Unit 1: DIFFRACTION

Q1.1. What do you mean by diffraction? State its types and differentiate between them.

(M.U. May 09, 11; Dec. 2009, 11, 15) (3 m)

As waves come into contact with obstacles (or openings), they bend around the edges of the obstacles if the obstacles' dimensions are equal to the waves' wavelength. Diffraction is the bending of waves around the edges of an obstacle (or opening).

Basically, the diffraction phenomenon has two main types:

(A) Fresnel Diffraction
(B) Fraunhoffer Diffraction

Difference between Fresnel diffraction and Fraunhoffer diffraction

Fresnel Diffraction	Fraunhoffer Diffraction
The source and slits are both at finite distance from the slit.	The source and slits are at infinite distance from the slit.
Both the incident and diffracted wavefronts are cylindrical or spherical.	Both the incident and diffracted wavefronts are plane.
The incident and diffracted rays are divergent.	The incident and diffracted rays are parallel.
Lenses are not required in actual experiment.	Lenses are used in experiment to achieve parallel wavefront.
Path difference between the rays forming the diffraction pattern depends on distance of slit from source as well as the screen and the angle of diffraction. Hence mathematical treatment is complicated.	Path difference between the rays forming the diffraction pattern depends only on the angle of diffraction. Hence mathematical treatment is comparatively easier.

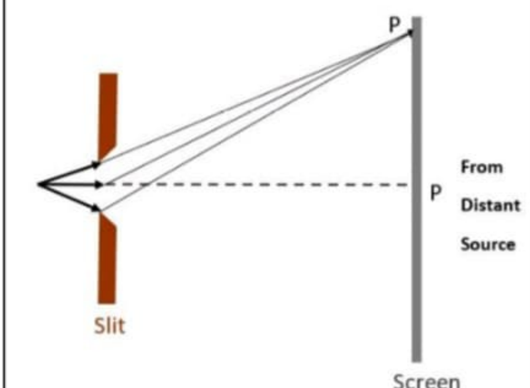

Figure 1.1.1: Fresnel Diffraction

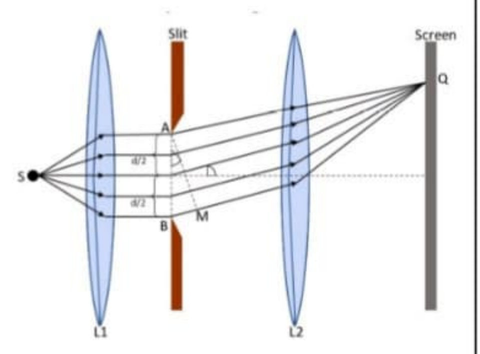

Figure 1.1.2: Fraunhofer Diffraction

Q1.2. Explain Fraunhoffer Diffraction at a single slit, obtain expression for the resultant intensity and derive expressions for maxima and minima for a single slit.

(M.U. May 2007) (5 m)

Let *AB* be a single narrow slit of width ***d*** on which a parallel beam of monochromatic light is incident as a plane wave front as shown in *Figure 1.2.1*.

The incident wave front is diffracted by the slit and is then focused on the screen by the lens L. According to Huygens principle, each point on AB acts as a source of secondary waves.

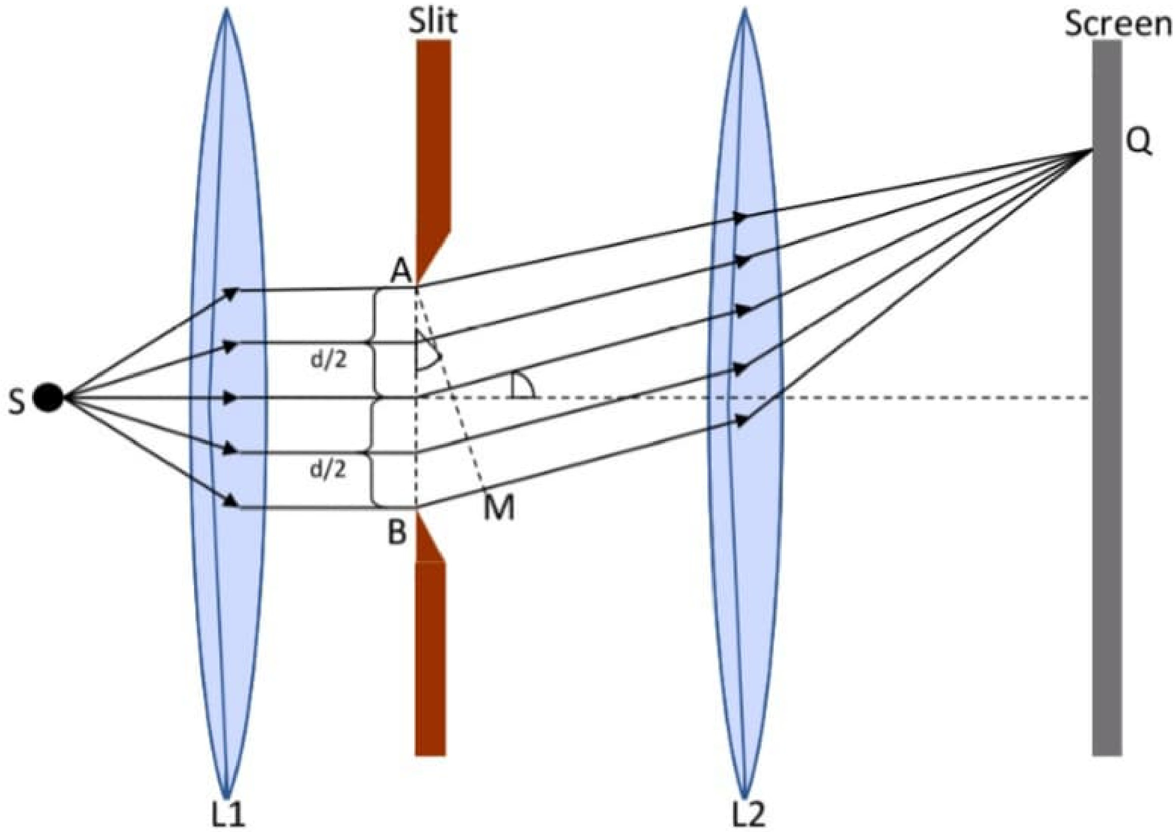

Figure 1.2.1: Fraunhofer Diffraction at a single slit

As all the points on AB are in phase, the point sources will be coherent. Hence the light from one portion of the slit can interfere with light from another portion and the resultant intensity on the screen will depend on the direction θ of the diffracted waves.

When the waves traveling in straight direction without diffraction, they are in the same phase and after covering equal optical path lengths their superposition produces zero order central maxima where the dotted line meets the screen. Lets refer this point as P.

Let us consider another point Q on the screen. The waves that leave the slit at an angle θ reach the point Q. The point Q will be dark or bright depending upon the path difference between the waves arriving at Q from different points on the wave front. The total path difference between the waves that are travelling from point A and point B on the slit is :-

path difference = $d\ sin\theta$

This path difference corresponds to a total phase difference of :-

$$\phi = \frac{2\pi}{\lambda} d \sin\theta \qquad \text{...............(1)}$$

Derivation for Intensity variation :-

In order to determine the intensity distribution for the single slit diffraction pattern, we follow the graphical approach.

Consider the slit is divided into a large number (N) of narrow strips of equal width Δy.

Each strip acts as a source of coherent radiation and the light emanating from it can be represented by a short phasor.

The path difference between the rays diffracted from its upper and lower edges :-

path difference for a strip= $dy.sin\theta$

hence the corresponding phase difference between them is,

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta y \sin\theta \qquad \text{...............(2) such that } \phi = N\Delta\phi$$

(i) At the centre of diffraction pattern θ=0 hence net phase difference is zero, and the phasors in this case are laid end to end as shown in *Figure 1.2.2*

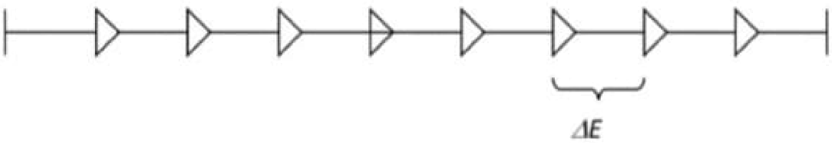

Figure 1.2.2: Resultant amplitude for diffraction angle=0

The amplitude of the resultant has its maximum value E_m given as :-

$E_m = N\Delta E$ (slit is divided into N no. of parts)

(ii) At a value of θ other than zero, $\Delta\phi$ has a finite value. The amplitude E_θ of the resultant is the vector sum of phasors, and hence given by the length of chord as shown in *Figure 1.2.3*. It can be seen that E_θ is less than E_m.

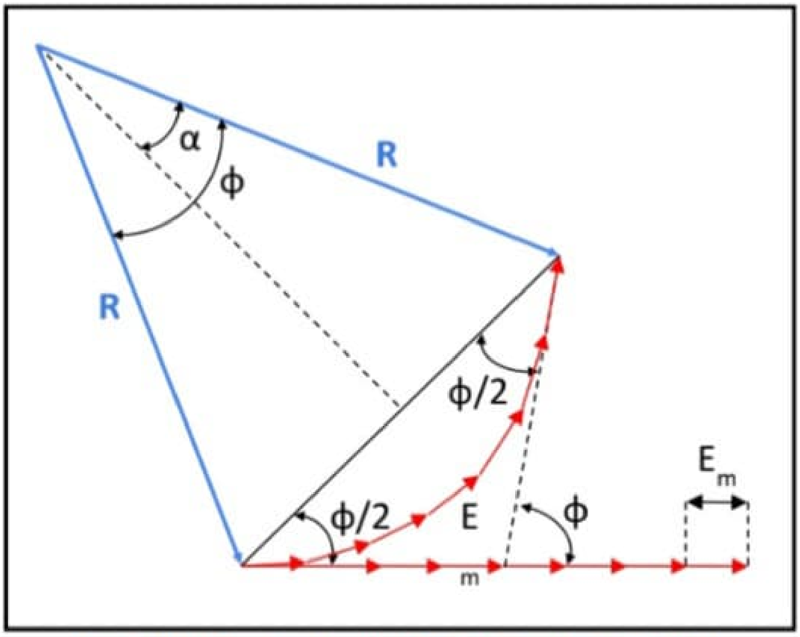

Figure 1.2.3: Resultant amplitude for diffraction angle=θ

The resultant amplitude E_θ equals the length of chord and E_m is the length of an arc as shown in the figure and ϕ is the measure of total phase difference between initial and final phasors and as opposite angles of quadrilateral are supplementary ϕ is also the angle of the sector formed between the two radii R.

Inside this sector the perpendicular drawn from the centre of the circle to the chord bisects the chord, hence

$$\sin\phi/2 = \frac{E_\theta/2}{R}$$

$$\therefore E_\theta = 2R\sin\phi/2$$

$$\text{But } \phi = \frac{E_m}{R} = \frac{arc}{radius}$$

$$E_\theta = 2\left(\frac{E_m}{\phi}\right)\sin\frac{\phi}{2}$$

Using all the above we can write expression for resultant amplitude at pt Q

$$\boxed{E_\theta = E_m\left[\frac{\sin\phi/2}{\phi/2}\right]} \quad \text{..............} \quad (3)$$

The intensity I_θ is proportional to amplitude square E_θ^2,

$$I_\theta = I_m\left[\frac{\sin\phi/2}{\phi/2}\right]^2 \qquad \text{..................(4)}$$

$$\text{Let } \frac{\phi}{2} = \alpha = \frac{\pi d\sin\phi}{\lambda} \qquad \text{...................(5)}$$

Hence,

$$\boxed{I_\theta = I_m\left[\frac{\sin^2\alpha}{\alpha}\right]} \qquad \text{..................(6)}$$

The following graph shows the intensity variation with respect to angle ϕ in a single slit diffraction as shown in *Figure 1.2.4.*

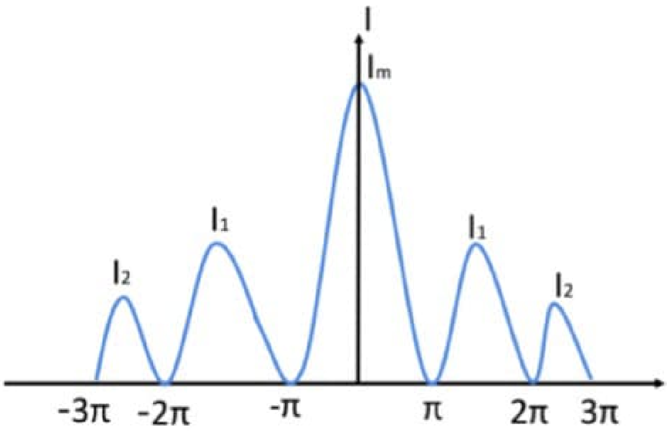

Figure 1.2.4: Intensity variation with respect to angle φ

Conditions for maxima and minima :-

(a) Principle Maximum:

The resultant amplitude in diffraction pattern is given by

$$E_\theta = E_m\left[\frac{\sin\alpha}{\alpha}\right]$$

For E_θ to be maximum, we need $\alpha = 0$ i.e $\alpha = \frac{\pi}{\lambda}d\sin\theta = 0$

$$\therefore \sin\theta = 0 \quad i.e.\ \theta = 0.$$

Thus principal maxima is obtained at $\theta = 0$.

(b) Minimum intensity positions (minima):

The intensity $I_\theta = I_m\left[\frac{\sin^2\alpha}{\alpha^2}\right]$ will be zero where $\sin\alpha = 0$ but $\alpha \neq 0$

The values of α which satisfy that equation are :-

$\alpha = n\pi$ where $n = \pm 1, \pm 2, \pm 3, \ldots\ldots$

$$\alpha = \frac{\pi}{\lambda}d\sin\theta = n\pi$$

Hence, the condition for minima is

$d \sin\theta = n\lambda$, where $n = \pm 1, \pm 2, \pm 3, \ldots\ldots$

(Since θ becomes zero which corresponds to the principal maximum. The positions of minima are on either side of principal maximum.)

(c)Secondary maxima

Analysis shows that the secondary maxima lie approximately half way between the minima. i.e.

$$\alpha = \pm\left(n + \frac{1}{2}\right)\pi \qquad\qquad n = 1, 2, 3, \ldots\ldots$$

$$d \sin\theta = (2n+1)\frac{\lambda}{2}$$

Substituting this value of α in I_θ.

$$I_\theta = I_m\left[\frac{\sin^2\alpha}{\alpha^2}\right] \text{ we get,}$$

$$\frac{I_\theta}{I_m} = \left[\frac{\sin\left(n+\frac{1}{2}\right)\pi}{\left(n+\frac{1}{2}\right)\pi}\right]^2$$

$$= \frac{1}{\left(n+\frac{1}{2}\right)^2\pi^2} \qquad m = 1, 2, 3, \ldots\ldots$$

$$\frac{I_\theta}{I_m} = 0.045, 0.016, 0.0083, \ldots\ldots$$

Thus, the successive maxima decrease in intensity rapidly.

Q1.3. What is Diffraction Grating? Explain the construction of diffraction grating. Determination of Wavelength of Light using Grating.

(M.U. May 2008, 11, 13, 16, 17; Dec. 2009, 12, 17, Nov. 2018) (5 m)

The diffraction grating is an arrangement consisting of a large number of parallel slits of equal width separated from one another by equal opaque spaces.

A diffraction grating is formed by ruling a plane glass plate with fine lines using a diamond point. Ideally a diffraction grating will have 5000 to 15000 lines per inch.

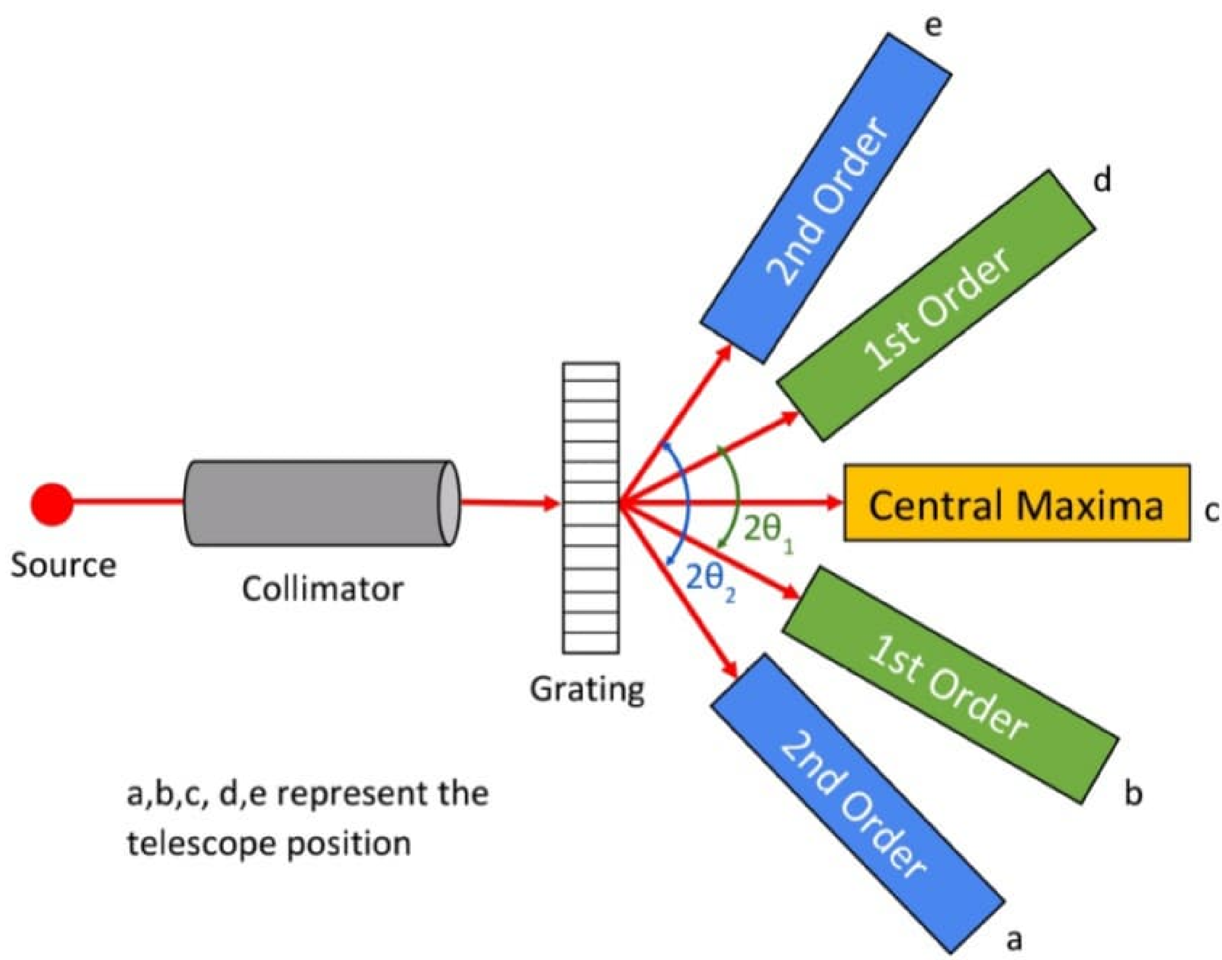

Figure 1.3.1: Determination of wavelength using diffraction grating

The diffraction formula for a principal maximum in a grating's diffraction is given by

$$(a+b)\sin\theta = n\lambda$$

Where; (a+b) = grating element

n = order of spectrum

λ= wavelength of incident light.

A diffraction grating is often used in laboratories to determine the unknown wavelength of light. The grating spectrum of the given source of monochromatic light is obtained by using a spectrometer. The arrangement is as shown in *Figure 1.3.1*

1. First the source for which wavelength is to be determined, the collimator and the telescope are kept in one line subsequently, the spectrometer is adjusted for parallel rays using a prism.
2. Then the prism is replaced by a diffraction grating on a prism table. One has to make sure that the grating is kept perpendicular to the light rays in order to achieve normal incidence.
3. The 0^{th} order spectrum can be seen from the telescope, when It is directly in line with the incident light.
4. As the telescope is moved in clockwise and anticlockwise direction with respect to its 0^{th} order position, higher order spectrums are seen.
5. The angle between the zero-order position and n^{th} order position is called as the diffraction angle for that order and denoted by θ_n.
6. For first and second order spectrums the diffraction angle is measured using the spectrometer.
7. The unknown wavelength is calculated using the grating formula for both the orders:

$$\lambda = \frac{(a+b)\sin\theta_n}{n}$$

Where, (a+b) is the grating element in centimeters,

which is calculated using the formula:

(a+b) = 1/N

Where, N is the number of lines per cm on the diffraction grating, which is mentioned on the grating itself by the manufacturer.

Mean wavelength from both the orders is the unknown wavelength of the source which is now determined.

Q1.4. Explain Rayleigh's Criterion of Resolution.

(M.U. May 2010, 11, 13, 14, 15; Dec. 2016, 17) (3 m)

Rayleigh's criterion of resolution states that:

When the central maximum of the diffraction pattern of one source falls over the 1st minimum of the diffraction pattern of another source, which is placed close to the previous source, the two-point sources of light are said to have been just resolved.

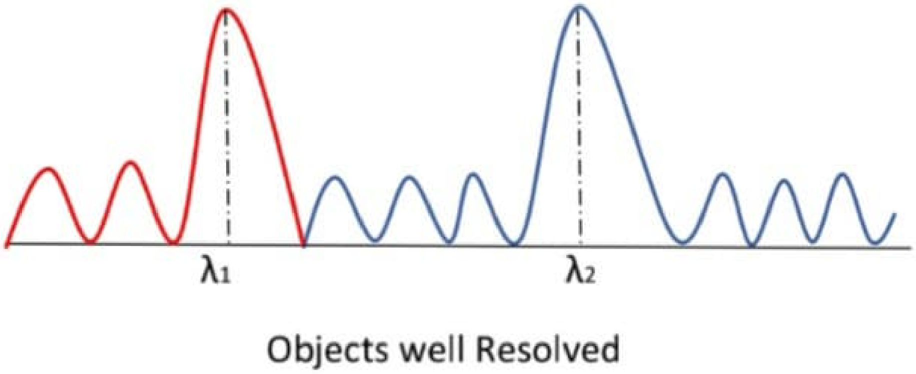

Figure 1.4.1a: Well resolved

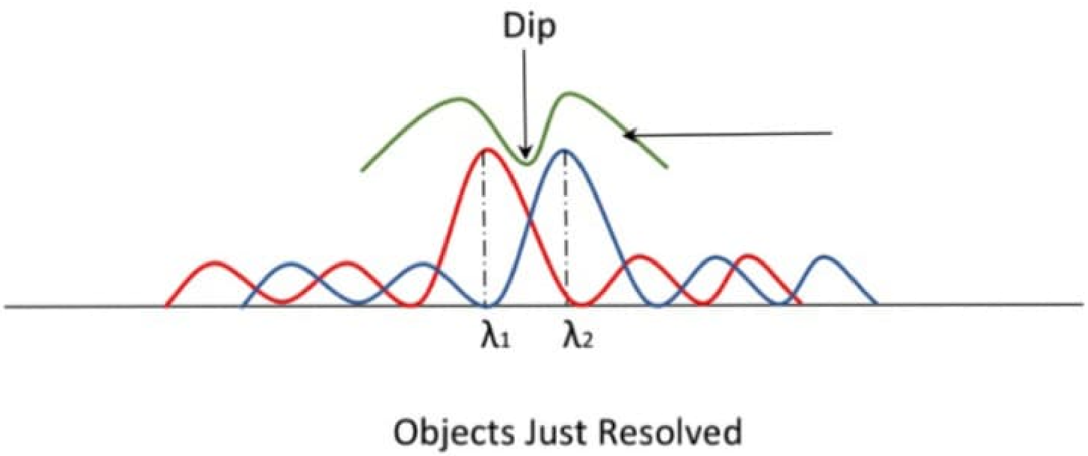

Figure 1.4.1b: Just resolved

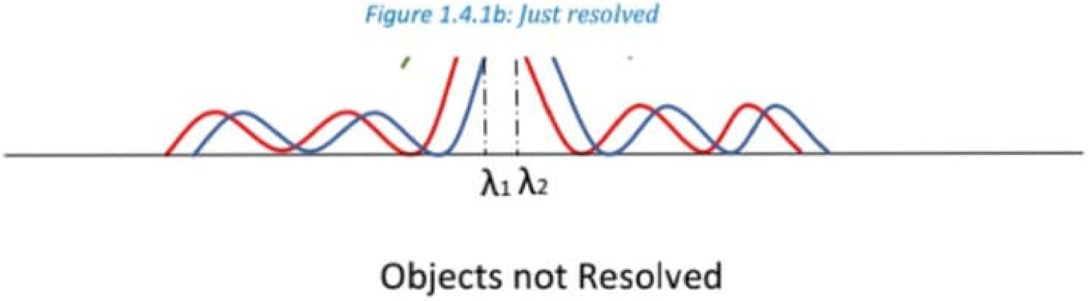

Figure 1.4.1c: Not resolved

Q1.5. What is Resolving Power of an optical Instrument? Obtain an expression for resolving power of a diffraction grating.

(M.U. May 2010, 11, 13, 14, 15; Dec. 2016, 17) (3 m)

If the accompanying diffraction patterns are distinguishable from each other, an optical instrument is said to be able to resolve two-point objects. **Resolving power** refers to an instrument's capacity to generate only different diffraction patterns of two close objects.

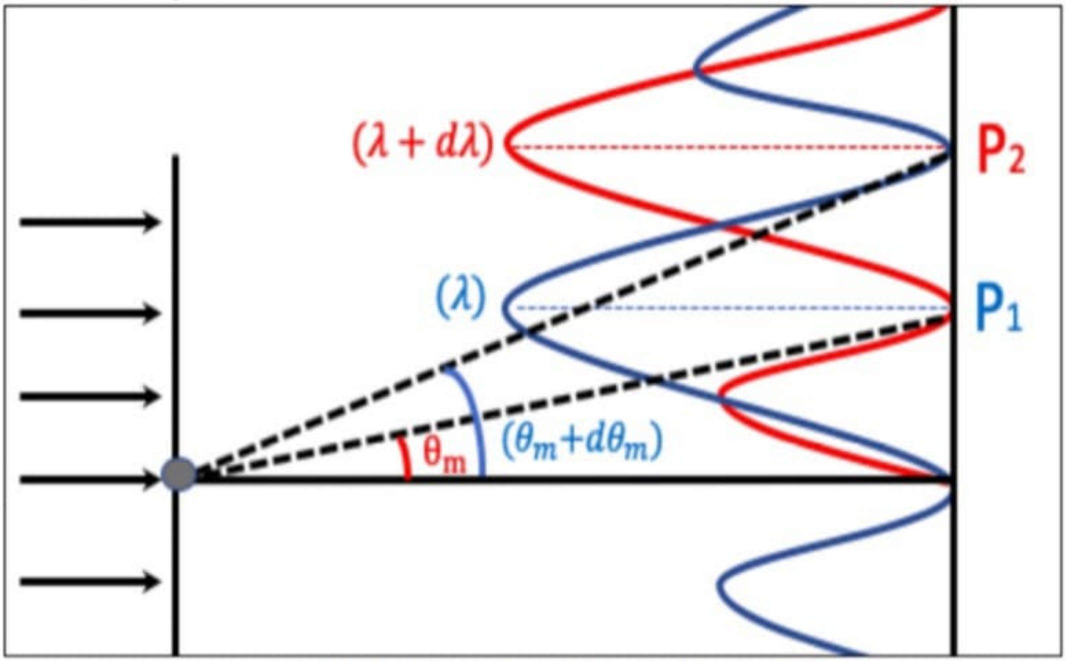

Figure 1.5.1: Resolving Power of an optical Instrument

Resolving Power of a grating:

A grating is capable of resolving the image of slit formed by the two spectral lines of wavelength λ and $(\lambda + d\lambda)$ as shown in *Figure 1.5.1*.

The resolving power of a grating is defined as the smallest wavelength $d\lambda$ for which the spectral lines can be first resolved at the wavelength λ, is mathematically given as $R.P = \frac{\lambda}{d\lambda}$ ---------(1a)

Let XY be the grating surface and MN is the field of view of the telescope.

P_1 is the m[th] primary maximum of spectral line wavelength λ at an angle of diffraction θ_m.

P_2 is the m^{th} primary maximum of a second spectral line of wavelength $\lambda+d\lambda$ at an angle of diffraction $\theta_m+d\theta_m$.

Conditions for maxima P_1 and P_2 are:

$$(a+b)\sin(\theta_m+d\theta_m) = m(\lambda+d\lambda)\text{--------}(1)$$

$$(a+b)\sin(\theta_m) = m\lambda\text{------}(2)$$

Distance between two maxima can be calculated by subtracting (1)-(2) we get:

$$P1\ \mathrm{P2} = (a+b)\sin(\theta_m+d\theta_m) - (a+b)\sin(\theta_m) = m(\lambda+d\lambda) - m\lambda$$

Which results into

$$P1\ \mathrm{P2} = m(d\lambda)\text{-------}(3)$$

According to Rayleigh's criterion $P1\ \mathrm{P2}$ will also be equal to the distance between the principal maxima and first minima of source λ

Principal Maxima condition: $(a+b)\sin(\theta_m) = 0$---------(4)

First Minima condition: $(a+b)\sin(\theta_m) = \frac{\lambda}{N}$----------(5)

Thus, we get $P1\ \mathrm{P2}$ by subtracting (5)-(4):

$$P1\ \mathrm{P2} = \frac{\lambda}{N}\text{-------}(6)$$

Comparing equation (6) and (3):

$$md\lambda = \frac{\lambda}{N}$$

Then by the definition of resolving power of grating in equation (1a) we obtain the expression for resolving power of grating in terms of number of lines N for the grating as:

$$\therefore R.P = \frac{\lambda}{d\lambda} = mN$$

Q1.6. Why is diffraction not evident in daily life?

(M.U. May 2008) (3 m)

In daily life the objects that we come across are very small. Objects of size of the order of wavelength of light i.e. around 0.1 micro meter are needed for diffraction to be observed, hence it is not evident in daily life easily.

Formula List for Diffraction

1. Grating formula

$$(a+b)\sin\theta = n\lambda$$

Where;

- $(a+b)$= Grating element
- θ = Angle of Diffraction
- n = order of Diffraction
- λ = Wavelength of wave getting Diffracted

2. Grating Element

$$(a+b) = \frac{1}{N}$$

Where;

- $(a+b)$= Grating element
- N = Number of lines per unit length of Grating

3. Resolving power of Grating (R.P.)

$$\text{R.P.} = \frac{\lambda}{d\lambda} = mN$$

Where;

- N = Number of lines per unit length of Grating
- m = order of Diffraction
- λ = Mean Wavelength of sources to be resolved
- d λ = Difference between Wavelength of sources to be resolved

DIFFRACTION PROBLEMS

Q1. A diffraction grating used at normal incidence gives a yellow line (λ =6000A^0)in a certain spectral order superimposed on a blue line (λ =4800A^0) of next higher order. If the angle of diffraction is sin^{-1}(3/4) calculate the grating element.

Given:- λ_1=6000A^0=6x10^{-5} cm; λ_2=4800A^0=4.8 x 10^{-5}cm ; $\theta = \sin^{-1}\left(\frac{3}{4}\right)$

Formula:- $(a + b)\sin\theta = n\lambda$; n=1,2,3,4 .....

Solution:- for given $(a + b)$ and θ ; n α 1/ λ

$$(a + b)\sin\theta = n\ \lambda_1$$
$$(a + b)\sin\theta = (n + 1)\ \lambda_2$$
$$n\,\lambda_1 = (n + 1)\,\lambda_2$$
$$\frac{\lambda_1}{\lambda_2} = 1 + \frac{1}{n}$$

Therefore, $n = 4$

$$(a + b) = \frac{n\lambda_1}{\sin\theta} = \frac{4\times 6\text{x }10^{-5}}{\frac{3}{4}} = 32 \times 10^{-5}\text{ cm}$$

Ans:- The grating element is 3.2 x 10^{-4} cm .

Q2. Monocromatic light of wavelength 6560A falls normally on a grating 2 cm wide. The frist order spectrum is produced at an angle of 16°17' from the normal. Calculate the total number of lines on the grating.

Given:- λ=6560A=6560 x 10^{-8}cm; width = 2cm ; n=1 ; θ=16.28°

Formula:- $(a + b)\sin\theta = n\lambda$

$$a + b = \frac{1}{(N)Number\ of\ lines\ per\ cm}$$

Total number of lines= N x width

Solution:- (a+b)=$\frac{n\lambda}{sin\theta}$ =$\frac{6560}{sin16.28} \times 10^{-8}$ = 2.34 x 10^{-4} cm

Number of lines per cm = $\frac{1}{a+b}$ =4273

Total no. of lines = 4273 x 2 = 8547

Ans:- There will be 8547 lines on the grating.

Q3.A parallel beam of light is incident on a plane transmission grating having 3000 lines/cm. A third order diffraction is observed at 30°. calculate the wavelength of the line.

Given:-a+b = 1/3000 ; n=3 ; θ=30

Formula:- $(\mathrm{a} + \mathrm{b})\sin\theta = n\lambda$, n=1,2,3...

Solution:- λ = $\frac{a+b}{n} \times sin\theta$

= $\frac{1}{3000\times3} \times sin30$

= $\frac{1}{9000\times2}$= 5.555×10^{-5} cm

Ans:- The wavelength of the line is 5.555×10^{-5} cm.

Q4. The visible spectrum ranges from 4000A^0 to 7000A^0 . Find the angular breath of the first order visible spectrum produced by a plane grating having 6000 lines/cm when light is incident normally on the grating.

Given:- l1 = 4000A=4×10^{-5} cm l2= 7000A=7x10^{-5} cm n=1
a+b=1/6000lines per cm

Formula:- $(\mathrm{a} + \mathrm{b})\sin\theta = n\lambda$

Solution:- (a +b)sin θ_1= λ_1

$\theta_1 = \sin^{-1}\frac{\lambda 1}{a+b}$ = $\sin^{-1}(4\times10^{-5}\times 6000)$ = 13.88°

(a +b)sin $\theta_2 = \lambda_2$

$\theta_2 = \sin^{-1}\frac{\lambda 2}{a+b}$ = $\sin^{-1}(7\times10^{-5}\times 6000)$ = 24.83°

$\theta_2 - \theta_1$ =24.83 - 13.88 = 10.95®

Ans :- The Angular separation = 10.95®

Q5. In plane transmission grating the angle of diffraction for the second order principal maxima for the wavelength 5 x 10^{-5} cm is 35®. Calculate the number of lines/cm on the diffraction grating.

Given:- l=5x10^{-5}cm ; θ =35® ; n=2

Formula:- $(a + b)\sin\theta = n\lambda$; $\frac{1}{a+b}$= number of lines/cm

Solution:- a+b=$\frac{n\lambda}{sin\theta}$ = $\frac{2\times5\times10^{-5}}{sin35°}$ = 1.74 x 10^{-4}

Number of lines per cm = $\frac{1}{a+b}$ = $\frac{1}{1.74\times10^{-4}}$ = 5735

Ans:- The number of lines pre cm is 5735.

Q6. A grating has 620 ruling/mm and is 0.5 mm wide. What is the smallest wavelength interval that can be resolved in the third order at λ =481mm?

Given:- N = 620 x 0.5=310 ; λ =481x10^{-9}m ; m=3

Formula:-$\frac{\lambda}{d\lambda} = mN$

Solution:- d λ = $\frac{\lambda}{mN}$ = $\frac{481\times10^{-9}}{3\times310}$ = 0.5172 x 10^{-9} m

dλ = 0.5172 A°

Ans:- The smallest wavelength interval is 0.5172 A°

Q7. Calculate the minimum number of lines required on grating that can just resolve the two sodium lines λ 1=5890A and λ 2=5893 A in third order.

Given:- λ_1=5890 x 10^{-8} cm, λ_2=5893 x 10^{-8}cm , m=3

Formula:- Resolving power $=\frac{\lambda}{d\lambda} = mN$

Solution:- $\lambda = \frac{\lambda_1+\lambda_2}{2} = \frac{(5890+5896)10^{-8}}{2}$ = 5893 x 10^{-8} cm

d= (5896-5890)x10^{-8} = 6x10^{-8} cm

$$N = \frac{\lambda}{m\,d\lambda} = \frac{5893 \times 10^{-8}}{3 \times 6 \times 10^{-8}} = 327$$

Ans:- Minimum of 327 lines are required on the grating.

Q8. Calculate the maximum order of diffraction maxima seen from plane transmission grating with 2500 lines per inch if light of wavelength 6900 A falls normally on it.

Given:- N=$\frac{1}{a+b}$ = 2500lines/inch = 2500x 2.52 x 10^{-2} = 63 lines/m

λ= 6900A=6900x10^{-10} m

Formula:- $(a+b)\sin\theta = n\lambda$

Solution:- for n= n_{max}, sin θ =1

$$n_{max} = \frac{a+b}{\lambda} = 2.3$$

Ans:- Maximum order of diffraction is 2