EXERCISE-1

Single Correct Choice Type

1. Solution set of the inequality, $2-\log_2(x^2+3x)\geq 0$ is :
(a) $[-4, 1]$
(b) $[-4,-3)\cup(0, 1]$
(c) $(-\infty,-3)\cup(1,\infty)$
(d) $(-\infty,-4)\cup[1,\infty)$

2. For $N>1$, the product $\frac{1}{\log_2 N}\cdot\frac{1}{\log_N 8}\cdot\frac{1}{\log_{32} N}\cdot\frac{1}{\log_N 128}$ simplifies to :
(a) $\frac{3}{7}$
(b) $\frac{3}{7\ln 2}$
(c) $\frac{3}{5\ln 2}$
(d) $\frac{5}{21}$

3. For $0<A<\frac{\pi}{2}$, the value of $\log_{100}\left(\frac{\cot^2 A\cdot\cos^2 A}{\cot^2 A-\cos^2 A}\right)$ is equal to :
(a) 0
(b) 1
(c) 2
(d) 3

4. Number of values of x satisfying the equation $5\cdot 3^{\log_3 x}-2^{1-\log_2 x}=3$, is:
(a) 0
(b) 1
(c) 2
(d) 3

5. The sum $\sum_{K=1}^{99}\log\sqrt[3]{\frac{K}{K+1}}$ is equal to :
(Assume base of logarithm is 10)
(a) 2
(b) -2
(c) $\frac{-2}{3}$
(d) $\frac{2}{3}$

6. If $\log_2(a^3 b)=x$ and $\log_2\left(\frac{3a}{b}\right)=y$, then the value of $\log_2 a$ is equal to :
(a) $\frac{x+y}{4}-\log_2\sqrt[4]{3}$
(b) $\frac{x+y}{4}$
(c) $\frac{x-y}{4}$
(d) $\frac{x-y}{4}-\log_2\sqrt[4]{3}$

7. If $b=\sqrt{3+2\sqrt{2}}+\sqrt{3-2\sqrt{2}}$, then the value of $\log_b 64$ is equal to :
(a) 3
(b) 4
(c) 5
(d) 6

8. If $\log_2[\log_3(\log_3 b)]=\log_3[\log_2(\log_2 a)]=0$, then the ratio of a to b in simplest form, is :
(a) $\frac{4}{25}$
(b) $\frac{1}{9}$
(c) $\frac{4}{27}$
(d) $\frac{27}{4}$

9. If $\log_{2\sqrt{2}}(32\sqrt[5]{4}) = \alpha + \beta$ where α is an integer and $\beta \in [0, 1)$, then α is :

(a) 3 (b) 4
(c) 5 (d) 6

10. Given $\log_{10} 2 = a$, $\log_{10} 3 = b$. If $9^x = 15$, the value of x in terms of a and b is :

(a) $\dfrac{1+a-b}{2a}$ (b) $\dfrac{1+b-a}{2a}$
(c) $\dfrac{1+a-b}{2b}$ (d) $\dfrac{1+b-a}{2b}$

11. If $\log_3 5 = x$ and $\log_{25} 11 = y$ then the value of $\log_3\left(\dfrac{11}{3}\right)$ in terms of x and y is :

(a) $2xy + 1$ (b) $2xy - 1$
(c) $\dfrac{2y-x}{x}$ (d) $x^2 y+1$

12. Let x and y are solutions of the system of equations

$$\begin{cases} \log_8(1) + \log_3(x+2) = \log_3(3-2y) \\ 2^{x+y} - 8^{3-y} = 0 \end{cases}$$

The value of $(y-x)$, is :

(a) -3 (b) 5
(c) 11 (d) none of these

13. If $2^a = 4^b = 8^c$ and $abc = 288$ then $\left(\dfrac{1}{2a}+\dfrac{1}{4b}+\dfrac{1}{8c}\right)$ is equal to :

(a) $\dfrac{11}{48}$ (b) $\dfrac{11}{24}$
(c) $\dfrac{11}{96}$ (d) $\dfrac{13}{96}$

14. If $\log_a(10) + \log_a(10^2) + \ldots\ldots + \log_a(10^{10}) = 110$, then a equals:

(a) $\sqrt{10}$ (b) 10
(c) 20 (d) $10^{1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots+\frac{1}{10}}$

15. If the value of the product $P = 3 \cdot 3^{\log_4 3} \cdot 3^{\log_4 3^{\log_4 3}} \cdot 3^{\log_4 3^{\log_4 3^{\log_4 3}}} \ldots \infty$ is $a^{\log_b c}$ where $a, b, c \in Q$, then b equals :

(a) $\dfrac{3}{4}$ (b) $\dfrac{4}{3}$
(c) $\dfrac{3}{2}$ (d) $\dfrac{2}{3}$

16. The number of solutions of the equation

$\log(x^{2016}+1)+\log(1+x^2+x^4+\ldots\ldots x^{2014}) = \log 2016 + 2015\log x$

is equal to :

(a) 1
(b) 2
(c) 3
(d) infinite

17. If $2^{\log_3 p} = 5$, then $p^{\log_5 4}$ equals :

(a) 1
(b) 3
(c) 9
(d) 25

18. If $N_1 = 5^7+5^7+5^7+5^7+5^7$

and $N_2 = 10^3+10^3+10^3+10^3+10^3+10^3+10^3+10^3+10^3+10^3$

then the value of $\log_{250} N_1N_2$ is equal to :

(a) 4
(b) 8
(c) 10
(d) 21

19. If a student has written a formulae of logarithm by mistake as $a^{\log_x a} = x$, where $a > 0,\ x > 0, \neq 1, a \neq x$ and the student has got the correct answer then the value of $\frac{a}{x}$ is equal to :

(a) a
(b) a^2
(c) a^3
(d) $\frac{1}{a^2}$

20. If $\log_{0.04}(x-1) \geq \log_{0.2}(x-1)$, then x belongs to the interval :

(a) $(1, 2]$
(b) $(-\infty, 2]$
(c) $[2, \infty)$
(d) $(0, \infty)$

21. If the set of values of x for which $\sqrt{\log_2 \frac{1}{x}} = 2 + f$ where $f \in [0, 1)$ is $\left(\frac{1}{4^m}, \frac{1}{4^n}\right]$, then $(4m+n)$ equals :

(a) 4
(b) 6
(c) 10
(d) 20

22. The number of integral solution in set of $x \in (-10, 10)$ which satisfy the inequality :

$$\log_{\frac{1}{2}}|(x)(x-3)| \geq \log_2\left|\frac{x}{x^3-2x^2-2x-3}\right|, \text{ is}$$

(a) 9
(b) 11
(c) 13
(d) 15

23. The number $N = (\text{antilog}_5(\log_{\sqrt{5}}(14)))\cdot(\log_9(\text{antilog}_3(2^{-1})))$, then $\sqrt{N}$ is relatively prime with :

(a) 30
(b) 42
(c) 70
(d) 105

24. Number of solution satisfying the equation $|||x-1|+2|+2|=10$, is :
(a) 0
(b) 2
(c) 4
(d) 8

25. Let $a = 27^{\log_{\sqrt{3}} \sqrt[6]{4^k+6}}$, $b = \dfrac{1}{5^{\log_{\sqrt{5}} \sqrt{2^k-1}}}$ and $ab = 10$. Then the sum of the squares of the real value(s) of k is :
(a) 4
(b) 5
(c) 9
(d) 10

26. Let $p = \sqrt[3]{11+2\sqrt{30}} + \sqrt[3]{11-2\sqrt{30}}$, then the value of $\log_5(p^3 - 3p + 3)$ is equal to :
(a) -2
(b) 0
(c) 1
(d) 2

27. Let N be the number which is divisible by 3, whose logarithm to the base 3 having characteristic 3. Then number of integers of N is equal to :
(a) 16
(b) 17
(c) 18
(d) 19

28. The solution set of the equation $|x-1| + |x| = 2$ is :
(a) a null set
(b) a singleton
(c) a set consisting of exactly two elements
(d) a set consisting of more than two elements

29. A rational number which is 50 times its own logarithm to the base 10 is :
(a) 1
(b) 10
(c) 100
(d) 1000

30. The minimum value of 'c' such that $\log_b(a^{\log_2 b}) = \log_a(b^{\log_2 b})$ and $\log_a(c-(b-a)^2) = 3$ where $a, b \in N$ is :
(a) 2
(b) 4
(c) 8
(d) 10

31. Let (x_0, y_0) be the solution of the equations $[7(x+2)]^{\ln 7} = (2y)^{\ln 2}$ and $(x+2)^{\ln 2} = (7)^{\ln y}$, then x_0 is :
(a) $\dfrac{1}{7}$
(b) $\dfrac{5}{7}$
(c) $\dfrac{1}{2}$
(d) $\dfrac{-13}{7}$

32. Let A denotes the $\text{Antilog}_{32}(0.\breve{8})$ and B denotes the number of the integers which have the characteristic 4 when the base of log is 5 and C denotes the value of $-\log_7\left(\log_3 \sqrt{\sqrt[7]{9}}\right)$ then unit digit of $(A+B+C)$ is :
(a) 0
(b) 1
(c) 7
(d) 9

33. Let $N = 10^{3\log 2 - 2\log(\log 10^3) + \log((\log 10^6)^2}$ where base of the logarithm is 10. The characteristic of the logarithm of N to the base 3, is equal to :

(a) 2 (b) 3

(c) 4 (d) 5

34. Let $a = \dfrac{\log_{27} 8}{\log_3 2}$, $b = \left(\dfrac{1}{2^{\log_2 5}}\right)\left(\dfrac{1}{5^{\log_5 (0.1)}}\right)$ and $c = \dfrac{\log_4 27}{\log_4 3}$, then the value of $(a+b+c)$, is :

(a) 1 (b) $\dfrac{4}{3}$

(c) $\dfrac{5}{3}$ (d) $\dfrac{2}{3}$

35. Let $P = \log_2 17 \cdot \log_{0.2}(2) \cdot \log_3(0.2)$ then P lies between two successive integers whose sum is :

(a) 3 (b) 5

(c) 7 (d) 1

36. If $x = x_0$ is solution of the equation $(2x)^{\log_5 2} - (3x)^{\log_5 3} = 0$, then the value of $\left(x_0 + \dfrac{1}{x_0}\right)$ is equal to :

(a) $\dfrac{37}{6}$ (b) $\dfrac{\log_5 2 + \log_5 3}{\log_5 3 - \log_5 2}$

(c) $\log_2 3$ (d) 2

37. Let x denotes the logarithm of number $\dfrac{5}{3}$ to the base 0.6 and y denotes the number whose logarithm to the base 0.125 is $\dfrac{-1}{3}$, then xy equals :

(a) – 1 (b) 1

(c) 2 (d) – 2

38. The number of solution satisfying equation $x^{\log_x 2} = \log_3(x+y)$ and $x^2 + y^2 = 65$ is equal to :

(a) 0 (b) 1

(c) 2 (d) Infinite many solution

39. If $5^{a_1} = 6, 6^{a_2} = 7, 7^{a_3} = 8 \ldots\ldots 624^{a_{620}} = 625$, then $\prod_{r=1}^{620} a_r$ equals :

(a) 4 (b) 5

(c) 3 (d) 2

40. The value of $N = 2\log_6^3 3(1 + 3\log_3^2 2) - \log_6^3\left(\dfrac{3}{2}\right)$ is equal to :

(a) 0 (b) 1

(c) 2 (d) –1

EXERCISE-2

Linked Comprehension Type

Paragraph for Question No. 1 to 3

A denotes the product xyz where x, y and z satisfy

$$\log_3 x = \log 5 - \log 7$$
$$\log_5 y = \log 7 - \log 3$$
$$\log_7 z = \log 3 - \log 5$$

B denotes the sum of square of solution of the equation

$$\log_2(\log_2 x^6 - 3) - \log_2(\log_2 x^4 - 5) = \log_2 3$$

C denotes characteristic of logarithm

$$\log_2(\log_2 3) - \log_2(\log_4 3) + \log_2(\log_4 5) - \log_2(\log_6 5) + \log_2(\log_6 7) - \log_2(\log_8 7)$$

1. The value of $A + B + C$ is equal to :
   (a) 18 (b) 34
   (c) 32 (d) 24
2. The value of $\log_2 A + \log_2 B + \log_2 C$ is equal to :
   (a) 5 (b) 6
   (c) 7 (d) 4
3. The value of $|A - B + C|$ is equal to :
   (a) -30 (b) 32
   (c) 28 (d) 30

Paragraph for Question No. 4 to 6

Let α, β and γ be the roots of the equation

$$\log_2\left(\frac{(x^2 - 5x + 6)^{1/3}(x-5) + 1}{|x+1|}\right) + \log_2(2|x+1|) = 1$$

where $\alpha < \beta < \gamma (\log_{10} 2 = 0.3010)$.

4. The number of digits in the number $(\gamma)^{20(\alpha+\beta)}$ is :
   (a) 48 (b) 67
   (c) 70 (d) 89
5. If $\log_5 N = \beta + p$ where $p \in [0, 1)$ then largest integral value of N is :
   (a) 31 (b) 80
   (c) 242 (d) 624
6. If $\log_7 M = \alpha + q_1$ and $\log_3 M = \gamma + q_2$, where $q_1, q_2 \in [0, 1)$, then number of integral values of M is :
   (a) 97 (b) 98
   (c) 99 (d) 100

Paragraph for Question No. 7 to 9

Let x denotes the antilog of 0.5 to the base 256.

y denotes the number of digit in 5^{25} (Given $\log_{10} 2 = 0.3010$)

and z denotes the number of positive integers, which have the characteristic 2, when the base of logarithm is 4.

7. $(x + y + z)$ is divisible by :

(a) 3 (b) 4
(c) 40 (d) 41

8. $\frac{xy}{z}$ is equal to :

(a) 4 (b) 5
(c) 6 (d) 7

9. Characteristic of number $(x + 2y + z)$ to the base 3 is :

(a) 3 (b) 4
(c) 5 (d) 6

EXERCISE-3

Multiple Correct Choice Type

1. Let $a = 5^{\log_5(\sin x)}$ and $b = 7^{\log_7(\cos x)}$ then $\frac{a}{b}$ can be equal to :

(a) tan 2°　　(b) tan 4°
(c) tan 5°　　(d) tan 1°

2. Identify which of the following statement(s) is(are) correct for the equation

$$\log_{(x^2+2x-3)}(4-\log_2(|x-1|+|x-3|)) = 0$$

(a) The equation has exactly one solution
(b) The equation has exactly two solutions
(c) The sum of all the solutions of the equation is 6
(d) The sum of all the solutions of the equation is 4

3. Let L be the number of digits in 3^{40} before decimal and M be the number of zeroes in 3^{-40} after decimal before a significant digit, then :

(a) $L + M = 39$　　(b) $L - M = 2$
(c) $L - M = 1$　　(d) $L + M = 38$

(Use : $\log_{10} 3 = 0.4771$)

4. If $\log_3(\log_2 x) + \log_{\frac{1}{3}}\left(\log_{\frac{1}{2}} y\right) = 1$ and $xy^2 = 9$, then :

(a) $xy = 81$
(b) $xy = 9$
(c) number of possible ordered pairs (x, y) is 2
(d) number of possible ordered pairs (x, y) is 1

5. Let $P(x) = x^{50} - x^3 + 2x - 1$. If $R(x)$ is the remainder when $P(x)$ is divided by (x^2-1) then :

(a) $R(3) = 3$　　(b) $R(2) = 5$
(c) $R(7) = 7$　　(d) $R(6) = 13$

6. Consider an equation $3^{x^2} - 2\cdot 3^{x^2+1} - 7^{\log_7 3^{x^2-2}} + k = 0$. Identify which of the following statement(s) is(are) correct?

(a) If $k = 46$, then equation has exactly two solutions
(b) If $k = \frac{46}{9}$, then equation has exactly one solution
(c) If $k = 0$, then equation has no solution.
(d) If $k = -1$, then equation has exactly two solutions

7. Consider $|x-2e^{\ln 3}|+|x+2e^{\ln 3}|=12$, then identify which of the following statement(s) is(are) correct?

(a) The number of integral solutions of the equation is 7

(b) The number of integral solutions of the equation is 13

(c) The sum of all the integral solutions of the equation is 0

(d) The sum of all the integral solutions of the equation is 28

8. If $\log_x(y^{\log_3 x}) = \log_y(x^{\log_3 x})$, where $x, y \in N$ and $\log_x(z-(x-y)^2)=4$, then :

(a) $xy = 1$

(b) $x - y = 0$

(c) minimum value of $x + 3y$ is 8

(d) minimum value of $z = 16$

EXERCISE-4

Match the Column Type

1.

	Column-I	Column-II	
(a)	The value of $2^{\log_3 5} - 5^{\log_3 2} + 3^{\log_2 7} - 7^{\log_2 3} + 6^{\log_6 11} - 4^{\log_2 3}$ is equal to	(p)	0
(b)	The value of $\tan 225° \cot 405° + \tan 765° \cot 675°$ is equal to	(q)	1
(c)	The number $N = \dfrac{\log_5 250}{\log_{50} 5} - \dfrac{\log_5 10}{\log_{1250} 5}$, when simplified reduces to a natural number, then N equals	(r)	2
(d)	The value of $\log_{10}\left(\dfrac{1}{1+x^{m-n}+x^{p-n}} + \dfrac{1}{1+x^{n-p}+x^{m-p}} + \dfrac{1}{1+x^{p-m}+x^{n-m}}\right)$ is equal to	(s)	3
		(t)	4

2.

	Column-I	Column-II	
(a)	If $N = \log_{\frac{1}{5}} 7 \cdot \log_{(2-\sqrt{3})} 5 \cdot \log_{\frac{1}{9}} (2+\sqrt{3}) \cdot \log_{\sqrt{\frac{1}{7}}} 3$ then N is coprime with	(p)	2
(b)	If $M = \dfrac{6}{\log_3 1500} + \dfrac{6}{\log_5 1500} + \dfrac{12}{\log_{10} 1500}$ then M is less than or equal to	(q)	3
(c)	If $P = \log_{\sqrt[5]{3.5}} (1+2+3 \div 6)$ then P is twin prime with	(r)	5
(d)	If $\left(\sqrt[6]{x^9 y^{-8}}\right)^{-3} = x^a \cdot y^b$ then $(2a+4b)$ is greater or equal to	(s)	6
		(t)	7

EXERCISE-5

Integer Answer Type

1. Find the sum of all the integral solution(s) of the equation $3^{|x|} = \left(\frac{3}{(\sqrt{3})^{|x-2|}}\right)^2$.

2. If the value of x which satisfies the equation $2\log_3\sqrt{3^{1-x}+2} = 1+\log_3(4\cdot 3^x - 1)$ is given by, $1-\log_3 k$, then find the value of k.

3. If $x = \alpha$ is the solution of the equation $|2+\log_2 7x| - \log_2(x-1) = 5$, then find the value of $(65)^{\frac{1}{3}\log_{\alpha^2+1}\alpha}$.

4. If the least integral value satisfying the equation $\log_3\sqrt{x^2-4x+4} = 2^{\log_2(\log_3(|x|-2))}$ is α, then find the number of zeroes after decimal and before first significant digit in the number of $(\alpha)^{-4\alpha}$.

5. The product of all values of x satisfying the equation $|x+1|-|x|+3|x-1| = x+2$ is.

6. If $x = \log_2 3 - 4\log_{\left(\ln\frac{5}{4}\right)^3}$, $y = \log_{\sqrt{3}}\left(\ln\frac{5}{4}\right)$ and $z = \log_2(\log_{(5/4)} e)$ then find the absolute value of $(xy+2z)$.

7. If $\log_x y + \log_y x^2 = 3$, then find the sum of all possible values of $\log_x y^3$.

8. Let $f(x) = e^{\left(\frac{2^{-x}}{1+2^{-x}}\right)}$. If $\prod_{r=-10}^{10} f(\ln 2^r) = e^{\lambda}$, then find the value of $(2\lambda - 15)$.

9. Let x is the value of satisfying the equation $5^{\log_{10}x} = 50 - x^{\log_{10}5}$. Then find the value of $\frac{x}{50}$.

10. Find the integral value of x satisfying the equation $|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$.

ANSWER KEY

Single Correct Choice Type

1. (b)	2. (d)	3. (a)	4. (b)	5. (c)	6. (a)	7. (b)	8. (c)
9. (a)	10. (d)	11. (b)	12. (d)	13. (c)	14. (a)	15. (b)	16. (a)
17. (c)	18. (a)	19. (b)	20. (c)	21. (d)	22. (a)	23. (a)	24. (b)
25. (d)	26. (d)	27. (c)	28. (c)	29. (c)	30. (c)	31. (d)	32. (c)
33. (b)	34. (c)	35. (b)	36. (a)	37. (d)	38. (b)	39. (a)	40. (b)

Linked Comprehension Type

1. (b)	2. (a)	3. (d)	4. (c)	5. (d)	6. (d)	7. (d)	8. (c)
9. (b)							

Multiple Correct Choice Type

1. (a,b,c,d)	2. (a,c)	3. (a,c)	4. (a,d)	5. (a,c)	6. (a,b,c)	7. (b,c)
8. (b,c,d)						

Match the Column Type

1. $(a \to r)$; $(b \to p)$; $(c \to r)$; $(d \to p)$
2. $(a \to p,q,r,s,t)$; $(b \to s,t)$; $(c \to q,t)$; $(d \to p,q,r,s,t)$

Integer Answer Type

1. 3	2. 4	3. 2	4. 9	5. 1	6. 8	7. 9	8. 6
9. 2	10. 9						

○○○

EXERCISE-1

Single Correct Choice Type

1. $p, q, r \in R$ such that $p, q, r \neq 0$. If α is a root of $p^2x^2 + qx + r = 0$, β is a root of $p^2x^2 - qx - r = 0$ and $\alpha > \beta > 0$, then the equation $p^2x^2 + 2qx + 2r = 0$ has a root δ, then which of the following is always true?
   (a) $\alpha > \delta > \beta$
   (b) $\delta > \alpha > \beta$
   (c) $\alpha > \beta > \delta$
   (d) None of these

2. If α and β are the real roots of the equation $x^3 - px - 4 = 0$, $p \in R$ such that $2\alpha + \beta = 0$ then the value of $2\alpha^3 + \beta^3$ equals :
   (a) 6
   (b) 9
   (c) 12
   (d) 48

3. A monic cubic polynomial $P(x)$ satisfying the condition $P(1) = 1$, $P(3) = 17$ and $P(5) = 49$ then the value of $P(2)$, is :
   (a) 5
   (b) 10
   (c) 11
   (d) 12

4. If α, β, γ are the roots of $x^3 - x^2 - 2 = 0$ then the value of $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ is equal to :
   (a) -4
   (b) -5
   (c) -6
   (d) -8

5. If α, β, γ are roots of the equation $x^3 + ax + 10 = 0$ and value of $(\alpha+\beta)^3 + (\beta+\gamma)^3 + (\gamma+\alpha)^3$ is $10k$, $k \in R$ then k is equal to :
   (a) 3
   (b) $\frac{3}{2}$
   (c) -3
   (d) $\frac{-3}{2}$

6. Number of positive integral solutions of x which satisfy the inequality
   $\frac{(1-\sin x)(x-2\sqrt{2})(x-3)}{(\sqrt{3}-x)(x-5)(2^x+2^{-x})} > 0$ is equal to :
   (a) 2
   (b) 3
   (c) 4
   (d) more than 4

7. For quadratic equation $(k^2 - 3k + 2)x^2 + 4x + k^2 - k = 0$ sum of all values of k for which given equation has one root infinity.
   (a) 0
   (b) 1
   (c) 2
   (d) 3

8. If $\sin\theta$ and $\cos\theta$ are two distinct roots of $ax^2+bx+c=0\ \forall\ a, b, c\in R, a\neq 0$ then which of the following is correct?

(a) $a^2-b^2=2ac$
(b) $b^2-a^2=2ac$
(c) $c^2-a^2=2ab$
(d) $a^2-c^2=2ab$

9. Sum of all real values of x satisfying the equation $|x-1|^{\log_3^2 x-3\log_3 x+3}=(x-1)$ is :

(a) 12
(b) 13
(c) 14
(d) 15

10. Let $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\alpha-\gamma)=3$, where $\alpha, \beta, \gamma\in[0,\pi]$. If α, β, γ are roots of $x^3+px^2+qx+r=0$ then which of the following must be true?

(a) $pq=9r^2$
(b) $pqr=9$
(c) $pq=9r$
(d) $q=9rp$

11. If $|x-1|-|x-2|+|x-4|=\lambda$ where $x, \lambda\in R$ has four distinct solutions then λ equals :

(a) 3
(b) $\frac{5}{2}$
(c) 2
(d) 1

12. If α and β are roots of quadratic $x^2+(k-5)x+3=0$ and $\alpha<1<\beta$ and $k\in I$. If largest possible value of k is n then $[\log_{(n^2+2)}(n^2+n+1)]$ is equal to :

(a) 0
(b) 1
(c) 2
(d) 4

[Note: $[k]$ denotes greatest integer function less than or equal to k.]

13. If equations $2x^2+3x+4=0$ and $4ax^2+5bx+6c=0$, where $a, b, c\in N$ have a common root then minimum value of $(a+b+c)$ is :

(a) 15
(b) 9
(c) 47
(d) 53

14. Let $f(x)=x^3-2x+2$. If a, b and c such that $|f(a)|+|f(b)|+|f(c)|=0$, then the value of $\left(f\left(a^2+\frac{2}{a}\right)\cdot f\left(b^2+\frac{2}{b}\right)\cdot f\left(c^2+\frac{2}{c}\right)\right)^{\frac{1}{3}}$ is :

(a) 1
(b) 2
(c) 6
(d) none of these

15. $\log_3\log_2 y$ equals, where $y=6+\cfrac{16}{6+\cfrac{16}{6+\cfrac{16}{6+\ldots\infty}}}$

(a) 1
(b) $\log_3 4$
(c) 2
(d) $\log_3 5$

16. The range of $y = \frac{\sec^2 x - |\sec x| + 4}{\sec^2 x + |\sec x| + 4}$ is :

(a) $\left[\frac{3}{5}, 1\right)$
(b) $\left[\frac{1}{2}, 1\right)$
(c) $\left[\frac{2}{3}, 1\right)$
(d) $\left[\frac{1}{3}, 1\right)$

17. The equations $3x^2 + bx + c = 0$ $(b, c \in R)$ and $x^2 - 3x + 5 = 0$ have a common root, then the root of equation $x^2 - cx + b = 0$ are :

(a) rational and distinct
(b) equal
(c) irrational
(d) not real

18. If the equation $x^2 + 2x + 5 = 0$ has two roots α and β then the equation whose roots are $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$ is :

(a) $3x^2 + 7x + 10 = 0$
(b) $2x^2 - 8x + 10 = 0$
(c) $x^2 - 7x + 15 = 0$
(d) $5x^2 + 12x + 20 = 0$

19. Number of real roots of the equation $||x-1|-3| = \sqrt{5}$ is/are :

(a) 0
(b) 2
(c) 4
(d) none of these

20. Let α and β be two real roots of the equation $x^2 - ax + a - 4 = 0$ if $\alpha < 0$ and $\beta > 2$, then number of positive integral values of 'a' is/are :

(a) 0
(b) 1
(c) 2
(d) more than 2

21. The minimum value of $3x^2 + 2xy + y^2 - 2x - 6y + 13 = 0$ where $x, y \in R$, is :

(a) – 1
(b) 0
(c) 1
(d) 2

22. The number of solution of the equation $2\cos\left(\frac{x}{4}\right) = 4^x + 4^{-x}$ is :

(a) 0
(b) 1
(c) 2
(d) 3

23. The number of ordered pair of the form (a, b) for which $a(x-1)^2 + b(x^2 - 3x + 2) + x - a^2 = 0$ becomes an identity are :

(a) 0
(b) 1
(c) 2
(d) 3

24. If α and β are the roots of the equations $\sin^2 x + p\sin x + q = 0$ and $\cos^2 x + r\cos x + s = 0$, then the value of $p^2 + r^2 - 2\cos(\alpha - \beta)$ is equal to :

(a) 0
(b) 1
(c) 2
(d) 3

25. The value of m for which the equations $3x^2 - 2mx - 4 = 0$ and $x^2 - 4mx + 2 = 0$ have a common root is :

(a) $\pm\frac{1}{2}$
(b) $\pm\frac{1}{\sqrt{2}}$
(c) $\pm\frac{1}{\sqrt{3}}$
(d) ± 2

26. The equation whose roots are reciprocal of the roots of the equation $x^2 + 5x + 2 = 0$ is $x^2 + ax + b = 0$, then value of $(a + b)$ is equal to :

(a) 5
(b) 3
(c) 1
(d) 7

27. If the equations $2ax^2 - 3bx + 4c = 0$ and $3x^2 - 4x + 5 = 0$ have a common root, then $\frac{5a+b}{b+6c}$ is equal to (where $a, b, c \in R - \{0\}$) :

(a) -1
(b) 1
(c) 3
(d) 5

28. If both the roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ exceed 3 then :

(a) $a < \frac{1}{2}$
(b) $a > \frac{1}{2}$
(c) $a < 1$
(d) $a > \frac{11}{9}$

29. The number of integral values of 'a' for which the quadratic equation $x^2 + (a+19)x + 19a + 1 = 0$ has integral roots, are :

(a) 0
(b) 1
(c) 2
(d) 3

30. Let α, β are roots of $x^2 - 3x + 5 = 0$ then value of $\frac{5\alpha \cdot \beta^{2016} + 3\alpha^{2017} \cdot \beta + \alpha \cdot \beta^{2018}}{\alpha^{2016} + \beta^{2016}}$ is :

(a) 5
(b) 3
(c) 15
(d) 5/3

31. If the equation $(|x|+2)^2 - (2p-3)(|x|+2) - (p-5) = 0$ has exactly three distinct real roots then number of value(s) of p is(are) :

(a) 0
(b) 1
(c) 2
(d) infinite

32. If $ax^2 + bx + 6 = 0$ does not have two distinct real roots, $a \in R$, $b \in R$, then the least value of $(3a + b)$ is :

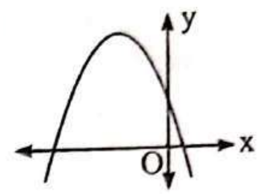
(a) 4
(b) -1
(c) 1
(d) -2

33. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real less than 3 then largest integral value of 'a' is :

(a) -1
(b) 0
(c) 1
(d) 2

34. A quadratic function $f(x)$ satisfies $f(x) \geq 0$ for all real x. If $f(2)=0$ and $f(4)=12$ then the value of $f(6)$ is :
(a) 12
(b) 24
(c) 48
(d) 64

35. The graph of a quadratic polynomial $y = ax^2 + bx + c$ is as shown in the adjacent figure. Which of the following quantities is(are) positive?



(a) $b - c$
(b) bc
(c) $c - a$
(d) ab^2

36. If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then :
(a) $-5 < a < 2$
(b) $a < -5$
(c) $a > 5$
(d) $2 < a < 5$

37. If α is the root of the equation $x^2 - x + 2 = 0$ then the value of $\dfrac{6(-\alpha^3 + 2\alpha^2 - \alpha)}{\alpha^5 - 3\alpha^4 + 3\alpha^3 - \alpha^2}$ is equal to :
(a) 3
(b) 6
(c) 9
(d) 12

38. Number of integral values of 'a' for which the equation $\sin^4 x - 2\cos^2 x + a^2 = 0$ has a solution, is :
(a) 1
(b) 2
(c) 3
(d) 4

39. If the roots of the equation $x^2 + bx + c = 0$ and $x^2 - cx + b = 0$ differ by the same quantity, then the value of $(b + c)$, is :
(a) 4
(b) 1
(c) 0
(d) -4

40. If the equation $x^4 - (2x^2 + 1)a + a^2 = 0$ has four distinct real roots then the least integral value of a is :
(a) 0
(b) 1
(c) 2
(d) 3

41. Let α, β, γ are roots of the equation $x^3 + x^2(3\sin\theta) + 4x\cos\theta - 1 = 0$, where $\theta \in R$, then maximum value of $(1-\alpha)(1-\beta)(1-\gamma)$ is :
(a) 1
(b) 3
(c) 4
(d) 5

42. If the equations $2ax^2 - 3bx + 4c = 0$ and $3x^2 - 4x + 5 = 0$ have a common root then $\dfrac{a+b}{b+c}$ is equal to : (where $a, b, c \in R - \{0\}$)
(a) $\dfrac{1}{2}$
(b) $\dfrac{3}{35}$
(c) $\dfrac{34}{31}$
(d) $\dfrac{29}{23}$

43. If α, β are the roots of the equation $3x^2 - 12x + \frac{\lambda}{3} = 0$ such that $1 < \alpha < 2$ and $2 < \beta < 3$ then number of positive integral value of λ are :

(a) 8 (b) 10
(c) 9 (d) 14

44. Let $y = P(x)$ be polynomial of degree 4 such that $P(1) = 2$ and attains minimum value 1 at both $x = 0$ and $x = 2$. Then $P'(1)$ is :

(a) -1 (b) 2
(c) 0 (d) 1

45. If α, β are roots of the equation $x^2 + 2x - 1 = 0$, then the value of

$$\frac{8\alpha\beta^4(\beta+2)^4 + 8\beta\alpha^4(\alpha+2)^4}{\left(\alpha - \frac{1}{\alpha}\right)^2 + \left(\beta - \frac{1}{\beta}\right)^2} \text{ is :}$$

(a) -2 (b) 0
(c) 2 (d) 3

46. If two equations $x^2 - cx + d = 0$ and $x^2 - ax + b = 0$ have one common roots and the second equation has equal roots then $2(b + d) =$

(a) 0 (b) $a + c$
(c) ac (d) $-ac$

47. If the polynomial $f(x) = x^2 + (p^2 + q)x + q - 9$ is negative of the polynomial $g(x) = -x^2 - 5px + q - 3$ for every $x \in R$, then :

(a) $p = 2, q = 6$ (b) $p = 6, q = 2$
(c) $p = -3, q = 6$ (d) none of these

48. If α and β are the roots of the equation $x^2 + 3x + 1 = 0$ then the value of $\left(\frac{\alpha^2}{\beta+3}\right)^2 + \left(\frac{\beta^2}{\alpha+3}\right)^2$ is :

(a) 7 (b) 9
(c) 16 (d) 17

49. The value of a for which the quadratic expression $ax^2 + (a-2)x - 2$ is negative for exactly two integral values of x, belongs to :

(a) $[-1, 1]$ (b) $[1, 2)$
(c) $[-2, -1)$ (d) $[2, 4)$

50. For the equation, $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to :

(a) 1/3 (b) 1
(c) 3 (d) 2/3

EXERCISE-2

Linked Comprehension Type

Paragraph for Question No. 1 to 2

Consider, $f(x) = 3(|a^2 - 3a - 2| + 4)x^2 - 12x + 4$, $a \in R$. Let α, β be the zeroes of the polynomial $f(x)$.

1. The maximum value of $(1 + \alpha)(1 + \beta)$ is :

(a) $\frac{1}{3}$ (b) $\frac{7}{3}$

(c) $\frac{7}{12}$ (d) $\frac{5}{12}$

2. If the maximum value of $f(x)$ is $g(a)$ for $0 \le x \le 2$ then minimum value of $g(a)$ is :

(a) 1 (b) 4

(c) 28 (d) 32

Paragraph for Question No. 3 to 4

For $a, b \in R - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right) \forall x \in R$.

Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

3. The value of $(a + b)$ is equal to :

(a) 4 (b) 5

(c) 6 (d) 7

4. The minimum value of $f(x)$ in $\left[0, \frac{3}{2}\right]$ is equal to :

(a) $\frac{-33}{8}$ (b) 0

(c) 4 (d) – 2

Paragraph for Question No. 5 to 7

Consider the quadratic polynomial $f(x) = x^2 - 4ax + 5a^2 - 6a$

5. The non - zero values of 'a' for which roots of $f(x) = 0$ are equal in magnitude and opposite in sign, is :

(a) 1 (b) 2

(c) 3 (d) none of these

6. The number of value(s) of a for which the equation $f(x) = 0$ has exactly one root equals to zero, is :

(a) 0 (b) 1

(c) 2 (d) 3

7. The largest integral value of a for which range of $f(x)$ is $[-5, \infty)$ for every real x, is :

(a) 3
(b) 5
(c) 7
(d) 10

Paragraph for Question No. 8 to 10

Consider a rational function $f(x) = \dfrac{x^2 - 3x - 4}{x^2 - 3x + 4}$ and a quadratic function $g(x) = x^2 - (b+1)x + b - 1$, where b is a parameter.

8. The sum of integers in the range of $f(x)$, is :

(a) -5
(b) -6
(c) -9
(d) -10

9. If both roots of the equation $g(x) = 0$ are greater than -1, then b lies in the interval :

(a) $(-\infty, -2)$
(b) $\left(-\infty, \dfrac{-1}{4}\right)$
(c) $(-2, \infty)$
(d) $\left(\dfrac{-1}{2}, \infty\right)$

10. The largest natural number b satisfying $g(x) > -2 \ \forall \ x \in R$, is :

(a) 1
(b) 2
(c) 3
(d) 4

EXERCISE-3

Multiple Correct Choice Type

1. If the equations $x^2-(p+q)x+4=0$; $x^2-2qx+8=0$ and $x^2-(2p+q)x+12=0$ have exactly one common root then which of the following is(are) CORRECT?
   (a) $p=2$
   (b) $q=3$
   (c) Common root is 4
   (d) Sum of all uncommon root is 6

2. The value of a for which the quadratic expression :
   $ax^2+(a-2+3^{\sqrt{\log_3 5}}-5^{\sqrt{\log_5 3}})x+(5^{\log_5 3}-3^{\log_3 5})$ is negative for exactly two integral values of x, can be :
   (a) 0
   (b) 1
   (c) $\frac{3}{2}$
   (d) 2

3. If α, β are the roots of the equation $x^2-kx+1=0$, $(k>2)$ then the value of $\sin(\alpha^2+1)+\sin\left(\frac{k^2}{1+\beta^2}\right)$ can be :
   (a) $2\sin\left(\frac{k^2}{2}\right)$
   (b) $2\sin(k\alpha)$
   (c) $2\sin\left(\frac{k}{\beta}\right)$
   (d) $2\sin(2\alpha^2)$

4. The value(s) of p for which the inequality
   $\log_{(p^2+1)}(3x^2-2x-p+6)\ge\cos^2\theta\,(1-4\sin^2\theta)^2-\cos^2 3\theta$ is satisfied for all real values of x and θ is(are) :
   (a) 0
   (b) 1
   (c) 4
   (d) 5

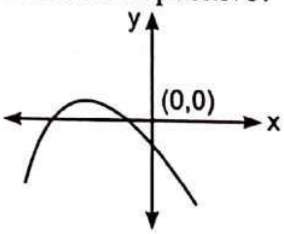
5. Consider $f(x)=x^2-(b+1)x+2(a-5)$ where $a, b\in N$. Identify which of the following statement(s) is(are) correct?
   (a) If $f(x)=0$ has two distinct real roots for all values of 'b', then sum of values of 'a' is 15.
   (b) If $f(x)=0$ has two distinct real roots for all values of 'b', then sum of values of 'a' is 10.
   (c) If $f(-1)>0$ for all values of 'a' then least value of 'b' is 7.
   (d) If $f(-1)>0$ for all values of 'a' then least value of 'b' is 6.

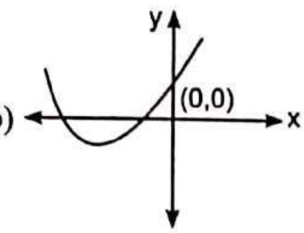
6. The value of 'k' for which both roots of the equation $4x^2-2x+k=0$ are completely in $(-1, 1)$, then k may be equal to :
   (a) -1
   (b) 0
   (c) 2
   (d) -3

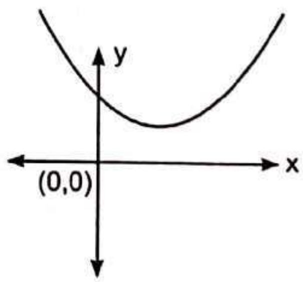
7. Which of the following number(s) lie(s) in the range of the functio $y=\dfrac{x^2+14x+9}{x^2+2x+3}$, $x \in R$?

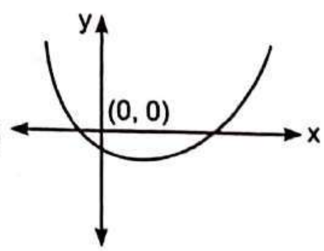
(a) 2
(b) $\dfrac{7}{2}$
(c) – 4
(d) – 7

8. For which of the following graphs of quadratic expression $y = ax^2 + bx + c$, ther product abc is positive?

(a) 

(b) 

(c) 

(d) 

9. Let $P(x)=x^4-8x^2+21$ and $Q(y)=y^2-6y+10$, $x,\ y\in R$. If $P(x)\cdot Q(y)=5$ then possible value(s) of $x+y$ is(are) :

(a) 1
(b) 3
(c) 5
(d) 7

10. Number of solutions of the equation $|x+1|+|x+2|+|x+3|=a$, where $x\in[-4,4]$ and 'a' is a parameter can be :

(a) 0
(b) 1
(c) 2
(d) 4

EXERCISE-4

Match the Column Type

1. The expression $y = ax^2 + bx + c$ ($a, b, c \in R$ and $a \neq 0$) represents a parabola which cuts the x-axis at the points which are roots of the equation $ax^2+bx+c=0$. Column-II contains values which correspond to the nature of roots mentioned in Column-I.

	Column-I		Column-II
(a)	For $a = 1, c = 4$, if both roots are greater than 2 then b can be equal to	(p)	4
(b)	For $a = -1, b = 5$, if roots lie on either side of -1 then c can be equal to	(q)	8
(c)	For $b = 6$, $c = 1$, if one root is less than -1 and the other root greater than $\frac{-1}{2}$ then a can be equal to	(r)	10
		(s)	no real value

2.

	Column-I		Column-II
(a)	If $\alpha, \beta \in (0, \pi)$ and $\alpha \neq \beta$ satisfy the equation $\frac{1-\cos 2\theta}{\sin\theta} = \frac{1}{2}$ then the value of $\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right)$ is equal to	(p)	0
(b)	If the expression $\frac{x^2 + (2m+3)x + (m^2+3)}{\sqrt{x^2 + (2m+1)x + m^2 + 2}}$ is non-negative $\forall\, x \in R$, then the possible values of m can be equal to	(q)	8
(c)	If the parabola $y = 5x^2 + x - 3$ lies above the parabola $y = 2x^2 + 6x - 1$, then integral values of x can be equal to	(r)	1
(d)	The number of real solutions of the equation $x^{2\log_x(x+3)} = 16$ is equal to	(s)	-1
		(t)	2

EXERCISE-5

Integer Answer Type

1. Let a, b, c and d be non-zero real numbers. If c and d are roots of the equation $x^2 + ax + b = 0$ and a and b are the roots of the equation $x^2 + cx + d = 0$, then find the absolute value of $(a + b + c + d)$.
2. Let k be the value of a for which the equation $(a^2 + 4a + 3)x^2 + (a^2 - a - 2)x + a(a + 1) = 0$ has more than two roots. If the expression $x^2 + (3m + 1)x + m^2 - 4 < 0$ $\forall\ x \in (k, k + 1)$, then find the number of integral values of m in the range of m.
3. Let α, β are real roots of equation $x^2 + ax + b = 0$; where $a \in R$ and $b \neq 0$. If another equation has two roots $\left(\alpha + \frac{1}{\alpha}\right)$ and $\left(\beta + \frac{1}{\beta}\right)$ such that sum of its roots is equal to product of roots, then range of b is $[m, M]$. Find the value of mM.
4. Let S be the sum of all possible values of 'a' for which the equation $x^2 + 2ax + 4a + 8 = 0$ has both integral roots. Find the value of $(S - 4)$.
5. Let α and β are real roots of the equation $3x^2 - (a^2 + 1)x + a^2 - 3a + 2 = 0$ such that $\log_\alpha \beta < 0$. If the range of a is (m, ∞) find m.
6. If the equation $x^2 + 2\alpha x + \alpha^2 - 1 = 0$ and $x^2 + 2\beta x + \beta^2 - 1 = 0$ have a common root $(\alpha \neq \beta)$ then the value of the expression $2\alpha^2 - 4\alpha\beta - |\alpha - \beta| + |\alpha - \beta|\beta^2$, is :
7. If the range of values of a for which the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ lie between the roots of the equation $x^2 - 2(a + 1)x + a(a - 1) = 0$ is (p, q), find the value of $\left(q + \frac{1}{p^2}\right)$.
8. Find the largest natural number 'a' for which the maximum value of $f(x) = a - 1 + 2x - x^2$ is smaller than the minimum value of $g(x) = x^2 - 2ax + 10 - 2a$.
9. Find sum of all integral values of a in $[1, 100]$ for which the equation $x^2 - (a - 5)x + \left(a - \frac{15}{4}\right) = 0$ has atleast one root greater than zero.
10. If sum of maximum and minimum value of $y = \log_2(x^4 + x^2 + 1) - \log_2(x^4 + x^3 + 2x^2 + x + 1)$ can be expressed in form $((\log_2 m) - n)$, where m and 2 are coprime then compute $(m + n)$.
11. If $1 - \log_x 2 + \log_{x^2} 9 - \log_{x^3} 64 < 0$, then range of x is (a, b). Find the minimum value of $(a + 9b)$.

12. If α, β are roots of the equation $2x^2+6x+b=0$ where $b<0$, then find the least integral value of $\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)$.

13. Find the number of integral values of a in the interval [0, 100] so that the range of function $f(x)=\frac{x+a}{x^2+1}$ contains the interval [0, 1].

14. Find the number of integral values of k for which the equation $x^2-4|x|+3-|k-1|=0$ has four distinct real roots.

15. Let $f: R \rightarrow R$ be defined as $f(x)=x^2-2x+a$. Also $g: R \rightarrow R$ be defined as $g(x)=|x-1|$.
If the equation $f(x)=g(x)$ has four distinct real roots then the range of 'a' is (p, q).
Find the value of $(5p+4q)$.

16. Let $f(x)=kx^2+x(3-4k)-12$. If the set of values of k for which $f(x)<0$ $\forall x \in (-3, 3)$ and $f(-4)>0$ is $(p, q]$ then find the value of $(4p+3q)$.

17. Find the number of integral values of a so that the inequation $x^2-2(a+1)x+3(a-3)(a+1)<0$ is satisfied by atleast one $x \in R^+$.

18. Let $\frac{y+3}{2y+5}=\sin^2 x+2\cos x+1$, where $x \in R$. If the range of y is $(-\infty, a] \cup [b, \infty)$, then find the value of $(3a-5b)$.

19. If both roots of the equation $x^2+3x+a-5=0$ are negative and distinct, then find the sum of integral values of a.

20. Find the number of integral values of a for which $(x-3a)(x-a-3) \leq 0$ for all $x \in [1,3]$.

ANSWER KEY

Single Correct Choice Type

1. (a)	2. (c)	3. (b)	4. (a)	5. (a)	6. (a)	7. (d)	8. (b)
9. (d)	10. (c)	11. (b)	12. (a)	13. (d)	14. (c)	15. (a)	16. (a)
17. (c)	18. (d)	19. (c)	20. (d)	21. (d)	22. (b)	23. (b)	24. (c)
25. (b)	26. (b)	27. (b)	28. (d)	29. (c)	30. (c)	31. (a)	32. (d)
33. (c)	34. (c)	35. (c)	36. (a)	37. (a)	38. (c)	39. (d)	40. (c)
41. (d)	42. (c)	43. (a)	44. (c)	45. (a)	46. (c)	47. (a)	48. (a)
49. (b)	50. (c)						

Linked Comprehension Type

1. (b)	2. (c)	3. (b)	4. (d)	5. (d)	6. (b)	7. (b)	8. (b)
9. (d)	10. (b)						

Multiple Correct Choice Type

1. (a,b,c,d)	2. (b,c)	3. (b,c)	4. (b,c)	5. (a,c)	6. (a,b)
7. (a,b,c)	8. (b,d)	9. (a,c)	10. (a,b,c)		

Match the Column Type

1. $(a \to s)$; $(b \to q, r)$; $(c \to p)$
2. $(a \to q)$; $(b \to p, s)$; $(c \to q, s)$; $(d \to p)$

Integer Answer Type

1. 2	2. 4	3. 1	4. 8	5. 2	6. 6
7. 17	8. 1	9. 5011	10. 5	11. 25	12. 10
13. 100	14. 5	15. 10	16. 6	17. 5	18. 4
19. 13	20. 1				

○○○

EXERCISE-1

Single Correct Choice Type

1. If $ax^2 + bx + c = 0$ and $x^2 + 3x + 5 = 0$ have a root in common where $a, b, c \in (1, 12)$ and $a, b, c \in I$ then sum of first ten terms of a sequence with first three terms as a, b, c is equal to :
   (a) 100 (b) 150
   (c) 200 (d) 300

2. The sum to infinity of the series $\frac{3}{1^3} + \frac{5}{1^3+2^3} + \frac{7}{1^3+2^3+3^3} + \ldots\ldots$ to ∞, is :
   (a) 3 (b) 4
   (c) 5 (d) 6

3. If $(11)^n + 2\cdot 11^{n-1}\cdot 10 + 3\cdot 11^{n-2}\cdot 10^2 + 4\cdot 11^{n-3}\cdot 10^3 + \ldots\ldots \infty = k(11)^n$ then k equals to :
   (a) 110 (b) 111
   (c) 120 (d) 121

4. Let $a_1, a_2, a_3 \ldots\ldots$ form an increasing G.P. If $9a_3, 5a_5, a_7$ are in arithmetic progression then common ratio of the geometric progression is :
   (a) 1 (b) 2
   (c) 3 (d) 4

5. If α, β, γ are the positive real roots of the equation $x^3 - 6x^2 + ax + b = 0$ then the minimum value of $\frac{3}{\alpha+1} + \frac{3}{\beta+1} + \frac{3}{\gamma+1}$ is :
   (a) 1 (b) 3
   (c) 9 (d) 27

6. If $a_1, a_2, a_3 \ldots\ldots\ldots\ldots a_{10}$ are A.M.'s between 5 and 35 then the maximum value of $(a_1 \cdot a_2 \cdot a_3 \ldots\ldots \cdot a_{10})$ is :
   (a) $(10)^{10}$ (b) $(20)^{10}$
   (c) $(30)^{10}$ (d) $(40)^{10}$

7. Let S_n be the sum of n terms of sequence $S_n = \frac{3}{1^3} + \frac{5}{1^3+2^3} + \frac{7}{1^3+2^3+3^3} \ldots\ldots\ldots$ (n terms). If $S_{50} = \lambda\left(\frac{2600}{2601}\right)$ then the value of λ is equal to :
   (a) 2 (b) 3
   (c) 4 (d) 5

8. Consider the equation $3x^4 - 18x^3 + px^2 - 8qx + 3q = 0$. The equation has only positive real roots then the value of $\frac{p}{q}$ is :
   (a) $\frac{1}{8}$ (b) 4 (c) $\frac{1}{4}$ (d) 8

9. The sum of the infinite series $\frac{3}{11}+\frac{33}{11^2}+\frac{333}{11^3}+\ldots\ldots$ is :

(a) 3
(b) $\frac{33}{10}$
(c) $\frac{30}{11}$
(d) none of these

10. Let α and β are roots of equation $7x^2-5x-1=0$, then $\lim\limits_{n\to\infty}\sum\limits_{r=0}^{n}\left(\frac{1}{(7\alpha-5)^r}+\frac{1}{(7\beta-5)^r}\right)$ is :

(a) 9
(b) -3
(c) 3
(d) $\frac{19}{13}$

11. In a ΔABC, sides a, b, c are in G.P. and angles A, B, C are in A.P. If area of ΔABC $=3\sqrt{3}$ then $(R+r)$ is equal to :

(a) -2
(b) 0
(c) 2
(d) 3

12. For an increasing A.P. : $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$, if $a_1+a_3+a_5=-12$ and $a_1a_3a_5=80$ then which of the following does not hold?

(a) $a_1=-10$
(b) $a_2=-1$
(c) $a_3=-4$
(d) $a_5=2$

13. If $\theta_1, \theta_2, \theta_3, \ldots\ldots\ldots, \theta_n$ are in A.P. whose common difference is d then $\sin d\cdot[\sec\theta_1\cdot\sec\theta_2+\sec\theta_2\sec\theta_3+\sec\theta_3\sec\theta_4+\ldots\ldots+\sec(\theta_{n-1})\sec\theta_n]$ is :

(a) $\cot\theta_n-\cot\theta_1$
(b) $\tan\theta_n-\tan\theta_1$
(c) $\tan\theta_1-\tan\theta_n$
(d) none of these

14. The sum to infinity of the series $\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots\ldots$ is equal to :

(a) 2
(b) 5/2
(c) 3
(d) none of these

15. The sum of the series $\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\ldots\ldots\ldots\infty$ terms, is :

(a) $\frac{1}{2}$
(b) $\frac{2}{3}$
(c) $\frac{3}{4}$
(d) $\frac{5}{6}$

16. Find the sum of the infinite series $\frac{1}{9}+\frac{1}{18}+\frac{1}{30}+\frac{1}{45}+\frac{1}{63}+\ldots\ldots$:

(a) $\frac{1}{3}$
(b) $\frac{1}{4}$
(c) $\frac{1}{2}$
(d) $\frac{2}{3}$

17. Let $a_n=16, 4, 1, \ldots$ be a geometric sequence. Define P_n as the product of the first n terms. The value of $\sum\limits_{n=1}^{\infty}\sqrt[n]{P_n}$:

(a) 8
(b) 16
(c) 32
(d) 64

18. Let $s_1, s_2, s_3 ......$ and $t_1, t_2, t_3 ......$ are two arithmetic sequences such that $s_1 = t_1 \neq 0$; $s_2 = 2t_2$ and $\sum_{i=1}^{10} s_i = \sum_{i=1}^{15} t_i$. Then the value of $\frac{s_2 - s_1}{t_2 - t_1}$ is :

(a) 8/3
(b) 3/2
(c) 19/8
(d) 2

19. Let a_r be the r^{th} term of an A.P. If $a_{11} = 45$ then the common difference that would make the value of $a_2 a_6 a_{11}$ least is equal to :

(a) 14
(b) 7
(c) 4
(d) 3

20. Let $a_1, a_2, a_3, ..., a_{10}$ are in G.P. with $a_{51} = 25$ and $\sum_{i=1}^{101} a_i = 125$, then the value of $\sum_{i=1}^{101}\left(\frac{1}{a_i}\right)$ equals :

(a) 5
(b) $\frac{1}{5}$
(c) $\frac{1}{25}$
(d) $\frac{1}{125}$

21. If $a_1 + 2a_2 + 3a_3 + 4a_4 = 1000$ where $a_i > 0\ \forall\ i = 1, 2, 3, 4$ and maximum value of $a_1 \cdot a_2^2 \cdot a_3^3 \cdot a_4^4 = p^q$, $p, q \in N$, then least value of $(p + q)$ is :

(a) 30
(b) 50
(c) 90
(d) 100

22. If a, x, y, z, b are in A.P., then $x + y + z = 15$ and if a, x, y, z, b are in H.P., then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$. Numbers a, b are :

(a) 8,2
(b) 11,3
(c) 9,1
(d) none of these

23. Let $a_1, a_2, a_3, ... a_{101}$ are in A.P. such that $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + ... + \frac{1}{a_{100} a_{101}} = 10$ and $a_2 + a_{100} = 50$, then the value of $(a_1 - a_{101})^2$ is equal to :

(a) 2460
(b) 2500
(c) 5050
(d) 4950

24. It is given that $\frac{1}{2^n \sin\alpha}$, 1, $2^n \sin\alpha$, are in A.P. for some value of α where $\alpha \in \left(0, \frac{\pi}{2}\right)$.

Let say for $n = 1$, α satisfying the above A.P. is α_1, for $n = 2$, the value is α_2 and so on.

If $S = \sum_{i=1}^{\infty} \sin\alpha_i$, then the value of S is :

(a) 1
(b) $\frac{1}{2}$
(c) 2
(d) none of these

25. If S is sum of $\frac{5}{1^2 \cdot 2^2 \cdot 3}+\frac{8}{2^2 \cdot 3^2 \cdot 4}+\frac{11}{3^2 \cdot 4^2 \cdot 5}+\frac{14}{4^2 \cdot 5^2 \cdot 6}+\ldots\ldots \infty$ terms, then S is equal to :

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

26. If x, y, z are positive reals satisfying $xyz = 32$ then the minimum value of $x^2+4xy+4y^2+2z^2$ is :

(a) 92 (b) 96
(c) 48 (d) 44

27. If $x^3-px^2+qx-27=0$ has three positive roots where $p, q>0$ and $p+k+l^2=4l+10$ where $k, l \in R$, then the greatest value of k is:

(a) 1 (b) 5
(c) 6 (d) 7

28. If α, β and γ are the negative roots of the equation $27x^3+216x^2+36px+32p=0$, $p \in R-\{0\}$, then $\left|\frac{p}{\gamma}\right|$ equals :

(a) 2 (b) 6
(c) 8 (d) 64

29. Four geometric means are inserted between the numbers $2^{11}-1$ and $2^{11}+1$. The product of these geometric means is :

(a) $2^{44}-2^{23}+1$ (b) $2^{44}-2^{22}+1$
(c) $2^{22}-2^{11}+1$ (d) $2^{22}-2^{12}+1$

30. If $f(x)=\tan x+2\tan^2 x+3\tan^3 x+4\tan^4 x+\ldots\ldots\infty$

where $x \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, then number of solution(s) of the equation $f(x)=2$, is(are) :

(a) 1 (b) 2
(c) 3 (d) 4

31. If $4a^2+9b^2+16c^2=2(3ab+6bc+4ca)$, then a, b, c are in (where $a, b, c \in R-\{0\}$):

(a) A.P. (b) G.P.
(c) H.P. (d) A.G.P.

32. Given a sequence of 4 numbers, first three of which are in A.P. and the last three in G.P. with common ratio $\frac{3}{2}$. If 1 is subtracted from the last number, all the four numbers in order form an A.P., then the value of sum of smallest and largest of the four numbers is :

(a) 10 (b) 13
(c) 12 (d) 11

33. If $T_n = \dfrac{(2n+1)(2n^2+2n-1)}{(n-1)^2 n^2 (n+1)^2 (n+2)^2}$, $n \in N$, $n \geq 2$, then $\lim\limits_{n\to\infty} \sum\limits_{r=2}^{n} T_r$ is equal to :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{9}$

34. For $a, b > 0$, let g_1, g_2, g_3 and g_4 be geometric means between a and b, then the roots of the equation $(g_2 g_3)x^2 - \left(\dfrac{g_2}{g_1+g_3}\right)x - g_1 g_4 = 0$ are :

(a) both negative (b) both positive
(c) one negative and one positive (d) imaginary

35. If n arithmetic means $a_1, a_2, \ldots\ldots, a_n$ are inserted between 50 and 200 and n harmonic means $h_1, h_2, \ldots\ldots h_n$ are inserted between the same two numbers, then $a_2 h_{n-1}$ is equal to :

(a) 500 (b) $\frac{10000}{n}$
(c) 10000 (d) $\frac{250}{n}$

36. The sum of infinity of the series $\frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \frac{9}{400} + \ldots\ldots \infty$ is :

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) 2

37. If in a ΔABC, $\cos A + 2\cos B + \cos C = 2$, then $\sin A, \sin B, \sin C$ are in :

(a) H.P. (b) G.P.
(c) increasing order (d) A.P.

38. The sum to infinite terms of the series $\dfrac{1}{1-\frac{1}{4}} + \dfrac{1}{(1+3)-\frac{1}{4}} + \dfrac{1}{(1+3+5)-\frac{1}{4}} + \ldots\ldots \infty$ is :

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 3

39. Let $S_n = \dfrac{1}{1^3} + \dfrac{1+2}{1^3+2^3} + \ldots\ldots + \dfrac{1+2+\ldots\ldots+n}{1^3+2^3+\ldots\ldots+n^3}$. Then S_n is not greater than :

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) 1

40. The sum to infinity of the series $1 + \left(1+\frac{1}{2}\right)\left(\frac{1}{3}\right) + \left(1+\frac{1}{2}+\frac{1}{2^2}\right)\left(\frac{1}{3^2}\right) + \ldots\ldots \infty$ is :

(a) $\frac{12}{5}$ (b) $\frac{9}{5}$ (c) $\frac{9}{10}$ (d) 3

EXERCISE-2

Linked Comprehension Type

Paragraph for Question No. 1 to 2

Let $S = \left\{ \lambda \,\middle|\, \sqrt{\dfrac{11\lambda - 2\lambda^2 - 8}{2\lambda}} \in I, \lambda > 0 \right\}$. All the elements of set S are the roots of the equation $x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_n = 0$ where n denotes the number of elements in S.

1. The value of $(a_1 + a_n)$ equals :

(a) 6 (b) 16 (c) 22 (d) 26

2. If $(n-2)$ is the 3^{rd} term of a G.P. such that sum of its infinite term is equal to a_n then its first term is equal to :

(a) a_n (b) $\sqrt{a_n}$

(c) $2\sqrt{a_n}$ (d) $\sqrt[4]{a_n}$

Paragraph for Question No. 3 to 4

Let a^2, $a(b-3)$, $4b$ be the first three terms of an increasing A.P. where $a, b \in I$ and $5 \le b \le 15$.

3. The value of $\left(\dfrac{b}{a}\right)$ is equal to :

(a) 1 (b) $\dfrac{3}{2}$ (c) $\dfrac{5}{2}$ (d) 7

4. If S_n is the sum of first n terms of the given A.P. then $\left(\dfrac{S_7 - S_4}{19}\right)$ is equal to :

(a) 12 (b) 3

(c) 4 (d) 6

Paragraph for Question No. 5 to 6

Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ be n terms in A.P. such that a_1 and $a_1 + a_2$ are prime numbers, $a_7 = 11. (a_2 \ge a_1)$. Let $T_n = 2\sum_{r=1}^{n} a_r$ and $S_n = \sum_{r=1}^{n} \dfrac{1}{T_r + 20}$:

5. a_{10} equals :

(a) 14 (b) 17

(c) 20 (d) 25

6. Which of the following is correct?

(a) $S_\infty = \frac{1}{4}$ (b) $S_\infty = \frac{1}{5}$ (c) $S_\infty = \frac{1}{20}$ (d) $S_\infty = \frac{3}{20}$

Paragraph for Question No. 7 to 8

The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its roots, the product of its roots and the sum of its coefficient are all equal. If the y-intercept of the graph of $y = P(x)$ is 2, then.

7. Minimum value of $cx^2 + 2ax - b$ is equal to :

(a) -7 (b) -1 (c) 2 (d) 7

8. If sum of first 10 terms of an A.P. is $c + 10$ and sum of odd numbered terms lying in first 10 terms is $b + 17$, then a is equal to :

(a) sum of first 5 terms (b) sum of first 10 terms

(c) sum of first 15 terms (d) sum of first 20 terms

Paragraph for Question No. 9 to 10

Let $a, b, b + 2a, \ldots\ldots$ be a geometric progression and S_n denotes the sum of its first n terms. Also, $S_{2n+1} - S_{2n} = 2 \ \forall \ n \in N$.

9. The sum of the series $\frac{1}{a} - \frac{1}{b^2} + \frac{1}{a^3} - \frac{1}{b^4} + \ldots\ldots$ up to infinite, is :

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

10. The value of $\sum_{n=1}^{\infty} \frac{n}{n^4 - ab}$ is :

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

EXERCISE-3

◈ Multiple Correct Choice Type

1. Let $f(x) = ax^2 + 2bx + c$ and $g(x) = x^2 + 2p^2x + 1$. If $f(x) = 0$ and $g(x) = 0$ have o one common root α and a, b, c are in A.P. ($p^2 \neq 1$). Then :

(a) $\alpha = -1$

(b) $\alpha = \frac{-c}{a}$

(c) $\lim\limits_{x \to \frac{-a}{c}} (1 + g(x))^{\frac{1}{x + \frac{a}{c}}} = e^{\frac{c^2 - a^2}{ca}}$

(d) $\lim\limits_{x \to \frac{-a}{c}} (1 + g(x))^{\frac{1}{x + \frac{a}{c}}} = e^{\frac{a^2 - c^2}{ac}}$

2. If $P(m) = (m^2 + m + 1)a^2 - 2(m^2 + 1)a + m^2 - m + 1 - b$ and $P(1) = P(2) = P(-3) = 0$ then

(a) $a + b = a^2 + b^2$

(b) $a - b = a^2 - b^2$

(c) If $a, A_1, A_2, A_3, \ldots\ldots\ldots\ldots A_{10}, b$ are in A.P. then $A_1 + A_2 + A_3 \ldots\ldots + A_{10} = 5$

(d) If $a, A_1, A_2, \ldots\ldots\ldots\ldots A_{10}, b$ are in A.P. then $A_1 + A_2 + A_3 \ldots\ldots + A_{10} = 10$

3. If x_1, x_2 and x_3 are positive roots of the equation $x^3 - 6x^2 + 3px - 2p = 0$, $p \in$ then :

(a) x_1, x_2, x_3 are in A.P.

(b) x_1, x_2^2, x_3^3 are in G.P.

(c) $x_1, 2x_2, 3x_3$ are in A.P.

(d) $x_1, x_1x_2, x_1x_2x_3$ are in G.P.

4. If the roots of the equation $x^4 - 10x^3 + bx^2 - cx + 24 = 0$ are in A.P. then which the following is(are) **CORRECT**?

(a) All roots are different integers

(b) $c - b = 15$

(c) b and c are rational

(d) roots are real

5. Let the equation $x^5 - 10x^4 + ax^3 - bx^2 + cx = 0$ has 5 distinct real roots which a in A.P. If the sum of the positive roots is maximum then which of the following mu be true ?

(a) equation has exactly one negative root.

(b) equation has no negative root.

(c) $a = 20$

(d) $b = 48$

6. Let $< h_n >$ and $< g_n >$ be harmonic and geometric sequences respectively. $h_1 = g_1 = \frac{1}{2}$ and $h_{10} = g_{10} = \frac{1}{1024}$ then :

(a) $h_{50} > g_{50}$

(b) $h_8 > g_8$

(c) $\sum\limits_{i=1}^{10} h_i > \sum\limits_{i=1}^{10} g_i$

(d) $\sum\limits_{i=11}^{50} h_i > \sum\limits_{i=11}^{50} g_i$

7. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are in A.P. such that $4\alpha_5 = 5\alpha_4$ and $\sum_{i=1}^{5}\alpha_i$ = sum of interior angles of a pentagon, then :

(a) $\sum_{i=1}^{5}\cos(\alpha_i) = -1$

(b) $\sum_{i=1}^{5}\sin(\alpha_i) = \sqrt{5}$

(c) $\prod_{i=1}^{5}\sin(\alpha_i) = 0$

(d) $\prod_{i=1}^{5}\cos(\alpha_i) = \frac{-1}{16}$

8. Let $S_n = \sum_{r=1}^{n}\frac{2r^3 - 3r^2 - 3r - 1}{3^{r+1}}$. Identify which of the following statement(s) is(are) correct?

(a) If $S_{10} = \frac{3^p - q^3}{3^{11}}$, then $(p + q)$ is equal to 12.

(b) If $S_{10} = \frac{3^p - q^3}{3^{11}}$, then $(p + q)$ is equal to 21.

(c) $\lim_{n\to\infty} S_n = \frac{2}{3}$

(d) $\lim_{n\to\infty} S_n = \frac{1}{3}$

9. If $\lambda = \sqrt{4 + \sqrt{\frac{37b - 81b^4 - 3}{b}}}$ and $f(x) = (x - p + 5)(x - p)$ where $p, \lambda \in R$ and $b > 0$ then :

(a) sum of integral values of λ is 9

(b) sum of integral values of λ is 5

(c) complete set of values of p for which $f(\lambda) < 0$ for all possible values of λ is (3, 7)

(d) complete set of values of p for which $f(\lambda) < 0$ for all possible values of λ is (4, 7)

10. If $\theta_1, \theta_2, \ldots, \theta_n$ satisfy the equation $\sin\theta + \frac{2\sin\theta}{1+\sqrt{\sin\theta}} + \frac{3\sin\theta}{(1+\sqrt{\sin\theta})^2} + \ldots + \infty = \frac{9}{4}$ where $\theta_i \in [0, 4\pi]\ \forall\ i \in N$ then :

(a) $\sum_{i=1}^{n}\sin\theta_i = 1$

(b) $\sum_{i=1}^{n}\sin\theta_i = \frac{1}{4}$

(c) $\sum_{i=1}^{n}\theta_i = 6\pi$

(d) $\sum_{i=1}^{n}\theta_i = 3\pi$

11. Let product of two natural numbers α, β is 192. Their greatest common divisor and least common multiple is represented by p and q. If ratio of A.M. and H.M. of p and q is $\frac{169}{48}$, then $\alpha + \beta$ can be :

(a) 52
(b) 28
(c) 98
(d) 32

12. The first three terms of a geometric sequence is x, y, z and these have the sum equal to 42. If the middle term y is multiplied by $\frac{5}{4}$, then numbers x, $\frac{5y}{4}$, z now form an A.P. If the largest possible value and smallest possible value of x are x_1, x_2 then :

(a) sum of infinite terms of G.P. with x_1, x_2 as first two terms is 32.
(b) sum of infinite terms of G.P. with x_1, x_2 as first two terms is 18.
(c) $x_1 + x_2 = 30$
(d) ratio of A.M. and G.M. of x_1, x_2 is $\frac{5}{4}$

13. If a, b are roots of quadratic equation $(2-b)x^2 + (b^2-3b)x + b^2 = 0$; $b \neq 2$; $b \neq 0$ then

(a) a, b, ab are in A.P.
(b) ab, a, b are in A.P.
(c) 2^{ab}, 2^a, 2^b are in G.P.
(d) $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{ab}$ are in H.P.

14. If $5 \cdot 2^8 \cdot 3^{16}$ is one of the terms of a G.P. whose first term is 5 and all its terms are natural number then possible common ratio of the G.P. is :

(a) 6
(b) 12
(c) 18
(d) $(324)^2$

15. If x, y, z are three distinct real numbers such that x, 12, y are in H.P. and x, 12, z, y are in increasing A.P. then :

(a) $x + z = 18$.
(b) $x + z = 24$.
(c) maximum value of $\sqrt{(x-3)\sin\alpha - (y-10)\cos\beta + 2}$ is 4 where $\alpha, \beta \in R$.
(d) maximum value of $\sqrt{(x-3)\sin\alpha - (y-10)\sin\alpha}$ is 10 where $\alpha \in R$.

EXERCISE-4

Match the Column Type

1.

	Column-I		Column-II
(a)	If a, b, c and d are four non zero real numbers such that $(d+a-b)^2+(d+b-c)^2=0$ and the roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are real and equal then	(p)	$a+b+c=0$
(b)	If a, b, c are real non-zero positive numbers such that $\log a$, $\log b$, $\log c$ are in A.P. then	(q)	a, b, c are in A.P.
(c)	If the equation $ax^2+bx+c=0$ and $x^3-3x^2+3x-1=0$ have a common real root then	(r)	a, b, c are in G.P.
(d)	Let a, b, c be positive real numbers such that the expression $bx^2+(\sqrt{(a+c)^2+4b^2})x+(a+c)$ is non negative $\forall\ x \in R$ then	(s)	a, b, c are in H.P.

2.

	Column-I		Column-II
(a)	In an A.P. the series containing 99 terms, the sum of all the odd numbered terms is 2550. The sum of all the 99 terms of the A.P. is	(p)	5010
(b)	f is a function for which $f(1)=1$ and $f(n)=n+f(n-1)$ for each natural number $n \geq 2$. The value of $f(100)$ is	(q)	5049
(c)	Suppose, $f(n)=\log_2(3)\cdot\log_3(4)\cdot\log_4(5)......\log_{n-1}(n)$ then the sum $\sum_{k=2}^{100} f(2^k)$ equals	(r)	5050
(d)	Concentric circles of radii 1, 2, 3......100 cm are drawn. The interior of the smallest circle is coloured red and the annular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. The total area of the green regions in sq. cm is $k\pi$ then 'k' equals	(s)	5100

EXERCISE-5

Integer Answer Type

1. If $(s-a)$, $(s-b)$, $(s-c)$, $(s-d)$ are positive numbers such that $256(s-a)(s-b)(s-c)$ $(s-d)=s^4$. Where $a+b+c+d=3s$ and $a=3$ then find the value of $\left[\frac{ad}{b+c}+s\right]$.

   [**Note** : $[k]$ denotes greatest integer function less than or equal to k.]

2. Let A be a point inside a regular polygon of 10 sides. Let $p_1, p_2, \ldots\ldots\ldots\ldots\ldots\ldots p_{10}$ be the distances of A from the sides of the polygon. If each side is of length 2 then find the minimum possible integral value of $\frac{1}{p_1}+\frac{1}{p_2}+\ldots\ldots\ldots\ldots\ldots\ldots\frac{1}{p_{10}}$.

3. If $\sec(\alpha-2\beta)$, $\sec\alpha$ and $\sec(\alpha+2\beta)$ are distinct terms in arithmetic progression, then find the value of $\frac{\cos^2\alpha}{\cos^2\beta}$.

4. If x, y, z are positive reals such that $x^2+y^2+z^2=7$; $xy+yz+zx=4$ then the minimum value of xy is $\frac{p}{q}$ (p, q are relatively prime). Find the value of $(p+q)$.

5. If ab^2c^3, $a^2b^3c^4$, $a^3b^4c^5$ are in A.P. where $a>0$, $b>0$, $c>0$. Then find the minimum value of $(a+b+c)$.

6. Let 4^{th} term of an arithmetic progression is 6 and m^{th} term is 18, if the A.P. has integral terms only then find the number of such A.P.'s.

7. Find the minimum value of the expression $\frac{2a}{b}+\sqrt{\frac{6b}{c}}+\sqrt[3]{\frac{9c}{a}}$ where $a>0, b>0, c>0$.

8. Let $a, ar_1, ar_1^2 \ldots\ldots \infty$ and $a, ar_2, ar_2^2, \ldots\ldots \infty$ are two different infinite G.P. of positive numbers with the same first term. The sum of first series is r_1 and sum of second series is r_2. If $(r_1-r_2)=\frac{1}{2}$ then find the value of $16a$.

9. If a, b, c are positive numbers in A.P. and $(a+2b-c)(4b+2c-2a)(c+a-b)=\lambda abc$, then find the value of λ.

10. Let x be the G.M. of two positive numbers and y and z be two A.M.'s inserted between the numbers then $(2y-z)\frac{(2z-y)}{x^2}$ equals

11. If the roots of the equation $ax^3+bx^2-x+1=0$ are in H.P. then $\left|\frac{b_{\max}}{a_{\min}}\right|$ equals

12. Let $a_1, a_2, a_3, \ldots..$ be terms of an arithmetic progression such that $\dfrac{a_1+a_2+\ldots\ldots+a_p}{a_1+a_2+\ldots\ldots+a_q}=\dfrac{p^2}{q^2}$, $p \neq q$. If $\dfrac{a_6}{a_{21}}=\dfrac{m}{n}$ (where m and n are in their lowest form), then find the value of $(4m-n)$.

13. Let $a+ar+ar^2+ar^3+\ldots\ldots\infty$ be a geometric sequence whose sum is 3 and whose first term is number of non negative integral solutions of $\log_{2x+3} x^2 < \log_{2x+3}(2x+3)$ then find the value of $\dfrac{3}{2}+r+r^2+r^3\ldots\ldots\infty$.

14. Let S_n be the sum of first n terms of an A. P. and P_n be the sum of next n terms of the A.P. such that $P_n = 3S_n$. Let T_n be the n^{th} term of the A.P. . If $S_n = 9T_1 + T_n$ then find n.

15. Suppose $\{a_1, a_2, a_3, \ldots\ldots\}$ is a geometric sequence of real numbers. The sum of the first n terms is denoted by S_n. If $S_{10}=10$ and $S_{30}=70$, then find the value of $\dfrac{S_{40}}{50}$.

16. If a, b, c, d, e are positive real numbers such that $a+b+c+d+e=15$ and $ab^2c^3d^4e^5=(120)^3\cdot 50$, then find the value of $a^2+b^2+c^2+d^2-e^2$.

17. If $\quad x=\cos 2\theta-2\cos^2 2\theta+3\cos^3 2\theta-4\cos^4 2\theta+\ldots\ldots\infty$

and $\quad y=\cos 2\theta+2\cos^2 2\theta+3\cos^3 2\theta+4\cos^4 2\theta+\ldots\ldots\infty$

where $\theta \in \left(0, \dfrac{\pi}{4}\right)$, then find least integral value of $\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$.

18. Consider a sequence $a_1, a_2, a_3, \ldots\ldots$ of real numbers and let the sequence $T_1, T_2, T_3\ldots$ defined by $T_r = a_{r+1}-a_r$ be such that its terms are in A.P. with common difference 2. If $T_1=2$ and $a_1=3$, then find the value of $\left(365-\sum_{n=1}^{10} a_n\right)$.

19. A set of consecutive positive integers starting with 1 is written. If one number is deleted except extreme numbers and thus the A.M. of the remaining numbers is found to be 10. If deleted number is k, then find $(k-5)$.

20. If a, b, c are unequal real numbers and a, b, c are in G.P. and $a+b+c=bx$, then find the least positive integral value of x.

ANSWER KEY

Single Correct Choice Type

1. (c) 2. (b) 3. (d) 4. (c) 5. (b) 6. (b) 7. (c) 8. (d)
9. (b) 10. (a) 11. (d) 12. (b) 13. (b) 14. (a) 15. (a) 16. (a)
17. (c) 18. (c) 19. (b) 20. (b) 21. (a) 22. (c) 23. (a) 24. (a)
25. (b) 26. (b) 27. (b) 28. (b) 29. (a) 30. (b) 31. (c) 32. (d)
33. (d) 34. (c) 35. (c) 36. (a) 37. (d) 38. (b) 39. (c) 40. (b)

Linked Comprehension Type

1. (a) 2. (c) 3. (c) 4. (a) 5. (a) 6. (b) 7. (a) 8. (a)
9. (d) 10. (c)

Multiple Correct Choice Type

1. (b,c) 2. (a,b,c) 3. (a,b,c,d) 4. (a,b,c,d) 5. (a,c) 6. (a,d)
7. (a,c,d) 8. (b,d) 9. (b,c) 10. (a,c) 11. (a,b) 12. (a,c,d)
13. (b,c) 14. (c,d) 15. (b,c)

Match the Column Type

1. $(a \to q, r, s)$; $(b \to r)$; $(c \to p)$; $(d \to q)$
2. $(a \to q)$; $(b \to r)$; $(c \to q)$; $(d \to r)$

Integer Answer Type

1. 5 2. 4 3. 2 4. 5 5. 3 6. 9 7. 6 8. 3
9. 8 10. 1 11. 9 12. 3 13. 2 14. 4 15. 3 16. 5
17. 5 18. 5 19. 5 20. 4