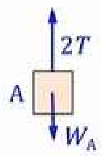
FBD of P2



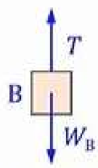
FBD of P3



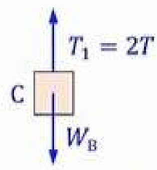
FBD of P4



FBD of block A



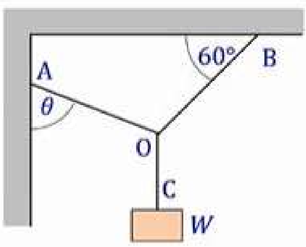
FBD of block B



FBD of block C

Example 05.

A box of weight W is held in equilibrium with the help of three strings OA, OB and OC as shown in the figure. Draw FBD of the box and the knot O.

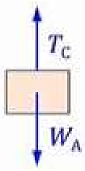


Note:

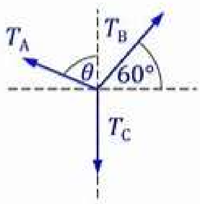
A knot of ideal strings is massless.

Solution.

Let the tensile forces in the strings OA, OB and OC be T_A, T_B and T_C.



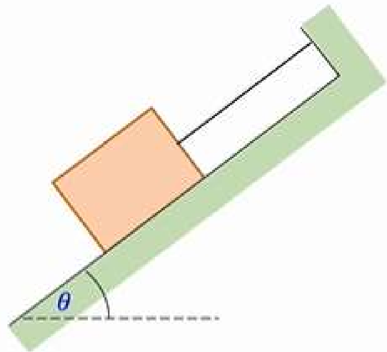
FBD of box



FBD of the knot O.

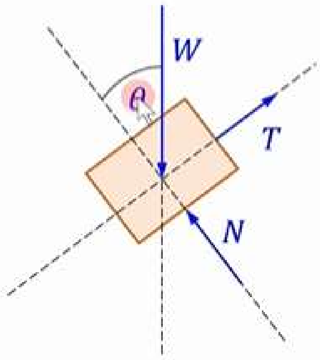
Example 06.

A box of weight W is held in equilibrium on a fixed frictionless inclined plane with the help of a string. Draw FBD of the box.



Solution.

Let tensile force in string be T and force of normal reaction from the inclined plane be N.



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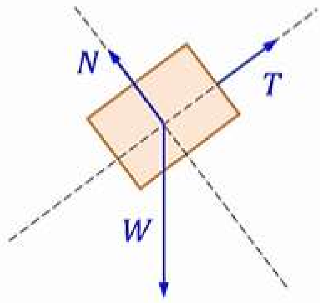
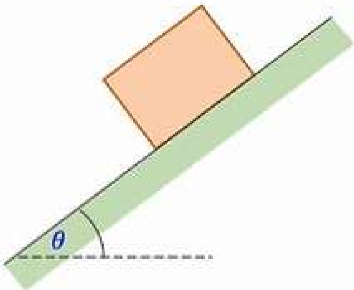


Example 07.

A box of weight W is sliding down a fixed inclined plane. Draw FBD of the box.

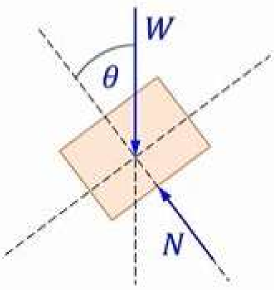
(a) Inclined plane is frictionless.

(b) Inclined plane is not frictionless.

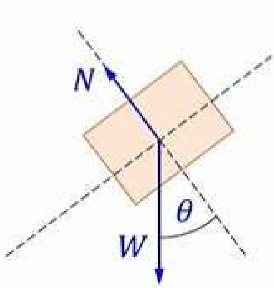


Solution.

(a) Inclined plane is frictionless.

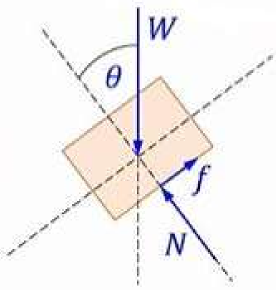


OR

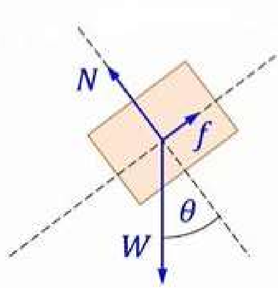


(b) Inclined plane is not frictionless.

Let f be the frictional force.



OR



Translational Equilibrium:

A body at rest or moving with a constant velocity is said to be in translational equilibrium.

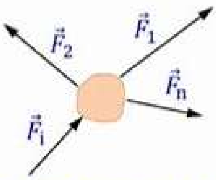
State of rest is known as static equilibrium and state of uniform velocity motion as dynamic equilibrium.

A body in translational equilibrium, has no acceleration.

$$\vec{a} = \vec{0}$$

Condition for translational equilibrium:

$\Sigma\vec{F}$ = $m\vec{0} = \vec{0}$



=

Free Body Diagram (FBD) **Kinetic Diagram (KD)**

If a body is in translational equilibrium, resultant of all the external forces $\vec{F}_1, \vec{F}_2, \ldots \vec{F}_i \ldots\ldots \vec{F}_n$ acting simultaneously on it will be a null vector.

$$\sum_{i=1}^{n} \vec{F}_i = \vec{0} \quad \text{Or} \quad \Sigma\vec{F} = \vec{0}$$

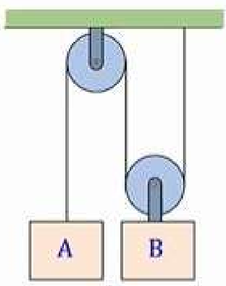
If the force are expressed in Cartesian components, we have :

$$\sum_{i=1}^{n} F_{ix} = 0 \qquad \sum_{i=1}^{n} F_{iy} = 0 \qquad \sum_{i=1}^{n} F_{iz} = 0$$

$$\text{Or} \quad \Sigma F_x = 0 \qquad \text{Or} \quad \Sigma F_y = 0 \qquad \text{Or} \quad \Sigma F_z = 0$$

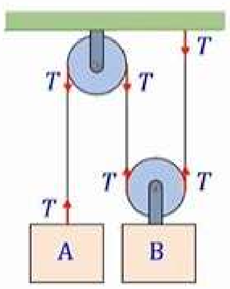
Example 01.

Two blocks A and B of masses m_A and m_B are suspended by a system of pulleys as shown. Find relation between m_A and m_B.

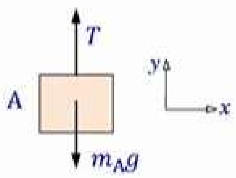


Solution.

Transmission of string tension.



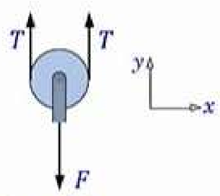
TE of the block A.



$\Sigma F_y = 0$

$\Rightarrow T = m_A g$ ...... (1)

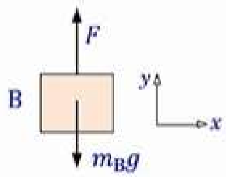
TE of the pulley supporting the block B.



$\Sigma F_y = 0$

$\Rightarrow F = 2T$ .... (2)

TE of the block B.

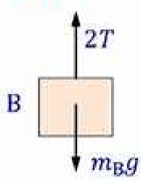


$\Sigma F_y = 0$

$\Rightarrow F = m_B g$ .... (3)

From the eq.. (1), (2) and (3), we get: $\Rightarrow 2m_A = m_B$

Note.

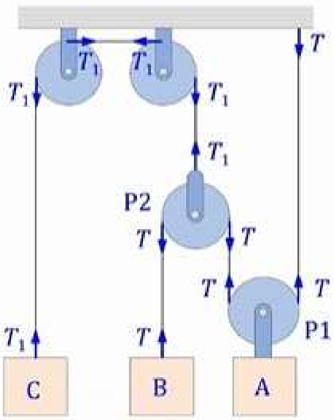


Example 02.

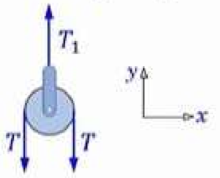
The setup is in equilibrium. If masses of the blocks A is m_A, find mass of the blocks B and C.

Solution.

Transmission of string tensions.

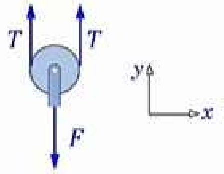


TE of the pulley P2.



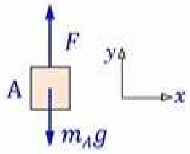
$\Sigma F_y = 0 \Rightarrow T_1 = 2T$ ...... (1)

TE of the pulley P1.



$\Sigma F_y = 0 \Rightarrow F = 2T$ ...... (2)

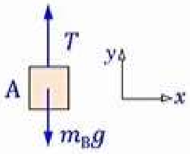
TE of the block A.



$\Sigma F_y = 0$

$\Rightarrow F = m_A g$ ...... (3)

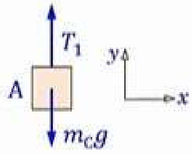
TE of the block B.



$\Sigma F_y = 0$

$\Rightarrow T = m_B g$ ...... (4)

TE of the block C.



$\Sigma F_y = 0$

$\Rightarrow T_1 = m_C g$ ...... (5)

From the eq.. (1), (2), (3), (4) and (5), we get:

$$\Rightarrow m_B = \frac{m_A}{2}$$

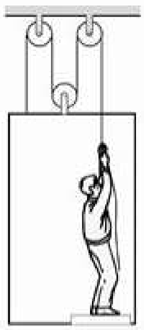
$$\Rightarrow m_C = m_A$$

Example 03.

A 70 kg man standing on a weighing machine in a 50 kg lift pulls on the rope, which supports the lift as shown in the figure.

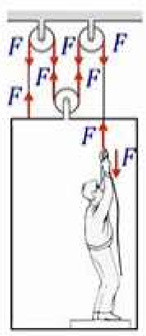
(a) Find the force with which the man should pull on the rope to keep the lift stationary.

(b) Find the weight of the man as shown by the weighing machine.

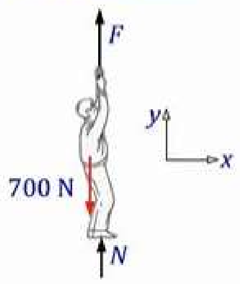


Solution.

Transmission of string tension.



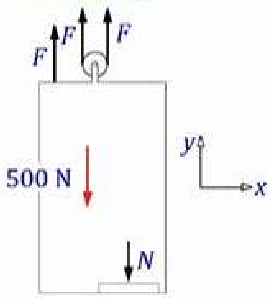
TE of the man.



$\Sigma F_y = 0$

$\Rightarrow F + N = 700$

TE of the lift.



$\Sigma F_y = 0$

$\Rightarrow 3F = N + 500$

(a) Force of the man on the rope: F

From the previous two equations:

$F = 300$ N

(b) Weight of the man shown by the weighing machine is the normal reaction of man on the weighing machine: N

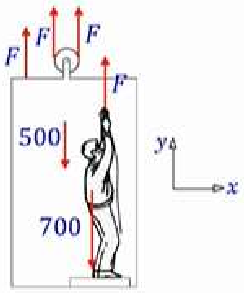
From the two equations:

$F = 400$ N

Alternate Approach:

When the man maintains the setup in equilibrium, there is no relative movements (in fact acceleration) between the man and the lift; therefore, we can assume all of three a single rigid body.

TE of this composite rigid body.

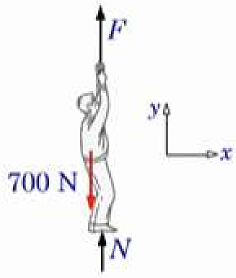


$\Sigma F_y = 0$

$\Rightarrow 4F = 500 + 700$

$\Rightarrow F = 300$ N

TE of the man.



$\Sigma F_y = 0 \quad \Rightarrow F + N = 700$

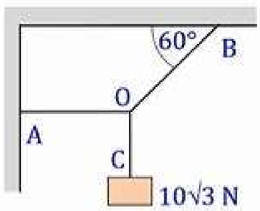
Substituting value of F.

$\Rightarrow N = 400$ N

Note:
Here, we have use this idea of composite body (constituent bodies have no relative acceleration) on the basis of intuitive understanding. But in the next lecture, we will discus the laws of physics behind this.

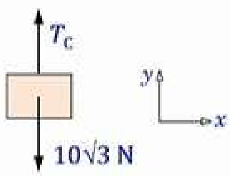
Example 04.

A box of weight $10\sqrt{3}$ N is held in equilibrium with the help of three strings OA, OB and OC as shown in the figure. The string OA is horizontal. Find the tensions in both the strings.



Solution.

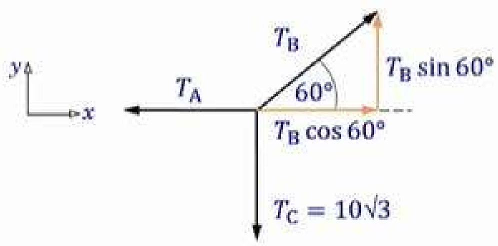
Conditions of (TE) of the box:



$\Sigma F_y = 0 \quad \Rightarrow T_C - 10\sqrt{3} = 0$

$\Rightarrow T_C = 10\sqrt{3}$ .... (1)

Conditions of TE of the knot:



$\Sigma F_y = 0 \quad \Rightarrow T_B \sin 60° = T_C$

$\Rightarrow T_B \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

$\Rightarrow T_B = 20$ N .... (2)

$\Sigma F_x = 0 \quad \Rightarrow T_A = T_B \cos 60°$

$\Rightarrow T_A = 20 \times \frac{1}{2}$

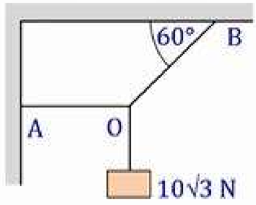
$\Rightarrow T_A = 10$ N ......(3)

Here, the calculations can be performed by using triangle law of vector addition in stead of analytical method (cartesian components).

Known as graphical method, this approach, usually provides great simplification.

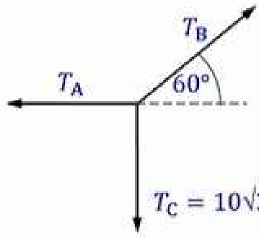
Graphical Method.

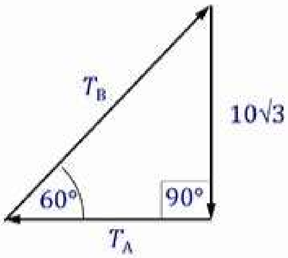
This method is based on the triangle law of vector addition.



The physical situation of the given setup reveals that for the translational equilibrium of the knot O, the vector sum of all the three forces acting on it must be a null vector.

$$\Sigma\vec{F} = \vec{0} \qquad \Rightarrow \vec{T}_A + \vec{T}_B + \vec{T}_C = 0$$





From TE of the box, we get : $T_C = 10\sqrt{3}$ N

From the geometry of the right-angled triangle, we get

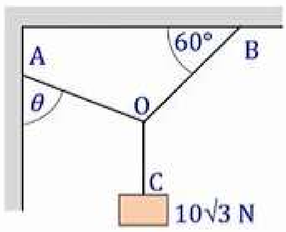
$$T_A = 10\sqrt{3}\cot 60° = 10\sqrt{3} \times \frac{1}{\sqrt{3}} = 10 \text{ N}$$

$$T_B = 10\sqrt{3}\,\text{cosec}\, 60° = 10\sqrt{3} \times \frac{2}{\sqrt{3}} = 20 \text{ N}$$

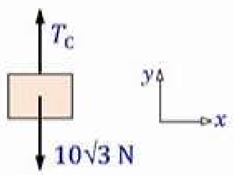
Note:
The graphical method, usually provides great simplification in dealing with TE with three simultaneous forces.

Example 05.

A box of weight $10\sqrt{3}$ N is held in equilibrium with the help of three strings OA, OB and OC as shown in the figure. You can change the location of the point A on the wall and hence vary the angle θ. Find the minimum tensile force in the string OA and the corresponding value of the angle θ.



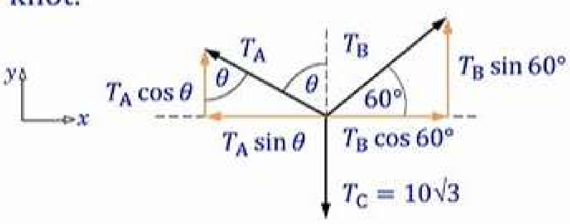
Solution. Analytical Method. Applying conditions of (TE) of the box:



$$\Sigma F_y = 0 \Rightarrow T_C - 10\sqrt{3} = 0$$

$$\Rightarrow T_C = 10\sqrt{3} \text{ ...... (1)}$$

Applying conditions of TE of the knot:



$$\Sigma F_x = 0 \Rightarrow T_A \sin\theta = T_B \cos 60°$$

$$\Rightarrow T_B = 2T_A \sin\theta \text{ .... (2)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow T_A \cos\theta + T_B \sin 60° = 10\sqrt{3} \text{ ..... (3)}$$

Substituting T_B from eq. (2) in eq. (3) we have

$$T_A = \frac{10\sqrt{3}}{\sqrt{3}\sin\theta + \cos\theta} \text{ .... (4)}$$

The above equation expresses the tensile force T_A as function of the angle θ.

According to this function, for the minimum value of T_A, the denominator must be maximum.

For an extremum i.e. either a maxima or minima:

$$\frac{d}{d\theta}\left(\sqrt{3}\sin\theta + \cos\theta\right) = 0$$

$$\Rightarrow \sqrt{3}\frac{d}{d\theta}\sin\theta + \frac{d}{d\theta}\cos\theta = 0$$

$$\Rightarrow \sqrt{3}\cos\theta - \sin\theta = 0$$

$$\Rightarrow \tan\theta = \sqrt{3}$$

$$\Rightarrow \theta = 60°$$

Substituting this value in eq. (4):

$$T_{Amin} = \frac{10\sqrt{3}}{\sqrt{3}\sin 60° + \cos 60°}$$

$$\Rightarrow T_{Amin} = \frac{10\sqrt{3}}{\sqrt{3}\times\frac{\sqrt{3}}{2} + \frac{1}{2}}$$

$$\Rightarrow T_{Amin} = 5\sqrt{3}\text{ N}$$

Another way of calculation.

$$D = \sqrt{3}\sin\theta + 1\times\cos\theta$$

Let us substitute:

$r\cos\varphi = \sqrt{3}$ and $r\sin\varphi = 1$

$$\Rightarrow D = r\cos\varphi\sin\theta + r\sin\varphi\cos\theta$$

$$\Rightarrow D = r(\cos\varphi\sin\theta + \sin\varphi\cos\theta)$$

$$\Rightarrow D = r\sin(\theta + \varphi)$$

Substituting r and φ :

$$D_{max} = r = \sqrt{\left(\sqrt{3}\right)^2 + 1} = 2 \quad \text{and} \quad \tan\varphi = \frac{1}{\sqrt{3}} \Rightarrow \theta = 60°$$

Substituting this maximum value of denominator in eq. (4), we get:

$$T_{Amin} = \frac{10\sqrt{3}}{D_{max}} = \frac{10\sqrt{3}}{2} = 5\sqrt{3}\text{ N}$$

Note:

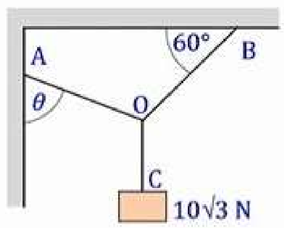
$r = a\sin\theta + b\cos\theta$

$\Rightarrow r_{max} = \sqrt{a^2 + b^2}$

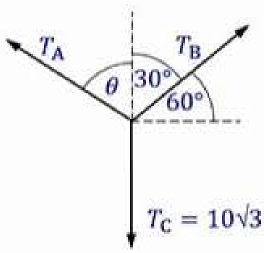
This idea will be very quick, if only r_{max} has to be found.

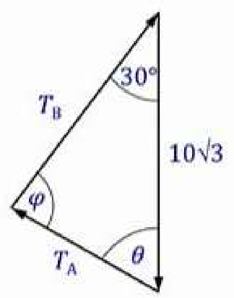
Graphical Method.

The physical situation of the given setup reveals that for the knot O to be in equilibrium, the vector sum of all the three forces acting on it must be a null vector.



$$\Sigma\vec{F} = \vec{0} \qquad \Rightarrow \vec{T}_A + \vec{T}_B + \vec{T}_C = 0$$





From TE of the box, we get : $T_C = 10\sqrt{3}$ N

From the figure, it is clear that for the vector T_A to be minimum, it must be perpendicular to the vector T_B. Therefore angle θ must be equal to 60° and $\varphi = 90°$.

$$T_{Amin} = 10\sqrt{3}\sin 30° = 5\sqrt{3}\text{ N}$$

Or

$$T_{Amin} = 10\sqrt{3}\cos\theta = 10\sqrt{3}\cos 60° = 5\sqrt{3}\text{ N}$$

Note:

Here, you can realize the simplicity brought in calculations by the graphical method.

In situations, where right-angled triangle is not formed, sin rule or Lami's theorem has to be used.

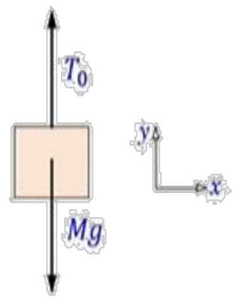
Example. 01

A block of mass M is suspended with the help of a uniform rope of mass m and length l from the ceiling as shown in the figure. Find expression for the tensile force in the rope at a distance x above the lower end.

Solution.

Tensile forces at lowest end of rope:

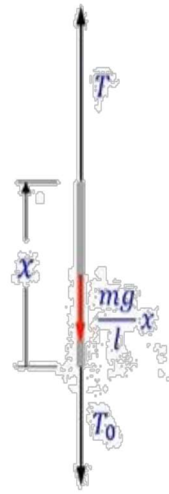
TE of block.



$\Sigma F_y = 0 \quad \Rightarrow T_0 = Mg$ ....(1)

Tensile forces at distance x above the lower end:

TE of this portion of the rope.



Mass of x length of the rope $= \frac{m}{l}x$

Weight of x length of the rope $= \frac{mg}{l}x$

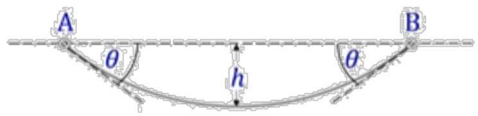
$\Sigma F_y = 0 \quad \Rightarrow T = T_0 + \frac{mx}{l}g$ ....(2)

From the above two eq. (1) and (2)

$\Rightarrow T = Mg + \frac{mg}{l}x$

Example. 02

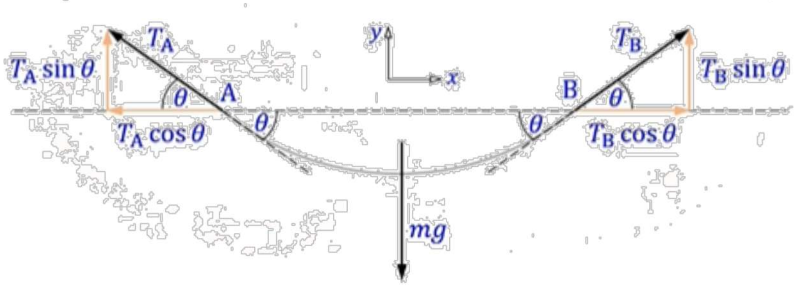
A uniform rope of mass m and length l is suspended from two fixed nails A and B that are in the same horizontal level. The tangents to the rope make an angle θ with the horizontal at the nails as shown in the figure.



(a) Find tensile forces in the rope at the end and the lowest points.

Solution.

Tensile forces at the nails: TE of the full rope.



$\Sigma F_x = 0 \quad \Rightarrow T_A \cos\theta = T_B \cos\theta$

$\Rightarrow T_A = T_B$

$\Sigma F_y = 0 \quad \Rightarrow T_A \sin\theta + T_B \sin\theta = mg$

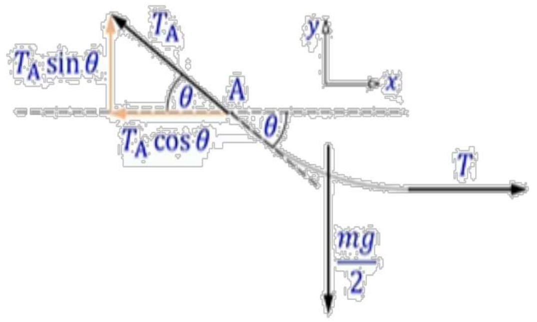
From the above two eq.

$\Rightarrow T_A = T_B = \frac{mg}{2\sin\theta}$

Tensile force at the lowest point: TE of the half rope.

Here both the halves are in identical conditions due to symmetry, so consider TE of any of the halves.

TE of the left half.



$\Sigma F_x = 0 \qquad \Rightarrow T = T_A \cos\theta$

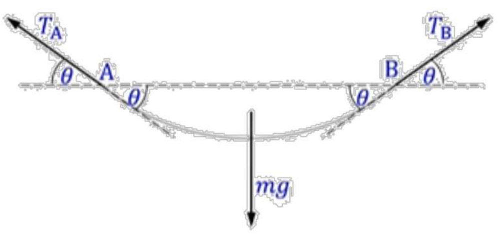
$\Sigma F_y = 0 \qquad \Rightarrow T_A \sin\theta = \frac{mg}{2}$

From the above two equations, we get:

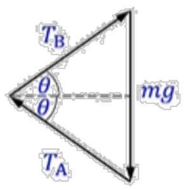
$$T = \frac{mg}{2}\cot\theta$$

Graphical Method:

TE of the full rope:

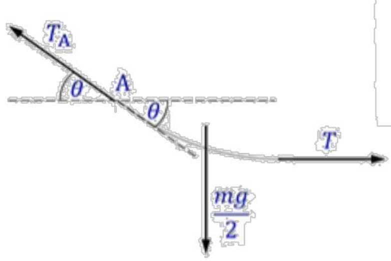


$m\vec{g} + \vec{T}_A + \vec{T}_B = \vec{0}$

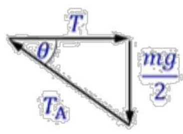


$\Rightarrow T_A = T_B = \frac{mg}{2\sin\theta} = \frac{mg}{2}\operatorname{cosec}\theta$

TE of the half rope:



$\frac{m\vec{g}}{2} + \vec{T}_A + \vec{T} = \vec{0}$



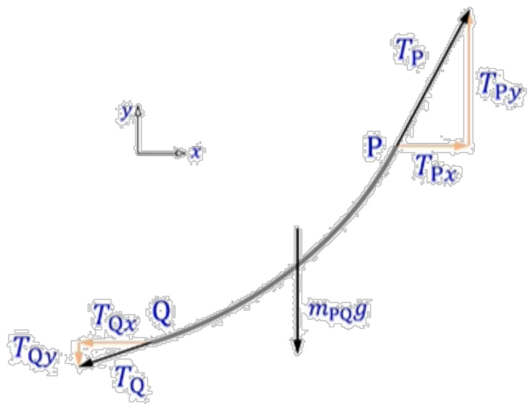
$\Rightarrow T = \frac{mg}{2}\cot\theta$

Note:
To analyse situations, where sum of three vectors is a null vector, graphical method should be preferred.

(b) What do you conclude for the vertical and horizontal components of the tensile force at different places of the rope?

Solution.

TE of a portion PQ of the rope.



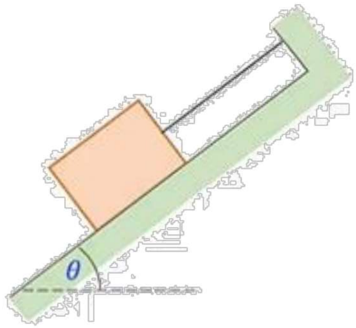
$\Sigma F_x = 0$

$\Rightarrow T_{Px} = T_{Qx}$

Horizontal component of the tensile forces in a rope suspended from its ends remains uniform in the rope.

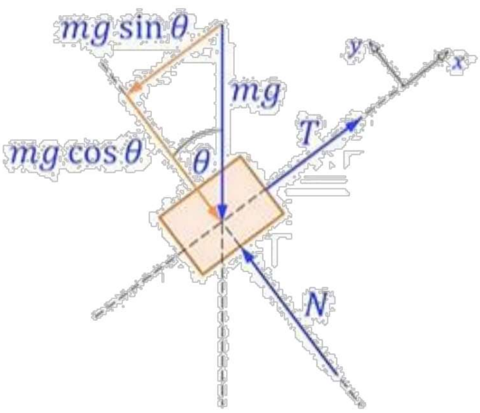
Example 03.

A box of mass m is held in equilibrium on a fixed frictionless inclined plane with the help of a string. Find tensile force in the string and the force of normal reaction between the box and the inclined plane.



Solution.

TE of the box.

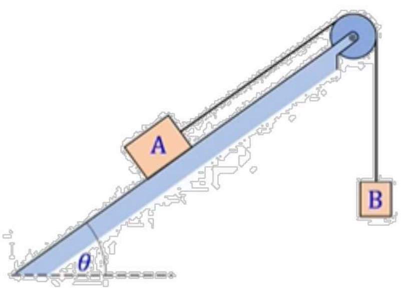


$\Sigma F_x = 0 \qquad \Rightarrow T = mg \sin\theta$

$\Sigma F_y = 0 \qquad \Rightarrow N = mg \cos\theta$

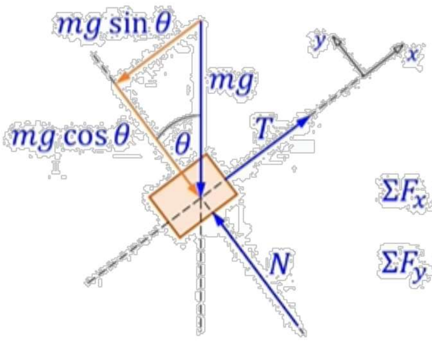
Example 04.

Block A of mass m placed on a frictionless inclined plane is connected by a string with another block B of mass M as shown in the figure. If the setup is in equilibrium, express M in terms of m and the angle θ of inclination.



Solution.

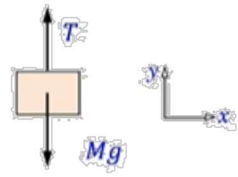
TE of the block A.



$\Sigma F_x = 0 \Rightarrow T = mg\sin\theta$ ………… (1)

$\Sigma F_y = 0 \Rightarrow N = mg\cos\theta$ ………… (2)

TE of the block B.



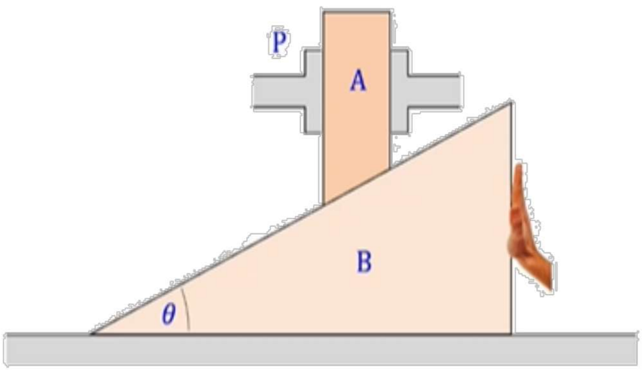
$\Sigma F_y = 0 \quad \Rightarrow T = Mg$ ………… (3)

From eq. (1) and (3), we get:

$M = m\sin\theta$

Example 05.

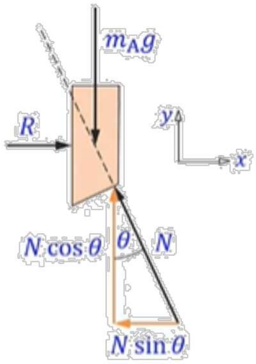
Rod A can slide vertically up and down in a fixed guide P and a wedge B horizontally on the floor. All the surfaces in contact are frictionless. Masses of rod and the wedge are m_A and m_B. The setup is held at rest by applying an appropriate push as shown in the figure.



(a) Find the push F by the hand to maintain the setup at rest.
(b) Find the net reaction force R by the fixed guide on the rod.
(c) Find the mutual normal reaction N between the rod and the wedge.

Solution.

TE of the rod.



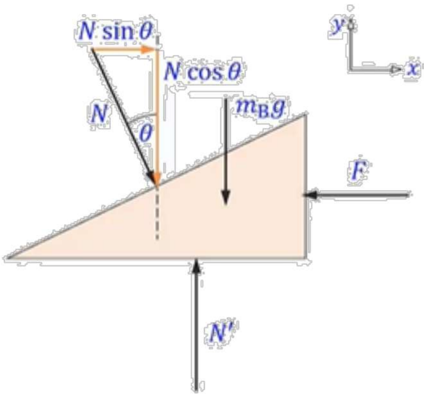
$\Sigma F_x = 0$

$\Rightarrow R = N\sin\theta$ .... (1)

$\Sigma F_y = 0$

$\Rightarrow N\cos\theta = m_A g$ .... (2)

TE of the wedge.



$\Sigma F_x = 0$

$\Rightarrow F = N\sin\theta$ .... (3)

$\Sigma F_y = 0$

$\Rightarrow N' = N\cos\theta + m_B g$ ...... (4)

From eq. (1) and (2)

$$R = m_A g\tan\theta$$

From eq. (2) and (3)

$$F = m_A g\tan\theta$$

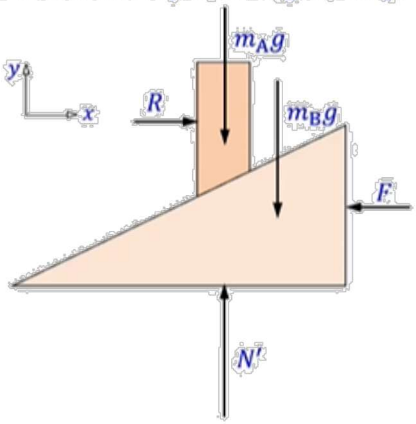
From eq. (2)

$$N = m_A g\sec\theta$$

Alternate approach:

There is no relative motion also no relative acceleration between the rod and the wedge, thus, we can treat them as a single rigid body.

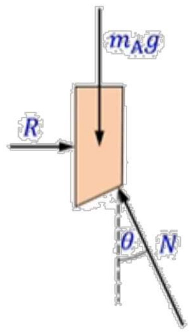
TE of this composite rigid body:



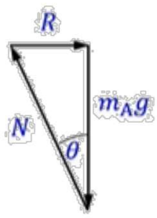
$\Sigma F_x = 0 \quad \Rightarrow R = F$ .... (1)

$\Sigma F_y = 0 \quad \Rightarrow N' = m_A g + m_B g$ ..... (2)

TE of the rod.



$$m_A\vec{g} + \vec{N} + \vec{R} = \vec{0}$$



$R = m_A g\tan\theta$ ...... (3)

$N = m_A g\sec\theta$ ...... (4)

From eq. (1) and (3)

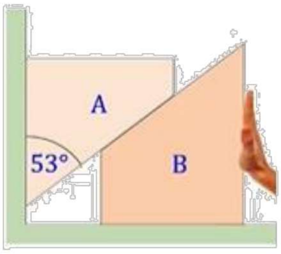
$$R = F = m_A g\tan\theta$$

From eq. (4)

$$N = m_A g\sec\theta$$

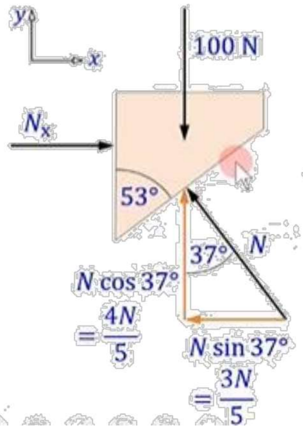
Example 06.

In the setup shown, two identical wedges each of mass 10 kg are held standstill against corner of a room. Calculate the necessary horizontal force applied on B to maintain the setup standstill. All surfaces in contact are frictionless. ($g = 10\ m/s^2$)



Solution. N: Mutual normal reaction, N_x: normal reaction by the wall and N_y: normal reaction by the floor.

TE of the wedge A:



$\Sigma F_y = 0$

$\Rightarrow \frac{4N}{5} = 100$

$\Rightarrow N = 125\text{ N}$ .... (1)

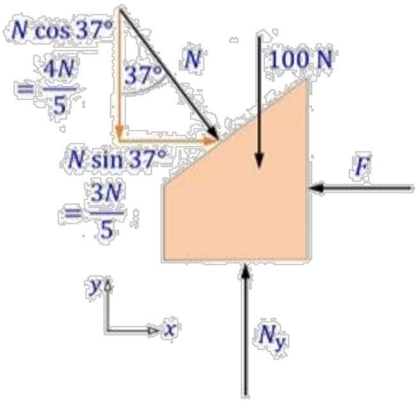
$\Sigma F_x = 0$

$\Rightarrow N_x = \frac{3N}{5}$

Now from eq. (1)

$\Rightarrow N_x = 75\text{ N}$ .... (2)

TE of the wedge B:



$\Sigma F_x = 0$

$\Rightarrow F = \frac{3N}{5}$

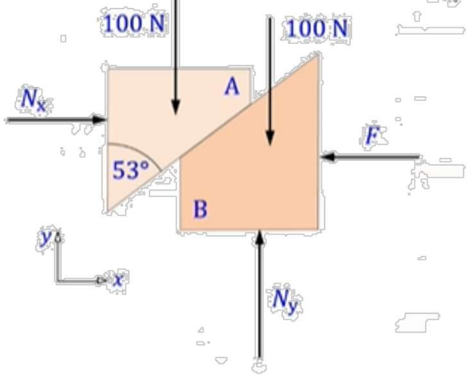
Now from eq. (1)

$\Rightarrow F = 75\text{ N}$

Alternate approach:

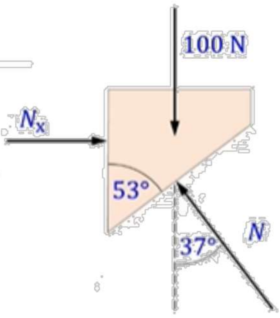
There is no relative motion also no relative acceleration between both the wedges, thus, we can treat them as a single rigid body.

TE of this composite rigid body:

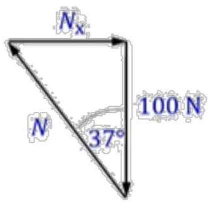


$\Sigma F_x = 0 \quad \Rightarrow F = N_x$ .... (1)

TE of the wedge A:



$m_A\vec{g} + \vec{N} + \vec{N_x} = \vec{0}$



From eq. (1) and the above figure:

$\Rightarrow F = N_x = 100 \tan 37° = 100 \times \frac{3}{4}$

$\Rightarrow F = 75\text{ N}$

Dynamics of Particles: Translation motion of accelerated bodies.

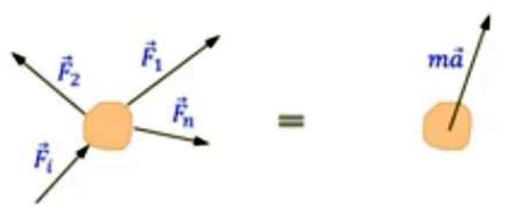
According to Newton's second law, forces acting on a body are considered as cause and rate of change in momentum or product of mass and acceleration as effect.

$$\Sigma\vec{F} = m\vec{a}$$

Strategy to use the law:

To write the equation of motion, it is recommended to draw the free body diagram, put a sign of equality and then draw the kinetic diagram.

Free Body Diagram (FBD) **Kinetic Diagram (KD)**



$$\Sigma\vec{F} = m\vec{a}$$

The equations obtained can either be solved by using triangle or polygon law of vector addition or by analytical method.

Triangle law may give very quick solutions in situations where only two forces are acting or some of the forces can be combined to reduce to a two force situation.

The analytical method is the most generalized one. In terms of Cartesian components, the analytical method yield the following equations.

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z$$

Ground or Laboratory Frame as Inertial Frame:

Newton's laws are valid only in inertial frames, so which frame we should prefer as a satisfactory inertial frame?

At present, we are interested in motion of terrestrial (earthly) bodies. For this purpose; a reference frame attached with the ground or a laboratory on the ground can be assumed a satisfactory inertial frame.

Free Space:

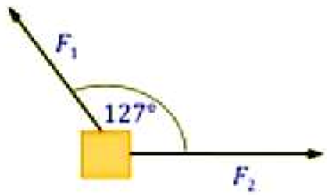
A region of space, where no force other then those we desire, can act on a body under consideration.

In mechanics, there is no interpretation of the term 'free space" other then the above mentioned one.

This term does not refer to vacuum or space.

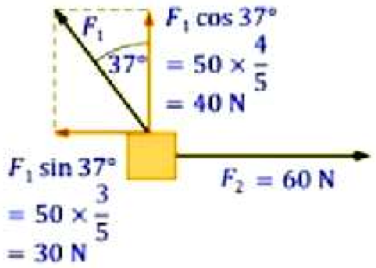
Example. 01

Two forces F_1 and F_2 of magnitudes 50 N and 60 N act simultaneously on a body of mass $m = 5$ kg as shown in the figure. Assuming the body in free space, find its acceleration vector?

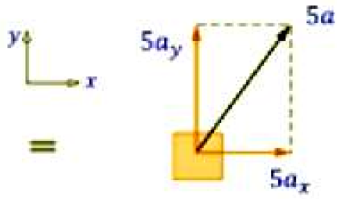


Solution.

Free Body Diagram (FBD)



Kinetic Diagram (KD)



$\Sigma F_x = ma_x \quad \Rightarrow 60 - 30 = 5a_x$

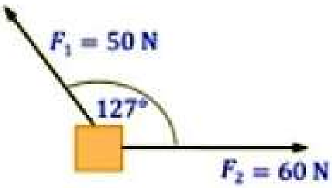
$\Rightarrow a_x = 6 \text{ m/s}^2$

$\Sigma F_y = ma_y \quad \Rightarrow 40 = 5a_y$

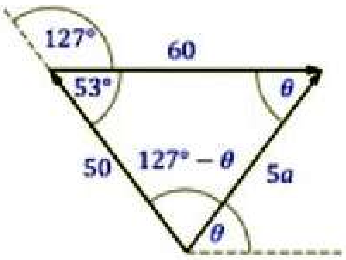
$\Rightarrow a_y = 8 \text{ m/s}^2$

$\vec{a} = (6\hat{\imath} + 8\hat{\jmath}) \text{ m/s}^2$

Graphical Method:



$\vec{F}_1 + \vec{F}_2 = 5\vec{a}$



Applying sin rule, we get:

$$\frac{5a}{\sin 53°} = \frac{50}{\sin\theta} = \frac{60}{\sin(127° - \theta)}$$

From the last two terms, we get:

$$5\sin\{90° + (37° - \theta)\} = 6\sin\theta$$

$$\Rightarrow 5\cos(37° - \theta) = 6\sin\theta$$

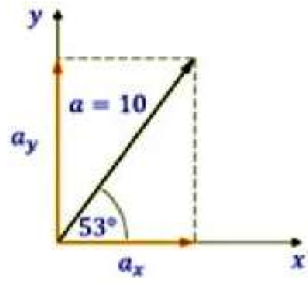
$$\Rightarrow 5\left(\frac{4}{5}\cos\theta + \frac{3}{5}\sin\theta\right) = 6\sin\theta$$

$$\Rightarrow 4\cos\theta = 3\sin\theta$$

$$\Rightarrow \tan\theta = \frac{4}{3} \quad \Rightarrow \theta = 53°$$

Now from the first two terms, we get:

$$\frac{5a}{\sin 53°} = \frac{50}{\sin 53°} \quad \Rightarrow a = 10$$



$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$

$= (10\cos 53°)\hat{\imath} + (10\sin 53°)\hat{\jmath}$

$= \left(10 \times \frac{3}{5}\right)\hat{\imath} + \left(10 \times \frac{4}{5}\right)\hat{\jmath}$

$= (6\hat{\imath} + 8\hat{\jmath}) \text{ m/s}^2$

Example 02.

A helium filled rigid spherical shell of mass m_1, when released in the air, starts ascending with an initial acceleration a. Another helium filled shell of the identical shape and size but of an unknown mass m_2, when released in the air, starts descending with an initial acceleration of the same magnitude as the previous one. Find mass of the second shell. Air drag is proportional to some unknown power of velocity.

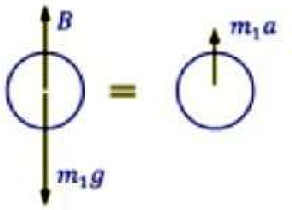
Solution.

Immediately after the release, velocity is zero so the air drag.

So, only two forces, the buoyant force and the weight, act on a shell immediately after they released.

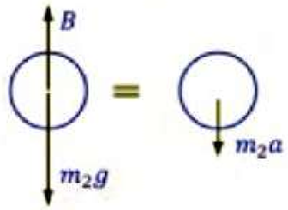
Buoyant force B on both the shells are equal as they displace the same amount of air due to equal volume.

For the first shell:



$$B - m_1 g = m_1 a \quad \text{.........} \ (1)$$

For the second shell:



$$m_2 g - B = m_2 a \quad \text{.........} \ (2)$$

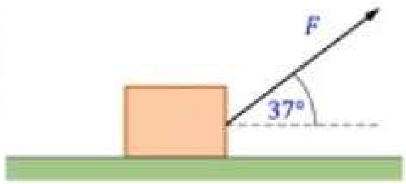
From the eq. (1) and (2), we get:

$$m_2 = m_1 \left(\frac{g + a}{g - a}\right)$$

Acceleration a cannot exceed g.

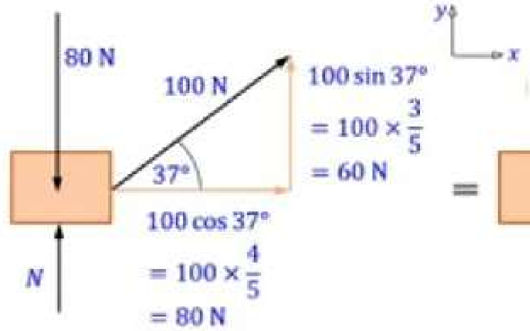
Example 03.

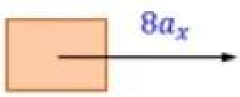
A block of mass 8 kg is being pulled by a constant force F of magnitude 100 N. If the horizontal floor is frictionless, find the acceleration of the block.



Solution.

Force acting on the block, the applied force F =100 N, the force of normal reaction from the ground N and weight 80 N are shown here:





The vertical component of the force F is smaller then the weight of the block, so the block will remain always in contact with the floor.

$$\Sigma F_x = m a_x \quad \Rightarrow 80 = 8 a_x$$

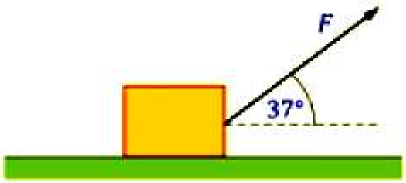
$$\Rightarrow a_x = 10 \text{ m/s}^2$$

$$\Sigma F_y = 0 \quad \Rightarrow N + 60 - 80 = 0$$

$$\Rightarrow N = 20 \text{ N}$$

Example 04.

A block of mass 2.4 kg is being pulled by a force of fixed direction but magnitude varying with time t as $F = 10t$ N. The horizontal floor is frictionless.

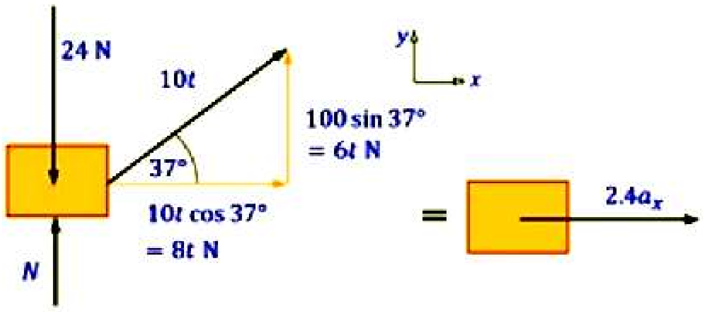


(a) When will the block leave the floor?

(b) Find speed of the block, when it is leaving the floor? *(c)* Find distance slid by the block on the floor.

Solution.

Force acting on the block, the applied force $F = 10t$ N, the force of normal reaction from the ground N and weight 24 N are shown here:



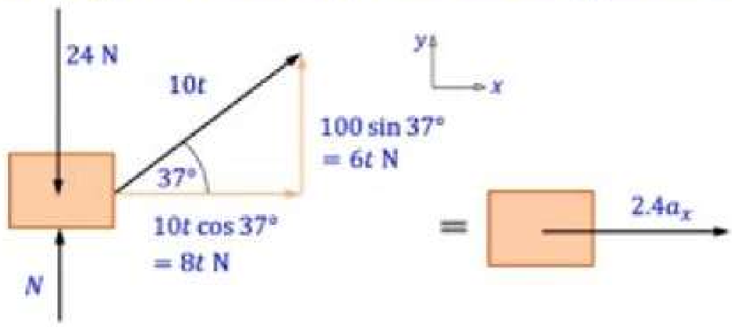
(a) When will the block leave the floor?

The moment, when the block leaves the floor, the force of normal reaction N will vanish and vertical component of acceleration be zero.

$$\Sigma F_y = 0 \qquad \Rightarrow 6t - 24 = 0$$

$$\Rightarrow t = 4 \text{ s}$$

(b) Find speed of the block, when it is leaving the floor?



$$\Sigma F_x = ma_x \qquad \Rightarrow 8t = 2.4a_x$$

$$\Rightarrow a_x = \frac{10t}{3} \text{ m/s}^2$$

Acceleration is function of time, therefore:

$$a_x = \frac{dv_x}{dt} \qquad \Rightarrow \frac{dv_x}{dt} = \frac{10t}{3} \qquad \Rightarrow dv_x = \frac{10t}{3} dt$$

$$\Rightarrow \int_0^{v_x} dv_x = \frac{10}{3}\int_0^t t dt$$

$$\Rightarrow v_x = \frac{5}{3}t^2 \qquad \text{.................. ....(1)}$$

Substituting $t = 4$ s, we get the desired speed.

$$\Rightarrow v_x = \frac{5}{3} \times 4^2$$

$$\Rightarrow v_x = 26.67 \text{ m/s}$$

(c) Find distance slid by the block on the floor.

Eq. (1) obtained in the previous part gives speed as function of time.

$$\Rightarrow v_x = \frac{5}{3}t^2 \quad \text{.............. ....(1)}$$

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t} \qquad \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{5}{3}t^2$$

$$\Rightarrow \mathrm{d}x = \frac{5}{3}t^2\mathrm{d}t$$

$$\Rightarrow \int_0^x \mathrm{d}x = \frac{5}{3}\int_0^4 t^2\mathrm{d}t$$

$$\Rightarrow x = \int_0^4 \frac{5}{3}t^2\mathrm{d}t$$

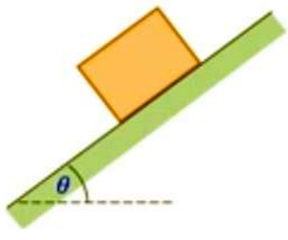
$$\Rightarrow x = 35.56 \text{ m}$$

Note:

In this problem, major part of the solution was from kinematics. Use of Newton's laws was limited to find time, when the block leaves the floor and horizontal component of acceleration.

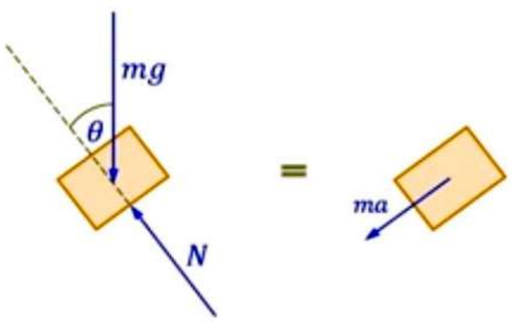
Example 05.

A block of mass m is sliding down a fixed frictionless inclined plane of inclination θ. Find acceleration of the block and the force of normal reaction.



Solution.

Analytical Method.



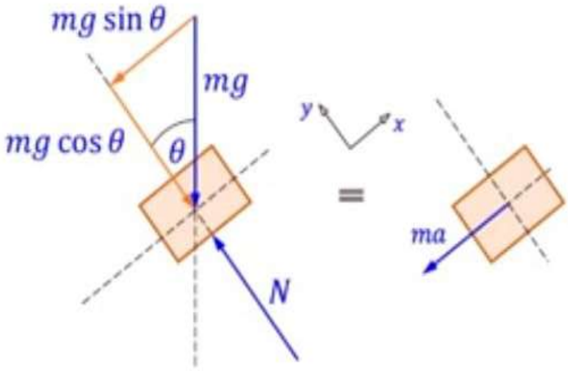
How to orient the coordinate axes?

Orientation of the coordinate axes should be so chosen that maximum number of vectors in LHS as well RHS fall along the axes. In this way, minimum number of vectors has to be resolved into their components.

N and ma must be resolved into their components.

Only mg has to be resolved into its components.

Here in this problem, this choice of orientation is more appropriate then the first one.

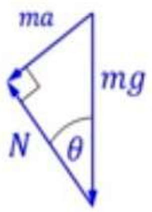


$$\Sigma F_x = ma_x \quad \Rightarrow mg\sin\theta = ma$$
$$\Rightarrow a = g\sin\theta$$

$$\Sigma F_y = ma_y \quad \Rightarrow N - mg\cos\theta = 0$$
$$\Rightarrow N = mg\cos\theta$$

Graphical Method.

$$m\vec{g} + \vec{N} = m\vec{a}$$

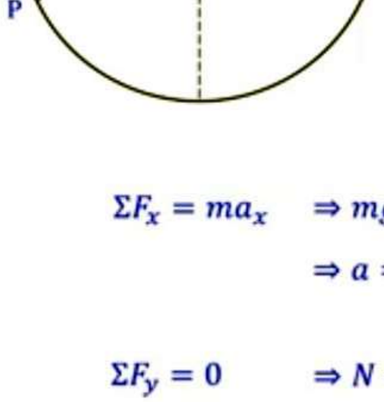


$$\Rightarrow mg\sin\theta = ma$$
$$\Rightarrow a = g\sin\theta$$
$$\Rightarrow N = mg\cos\theta$$

Note:
Orientation of the coordinate axes should be so chosen that maximum number of vectors in LHS as well RHS fall along the axes. In this way, minimum number of vectors has to be resolve into their components.

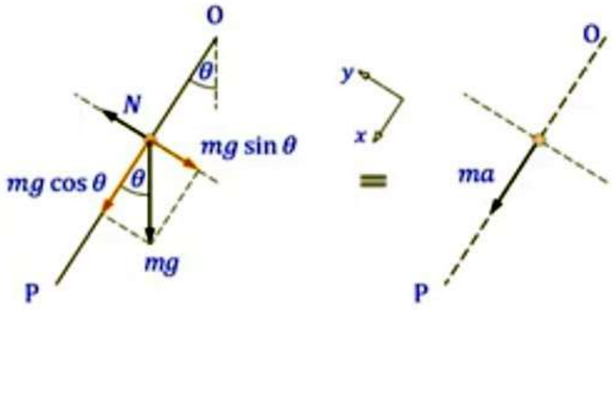
Example 06.

A circular wire frame of radius r is fixed in a vertical plane. From the top point O of the frame, a straight rigid wire OP is fixed in the frame along a chord as shown in the figure. A bead B can slide on the wire OP without friction. Find time of descend of the bead on the wire OP.



Solution.

Acceleration of the bead on the wire OP:

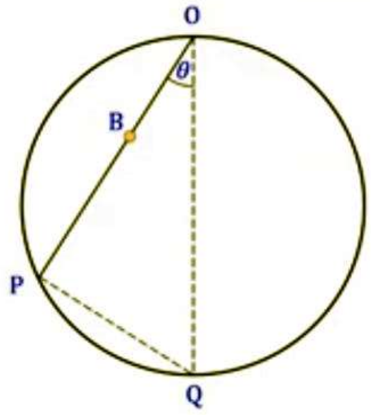


$$\Sigma F_x = ma_x \quad \Rightarrow mg\cos\theta = ma$$
$$\Rightarrow a = g\cos\theta \quad \text{.......................} (1)$$

$$\Sigma F_y = 0 \quad \Rightarrow N - mg\sin\theta = 0$$
$$\Rightarrow N = mg\sin\theta \quad \text{.................} (2)$$

Time of descend of the bead on the wire OP:

Construct another chord PQ as shown.



ΔOPQ is a right angle triangle as angle subtended by a diameter on the circle is a right angle.

Denoting length OP by l:

$$l = 2r\cos\theta \quad \text{......................} \quad (3)$$

Eq. (1) reveals that the acceleration is a uniform one. Thus:

$$\Delta x = ut + \frac{1}{2}at^2 \quad \Rightarrow l = \frac{1}{2}at^2$$

Substituting expressions of a and l from eq. (1) and (3) respectively, we get:

$$\Rightarrow 2r\cos\theta = \tfrac{1}{2}(g\cos\theta)t^2$$

$$\Rightarrow t = 2\sqrt{\frac{r}{g}}$$

The time is independent of the angle θ.

System of Particles:

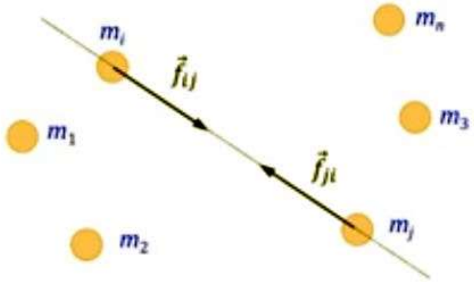
A system of particles is a well-defined collection of several or large number of particles, which may or may not apply forces of mutual interaction on each other.

By the term "particle", we mean either a material point or an extended body, only translational motion of which is to be considered.

As a schematic representation, consider a system of n particles of masses $m_1, m_2, ... m_i ... m_j$ .... and m_n respectively. They may be actual particles or rigid bodies in translation motion.

Some of them may interact with each other and some of them may not.

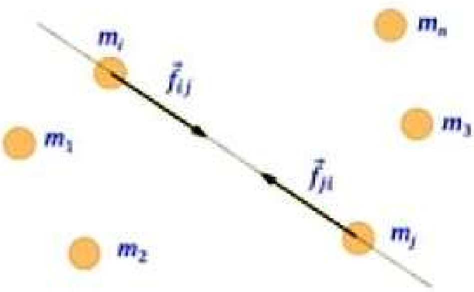
The particles, which interact with each other, apply forces on each other. The forces of interaction $\vec{f}_{ij}$ and $\vec{f}_{ji}$ between a pair of i^{th} and j^{th} particles are shown in the figure.



A System of Particles.

Internal and External Forces:

A System of Particles.



Internal Forces:

The forces of mutual interaction between the particles of a system are internal forces of the system.

The internal forces always exist in pairs of forces of equal magnitudes and opposite directions. In other words each pair is a Newton's third law pair.

Sum of all the internal forces of a system is a null vector.

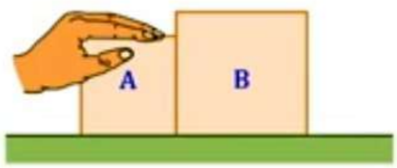
$$\Sigma\vec{f}_{ij} = \vec{0}$$

External Forces:

All those forces that are applied on a particle of a system by a body not included in the system.

Example. 07

Boxes A and B of masse m_A and m_B placed on a frictionless horizontal floor are being pushed horizontally by a force F as shown in the figure. Identify all the internal and external forces for the system of blocks A and B.



Solution.

All the forces acting on or within the system are listed here and shown in the adjacent figure.

$m_A g \stackrel{\text{def}}{\equiv}$ Weight of the block A

$m_B g \stackrel{\text{def}}{\equiv}$ Weight of the block B

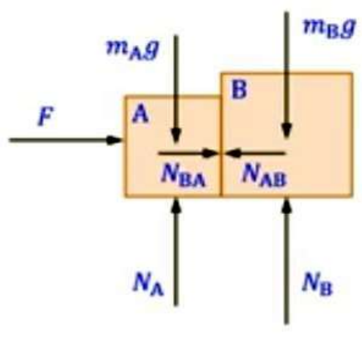
$N_A \stackrel{\text{def}}{\equiv}$ Normal reaction of the floor on block A

$N_B \stackrel{\text{def}}{\equiv}$ Normal reaction of the floor on block B

$N_{AB} \stackrel{\text{def}}{\equiv}$ Normal reaction of the the block B on A

$N_{BA} \stackrel{\text{def}}{\equiv}$ Normal reaction of the the block A on B

$F \stackrel{\text{def}}{\equiv}$ Horizontal push of the hand



Internal Forces:

N_{AB} and N_{BA}

External Forces:

$m_A g$

$m_B g$

N_A

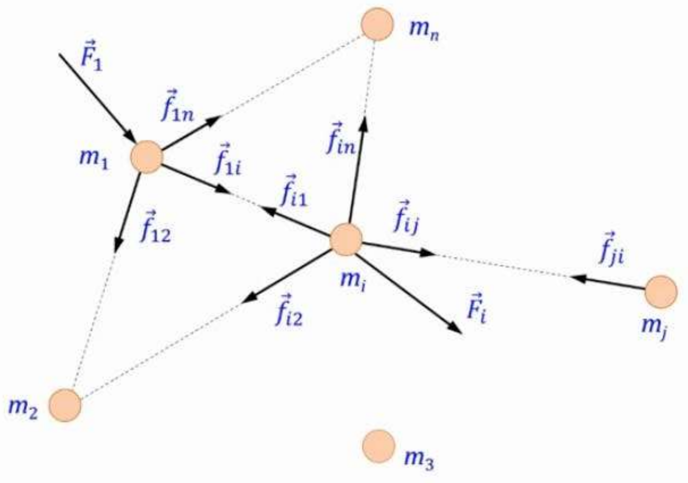
N_B

F

NLM SYSTEM OF PARTICLES
LECTURE - 6

Force in a System of Particles:

Consider a system of n particles of masses $m_1, m_2, \ldots\, m_i \ldots m_j \ldots\ldots$ and m_n respectively. They may be actual particles or rigid bodies in translation motion.



$$\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \vec{f}_{ij} = \vec{0}$$

Internal Forces:

$\vec{f}_{ij} \stackrel{\text{def}}{=}$ Force on i^{th} particle by j^{th} particle.

An internal force is due to mutual interaction between particles (bodies) included in the system.

Net resultant of all the internal forces is a null vector.

External Forces:

$\vec{F}_i \stackrel{\text{def}}{=}$ Net external force on i^{th} particle

An external force is exerted by a body not included in the system on a body included in the system.

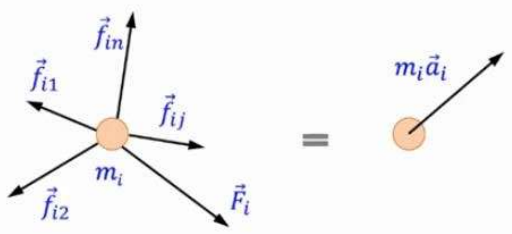
Here all the internal and external forces are not shown for the better readability of the diagram.

Newton's Second Law on i^{th} particle a of the System:

Let particles of masses $m_1, m_2, \ldots\, m_i \ldots m_j \ldots\ldots$ and m_n are moving with accelerations $\vec{a}_1, \vec{a}_2 \ldots\, \vec{a}_i \ldots\, \vec{a}_j \ldots\, \vec{a}_n$ respectively.

To write the equation of motion for a system of n particles, we have to apply Newton's second law on each individual particle of the system.

Let us consider the i^{th} particle with all the forces acting on it as shown.



The forces $\vec{f}_{i1}, \vec{f}_{i2} \ldots\, \vec{f}_{ij} \ldots\, \vec{f}_{in}$, though are internal forces for the system, yet they are external forces on the i^{th} particle in addition to the external force $\vec{F}_i$.

$$\vec{F}_i + \sum_{j=1, j\neq i}^{n} \vec{f}_{ij} = m_i\vec{a}_i$$

In this way, we can write equation for Newton's second law for every particle.

$$\sum_{i=1}^{n} \vec{F}_i = \sum_{i=1}^{n} m_i \vec{a}_i$$

The LHS of the above equation is the sum of all the external forces on the system and the RHS is sum of all effective forces (product masses of all the particles and their accelerations).

Let us write the equation in simpler notations.

$$\Sigma \vec{F}_i = \Sigma m_i \vec{a}_i$$

The equations obtained can either be solved by using triangle or polygon law of vector addition or by analytical method.

The analytical method is the most generalized one. In terms of Cartesian components, the analytical method yield the following equations.

$$\Sigma F_{ix} = \Sigma m_i a_{ix} \qquad \Sigma F_{iy} = \Sigma m_i a_{iy} \qquad \Sigma F_{iz} = \Sigma m_i a_{iz}$$

An important result:

If all the particles (bodies) of a system have equal accelerations say $\vec{a}$
i.e. there is no relative acceleration between them, we can write

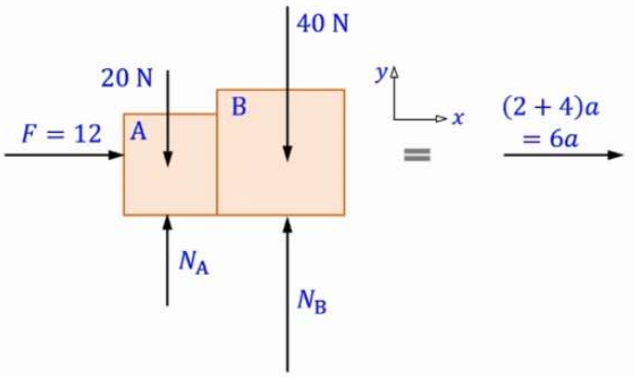
$$\Sigma \vec{F}_i = (\Sigma m_i)\vec{a}$$

This suggests that, if all particles (bodies) of a system are moving with equal accelerations, the system can be assumed to behave like a single composite rigid body.

You may recall that this idea we have thought intuitively and used in analysing statics (equilibrium) of system two or more bodies, now will use this idea to analyse dynamics of system of two or more bodies.

Alternate method: $\vec{a}_i = \vec{a} \quad \Rightarrow \Sigma\vec{F}_i = \Sigma m_i \vec{a}_i = (\Sigma m_i)\vec{a}$

First consider the system as a single rigid body:



$\Sigma F_x = (\Sigma m_i) a_x \quad \Rightarrow 12 = 6a$

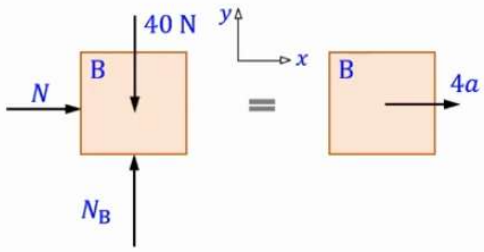
$\Rightarrow a = 2\ \text{m/s}^2$ .... (1)

$\Sigma F_y = (\Sigma m_i) a_y \quad \Rightarrow N_A + N_B = 20 + 40$

$\Rightarrow N_A + N_B = 60\ \text{N}$ .... (2)

After finding the acceleration, we can apply NLM on any one of the blocks to find *N*.

For the block B:



$\Sigma F_x = m a_x \quad \Rightarrow N = 4a = 4 \times 2 = 8\ \text{N}$

System of Particles:

A system of particles is a well-defined collection of several or large number of particles, which may or may not apply forces of mutual interaction on each other.

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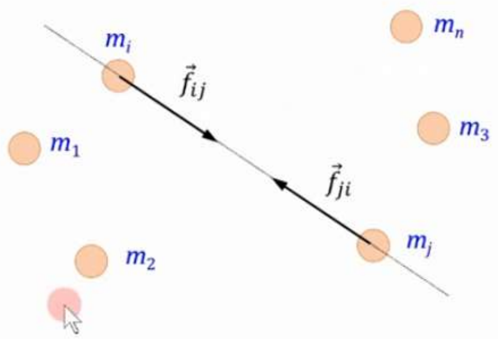
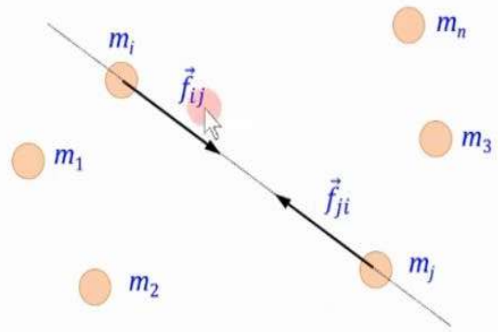
The particles, which interact with each other, apply forces on each other. The forces of interaction $\vec{f}_{ij}$ and $\vec{f}_{ji}$ between a pair of i^{th} and j^{th} particles are shown in the figure.



A System of Particles.

Internal and External Forces:



A System of Particles.

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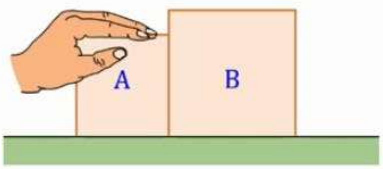
$$\Sigma \vec{f}_{ij} = \vec{0}$$

External Forces:

All those forces that are applied on a particle of a system by a body not included in the system.

Example 01.

Boxes A and B of masse m_A and m_B placed on a frictionless horizontal floor are being pushed horizontally by a force F as shown in the figure. Identify all the internal and external forces for the system of blocks A and B.



Solution.

All the forces acting on or within the system are listed here and shown in the adjacent figure.

$m_A g \stackrel{\text{def}}{=}$ Weight of the block A

$m_B g \stackrel{\text{def}}{=}$ Weight of the block B

$N_A \stackrel{\text{def}}{=}$ Normal reaction of the floor on block A

$N_B \stackrel{\text{def}}{=}$ Normal reaction of the floor on block B

$N_{AB} \stackrel{\text{def}}{=}$ Normal reaction of the the block B on A

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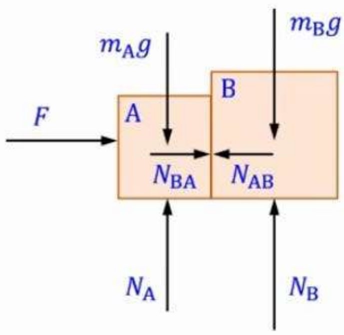
$F \stackrel{\text{def}}{=}$ Horizontal push of the hand



Internal Forces:

N_{AB} and N_{BA}

External Forces:

$m_A g$

$m_B g$

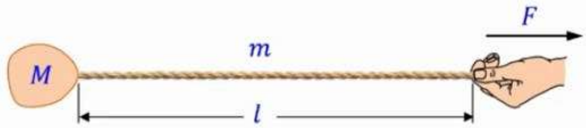
N_A

N_B

F

Example. 02

A body of mass M is being pulled in free space with the help of a uniform rope of mass m and length l. The rope is pulled by a force F as shown in the figure. Find an expression for the tensile force T at a distance x from the body.



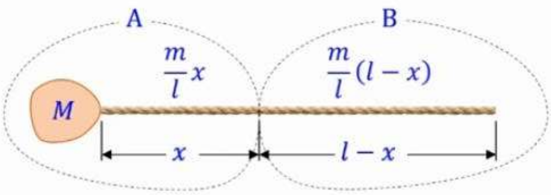
Solution.

The rope and the body, both are moving with the same acceleration; therefore, we can find their acceleration treating them like a single rigid body.

$$\Sigma F_x = (\Sigma m_i)a_x \quad \Rightarrow F = (m+M)a$$

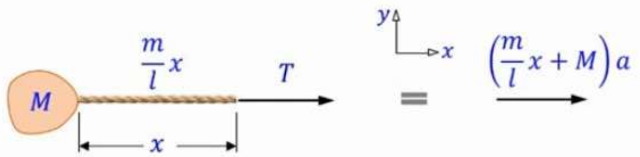
$$\Rightarrow a = \frac{F}{(m+M)} \quad \text{.....................(1)}$$

Now applying NLM to any of the following two parts A or B, we can find T. Here T is an internal force for the system consisting of the parts A and B.



Mass of a portion of length x of the rope $= \frac{m}{l}x$

NLM on part A containing rope of length x:



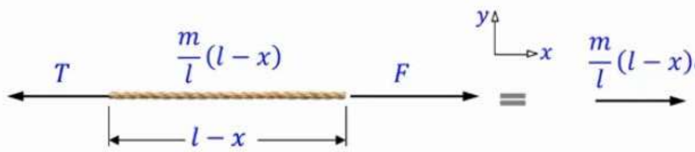
$$\Sigma F_x = (\Sigma m_i)a_x \quad \Rightarrow T = \left(\frac{m}{l}x + M\right)a$$

Substituting a from eq. (1), we get

$$\Rightarrow T = \left(\frac{m}{l}x + M\right)\frac{F}{(m+M)}$$

$$\Rightarrow T = \frac{mF}{(m+M)l}x + \frac{MF}{(m+M)}$$

NLM on part B containing rope of length $(l - x)$:



$$\Sigma F_x = (\Sigma m_i)a_x \quad \Rightarrow F - T = \frac{m}{l}(l-x)a$$

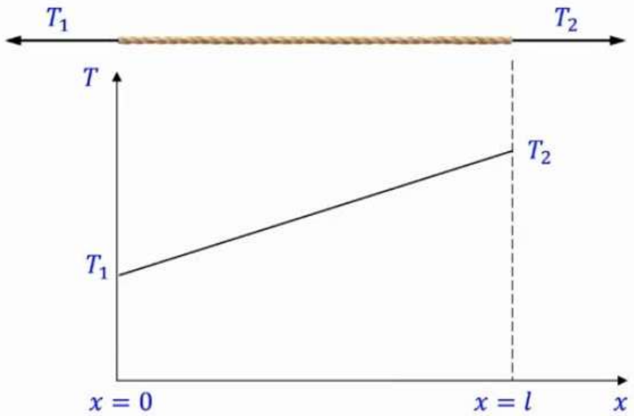
Substituting a from eq. (1), we get

$$\Rightarrow T = \frac{mF}{(m+M)l}x + \frac{MF}{(m+M)}$$

Note:
To find an internal force by using NLM, consider that body or part of system on which lesser number of forces are acting.

Alternate method:

This method is based on a fact that in a uniform rope, acceleration of every part of which is uniform, tensile force varies linearly with distance from one of the ends.



Tension at the left end that is the force with which the rope is pulling the body of mass M.

$$T_1 = Ma = \frac{MF}{(m+M)}$$

Tension at the right end that is the force with which the rope is being pulled.

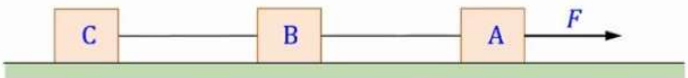
$$T_2 = F$$

$$T = \frac{T_2 - T_1}{l}x + T_1$$

$$\Rightarrow T = \frac{mF}{(m+M)l}x + \frac{MF}{(m+M)}$$

Example. 03

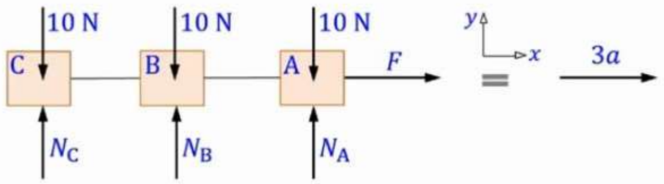
Three identical blocks A, B and C, each of mass 1.0 kg are connected by light strings as shown in the figure. If the block A is pulled by an unknown force F, the tension in the string connecting blocks A and B is measured to be 4.0 N.



Calculate magnitude of the force F and tension in the string connecting blocks B and C.

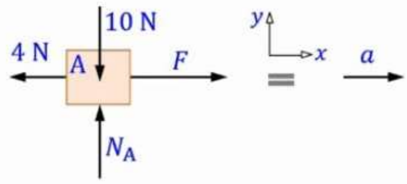
Solution.

All the three blocks move with equal accelerations, therefore, considering them a single rigid body we can use the same strategy as we have used in the previous example.



$\Sigma F_x = (\Sigma m_i)a_x \quad \Rightarrow F = 3a$ .... (1)

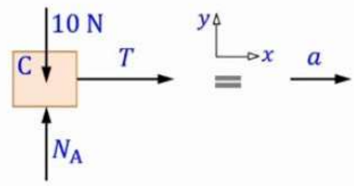
NLM for the block A:



$$\Sigma F_x = ma_x$$

$$\Rightarrow F - 4 = a \quad .... (2)$$

NLM for the block C:



$$\Sigma F_x = ma_x$$

$$\Rightarrow T = a \quad .... (3)$$

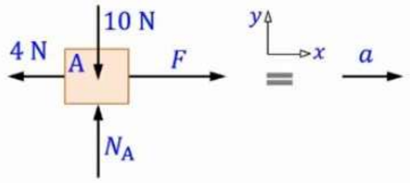
From the eq. (1), (2) and (3), we get: $F = 6$ N, $T = 2$ N,

Alternate method: Apply NLM of each body individually.

NLM for the block A:

$\Sigma F_x = ma_x$

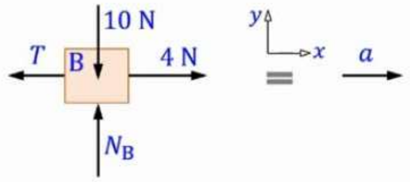
$\Rightarrow F - 4 = a$ .... (1)



NLM for the block B:

$\Sigma F_x = ma_x$

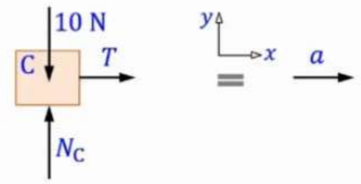
$\Rightarrow 4 - T = a$ .... (2)



NLM for the block C:

$\Sigma F_x = ma_x$

$\Rightarrow T = a$ .... (3)



From eq. (2) and (3), we get:

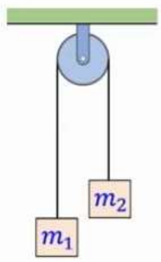
$T = 2\ \text{N}$ and $a = 2\ \text{m/s}^2$

Substituting value of a in eq. (1), we get:

$F = 6\ \text{N}$

Example. 04

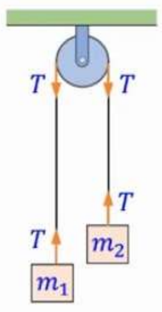
The system shown in the figure is released from rest. Assuming mass m_2 more than the mass m_1, find the accelerations of the blocks and the tension in the string.



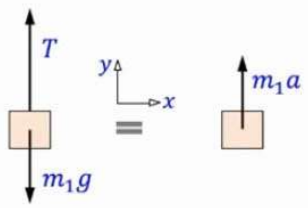
Solution.

Constraint: Magnitude of accelerations of both the blocks are equal say a.

Transmission of tension in the string:



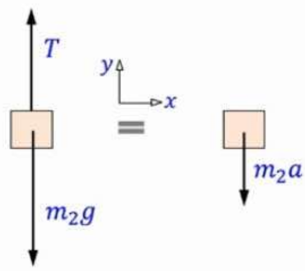
NLM for the block of mass m_1:



$\Sigma F_y = ma_y$

$\Rightarrow T - m_1 g = m_1 a$ .... (1)

NLM for the block of mass m_2:



$\Sigma F_y = ma_y$

$\Rightarrow m_2 g - T = m_2 a$ .... (2)

From eq. (1) and (2), we get:

$$T = \frac{2m_1 m_2}{m_1 + m_2} g$$

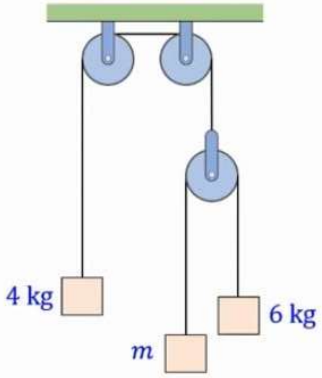
$$a = \frac{m_2 - m_1}{m_2 + m_1} g$$

Note:

This system of pulleys is known as simple Atwood's machine.

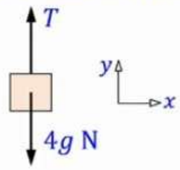
Example. 05

In the setup shown, what should be value of the unknown mass m to maintain the 4 kg block in equilibrium?



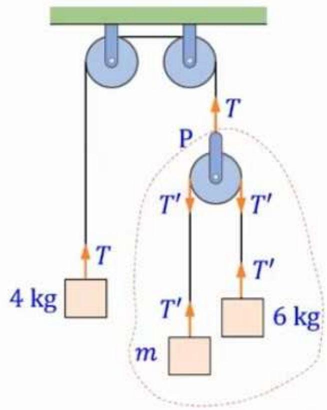
Solution.

TE of the 4 kg block:



$\Sigma F_y = ma_y \Rightarrow T = 4g$ N .... (1)

Transmission of tensions in the strings:



For TE of the 4 kg block, the pulley P must be in TE.

$\Rightarrow T = 2T'$ .... (2)

TE of the pulley P suggests to consider the part of the setup shown in the enclosure as a simple Atwood's machine, therefore:

$$T' = \frac{2m_1m_2g}{m_1+m_2} = \frac{2m \times 6g}{m+6} \text{ .... (3)}$$

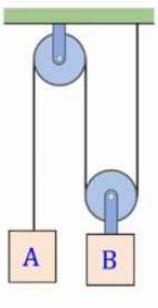
Making substitutions from eq. (1) and (3) in (2), we get:

$$4g = 2 \times \frac{2m \times 6g}{m+6}$$

$$\Rightarrow m = \frac{6}{5} \text{ kg}$$

Example 06.

In the setup shown, blocks A and B are of equal masses. Find accelerations of the blocks.



Important Discussion:

In this setup, there are three unknowns, string tension and accelerations of the two blocks. Therefore, we need three independent equations to solve the problem.

By applying NLM, we can write two independent equations, one for each block. The third independent equation that is the relation between accelerations of the blocks, we can write using constraints motion.

In example 4 of simple Atwood's machine, equality of acceleration moduli was the constraint relation. Actually it was so obvious that we have used it unknowingly. But this will not always going be the same as in this example.

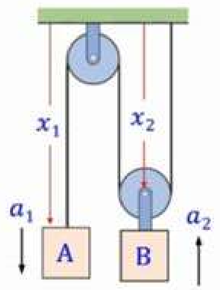
Conclusion:

Dealing with dynamics of systems of interconnected bodies, we have to always use the required constraint relations. Therefore, it is recommended to write it first, before proceeding with equations of NLM.

For writing constraint relations, we practice the methods learnt for them till so far. In chapter work and energy we will learn another very interesting as well as simpler method.

Solution.

Constraint relation:



Sum of all the variable portions of the string is a constant, therefor.

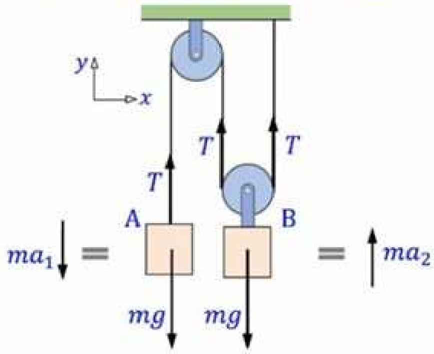
$x_1 + 2x_2 = l$

$\overset{(\ddot{\ })}{\Longrightarrow} \ \ddot{x}_1 + 2\ddot{x}_2 = 0$

$\Rightarrow a_1 - 2a_2 = 0$

$\Rightarrow a_1 = 2a_2$ ...... (1)

NLM on each of the blocks:



For the block A:

$mg - T = ma_1$ ...... (2)

For the block B:

$2T - mg = ma_2$ ...... (3)

From eq. (1), (2) and (3), we get:

$a_1 = \dfrac{2g}{5}$ and $a_2 = \dfrac{g}{5}$