Fluid (JEE Advance 2012- 2023)

* 14. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are)

(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

(B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(C) the light sphere is partially submerged.

(D) the light sphere is completely submerged.

Sol. [redacted]

2. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

(A) more than half filled if ρ_c is less than 0.5
(B) more than half filled if ρ_c is less than 1.0
(C) half filled if ρ_c is less than 0.5
(D) less than half filled if ρ_c is less than 0.5

Ans. [redacted]

V_0

*17. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists.

	List I		List II
P.	Lift is accelerating vertically up.	**1.**	d = 1.2 m
Q.	Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	**2.**	d > 1.2 m
R.	Lift is moving vertically up with constant speed.	**3.**	d < 1.2 m
S.	Lift is falling freely.	**4.**	No water leaks out of the jar

Code:

(A) P-2, Q-3, R-2, S-4
(B) P-2, Q-3, R-1, S-4
(C) P-1, Q-1, R-1, S-4
(D) P-2, Q-3, R-1, S-1

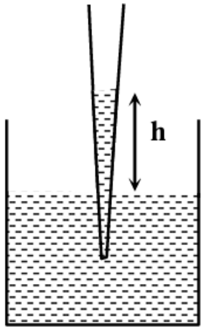
*2. A glass capillary tube is of the shape of truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity)

(A) $\frac{2S}{b\rho g}\cos(\theta-\alpha)$

(B) $\frac{2S}{b\rho g}\cos(\theta+\alpha)$

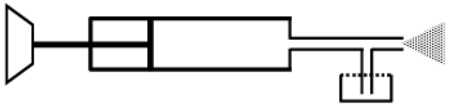
(C) $\frac{2S}{b\rho g}\cos(\theta-\alpha/2)$

(D) $\frac{2S}{b\rho g}\cos(\theta+\alpha/2)$



Paragraph for Questions 15 & 16

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



*15. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of

(A) 0.1 ms^{-1} (B) 1 ms^{-1} (C) 2 ms^{-1} (D) 8 ms^{-1}

*16. If the density of air is ρ_a and that of the liquid ρ_ℓ, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(A) $\sqrt{\frac{\rho_a}{\rho_\ell}}$ (B) $\sqrt{\rho_a\rho_\ell}$ (C) $\sqrt{\frac{\rho_\ell}{\rho_a}}$ (D) ρ_ℓ

*14. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If P(r) is the pressure at $r(r<R)$, then the correct option(s) is(are)

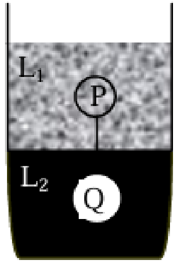
(A) $P(r=0)=0$

(B) $\frac{P(r=3R/4)}{P(r=2R/3)}=\frac{63}{80}$

(C) $\frac{P(r=3R/5)}{P(r=2R/5)}=\frac{16}{21}$

(D) $\frac{P(r=R/2)}{P(r=R/3)}=\frac{20}{27}$

*10. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then



(A) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_1}{\eta_2}$

(B) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_2}{\eta_1}$

(C) $\vec{V}_P . \vec{V}_Q > 0$

(D) $\vec{V}_P . \vec{V}_Q < 0$

*17. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1cm and 0.5cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity η = 3 poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity η = 2 poiseulles. The ratio of the terminal velocities of P and Q is

Sol. [redacted]

*Q.8 A drop of liquid of radius R = 10^{-2} m having surface tension $S = \dfrac{0.1}{4\pi}$ Nm^{-1} divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of α is

Sol. [redacted]

*Q.2 Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true?

(A) The resistive force of liquid on the plate is inversely proportional to h

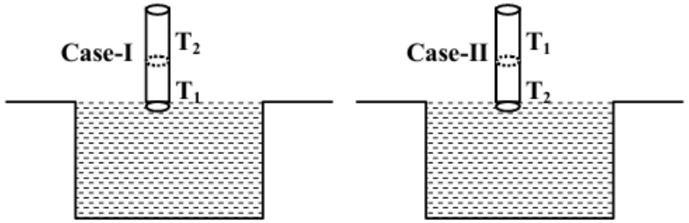
(B) The resistive force of liquid on the plate is independent of the area of the plate

(C) The tangential (shear) stress on the floor of the tank increases with u_0

(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

*Q.3 A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0^0 and 60^0, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct?

[Surface tension of a water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2]



A. For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

B. For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

C. For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

D. The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

*Q.3 A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true?

(A) For a given material of the capillary tube, h decreases with increase in r

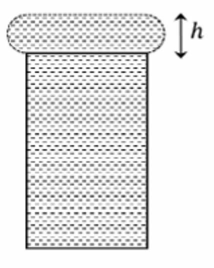
(B) For a given material of the capillary tube, h is independent of σ

(C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases

(D) h is proportional to contact angle θ

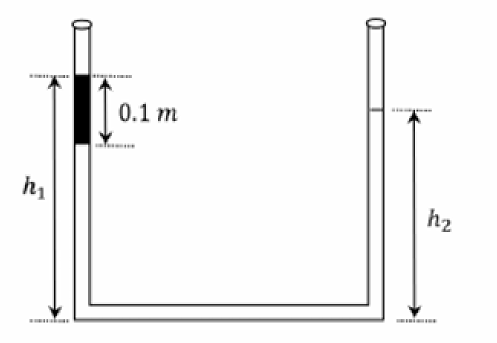
Sol. [redacted]

*14. When water is filled carefully in a glass, one can fill it to a height h above the rim of the glass due to the surface tension of water. To calculate h just before water starts flowing, model the shape of the water above the rim as a disc of thickness h having semicircular edges, as shown schematically in the figure. When the pressure of water at the bottom of this disc exceeds what can be withstood due to the surface tension, the water surface breaks near the rim and water starts flowing from there. If the density of water, its surface tension and the acceleration due to gravity are $10^3 kg\ m^{-3}$, $0.07\ Nm^{-1}$ and $10\ ms^{-2}$, respectively, the value of h (in mm) is ________.



Sol. [redacted]

*6 An open-ended U-tube of uniform cross-sectional area contains water (density $10^3 kg\ m^{-3}$). Initially the water level stands at 0.29 m from the bottom in each arm. Kerosene oil (a water-immiscible liquid) of density 800 kg m^{-3} is added to the left arm until its length is 0.1 m, as shown in the schematic figure below. The ratio $\left(\frac{h_1}{h_2}\right)$ of the heights of the liquid in the two arms is

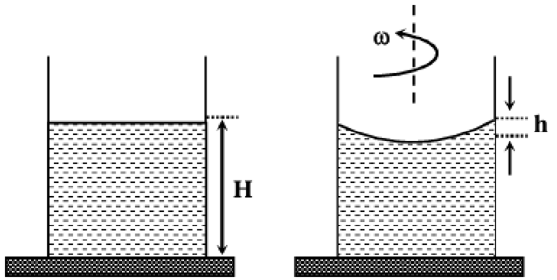


(A) $\frac{15}{14}$ (B) $\frac{35}{33}$

(C) $\frac{7}{6}$ (D) $\frac{5}{4}$

Sol. [redacted]

Q.7 A beaker of radius r is filled with water $\left(\text{refractive index } \frac{4}{3}\right)$ up to a height H as shown in the figure on the left. The beaker is kept on a horizontal table rotating with angular speed ω. This makes the water surface curved so that the difference in the height of water level at the center and at the circumference of the beaker is $h(h << H, h << r)$, as shown in the figure on the right. Take this surface to be approximately spherical with a radius of curvature R. Which of the following is/are correct? (g is the acceleration due to gravity)



(A) $R = \frac{h^2 + r^2}{2h}$

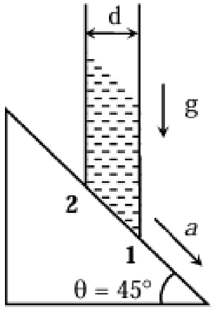
(B) $R = \frac{3r^2}{2h}$

(C) Apparent depth of the bottom of the beaker is close to $\frac{3H}{2}\left(1 + \frac{\omega^2 H}{2g}\right)^{-1}$

(D) Apparent depth of the bottom of the beaker is close to $\frac{3H}{4}\left(1 + \frac{\omega^2 H}{4g}\right)^{-1}$

Sol. [redacted]

*16. A cylindrical tube, with its base as shown in the figure, is filled with water. It is moving down with a constant acceleration a along a fixed inclined plane with angle $\theta = 45°$. P_1 and P_2 are pressures at points **1** and **2**, respectively located at the base of the tube. Let $\beta = (P_1 - P_2) / (\rho g d)$, where ρ is density of water, d is the inner diameter of the tube and g is the acceleration due to gravity. Which of the following statement(s) is(are) correct?



(A) $\beta = 0$ when $a = g/\sqrt{2}$

(B) $\beta > 0$ when $a = g/\sqrt{2}$

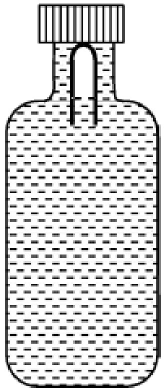
(C) $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when $a = g/2$

(D) $\beta = \frac{1}{\sqrt{2}}$ when $a = g/2$

Question Stem

A Soft plastic bottle, filled with water of density 1 gm/cc, carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm/cc. Initially the bottle is sealed at atmospheric pressure $p_0 = 10^5$ Pa so that the volume of the trapped air is $v_0 = 3.3$ cc. When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure $p_0 + \Delta p$ without changing its orientation. At this pressure, the volume of the trapped air is $v_0 - \Delta v$.
Let Δv = X cc and $\Delta p = Y \times 10^3$ Pa.

Find x and y



*Q.11 A bubble has surface tension S. The ideal gas inside the bubble has ratio of specific heats $\gamma = \frac{5}{3}$. The bubble is exposed to the atmosphere and it always retains its spherical shape. When the atmospheric pressure is $P_{\alpha 1}$, the radius of the bubble is found to be r_1 and the temperature of the enclosed gas is T_1. When the atmospheric pressure is $P_{\alpha 2}$, the radius of the bubble and the temperature of the enclosed gas are r_2 and T_2, respectively.

Which of the following statement(s) is(are) correct?

(A) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{r_1}{r_2}\right)^5 = \frac{P_{\alpha 2} + \frac{2S}{r_2}}{P_{\alpha 1} + \frac{2S}{r_1}}$

(B) If the surface of the bubble is a perfect heat insulator, then the total internal energy of the bubble including its surface energy does not change with the external atmospheric pressure.

(C) If the surface of the bubble is a perfect heat conductor and the change in atmospheric temperature is negligible, then $\left(\frac{r_1}{r_2}\right)^3 = \frac{P_{\alpha 2} + \frac{4S}{r_2}}{P_{\alpha 1} + \frac{4S}{r_1}}$

*Q.11 An incompressible liquid is kept in a container having a weightless piston with a hole. A capillary tube of inner radius 0.1 mm is dipped vertically into the liquid through the airtight piston hole, as shown in the figure. The air in the container is isothermally compressed from its original volume V_0 to $\frac{100}{101}V_0$ with the movable piston. Considering air as an ideal gas, the height (h) of the liquid column in the capillary above the liquid level in cm is_______.

[Given: Surface tension of the liquid is 0.075 N m^{-1}, atmospheric pressure is 10^5 N m^{-2}, acceleration due to gravity (g) is 10 m s^{-2}, density of the liquid is 10^3 kg m^{-3} and contact angle of capillary surface with the liquid is zero]