

$$p_x = \frac{nx'}{\sqrt{1+x'^2+y'^2}} \quad \text{and} \quad p_y = \frac{ny'}{\sqrt{1+x'^2+y'^2}} \quad \text{--- (2)}$$

Find  $x'$  and  $y'$ .

From (1),

$$p_x^2(1+x'^2+y'^2) = n^2x'^2.$$

Solve for  $y'^2$ :

$$y'^2 = \frac{n^2x'^2 - p_x^2 - p_x^2x'^2}{p_x^2}$$

$$= \frac{x'^2(n^2 - p_x^2) - p_x^2}{p_x^2}$$

$$\boxed{y'^2 = x'^2 \left[ \left( \frac{n}{p_x} \right)^2 - 1 \right] - 1} \quad \text{--- (a)}$$

$$\text{Similarly, } x'^2 = \frac{y'^2(n^2 - p_y^2) - p_y^2}{p_y^2} \quad \text{--- (b)}$$

$$\text{or simply } \boxed{x'^2 = y'^2 \left[ \left( \frac{n}{p_y} \right)^2 - 1 \right] - 1}$$

Now substitute (b) into (a) and

expand and solve for  $y'$ .