# Unit 3: ELECTRODYNAMICS

## Q3.1. Describe three different types of Coordinate Systems

In electromagnetics, most of the quantities are either functions of space or time. In order to describe the spatial variations of these quantities, all the points in space must be defined uniquely using an appropriate coordinate system.

The most useful three coordinate systems for this purpose are:-

1. Cartesian, or rectangular, coordinates
2. Cylindrical, or circular, coordinates
3. Spherical, or polar, coordinates.

1. Cartesian or Rectangular Coordinates (x, y, z)

A point P in Cartesian coordinates is represented as P(x, y, z) as shown in *Figure 3.1.1*

The ranges of coordinate variables are

$-\infty < x < \infty$

$-\infty < y < \infty$

$-\infty < z < \infty$

A vector $\vec{A}$ in the Cartesian coordinate system is written as,

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Where, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along the x, y and z directions respectively.

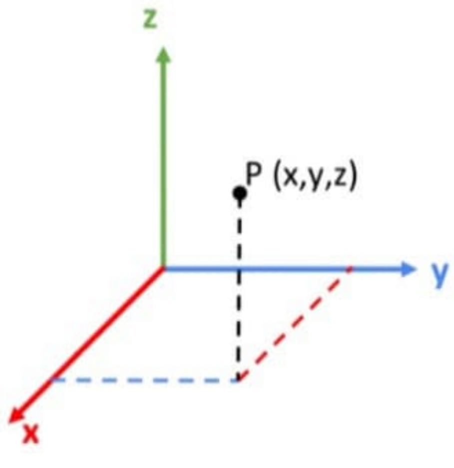

*Figure 3.1.1: Cartesian coordinates*

2. Cylindrical Coordinates P (r, ϕ, z)

A point P in cylindrical coordinates is represented as P(r, ϕ, z) as shown in *Figure 3.1.2* ,

The Ranges of Coordinates are,

$$0 \leq r < \infty$$

$$0 \leq \emptyset < 2\pi$$

$$-\infty < z < \infty$$

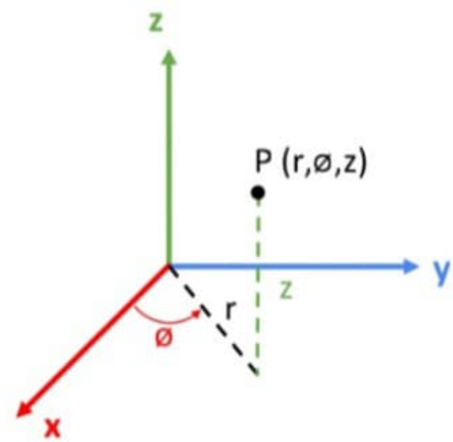

Figure 3.1.2: Cylindrical coordinates

Where,

r=Radius of the cylinder passing through P

ϕ=Angle from the X-axis in the xy-plane, known as azimuthal angle

z= same as in Cartesian coordinates

A vector '$\vec{A}$' in cylindrical coordinate system is written as,

$$\overrightarrow{A} = A_r\hat{r} + A_{\emptyset}\hat{\emptyset} + +A_z\hat{z}$$

Where $\hat{r}, \hat{\emptyset}$ and $\hat{z}$ are the unit vectors along the r, ø and z directions respectively.

3.Spherical or Polar Coordinates (r,θ,ø)

A point P in spherical coordinates is represented as P(r,θ,ϕ) as shown in *Figure 3.1.3* . The ranges of coordinate variables are,

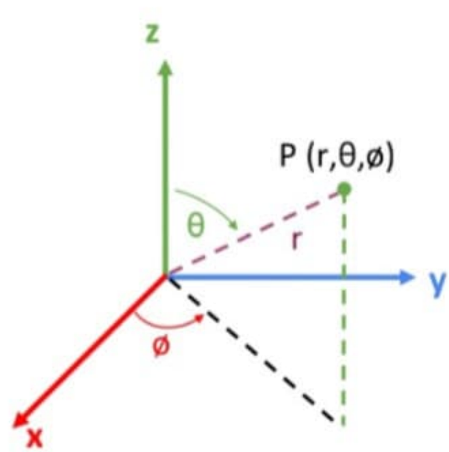

Figure 3.1.3: Spherical coordinates

$$0 \leq r < \infty$$

$$0 < \theta < \pi$$

$$0 \leq \emptyset < 2\pi$$

Where,

r= Distance of the point from the origin

θ = Angle between the z-axis and P

ϕ =Angle from the X-axis in the xy-plane, known as azimuthal angle

A vector ' $\vec{A}$ ' inspherical coordinate system is written as,

$$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$$

Where, $\hat{r}, \hat{\theta} and \hat{\phi}$ are the unit vectors along the r,θ,and ø directions respectively.

## Q3.2. Obtain relation between Cartesian and Spherical coordinates and also between Cartesian and Cylindrical coordinates

Relation between Cartesian(x,y,z) and Spherical (r,θ,ϕ) Coordinates

r=$\sqrt{x^2+y^2+z^2}$ θ=$\tan^{-1}(\frac{\sqrt{x^2+y^2}}{z})$ ϕ=$\tan^{-1}\frac{y}{x}$

And

x=r sinθ cosϕ y= r sinθ sinϕ z=r cosθ

Relations between Cartesian (x, y, z) and Cylindrical (r, ø, z) Coordinates

r=$\sqrt{x^2+y^2}$ ϕ=$\tan^{-1}\frac{y}{x}$ z = z

And

x=r cosϕ y= r sinϕ z = z

## Q3.3. Define a field. What are scalar and vector fields

**Field:** Behaviour of a physical quantity in a given region is described by its value at each point in the region, the function describing this value is called a field.

**Scalar field:** A scalar field is something that has a particular value at every point in space. An example of a scalar field is temperature.

**Vector field:** A vector field is something that has a particularvalue and direction at every point in space.An example of a vector field is Electric field.

## Q3.4. What is ∇ (DEL) Operator

The collection of partial derivative operators is called DEL operator. Hence DEL can be viewed as the derivative in multi-dimensional space.DEL operator is defined as a vector differential operator. A DEL operator is not a vector in itself, but when acts on a scalar function, it becomes a vector.DEL is not simply a vector; it is a vector operator. Whereas a vector is a quantity with both a magnitude and direction, DEL does not have a precise until it is allowed to operate on something.In Cartesian coordinate system Del operator is given as:

$$\vec{\nabla}=\hat{\mathrm{i}}\frac{\partial}{\partial \mathrm{x}}+\hat{\mathrm{j}}\frac{\partial}{\partial \mathrm{y}}+\hat{\mathrm{k}}\frac{\partial}{\partial \mathrm{z}}$$

This operator is useful or significant in defining

- Gradient of a scalar V: $(\vec{\nabla}V)$
- Divergence of a vector A:$(\vec{\nabla}.\vec{A})$
- Curl of a vector A: $(\vec{\nabla}\times\vec{A})$

## Q3.5. What is gradient of a scalar field? Express in Cartesian form

The gradient of a scalar field provides a vector field that states how the scalar value is changing throughout space – a change that has both a magnitude and direction.

The physical meaning of the gradient of a scalar is that it represents the steepness of the slope or line. For example, let T be the scalar function of temperature then the first term of the gradient defines rate of change of temperature along x axis, second term defines change along y axis and third term along z axis. It is mathematically represented as:

$$\vec{\nabla}\mathrm{T}=\hat{\mathrm{i}}\frac{\partial \mathrm{T}}{\partial \mathrm{x}}+\hat{\mathrm{j}}\frac{\partial \mathrm{T}}{\partial \mathrm{y}}+\hat{\mathrm{k}}\frac{\partial \mathrm{T}}{\partial \mathrm{z}}$$

Thus, the Gradient is also called the directional derivative of a scalar function.

## Q3.6. What is Curl of a Vector field ? Express in Cartesian form

Curl of a Vector Field (Curl A) in Cartesian Coordinate system is given as:

$$\vec{\nabla}\times\vec{A}=\text{determinant}\left|\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}\right|$$

The direction of the curl is the axis of rotation, as determined by the right hand thumb rule and the magnitude of the curl is the magnitude of rotation. If the curl of a vector field $\vec{A}$ is zero, then the vector field $\vec{A}$ is said to be irrotational. In such cases, the circulation of $\vec{A}$ around a closed path is zero; it implies that the line integral of $\vec{A}$ is independent of the chosen path. Hence an irrotational field is also known as a conservative field.

**Q3.7. Explain the physical significance of Divergence.**

**(M.U. Dec 2019) (5 m)**

Divergence of a vector 'A' $(\nabla . A)$ is the measure of the extent to which the vector 'A' spreads. In other words, $\nabla . A$ indicates how much vector A diverges (spreads out) from a given point.

$(\nabla . A)$indicates how much vector A diverges (spreads out) from a given point P.

$\int_0^S (\nabla . A)\, ds$ indicates how much vector A diverges (spreads out) from a given surface 'S'.

$\int_0^V (\nabla . A)\, dV$ indicates how much vector A diverges (spreads out) from a given Volume 'V'.

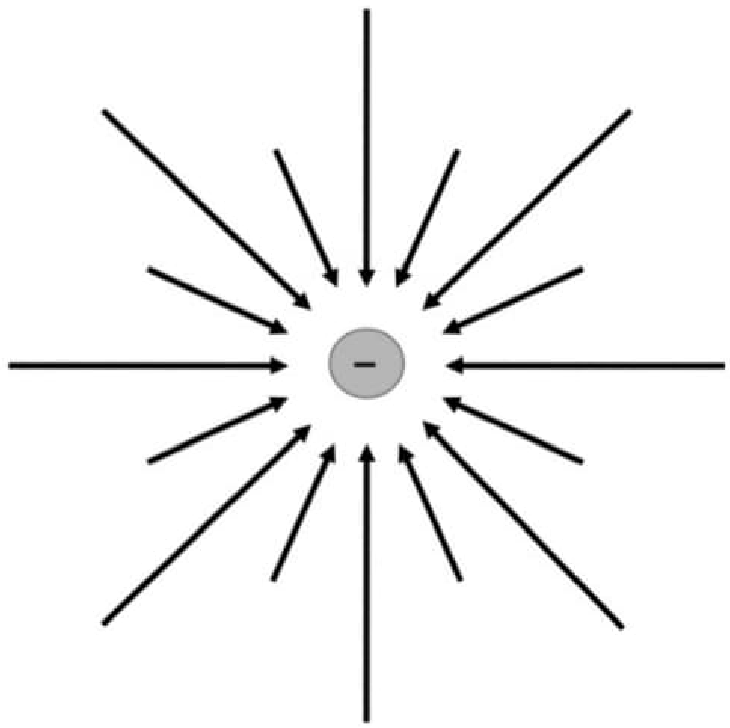

Figure 3.7.1: Negative divergence

The value of the divergence could be positive, negative or zero. Depending on its value following conclusions can be derived.

1. $(\nabla. A)$= Negative indicates that the vector A is closing in on point P. Then point P where divergence is found negative is called the sink for vector field A as shown in Figure 3.7.1.

2. $(\nabla. A)$= Positive indicates that the vector A is spreading out from point P. Then point P where divergence is found positive is called the source for vector field A as shown in Figure 3.7.2.

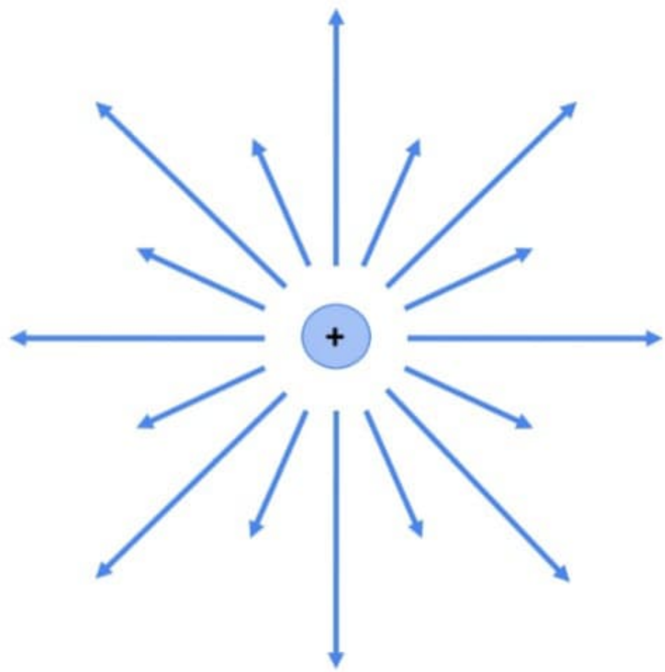


Figure 3.7.2: Positive divergence

3. $(\nabla. A)$=0 then A vector field is called solenoidal, incompressible vector field or divergence less vector field. This means that point 'P' is neither a source nor a sink. It also indicates the amount of field flux entering is equal to amount of the field flux leaving the point 'P' as shown in Figure 3.7.3.

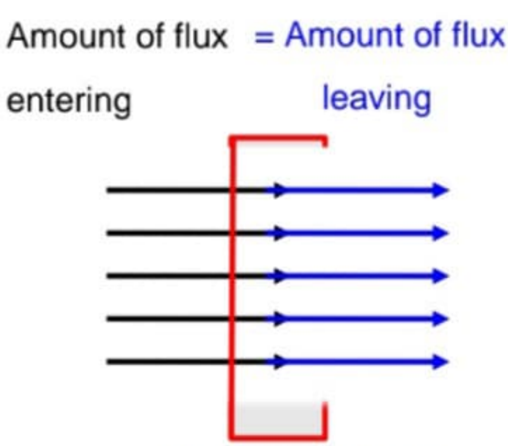


Figure 3.7.3: Zero divergence

**Q3.8. What is Divergence of a Vector field? Express in Cartesian form**

**(M.U. May 2017) (3 m)**

Divergence of a vector field $\vec{A}$in Cartesian coordinate system is given as:

$$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

When divergence is applied to a vector function it yields a scalar. Divergence of a vector field $\vec{A}$ is a measure of how much a vector field converges to or diverges from a given point. In simple terms it is a measure of the outgoingness of a vector field.A vector field with constant zero divergence is called solenoid or divergence less or incompressible ($\vec{\nabla} . \vec{A} = 0$). In such cases no net flow can occur across any closedsurface.

**Q3.9. Show that divergence of the curl of a vector is zero.**

**(M.U. Dec. 2017, 19) (3 m)**

**Ans.** Let $A = A_x i + A_y j + A_z k$

$$A = \nabla \times A = \text{determinant} \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) i - \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) j + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) k$$

Now, div(curl A) = $\nabla . (\nabla \times A)$

$$= \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$= 0$$

Hence divergence of a curl vanishes.

**Q3.10. State two important Theorem for electrodynamics.**

1. Divergence Theorem.
   This theorem states that volume integral of divergence of Vector $\vec{A}$ taken over volume 'V' equals surface integral of Vector $\vec{A}$ taken over surface 'S' enclosing the volume.

$$\int_0^V \vec{\nabla}.\vec{A}\, dv = \oint_0^S \vec{A}.\vec{dS}$$

2. Stokes Theorem

   The Surface integral of curl of a vector $\vec{A}$ over an open surface 'S' equals line integral of a vector $\vec{A}$ over the enclosed curve '$l$' surrounding surface area 'S'.

$$\int_0^s \overrightarrow{(\vec{\nabla}x\vec{A})}.\, dS = \oint_0^l \vec{A}.\vec{dl}$$

**Q3.11. Derive point as well as integral form of Maxwell's equations (M.U. Dec. 2017) (5 m)**

1. Maxwell's First Equation (Gauss Law for Electric Field).

   **Statement:**Electric flux passing through any closed surface 'S' is equal to the total charge enclosed by the surface. Mathematically

   $$ø = Q_{enclosed}/\mathcal{E}_0 \text{-----------}(1)$$

   Electric field E is electric flux ø per unit area, thus flux can be written as:

   $$ø = \oint_0^S \vec{E}.\vec{dS} \text{-------------}(2)$$

   And we know that change enclosed inside a closed volume V is

   $$Q_{enclosed} = \int_0^V \rho\, dV; \text{ -----------}(3)$$

Where, $\rho = \frac{\text{Charge}}{\text{Volume}}$ change density

   Using (1) ,(2) and (3) we get ;

   $$\oint_0^S \vec{E}.\vec{dS} = \int_0^V \rho\, dV/\mathcal{E}_0 \text{------------}(4)$$

   By Divergence Theorem, $\oint_0^S \vec{E}.\vec{dS} = \int_0^V \vec{\nabla}.\vec{E}\, dV$ ------------(5)

   Therefore, $\int_0^V \vec{\nabla}.\vec{E}\, dV = \int_0^V \frac{\rho}{\mathcal{E}_0} dV$

   Hence, $\overline{\vec{\nabla}.\vec{E}} = \rho/\mathcal{E}_0$ . This is Maxwell's first equation.

## 2. Maxwell's Second Equation (Gauss Law for Magnetic Field).

**Statement:** In a magnetic field, the magnetic lines are closed on themselves. Hence, total magnetic flux is zero. This in other words means that there is no magnetic monopole. Mathematically :

$$\phi_B = 0$$

Magnetic field B is magnetic flux ø per unit area, hence magnetic flux is:

$$\phi_B = \oint_0^S \vec{B}.\vec{dS}$$

Where, B is magnetic flux density

By Divergence Theorem, $\oint_0^S \vec{B}.\vec{dS} = \int_0^V \vec{\nabla}.\vec{B}\ dV$

Using above three equations we get $\int_0^V \vec{\nabla}.\vec{B}\ dV = 0$

If any volume integral is zero than the integrand must be zero

Hence $\vec{\nabla}.\vec{B} = 0$. This is Maxwell's Second equation.

## 3. Faraday's Law (Maxwell's third equation for steady fields).

**(M.U. Nov. 2018) (5 m)**

**Statement:** In a static electric field the work done in moving the test charge once around the closed path is equal to zero. Mathematically;

$$\oint_0^l \vec{E}.\vec{dl} = 0 \text{------------}(1)$$

{Because ; Work done = 0; Force x displacement =0
Force = charge x E (i.e. E is the force on unit charge)}

By Stoke's Law, $\oint_0^l \vec{E}.\vec{dl} = \oint_0^S (\vec{\nabla} \times \vec{E}).\vec{dS}$

Therefore, $\vec{\nabla} \times \vec{E} = 0$. This is Maxwell's Third equation.

## 4. Ampere's law (Maxwell's fourth equation for steady fields).

**Statement:** The line integral of magnetic field $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path.

$$\oint \vec{H}.\vec{dl} = I$$

But we know that, $\vec{B} = \mu_0\vec{H}$ for free space, hence

$$\oint_0^c \vec{B}.\vec{dl} = \mu_0 I$$

Let J be the current density i.e. current per unit cross sectional area and therefore current enclosed inside a surface S is written in terms of J as:

$$I= \int_0^s \vec{J}.\overrightarrow{ds}$$

Hence from above equations we get;

$$\oint_0^c \vec{B}.\vec{dl} = \mu_0 \int_0^s \vec{J}.\overrightarrow{ds}$$

Using strokes theorem $\oint_0^c \vec{B}.\vec{dl} = \int_0^s (\vec{\nabla} \times \vec{B}).\overrightarrow{ds}$

Therefore, $\vec{\nabla} \times \vec{B} = \mu_0 J$. This is Maxwell's Fourth equation.

Maxwell's Equations for Static fields:

| S.NO. | DIFFERENTIAL FORM | INTEGRAL FORM | SIGNIFICANCE |
|---|---|---|---|
| 1. | $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$ | $\oint_0^S \vec{E}.\overrightarrow{dS} = Q/\varepsilon_0$ | Gauss Law of Electrostatics |
| 2. | $\vec{\nabla}.\vec{B} = 0$ | $\oint_0^S \vec{B}.\overrightarrow{dS} = 0$ | Gauss Law of Magneto statics |
| 3. | $\vec{\nabla} \times \vec{E} = 0$ | $\oint_0^l \vec{E}.\vec{dl} = 0$ | Faraday's Law |
| 4. | $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ | $\oint_0^l \vec{B}.\vec{dl} = \mu_0 \vec{I}$ | Ampere's Circuital Law |

**Q3.12. What is continuity equation.**

Consider a small volume element $\Delta V$ located inside a conducting medium the current density (J) has the direction of current flow.

If there is no source or sink of charge inside $\Delta V$, the current is steady and continuous so divergence of the current in that volume is zero;

$$\int_0^V \vec{\nabla}.\vec{J}dv = 0 \text{---------} (1)$$

Using the Divergence theorem, $\int_0^V \vec{\nabla}.\vec{J}dv = \oint_0^s \vec{J}.\overrightarrow{ds} = 0$

On the contrary the case where the current is not steady, difference between the current flowing into the volume and that flowing out of the volume mustbe equal to rate of change of electric charge inside the volume. Mathematically this is ;

$$\int_0^V \vec{\nabla}.\vec{J}dv = -\int \frac{\partial \rho}{\partial t} dv$$

When the net flow of current is outward$\nabla.J$is positive; but at the same time the charge inside the volume decreases giving a negative rate ;

Rate of decrease of charge $(-\frac{\partial \rho}{\partial t})$ this is expressed by continuity equation

$$\vec{\nabla}.\vec{J} = -\frac{\partial \rho_V}{\partial t}$$

This equation is the famous continuity equation that relates the current density to the charge density.

## Q3.13. Obtain Maxwell's third and fourth equation for time varying fields in differential and integral forms

### Maxwell's Third Equation with time varying field:

**Statement:** A time varying magnetic field produces an electromotive force which may establish a current in a suitable closed circuit.

Electromotive force induced in closed loop is negative rate of change of magnetic flux ø given as the lens law:

$$e = -\frac{d\text{ø}}{dt}$$

$$\text{ø} = \oint_0^S \vec{B}.ds$$

Therefore, $e = -\oint_0^S \left(\frac{d\vec{B}}{dt}\right).dS$--------------------(1)

The electromotive force is the work done in carrying unit charge around a closed loop.

Therefore, e = $\oint_0^l \vec{E}.\vec{dl}$ ---------------(2)

Using (1) and (2) we get;

$$\oint_0^l \vec{E}.\vec{dl} = -\oint_0^S \left(\frac{d\vec{B}}{dt}\right).dS \text{ --------------(3)}$$

By Stoke's Theorem,

$$\oint_0^l \vec{E}.\vec{dl} = \int_0^S (\vec{\nabla}\times\vec{E}).\vec{dS} \text{ --------------(4)}$$

Using (3) and (4) we get ,

$$\int_0^S (\vec{\nabla}\times\vec{E}).\vec{dS} = -\oint_0^S \left(\frac{d\vec{B}}{dt}\right).dS$$

$$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$$

This is the Maxwell's third equation for time varying fields.

## Maxwell's Fourth Equation with time varying field:

**Statement for Ampere's law:** The line integral of magnetic field $\overrightarrow{H}$ around a closed path is exactly equal to the direct current enclosed by that path.

$$\oint \vec{H}.\vec{dl} = I$$

This leads to Maxwell's fourth Equation $\vec{\nabla}\times\vec{B} = \mu_0 J$. -------(5)

Taking the divergence on both sides of above equation (5) gives;

$$\vec{\nabla}.(\vec{\nabla}\times\vec{B}) = \mu_0\vec{\nabla}.\vec{J}$$

As the divergence of a curl is always zero; hence the above equation implies that $\vec{\nabla}.\vec{J} = 0$ ; which is not true for non steady fields as it contradicts the continuity equation.

Thus for the case of non steady field, let us assume ,

$$\vec{\nabla}\times\vec{B} = \mu_0(\vec{J}+\overrightarrow{G})\text{----(6)}$$

Where, G is some unknown function. Taking the divergence of eq.(6) again

$$\vec{\nabla}.(\vec{\nabla}\times\vec{B}) = \mu_0(\vec{\nabla}.\vec{J}+\vec{\nabla}.\vec{G})=0$$

We get;

$$\nabla . J = -\nabla . G \text{--------}(7)$$

From the continuity equation(refer Q10.) and Eq. (7) we have;

$$\nabla . G = \frac{\partial \rho}{\partial t} \text{------------}(8)$$

From Maxwell's first Equation;

Electrostatics Gauss law: $\rho = \varepsilon_0(\nabla . E)$---------(9)

Putting (9) in Eq.(8) we get$\nabla . G = \varepsilon_0 \nabla . \frac{\partial E}{\partial t}$-------------(10)

Thus $G = \varepsilon_0 \frac{\partial E}{\partial t}$-------------(11)

Putting Eq.(11) in Eq. (6) We get ;

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

This is the Maxwell's Fourth equation for time varying fields.

Maxwell's Equations for Time-Varying Fields:

| S.NO. | DIFFERENTIAL FORM |
|---|---|
| 1 | $\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$ |
| 2 | $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ |

**Q3.14. State the significance of Maxwells Equations**.

**(M.U. May 2019) (5 m)**

1. Maxwell's equations are set of four complicated equations that describe the world of electromagnetism in a concise way.
2. Maxwell's equations describe how electric and magnetic field propagate, interact and how are they influenced by objects.
3. Maxwell's equations unite electromagnetism and optics. Since Maxwell's equations electricity, magnetism and light are understood as aspect of single objective the electromagnetic field.

4. In short Maxwell's equations for the first time summarized the fundamentals of electricity and magnetism in the most elegant way, forming a theory of electrodynamics.
5. Maxwell's equations are critical in understanding working of antennas, waveguide and satellite communication

Beauty of these equations makes these one of the greatest intellectual achievements of mankind.

Formula List for Electrodynamics

1. Divergence of a vector

$$(\vec{\nabla}.\vec{A}) = \hat{\imath}\frac{\partial}{\partial x}A_x + \hat{\jmath}\frac{\partial}{\partial y}A_y + \hat{k}\frac{\partial}{\partial z}A_z$$

Where;

- $\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ is the del operator
- $\vec{A}$= Any arbitrary vector
- $\vec{\nabla}.\vec{A}$= divergence of vector A

2. Curl of a vector

$$(\vec{\nabla} \times \vec{A}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Where;

- $\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ is the del operator
- $\vec{A}$= Any arbitrary vector
- $(\vec{\nabla} \times \vec{A})$= curl of vector A

3. Gradient of a Scalar function

$$\vec{\nabla}f(x,y,z) = \hat{\imath}\frac{\partial}{\partial x}f(x,y,z) + \hat{\jmath}\frac{\partial}{\partial y}f(x,y,z) + \hat{k}\frac{\partial}{\partial z}f(x,y,z)$$

Where;

- $\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ is the del operator
- $f(x, y, z)$= Any arbitrary function of x, y and z.
- $\vec{\nabla} f(x, y, z)$= Gradient of a function f(x,y,z)

4. Conversion of Cartesian co-ordinates to cylindrical

$$r = \sqrt{x^2 + y^2}$$
$$\emptyset = tan^{-1}\left(\frac{y}{x}\right)$$
$$z = z$$

Where;

- $r, \emptyset, z$ = The Cylindrical co-ordinates
- $x, y, z$ = The Cartesian co-ordinates

5. Conversion of Cartesian co-ordinates to Spherical

$$\mathrm{r} = \sqrt{\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2}$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{\mathrm{x}^2 + \mathrm{y}^2}}{\mathrm{z}}\right)$$
$$\emptyset = \tan^{-1}\left(\frac{\mathrm{y}}{\mathrm{x}}\right)$$

Where;

- $r, \theta, \emptyset$ = The Spherical co-ordinates
- $x, y, z$ = The Cartesian co-ordinates

**ELECTRODYNAMICS PROBLEMS**

**Q1.** Convert the point P(1,3,5) from Cartesian to Cylindrical and Spherical polar coordinates

**Given:- X= 1; Y= 3; Z=5**

**Formula:-** Cylindrical- Cartesian $r = \sqrt{x^2 + y^2}; \emptyset = tan^{-1}\left(\frac{y}{x}\right); \;\; z = z$

Spherical-Cartesian;

$r = \sqrt{x^2 + y^2 + z^2};\ \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right); \emptyset = tan^{-1}\left(\frac{y}{x}\right)$

**Solution:-**

In cylindrical polar coordinates, (r, $\emptyset, z$)

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 3^2} = \sqrt{10} = 3.162$$

$$\emptyset = tan^{-1}\left(\frac{y}{x}\right) = tan^{-1}(3) = 71.57$$

Z = 5

In Spherical polar coordinates (r,$\theta, \emptyset$)

r=$\sqrt{x^2 + y^2 + z^2} = \sqrt{1^2 + 3^2 + 5^2} = \sqrt{35}$ =5.916

$$\emptyset = tan^{-1}\left(\frac{y}{x}\right) = tan^{-1}(3) = 71.57$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \tan^{-1}\left(\frac{\sqrt{1^2+3^2}}{5}\right) = 32.3$$

**Ans:- Coordinates of P in cylindrical coordinates are (3.162, 71.57, 5) and in spherical polar coordinates are (5.916, 71.57, 32.3)**

**Q2.** Given vector $\vec{A}(x,y,z) = y\hat{\imath} + (x+z)\hat{\jmath}$ in Cartesian coordinate system at point P(-2,6,3). Convert the vector $\vec{A}$ into cylindrical and spherical coordinates.

**Given:-** $\vec{A} = y\hat{\imath} + (x+z)\hat{\jmath}$, x=-2, y=6, z=3

**Formula:-** Cylindrical- Cartesian $r = \sqrt{x^2+y^2}$; $\emptyset = tan^{-1}\left(\frac{y}{x}\right)$; $z = z$

Spherical-Cartesian;

$r = \sqrt{x^2+y^2+z^2}$; $\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$; $\emptyset = tan^{-1}\left(\frac{y}{x}\right)$

**Solution:-**

Cylindrical coordinates:

$$r = \sqrt{(-2)^2+6^2} = 6.32$$

$$\emptyset = \frac{6}{-2} = 108.43$$

Z=3

Spherical coordinates:

$$r = \sqrt{(-2)^2+6^2+3^2} = 7$$

$$\theta = \frac{\sqrt{(-2)^2+6^2}}{3} = 64.62$$

$$\emptyset = \frac{6}{-2} = 108.43$$

**Ans:-** $\vec{A} = (6.32\,\hat{r},\ 108.43\,\hat{\emptyset},\ 3\,\hat{z})$ **is cylindrical coordinates**

$\vec{A} = (7\,\hat{r},\ 64.62\,\hat{\theta},\ 108.43\,\hat{\emptyset})$ **is spherical coordinates.**

**Q3.** Find the divergence and curl of the field $F = 30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k}$ in Cartesian coordinates.

**Given:-** $F = 30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k}$

**Formula:-**Divergence $(\vec{\nabla}.\vec{A})$= $\hat{\imath}\frac{\partial}{\partial x}A_x + \hat{\jmath}\frac{\partial}{\partial y}A_y + \hat{k}\frac{\partial}{\partial z}A_z$

Curl: $(\vec{\nabla} \times \vec{A})$=$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

***Solution:-***

Divergence, $\vec{\nabla}.\vec{F} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).(30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k})$

$$=\frac{\partial}{\partial x}(30) + \frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(2xz^2) = 2x + 10xz$$

$$\vec{\nabla}.\vec{F} = 2x(1 + 5z)$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 30 & 2xy & 5xz^2 \end{vmatrix} = -5z^2\,\hat{\jmath} + 2y\,\hat{k}$$

**Ans:- Divergence of field is $2x(1 + 5z)$ and its curl $= -5z^2\,\hat{\jmath} + 2y\,\hat{k}$**

**Q4.** If $\emptyset(x, y, z) = 3(x^2y - y^2x)$, Calculate gradient.

**Given:-**$\emptyset(x, y, z) = 3(x^2y - y^2x)$

**Formula:-**

**Solution:-**

Grad $\emptyset = \nabla\emptyset = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\{3(x^2y - y^2x)\}$

$$= \hat{\imath}(6xy - 3y^2) + \hat{\jmath}(3x^2 - 2yx) + \hat{k}(0)$$

Therefore, $\nabla\emptyset = \hat{\imath}(6xy - 3y^2) + \hat{\jmath}(3x^2 - 2yx)$

**Ans:- The gradient is , $\nabla\emptyset = \hat{\imath}(6xy - 3y^2) + \hat{\jmath}(3x^2 - 2yx)$.**

**Q5.** A region is specified by the potential function given by $\emptyset = 4x^2 + 3y^2 - 9z^2$. Calculate electric field strength.

**Given:-**$\emptyset = 4x^2 + 3y^2 - 9z^2$.

**Formula:-**$\vec{E} = -grad(potential\ function)$

**<u>Solution:-</u>**

$$\begin{aligned}\vec{E} &= -grad(potential\ function)\\ &= -grad\ \emptyset = -\nabla[4x^2 + 3y^2 - 9z^2]\\ &= -\left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(4x^2 + 3y^2 - 9z^2)\end{aligned}$$

Therefore, $\vec{E} = -8x\hat{\imath} - 6y\hat{\jmath} + 18z\hat{k}$

**<u>Ans:- The electric field strength is $\vec{E} = -8x\hat{\imath} - 6y\hat{\jmath} + 18z\hat{k}$.</u>**