# Chapter 2

# Newton's Laws of Motion

Ideas of linear momentum, inertia, force, inertial and non-inertial reference frames, physical and pseudo forces, the three laws of Newton, applications of these laws in analysing translational equilibrium and translational motion, concept of friction and resistive forces, laws of dry friction, applications of Newton's laws in analysing dynamics of circular and curvilinear translation.

"I learned very early the difference between knowing the name of something and knowing something."

**Richard P. Feynman**
(11 May 1918 – 15 February 1988)

## Back to Basics

**Linear momentum:**

It is the amount of translational or linear motion of a body.

Linear momentum $\vec{p}$ of a body in translational motion is equal to product of mass $m$ and velocity vector $\vec{v}$ of the body.

$$\vec{p} = m\vec{v}$$

Usually the term linear is dropped and we use the term momentum.

**Inertia:**

Inertia of a material body is its tendency to oppose any change in its momentum.

Mass of a body is the measure of its inertia to translational motion.

**Force:**

The concept of force explains mutual interaction between two material bodies as the action of one body on another in the form of push or pull.

A mutual interaction between two bodies, which creates force on one body, also creates force on the other body. Force on the body under study is known as *action* and that on the other body is known as *reaction*.

**The first law:**

Every material body preserves its state of rest or of uniform motion in a straight line, unless it is compelled to change that state by external forces impressed on it.

**The second law:**

The rate of change in momentum of a body is equal to and occurs in the direction of the net applied force.

A body of mass $m$ in translational motion with velocity $\vec{v}$, if acted upon with a net external force $\Sigma\vec{F}$, the second law suggests:

$$\Sigma\vec{F} = \frac{\mathrm{d}}{\mathrm{d}t}(m\vec{v})$$

If mass of the body is a constant, the above equation relates acceleration $\vec{a}$ of the body with the net force $\Sigma\vec{F}$ acting on it.

$$\Sigma\vec{F} = m\vec{a}$$

**The third law:**

The action and reaction originating from a mutual interaction between two bodies are equal in magnitude, opposite in direction and identical in kind.

These laws are fundamental in nature. The first law tells us under what conditions there is no net external force, the second law shows how to measure a force when it exists and the third law reminds us that a force is interaction between two bodies.

**Inertial and non-inertial reference frames:**

A reference frame that is in the state of absolute rest or of uniform motion in a straight line is an inertial reference frame and a reference frame that is in the state of accelerated motion is a non-inertial reference frame.

The three laws of Newton are valid only in inertial reference frames.

**Physical and pseudo force:**

A physical force or real force is result of interaction between two bodies, whereas a pseudo force is a false force. The idea of pseudo force is used to account for the observed acceleration of a body in a non-inertial frame due to acceleration of the non-inertial frame.

Pseudo force on a body equals the product of the mass of the body and negative of acceleration vector of the non-inertial frame.

**Translational equilibrium:**

An un-accelerated body is said to be in the state of translational equilibrium.

For a body to be in translational equilibrium, no net force must act on it i.e. vector sum of all the forces acting on it must be a null vector.

$$\Sigma\vec{F} = \vec{0}$$

**Friction:**

Whenever surfaces in contact and pressing each other slide or tend to slide over each other, opposing forces acting tangentially to the surfaces in contact are generated. These tangential forces opposing sliding or tendency of sliding between the surfaces, are called *frictional forces*. Frictional forces on both the bodies constitute a third law action-reaction pair.

Frictional force between two solid un-lubricated surfaces is known as dry friction or Coulomb friction.

**Static friction:**

When a force of normal reaction $N$ exists between two surfaces, and there is tendency of sliding between them, the force of static friction $f_s$ acts before sliding initiates. It balances the net available force $F$ creating the tendency of sliding and can have a value up to a limiting value known as limiting friction $f_{sm}$.

$$f_s = F \le f_{sm}$$

The limiting friction is proportional to the normal reaction.

$$f_{sm} = \mu_s N$$

The constant of proportionality $\mu_s$ is known as *coefficient of static friction*.

**Kinetic friction:**

It opposes sliding between the surfaces. When a force of normal reaction $N$ exists between the two surfaces that are sliding on each other, the force of kinetic friction $f_k$ acts on a surface opposite to its velocity relative to the other surface. Kinetic friction is proportional to the normal reaction.

$$f_k = \mu_k N$$

The constant of proportionality $\mu_k$ is known as *coefficient of kinetic friction*.

**Dynamics of curvilinear translation and circular motion:**

Components of the net external force towards the centre of curvature of a path and along the tangent to the path are known as the normal component $F_n$ of the force and the tangential force $F_\tau$ respectively. The normal component of the force is responsible for change in direction only whereas the tangential force is responsible for change in speed only.

$$F_n = ma_n = \frac{mv^2}{\rho} = m\omega_v^2 \rho \qquad \text{and} \qquad F_\tau = ma_\tau = m\frac{dv}{dt}$$

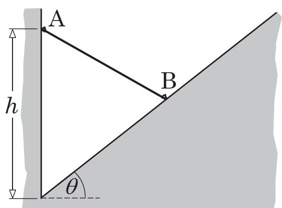
Here $\omega_v$ is the time rate of rotation of the velocity vector.

In circular motion, normal component of the force is usually called centripetal force.

## Multiple Choice Questions

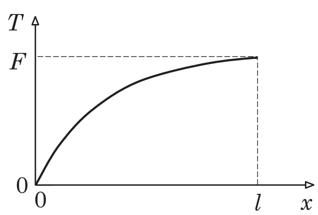
1. A bead released from one end of a frictionless rigid wire frame fixed in a vertical plane slides periodically on the frame. Size of the bead is negligible as compared to the radius of curvature of the turns, which is negligible as compared to the length of straight portions of the frame. If linear dimensions of the frame are made four times, by what factor will frequency of periodic motion of the bead change?

   *(a)* 4   *(b)* 2
   *(c)* 1/4   *(d)* 1/2



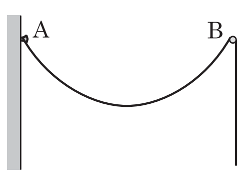

2. In the setup shown a thread is taut between a nail A on a wall and a nail B on an inclined plane as shown in the figure. You can change the length of the thread by changing location of the nail B. A bead that can slide on the thread without friction is released from the nail A. What should the length of the thread be so that the bead reaches the nail B in shortest time?

   *(a)* $h$   *(b)* $h\cos\theta$
   *(c)* $h\sec(0.5\theta)$   *(d)* $h\cos\theta\sec(0.5\theta)$

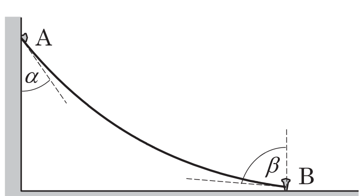

3. A rope of length $l$ placed straight on a frictionless horizontal floor is pulled longitudinally by a force $F$ from one of its ends. Tensile force $T$ developed in the rope at a distance $x$ from the rear end varies with $x$ as shown in the graph. What can you certainly conclude about density of the rope?

   *(a)* It is uniform.
   *(b)* It decreases with distance $x$ from the rear end.
   *(c)* It increases with distance $x$ from the rear end.
   *(d)* It is maximum in the middle and decreases towards the ends.




4. A uniform rope of length 13 m tied on a peg A on a wall passes over a frictionless peg B fixed in level with the peg A as shown in the figure. If in equilibrium, length of the rope hanging between the pegs is 8 m, the angle which the rope makes with the wall at the peg A is closest to

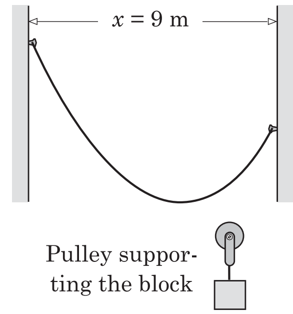
   *(a)* 30°   *(b)* 37°
   *(c)* 45°   *(d)* 53°




5. A uniform rope is tied between a nail A on the wall and a nail B on the ground. The rope without touching the ground anywhere assumes a curved shape known as "catenary". Tangents at the ends A and B of this catenary make angles $\alpha$ and $\beta$ with the vertical respectively. Which of the following conclusions can you make?

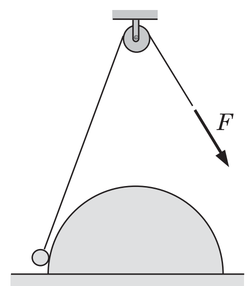
   *(a)* Horizontal component of the tensile force in the rope is uniform.
   *(b)* Vertical component of the tensile force increases with height.
   *(c)* Angle $\alpha$ can be greater than angle $\beta$.
   *(d)* Angle $\alpha$ cannot assume a value of 0°.

6. A light inextensible string of length 15 m is hanging between two pegs that are 9 m horizontally apart as shown. A small frictionless pulley of weight $W_p$ = 4.8 N supporting a block of weight $W_b$ = 8.0 N is gently placed on the string and allowed to move gradually to a place on the string, where it can stay in equilibrium. Radius of the pulley-wheel is negligible as compared to the length of the string. Which of the following conclusions can you make when the pulley is in equilibrium?



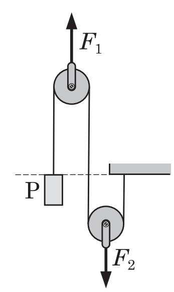

*(a)* Level difference of the pegs is 4.0 m.
*(b)* Tensile force in the string depends on the level difference of the pegs.
*(c)* Tensile force in the string is 8.0 N irrespective of the level difference of the pegs.
*(d)* Lengths of string segments on left and right sides of the pulley are 10 m and 5.0 m respectively.

7. A small metal ball is being pulled gradually on a fixed frictionless hemisphere as shown in the figure. Radii of the ball and that of the pulley are much smaller than that of the hemisphere. As the ball slides from the bottom to a position close to the top of the hemisphere, how do the magnitudes of pulling force $F$ and contact force $R$ between the ball and the hemisphere change?



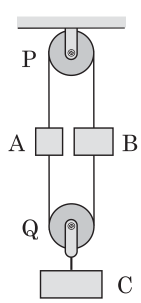

*(a)* $F$ increases and $R$ decreases.
*(b)* $F$ decreases and $R$ increases.
*(c)* $F$ decreases and $R$ remains unchanged.
*(d)* $F$ remains unchanged and $R$ decreases.

8. A block is suspended from a uniform rope that passes through two ideal pulleys. The other end of the rope is tied to a fixed support as shown in the figure. If the pulleys are pulled vertically with forces $F_1$ = 110 N and $F_2$ = 90 N, the system stays at rest. Assuming linear mass density of the rope 0.25 kg/m and the acceleration of free fall 10 m/s$^2$, find the length of the rope. Pulleys are not small.

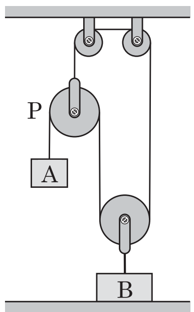



| | |
|---|---|
| *(a)* 4 m | *(b)* 6 m |
| *(c)* 8 m | *(d)* Insufficient information. |

9. Two blocks A and B of masses 3 kg and 6 kg respectively are suspended at the ends of a light inextensible cord. The cord passes over an ideal pulley P fixed to the ceiling. Ends of another light inextensible cord are attached at the bottoms of the blocks. This cord supports another ideal pulley Q from which a block C of mass 4 kg is suspended as shown in the figure. Initially the system is held motionless and then released. Which of the following conclusions can you make? Acceleration due to gravity is 10 m/s$^2$.

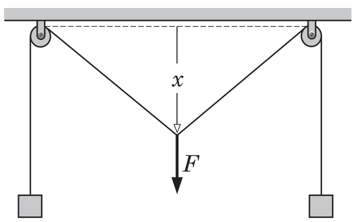



*(a)* All the blocks remain motionless.
*(b)* Acceleration magnitudes of blocks A and B is 10/3 m/s$^2$ while C remains motionless.
*(c)* Tensile forces in the upper and the lower cords are 40 N and 20 N respectively.
*(d)* Tensile forces in the upper and the lower cords are 60 N and 20 N respectively.

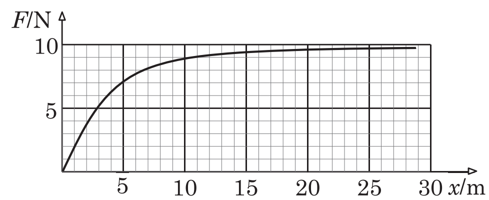



**10.** In the setup shown, the string is inextensible and light, there is no friction at the axles of the pulleys but friction between the string and the pulley wheels is sufficient to prevent slipping. All the pulleys are light except the pulley P, which has whole of its mass $m$ concentrated in its axle. Initially the block B of mass $M$ is on the ground and the block A is held at rest. How much minimum mass should the block A have, so that after releasing it, the block B starts rising?

*(a)* $0.5M$ *(b)* $0.5M - m$
*(c)* Whatever mass the block A has, the block B will not rise.
*(d)* Pulley P moves downwards without rotation of its wheel while wheels of all the other pulleys rotate.




**11.** A light inextensible cord supporting two identical loads at its ends passes over two ideal small pulleys fixed to a horizontal ceiling. If the midpoint of the cord between the pulleys is gradually pulled downwards, the pulling force $F$ varies with displacement $x$ of the midpoint according to the following graph. Acceleration of free fall is 10 m/s². The distance between the pulleys is closest to




*(a)* 5 m *(b)* 10 m
*(c)* 15 m *(d)* 20 m

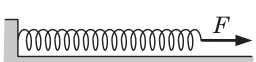


**Figure-1**

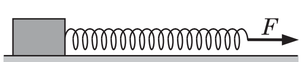


**Figure-2**

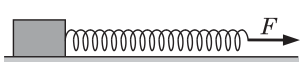


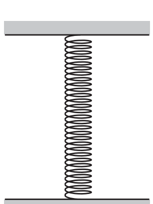
**Figure-3**

**12.** Consider three identical massless springs. The left ends of the first, second and the third springs are affixed to a wall, to a block of mass $m$ placed on a horizontal floor (not frictionless) and to a block of mass $m$ placed on a horizontal frictionless floor respectively as shown in the figures. The right end of each spring is pulled by a gradually increasing force. When magnitude of the force becomes $F$, extensions in these springs become $x_1$, $x_2$ and $x_3$ respectively. Which of the following statements is correct?

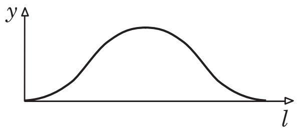
*(a)* $x_1 = x_2 = x_3$ *(b)* $x_1 > x_2 > x_3$
*(c)* $x_1 > x_2 = x_3$ *(d)* $x_1 = x_2 > x_3$

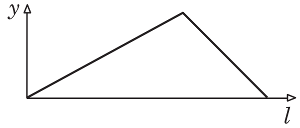
**13.** Length of a one-metre long uniform spring of mass 50.0 g increases by 2.00 cm due to its own weight, if it is suspended from a fixed support. How much load should be suspended from the lower end of this spring, so that total extension becomes 10.0 cm.

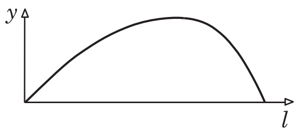
*(a)* 100 g *(b)* 125 g
*(c)* 200 g *(d)* Insufficient information

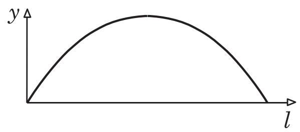


**14.** A light uniform spring is tied between the ceiling and the floor keeping the spring vertical as shown in the figure. A bead of finite mass is glued at a distance $l$ from the upper end of the spring and then allowed to move gradually downwards. The bead shifts a distance $y$ to come in equilibrium. This experiment is repeated for different values of $l$. Which of the following graph shows the best dependence of $y$ on $l$?

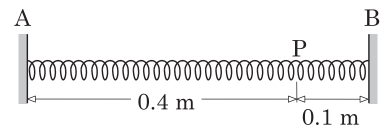
*(a)* 


*(b)* 


*(c)* 


*(d)* 


**15.** A light spring of force constant 1.00 N/cm is stretched between two fixed supports A and B. A point P on the spring is pulled parallel to the spring towards the support B by a gradually increasing force. When this force becomes 5.00 N, what is magnitude of displacement of the point P?




*(a)* 8.00 mm
*(b)* 22.2 mm
*(c)* 40.0 mm
*(d)* Relaxed length of the spring is required.

**16.** Three rubber cords, each of force constant $k$ are joined with each other at one of their ends. The free ends of the cords are pulled maintaining them always in a plane. For a particular set of pulling forces an arrangement shown in the figure-I is obtained and for another set of pulling of forces arrangement shown in the figure-II is obtained. In the arrangement of figure-I, extended length of each cord is $L_1$; and in the arrangement of figure-II, extended lengths of each cord is $L_2 > L_1$.

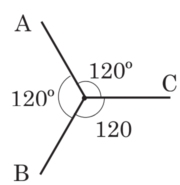



**Figure - I**

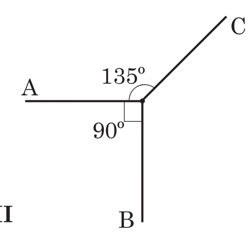
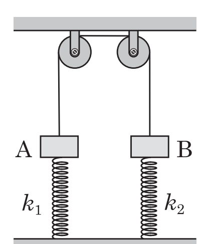



**Figure - II**

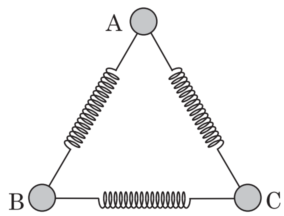
With the help of the given information, check validity of the following statements.

*(a)* $l_A = l_B < l_C$ and $k_A = k_B < k_C$
*(b)* $l_A = l_B < l_C$ and either $k_A = k_B > k_C$ or $k_A = k_B < k_C$
*(c)* $l_A = l_B < l_C$ if $k_A = k_B > k_C$ or $l_A = l_B > l_C$ if $k_A = k_B < k_C$
*(d)* $l_A < l_B < l_C$ if $k_A > k_B > k_C$ or $l_A > l_B > l_C$ if $k_A < k_B < k_C$

**17.** Two blocks A and B each of mass $m$ are connected by an ideal string that passes over two fixed ideal pulleys. The blocks are also connected with the ground by springs of force constants $k_1$ and $k_2$ ($k_1 > k_2$). When both the springs are relaxed, the block A is pulled down a distance $x$ and released. Acceleration magnitudes of the blocks A and B immediately after the release are $a_1$ and $a_2$ respectively. Mark the correct options.

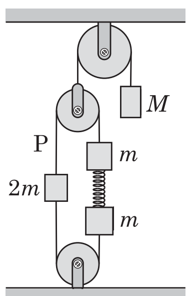



*(a)* $a_1 > a_2$
*(b)* $a_1 \leq a_2$
*(c)* $a_1 > a_2$ if $k_1 > k_2 + 2mg/x$
*(d)* $a_1 = a_2$ if $k_1 \leq k_2 + 2mg/x$




18. Three identical balls A, B and C each of mass $m$ connected by three identical springs when placed on a frictionless horizontal floor occupy corners of an equilateral triangle as shown in the figure. This assembly is suspended by attaching the ball A to the ceiling with the help of a light thread. What can you predict regarding accelerations of the balls immediately after the thread is cut? Acceleration of free fall is $g$.

(a) All the balls will have downward acceleration $g$.
(b) All the balls have acceleration greater than $g$ in different directions.
(c) Acceleration of the upper ball is $3g$ downwards and the accelerations of the lower balls are vanishingly small.
(d) Acceleration of the upper ball is $3g$ downwards and the lower balls move apart with finite accelerations.




19. In the setup shown, the pulleys, the cords and the spring are ideal and masses of the loads are indicated in the figure. Initially the system is in equilibrium. What should be the range of mass $M$ so that acceleration of the load of mass $2m$ becomes greater than acceleration of free fall immediately after the cord is cut at point P?

(a) $M > 4m$ (b) $M > 6m$
(c) $M > 8m$ (d) $M > 14m$

20. While a block is sliding on a horizontal floor, a constant horizontal force is applied on it opposite to its direction of motion. Which of the following can be a correct velocity-time graph for the ensuing motion of the block?

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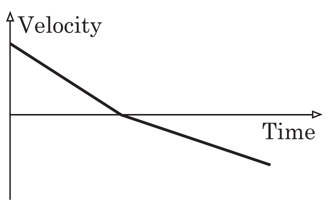


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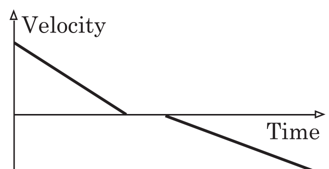


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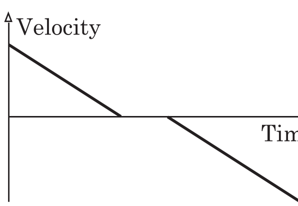


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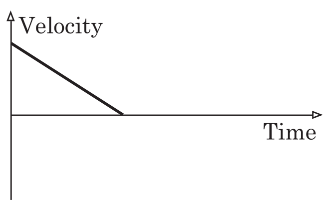
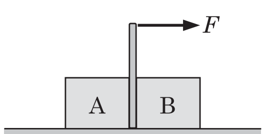
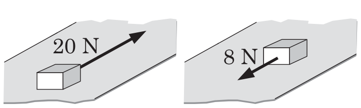



21. A rod is inserted between two identical blocks A and B placed close to each other on a horizontal floor, which is not frictionless. If the upper end of the rod is pulled horizontally by a gradually increasing force $F$, which of the blocks will start sliding first?

(a) Block A
(b) Block B
(c) Both will start sliding simultaneously.
(d) It is a matter of chance and hence either one may start sliding first.




22. A force of 20 N along the line of the greatest slope is required to slide a block upwards with constant speed and a force of 8 N along the line of the greatest slope is required to slide the block downwards with constant speed on an inclined plane. The force of kinetic friction between the block and the plane is

| | |
|---|---|
| *(a)* 8 N | *(b)* 12 N |
| *(c)* 14 N | *(d)* None of these |

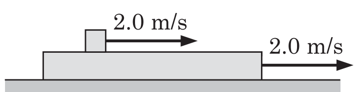
**23.** A block of mass 5 kg rests on a horizontal floor. When a constant horizontal force is applied on the block for a time interval 5 s, the block slides on the floor a distance 5 m under the action of the force and after the removal of the force, it further slides a distance 1 m before coming to a stop. Acceleration due to gravity is 10 m/s$^2$.

*(a)* Magnitude of the applied force is 12 N.
*(b)* The maximum speed acquired by the block is 2 m/s.
*(c)* Coefficient of friction between the block and the floor is 0.2.
*(d)* The block continues to move for 1 s after the removal of the force.

**24.** Near a station, a train is retarding at 2.0 m/s$^2$ to stop. When its speed is 36 km/h a passenger standing in the corridor of a bogie, puts his suitcase on the floor. The floor was a little bit slippery so the suitcase began to slide and finally stopped in the train after sliding 12 m on the floor relative to the train. According to this situation, which of the following statements are correct?

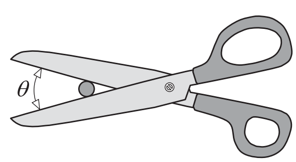
*(a)* The train stopped before the suitcase stopped sliding.
*(b)* Speed of the suitcase relative to the bogie decreases monotonically.
*(c)* Speed of the suitcase relative to the bogie first increases then decreases.
*(d)* Coefficient of friction between the suitcase and the floor of the bogie is close to 0.135.

**25.** A block rests on a long plank that is moving with a constant velocity 2.0 m/s on a horizontal floor. Coefficient of friction between the block and the plank is 0.10. If the plank starts decelerating uniformly and stops in 0.50 s, what is the distance slid by the block on the plank? Acceleration of free fall is 10 m/s$^2$.



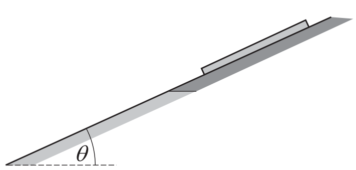

| | |
|---|---|
| *(a)* 0.50 m | *(b)* 0.75 m |
| *(c)* 1.0 m | *(d)* 1.5 m |

**26.** A pair of scissors is used to cut a wire of circular cross section held vertically. To reduce required force the wire must be placed close to the hinge; but if it is placed close to the hinge, it slides away from the hinge until the angle between the blades becomes $\theta$. Find the coefficient of friction between the blades and the wire.




| | |
|---|---|
| *(a)* $\cot(0.5\theta)$ | *(b)* $\tan(0.5\theta)$ |
| *(c)* $0.5\tan\theta$ | *(d)* Insufficient information |

**27.** A homogeneous bar of mass $m$ is released from rest on a fixed inclined plane of inclination $\theta$ above the horizontal as shown. The upper part of the plane shown in dark grey is not frictionless and the lower part shown in light grey is frictionless. Coefficient of friction between the bar and the upper part of the plane is $\mu$. What will be the maximum tensile force in the bar while it is sliding on the plane? Acceleration of free fall is $g$.

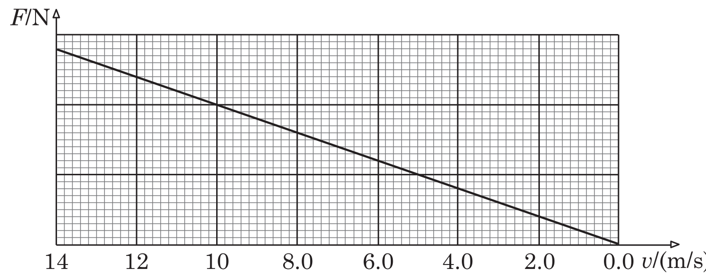



| | |
|---|---|
| *(a)* Zero | *(b)* $0.25\mu mg\cos\theta$ |
| *(c)* $0.5\mu mg\cos\theta$ | *(d)* $\mu mg\cos\theta$ |

**28.** Two balls of equal volume and masses $m_1$ and $m_2$ $(m_2 > m_1)$ connected with a thin light thread are dropped from a certain height. Viscous drag of air on a ball depends on its velocity and buoyant force is equal to the weight of air displaced by the ball. When the balls acquire a uniform velocity after a sufficient time from the instant they were dropped, what is the tensile force in the thread? Acceleration due to gravity is $g$.

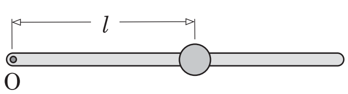
*(a)* Zero
*(b)* $(m_2 - m_1)\,g$
*(c)* $0.5(m_2 + m_1)\,g$
*(d)* $0.5(m_2 - m_1)\,g$

**29.** A block of mass 1.0 kg is given an initial velocity 20 m/s by a sharp hit on a horizontal floor lubricated with oil. The block moves in a straight line and stops due to viscous drag of the oil film. How the force $F$ of viscous drag varies with velocity $v$ of the block is shown in the following graph for a velocity interval [14 m/s, 0.0 m/s]. However, the experimenter forgot to label the ordinate.




After an instant, when the block was moving with 8 m/s, it slides 20 m and stops. How far does the block slide from the beginning to the instant it stops sliding?

*(a)* 50 m
*(b)* 75 m
*(c)* 100 m
*(d)* More information is required.




**30.** A massive bead is threaded on a long light rod, one end of which is pivoted to a fixed point O. Initially, the rod is held horizontally and the bead is at a distance $l$ from the pivot. Coefficient of friction between the rod and the bead is $\mu$. Which of the following statements correctly describe relation between angle $\theta$ that the rod makes with the horizontal after the rod is released and time $t$?

*(a)* If $\mu$ is negligible, $\theta \to \tan^{-1}\left(\frac{gt^2}{2l}\right)$.

*(b)* If $\mu$ is finite, $\theta \to \tan^{-1}\left(\frac{gt^2}{2l}\right)$.

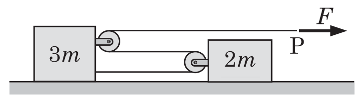
*(c)* If $\mu$ is finite, $\theta$ is smaller than $\tan^{-1}\left(\frac{gt^2}{2l}\right)$ by a finite amount.

*(d)* None of the above statements is correct.

**31.** A ball dropped from a high altitude acquires a terminal velocity before hitting the ground, where it bounces off elastically. If air resistance depends on the speed of the ball, what will its acceleration be immediately after the first bounce?

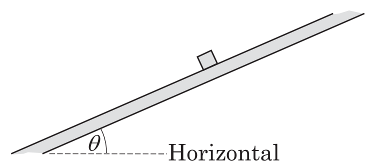
*(a)* Zero
*(b)* $g\downarrow$
*(c)* $2g\downarrow$
*(d)* $3g\downarrow$

32. In the setup shown, blocks of masses $3m$ and $2m$ are placed on a frictionless horizontal ground and the free end P of the thread is being pulled by a constant force $F$. Find acceleration of the free end P.



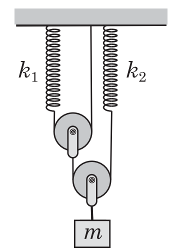

(a) $F/(5m)$ (b) $2F/m$
(c) $3F/m$ (d) $5F/m$

33. A small block is sliding on a frictionless inclined plane that is moving upward with a constant acceleration. If the block remains at a level height, what is the acceleration of the inclined plane? Acceleration due to gravity is $g$.



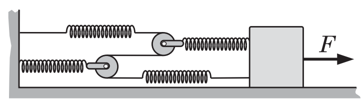

(a) $g\tan\theta$ (b) $g\cot\theta$
(c) $g\sin^2\theta$ (d) $g\tan^2\theta$

34. In the arrangement shown, the springs are light and have stiffness $k_1$ =100 N/m and $k_2$ = 200 N/m and the pulleys are ideal. An 8.0 kg block suspended from the lower pulley is initially held at rest maintaining the strings straight and springs relaxed. Now the force supporting the block is gradually reduced to zero. How far does the block descend during the process of reduction of the force? Acceleration due to gravity is 10 m/s$^2$.



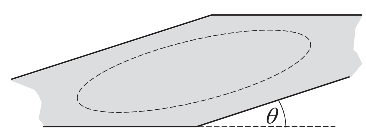

(a) 10 cm (b) 15 cm
(c) 1.6 m (d) None of these

35. In the setup shown, a block is placed on a frictionless floor, the cords and pulleys are ideal and each spring has stiffness $k$. The block is pulled away from the wall. How far will the block shift, while the pulling force is increased gradually from zero to a value $F$?




(a) $\dfrac{2F}{5k}$ (b) $\dfrac{10F}{3k}$
(c) $\dfrac{8F}{9k}$ (d) $\dfrac{10F}{9k}$

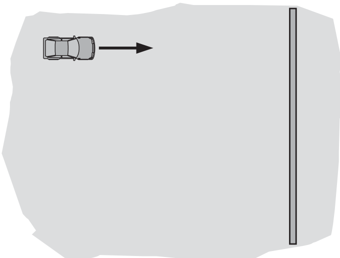
36. A large parking place has uniform slope of angle $\theta$ with the horizontal. A driver wishes to drive his car in a circle of radius $R$, at constant speed. Coefficient of static friction between the tyres and the ground is $\mu$. What greatest speed can the driver achieve without slipping? Assume entire load of the car on the front wheels.




(a) $\sqrt{gR\tan\theta}$ (b) $\sqrt{gR\cot\theta}$
(c) $\sqrt{gR(\sin\theta+\mu\cos\theta)}$ (d) $\sqrt{gR(\mu\cos\theta-\sin\theta)}$

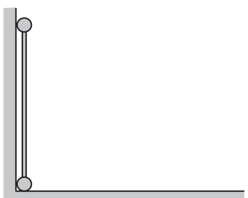
37. Consider a circular turn on a highway, where angle of banking is more than angle of repose. A car on this turn can move with a constant speed $v_0$ without help of friction. Taking advantage of friction, it can achieve a maximum speed $v_{max}$ but cannot reduce its speed lower than a minimum speed $v_{min}$. Which of the following conclusions can you draw from the given information?

(a) $v_0 < 0.5(v_{max} + v_{min})$ (b) $v_0 = 0.5(v_{max} + v_{min})$
(c) $v_0 > 0.5(v_{max} + v_{min})$ (d) It depends on coefficient of static friction.



**38.** A car is moving with speed 108 km/h on a large uniform horizontal pavement perpendicularly towards a wall. To avoid collision, the driver takes a turn and finally runs the car parallel to the wall with the same speed. Coefficient of friction between the tyres and the pavement is 0.6, acceleration of free fall is 10 m/s$^2$ and all the load of the car is concentrated on the front axle. If the turning process is the fastest one on this pavement, check validity of the following statements.

*(a)* The car follows a circular path during the turn.
*(b)* The car follows a parabolic path during the turn.
*(c)* Minimum radius of curvature of the path is 75 m.
*(d)* Initial distance of the wall must be greater than $75\sqrt{2}$ m.



**39.** A dumbbell is constructed by fixing small identical balls at the ends of a light rod of length $l$. The dumbbell stands vertically in the corner formed by a frictionless wall and frictionless floor. After the bottom end is slightly pushed towards the right, the dumbbell begins to slide. Value of which of the following quantities vanish, when the upper ball is leaving the wall.

*(a)* Tensile force in the rod
*(b)* Acceleration of the lower ball
*(c)* Acceleration of the upper ball
*(d)* Normal reaction from floor on the lower ball

**40.** A particle is moving with constant angular velocity on a circular path of radius $R$ in the $x$-$y$ plane. If observed from a reference frame moving with constant velocity along the $z$-axis, the particle will appear moving on a helical path of constant pitch $h$. Making use of the given information, what expression can be deduced for radius of curvature of the helical path.

*(a)* $\sqrt{R^2 + \dfrac{h^2}{4\pi^2}}$ *(b)* $R + \dfrac{h^2}{4\pi^2 R}$

*(c)* $\sqrt{R^2 + h^2}$

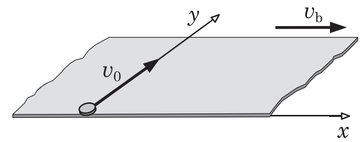
*(d)* Cannot be calculated from the given information.

**41.** Imagine that mass, which governs acceleration of the bodies and their mutual gravitational interaction, might sometimes be negative. Two particles A and B of masses $m_1$ and $m_2$ are initially at rest some distance apart in free space relative to an inertial frame. What would happen after they are released?

| The masses | Observation |
|---|---|
| *(a)* $m_1 < 0$ and $m_2 < 0$ | *(p)* They move towards each other. |
| *(b)* $m_1 > 0$, $m_2 < 0$ and $\|m_1\| = \|m_2\|$ | *(q)* They move away from each other. |
| *(c)* $m_1 > 0$, $m_2 < 0$ and $\|m_1\| > \|m_2\|$ | *(r)* Eventually B will escape away from A. |
| *(d)* $m_1 < 0$, $m_2 > 0$ and $\|m_1\| > \|m_2\|$ | *(s)* B follows A and finally collides with A. |
| | *(t)* B follows A with a constant separation. |

**Questions 42 to 44 are based on the following physical situation.**

A horizontal conveyor belt is running at a constant speed $v_b = 3.0$ m/s. A small disc enters the belt moving horizontally with a velocity $v_0 = 4.0$ m/s that is perpendicular to the velocity of the belt. Coefficient of friction between the disc and the belt is 0.50.




42. What can you predict regarding the path of the disc?

*(a)* It is a parabola relative the belt.
*(b)* It is a straight line relative to the belt.
*(c)* It is a parabola relative to the ground.
*(d)* It is a straight line relative to the ground.

43. What should the minimum width of the belt be so that the disc always remains on the belt?

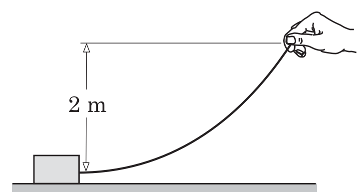
*(a)* 0.9 m *(b)* 1.6 m
*(c)* 2.0 m *(d)* 2.5 m

44. What is the minimum speed of the disc relative to the ground?

*(a)* 0.0 m/s *(b)* 1.8 m/s
*(c)* 2.4 m/s *(d)* 3.0 m/s

**Questions 45 to 47 are based on the following physical situation.**

Lower end of a uniform inextensible rope of mass 2 kg and length 4 m is attached to a block of mass 7.5 kg placed on a horizontal floor. Coefficient of friction between the block and the floor is 0.5. The upper end of the rope is held 2 m above the lower end so that the tangent at the lower end remains horizontal as shown in the figure. In this situation, the block stays standstill on the floor. Acceleration due to gravity is 10 m/s$^2$.




45. The upper end must be pulled at an angle that is closest to

*(a)* 60° above the horizontal *(b)* 53° above the horizontal
*(c)* 45° above the horizontal *(d)* Insufficient information

46. Frictional force between the block and the floor is closest to

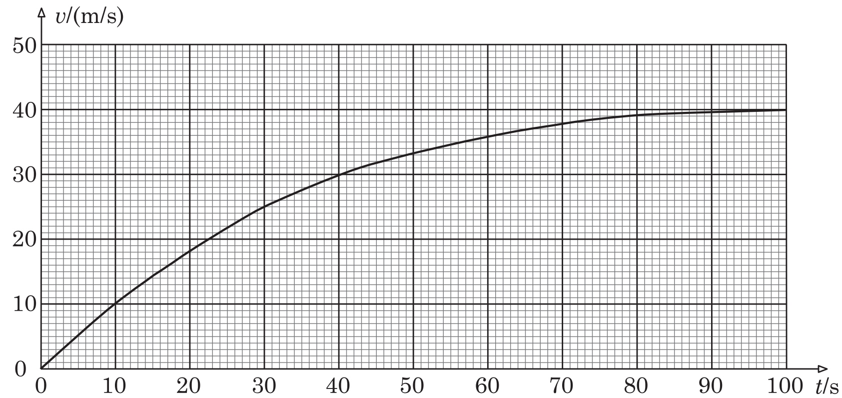
*(a)* 15 N *(b)* 20 N
*(c)* 30 N *(d)* 37.5 N

47. The upper end of the rope is now slowly shifted downwards and simultaneously away from the block maintaining the tangent at lower end horizontal. When the block begins sliding, at what height above the lower end is the upper end?

*(a)* 0.5 m *(b)* 0.75 m
*(c)* 1.0 m *(d)* 1.5 m

**Questions 48 to 50 are based on the following physical situation.**

Velocity-time relation of a four-wheel drive car running without any gearshift on a straight level road is shown in the following graph. The engine is running at constant throttle and the wheels are maintained always at the verge of slipping. Air drag on the car depends on its speed; therefore, it can be neglected for initial few seconds. The graph can satisfactorily be treated as a straight line in every 10 s interval.




The driver shifts gears once after acquiring some speed. Gear shifting requires a period of 0.5 s, during which the engine remains declutched so that the car decelerates due to air drag. After the gearshift, clutches are released and the car follows the same velocity profile as shown in the graph with a delay of 1.0 s. Therefore, the car acquires the final velocity 1.0 s later than that acquired without a gearshift. Acceleration of free fall is 10 m/s$^2$.

**48.** Coefficient of friction between the tyres and the road is closest to

*(a)* 0.04 *(b)* 0.05
*(c)* 0.10 *(d)* 0.20

**49.** Minimum time after the car starts, when the gear was shifted is closest to

*(a)* 10 s *(b)* 25 s
*(c)* 30 s *(d)* 35 s

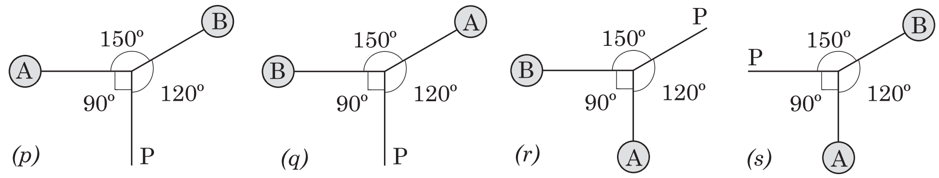
**50.** Due to gearshift, how much lesser distance does the car cover during the first 100 s, as compared to the case of no gearshift?

*(a)* 12.5 m *(b)* 15 m
*(c)* 18 m *(d)* 30 m

## Build-up your understanding

**1.** Under simultaneous action of two forces, a stationary particle starts moving parallel to a vector $\hat{i} - \hat{j}$. If one of the force is $\left(3\hat{i} - \hat{j} - \hat{k}\right)$N and the other has smallest possible magnitude, find the other force.

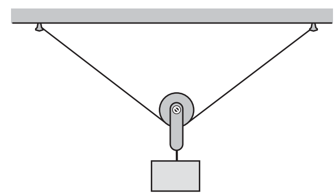
**2.** Two small discs A and B of masses 1 kg and 2 kg are connected by a light cord that is connected at its midpoint to another light cord. This assembly is placed on a frictionless horizontal floor in such a way that the segments of the cord connecting the disc and the new cord remain straight making angles of 90º, 120º and 150º. For this description, following four arrangements are suggested.



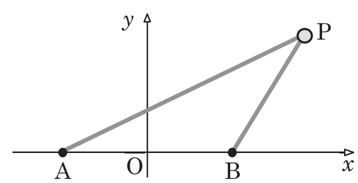

The free end P of the new cord is pulled by a 10 N force and both the discs begin to move with accelerations of equal magnitudes.

*(a)* Which of the above arrangements satisfy the given condition?
*(b)* What is the magnitude of the acceleration?

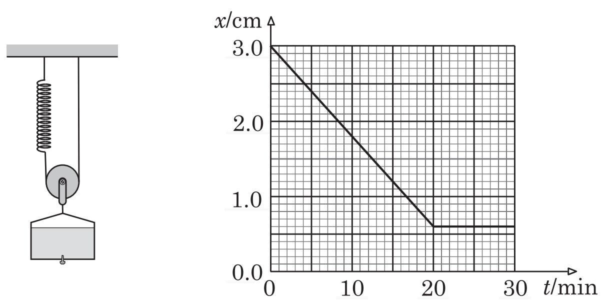
**3.** A light and inextensible string of length $l = 20$ m tied between two nails, supports a frictionless pulley of weight $W_p = 10\sqrt{2}$ N from which a block of weight $W_b = 10\sqrt{2}$ N is also suspended as shown in the figure. The nails are fixed in a level a distance $x = 10\sqrt{2}$ m apart. Radius of the pulley is $r = 10$ cm. How much normal reaction per unit of its length does the string apply on the pulley?



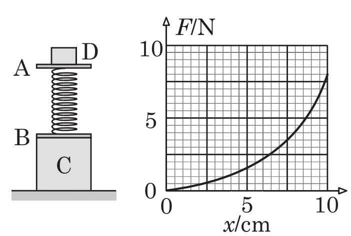
**4.** Two identical elastic cords of negligible relaxed lengths are tied at one of their ends to fixed nails A and B that are equidistant from the origin O. The other ends of the strings are tied to a small ball. To hold the ball in equilibrium at a point P (4 m, 3 m), a force of magnitude $F = 1000$ N is required. Assuming free space conditions, find force constant of the cords?

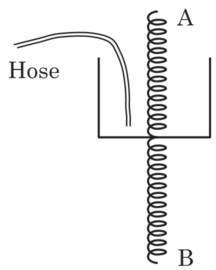



**5.** A wide container filled with water is suspended with the help of a light spring of stiffness 1000 N/m, a light inextensible cord and an ideal pulley. Initially, when the system is in equilibrium, the plug inserted in an orifice at the centre of the bottom of the container is pulled out. The extension in the spring is recorded and shown in the adjoining graph. Find the time rate of flow of water from the orifice.



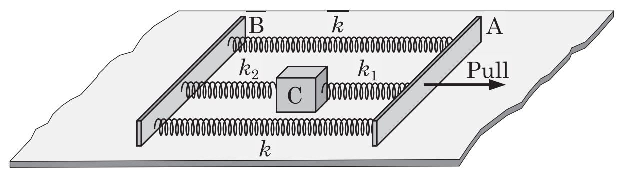

**6.** Two light discs A and B are attached at the ends of a spring of force constant 100 N/m. The assembly is placed on a rubber pad C, which is placed on the floor. When a load D is placed on the disc A, the disc shifts downwards and stays there in equilibrium simultaneously the rubber pad is compressed and the disc B shifts downwards by a distance $x$. The restoring force $F$ of the rubber pad varies with its compression $x$ according to the given graph. Acceleration due to gravity is 10 m/s$^2$. Find mass of the load, so that when equilibrium is established, the disc A shifts 10 cm downwards.



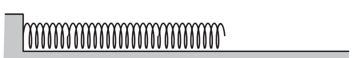



7. Two identical springs each of force constant $k = 10$ N/m are attached at the midpoint of the bottom of an inertia-less cup, one from the inside and the other from the outside. Free ends A and B of the springs are held maintaining the springs vertical. The ends A and B of the springs are made to move simultaneously with constant velocities $v_A = 10.0$ cm/s upwards and $v_B = 8.0$ cm/s downwards respectively. The moment when the ends start moving, the cup starts collecting water at the rate $r = 1.0$ g/s from a hose with a negligible velocity relative to the cup. Find velocity of the cup. Acceleration due to gravity is $g = 10$ m/s$^2$.

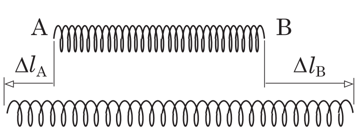
8. An assembly placed on a frictionless floor, consists of two light bars A and B, four light springs and a block C of mass $m$. Stiffness of each springs is shown in the figure. Initially all the springs are relaxed and the bars are parallel to each other. The bar A is pulled from its midpoint towards the right by a force that increases gradually from zero. When the block acquires an acceleration $a$, by what amount will the separation between the bars increase?



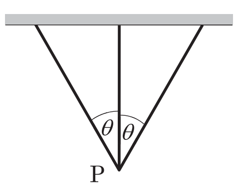

9. A uniform massless spring if extended or compressed, distance between every two consecutive turns (pitch) changes by the same amount. A spring obeying this property and Hooke's law is called a linear spring.



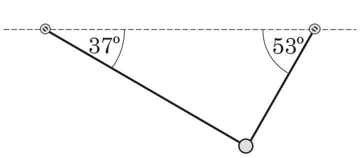
*(a)* A linear spring of relaxed length $l_0 = 30$ cm is attached at one end to a wall. If the other end is pulled away from the wall with a force $F = 60$ N, $p = 6^{th}$ turn from the wall reaches a position, where $q = 8^{th}$ turn of relaxed spring was. Find force constant of the spring.




*(b)* The ends A and B of a linear spring of relaxed length $l_0$ are so pulled that the ends shift by distances $\Delta l_A = 5$ cm and $\Delta l_B = 25$ cm as shown in the figure. Find shift of a point of the spring that was at a distance $l_0/n$ ($n = 3$) from the end A in relaxed spring.



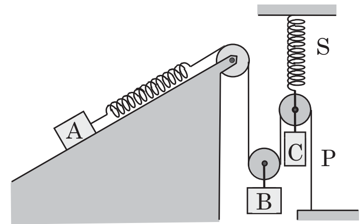

10. Three segments cut from a long elastic light cord are knotted at point P. The other ends of the cords are attached to the ceiling so that all the segments are in a vertical plane and angles between the outer and the middle segments each being $\theta$ as shown in the figure. A load of mass $m$ is suspended from the knot P. If extensions in the cords are negligible as compared to their relaxed lengths, find the tensile force $T$ developed in the middle cord. Acceleration due to gravity is $g$.



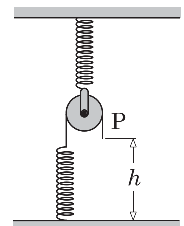

11. A light elastic cord is tied between two nails in the same level 100 cm apart. Distance between the nails is equal to the relaxed length of the cord. A bead is glued somewhere on the cord and then released. When equilibrium is established, elongated sections of the cord make angles 37° and 53° with the horizontal. At what distance from the left nail was the bead glued?

**12.** The system shown in the figure is in equilibrium. The blocks A, B and C are of equal masses, the inclined face is frictionless, the pulleys are light and frictionless, the thread is light and inextensible, the springs are light and the blocks B and C are connected to the axles of the respective pulleys by rigid links. Find accelerations of the blocks A, B and C, immediately after

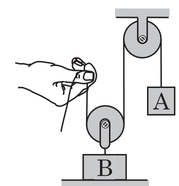
*(a)* the thread is cut at point P. *(b)* the spring S is cut.



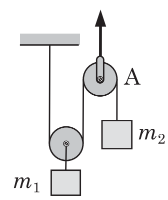

**13.** A light frictionless pulley is suspended with the help of a light spring. One end P of a light inextensible thread that passes over the pulley is free and the other end is tied to another light spring that is affixed to the ground at its lower end as shown in the figure. Stiffness of both the springs is $k$ = 500 N/m. The free end P is $h$ = 10.0 cm above the ground. What minimum pull at the end P will bring it to the ground?



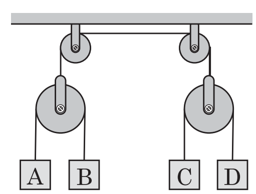

**14.** In the system shown, initially the block A of mass $m$ is hanging at rest and the block B of mass $2m$ is on the ground. The pulleys have negligible masses and negligible friction, and the thread is extremely light and almost inextensible. Acceleration due to gravity is $g$. The free end of the thread is pulled upwards with a constant force. When it acquires a speed $v$, find speeds of both the blocks.



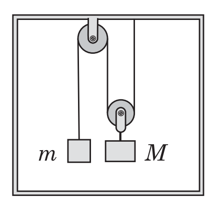

**15.** In the system shown, thread is inextensible, masses of the thread and that of the pulleys are negligible as compared to the loads to be lifted and friction at the axles of the pulleys is absent. Masses of the loads are $m_1$ = 1.0 kg and $m_2$ = 2.0 kg. Acceleration of free fall is $g$ = 10 m/s$^2$. If axle of the pulley A is pulled upwards with a force $F$ = 20 N, how much acceleration will it acquire?



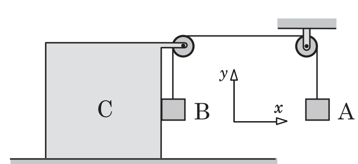

**16.** The system shown in the figure consists of four blocks A, B, C and D of masses $m$, $2m$, $2m$ and $4m$ respectively. The threads are inextensible, masses of the threads and the pulleys are negligible, and friction is absent at the axles of the pulleys. Initially the system is held motionless. Find accelerations of all the blocks after the system is set free. Acceleration of free fall is $g$.



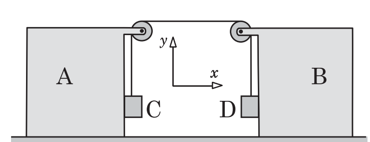

**17.** An arrangement setup inside a lift is shown in the figure. The pulleys and threads are ideal and masses of the blocks are $m$ and $M$ ($M > 2m$). Find minimum acceleration of the lift for which the thread remains taut and both the blocks accelerate in the same direction relative to the ground. Acceleration due to gravity is $g$.

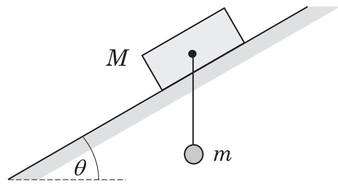



**18.** In the system shown, the blocks A, B and C are of equal mass, the pulleys are ideal and the cord is light and inextensible. There is no friction between the blocks B and C as well as between the horizontal floor and the block C. If the system is set free, find acceleration vectors of all the blocks. Acceleration of free fall is $g$ = 10 m/s$^2$.

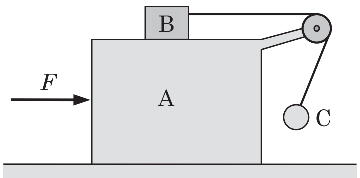



**19.** In the given arrangement, masses of the blocks A, B, C and D are $M_1$, $M_2$, $m_1$ and $m_2$ respectively, pulleys are ideal and cords are light and inextensible. There is no friction between any pair of surfaces in contact. If the system is set free, find $x$-components of accelerations of all the blocks. Acceleration of free fall is $g$ = 10 m/s$^2$.

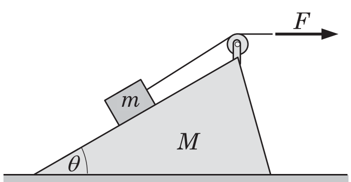




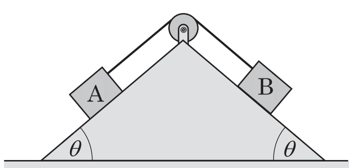

**20.** A ball of mass $m$ is suspended from a bar of mass $M$ with a light inextensible cord. The bar can slide on a frictionless slope of inclination $\theta$. Initially the bar is held motionless so that the ball also stays motionless as shown in the figure. Find accelerations of the bar and of the ball relative to the ground immediately after the bar is released. Acceleration due to gravity is $g$.



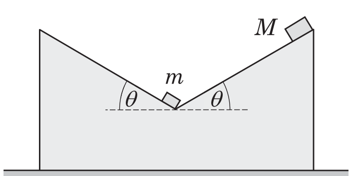

**21.** In the setup shown, magnitude of the force $F$ exerted on the block A is so adjusted that the block B and the ball C remain motionless relative to the block A without contact between A and C. All surfaces in contact are frictionless. Masses of these bodies are $m_A = 12$ kg, $m_B = 5$ kg and $m_C = 3$ kg respectively. Acceleration of free fall is $g = 10$ m/s$^2$. Find expression for the necessary force $F$.



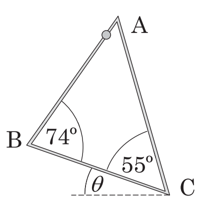

**22.** A block of mass $m$ placed on a wedge of mass $M$ is attached at one end of a light inextensible cord that passes over an ideal pulley affixed on the top of the wedge as shown in the figure. The free end of the thread is pulled by a constant horizontal force $F$. If friction between the block and the wedge as well as between the wedge and the floor is absent, find acceleration of the wedge. Acceleration due to gravity is $g$.




**23.** In the system shown, blocks A and B of masses $m_1$ and $m_2$ ($m_2 > m_1$) are placed on the frictionless inclined surfaces of a triangular wedge of mass $m_0$. The blocks are connected by a light inextensible cord that passes over an ideal pulley affixed at the top of the wedge. The wedge is placed on a horizontal frictionless floor. Initially the system is held motionless and then set free. Find acceleration of the wedge. Acceleration due to gravity is $g$.

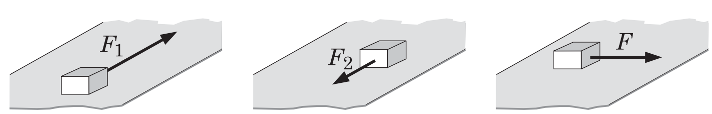



**24.** Top of a wedge made of a very light material has two frictionless inclined planes. A block of mass $m$ is placed at the bottom of the inclined planes and another block of mass $M$ is held motionless at the top of one of the inclined planes as shown in the figure. Find range of values of $m$ in terms of $M$ and the angle of inclination $\theta$ so that when the upper block is released, the lower block starts sliding up the wedge.



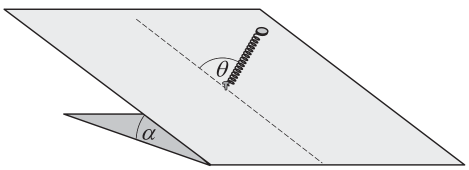

**25.** A rigid triangular frame ABC made of a thin rod is fixed in a vertical plane. Angles between the rod segments at the corner B and C are 74° and 55° respectively. A small bead starting from rest from corner A takes equal time to slide down the arms AB and AC. There is no friction between the bead and the arms. Find angle $\theta$, which the arm BC makes with the horizontal.

**26.** To drag a block up an inclined plane with a constant speed, a force $F_1$ has to be applied along the line of fastest descent, whereas to drag the block down the plane with a constant speed, a force $F_2$ has to be applied along the line of fastest descent as shown in the following figure. Find the minimum horizontal force $F$ parallel to the plane required to slide the block.

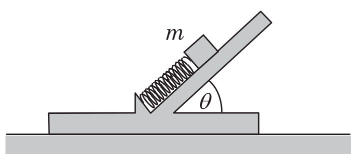


**27.** A small block is kept on an inclined plane of adjustable inclination. Coefficients of static and kinetic frictions between the block and the plane are $\mu_s$ and $\mu_k$ respectively. The angle of inclination of the plane is gradually increased from zero until the block starts sliding. How much speed will the block acquire in sliding down a distance $l$? Acceleration due to gravity is $g$.

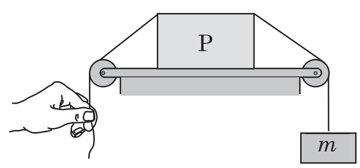
**28.** One end of a spring of stiffness $k$ is attached to a nail O on an inclined plane and the other end to a small disc of mass $m$ placed on the inclined plane. Inclination of the plane with the horizontal is $\alpha$ and coefficient of friction between the disc and the plane is slightly less than $\tan\alpha$. Find suitable expression for deformation $\Delta r$ in the spring when the disc is in equilibrium at an angular position $\theta$ from the line of greatest slope as shown in the figure. Acceleration due to gravity is $g$.




**29.** A catapult placed on horizontal ground can launch blocks at angle $\theta$ above the horizontal with the help of a spring as shown in the figure. Mass of a block to be launched is $m$, mass of the catapult is much smaller than $m$, the maximum force $F$ of the spring on the block is much greater than the gravitation pull of the earth on the block and coefficient of friction between the catapult and the ground is $\mu$. How much maximum horizontal acceleration can the block be imparted by the catapult?



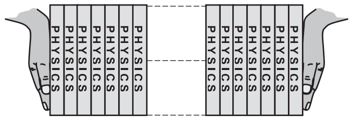

**30.** A light block P on a horizontal tabletop placed equidistant from the pulleys supports a load with the help of a cord as shown in the figure. If the minimum and the maximum forces applied by hand to keep the system in equilibrium are $F_{min} = 40$ N and $F_{max} = 90$ N, calculate mass $m$ of the load. Acceleration due to gravity $g = 10$ m/s$^2$.




**31.** A block given a velocity $u$ on horizontal ground (not frictionless) is observed a distance $s$ away a time $\tau$ later. Find coefficient of kinetic friction between the floor and the block. Acceleration of free fall is $g$.

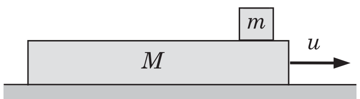
**32.** A block of mass $m$ made of chalk is projected with velocity $u$ along a horizontal floor. Coefficient of friction between the block and the floor is $\mu$. If due to wear, mass of the block decreases at a constant rate $r$ (a unit of mass in every unit of distance travelled), find expression for total distance travelled by the block. Acceleration due to gravity is $g$.

**33.** A block of mass $m = 5.0$ kg is sliding eastwards on a horizontal floor. When its velocity is $u = 8.0$ m/s, in addition to frictional force from the floor a westward force $F$ starts acting on it. Magnitude of this force varies with time according to equation $F = kt$, where $k = 5.0$ N/s and $t$ is time in seconds. Coefficient of friction between the block and the floor is $\mu = 0.3$. Draw a graph to show how the frictional force $f$ on the block varies with time. Acceleration of free fall is $g = 10$ m/s$^2$.

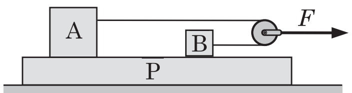



**34.** A boy lifts a stack of several identical books pressing hard with his hands. Coefficient of static friction between hand and a book is $\mu_{hb} = 0.40$, between the books is $\mu_{bb} = 0.25$ and mass of a book is $m = 400$ g. Now the boy starts decreasing the pressure gradually. When the horizontal component of the force applied by the boy becomes $F = 120$ N, the books are about to fall. Acceleration of free fall is $g = 10$ m/s$^2$.

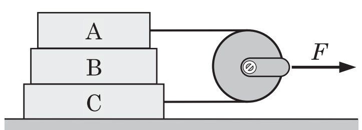
*(a)* How many books are there in the stack?
*(b)* Find frictional force between the third and the fourth book.



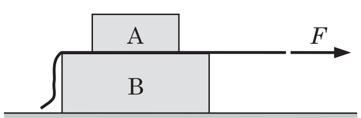

**35.** A large plank of mass $M$ is moving with a velocity $u$ on a horizontal frictionless floor. A block of mass $m$ is gently placed on the plank without any velocity. If the block slides a distance $l$ on the plank before it stops sliding, find coefficient of friction between the block and the plank.




**36.** In the setup shown, blocks A and B of masses 4.0 kg and 1.0 kg are placed on a platform P of mass 1.0 kg that is placed on a horizontal frictionless floor. The blocks are connected by a light inextensible string that passes round an ideal pulley. Coefficients of static and kinetic friction between the blocks and the platform are 0.16 and 0.10. On applying a horizontal force $F$ on the pulley, acceleration of the platform becomes 2.0 m/s$^2$. Find magnitude of the force $F$, frictional forces on the blocks and their accelerations. Acceleration due to gravity is 10 m/s$^2$.

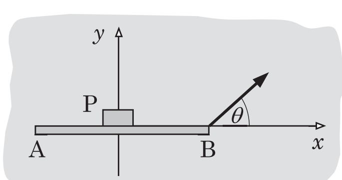



**37.** In the given setup, a bar B is sandwiched between bars A and C that are connected by a light inextensible thread, which passes round an ideal pulley. Mass of each bar is $m$, coefficient of friction between the bars is $\mu$ and the floor is frictionless. Acceleration due to gravity is $g$. If a horizontal force $F$ is applied on the pulley, find acceleration of the bar B.

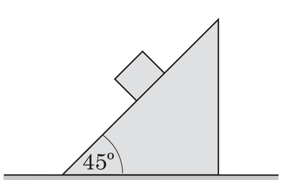



**38.** On a horizontal frictionless floor, a block A of mass $m$ rests on another stationary block B of mass $M$ ($m < M$) with a thin paper sheet of negligible mass inserted between them. The paper sheet is wide enough to prevent direct contact between the blocks. Coefficient of friction between each block and the paper sheet is $\mu$. Find necessary horizontal force $F$ applied on the paper sheet as shown in the figure to fulfil the following conditions. Acceleration due to gravity is $g$.

*(a)* The block B does not slide relative to the paper sheet.
*(b)* The paper sheet slides relative to B but does not slide relative to A.




**39.** A bar AB and a block P are placed on a frictionless horizontal floor with their adjacent vertical faces in contact as shown in the figure by a top view. Here the $x$-$y$ plane of a coordinate frame is in the plane of the floor. The bar is made to move horizontally along the floor in a straight line with a constant acceleration without rotation. Direction of motion of the bar makes an angle $\theta = \sin^{-1}(0.6)$ with the positive $x$-axis. Coefficient of friction between the block and the bar is $\mu = 0.75$. Find distance travelled by the block until the plank moves a distance $l = 40$ cm.




**40.** A block is released on the slant face of a wedge of equal mass placed on a horizontal floor. The slant face of the wedge makes an angle of 45° with the floor. Coefficient of friction between all surfaces in contact is the same. Find range of coefficient of friction $\mu$ for the following cases.

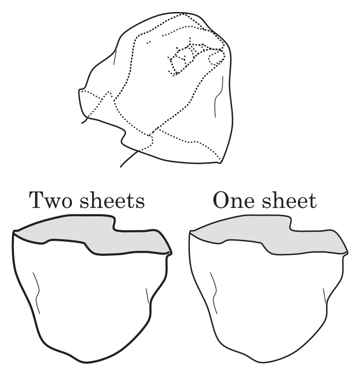
*(a)* Both the bodies remain motionless.
*(b)* The wedge remains motionless and the block slides down.
*(c)* The block slides down the wedge and the wedge slides on the floor.
*(d)* The block does not slide on the wedge but the wedge slides on the floor.

**41.** A wedge of mass $M = 3$ kg rests on a horizontal floor. Slant face of the wedge makes an angle $\theta = \sin^{-1}(0.6)$ with the horizontal. A block of mass $m = 5$ kg is released on the slant face. Coefficient of friction between the floor and the wedge is $\mu = 1/3$ and the slant face is friction-less. Find the force of normal reaction between the block and the slant face. Acceleration of free fall is $g = 10$ m/s$^2$.

**42.** One end of a uniform rope of mass $m = 3.0$ kg placed on the ground is pulled by a force $F = 10\sqrt{2}$ N in such a way that $\eta = 2/3$ of length of the rope remains at rests on the ground in a straight line and rest of the rope in the air in the vertical plane containing the portion of the rope on the ground. Find range of values of coefficient of friction between the rope and the ground. Acceleration of free fall is $g = 10$ m/s$^2$.

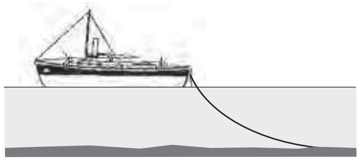
**43.** A train of mass $M = 1000$ ton is running with a uniform velocity on a level track. Resistance to motion is proportional to the mass being pulled and pulling force of the engine is a constant. The last carriage of mass $m = 200$ ton gets decoupled. The driver discovers it after travelling a distance $l = 100$ m and instead of applying brakes, he immediately shuts off the engine. When both the parts of the train come to rest, what is the distance $s$ between them?

**44.** A student starts an experiment with a stack of three identical sheets of paper. He crumples the stack against his fist as shown in the figure, then separates a single sheet from the other two without altering their crumpled (pseudo-conical) shape. This yields two objects of the same shape, but of different masses. He releases these objects simultaneously, and records their motion in still air. After the two objects acquire their terminal velocities, the heavier one moves $\eta = \sqrt{2}$ times farther than the lighter one in a particular time interval. If the force of air resistance is proportional to $v^x$, where $v$ is velocity and $x$ is a constant exponent, what should be value of the exponent $x$?

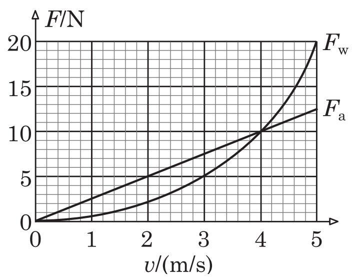



**45.** A ball projected vertically upwards with a velocity 20 m/s returns back on the ground with a velocity 16 m/s. If the air resistance is proportional to the speed of the ball, find airtime of the ball. Acceleration of free fall is 10 m/s$^2$.

**46.** A cricket ball during its motion in the air experiences a drag force proportional to square of its speed relative to the air. Immediately before a batsman hits a ball, the ball was moving horizontally in the air with speed $u = 20$ m/s and acceleration $a = \sqrt{164}$ m/s$^2$. After the hit, the ball starts rising vertically upwards with an initial velocity $v = 10$ m/s. Neglecting buoyant force, calculate acceleration of the ball immediately after the hit? Assume acceleration of free fall $g = 10$ m/s$^2$.



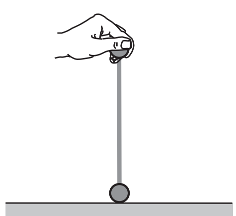
**47.** A boat of mass $m$ is anchored in the middle of a river, which is flowing with a constant velocity $v_0$. Horizontal component of the force exerted on the boat by the anchor chain is $T_0$. If the anchor chain suddenly breaks, determine time required for the boat to attain a velocity equal to $0.5v_0$. Assume that the drag force of the water is proportional to the velocity of the boat relative to the water and neglect air drag.



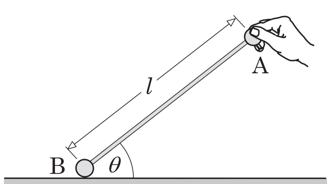

**48.** The given figure shows how forces of air resistance $F_a$ and water resistance $F_w$ on a sailboat vary with velocity $v$ of the boat relative to the respective media.

*(a)* What speed a sailboat will acquire in stagnant water, when the wind speed is 5 m/s everywhere? The direction of motion of the boat coincides with the direction of the wind.

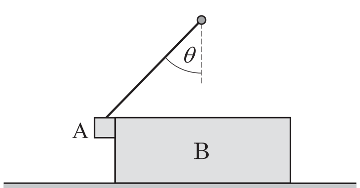
*(b)* What speed a sailboat will acquire in stagnant air, when the water is flowing with velocity 5 m/s everywhere? The direction of motion of the boat coincides with the direction of the water current.



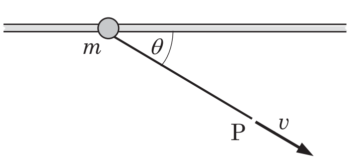
**49.** A dumbbell is constructed by affixing small balls each of mass $m$ at the ends of a light rod of length $l$. The dumbbell is held vertically with the lower ball resting on a horizontal floor as shown in the figure. The upper ball is released and given a horizontal velocity $v_0$ by a sharp hit. If the bottom ball immediately loses contact with the floor, what should the maximum length $l$ of the dumbbell be? Acceleration due to gravity is $g$.



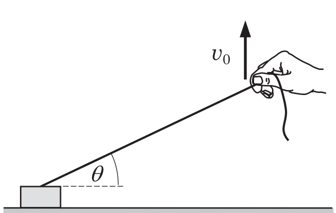

**50.** A dumbbell consists of two very small size balls A and B of masses $m$ and $M$ fixed at the ends of a light rigid rod of length $l$. The dumbbell is held motionless on a horizontal floor making an angle $\theta$ with the floor as shown in the figure. Find range of coefficient of friction $\mu$ between the floor and the ball B, so that the ball starts sliding immediately after the dumbbell is released.




**51.** In the setup shown, a small block A of mass $m$ suspended from a peg with the help of a light inextensible cord rests on a vertical face of a block B of mass $M$, which is held motionless on the horizontal ground. Inclination of the cord with the vertical is $\theta$ and all the surfaces in contact are frictionless. Find acceleration of the block B immediately after it is released. Acceleration due to gravity is $g$.

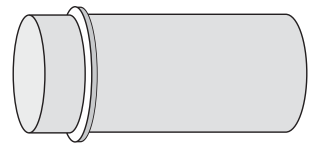



**52.** A bead of mass $m$, which can slide on a straight horizontal frictionless rod, is being pulled with the help of a light inextensible thread of length $l$ in such a manner that the end P of the thread, which is being pulled moves with a constant speed $v_0$ always directed along the thread. Find an expression for the tensile force $T$ in the thread at an instant when the thread makes angle $\theta$ with the rod.

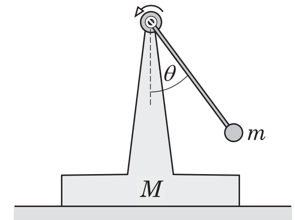



**53.** A light inextensible straight thread of length $l$ = 3.2 m is placed on a frictionless horizontal floor. A small block of mass $m$ = 3 kg placed on the floor is attached at one end of the thread. With how much constant speed $v_0$ the free end of the thread must be lifted vertically upwards so that the block leaves the floor, when the thread makes an angle $\theta = 30°$ with the floor. Acceleration of free fall is $g = 10\ \text{m/s}^2$.

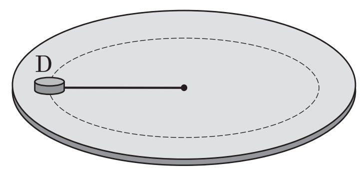
**54.** In a gravity free space a long uniform cylinder of radius $r = 30$ cm wears a thin uniform ring that can slide on the cylinder. When the ring is given an angular velocity $\omega = 10$ rad/s, it stops in $t_0 = 3.0$ s. Now the ring is given the same angular velocity and simultaneously projected along the rod with speed $u = 4.0$ m/s. How far will the ring move along the rod before it stops?



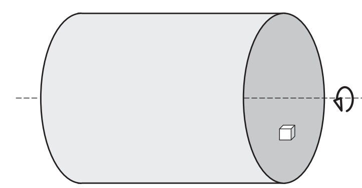
**55.** A motor is installed at the top of a pole rigidly fixed on a platform. A light rod of length $r = 1$ m is rigidly attached to the motor shaft at its one end and at the other end a small ball of mass $m$ is attached. The rod can be rotated in a vertical plane with the help of the motor. Total mass of the platform, the pole and the motor is $\eta = 4$ times the mass of the ball. The motor rotates the rod at a constant angular velocity. The platform is placed on a horizontal surface, where coefficient of friction is $\mu = 1/\sqrt{3}$. At which minimum angular velocity $\omega_0$ of the rod will the platform start sliding?



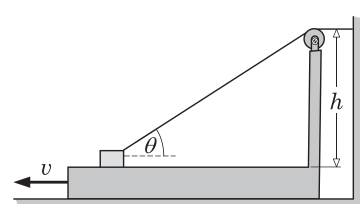

**56.** A disc D of mass $m = 4.0$ kg is tied at one end of a cord of length $l = 1.0$ m, other end of which is fixed at the centre of a large circular platform. The platform is maintained horizontal and rotated about its vertical axis of symmetry. The disc is placed on the platform as shown. How long after the disc is placed, will the cord break? The maximum tensile force the cord can withstand is $T_{\max} = 100$ N, and coefficient of kinetic friction between the disc and the platform is $\mu_k = 0.1$.




**57.** Consider a hollow cylinder of radius $r$ fixed in a laboratory with its axis horizontal. A small block is placed inside the cylinder as shown in the figure. Coefficient of friction between the block and the inner surface of the cylinder is $\mu$. Find the angular velocity at which if the cylinder is rotated about its axis, the block does not slide. Acceleration due to gravity is $g$.

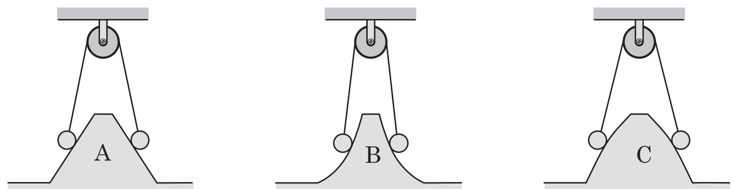


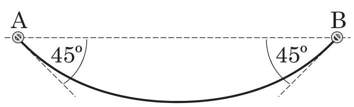
**58.** A small block of mass $m$ is placed on a platform that has a rectangular protrusion. The block is connected to one end of a light inextensible cord that passes over an ideal pulley fixed on the corner of the rectangular protrusion and finally attached to the wall. Segment of the rope between the pulley and the wall is horizontal and is at a height $h$ above the top face of the platform as shown in the figure. The platform is on a horizontal frictionless floor. Coefficient of friction between the block and top face of platform is $\mu$. The platform is pulled away from the wall so that it moves with a constant velocity $v$. Find the force $F$ pulling the platform when segment of the cord between the block and the pulley makes angle $\theta$ with the horizontal. Denote acceleration of free fall by $g$ and assume that the block does not leave the platform.



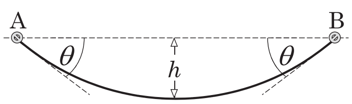
## Check your understanding

1. As shown in the following figure, consider three arrangements A, B and C, where the pulleys are ideal, strings are light and inextensible, balls attached at the ends of the strings are identical and surfaces where the balls rest are frictionless. Identify nature of equilibria (i.e. stable, unstable or neutral) of the arrangements.



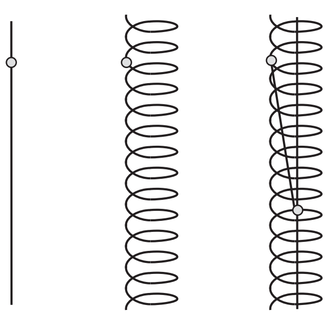




2. A long elastic cord obeying Hooke's law is stretched considerably due to its own weight. In one such case, a cord is held from one of its ends and hung vertically and in another case that is shown in the figure, the ends are attached to separate nails A and B in the same level so that the cord assume shape of a catenary. In which case is the cord stretched longer?

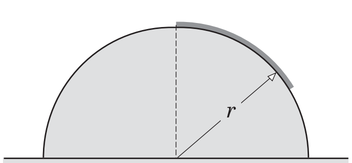



3. A uniform rope of length $l = 12$ m is suspended between two fixed nails A and B that are in the same horizontal level. If the rope makes an angle $\theta = \sin^{-1}(0.6)$ with the horizontal at the nails, find depth $h$ of the lowest point on the rope below the nails and radius of curvature $\rho$ of the rope at the lowest point.

4. If you hang vertically some length of a long uniform inextensible rope, the longest hanging length is $l_0 = 2$ m that does not break due to its own weight. Now you cut a length of the rope and place it straight on a horizontal frictionless table with a little length hanging over the edge so that it begins to slide down when released. What maximum length can this piece have so that it does not break during sliding?

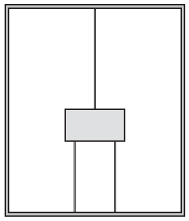


5. A bead slides down a height $h$ on a rigid thin vertical rod in time $T_1$, and the same height on a rigid helix in time $T_2$. Now the rod is rigidly fixed along the axis of the helix and the bead that can slide on the helix is tied with a light inextensible thread to another identical bead that can slide on the rod as shown in the figure. The assembly consisting of the beads and the thread is released from rest. How long will this assembly take to slide down the same height $h$? Neglect friction as well as air resistance and assume radius of the helix so small as compared to the length of the thread that the thread can be assumed almost vertical.




6. A uniform rope of length $l$ is held motionless on a frictionless hemisphere of radius $r$ with one end of the rope at the top of the hemisphere. The hemisphere is made immobile by gluing it on a horizontal floor. Find where on the rope, maximum tensile force is developed immediately after the rope is released.

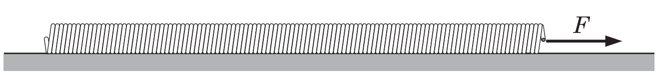
7. A block of mass $m = 15$ kg is suspended in an elevator with the help of three identical light elastic cords stretched vertically. One of them is tied to the ceiling and the other two are tied to the floor of the elevator as shown in the figure. When the elevator is stationary, the tensile force in each of the lower cords is $F = 7.5$ N. Acceleration due to gravity is $g = 10$ m/s$^2$.



   *(a)* For what acceleration of the lift, will the lower cords become relaxed?
   *(b)* If the elevator moves with an upward acceleration $a = 1.0$ m/s$^2$, how much tensile force will develop in the upper cord?
   *(c)* If the elevator moves with an upward acceleration $a = 2.0$ m/s$^2$, how much tensile force will develop in the upper cord?
   *(d)* For what acceleration of the lift will the upper cord become relaxed?

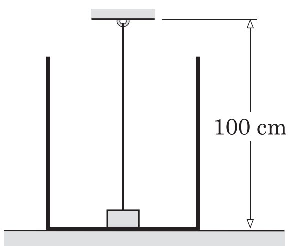
8. An elastic cord that obeys Hooke's law is stretched along the $y$-axis in the $x$-$y$ plane of a coordinate frame so that its ends A and B occupy positions (0, 2) and (0, –8) respectively. A point of the cord that coincides with the origin is marked by a permanent ink; let us call this mark C. Now both the ends are made to move simultaneously; the end A in positive $x$-direction with constant velocity 1.0 m/s and end B in the negative $y$-direction with a constant acceleration. If the mark C passes the point (4, –2), find acceleration of the end B. All the coordinates are in metres.

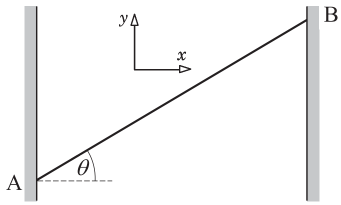
9. Turns of a uniform spring of relaxed length $l = 1.00$ m and force constant k = 500 N/m almost touch each other. A light glue is applied evenly between every adjacent turn. Breaking strenght of the glue is $F_b = 100$ N. The spring is placed on a frictionless horizontal floor and pulled from one of its ends. If the pulling force is gradually increased to a value $F = 200$ N, how much will the length of the spring become?



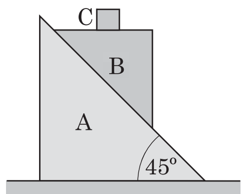

10. Two beetles are hanging from the lower end of a light elastic cord suspended from the ceiling. Stiffness of the cord is $k$, its relaxed length is $l_0$, mass of each beetle is $m$ and acceleration due to gravity is $g$. One of the beetles starts climbing the cord with a constant velocity $u$ relative to the ceiling while the other one keeps on hanging from the lower end. With what velocity and in which direction will the lower beetle be shifted? Ignore all disturbances caused by the initial acceleration of the beetle that is climbing the rope.

11. A small block placed at the bottom of an empty container is attached at its top with one end of an elastic cord of relaxed length 80 cm and force constant 100 N/m. The force constant of the cord increases with decrease in temperature. Its value at 0 °C becomes 4 times of its value at room temperature. The other end of the cord is attached to a fixed hook at height 100 cm above the top of the block as shown in the figure. In this situation, the cord is extended but its elastic force is not sufficient to lift the block. Up to what minimum height cold water of temperature 0 °C should be filled into the container so that the block leaves the bottom. Weight of the block in the cold water is 40 N. Assume no heat conduction in the cord along its length and no heat loss to the surroundings.

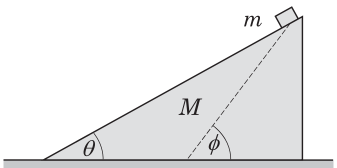




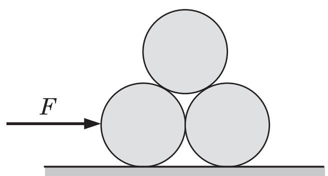

12. A cobweb thread of negligible relaxed length is stretched between points A and B on two opposite walls as shown in the figure. In this state, the thread makes an angle $\theta$ with the horizontal and its length and force constant are $l$ and $k$ respectively. A spider of mass $m$ crawls gradually from end A to B on this thread. Representing the horizontal by $x$-axis, the vertical by $y$-axis and the point A as the origin, find equation of path followed by the spider assuming that the thread obeys Hooke's law.




13. A wedge A of mass $M$ and base angle 45° is placed on a horizontal floor. Another wedge B of the same mass and base angle is set on the wedge A and a small block C of mass $m$ is placed on the upper horizontal face of the wedge B. Initially the system is held motionless as shown in the figure. All the surfaces in contact are frictionless. Find acceleration of the block C after the system is released. Acceleration of free fall is $g$.



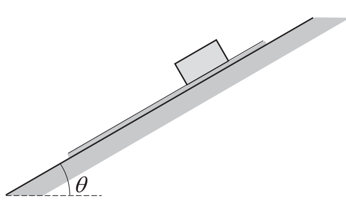

14. The wedge shown can slide without friction on a horizontal floor. Mass of the wedge is $M$ and its angle of inclination is $\theta = 30°$. A block of mass $m$ slides down the wedge without friction when released on its inclined face. If path of the block relative to the ground makes an angle of $\phi = 60°$ with the horizontal, find the ratio of mass of the block to that of the wedge.




15. Three identical cylinders each of mass $m = 10\sqrt{3}$ kg are arranged on a horizontal floor as shown in the figure with their axes horizontal. The floor as well as the cylinders are frictionless. You apply a constant horizontal force (directed to the right) on the left cylinder. For what range of $F$ will all three cylinders remain in contact with each other. Acceleration of free fall is $g = 10$ m/s$^2$.

16. Having lost his way, a hunter reached bank of a frozen river of width $l$ and decided to cross the river. However, due to very low friction he was unable to walk on the icy river. Assume a large horizontal area near the riverbank where friction coefficient between the shoes of the hunter and the ground is $\mu$; deduce suitable expression for the minimum time in which the hunter can reach the other bank. Acceleration of free fall is $g$.

17. A horizontal conveyor belt reverses its velocity almost instantly after every $\tau = 1.0$ s. Speed of the belt in each direction is $v_0 = 1.0$ m/s. Coefficient of friction between the belt and a block placed on the belt depends on the direction sliding. When the block slides to the right on the belt, the coefficient of friction is $\mu_r = 0.25$ and when it slides to the left on the belt, the coefficient of friction is $\mu_l = 0.50$. Find the average velocity of the block relative to the ground during a complete cycle of motion of the belt long time after the block is placed.

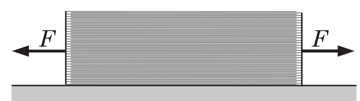



18. On a fixed inclined plane place a thin paper sheet of negligible mass and then a block on the paper and hold them motionless. Coefficient of friction between the block and paper is $\mu_1$ and coefficient of friction between the paper and the incline plane is $\mu_2$. Now release them simultaneously.

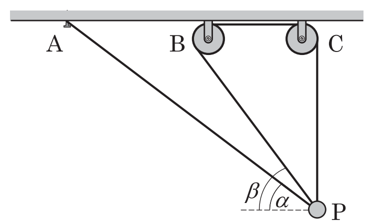
*(a)* If $\mu_2 \geq \mu_1 < \tan\theta$, find accelerations $a_b$ of the block and $a_p$ of the paper.
*(b)* If $\mu_1 > \mu_2 < \tan\theta$, find accelerations $a_b$ of the block and $a_p$ of the paper.

**19.** A well-known experiment to demonstrate property of inertia is to pull out a cloth without letting a glass placed on it to fall. The cloth spreads up to a length $l$ from an edge of a table and the glass is placed on the cloth at a distance $x$ ($x \le l$) from this edge. Coefficient of friction between the cloth and the glass is $\mu$ and that between the glass and the table is sufficient to prevent slipping of the glass on the table. The glass can be considered as a point particle and the cloth light. If the cloth is pulled with a constant speed, find range of this speed for a successful demonstration of the experiment.

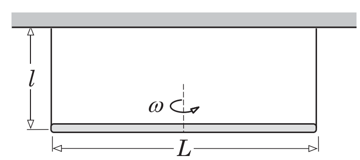
**20.** Two identical books each of $n$ = 400 pages are so meshed that their alternate papers overlap each other. Mass of overlapped portion of each paper is $m$ = 0.5g and coefficient of static friction between the papers is $\mu$ = 0.1. The arrangement is placed on a frictionless horizontal table and the books are pulled apart horizontally without rotation. Find minimum pulling force $F$ required. Acceleration due to gravity is $g$ = 10 m/s$^2$.



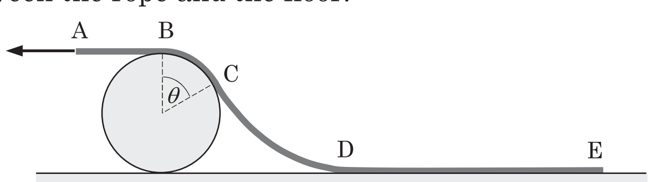

**21.** One end of a light inextensible cord is attached to a nail A on the ceiling. The cord passes through frictionless hole of a small bead P thereafter it passes round two ideal pulleys and finally the other end is attached to the bead. Initially the bead is held at rest with the cord segments AP and BP making angles $\alpha$ and $\beta$ with the horizontal while keeping the segment CP vertical as shown in the figure. Find acceleration of the bead immediately after it is released. Acceleration due to gravity is $g$.



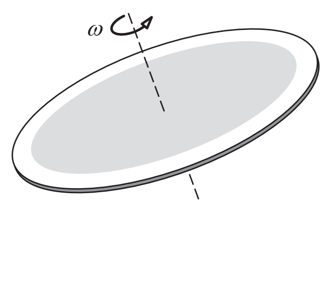

**22.** A rod of mass $m$ and length $L$ is suspended from the ceiling with the help of two light inextensible cords each of length $l$ so that the rod is horizontal. The rod is given an angular velocity $\omega$ about its central vertical axis. Find increment in the tensile force in a cord immediately after the rod is given the angular velocity.

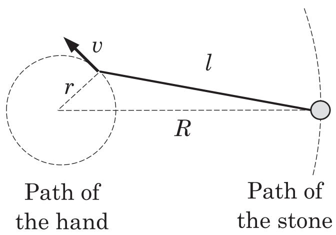



**23.** A cylinder is being pulled slowly with the help of a long uniform rope lying on a horizontal floor as shown in the figure. The pulling force applied at the end A is so adjusted that the length BC of the rope touching the cylinder always subtends angle $\theta = \sin^{-1}(0.8)$ at the centre of the cylinder and the length of the hanging portion CD always remains half of the length DE being dragged on the floor. Find coefficient of friction between the rope and the floor.

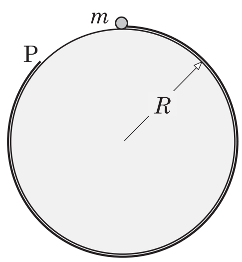



**24.** A disc of radius $R$ inclined at an angle $\theta$ with the horizontal is evenly covered with a thin layer of sand. Coefficient of friction between the sand and the disc is $\mu$. The disc is made to rotate about its central axis of symmetry with gradually increasing angular velocity. The angular velocity is increased so gradually that tangential acceleration of any of the sand particle is practically negligible. What is the angular velocity of the disc, when $\eta$ fraction of the sand has fallen off the disc? Acceleration due to gravity is $g$.

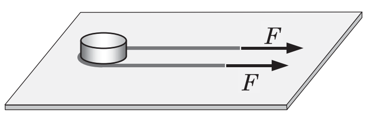





**25.** To whirl a stone tied to a cord, one has to move the free end of the cord on a circular path pulling the stone on a larger circular path. In this way a stone of mass $m$ is whirled on a horizontal circular path of radius $R$ with the help of a light inextensible cord of length $l$ by moving the free end on a circular path of radius $r$ with a uniform speed $v$ as shown in the figure. At sufficiently great speed $v$, tensile force in the cord becomes so large that the effect of gravity can be neglected but at this speed air resistance becomes considerable. Find suitable expression for the force of air resistance. Acceleration due to gravity is $g$.




**26.** A light inextensible cord is wrapped around a frictionless cylinder of radius $R$ fixed vertically on a floor that is lubricated with oil. A bead of mass $m$ is tied at one end of the cord and the other end P is free as shown by the top view of the situation. Force of viscous drag on the bead due to layer of the lubricating oil is proportional to the speed of the bead and the proportionality constant is $k$. If the free end of the cord is pulled with a constant speed $u$ along the cylinder, the bead eventually acquires a steady motion on a circular path. Find expressions for speed of the bead and radius of its circular path in its steady motion.




**27.** A uniform rope of mas $m$ and length $l$ is spread out on a horizontal frictionless surface, wrapping half turn around a fixed vertical frictionless cylinder of radius $r << l$. Initially, both the ends of the rope are pulled by equal forces each $F$ in magnitude keeping both the unwrapped segments parallel to each other and equal in length as shown in the figure. Suddenly pulling force at one end is removed without any change in the pulling force at the other end. After how much time from removal of the pulling force at an end, will the rope start losing contact with the cylinder?

**28.** A small disc of mass $m$ = 100 $g$ is placed on a large horizontal floor lubricated with oil. Force of viscous drag due to layer of lubricating oil is given by equation $\vec{f} = -k\vec{v}$, here $\vec{v}$ is velocity of the disc and $k$ = 0.04 kg/s. If a force of constant magnitude $F$ = 0.06 N rotating with a constant angular velocity $\omega$ = 0.3 rad/s is applied on the disc, the disc eventually acquires a steady motion on a circular path. Find speed of the disc in its steady motion.

**29.** A small bead starts sliding down a frictionless rigid helical wire frame. The axis of the helix is vertical, radius is $R$ and pitch is $h$.

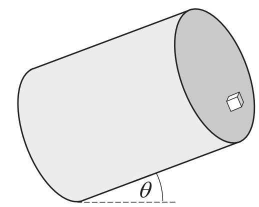
*(a)* In how much time, will the bead descend a height $H$?

*(b)* How much force will the wire frame exert on the bead when the bead has descended a height $H$? Acceleration of free fall $g$.

**30.** A small bead slides down a helical wire frame with a constant speed $v$ due to gravity. Axis of the helix is vertical, its turns are tilted by angle $\theta$ with the horizontal and its radius is $R$. Find expression for the coefficient of kinetic friction between the wire and the bead.

**31.** Starting from rest a motorcyclist moves on a horizontal un-banked circular track of radius $R$ = 28 m. Coefficients of friction between the tyres and the track is uniform everywhere. How much minimum distance must the motorcycle cover to achieve the maximum allowable speed? Treat the motorcycle as a point particle and use $\pi = 22/7$.

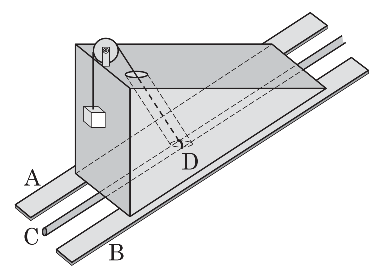
32. A small block is placed inside a hollow cylinder of radius $r$, axis of which is inclined at an angle $\theta$ with the horizontal as shown in the figure. Coefficient of friction between the block and the inner surface of the cylinder is $\mu$. Find angular velocity of the cylinder about its axis, so that the block does not slide. Acceleration due to gravity is $g$.




33. A block is placed on a large inclined plane that makes an angle $\theta$ with the horizontal. Coefficient of kinetic friction between the block and the plane is $\mu > \tan\theta$. The block is given a velocity $v_0$ at an angle $\phi_0$ with the line of fastest descent. Find expression for the time interval during which the block remains in motion. Acceleration due to gravity is $g$.

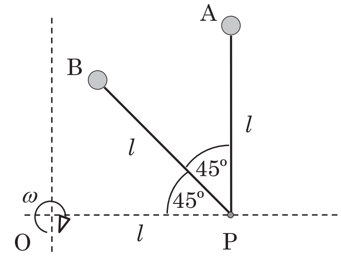
## Challenge your understanding

1. A wedge of mass $M$ can slide on two parallel rails A and B inclined at an angle $\theta$ to the horizontal. In the middle of the rails and parallel to them runs a frictionless rod C that wears a light ring D. The rod does not touch the wedge. A light inextensible cord attached with the ring runs through a hole in the wedge without touching it, then runs round an ideal pulley on the top of the wedge and finally attached to a block of mass $m$ that can slide along the vertical face of the wedge without friction as shown in the figure. Coefficient of friction between the wedge and the rails is $\mu$. Initially the system is held motionless and then released. Find the acceleration of the block. Acceleration due to gravity is $g$.




2. A boy is driving a model aircraft with the help of a light inextensible cord of length $L$. He does so by moving the free end of the cord with a constant speed in a horizontal circle of radius $r$. As a result, the aircraft moves in another horizontal circle of radius $R > r$ with a constant speed $v$ at an altitude $h$ above the plane containing circular path of the end held. The centres of both the circular paths lie on a vertical line. Considering the air drag, find lift force of air on the aircraft.

3. Two small identical discs A and B are tied to a nail P on a large horizontal platform with the help of identical light inextensible strings each of length $l = 25\sqrt{2}$ cm. The nail is fixed at distance $l$ from the centre O of the platform. Initially the threads tying the discs A and B are straight and make angles $\theta_0 = 45°$ as shown in the figure. Coefficient of friction between the discs and the platform is $\mu = 0.40$. Acceleration of free fall is $g = 10$ m/s$^2$. Now the platform starts rotating with gradually increasing angular velocity $\omega$ about its vertical central axis. Find expression to describe angle $\theta$ between the threads PA and PB as function of angular velocity of the platform and draw an approximate graph to show this relationship.



# Answers and Hints

## Multiple Choice Questions

1. *(d)*
2. *(d)*
3. *(b)*
4. *(b)*
5. *(a)*, *(b)* and *(d)*
6. *(c)*
7. *(c)*
8. *(c)*
9. *(b)* and *(d)*
10. *(c)* and *(d)*
11. *(b)*
12. *(a)*
13. *(a)*
14. *(d)*
15. *(a)*
16. *(a)*
17. *(c)* and *(d)*
18. *(c)*
19. *(b)*
20. *(a)* and *(d)*
21. *(b)*
22. *(c)*
23. *(a)*, *(b)*, *(c)* and *(d)*
24. *(a)*, *(c)* and *(d)*
25. *(d)*
26. *(b)*
27. *(b)*
28. *(d)*
29. *(a)*
30. *(a)* and *(b)*
31. *(c)*
32. *(d)*
33. *(d)*
34. *(b)*
35. *(d)*
36. *(d)*
37. *(a)*
38. *(b)*, *(c)* and *(d)*
39. *(a)* and *(b)*
40. *(b)*
41. *(a)*→*(q)* and *(r)*, *(b)*→*(t)*, *(c)*→*(s)* *(d)*→*(r)*
42. *(b)* and *(c)*
43. *(c)*
44. *(c)*
45. *(b)*
46. *(a)*
47. *(c)*
48. *(c)*
49. *(c)*
50. *(b)*

## Build-up your understanding

1. $\left(-\hat{i}-\hat{j}+\hat{k}\right)$ N

2. *(a)* $s$ *(b)* $\dfrac{10}{\sqrt{3}}$ m/s$^2$

3. $\dfrac{\left(W_{\text{p}}+W_{\text{b}}\right)l}{2r\sqrt{l^2-x^2}}=200$ N/m

   Hint: Since the pulley is ideal and the string is light, modulus of tensile force at every point of the string in contact with the pulley is the same. Therefore, modulus of normal reaction between the pulley and the string at every point in contact must be uniform in magnitude.

4. $\dfrac{F}{2\left|\overrightarrow{OP}\right|}=100$ N/m

   Hint: To simplify calculations make use of vector notations.

5. 4.0 g/s

6. 300 g

   Hint: Compression in the spring equals the difference of the downward shift of disc A say $h$ and compression $x$ of the rubber pad. Express the spring force that is equal to the weight $mg$ in terms of $h$ and $x$. Substitute given values in the equation obtained and plot it on the given graph.

7. $\dfrac{k\left(v_{\text{A}}-v_{\text{B}}\right)-rg}{2k}=0.95$ cm/s ↑

   Hint: Since the cup is inertia-less, resultant of all the forces acting on it must be a null vector.

8. $x=\dfrac{mak_2}{2k\left(k_1+k_2\right)+k_1k_2}$

9. *(a)* $k=\dfrac{pF}{(q-p)l_0}=600$ N/m

   Hint: Uniformity of change in length of every turn suggests that the fractional change in the length i.e. longitudinal strain of any portion of the spring is equal to that of the whole spring.

*(b)* $\Delta x = \dfrac{\Delta l_B - (n-1)\Delta l_A}{n} = 5 \text{ cm}$

In the direction of $\Delta \vec{l}_B$

Hint: Displacement of a point P on a linear spring relative to one of its ends is proportional to the length of the segment of the spring between the point P and that end of the spring.

10. $T = \dfrac{mg}{1 + 2\cos^3\theta}$

Hint: Since product of force constant of a segment of an elastic cord or spring and length of the segment is a constant, therefore force constant of the middle cord differs from that of the outer cords.

11. 59.17 cm

12. *(a)* $g/2$ down the plane, $g\downarrow, g\uparrow$

*(b)* 0, 0, $2g\downarrow$

Hint: No material body at rest can change its position by a finite amount in an infinitesimally small interval of time due to its inertia. Therefore, force of a spring connected with stationary material bodies at both of its ends cannot change by a finite amount in an infinitesimally small interval of time.

13. $\dfrac{kh}{5} = 10.0 \text{ N}$

Hint: Since the pulley has vanishingly small inertia, net forces on it must be vanishingly small; in addition, the pulley can acquire a finite velocity as well as can be stopped almost instantaneously.

14. $v_A = v_B = \dfrac{v}{3}$

Hint: While the free end is speeding up, accelerations of the blocks are equal, therefore both the blocks will always have equal speeds.

15. $\dfrac{3g}{2} - \dfrac{(m_1 + 4m_2)F}{4m_1m_2} = 0 \text{ m/s}^2$

16. $a_A = \dfrac{7g}{9}\uparrow;\ a_B = a_C = \dfrac{g}{9}\downarrow;\ a_D = \dfrac{5g}{9}\downarrow$

17. $a > \dfrac{(M-2m)g}{6m}\uparrow$ and $g > a > \dfrac{2(M-2m)g}{3M}\downarrow$

18. $\vec{a}_A = -2\hat{j} \text{ m/s}^2,\ \vec{a}_B = \left(4\hat{i} - 2\hat{j}\right) \text{ m/s}^2,\ \vec{a}_C = 4\hat{i} \text{ m/s}^2$

19. For blocks A and C:

$$\frac{2g\hat{i}}{(M_1 + m_1)\left(\dfrac{1}{M_1 + m_1} + \dfrac{1}{M_2 + m_2} + \dfrac{1}{m_1} + \dfrac{1}{m_2}\right)}$$

For blocks B and D:

$$\frac{-2g\hat{i}}{(M_2 + m_2)\left(\dfrac{1}{M_1 + m_1} + \dfrac{1}{M_2 + m_2} + \dfrac{1}{m_1} + \dfrac{1}{m_2}\right)}$$

20. $\dfrac{(m+M)g\sin\theta}{M + m\sin^2\theta}$ down the slope

and $\dfrac{(m+M)g\sin^2\theta}{M + m\sin^2\theta}\downarrow$

Hint: The bar, which accelerates from rest, cannot change its position in a vanishingly small time due to its inertia, hence immediately after the bar is released the cord remains vertical and tensile force in it decreases causing the ball to accelerate vertically downwards. Moreover, vertical component of acceleration of the bar cannot exceed the acceleration of free fall; the inextensible cord remains taut making acceleration of the ball equal to the vertical component of acceleration of the bar.

21. $F = \dfrac{(m_A + m_B + m_C)m_C g}{\sqrt{m_B^2 - m_C^2}} = 150 \text{ N}$

22. $\dfrac{F(1-\cos\theta) + mg\sin\theta\cos\theta}{M + m\sin^2\theta}\rightarrow$

23. $\dfrac{(m_2 - m_1)g\sin\theta\cos\theta}{m_0 + (m_1 + m_2)\sin^2\theta}\rightarrow$

24. $m < M\cos 2\theta$

25. 19°

26. $F = \sqrt{F_1F_2}$

27. $\sqrt{\dfrac{2(\mu_s - \mu_k)gl}{\sqrt{1+\mu_s^2}}}$

28. $\Delta r = \dfrac{2mg\sin\alpha}{k}\cos\theta$

29. $a_{max} = \dfrac{\mu F}{m\sqrt{1+\mu^2}}$

30. $m = \dfrac{\sqrt{F_{min}F_{max}}}{g} = 6$ N

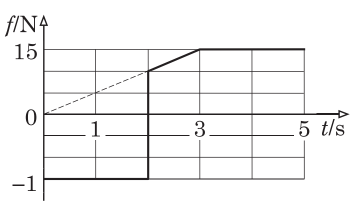
Hint: Limiting friction depends on the normal reaction, therefore a change in the pull on the rope changes the normal reaction and hence the limiting friction.

31. $\mu = \begin{cases} \dfrac{u^2}{2gs}; & 0 \le s \le \dfrac{u\tau}{2} \\ \dfrac{2(u\tau - s)}{g\tau^2}; & \dfrac{u\tau}{2} \le s \le u\tau \end{cases}$

And $s > u\tau$ is impossible.

32. $x = \dfrac{m}{r}$ or $x = \dfrac{u^2}{2\mu g}$ whichever is smaller

Hint: The block slides with a uniform retardation and its size gradually reduces due to wear. Therefore, it may completely vanish before it stops or may stop before it completely vanishes.

33. 


34. (a) $2\left(1+\dfrac{\mu_{bb}F}{mg}\right) = 17$

Hint: The limiting friction between the hand and book is greater than that between any two books, therefore slipping will occur between the end books and the stack of the inner books.

(b) 22 N

35. $\mu = \dfrac{Mu^2}{2gl(m+M)}$

36. 18 N, 1.0 N, 1.0 N, 2.0 m/s² and 8.0 m/s² respectively

Hint: Since pull of string on both the blocks are equal and limiting friction on the block B is smaller than that on block A, the block B slides whereas block A does not slide on the plank under the given conditions.

37. $= \begin{cases} \dfrac{F}{3m}; & F \le 6\mu mg \\ \dfrac{F+2\mu mg}{4m}; & 6\mu mg \le F \le 10\mu mg \\ 3\mu g; & F \ge 10\mu mg \end{cases}$

38. (a) $F < \mu mg\left(1+\dfrac{m}{M}\right)$

(b) $\mu mg\left(1+\dfrac{m}{M}\right) < F < 2\mu mg$

Hint: The paper is almost massless, therefore the net force on it must be vanishingly small.

39. $s = l\sin\theta\sqrt{1+\mu^2} = 0.3$ cm

Hint: Friction is not sufficient to prevent slipping between the block and the bar and normal reaction between the bar and the block is a constant, therefore a constant total contact force (resultant of the normal reaction from the bar and kinetic friction) acts on the block by the bar. In addition, starting from rest, the block moves in a straight line.

40. (a) $\mu > 1.0$ (b) $\sqrt{5}-2 < \mu < 1.0$

(c) $\mu < \sqrt{5}-2$ (d) The event is not possible.

41. $N = \dfrac{mMg(\cos\theta + \mu\sin\theta)}{M + m\sin\theta(\sin\theta - \mu\cos\theta)} = 37.5$ N

Hint: The wedge will also accelerate.

42. $\mu \ge \dfrac{1}{\eta}\sqrt{\left(\dfrac{F}{mg}\right)^2 - (1-\eta)^2} = \dfrac{1}{2}$

Hint: Horizontal component of the force $F$ balances the frictional force, which cannot exceed the limiting friction. Vertical component of the force $F$ equals the weight of the hanging portion.

43. $s = \dfrac{Ml}{M - m} = 125 \text{ m}$

44. 2

45. 3.6 s

46. $g + \left(\dfrac{v}{u}\right)^2 \sqrt{a^2 - g^2} = 12 \text{ m/s}^2$

Hint: Immediately before the ball is hit, force of air resistance and of gravity are mutually perpendicular and after the hit, both the forces are vertically downward.

47. $\dfrac{mv_0 \ln 2}{T_0}$

48. *(a)* 3.0 m/s

Hint: As the boat speeds up, its velocity relative to water increases and that with respect to wind decreases. Therefore, curve representing the air resistance must be inverted about a suitable line parallel to the ordinate to make it suitable for decreasing speed relative to air.

*(b)* 2.0 m/s

Hint: As the boat speeds up, its velocity relative to air increases and that with respect to water decreases. Therefore, curve representing the water resistance must be inverted about a suitable line parallel to the ordinate to make it suitable for decreasing speed relative to water.

49. $\dfrac{v_0^2}{2g}$

Hint: Immediately after the hit, the upper ball is in circular motion around the lower ball.

50. $\mu < \dfrac{m \sin 2\theta}{2(M + m \sin^2 \theta)}$

51. $\dfrac{mg \sin\theta \cos\theta}{m + M \cos^2\theta}$

Hint: The block A will move on circular path and initially it has only tangential acceleration, whose horizontal component is equal to the acceleration of block B.

52. $T = \dfrac{mv_0^2 \sin^2\theta}{l \cos^4\theta}$

53. $v_0 = \left(\sqrt{\dfrac{gl}{\sin\theta}}\right)\cos^2\theta = 6 \text{ m/s}$

Hint: When the block is leaving the floor, vertical component of the tensile force balances weight of the block and horizontal component provides necessary acceleration to the block.

54. $x = \dfrac{ut_0\sqrt{u^2 + (\omega r)^2}}{2\omega r} = 10 \text{ m}$

Hint: Friction at a point of the ring acts opposite to velocity of the point relative to the rod.

55. $\omega_0 = \sqrt{\dfrac{\mu(1+\eta)g}{r\sqrt{1+\mu^2}}} = 5 \text{ rad/s}$

56. $t = \dfrac{1}{\mu_k g}\sqrt{\dfrac{T_{max}}{ml}} = 5.0 \text{ s}$

Hint: The tensile force in the cord and the frictional force provide the centripetal and the tangential accelerations respectively.

57. $\omega \geq \sqrt{\dfrac{g\sqrt{1+\mu^2}}{\mu r}}$

58. $F = m\left\{\dfrac{v^2 \tan^3\theta + \mu gh}{h(\cos\theta + \mu \sin\theta)} - \dfrac{v^2 \tan^3\theta}{h}\right\}$

## Check your understanding

1. Arrangements A and B are in stable, whereas C is in unstable equilibrium.

2. In the second case.

Hint: You may use the idea of average tensile force developed in the cord.

3. $h = \dfrac{l(1 - \cos\theta)}{2\sin\theta} = 2 \text{ m}, \quad \rho = \dfrac{l \cot\theta}{2} = 8 \text{ m}$

4. $4l_0 = 8$ m

Hint: Tensile force in the sliding rope acquires its maximum value at the edge of the table when half of the rope has slid.

5. $\sqrt{\dfrac{T_1^2 + T_2^2}{2}}$

6. From the top at angular position

$$\sin^{-1}\left[\frac{r}{l}\left\{1-\cos\left(\frac{l}{r}\right)\right\}\right]$$

7. *(a)* $\dfrac{3F}{m} = 1.5 \text{ m/s}^2 \uparrow$

*(b)* $m\left(g+\dfrac{a}{3}\right)+2F = 170$ N

*(c)* $m(g+a) = 180$ N

*(d)* $3g + \dfrac{6F}{m} = 33 \text{ m/s}^2 \downarrow$

8. 0.8 m/s²

Hint: Ratio of lengths of segments AC and CB as well as ratios of their $x$ and $y$-components is a constant.

9. $l + \dfrac{F^2 - F_b^2}{2kF} = 1.15$ m

10. $\dfrac{mgu}{kl_0 + 2mg} \uparrow$

11. 60 cm

Hint: Denoting the relaxed length of the cord by $l$, its force constant by $k$, height of the hook above the block by $H$, relaxed length of the portion of cord above water by $l_1$ and height of water level by $h$, the following equations can be written.

For portion of string above water:

$$\frac{kl}{l_1}(H-h-l_1) = F$$

For portion of string under water:

$$\frac{4kl}{(l-l_1)}(h-l+l_1) = F$$

12. $y = \left(\tan\theta - \dfrac{mg}{kl\cos\theta}\right)x + \dfrac{mg}{kl^2\cos^2\theta}x^2$

13. $\dfrac{2(m+M)g}{(3M+2m)}$

Hint: Vertical component of acceleration of wedge B is equal to acceleration of block C and horizontal component of acceleration of wedge B is equal in magnitude and opposite in direction to acceleration of wedge A.

14. $\dfrac{m}{M} = \dfrac{\tan\phi - \tan\theta}{\tan\theta} = 2$

Hint: Express $\tan\phi$ in terms of acceleration of the block relative to the wedge $a$ and acceleration of the wedge $b$.

$$\tan\phi = \frac{a\sin\theta}{a\cos\theta - b}$$

Since there is no net force in horizontal direction on the system, you can write

$$m(a\cos\theta - b) = Mb$$

15. $\dfrac{mg}{\sqrt{3}} \le F \le mg\sqrt{3} \Rightarrow 100 \text{ N} \le F \le 300 \text{ N}$

Hint: For the given condition, the force $F$ has a range of values. At its minimum value, the lower cylinders tend to lose contact from each other and at the maximum value, the upper cylinder tends to lose contact from the front cylinder.

16. $2\sqrt{\dfrac{(\sqrt{2}+1)l}{\mu g}}$

17. $\dfrac{v_0^2}{\tau g}\left(\dfrac{1}{\mu_r} + \dfrac{1}{\mu_1}\right) = 0.2 \text{ m/s} \rightarrow$

18. *(a)* $a_b = g(\sin\theta - \mu_1\cos\theta)$ and $a_p = 0$

*(b)* $a_b = a_p = g(\sin\theta - \mu_2\cos\theta)$

Hint: The paper sheet is almost massless, therefore, the net force on it must be vanishingly small.

**19.** $v_c : \begin{cases} v_c > \sqrt{\dfrac{\mu g l^2}{2x}}; & x < \dfrac{l}{2} \\ v_c > \sqrt{2\mu g(l-x)}; & x > \dfrac{l}{2} \end{cases}$

**Hint:** For successful demonstration of the experiment, average velocity of the glass should not be greater than half the constant velocity of the cloth and hence displacement of the glass cannot exceed half the displacement of the cloth.

**20.** $F \geq n(2n-1)\mu mg = 39.9$ N

**Hint:** One paper has two pages.

**21.** $a = g\sin\theta$

Here $\theta = \tan^{-1}\left(\dfrac{\cos\alpha + \cos\beta}{1+\sin\alpha+\sin\beta}\right)$

**22.** $\dfrac{m\omega^2 L^2}{8l}$

**23.** $\mu = \dfrac{\cot\theta}{2} = \dfrac{3}{8}$

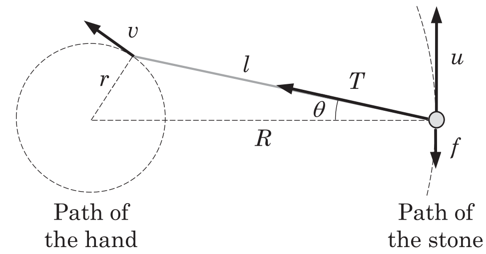
**Hint:** The event described in the problem is possible only when, either the rope slides on the cylinder and the cylinder rolls on the floor or the cylinder slides on the floor without rotation and the rope does not slide on the cylinder, making the whole rope in state of pure translational motion. In addition, at the point C, vertical component of the tensile force equals the weight of the portion CD and horizontal component equals the force of kinetic friction on the portion DE.

**24.** $\omega = \sqrt{\dfrac{g(\mu\cos\theta - \sin\theta)}{R\sqrt{1-\eta}}}$

**Hint:** Sand particle at the lowest position has maximum tendency to slide off the disc.

**25.** $\dfrac{mv^2R}{r^2}\sqrt{\dfrac{4R^2l^2}{(R^2+l^2-r^2)^2} - 1}$

**Hint:** Components of the tensile force in the cord towards the centre of the path of the stone provides the necessary centripetal force and the component along the tangent to the path counterbalances the force of air resistance.




**26.** $u\sqrt{1+\left(\dfrac{mu}{kR}\right)^2}$ and $R\sqrt{1+\left(\dfrac{mu}{kR}\right)^2}$

**27.** $\sqrt{\dfrac{ml}{3F}\left(1-\dfrac{\pi r}{l}\right)}$

**28.** $\dfrac{F}{\sqrt{(m\omega)^2 + k^2}} = 1.2$ m/s

**29.** *(a)* $\sqrt{\dfrac{2H(4\pi^2R^2+h^2)}{gh^2}}$

*(b)* $\dfrac{2\pi mgR\sqrt{4\pi^2R^2+h^2+16\pi^2H^2}}{(4\pi^2R^2+h^2)}$

**30.** $\dfrac{gR\tan\theta}{\sqrt{g^2R^2+v^4\cos^2\theta}}$

**31.** $\dfrac{\pi R}{4} = 22$ m

**32.** $\geq \begin{cases} \sqrt{\dfrac{g\sqrt{1+\mu^2}}{\mu r}}; & \mu > \tan\theta \\ \sqrt{\dfrac{g(\mu\cos\theta+\sin\theta)}{\mu r}}; & \mu < \tan\theta \end{cases}$

**33.** $\dfrac{v_0(\mu\cos\theta + \sin\theta\cos\phi_0)}{g(\mu^2\cos^2\theta - \sin^2\theta)}$

**Hint:** Assume line of the fastest descent as $y$-axis and horizontal as $x$-axis and express tangential and $y$-component of accelerations. Integrate these equations over the limits considering speed change from $v_0$ to zero and time change from zero to the desired time.

## Challenge your understanding

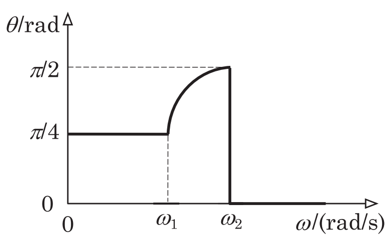
1. $$= \begin{cases} g\left\{\dfrac{(m+M)(\sin\theta-\mu\cos\theta)-\mu m}{m+M-\mu m\sin\theta}\right\}; & \mu < \dfrac{(m+M)\sin\theta}{m+(m+M)\cos\theta} \\ 0; & \mu > \dfrac{(m+M)\sin\theta}{m+(m+M)\cos\theta} \end{cases}$$

2. $$m\left(g+\frac{2v^2h}{R^2+L^2-r^2-h^2}\right)$$

3. $$\theta = \begin{cases} \dfrac{\pi}{4}; & 0<\omega<\omega_1 \\ \dfrac{\pi}{4}+\sin^{-1}\left(\dfrac{\mu g}{l\omega^2}\right); & \omega_1<\omega<\omega_2 \\ 0; & \omega_2<\omega \end{cases}$$

Here $\omega_1 = \sqrt{\dfrac{\mu g}{l}} = \sqrt{8\sqrt{2}}$ rad/s and

$$\omega_2 = \sqrt{\frac{\mu g}{l\sin\theta_o}} = 4 \text{ rad/s}$$




**Hint:** Since the discs can move on a circular path of radius $l$ with centre at the point P, therefore frictional force on them can only be along the tangent to this path.

Conditions of motion of the discs can be analysed with more convenience relative to a frame attached with the platform. Now you can find angular velocities $\omega_1$ and $\omega_2$ at which the disc A and B begins to slide respectively.

Till the angular velocity of the disc reaches the value $\omega_1$ none of the discs slide and angular separation between them remains $\pi/4$ radians. Since $\omega_1 < \omega_2$, the disc A begins to slide first and very slowly the thread attached to it turns through angle $\pi/4$ radians in clockwise direction and remains there almost motionless. When the angular velocity of the platform reaches value $\omega_2$, the disc B begins to slide and catches the disc A thereafter both of them move together.