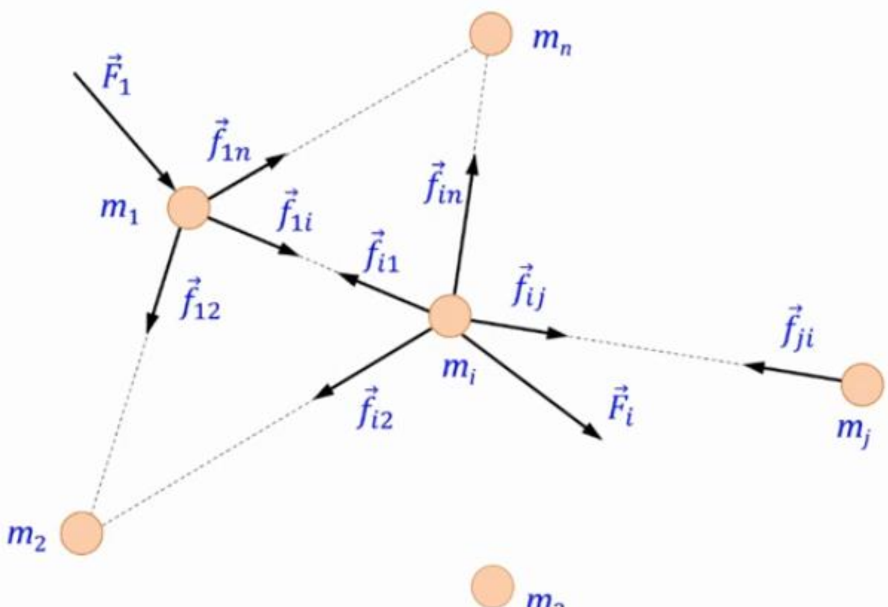
NLM SYSTEM OF PARTICLES
LECTURE - 6

Force in a System of Particles:

Consider a system of n particles of masses m_1, m_2, ... m_i ... m_j .... and m_n respectively. They may be actual particles or rigid bodies in translation motion.



$$\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} \vec{f}_{ij} = \vec{0}$$

Internal Forces:

$\vec{f}_{ij} \stackrel{\text{def}}{=}$ Force on i^{th} particle by j^{th} particle.

An internal force is due to mutual interaction between particles (bodies) included in the system.

Net resultant of all the internal forces is a null vector.

External Forces:

$\vec{F}_i \stackrel{\text{def}}{=}$ Net external force on i^{th} particle

An external force is exerted by a body not included in the system on a body included in the system.

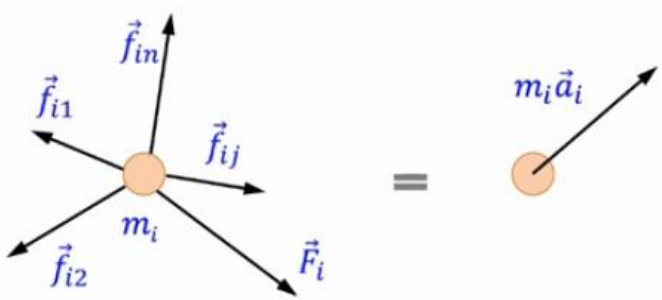
Here all the internal and external forces are not shown for the better readability of the diagram.

Newton's Second Law on i^{th} particle a of the System:

Let particles of masses m_1, m_2, ... m_i ... m_j .... and m_n are moving with accelerations $\vec{a}_1, \vec{a}_2$ ...... $\vec{a}_i$ ...... $\vec{a}_j$ ...... $\vec{a}_n$ respectively.

To write the equation of motion for a system of n particles, we have to apply Newton's second law on each individual particle of the system.

Let us consider the i^{th} particle with all the forces acting on it as shown.



The forces $\vec{f}_{i1}, \vec{f}_{i2}$ .... $\vec{f}_{ij}$ ..... $\vec{f}_{in}$, though are internal forces for the system, yet they are external forces on the i^{th} particle in addition to the external force $\vec{F}_i$.

$$\vec{F}_i + \sum_{j=1, j\neq i}^{n} \vec{f}_{ij} = m_i\vec{a}_i$$

In this way, we can write equation for Newton's second law for every particle.

$$\sum_{i=1}^{n} \vec{F}_i = \sum_{i=1}^{n} m_i \vec{a}_i$$

The LHS of the above equation is the sum of all the external forces on the system and the RHS is sum of all effective forces (product masses of all the particles and their accelerations).

Let us write the equation in simpler notations.

$$\Sigma \vec{F}_i = \Sigma m_i \vec{a}_i$$

The equations obtained can either be solved by using triangle or polygon law of vector addition or by analytical method.

The analytical method is the most generalized one. In terms of Cartesian components, the analytical method yield the following equations.

$$\Sigma F_{ix} = \Sigma m_i a_{ix} \qquad \Sigma F_{iy} = \Sigma m_i a_{iy} \qquad \Sigma F_{iz} = \Sigma m_i a_{iz}$$

An important result:

If all the particles (bodies) of a system have equal accelerations say $\vec{a}$
i.e. there is no relative acceleration between them, we can write

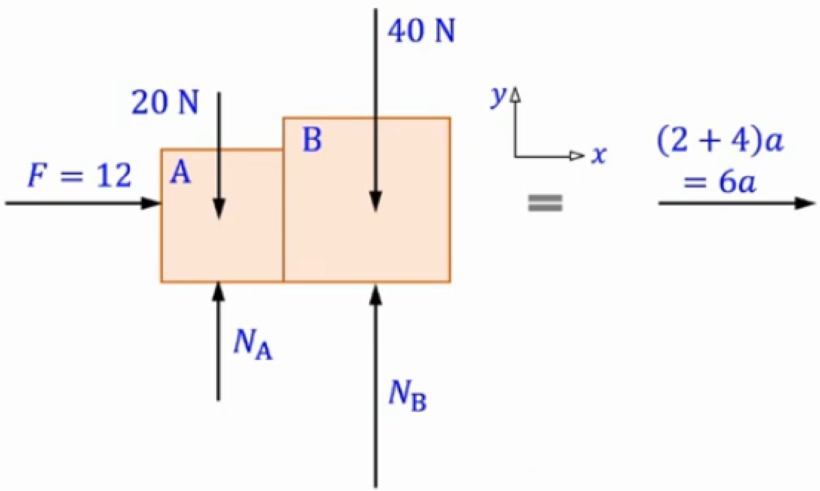
$$\Sigma \vec{F}_i = (\Sigma m_i)\vec{a}$$

This suggests that, if all particles (bodies) of a system are moving with equal accelerations, the system can be assumed to behave like a single composite rigid body.

You may recall that this idea we have thought intuitively and used in analysing statics (equilibrium) of system two or more bodies, now will use this idea to analyse dynamics of system of two or more bodies.

Alternate method: $\vec{a}_i = \vec{a} \quad \Rightarrow \Sigma\vec{F}_i = \Sigma m_i\vec{a}_i = (\Sigma m_i)\vec{a}$

First consider the system as a single rigid body:



$\Sigma F_x = (\Sigma m_i)a_x \quad \Rightarrow 12 = 6a$

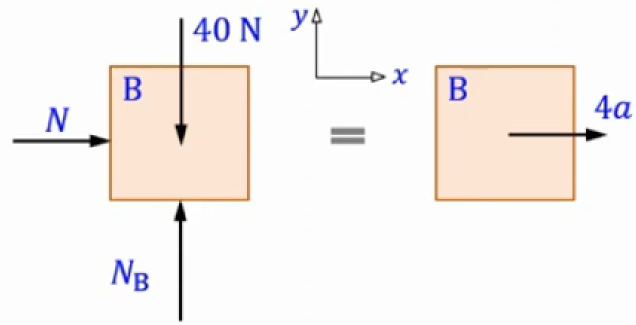
$\Rightarrow a = 2\ \mathrm{m/s^2}$ .... (1)

$\Sigma F_y = (\Sigma m_i)a_y \quad \Rightarrow N_A + N_B = 20 + 40$

$\Rightarrow N_A + N_B = 60\ \mathrm{N}$ .... (2)

After finding the acceleration, we can apply NLM on any one of the blocks to find N.

For the block B:



$\Sigma F_x = ma_x \quad \Rightarrow N = 4a = 4 \times 2 = 8\ \mathrm{N}$

System of Particles:

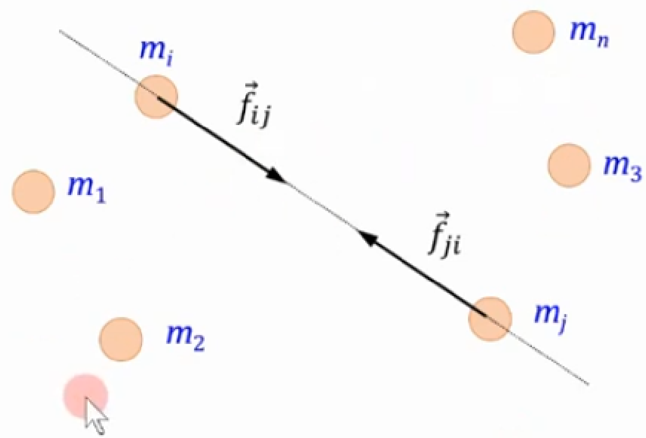
A system of particles is a well-defined collection of several or large number of particles, which may or may not apply forces of mutual interaction on each other.

By the term "particle", we mean either a material point or an extended body, only translational motion of which is to be considered.

As a schematic representation, consider a system of n particles of masses $m_1, m_2, \ldots.\ m_i\ \ldots\ m_j\ \ldots\ldots\ldots$ and m_n respectively. They may be actual particles or rigid bodies in translation motion.

Some of them may interact with each other and some of them may not.

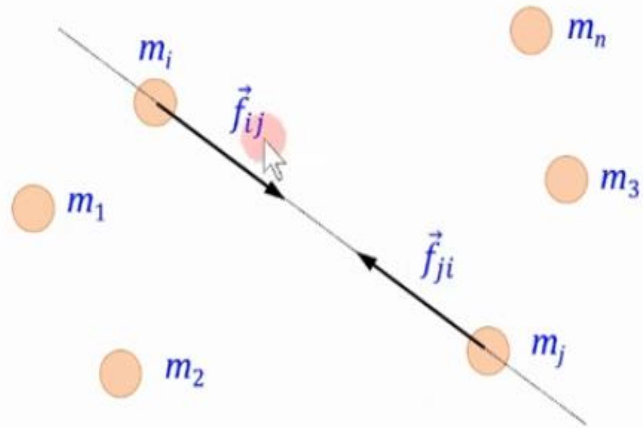
The particles, which interact with each other, apply forces on each other. The forces of interaction $\vec{f}_{ij}$ and $\vec{f}_{ji}$ between a pair of i^{th} and j^{th} particles are shown in the figure.



A System of Particles.

Internal and External Forces:

A System of Particles.



Internal Forces:

The forces of mutual interaction between the particles of a system are internal forces of the system.

The internal forces always exist in pairs of forces of equal magnitudes and opposite directions. In other words each pair is a Newton's third law pair.

Sum of all the internal forces of a system is a null vector.

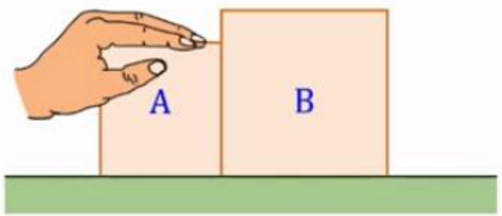
$$\Sigma \vec{f}_{ij} = \vec{0}$$

External Forces:

All those forces that are applied on a particle of a system by a body not included in the system.

Example 01.

Boxes A and B of masse m_{A} and m_{B} placed on a frictionless horizontal floor are being pushed horizontally by a force F as shown in the figure. Identify all the internal and external forces for the system of blocks A and B.



Solution.

All the forces acting on or within the system are listed here and shown in the adjacent figure.

$m_{\mathrm{A}}g \stackrel{\text{def}}{=}$ Weight of the block A

$m_{\mathrm{B}}g \stackrel{\text{def}}{=}$ Weight of the block B

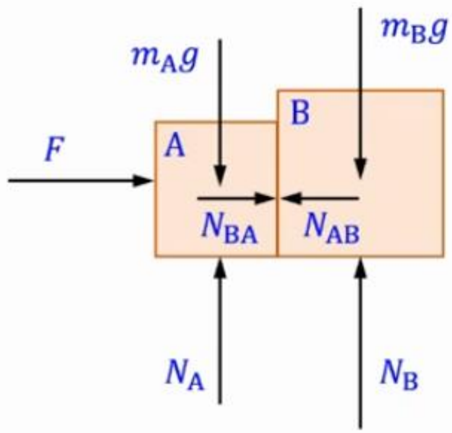
$N_{\mathrm{A}} \stackrel{\text{def}}{=}$ Normal reaction of the floor on block A

$N_{\mathrm{B}} \stackrel{\text{def}}{=}$ Normal reaction of the floor on block B

$N_{\mathrm{AB}} \stackrel{\text{def}}{=}$ Normal reaction of the the block B on A

$N_{\mathrm{BA}} \stackrel{\text{def}}{=}$ Normal reaction of the the block A on B

$F \stackrel{\text{def}}{=}$ Horizontal push of the hand



Internal Forces:

N_{AB} and N_{BA}

External Forces:

$m_{\mathrm{A}}g$

$m_{\mathrm{B}}g$

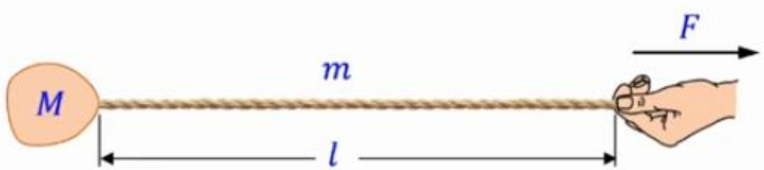
N_{A}

N_{B}

F

Example. 02

A body of mass M is being pulled in free space with the help of a uniform rope of mass m and length l. The rope is pulled by a force F as shown in the figure. Find an expression for the tensile force T at a distance x from the body.



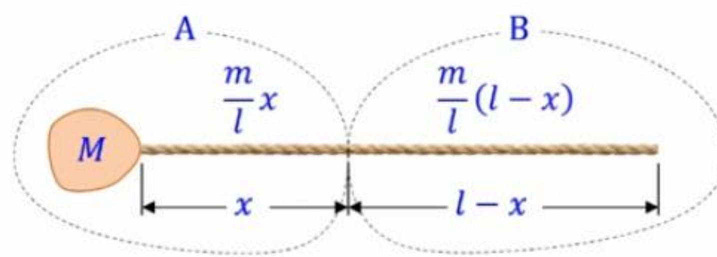
Solution.

The rope and the body, both are moving with the same acceleration; therefore, we can find their acceleration treating them like a single rigid body.

$$\Sigma F_x = (\Sigma m_i)a_x \qquad \Rightarrow F = (m + M)a$$

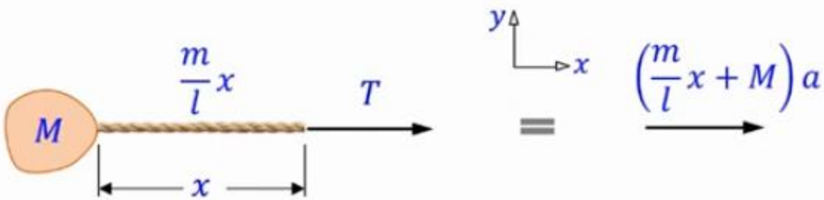
$$\Rightarrow a = \frac{F}{(m + M)} \quad \text{.......................}(1)$$

Now applying NLM to any of the following two parts A or B, we can find T. Here T is an internal force for the system consisting of the parts A and B.



Mass of a portion of length x of the rope $= \frac{m}{l}x$

NLM on part A containing rope of length x:



$$\Sigma F_x = (\Sigma m_i)a_x \quad \Rightarrow T = \left(\frac{m}{l}x + M\right)a$$

Substituting a from eq. (1), we get

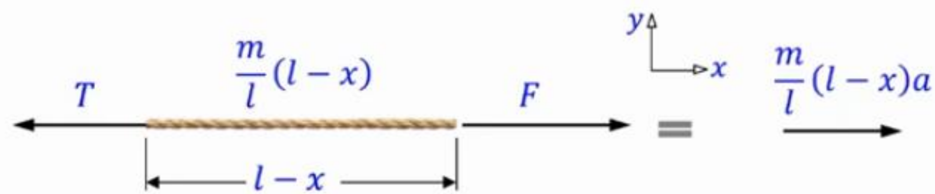
$$\Rightarrow T = \left(\frac{m}{l}x + M\right)\frac{F}{(m+M)}$$

$$\Rightarrow T = \frac{mF}{(m+M)l}x + \frac{MF}{(m+M)}$$

Note:

To find an internal force by using NLM, consider that body or part of system on which lesser number of forces are acting.

NLM on part B containing rope of length $(l - x)$:



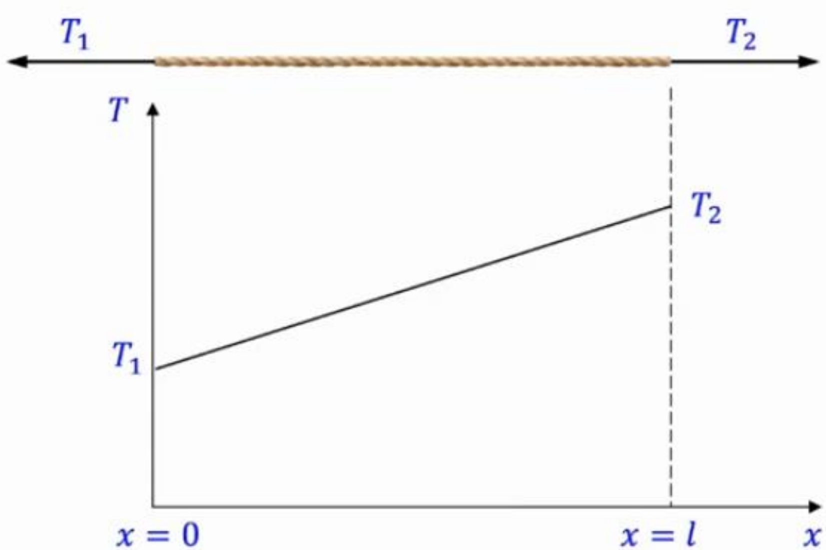
$$\Sigma F_x = (\Sigma m_i)a_x \quad \Rightarrow F - T = \frac{m}{l}(l-x)a$$

Substituting a from eq. (1), we get

$$\Rightarrow T = \frac{mF}{(m+M)l}x + \frac{MF}{(m+M)}$$

Alternate method:

This method is based on a fact that in a uniform rope, acceleration of every part of which is uniform, tensile force varies linearly with distance from one of the ends.



Tension at the left end that is the force with which the rope is pulling the body of mass M.

$$T_1 = Ma = \frac{MF}{(m+M)}$$

Tension at the right end that is the force with which the rope is being pulled.

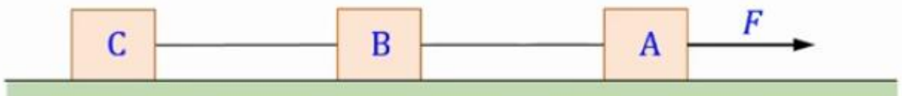
$$T_2 = F$$

$$T = \frac{T_2 - T_1}{l}x + T_1$$

$$\Rightarrow T = \frac{mF}{(m+M)l}x + \frac{MF}{(m+M)}$$

Example. 03

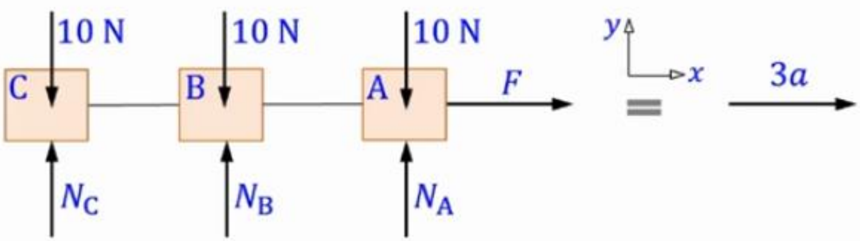
Three identical blocks A, B and C, each of mass 1.0 kg are connected by light strings as shown in the figure. If the block A is pulled by an unknown force F, the tension in the string connecting blocks A and B is measured to be 4.0 N.



Calculate magnitude of the force F and tension in the string connecting blocks B and C.

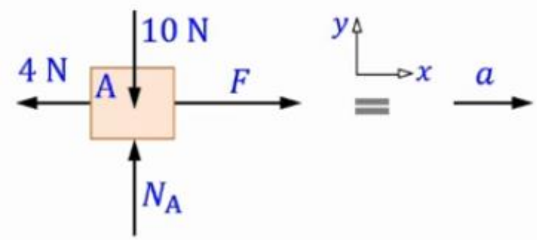
Solution.

All the three blocks move with equal accelerations, therefore, considering them a single rigid body we can use the same strategy as we have used in the previous example.



$$\Sigma F_x = (\Sigma m_i) a_x \qquad \Rightarrow F = 3a \quad \ldots\ldots\ldots\ldots\ldots (1)$$

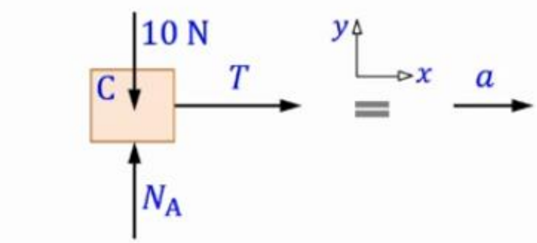
NLM for the block A:



$$\Sigma F_x = m a_x$$

$$\Rightarrow F - 4 = a \quad \ldots\ldots (2)$$

NLM for the block C:



$$\Sigma F_x = m a_x$$

$$\Rightarrow T = a \quad \ldots\ldots (3)$$

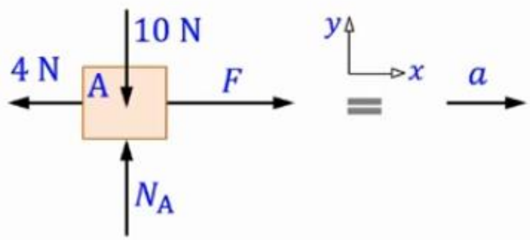
From the eq. (1), (2) and (3), we get: $F = 6$ N, $T = 2$ N,

Alternate method: Apply NLM of each body individually.

NLM for the block A:

$\Sigma F_x = ma_x$

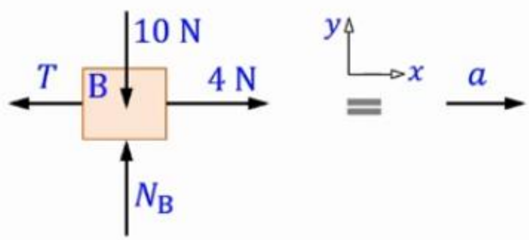
$\Rightarrow F - 4 = a$ .... (1)



NLM for the block B:

$\Sigma F_x = ma_x$

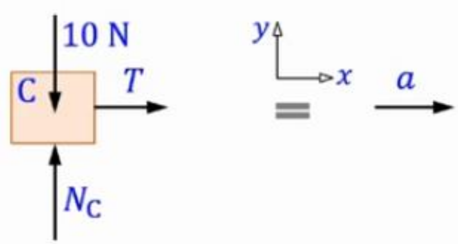
$\Rightarrow 4 - T = a$ .... (2)



NLM for the block C:

$\Sigma F_x = ma_x$

$\Rightarrow T = a$ .... (3)



From eq. (2) and (3), we get:

$T = 2\text{ N}$ and $a = 2\text{ m/s}^2$

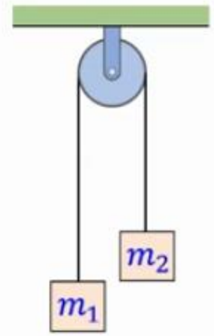
Substituting value of a in eq. (1), we get:

$F = 6\text{ N}$

AUROUS ACADEMY

Example. 04

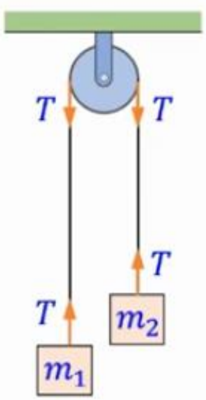
The system shown in the figure is released from rest. Assuming mass m_2 more than the mass m_1, find the accelerations of the blocks and the tension in the string.



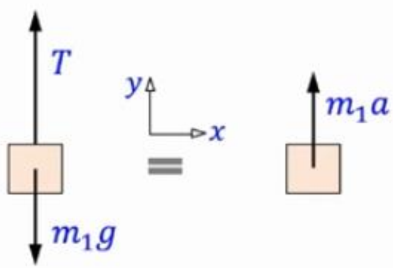
Solution.

Constraint: Magnitude of accelerations of both the blocks are equal say a.

Transmission of tension in the string:



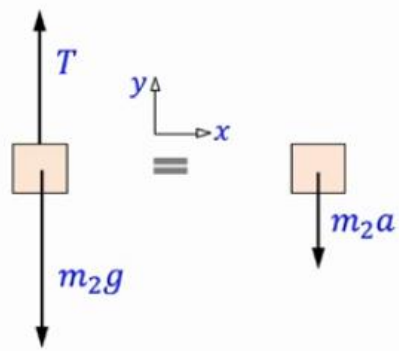
NLM for the block of mass m_1:



$$\Sigma F_y = m a_y$$

$$\Rightarrow T - m_1 g = m_1 a \quad .... (1)$$

NLM for the block of mass m_2:



$$\Sigma F_y = m a_y$$

$$\Rightarrow m_2 g - T = m_2 a \quad .... (2)$$

From eq. (1) and (2), we get:

$$T = \frac{2m_1 m_2}{m_1 + m_2} g$$

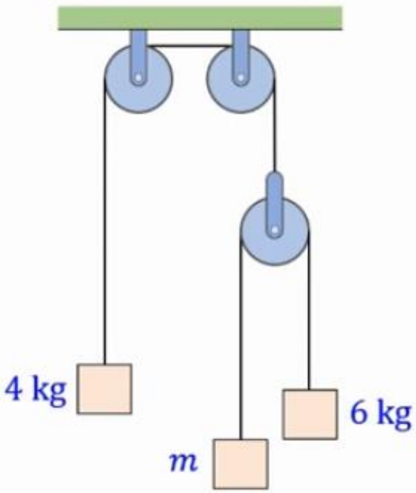
$$a = \frac{m_2 - m_1}{m_2 + m_1} g$$

Note:

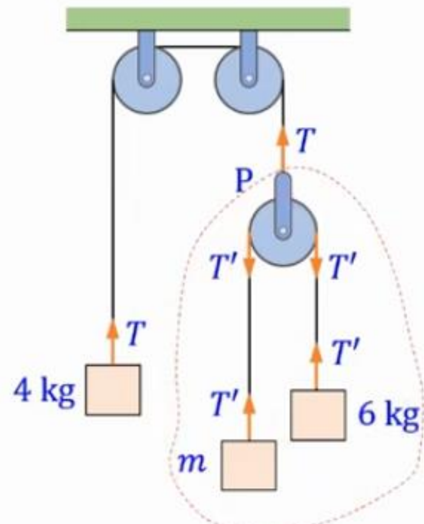
This system of pulleys is known as simple Atwood's machine.

Example. 05

In the setup shown, what should be value of the unknown mass m to maintain the 4 kg block in equilibrium?

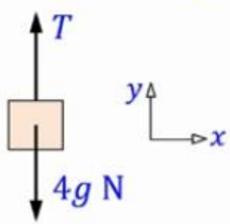


Transmission of tensions in the strings:



Solution.

TE of the 4 kg block:



$$\Sigma F_y = ma_y \;\Rightarrow T = 4g \text{ N} \text{ .... (1)}$$

For TE of the 4 kg block, the pulley P must be in TE.

$$\Rightarrow T = 2T' \text{ .... (2)}$$

TE of the pulley P suggests to consider the part of the setup shown in the enclosure as a simple Atwood's machine, therefore:

$$T' = \frac{2m_1 m_2 g}{m_1 + m_2} = \frac{2m \times 6g}{m + 6} \text{ .... (3)}$$

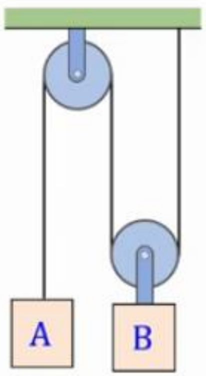
Making substitutions from eq. (1) and (3) in (2), we get:

$$4g = 2 \times \frac{2m \times 6g}{m + 6}$$

$$\Rightarrow m = \frac{6}{5} \text{ kg}$$

Example 06.

In the setup shown, blocks A and B are of equal masses. Find accelerations of the blocks.



Important Discussion:

In this setup, there are three unknowns, string tension and accelerations of the two blocks. Therefore, we need three independent equations to solve the problem.

By applying NLM, we can write two independent equations, one for each block. The third independent equation that is the relation between accelerations of the blocks, we can write using constraints motion.

In example 4 of simple Atwood's machine, equality of acceleration moduli was the constraint relation. Actually it was so obvious that we have used it unknowingly. But this will not always going be the same as in this example.

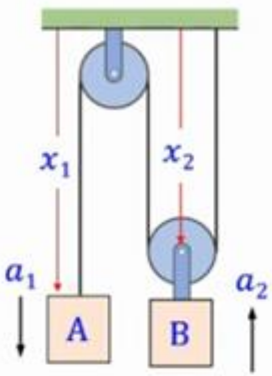
Conclusion:

Dealing with dynamics of systems of interconnected bodies, we have to always use the required constraint relations. Therefore, it is recommended to write it first, before proceeding with equations of NLM.

For writing constraint relations, we practice the methods learnt for them till so far. In chapter work and energy we will learn another very interesting as well as simpler method.

Solution.

Constraint relation:



Sum of all the variable portions of the string is a constant, therefor.

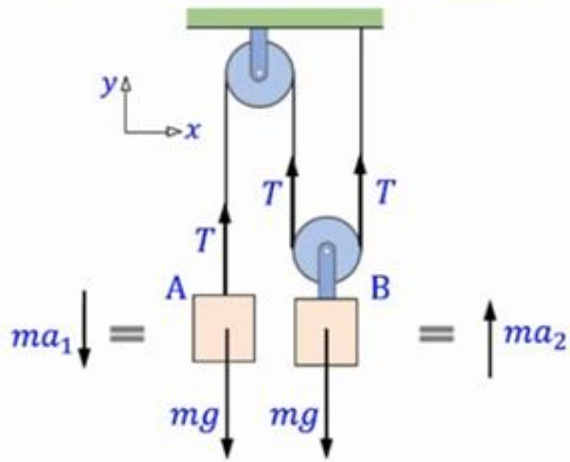
$$x_1 + 2x_2 = l$$

$$\xRightarrow{(\ddot{\ })} \ddot{x}_1 + 2\ddot{x}_2 = 0$$

$$\Rightarrow a_1 - 2a_2 = 0$$

$$\Rightarrow a_1 = 2a_2 \quad \text{......} (1)$$

NLM on each of the blocks:



For the block A:

$$mg - T = ma_1 \quad \text{......} (2)$$

For the block B:

$$2T - mg = ma_2 \text{......} (3)$$

From eq. (1), (2) and (3), we get:

$$a_1 = \frac{2g}{5} \quad \text{and} \quad a_2 = \frac{g}{5}$$