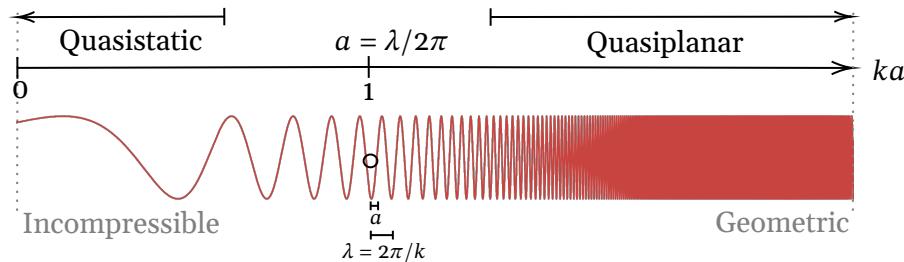


## Synopsis of dissertation

*Scattering and diffraction of acoustic waves in three problems with broken symmetry*

The effects of asymmetry in acoustic **scattering** and **diffraction** phenomena are investigated across the frequency spectrum, which is characterized by the parameter  $ka$ , where  $k$  is the wavenumber and  $a$  is the characteristic size of the scatterer or source of sound:



Calculated first is the **acoustic radiation force** exerted by progressive waves on asymmetric scatterers in the Rayleigh limit ( $ka \ll 1$ ). The Born approximation<sup>1</sup> in combination with Westervelt's far-field integral<sup>2</sup> leads to simple analytical expressions for the radiation force. Material asymmetry contributes to the force on the order of  $(ka)^6$ . For scatterers with symmetric material properties, Gor'kov's  $O[(ka)^4]$  force<sup>3</sup> is recovered. The result informs efforts to acoustically transport small particles over large distances.

Next considered is the effect of low- $ka$  asymmetric scattering in a **piezoelectric composite**. Constraints due to reciprocity and passivity are derived for the medium's constitutive relations, which fully couple continuum mechanics to electrodynamics. Limiting cases recover previous results that neglect magnetism,<sup>4</sup> piezoelectricity,<sup>5</sup> and elastodynamics.<sup>6</sup> The results guide the design of acoustic vector sensors.

Attention is finally turned to **vortex beams**, which are sound beams that break axisymmetry by an amount characterized by the orbital number  $\ell$ .<sup>7</sup> The vortex field radiated by a circular piston is represented for all  $ka$  in terms of Bessel beams. Closed-form analytical solutions are derived in the paraxial approximation ( $ka \gg 1$ ). In the limit that  $ka$  is infinite, the effects of diffraction are suppressed, leading to a geometric description of how the field depends on  $\ell$ . In the absence of orbital motion ( $\ell = 0$ ), the exact, paraxial, and ray solutions recover known descriptions of axisymmetric beams.<sup>8</sup> The analytical solutions and scaling laws derived simplify the modeling of vortex fields, which are used for particle manipulation and acoustic communications.

<sup>1</sup>P. M. Morse and K. U. Ingard, *Theoretical Acoustics*, (McGraw-Hill, 1968), Eq. (8.1.20).

<sup>2</sup>P. J. Westervelt, *J. Acoust. Soc. Am.* **29**, 26–29 (1957), Eq. (2).

<sup>3</sup>L. P. Gor'kov, *Sov. Phys. Dokl.* **6**, 773–775 (1962), Eq. (10).

<sup>4</sup>R. Pernas-Salomón and G. Shmuel, *Phys. Rev. Appl.* **14**, 1–19 (2020), Eqs. (18), (20), (33), and (42).

<sup>5</sup>M. B. Muhlestein et al., *Proc. R. Soc. A* **472**, 1–15 (2016), Eqs. (3.10)–(3.17).

<sup>6</sup>J. A. Kong, *P. IEEE* **60**, 1036–1046 (1972), Eqs. (8) and (23).

<sup>7</sup>B. T. Hefner and P. L. Marston, *J. Acoust. Soc. Am.* **103**, 2971 (1998).

<sup>8</sup>M. F. Hamilton, "Sound Beams," M. F. Hamilton and D. T. Blackstock, editors, *Nonlinear Acoustics*, 3rd edition, (Springer, 2024), Eqs. (8.19) and (8.37).