NLM
LECTURE - 3

Translational Equilibrium:

A body at rest or moving with a constant velocity is said to be in translational equilibrium.

State of rest is known as static equilibrium and state of uniform velocity motion as dynamic equilibrium.

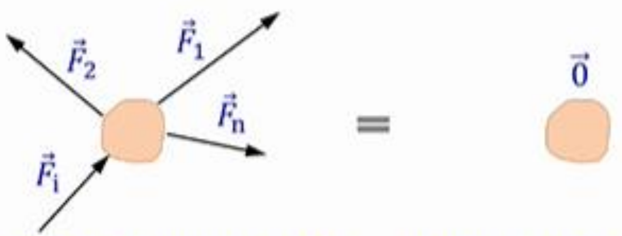
A body in translational equilibrium, has no acceleration.

$$\vec{a} = \vec{0}$$

Condition for translational equilibrium:

If a body is in translational equilibrium, resultant of all the external forces $\vec{F}_1$, $\vec{F}_2$, ... $\vec{F}_i$ .... $\vec{F}_n$ acting simultaneously on it will be a null vector.

$$\Sigma\vec{F} \quad = \quad m\vec{0} = \vec{0}$$



Free Body Diagram (FBD) **Kinetic Diagram (KD)**

$$\sum_{i=1}^{n} \vec{F}_i = \vec{0} \qquad \text{Or} \qquad \Sigma\vec{F} = \vec{0}$$

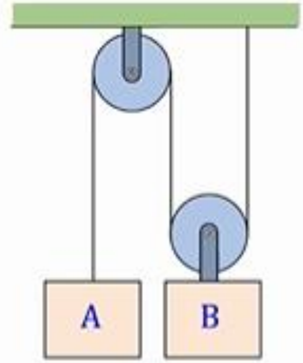
If the force are expressed in Cartesian components, we have :

$$\sum_{i=1}^{n} F_{ix} = 0 \qquad \sum_{i=1}^{n} F_{iy} = 0 \qquad \sum_{i=1}^{n} F_{iz} = 0$$

$$\text{Or} \quad \Sigma F_x = 0 \qquad \text{Or} \quad \Sigma F_y = 0 \qquad \text{Or} \quad \Sigma F_z = 0$$

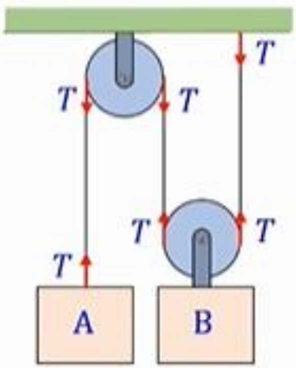
Example 01.

Two blocks A and B of masses m_{A} and m_{B} are suspended by a system of pulleys as shown. Find relation between m_{A} and m_{B}.

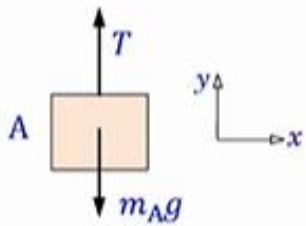


Solution.

Transmission of string tension.



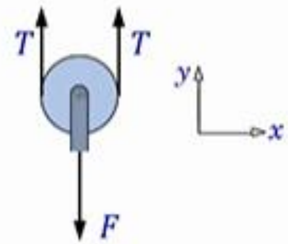
TE of the block A.



$\Sigma F_y = 0$

$\Rightarrow T = m_{\mathrm{A}}g$ ...... (1)

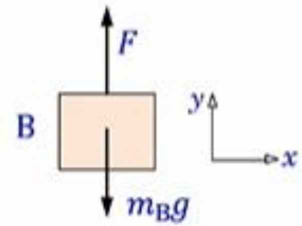
TE of the pulley supporting the block B.



$\Sigma F_y = 0$

$\Rightarrow F = 2T$ .... (2)

TE of the block B.

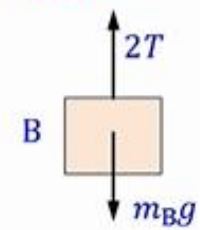


$\Sigma F_y = 0$

$\Rightarrow F = m_{\mathrm{B}}g$ .... (3)

From the eq.. (1), (2) and (3), we get: $\Rightarrow 2m_{\mathrm{A}} = m_{\mathrm{B}}$

Note.

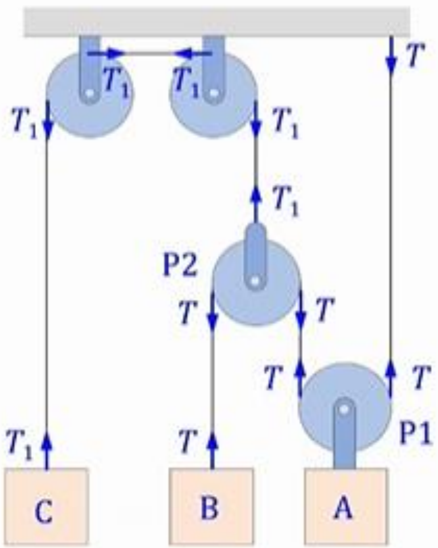


Example 02.

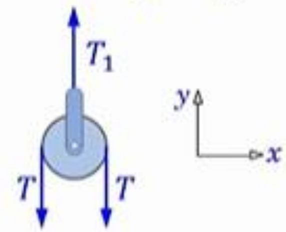
The setup is in equilibrium. If masses of the blocks A is m_A, find mass of the blocks B and C.

Solution.

Transmission of string tensions.

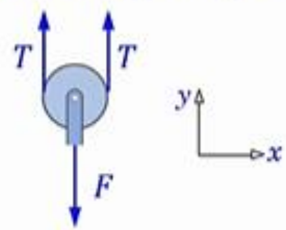


TE of the pulley P2.



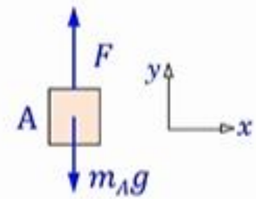
$\Sigma F_y = 0 \;\; \Rightarrow T_1 = 2T$ ...... (1)

TE of the pulley P1.



$\Sigma F_y = 0 \Rightarrow F = 2T$ ...... (2)

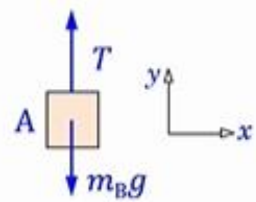
TE of the block A.



$\Sigma F_y = 0$

$\Rightarrow F = m_A g$ ...... (3)

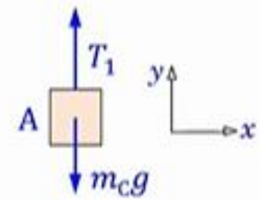
TE of the block B.



$\Sigma F_y = 0$

$\Rightarrow T = m_B g$ ...... (4)

TE of the block C.



$\Sigma F_y = 0$

$\Rightarrow T_1 = m_C g$ ...... (5)

From the eq.. (1), (2), (3), (4) and (5), we get:

$$\Rightarrow m_B = \frac{m_A}{2}$$

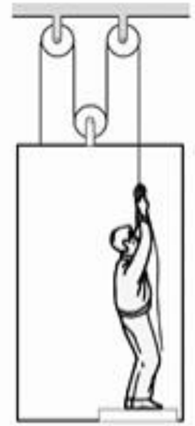
$$\Rightarrow m_C = m_A$$

Example 03.

A 70 kg man standing on a weighing machine in a 50 kg lift pulls on the rope, which supports the lift as shown in the figure.

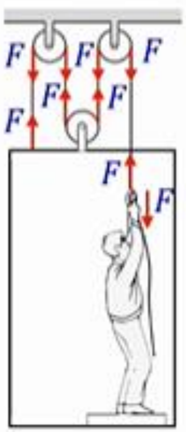
(a) Find the force with which the man should pull on the rope to keep the lift stationary.

(b) Find the weight of the man as shown by the weighing machine.

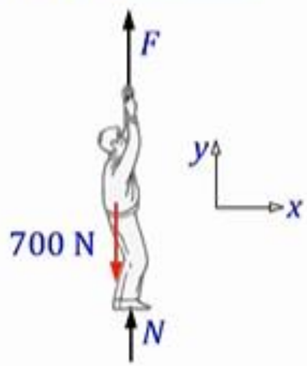


Solution.

Transmission of string tension.



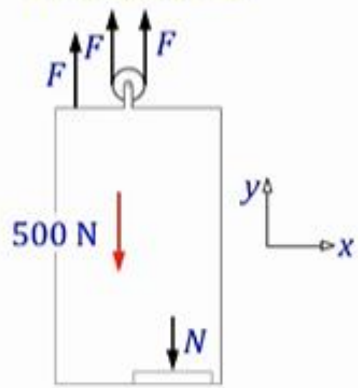
TE of the man.



$$\Sigma F_y = 0$$

$$\Rightarrow F + N = 700$$

TE of the lift.



$$\Sigma F_y = 0$$

$$\Rightarrow 3F = N + 500$$

(a) Force of the man on the rope: F

From the previous two equations:

$$F = 300 \text{ N}$$

(b) Weight of the man shown by the weighing machine is the normal reaction of man on the weighing machine: N

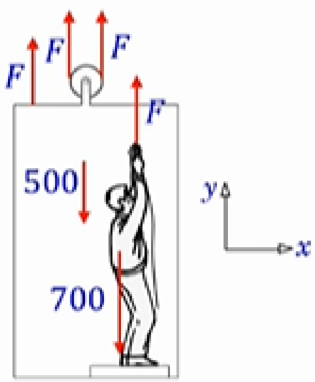
From the two equations:

$$F = 400 \text{ N}$$

Alternate Approach:

When the man maintains the setup in equilibrium, there is no relative movements (in fact acceleration) between the man and the lift; therefore, we can assume all of three a single rigid body.

TE of this composite rigid body.

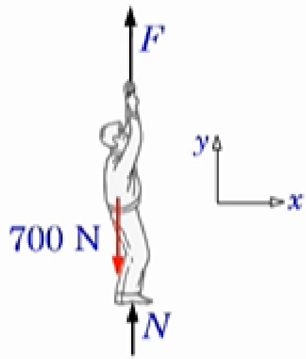


$\Sigma F_y = 0$

$\Rightarrow 4F = 500 + 700$

$\Rightarrow F = 300$ N

TE of the man.



$\Sigma F_y = 0 \quad \Rightarrow F + N = 700$

Substituting value of F.

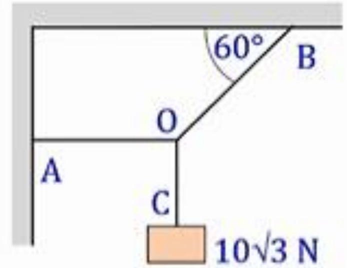
$\Rightarrow N = 400$ N

Note:

Here, we have use this idea of composite body (constituent bodies have no relative acceleration) on the basis of intuitive understanding. But in the next lecture, we will discus the laws of physics behind this.

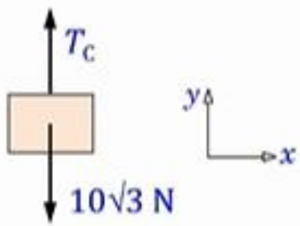
Example 04.

A box of weight $10\sqrt{3}$ N is held in equilibrium with the help of three strings OA, OB and OC as shown in the figure. The string OA is horizontal. Find the tensions in both the strings.



Solution.

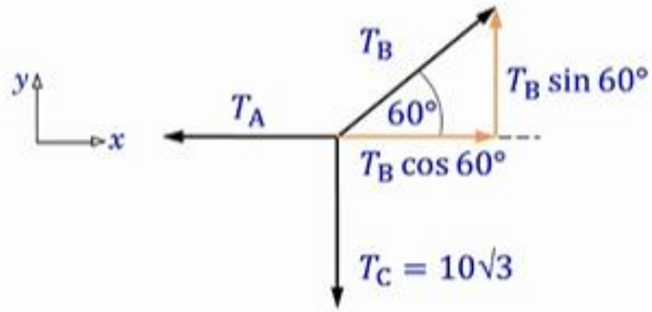
Conditions of (TE) of the box:



$$\Sigma F_y = 0 \quad \Rightarrow T_C - 10\sqrt{3} = 0$$

$$\Rightarrow T_C = 10\sqrt{3} \quad \text{........} \ (1)$$

Conditions of TE of the knot:



$$\Sigma F_y = 0 \quad \Rightarrow T_B \sin 60° = T_C$$

$$\Rightarrow T_B \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$$

$$\Rightarrow T_B = 20 \text{ N} \ \text{.......} \ (2)$$

$$\Sigma F_x = 0 \quad \Rightarrow T_A = T_B \cos 60°$$

$$\Rightarrow T_A = 20 \times \frac{1}{2}$$

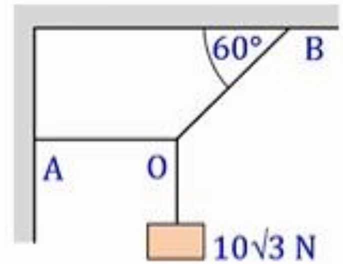
$$\Rightarrow T_A = 10 \text{ N} \quad \text{......}(3)$$

Here, the calculations can be performed by using triangle law of vector addition in stead of analytical method (cartesian components).

Known as graphical method, this approach, usually provides great simplification.

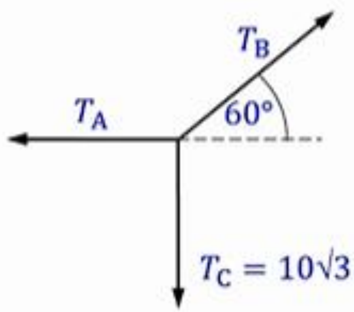
Graphical Method.

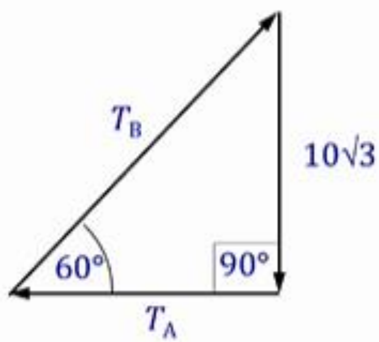
This method is based on the triangle law of vector addition.



The physical situation of the given setup reveals that for the translational equilibrium of the knot O, the vector sum of all the three forces acting on it must be a null vector.

$\Sigma\vec{F} = \vec{0} \qquad \Rightarrow \vec{T}_A + \vec{T}_B + \vec{T}_C = 0$





From TE of the box, we get : $T_C = 10\sqrt{3}$ N

From the geometry of the right-angled triangle, we get

$$T_A = 10\sqrt{3}\cot 60^\circ = 10\sqrt{3} \times \frac{1}{\sqrt{3}} = 10 \text{ N}$$

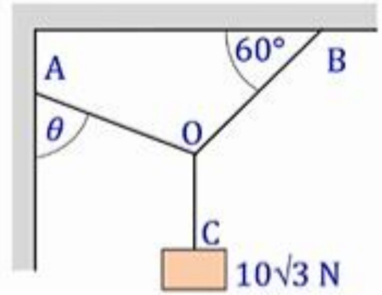
$$T_B = 10\sqrt{3}\,\mathrm{cosec}\, 60^\circ = 10\sqrt{3} \times \frac{2}{\sqrt{3}} = 20 \text{ N}$$

Note:

The graphical method, usually provides great simplification in dealing with TE with three simultaneous forces.

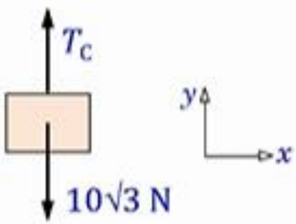
Example 05.

A box of weight $10\sqrt{3}$ N is held in equilibrium with the help of three strings OA, OB and OC as shown in the figure. You can change the location of the point A on the wall and hence vary the angle θ. Find the minimum tensile force in the string OA and the corresponding value of the angle θ.



Solution. Analytical Method.

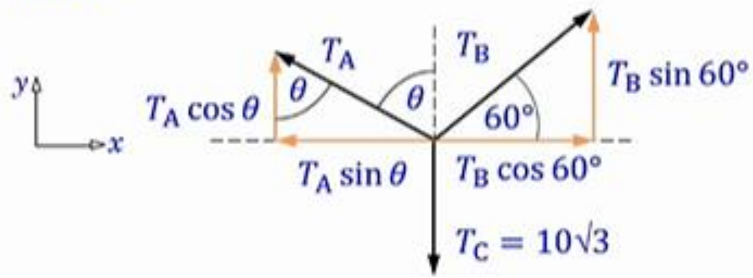
Applying conditions of (TE) of the box:



$$\Sigma F_y = 0 \Rightarrow T_C - 10\sqrt{3} = 0$$

$$\Rightarrow T_C = 10\sqrt{3} \text{ ...... (1)}$$

Applying conditions of TE of the knot:



$$\Sigma F_x = 0 \Rightarrow T_A \sin\theta = T_B \cos 60°$$

$$\Rightarrow T_B = 2T_A \sin\theta \text{ .... (2)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow T_A \cos\theta + T_B \sin 60° = 10\sqrt{3} \text{ ..... (3)}$$

Substituting T_B from eq. (2) in eq. (3) we have

$$T_A = \frac{10\sqrt{3}}{\sqrt{3}\sin\theta + \cos\theta} \text{ .... (4)}$$

The above equation expresses the tensile force T_A as function of the angle θ.

According to this function, for the minimum value of T_A, the denominator must be maximum.

For an extremum i.e. either a maxima or minima:

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sqrt{3}\sin\theta + \cos\theta\right) = 0$$

$$\Rightarrow \sqrt{3}\frac{\mathrm{d}}{\mathrm{d}\theta}\sin\theta + \frac{\mathrm{d}}{\mathrm{d}\theta}\cos\theta = 0$$

$$\Rightarrow \sqrt{3}\cos\theta - \sin\theta = 0$$

$$\Rightarrow \tan\theta = \sqrt{3}$$

$$\Rightarrow \theta = 60°$$

Substituting this value in eq. (4):

$$T_{\mathrm{Amin}} = \frac{10\sqrt{3}}{\sqrt{3}\sin 60° + \cos 60°}$$

$$\Rightarrow T_{\mathrm{Amin}} = \frac{10\sqrt{3}}{\sqrt{3}\times\frac{\sqrt{3}}{2} + \frac{1}{2}}$$

$$\Rightarrow T_{\mathrm{Amin}} = 5\sqrt{3}\ \mathrm{N}$$

Another way of calculation.

$$D = \sqrt{3}\sin\theta + 1\times\cos\theta$$

Let us substitute:

$r\cos\varphi = \sqrt{3}$ and $r\sin\varphi = 1$

$$\Rightarrow D = r\cos\varphi\sin\theta + r\sin\varphi\cos\theta$$

$$\Rightarrow D = r(\cos\varphi\sin\theta + \sin\varphi\cos\theta)$$

$$\Rightarrow D = r\sin(\theta + \varphi)$$

Substituting r and φ :

$D_{\max} = r = \sqrt{\left(\sqrt{3}\right)^2 + 1} = 2$ and $\tan\varphi = \frac{1}{\sqrt{3}} \Rightarrow \theta = 60°$

Substituting this maximum value of denominator in eq. (4), we get:

$$T_{\mathrm{Amin}} = \frac{10\sqrt{3}}{D_{\max}} = \frac{10\sqrt{3}}{2} = 5\sqrt{3}\ \mathrm{N}$$

Note:

$r = a\sin\theta + b\cos\theta$

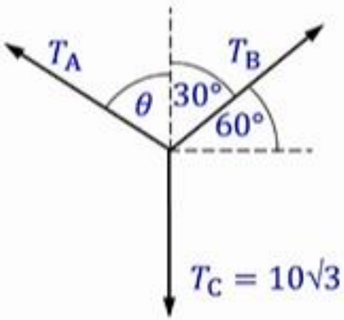
$\Rightarrow r_{\max} = \sqrt{a^2 + b^2}$

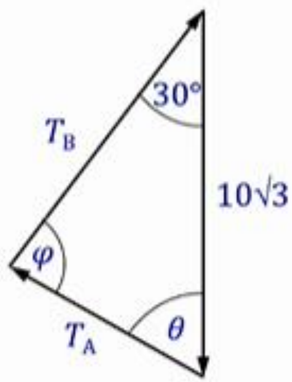
This idea will be very quick, if only $r_{\max}$ has to be found.

Graphical Method.

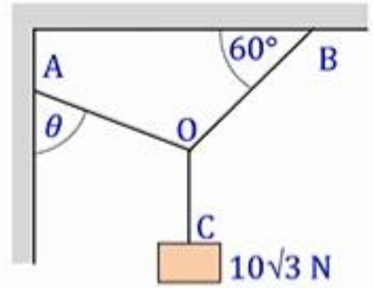
The physical situation of the given setup reveals that for the knot O to be in equilibrium, the vector sum of all the three forces acting on it must be a null vector.

$\Sigma\vec{F} = \vec{0}$ $\quad\Rightarrow \vec{T}_A + \vec{T}_B + \vec{T}_C = 0$





From TE of the box, we get : $T_C = 10\sqrt{3}$ N



From the figure, it is clear that for the vector T_A to be minimum, it must be perpendicular to the vector T_B. Therefore angle θ must be equal to 60° and $\varphi = 90°$.

$$T_{A\min} = 10\sqrt{3}\sin 30° \quad = 5\sqrt{3}\ \text{N}$$

Or

$$T_{A\min} = 10\sqrt{3}\cos\theta = 10\sqrt{3}\cos 60° \quad = 5\sqrt{3}\ \text{N}$$

Note:

Here, you can realize the simplicity brought in calculations by the graphical method.

In situations, where right-angled triangle is not formed, sin rule or Lami's theorem has to be used.