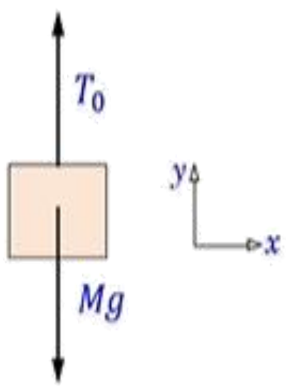
Example. 01

A block of mass M is suspended with the help of a uniform rope of mass m and length l from the ceiling as shown in the figure. Find expression for the tensile force in the rope at a distance x above the lower end.

Solution.

Tensile forces at lowest end of rope:

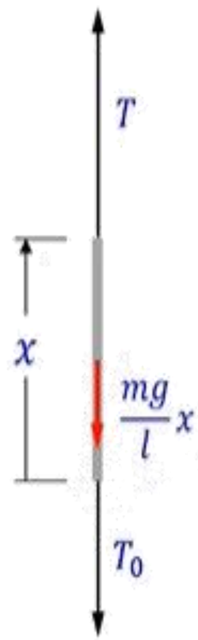
TE of block.



$$\Sigma F_y = 0 \quad \Rightarrow T_0 = Mg \quad ....(1)$$

Tensile forces at distance x above the lower end:

TE of this portion of the rope.



Mass of x length of the rope $= \dfrac{m}{l}x$

Weight of x length of the rope $= \dfrac{mg}{l}x$

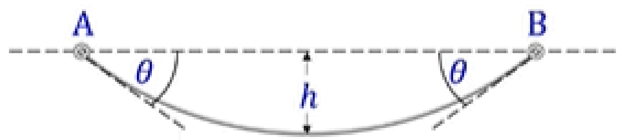
$$\Sigma F_y = 0 \quad \Rightarrow T = T_0 + \frac{mx}{l}g \quad ....(2)$$

From the above two eq. (1) and (2)

$$\Rightarrow T = Mg + \frac{mg}{l}x$$

Example. 02

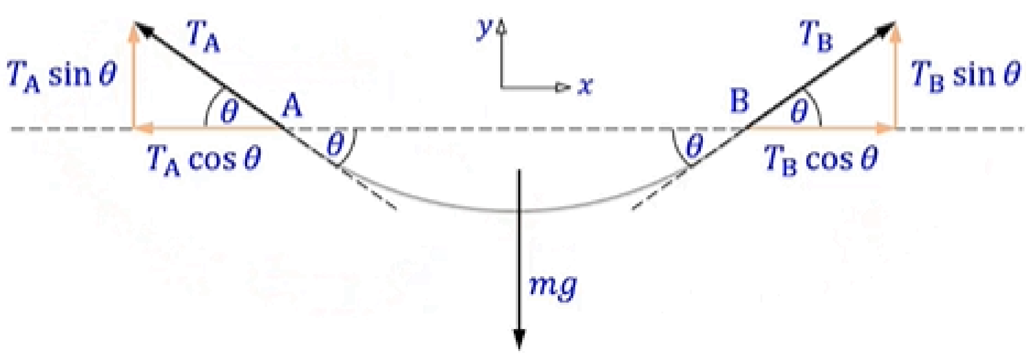
A uniform rope of mass m and length l is suspended from two fixed nails A and B that are in the same horizontal level. The tangents to the rope make an angle θ with the horizontal at the nails as shown in the figure.



(a) Find tensile forces in the rope at the end and the lowest points.

Solution.

Tensile forces at the nails: TE of the full rope.



$$\Sigma F_x = 0 \quad \Rightarrow T_A \cos\theta = T_B \cos\theta$$

$$\Rightarrow T_A = T_B$$

$$\Sigma F_y = 0 \quad \Rightarrow T_A \sin\theta + T_B \sin\theta = mg$$

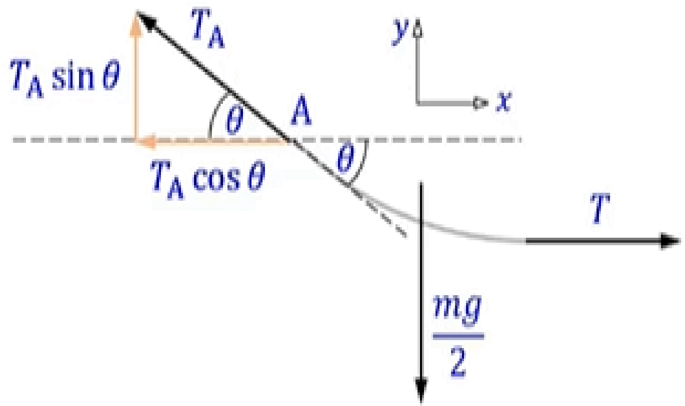
From the above two eq.

$$\Rightarrow T_A = T_B = \frac{mg}{2\sin\theta}$$

Tensile force at the lowest point: TE of the half rope.

Here both the halves are in identical conditions due to symmetry, so consider TE of any of the halves.

TE of the left half.



$$\Sigma F_x = 0 \qquad \Rightarrow T = T_A \cos\theta$$

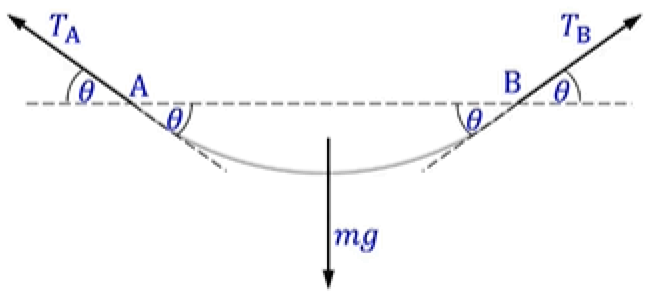
$$\Sigma F_y = 0 \qquad \Rightarrow T_A \sin\theta = \frac{mg}{2}$$

From the above two equations, we get:

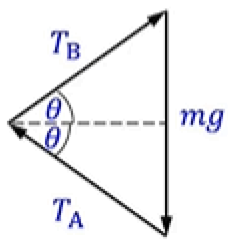
$$T = \frac{mg}{2} \cot\theta$$

Graphical Method:

TE of the full rope:

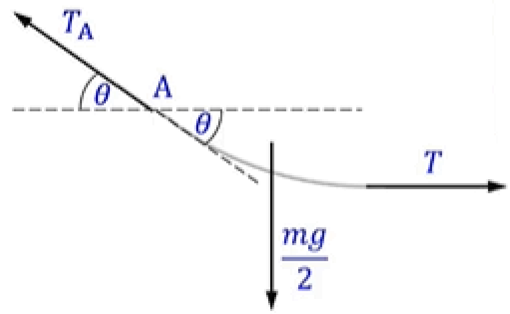


$$m\vec{g} + \vec{T}_A + \vec{T}_B = \vec{0}$$

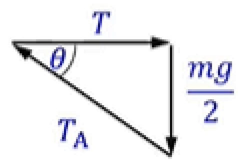


$$\Rightarrow T_A = T_B = \frac{mg}{2\sin\theta} = \frac{mg}{2}\operatorname{cosec}\theta$$

TE of the half rope:



$$\frac{m\vec{g}}{2} + \vec{T}_A + \vec{T} = \vec{0}$$



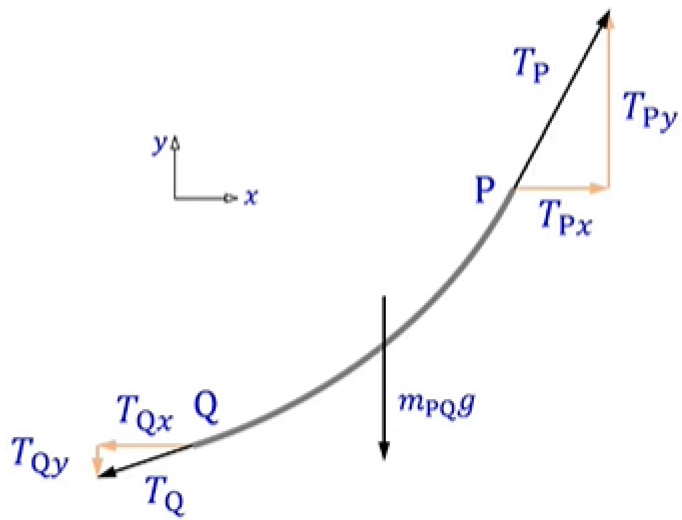
$$\Rightarrow T = \frac{mg}{2}\cot\theta$$

Note:
To analyse situations, where sum of three vectors is a null vector, graphical method should be preferred.

(b) What do you conclude for the vertical and horizontal components of the tensile force at different places of the rope?

Solution.

TE of a portion PQ of the rope.



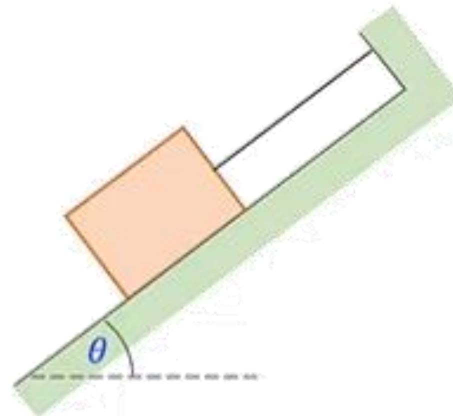
$$\Sigma F_x = 0$$

$$\Rightarrow T_{Px} = T_{Qx}$$

Horizontal component of the tensile forces in a rope suspended from its ends remains uniform in the rope.

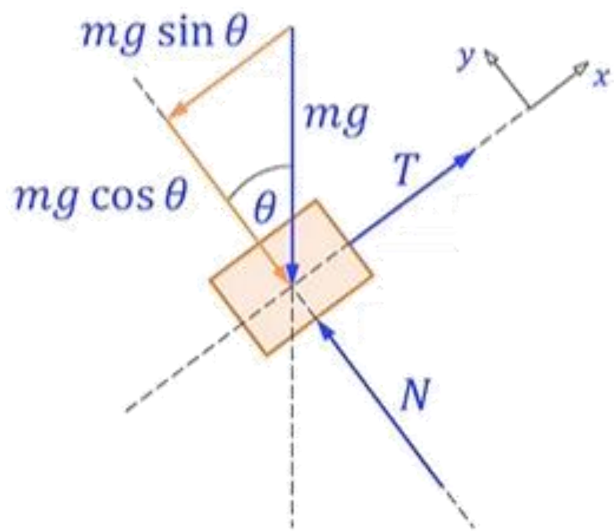
Example 03.

A box of mass m is held in equilibrium on a fixed frictionless inclined plane with the help of a string. Find tensile force in the string and the force of normal reaction between the box and the inclined plane.



Solution.

TE of the box.

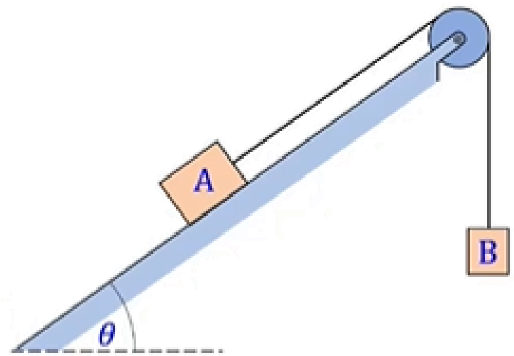


$\Sigma F_x = 0 \quad \Rightarrow T = mg \sin\theta$

$\Sigma F_y = 0 \quad \Rightarrow N = mg \cos\theta$

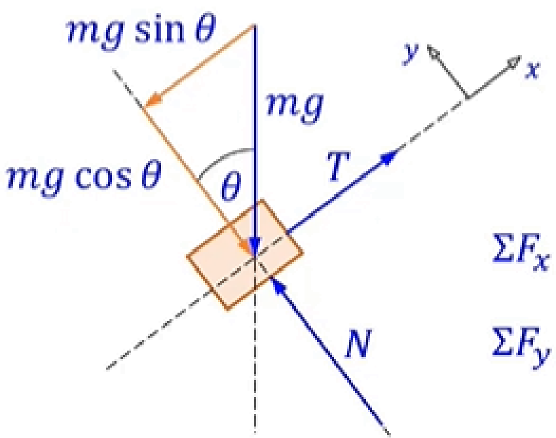
Example 04.

Block A of mass m placed on a frictionless inclined plane is connected by a string with another block B of mass M as shown in the figure. If the setup is in equilibrium, express M in terms of m and the angle θ of inclination.



Solution.

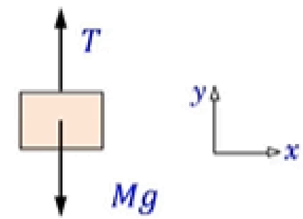
TE of the block A.



$$\Sigma F_x = 0 \Rightarrow T = mg\sin\theta \quad \text{..........} (1)$$

$$\Sigma F_y = 0 \Rightarrow N = mg\cos\theta \quad \text{..........} (2)$$

TE of the block B.



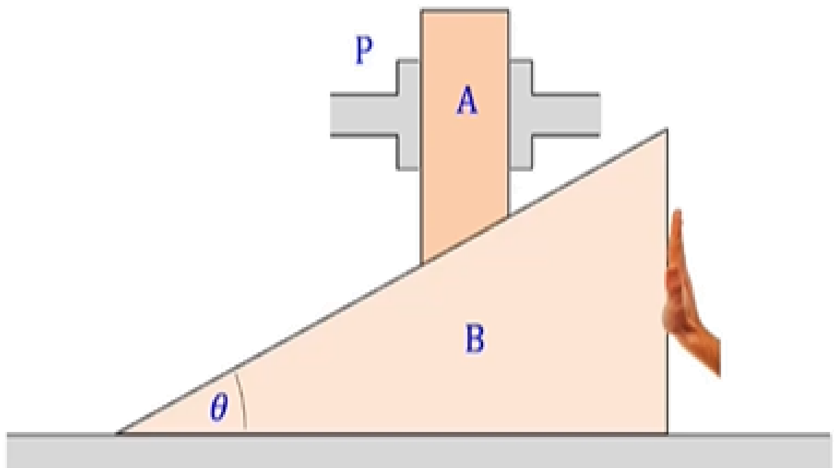
$$\Sigma F_y = 0 \quad \Rightarrow T = Mg \quad \text{..........} (3)$$

From eq. (1) and (3), we get:

$$M = m\sin\theta$$

Example 05.

Rod A can slide vertically up and down in a fixed guide P and a wedge B horizontally on the floor. All the surfaces in contact are frictionless. Masses of rod and the wedge are m_A and m_B. The setup is held at rest by applying an appropriate push as shown in the figure.



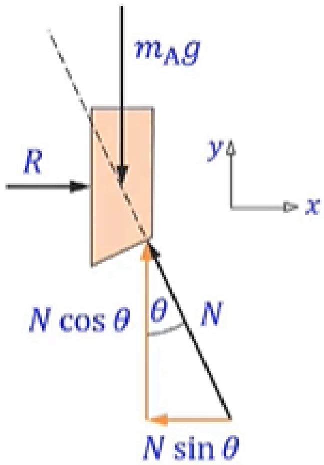
(a) Find the push F by the hand to maintain the setup at rest.

(b) Find the net reaction force R by the fixed guide on the rod.

(c) Find the mutual normal reaction N between the rod and the wedge.

Solution.

TE of the rod.



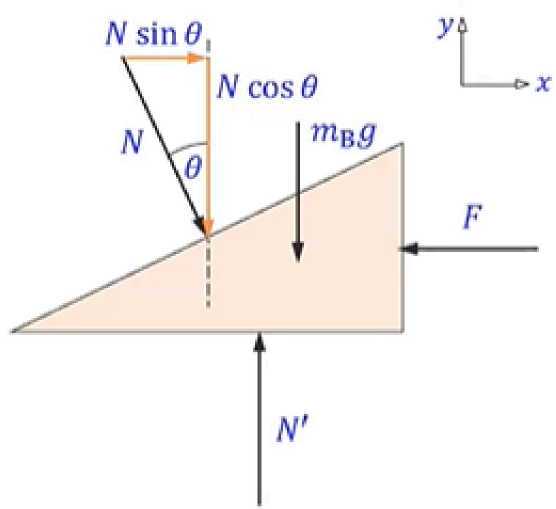
$\Sigma F_x = 0$

$\Rightarrow R = N\sin\theta$ .... (1)

$\Sigma F_y = 0$

$\Rightarrow N\cos\theta = m_A g$ .... (2)

TE of the wedge.



$\Sigma F_x = 0$

$\Rightarrow F = N\sin\theta$ .... (3)

$\Sigma F_y = 0$

$\Rightarrow N' = N\cos\theta + m_B g$ ...... (4)

From eq. (1) and (2)

$$R = m_A g \tan\theta$$

From eq. (2) and (3)

$$F = m_A g \tan\theta$$

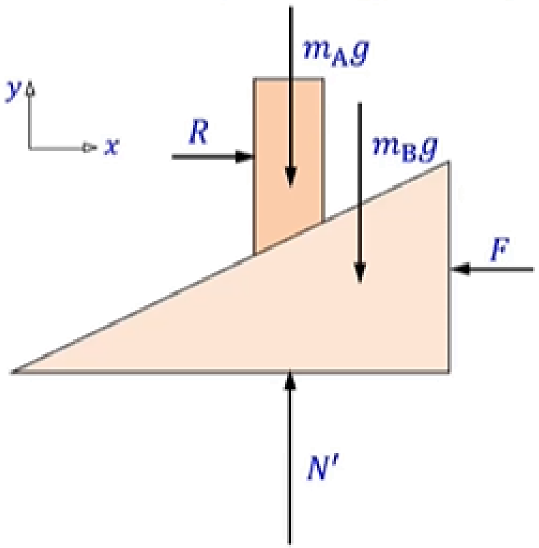
From eq. (2)

$$N = m_A g \sec\theta$$

Alternate approach:

There is no relative motion also no relative acceleration between the rod and the wedge, thus, we can treat them as a single rigid body.

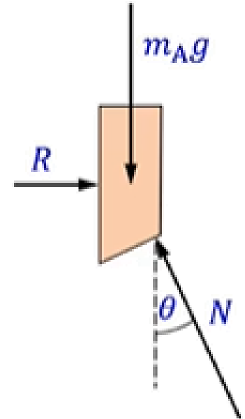
TE of this composite rigid body:



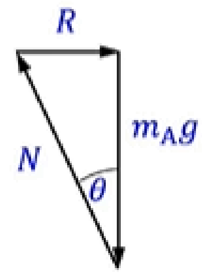
$$\Sigma F_x = 0 \quad \Rightarrow R = F \quad \text{..................} \ (1)$$

$$\Sigma F_y = 0 \quad \Rightarrow N' = m_A g + m_B g \ \text{.....} \ (2)$$

TE of the rod.



$$m_A \vec{g} + \vec{N} + \vec{R} = \vec{0}$$



$$R = m_A g \tan\theta \quad \text{.....} \ (3)$$

$$N = m_A g \sec\theta \quad \text{.....} \ (4)$$

From eq. (1) and (3)

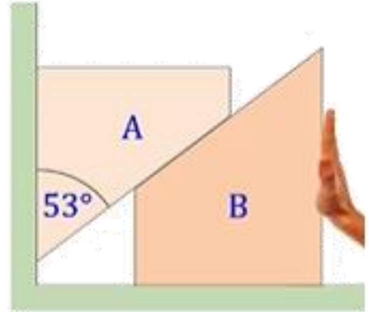
$$R = F = m_A g \tan\theta$$

From eq. (4)

$$N = m_A g \sec\theta$$

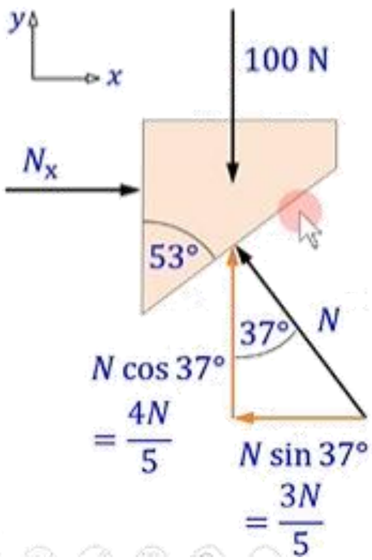
Example 06.

In the setup shown, two identical wedges each of mass 10 kg are held standstill against corner of a room. Calculate the necessary horizontal force applied on B to maintain the setup standstill. All surfaces in contact are frictionless. ($g = 10\ \mathrm{m/s^2}$)



Solution. N: Mutual normal reaction, N_x: normal reaction by the wall and N_y: normal reaction by the floor.

TE of the wedge A: $\Sigma F_y = 0$



$$\Rightarrow \frac{4N}{5} = 100$$

$$\Rightarrow N = 125\ \mathrm{N} \quad \text{........ (1)}$$

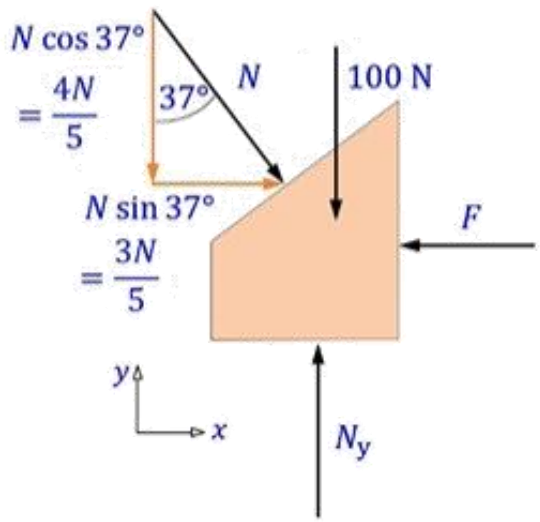
$$\Sigma F_x = 0$$

$$\Rightarrow N_x = \frac{3N}{5}$$

Now from eq. (1)

$$\Rightarrow N_x = 75\ \mathrm{N} \quad \text{........ (2)}$$

TE of the wedge B: $\Sigma F_x = 0$



$$\Rightarrow F = \frac{3N}{5}$$

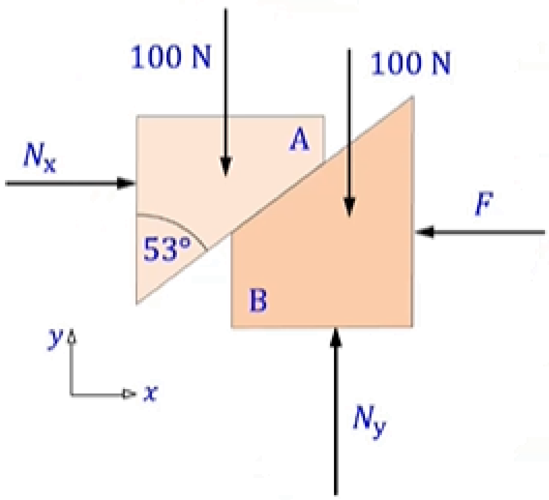
Now from eq. (1)

$$\Rightarrow F = 75\ \mathrm{N}$$

Alternate approach:

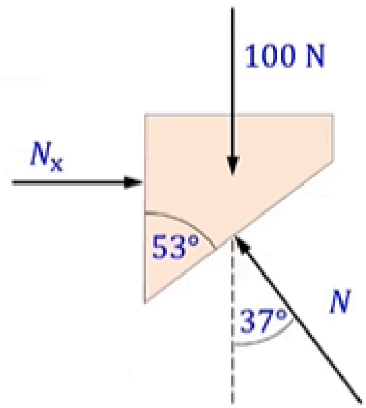
There is no relative motion also no relative acceleration between both the wedges, thus, we can treat them as a single rigid body.

TE of this composite rigid body:

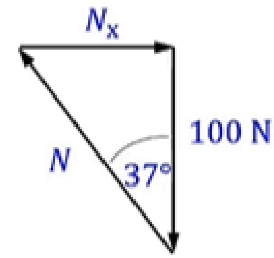


$$\Sigma F_x = 0 \quad \Rightarrow F = N_x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

TE of the wedge A:



$$m_A\vec{g} + \vec{N} + \overrightarrow{N_x} = \vec{0}$$



From eq. (1) and the above figure:

$$\Rightarrow F = N_x = 100 \tan 37° = 100 \times \frac{3}{4}$$

$$\Rightarrow F = 75\text{ N}$$