Source-driven homogenization theory for electro-momentum coupled scatterers

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Questions to be answered...

1. What is acoustic and electromagnetic bianisotropy?

2. What is electromomentum coupling and double bianisotropy?

3. What are the origins of double bianisotropy?

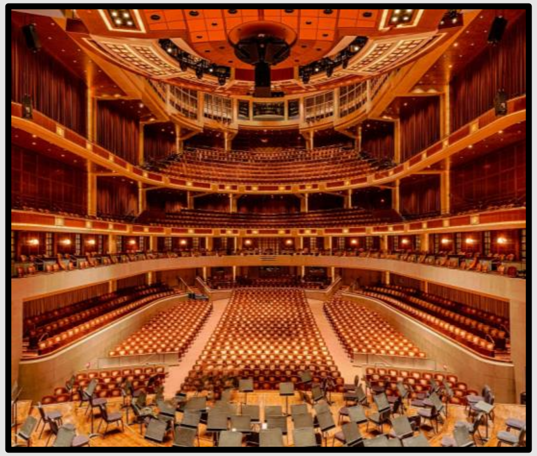
Acoustic bianisotropy (Willis coupling)

$$\varepsilon = -\beta p$$
$$\boldsymbol{\mu} = \rho \boldsymbol{u}$$

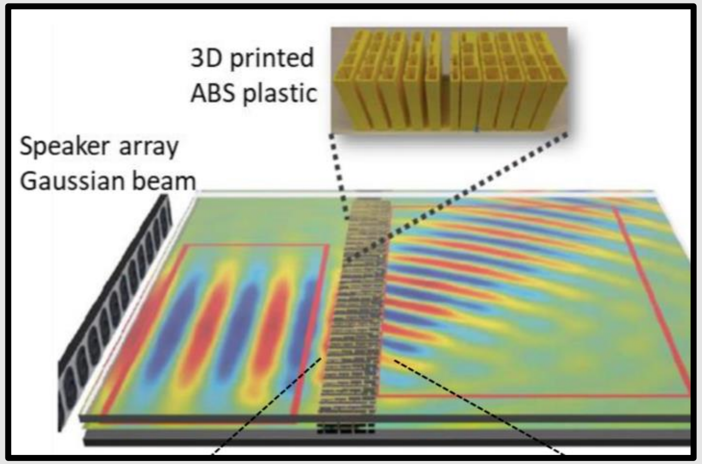
Isotropic

$$\varepsilon = -\beta p + \boldsymbol{\gamma} \cdot \boldsymbol{u}$$
$$\boldsymbol{\mu} = -\boldsymbol{\eta} p + \underline{\rho} \boldsymbol{u}$$

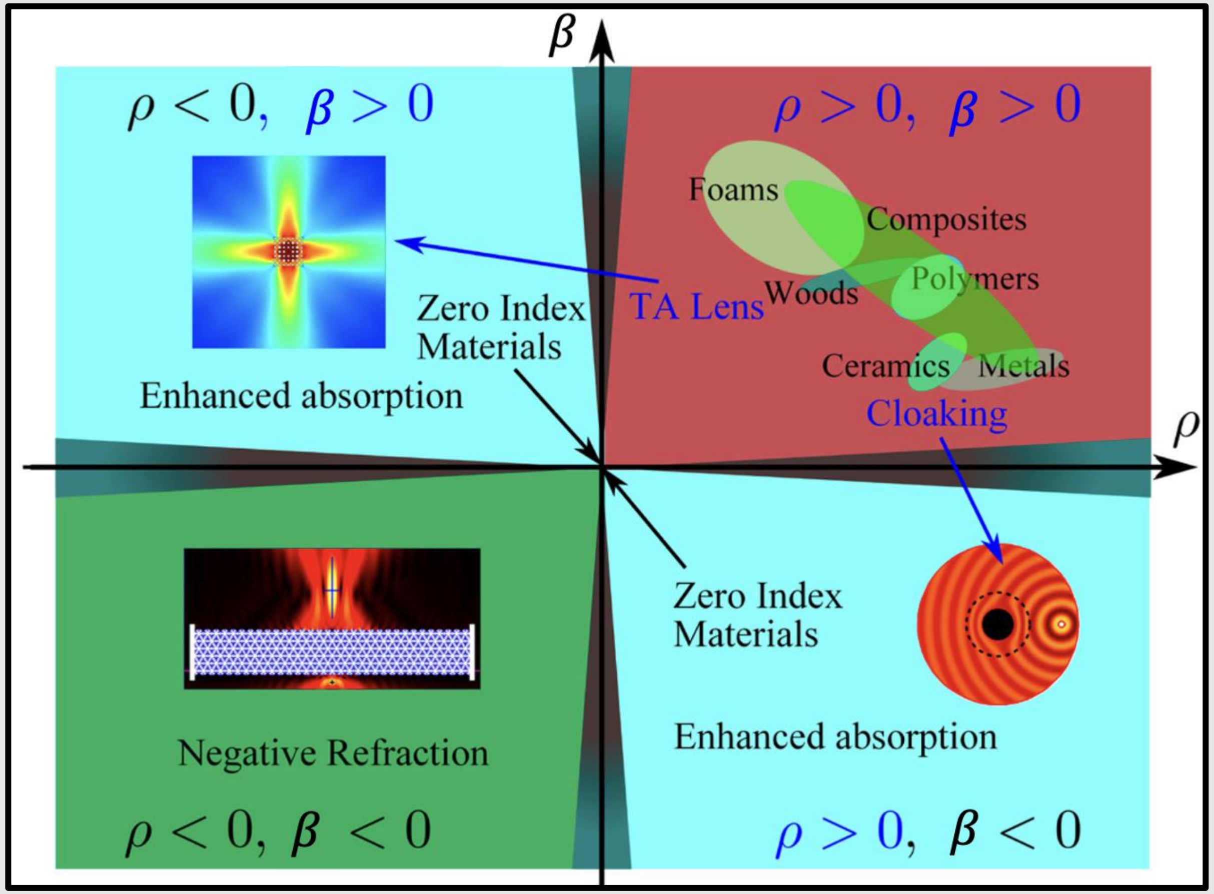
Bianisotropic



Meyerson Symphony Center



J. Li et al., Nat. Comm., **9** (2018)



M.R. Haberman & A. N. Norris, Acoustics Today, **12**, 3 (2016)

PHYSICAL REVIEW B **96**, 104303 (2017)

Origins of Willis coupling and acoustic bianisotropy in acoustic metamaterials through source-driven homogenization

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L. Quan et al.
Nat. Commun. **10**, (2019)

asymmetry

lattice effects

Electromagnetic bianisotropy

$$D = \epsilon E \qquad B = \mu H$$

Isotropic

$$D = \underline{\epsilon} E \qquad B = \underline{\mu} H$$

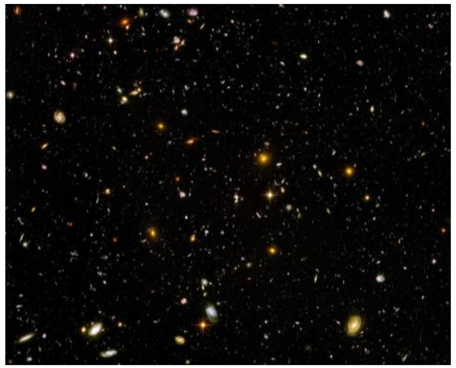
Anisotropic

$$D = \epsilon E + \zeta H \qquad B = \xi E + \mu H$$

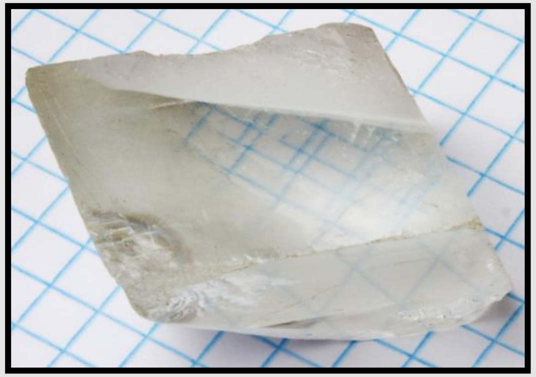
Biisotropic

$$D = \underline{\epsilon} E + \underline{\zeta} H \qquad B = \underline{\xi} E + \underline{\mu} H$$

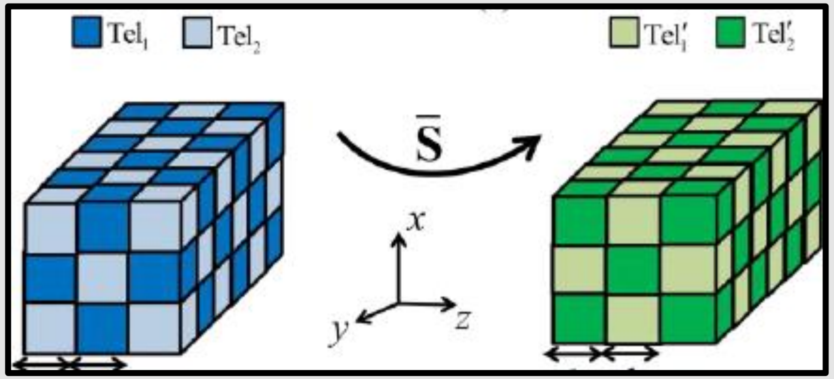
Bianisotropic



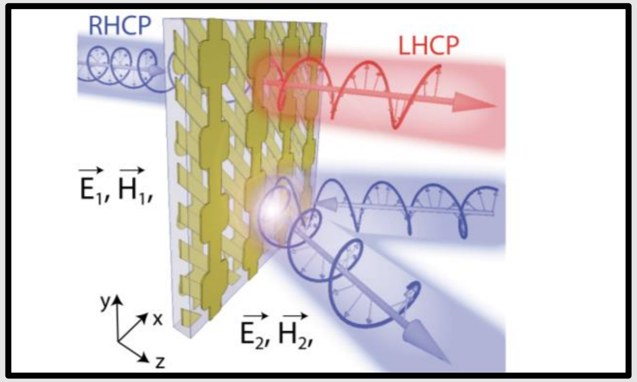
Hubble deep field, NASA



Birefringence, Wikipedia



Prudencio et al., IEEE, **62** (2014)



C. Pfeiffer et al., PRL, **113** (2014)

PHYSICAL REVIEW B **84**, 075153 (2011)

First-principles homogenization theory for periodic metamaterials

Andrea Alù*

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(Received 6 December 2010; revised manuscript received 6 June 2011; published 15 August 2011)

L. Ran et al. PIER, **51** (2005)

asymmetry

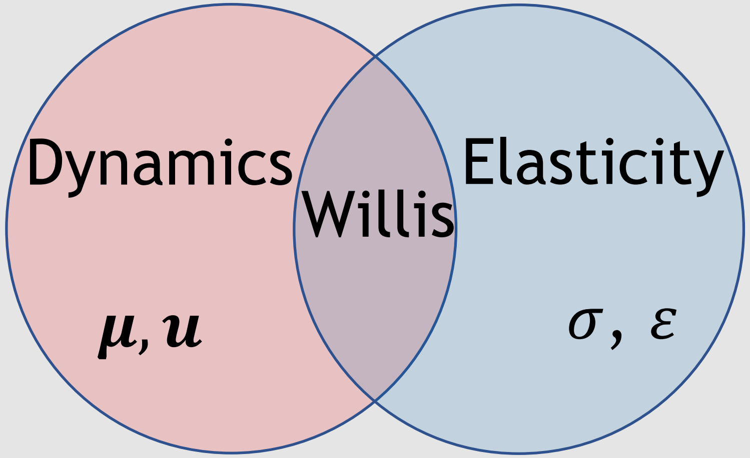
lattice effects

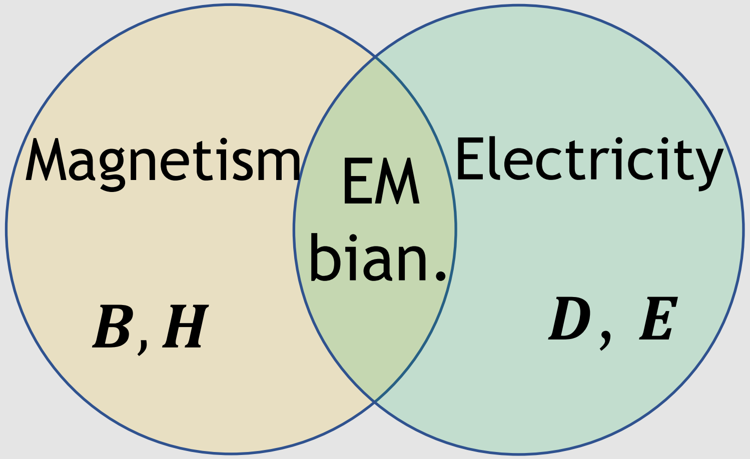
Generalized Willis coupling

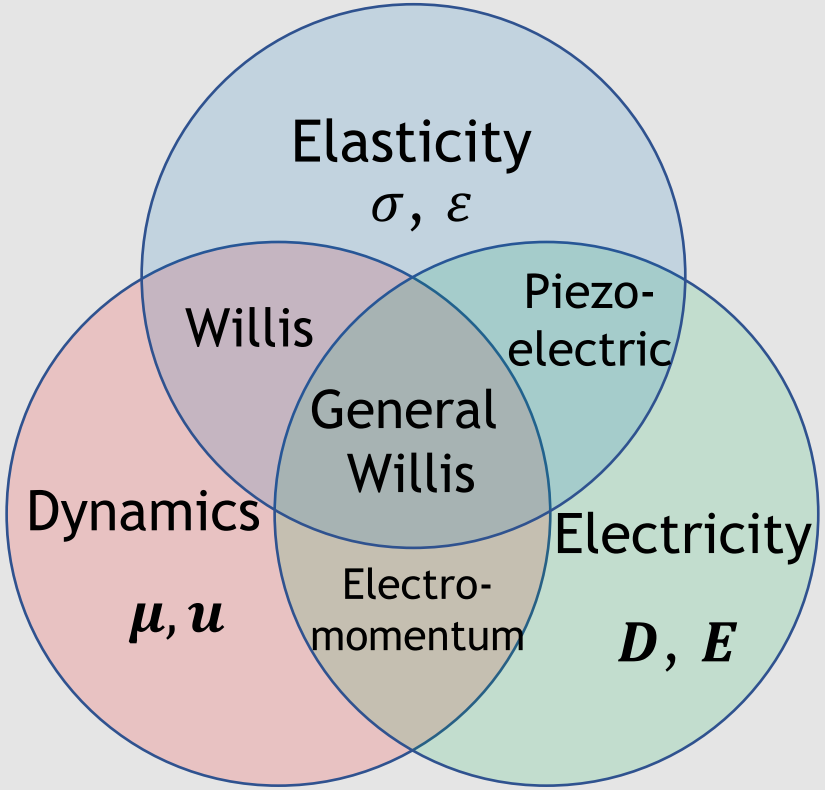
$$\boldsymbol{\varepsilon} = -\boldsymbol{\beta} p + \boldsymbol{\gamma} \cdot \boldsymbol{u}$$
$$\boldsymbol{\mu} = -\boldsymbol{\eta} p + \underline{\boldsymbol{\rho}} \boldsymbol{u}$$

$$\boldsymbol{D} = \underline{\boldsymbol{\epsilon}} \boldsymbol{E} + \underline{\boldsymbol{\zeta}} \boldsymbol{H}$$
$$\boldsymbol{B} = \underline{\boldsymbol{\xi}} \boldsymbol{E} + \underline{\boldsymbol{\mu}} \boldsymbol{H}$$

$$\begin{pmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\mu} \\ \boldsymbol{D} \end{pmatrix} = \begin{pmatrix} -\boldsymbol{\beta} & \boldsymbol{\gamma}^{\mathsf{T}} & \boldsymbol{d}^{\mathsf{T}} \\ -\boldsymbol{\eta} & \boldsymbol{\mu} & \underline{\boldsymbol{v}} \\ -\boldsymbol{d} & \underline{\boldsymbol{w}} & \underline{\boldsymbol{\epsilon}} \end{pmatrix} \begin{pmatrix} p \\ \boldsymbol{u} \\ \boldsymbol{E} \end{pmatrix}$$







Journal of the Mechanics and Physics of Solids **134** (2020) 103770

Symmetry breaking creates electro-momentum coupling in piezoelectric metamaterials

René Pernas-Salomón, Gal Shmuel*

Faculty of Mechanical Engineering, Technion–Israel Institute of Technology, Haifa 32000, Israel

Quasi-electrostatic approximation

$\frac{\lambda_{\text{light}}}{\lambda_{\text{sound}}} \sim 8.7 \times 10^5$ (air), $\sim 1.5 \times 10^5$ (water)

Double bianisotropy

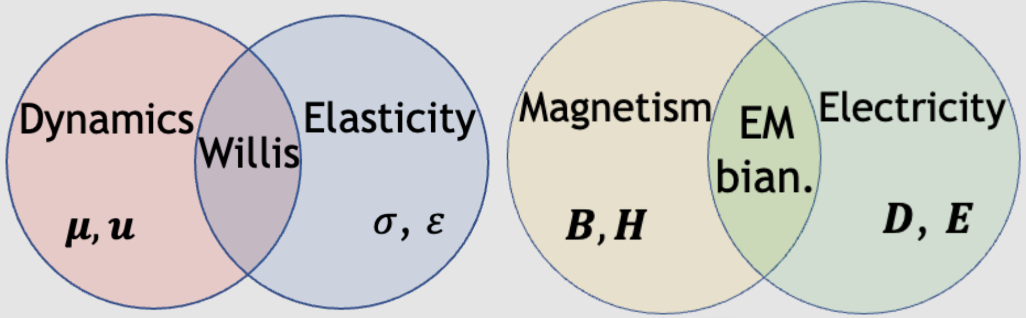
$$\varepsilon = -\beta p + \boldsymbol{\gamma} \cdot \boldsymbol{u} \qquad \boldsymbol{D} = \underline{\boldsymbol{\epsilon}}\boldsymbol{E} + \underline{\boldsymbol{\zeta}}\boldsymbol{H}$$

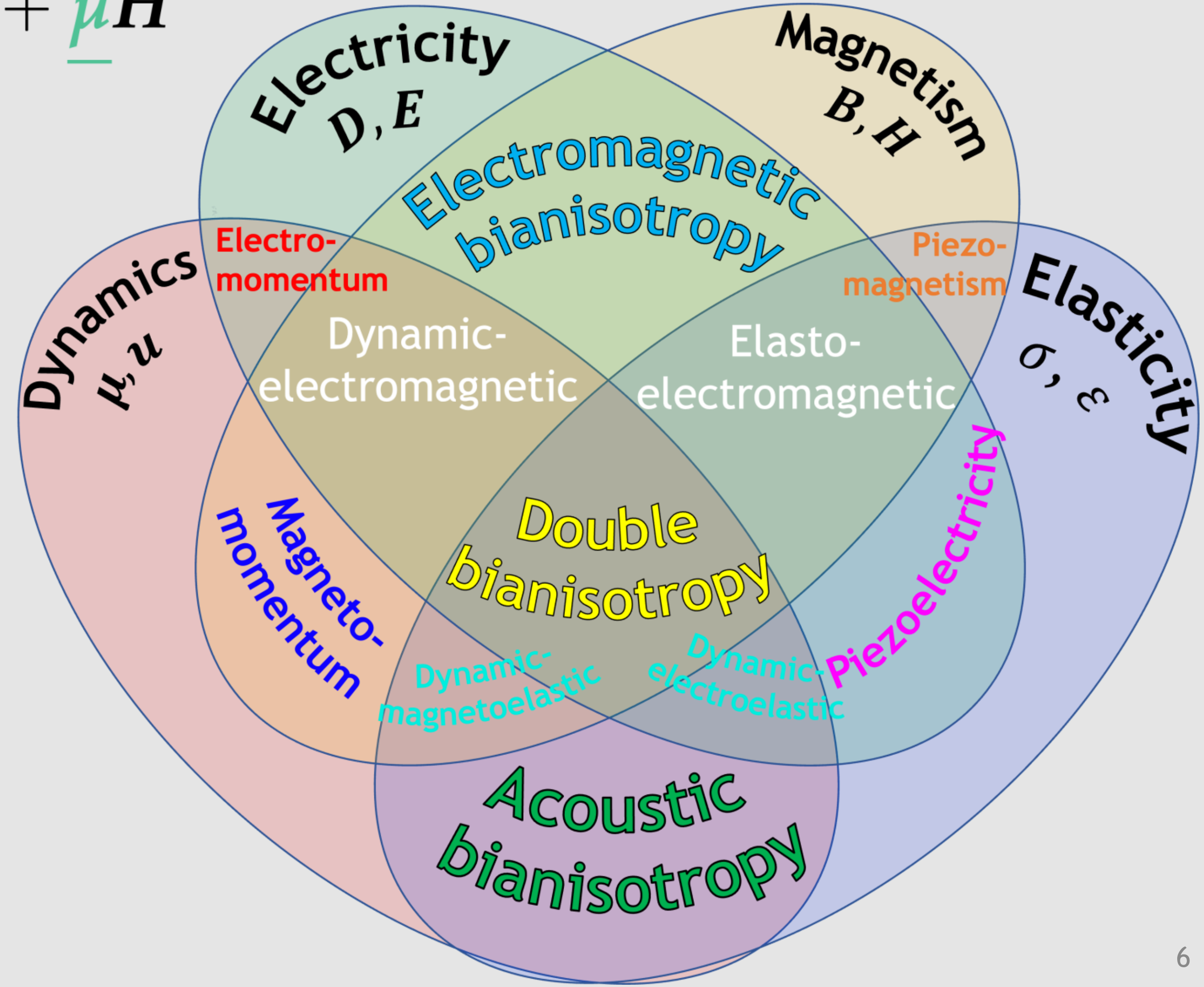
$$\boldsymbol{\mu} = -\boldsymbol{\eta} p + \underline{\boldsymbol{\rho}}\boldsymbol{u} \qquad \boldsymbol{B} = \underline{\boldsymbol{\xi}}\boldsymbol{E} + \underline{\boldsymbol{\mu}}\boldsymbol{H}$$

Why include $\boldsymbol{H}$?

- $\nabla \times \boldsymbol{H} = \boldsymbol{J} + \epsilon_0 \partial \boldsymbol{E}/\partial t$: reciprocity, passivity
- Complements previous wave studies[1, 2, 3]
- Reveals new emergent couplings[4]

$$\begin{pmatrix} \varepsilon \\ \boldsymbol{\mu} \\ \boldsymbol{D} \\ \boldsymbol{B} \end{pmatrix} = \begin{pmatrix} -\beta & \boldsymbol{\gamma}^{\mathsf{T}} & \boldsymbol{b}^{\mathsf{T}} & \boldsymbol{d}^{\mathsf{T}} \\ -\boldsymbol{\eta} & \underline{\boldsymbol{\rho}} & \underline{\boldsymbol{v}} & \underline{\boldsymbol{m}} \\ \boldsymbol{c} & \underline{\boldsymbol{w}} & \underline{\boldsymbol{\epsilon}} & \underline{\boldsymbol{\xi}} \\ \boldsymbol{e} & \underline{\boldsymbol{n}} & \underline{\boldsymbol{\zeta}} & \underline{\boldsymbol{\mu}} \end{pmatrix} \begin{pmatrix} p \\ \boldsymbol{u} \\ \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}$$





1 Sieck et al., Phys. Rev. B **96**, 104303 (2017)
2 Wallen et al., POMA **46**, 065002 (2022)
3 Lee et al., J. Appl. Phys. **132**, 125108 (2022)
4 J. Venn, *Symbolic Logic*, MacMillan (1881)

Double bianisotropy

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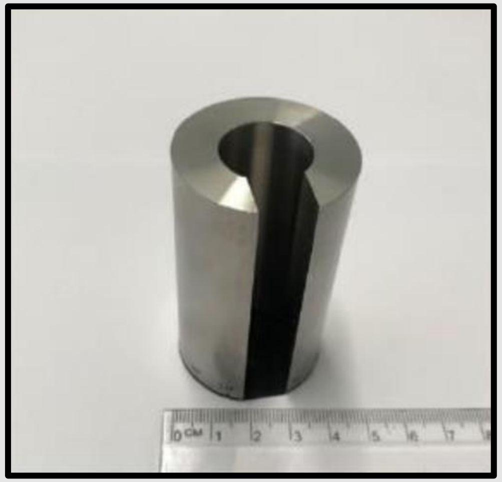
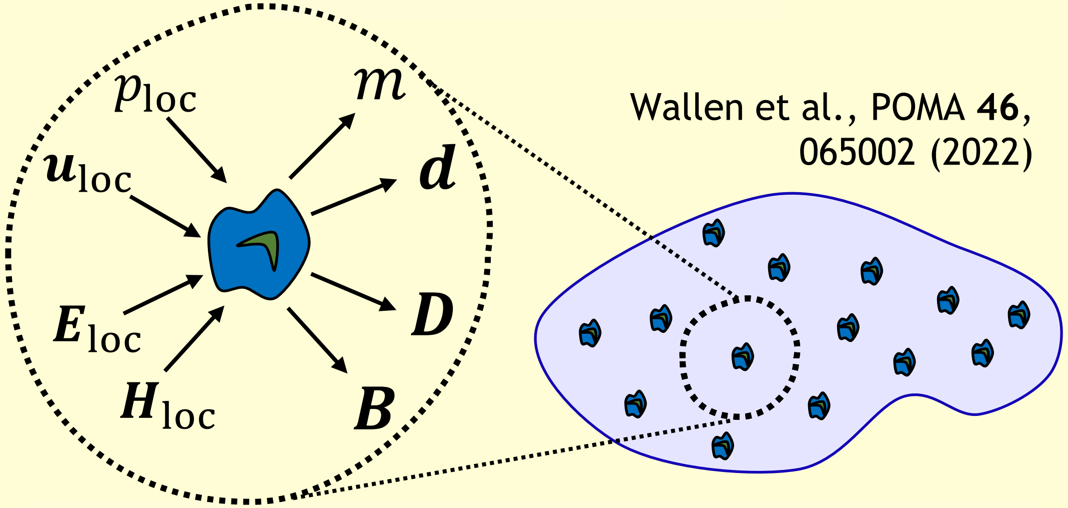


L. Quan et al.,
Nat. Commun. **10,** (2019)

L. Ran et al.,
PIER, **51** (2005)

General scatterer

$$\begin{pmatrix} m_0 \\ \boldsymbol{d}_0 \\ \boldsymbol{P}_0 \\ \boldsymbol{M}_0 \end{pmatrix} = \underline{\boldsymbol{\alpha}} \begin{pmatrix} p_{\text{loc}} \\ \boldsymbol{u}_{\text{loc}} \\ \boldsymbol{E}_{\text{loc}} \\ \boldsymbol{H}_{\text{loc}} \end{pmatrix}$$



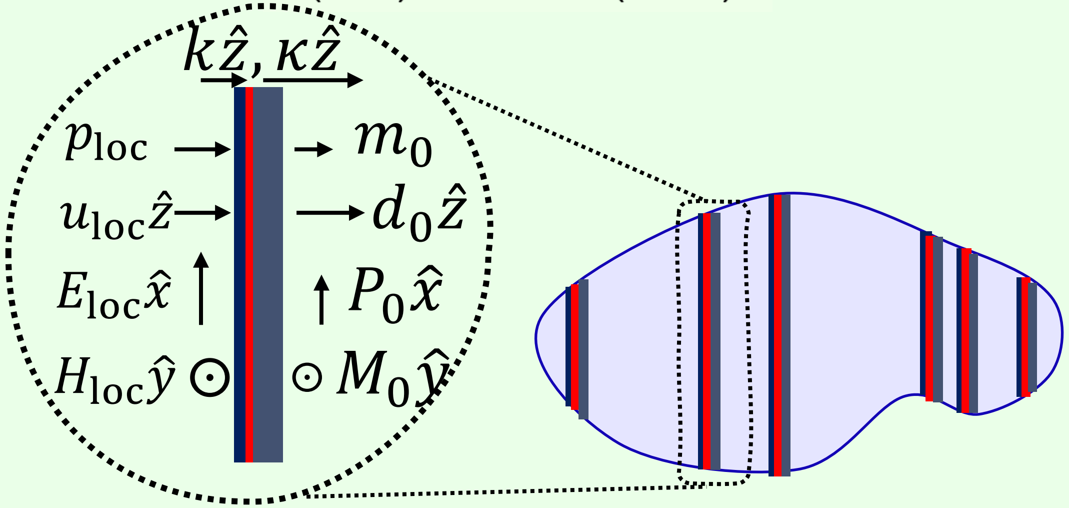
Wallen et al., POMA **46**, 065002 (2022)

$$\begin{pmatrix} \varepsilon_{\text{eff}} \\ \boldsymbol{\mu}_{\text{eff}} \\ \boldsymbol{D}_{\text{eff}} \\ \boldsymbol{B}_{\text{eff}} \end{pmatrix} = \begin{pmatrix} -\beta & \boldsymbol{\gamma}^{\mathsf{T}} & \boldsymbol{b}^{\mathsf{T}} & \boldsymbol{d}^{\mathsf{T}} \\ -\boldsymbol{\eta} & \underline{\boldsymbol{\rho}} & \underline{\boldsymbol{v}} & \underline{\boldsymbol{m}} \\ \boldsymbol{c} & \underline{\boldsymbol{w}} & \underline{\boldsymbol{\epsilon}} & \underline{\boldsymbol{\xi}} \\ \boldsymbol{e} & \underline{\boldsymbol{n}} & \underline{\boldsymbol{\zeta}} & \underline{\boldsymbol{\mu}} \end{pmatrix} \begin{pmatrix} p_{\text{eff}} \\ \boldsymbol{u}_{\text{eff}} \\ \boldsymbol{E}_{\text{eff}} \\ \boldsymbol{H}_{\text{eff}} \end{pmatrix}$$

Double bianisotropy

1D wave propagation

$$\begin{pmatrix} m_0 \\ d_0 \\ P_0 \\ M_0 \end{pmatrix} = [\alpha] \begin{pmatrix} p_{\mathrm{loc}} \\ u_{\mathrm{loc}} \\ E_{\mathrm{loc}} \\ H_{\mathrm{loc}} \end{pmatrix}$$



$$\begin{pmatrix} \varepsilon_{\mathrm{eff}} \\ \mu_{v,\mathrm{eff}} \\ D_{\mathrm{eff}} \\ B_{\mathrm{eff}} \end{pmatrix} = \begin{pmatrix} -\beta & \gamma & b & d \\ -\eta & \rho & v & m \\ c & w & \epsilon & \xi \\ e & n & \zeta & \mu \end{pmatrix} \begin{pmatrix} p_{\mathrm{eff}} \\ u_{\mathrm{eff}} \\ E_{\mathrm{eff}} \\ H_{\mathrm{eff}} \end{pmatrix}$$

General scatterer

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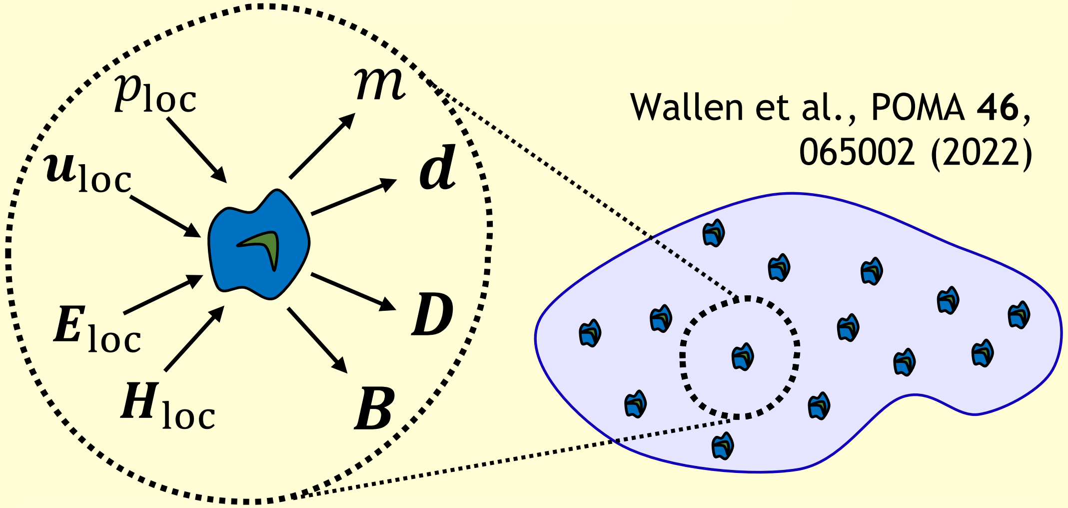

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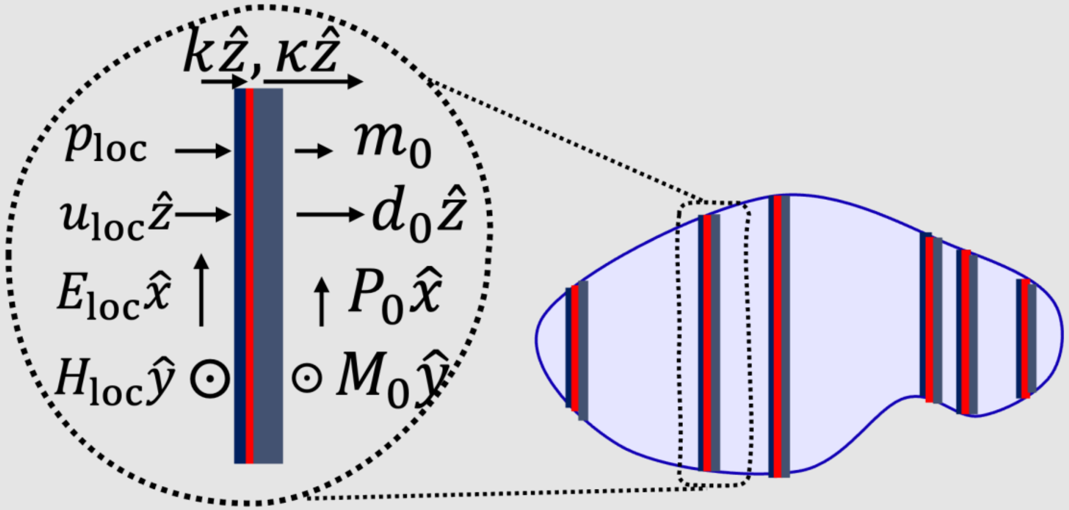
What is the origin of double bianisotropy?

$$\begin{pmatrix} m_0 \\ d_0 \\ P_0 \\ M_0 \end{pmatrix} = \begin{pmatrix} -\beta_0 \alpha_{pp} & -i(\rho_0\beta_0)^{1/2}\alpha_{pu} & (\beta_0\epsilon_0)^{1/2}\alpha_{pE} & 0 \\ -i(\rho_0\beta_0)^{1/2}\alpha_{pu} & \rho_0\alpha_{uu} & -i(\rho_0\epsilon_0)^{1/2}\alpha_{uE} & 0 \\ -(\beta_0\epsilon_0)^{1/2}\alpha_{pE} & i(\rho_0\epsilon_0)^{1/2}\alpha_{uE} & \epsilon_0\alpha_{EE} & 0 \\ 0 & 0 & 0 & \mu_0\alpha_{HH} \end{pmatrix} \begin{pmatrix} p_{\text{loc}} \\ u_{\text{loc}} \\ E_{\text{loc}} \\ H_{\text{loc}} \end{pmatrix}$$

- Conventional response



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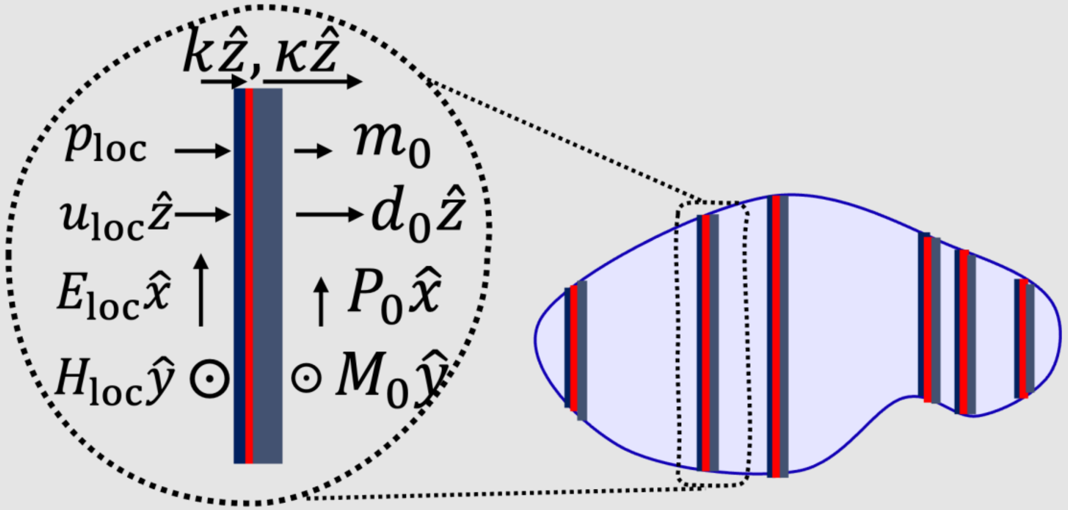
- Conventional response
- Acoustic[1] & electromagnetic[2] bianisotropy



[1] A. Alù, PRB **84**, 075153 (2011)
[2] Sieck et al., PRB **96**, 104303 (2017)

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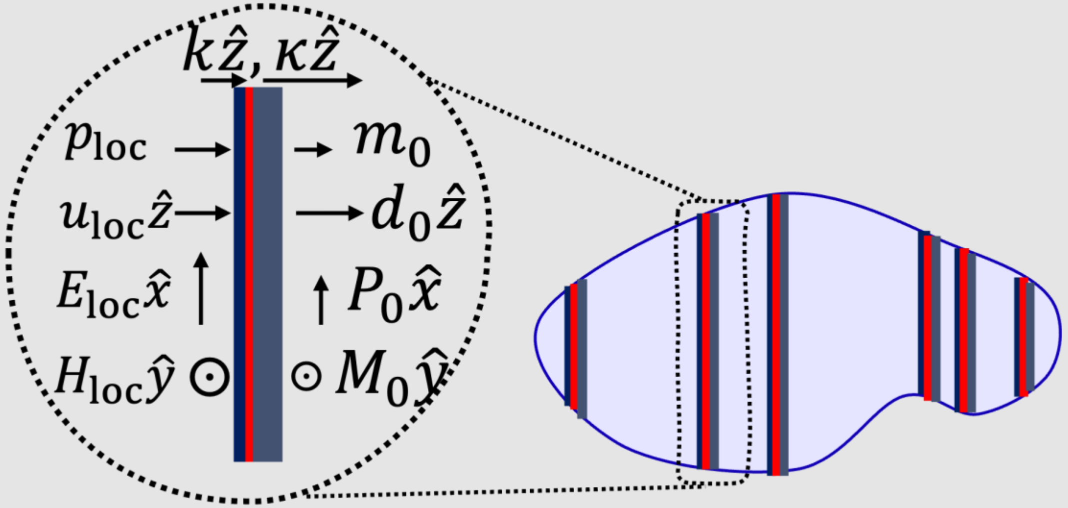
- Conventional response
- Acoustic[1] & electromagnetic[2] bianisotropy
- Piezoelectricity & piezomagnetism



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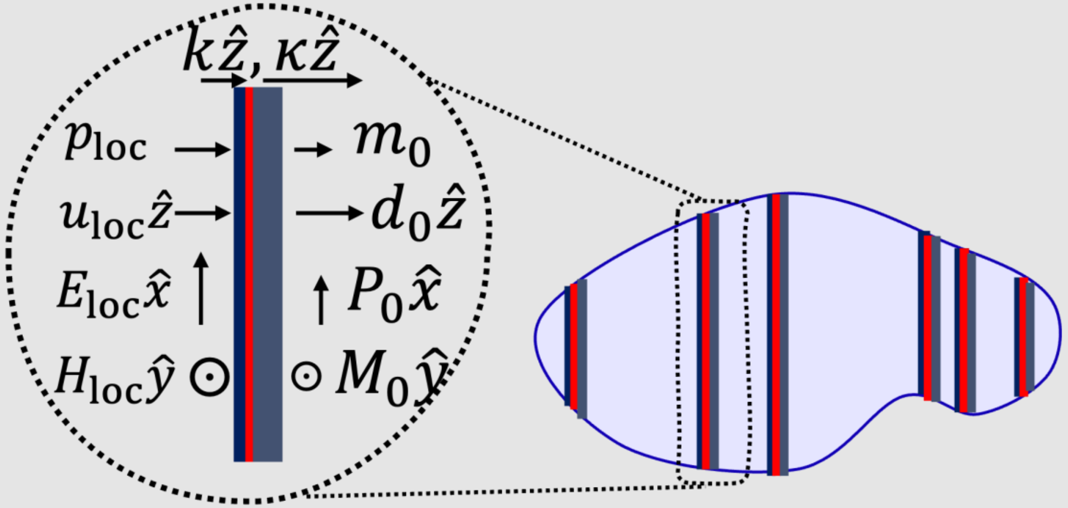
- Conventional response
- Acoustic[1] & electromagnetic[2] bianisotropy
- Piezoelectricity & piezomagnetism
- electromomentum & magnetomomentum



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- Conventional response
- Acoustic[1] & electromagnetic[2] bianisotropy
- Piezoelectricity & piezomagnetism
- electromomentum & magnetomomentum

Objective

Explore the **microscale** and **mesoscale** origins of the **macroscale** coupling between ε_{eff}, μ_{eff}, D_{eff}, and H_{eff}.

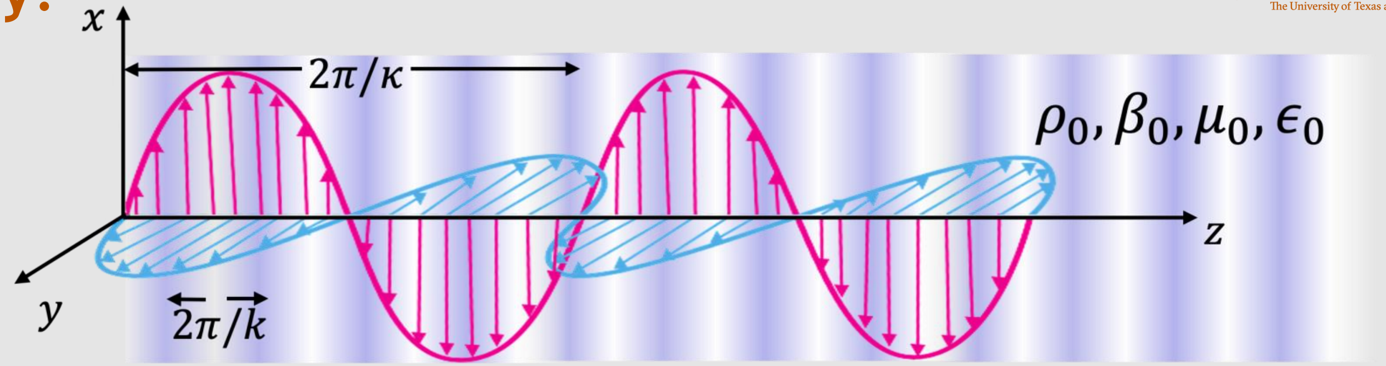
$$\begin{pmatrix} \varepsilon_{\text{eff}} \\ \mu_{v,\text{eff}} \\ D_{\text{eff}} \\ B_{\text{eff}} \end{pmatrix} = \begin{pmatrix} -\beta & \gamma & b & d \\ -\eta & \rho & v & m \\ c & w & \epsilon & \xi \\ e & n & \zeta & \mu \end{pmatrix} \begin{pmatrix} p_{\text{eff}} \\ u_{\text{eff}} \\ E_{\text{eff}} \\ H_{\text{eff}} \end{pmatrix}$$

1 A. Alù, PRB **84**, 075153 (2011)
2 Sieck et al., PRB **96**, 104303 (2017)

What is the origin of double bianisotropy?

$$\left.\begin{aligned} ik p_{\mathrm{ext}} &= i\omega\rho_0 u_{\mathrm{ext}} + f_{\mathrm{ext}} \\ ik u_{\mathrm{ext}} &= i\omega\beta_0 p_{\mathrm{ext}} + q_{\mathrm{ext}} \end{aligned}\right\} \exp i(kz - \omega t)$$

$$\left.\begin{aligned} i\kappa E_{\mathrm{ext}} &= i\omega\mu_0 H_{\mathrm{ext}} - K_{\mathrm{ext}} \\ i\kappa H_{\mathrm{ext}} &= i\omega\epsilon_0 E_{\mathrm{ext}} - J_{\mathrm{ext}} \end{aligned}\right\} \exp i(\kappa z - \omega t)$$



external fields obey governing equations

What is the origin of double bianisotropy?

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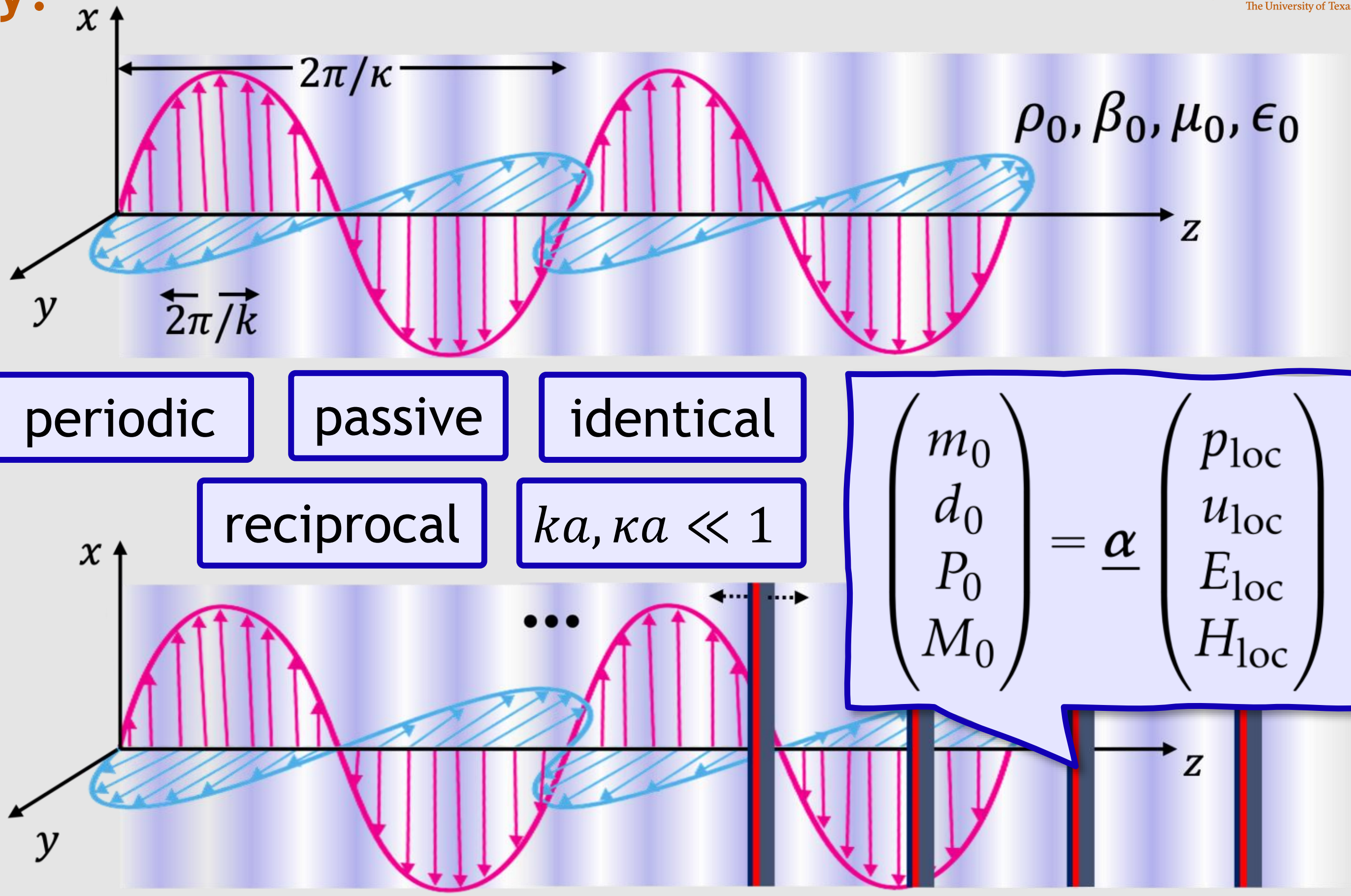

periodic

passive

identical

reciprocal

$ka, \kappa a \ll 1$

$$\begin{pmatrix} m_0 \\ d_0 \\ P_0 \\ M_0 \end{pmatrix} = \underline{\boldsymbol{\alpha}} \begin{pmatrix} p_{\mathrm{loc}} \\ u_{\mathrm{loc}} \\ E_{\mathrm{loc}} \\ H_{\mathrm{loc}} \end{pmatrix}$$

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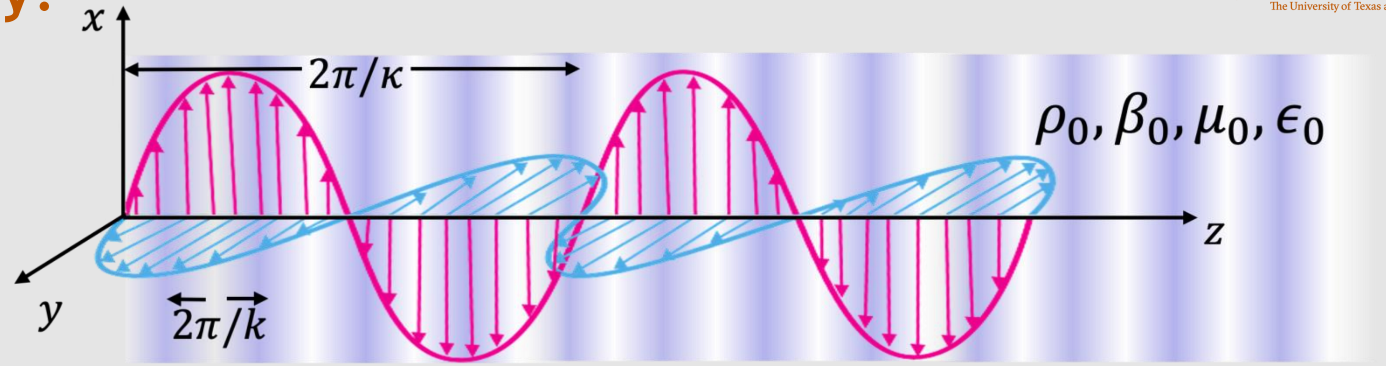

periodic

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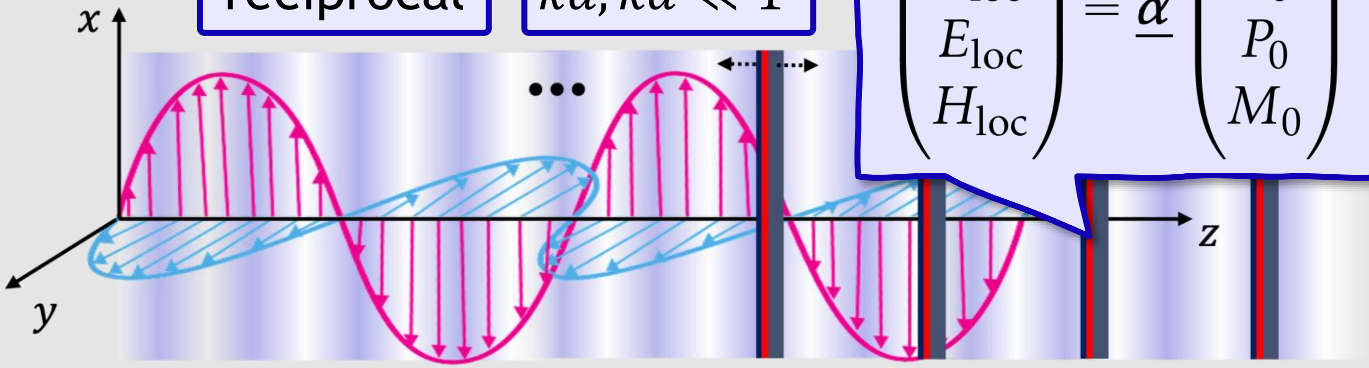
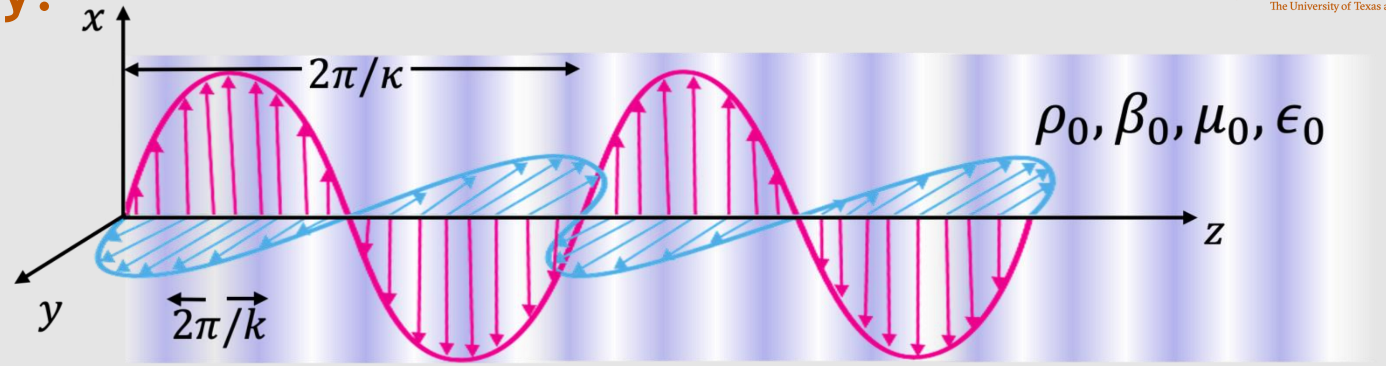


$$\begin{pmatrix} p_{\text{loc}} \\ u_{\text{loc}} \\ E_{\text{loc}} \\ H_{\text{loc}} \end{pmatrix} = \underline{\tilde{\alpha}} \begin{pmatrix} m_0 \\ d_0 \\ P_0 \\ M_0 \end{pmatrix}$$

What is the origin of double bianisotropy?

$$\left.\begin{aligned} ikp_{\text{ext}} &= i\omega\rho_0 u_{\text{ext}} + f_{\text{ext}} \\ iku_{\text{ext}} &= i\omega\beta_0 p_{\text{ext}} + q_{\text{ext}} \end{aligned}\right\} \exp i(kz-\omega t)$$

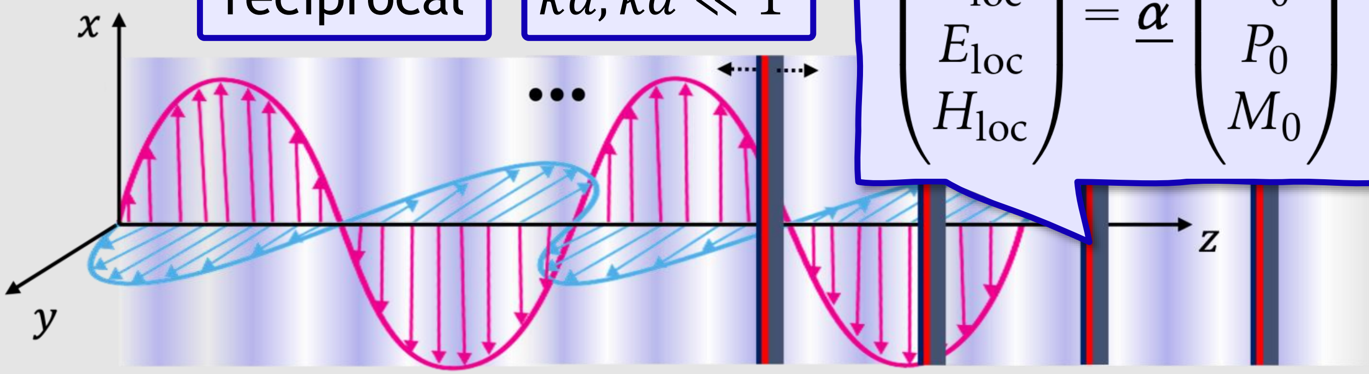
$$\left.\begin{aligned} i\kappa E_{\text{ext}} &= i\omega\mu_0 H_{\text{ext}} - K_{\text{ext}} \\ i\kappa H_{\text{ext}} &= i\omega\epsilon_0 E_{\text{ext}} - J_{\text{ext}} \end{aligned}\right\} \exp i(\kappa z-\omega t)$$



periodic | passive | identical

reciprocal | $ka, \kappa a \ll 1$

$$\begin{aligned} p_{\text{loc}} &= p_{\text{ext}} - \beta_0^{-1} S_{\text{ac}} m_0 + c_0 S_{\text{ac}} d_0 \\ u_{\text{loc}} &= u_{\text{ext}} - c_0 S_{\text{ac}} m_0 + \rho_0^{-1} S_{\text{ac}} d_0 \\ E_{\text{loc}} &= E_{\text{ext}} + \epsilon_0^{-1} S_{\text{em}} P_0 + C_0 S_{\text{em}} M_0 \\ H_{\text{loc}} &= H_{\text{ext}} + \underbrace{C_0 S_{\text{em}} P_0 + \mu_0^{-1} S_{\text{em}} M_0}_{\text{scattered fields}} \end{aligned}$$

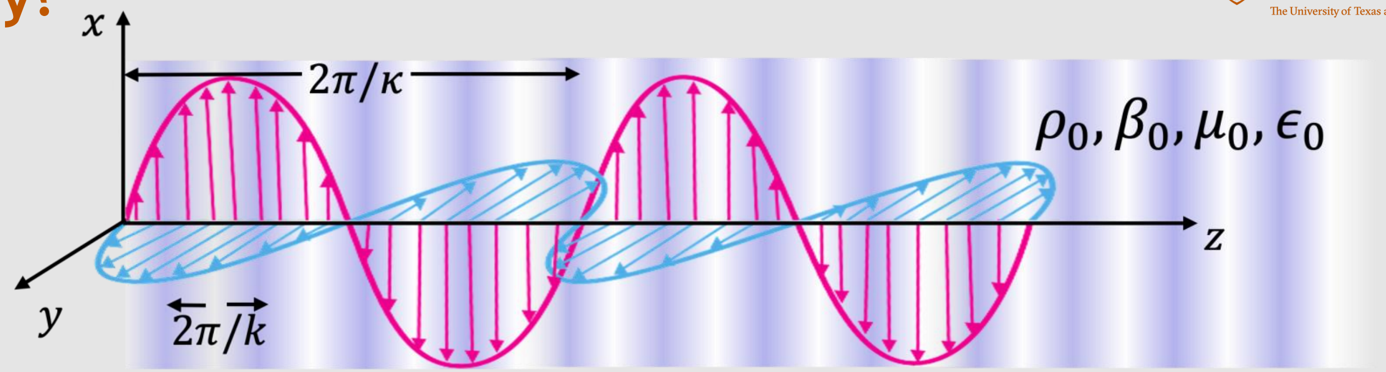


$$\begin{pmatrix} p_{\text{loc}} \\ u_{\text{loc}} \\ E_{\text{loc}} \\ H_{\text{loc}} \end{pmatrix} = \underline{\tilde{\alpha}} \begin{pmatrix} m_0 \\ d_0 \\ P_0 \\ M_0 \end{pmatrix}$$

What is the origin of double bianisotropy?

$$\left.\begin{aligned} ikp_{\mathrm{ext}} &= i\omega\rho_0 u_{\mathrm{ext}} + f_{\mathrm{ext}} \\ iku_{\mathrm{ext}} &= i\omega\beta_0 p_{\mathrm{ext}} + q_{\mathrm{ext}} \end{aligned}\right\} \exp i(kz-\omega t)$$

$$\left.\begin{aligned} i\kappa E_{\mathrm{ext}} &= i\omega\mu_0 H_{\mathrm{ext}} - K_{\mathrm{ext}} \\ i\kappa H_{\mathrm{ext}} &= i\omega\epsilon_0 E_{\mathrm{ext}} - J_{\mathrm{ext}} \end{aligned}\right\} \exp i(\kappa z-\omega t)$$



$$\begin{aligned} p_{\mathrm{loc}} &= p_{\mathrm{ext}} - \beta_0^{-1} S_{\mathrm{ac}} m_0 + c_0 S_{\mathrm{ac}} d_0 \\ u_{\mathrm{loc}} &= u_{\mathrm{ext}} - c_0 S_{\mathrm{ac}} m_0 + \rho_0^{-1} S_{\mathrm{ac}} d_0 \\ E_{\mathrm{loc}} &= E_{\mathrm{ext}} + \epsilon_0^{-1} S_{\mathrm{em}} P_0 + C_0 S_{\mathrm{em}} M_0 \\ H_{\mathrm{loc}} &= H_{\mathrm{ext}} + C_0 S_{\mathrm{em}} P_0 + \mu_0^{-1} S_{\mathrm{em}} M_0 \end{aligned}$$

$$\begin{pmatrix} p_{\mathrm{loc}} \\ u_{\mathrm{loc}} \\ E_{\mathrm{loc}} \\ H_{\mathrm{loc}} \end{pmatrix} = \underline{\tilde{\alpha}} \begin{pmatrix} m_0 \\ d_0 \\ P_0 \\ M_0 \end{pmatrix}$$

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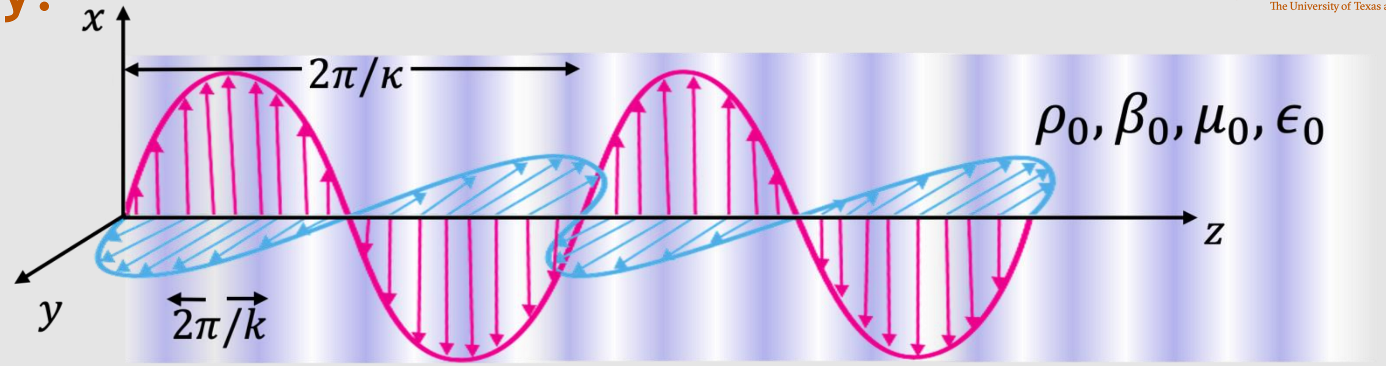

$$p_{\text{ext}} = V[-\beta_0^{-1}(\tilde{\alpha}_{pp} - S_{\text{ac}})\bar{m}_{\text{eff}} + c_0(-i\tilde{\alpha}_{pu} - S_{\text{ac}})\bar{d}_{\text{eff}} - i(\beta_0\epsilon_0)^{-1/2}\tilde{\alpha}_{pE}\bar{P}_{\text{eff}}]$$

$$u_{\text{ext}} = V[-c_0(i\tilde{\alpha}_{pu} - S_{\text{ac}})\bar{m}_{\text{eff}} + \rho_0^{-1}(\tilde{\alpha}_{uu} - S_{\text{ac}})\bar{d}_{\text{eff}} - i(\rho_0\epsilon_0)^{-1/2}\tilde{\alpha}_{uE}\bar{P}_{\text{eff}}]$$

$$E_{\text{ext}} = V[-i(\beta_0\epsilon_0)^{-1/2}\tilde{\alpha}_{pE}\bar{m}_{\text{eff}} - i(\rho_0\epsilon_0)^{-1/2}\tilde{\alpha}_{uE}\bar{d}_{\text{eff}} + \epsilon_0^{-1}(\tilde{\alpha}_{EE} - S_{\text{em}})\bar{P}_{\text{eff}} - C_0 S_{\text{em}}\bar{M}_{\text{eff}}]$$

$$H_{\text{ext}} = V[-C_0 S_{\text{em}}\bar{P}_{\text{eff}} - \mu_0^{-1}(S_{\text{em}} - \tilde{\alpha}_{HH})\bar{M}_{\text{eff}}]$$

where $\bar{m}_{\text{eff}} = m_0/V$, $\bar{d}_{\text{eff}} = d_0/V$, $\bar{P}_{\text{eff}} = P_0/V$, $\bar{M}_{\text{eff}} = M_0/V$

What is the origin of double bianisotropy?

$$\left.\begin{aligned} ik p_{\text{ext}} &= i\omega\rho_0 u_{\text{ext}} + f_{\text{ext}} \\ ik u_{\text{ext}} &= i\omega\beta_0 p_{\text{ext}} + q_{\text{ext}} \end{aligned}\right\} \exp i(kz-\omega t)$$

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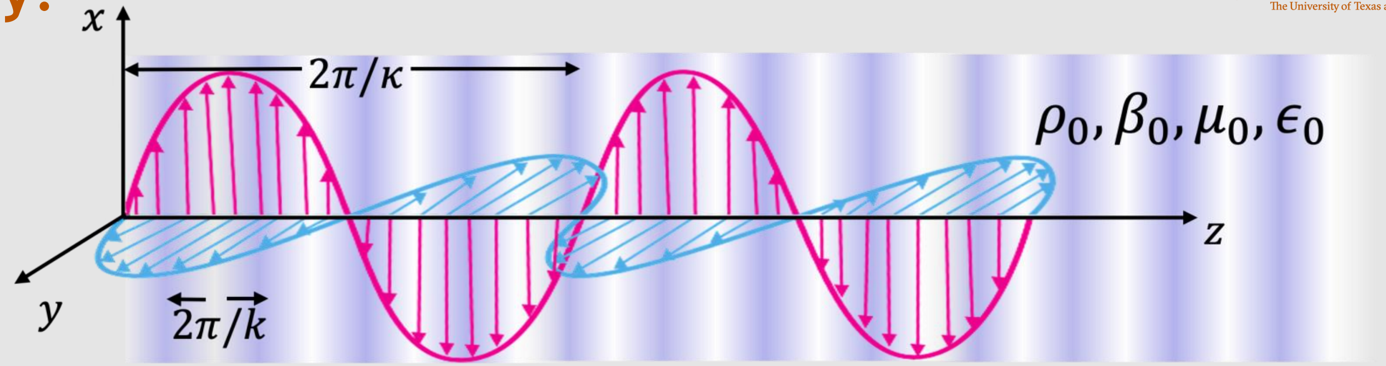

$$p_{\text{ext}} = V[-\beta_0^{-1}(\tilde{\alpha}_{pp} - S_{\text{ac}})\bar{m}_{\text{eff}} + c_0(-i\tilde{\alpha}_{pu} - S_{\text{ac}})\bar{d}_{\text{eff}} - i(\beta_0\epsilon_0)^{-1/2}\tilde{\alpha}_{pE}\bar{P}_{\text{eff}}]$$

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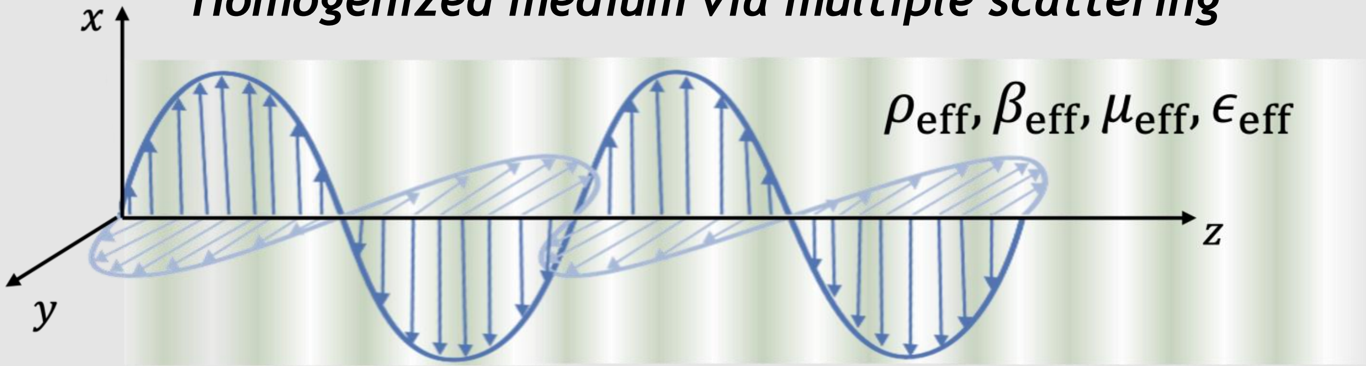
$$E_{\text{ext}} = V[-i(\beta_0\epsilon_0)^{-1/2}\tilde{\alpha}_{pE}\bar{m}_{\text{eff}} - i(\rho_0\epsilon_0)^{-1/2}\tilde{\alpha}_{uE}\bar{d}_{\text{eff}} + \epsilon_0^{-1}(\tilde{\alpha}_{EE} - S_{\text{em}})\bar{P}_{\text{eff}} - C_0 S_{\text{em}}\bar{M}_{\text{eff}}]$$

$$H_{\text{ext}} = V[-C_0 S_{\text{em}}\bar{P}_{\text{eff}} - \mu_0^{-1}(S_{\text{em}} - \tilde{\alpha}_{HH})\bar{M}_{\text{eff}}]$$

$$\left.\begin{aligned} \varepsilon_{\text{eff}} &= -\beta_0 p_{\text{eff}} + \bar{m}_{\text{eff}} \\ \mu_{v,\text{eff}} &= \rho_0 u_{\text{eff}} + \bar{d}_{\text{eff}} \\ D_{\text{eff}} &= \epsilon_0 E_{\text{eff}} + \bar{P}_{\text{eff}} \\ B_{\text{eff}} &= \mu_0 H_{\text{eff}} + \bar{M}_{\text{eff}} \end{aligned}\right\} \text{by definition}$$

Homogenized medium via multiple scattering



What is the origin of double bianisotropy?

$$
\begin{aligned}
ikp_{\text{ext}} &= i\omega\rho_0 u_{\text{ext}} + f_{\text{ext}} \\
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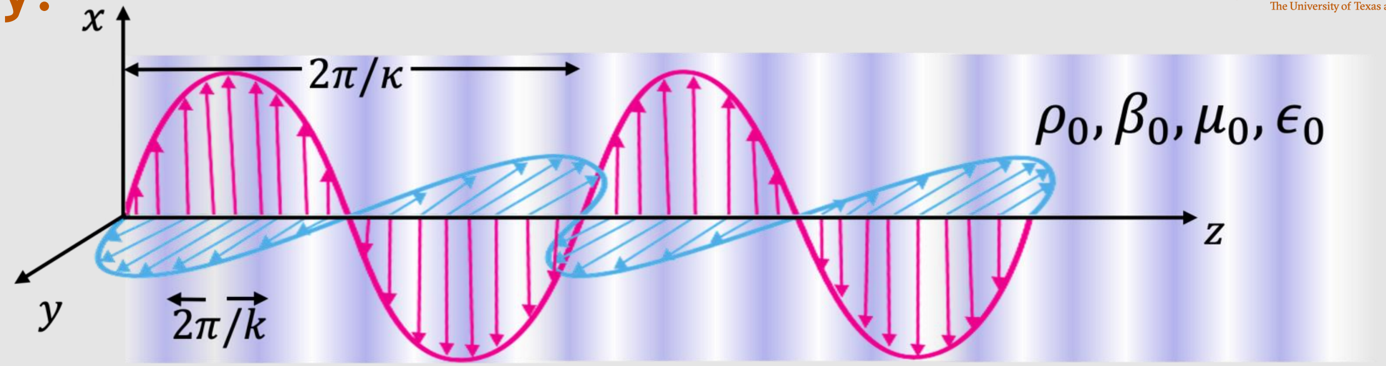
i\kappa H_{\text{ext}} &= i\omega\epsilon_0 E_{\text{ext}} - J_{\text{ext}}
\end{aligned}
$$



$$p_{\text{eff}} = p_{\text{ext}} + \frac{1}{1 - k_0^2/k^2}\left(-\frac{k_0^2}{k^2}\beta_0^{-1}\bar{m}_{\text{eff}} + \frac{k_0}{k}c_0\bar{d}_{\text{eff}}\right)$$

$$u_{\text{eff}} = u_{\text{ext}} + \frac{1}{1 - k_0^2/k^2}\left(-\frac{k_0}{k}c_0\bar{m}_{\text{eff}} + \frac{k_0^2}{k^2}\rho_0^{-1}\bar{d}_{\text{eff}}\right)$$

$$E_{\text{eff}} = E_{\text{ext}} + \frac{1}{1 - \kappa_0^2/\kappa^2}\left(\frac{\kappa_0^2}{\kappa^2}\epsilon_0^{-1}\bar{P}_{\text{eff}} + \frac{\kappa_0}{\kappa}C_0\bar{M}_{\text{eff}}\right)$$

$$H_{\text{eff}} = H_{\text{ext}} + \frac{1}{1 - \kappa_0^2/\kappa^2}\left(\frac{\kappa_0}{\kappa}C_0\bar{P}_{\text{eff}} + \frac{\kappa_0^2}{\kappa^2}\mu_0^{-1}\bar{M}_{\text{eff}}\right)$$

$$
\begin{aligned}
p_{\text{ext}} &= V[-\beta_0^{-1}(\tilde{\alpha}_{pp} - S_{\text{ac}})\bar{m}_{\text{eff}} + c_0(-i\tilde{\alpha}_{pu} - S_{\text{ac}})\bar{d}_{\text{eff}} - i(\beta_0\epsilon_0)^{-1/2}\tilde{\alpha}_{pE}\bar{P}_{\text{eff}}] \\
u_{\text{ext}} &= V[-c_0(i\tilde{\alpha}_{pu} - S_{\text{ac}})\bar{m}_{\text{eff}} + \rho_0^{-1}(\tilde{\alpha}_{uu} - S_{\text{ac}})\bar{d}_{\text{eff}} - i(\rho_0\epsilon_0)^{-1/2}\tilde{\alpha}_{uE}\bar{P}_{\text{eff}}] \\
E_{\text{ext}} &= V[-i(\beta_0\epsilon_0)^{-1/2}\tilde{\alpha}_{pE}\bar{m}_{\text{eff}} - i(\rho_0\epsilon_0)^{-1/2}\tilde{\alpha}_{uE}\bar{d}_{\text{eff}} + \epsilon_0^{-1}(\tilde{\alpha}_{EE} - S_{\text{em}})\bar{P}_{\text{eff}} - C_0 S_{\text{em}}\bar{M}_{\text{eff}}] \\
H_{\text{ext}} &= V[-C_0 S_{\text{em}}\bar{P}_{\text{eff}} - \mu_0^{-1}(S_{\text{em}} - \tilde{\alpha}_{HH})\bar{M}_{\text{eff}}]
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon_{\text{eff}} &= -\beta_0 p_{\text{eff}} + \bar{m}_{\text{eff}} \\
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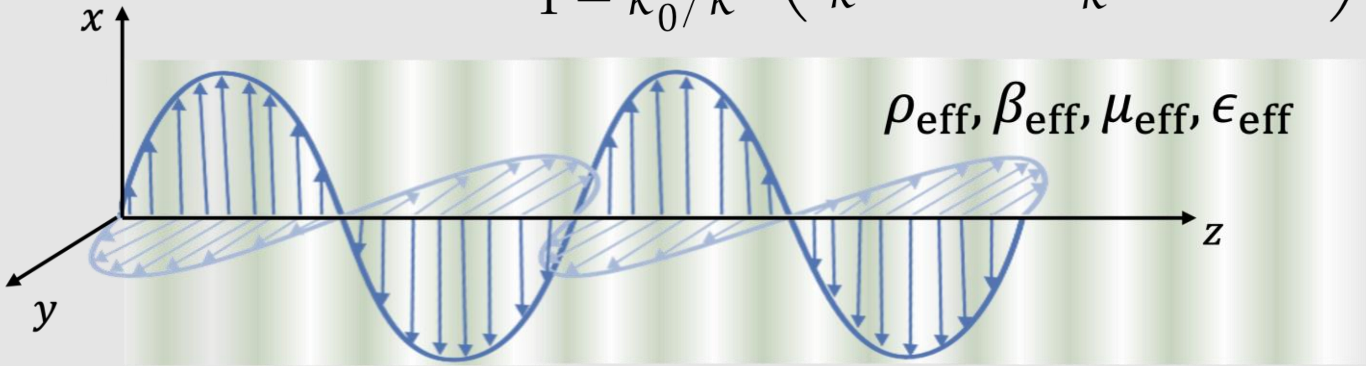
D_{\text{eff}} &= \epsilon_0 E_{\text{eff}} + \bar{P}_{\text{eff}} \\
B_{\text{eff}} &= \mu_0 H_{\text{eff}} + \bar{M}_{\text{eff}}
\end{aligned}
$$



What is the origin of double bianisotropy?

$$p_{\text{eff}} = p_{\text{ext}} + \frac{1}{1-k_0^2/k^2}\left(-\frac{k_0^2}{k^2}\beta_0^{-1}\bar{m}_{\text{eff}} + \frac{k_0}{k}c_0\bar{d}_{\text{eff}}\right)$$

$$u_{\text{eff}} = u_{\text{ext}} + \frac{1}{1-k_0^2/k^2}\left(-\frac{k_0}{k}c_0\bar{m}_{\text{eff}} + \frac{k_0^2}{k^2}\rho_0^{-1}\bar{d}_{\text{eff}}\right)$$

$$E_{\text{eff}} = E_{\text{ext}} + \frac{1}{1-\kappa_0^2/\kappa^2}\left(\frac{\kappa_0^2}{\kappa^2}\epsilon_0^{-1}\bar{P}_{\text{eff}} + \frac{\kappa_0}{\kappa}C_0\bar{M}_{\text{eff}}\right)$$

$$H_{\text{eff}} = H_{\text{ext}} + \frac{1}{1-\kappa_0^2/\kappa^2}\left(\frac{\kappa_0}{\kappa}C_0\bar{P}_{\text{eff}} + \frac{\kappa_0^2}{\kappa^2}\mu_0^{-1}\bar{M}_{\text{eff}}\right)$$

$$p_{\text{ext}} = V[-\beta_0^{-1}(\tilde{\alpha}_{pp} - S_{\text{ac}})\bar{m}_{\text{eff}} + c_0(-i\tilde{\alpha}_{pu} - S_{\text{ac}})\bar{d}_{\text{eff}} - i(\beta_0\epsilon_0)^{-1/2}\tilde{\alpha}_{pE}\bar{P}_{\text{eff}}]$$

$$u_{\text{ext}} = V[-c_0(i\tilde{\alpha}_{pu} - S_{\text{ac}})\bar{m}_{\text{eff}} + \rho_0^{-1}(\tilde{\alpha}_{uu} - S_{\text{ac}})\bar{d}_{\text{eff}} - i(\rho_0\epsilon_0)^{-1/2}\tilde{\alpha}_{uE}\bar{P}_{\text{eff}}]$$

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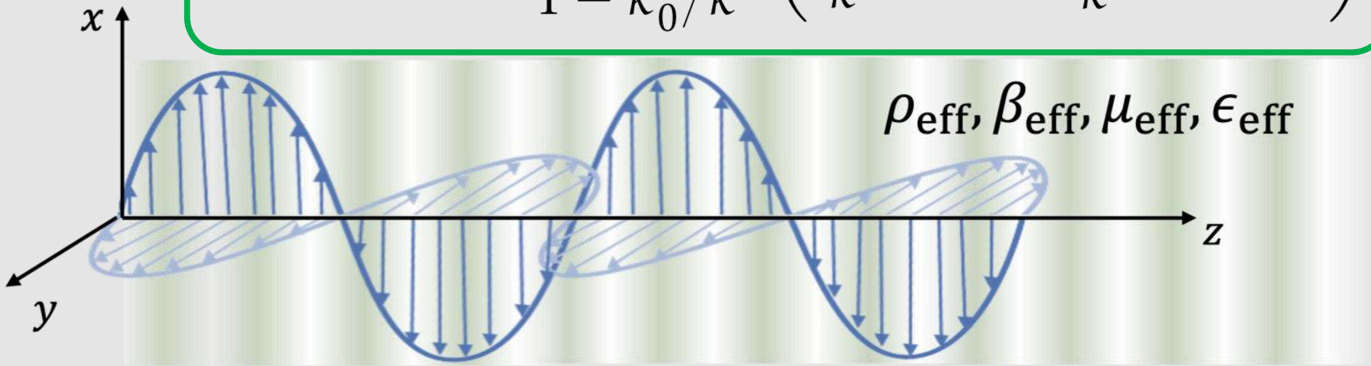


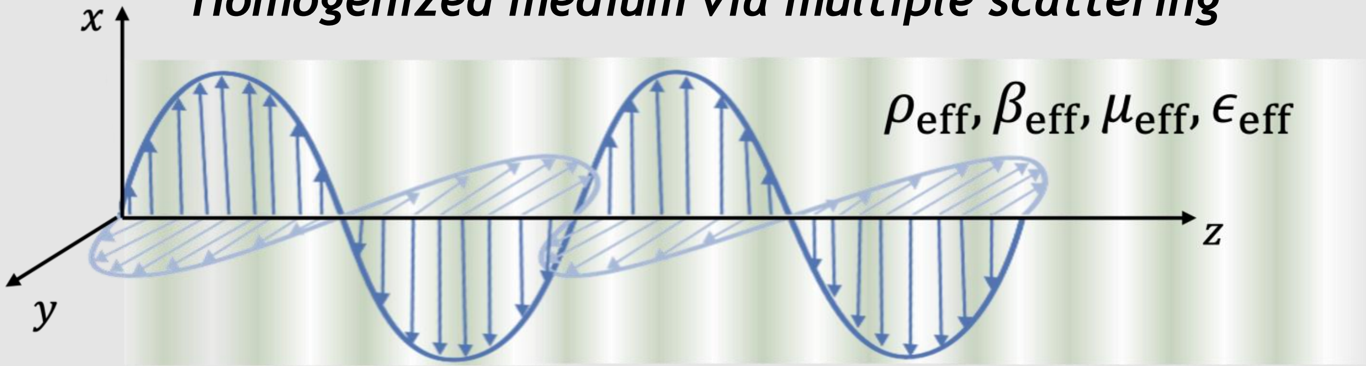
What is the origin of double bianisotropy?

$$\begin{pmatrix} \varepsilon_{\text{eff}} \\ \mu_{v,\text{eff}} \\ D_{\text{eff}} \\ B_{\text{eff}} \end{pmatrix} = \begin{pmatrix} -\beta_{\text{eff}} & \chi^{\text{o}}_{pu} - i\chi^{\text{e}}_{pu} & i\chi^{\text{o}}_{pE} - \chi^{\text{e}}_{pE} & -(\chi^{\text{o}}_{pH} - i\chi^{\text{e}}_{pH}) \\ -(\chi^{\text{o}}_{pu} + i\chi^{\text{e}}_{pu}) & \rho_{\text{eff}} & \chi^{\text{o}}_{uE} - i\chi^{\text{e}}_{uE} & -i\chi^{\text{o}}_{uH} + \chi^{\text{e}}_{uH} \\ i\chi^{\text{o}}_{pE} + \chi^{\text{e}}_{pE} & \chi^{\text{o}}_{uE} + i\chi^{\text{e}}_{uE} & \epsilon_{\text{eff}} & \chi^{\text{o}}_{EH} \\ \chi^{\text{o}}_{pH} + i\chi^{\text{e}}_{pH} & i\chi^{\text{o}}_{uH} + \chi^{\text{e}}_{uH} & \chi^{\text{o}}_{EH} & \mu_{\text{eff}} \end{pmatrix} \begin{pmatrix} p_{\text{eff}} \\ u_{\text{eff}} \\ E_{\text{eff}} \\ H_{\text{eff}} \end{pmatrix}$$

periodic | passive | identical

1D | reciprocal | $ka, \kappa a \ll 1$

Homogenized medium via multiple scattering



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Insights

- recovers acoustic & EM results[1,2]

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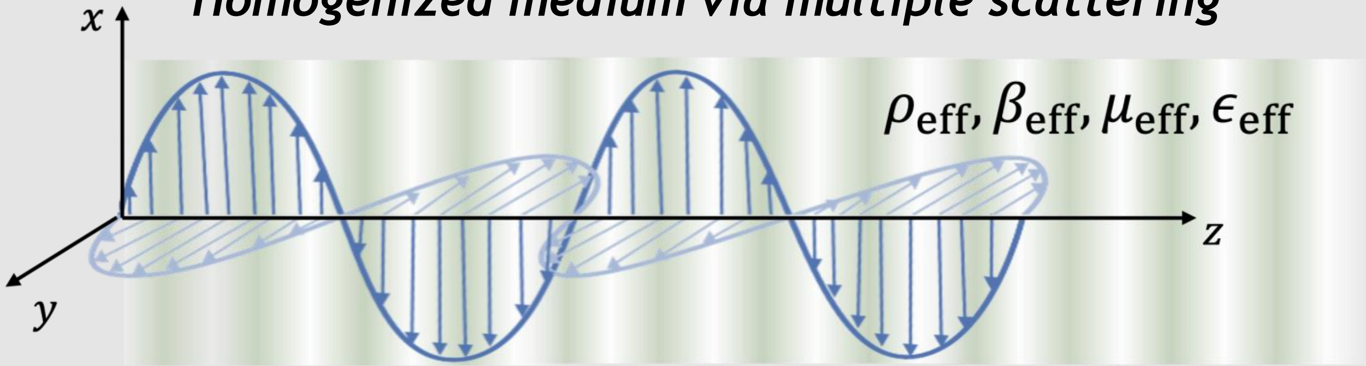

1 Sieck et al., Phys. Rev. B **96**, 104303 (2017)
2 A. Alù, Phys. Rev. B 84, 075153 (2011)

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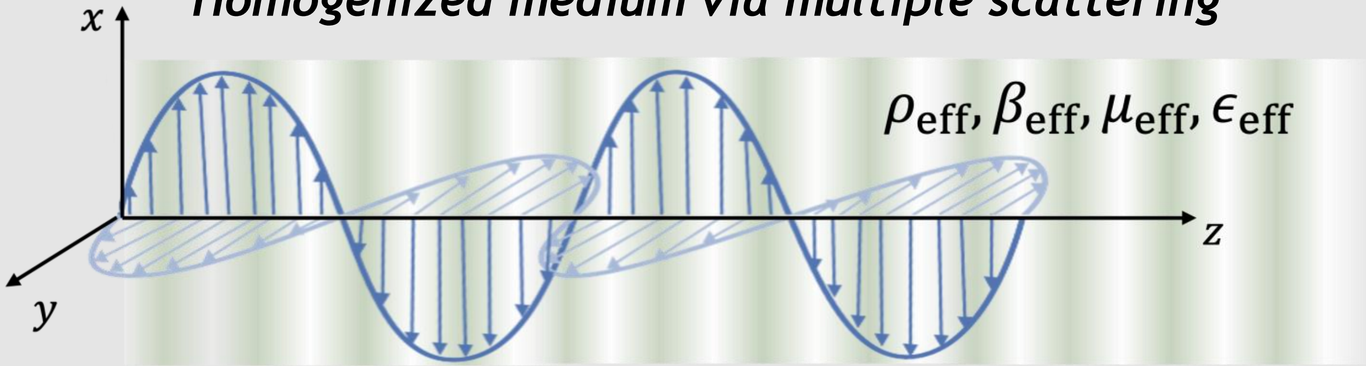
Insights

- recovers acoustic & EM results[1,2]
- satisfies reciprocity and passivity

Homogenized medium via multiple scattering



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Insights

- recovers acoustic & EM results[1,2]
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- describes origins of e-μ & m-μ

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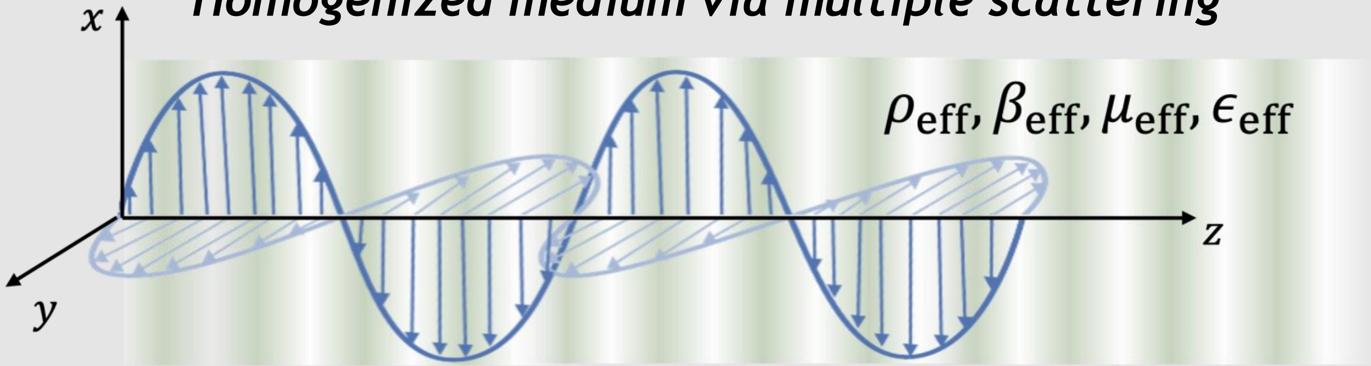
Homogenized medium via multiple scattering



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Insights

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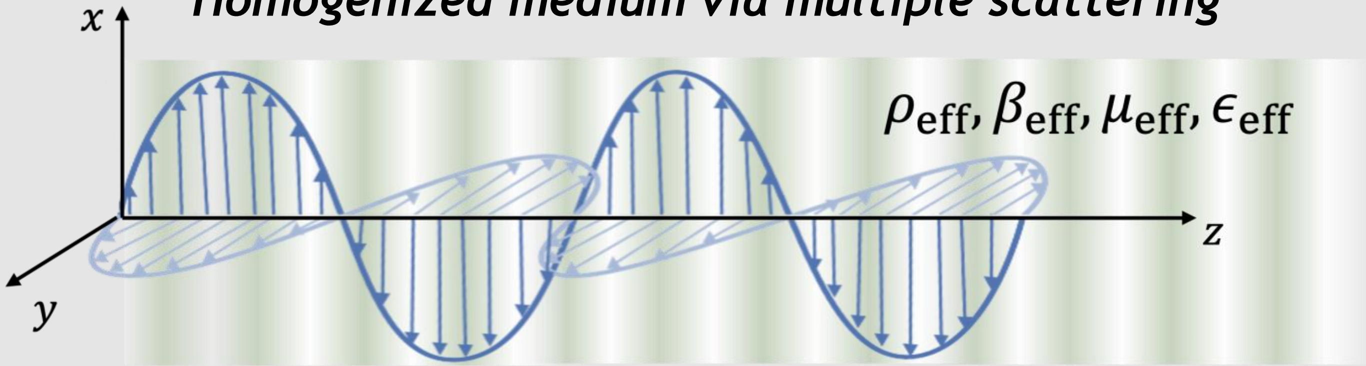
- satisfies reciprocity and passivity
- describes origins of e-μ & m-μ
- reveals emergent couplings

"Even when inclusions are perfectly centersymmetric and with no inherent bianisotropy, a form of magnetoelectric coupling is still expected... This additional coupling term is due to lattice effects and the nonzero value of [κ]."

—A. Alù.
[Phys. Rev. B **84**, 075153, 2011]

Homogenized medium via multiple scattering



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Insights

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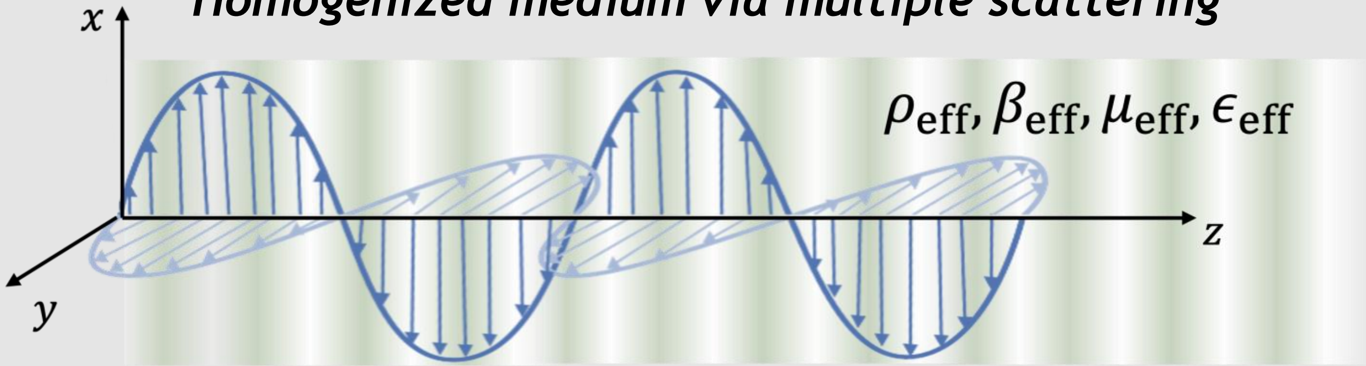
- describes origins of e-μ & m-μ
- reveals emergent couplings

"Even when inclusions are perfectly centersymmetric and with no inherent bianisotropy, a form of magnetoelectric coupling is still expected... This additional coupling term is due to lattice effects and the nonzero value of [κ]."

—A. Alù.
[Phys. Rev. B **84**, 075153, 2011]

Homogenized medium via multiple scattering



1 Sieck et al., Phys. Rev. B **96**, 104303 (2017)
2 A. Alù, Phys. Rev. B 84, 075153 (2011)

Summary

What is acoustic and electromagnetic bianisotropy?

$$\varepsilon = -\beta p + \gamma \cdot \boldsymbol{u} \qquad \boldsymbol{D} = \underline{\boldsymbol{\epsilon}}\boldsymbol{E} + \underline{\boldsymbol{\zeta}}\boldsymbol{H}$$

$$\boldsymbol{\mu} = -\boldsymbol{\eta} p + \underline{\boldsymbol{\rho}}\boldsymbol{u} \qquad \boldsymbol{B} = \underline{\boldsymbol{\xi}}\boldsymbol{E} + \underline{\boldsymbol{\mu}}\boldsymbol{H}$$

What is electromomentum coupling and double bianisotropy?

$$\begin{pmatrix} \varepsilon \\ \boldsymbol{\mu} \\ \boldsymbol{D} \end{pmatrix} = \begin{pmatrix} -\beta & \boldsymbol{\gamma}^{\mathsf{T}} & \boldsymbol{d}^{\mathsf{T}} \\ -\boldsymbol{\eta} & \boldsymbol{\mu} & \underline{\boldsymbol{v}} \\ -\boldsymbol{d} & \underline{\boldsymbol{w}} & \underline{\boldsymbol{\epsilon}} \end{pmatrix} \begin{pmatrix} p \\ \boldsymbol{u} \\ \boldsymbol{E} \end{pmatrix} \qquad \begin{pmatrix} \varepsilon \\ \boldsymbol{\mu} \\ \boldsymbol{D} \\ \boldsymbol{B} \end{pmatrix} = \begin{pmatrix} -\beta & \boldsymbol{\gamma}^{\mathsf{T}} & \boldsymbol{b}^{\mathsf{T}} & \boldsymbol{d}^{\mathsf{T}} \\ -\boldsymbol{\eta} & \underline{\boldsymbol{\rho}} & \underline{\boldsymbol{v}} & \underline{\boldsymbol{m}} \\ \boldsymbol{c} & \underline{\boldsymbol{w}} & \underline{\boldsymbol{\epsilon}} & \underline{\boldsymbol{\xi}} \\ \boldsymbol{e} & \underline{\boldsymbol{n}} & \underline{\boldsymbol{\zeta}} & \underline{\boldsymbol{\mu}} \end{pmatrix} \begin{pmatrix} p \\ \boldsymbol{u} \\ \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}$$

What are the origins of double bianisotropy?

$$\begin{pmatrix} \varepsilon_{\text{eff}} \\ \mu_{v,\text{eff}} \\ D_{\text{eff}} \\ B_{\text{eff}} \end{pmatrix} = \begin{pmatrix} -\beta_{\text{eff}} & \chi^{\text{o}}_{pu} - i\chi^{\text{e}}_{pu} & i\chi^{\text{o}}_{pE} - \chi^{\text{e}}_{pE} & -(\chi^{\text{o}}_{pH} - i\chi^{\text{e}}_{pH}) \\ -(\chi^{\text{o}}_{pu} + i\chi^{\text{e}}_{pu}) & \rho_{\text{eff}} & \chi^{\text{o}}_{uE} - i\chi^{\text{e}}_{uE} & -i\chi^{\text{o}}_{uH} + \chi^{\text{e}}_{uH} \\ i\chi^{\text{o}}_{pE} + \chi^{\text{e}}_{pE} & \chi^{\text{o}}_{uE} + i\chi^{\text{e}}_{uE} & \epsilon_{\text{eff}} & \chi^{\text{o}}_{EH} \\ \chi^{\text{o}}_{pH} + i\chi^{\text{e}}_{pH} & i\chi^{\text{o}}_{uH} + \chi^{\text{e}}_{uH} & \chi^{\text{o}}_{EH} & \mu_{\text{eff}} \end{pmatrix} \begin{pmatrix} p_{\text{eff}} \\ u_{\text{eff}} \\ E_{\text{eff}} \\ H_{\text{eff}} \end{pmatrix}$$

https://forms.gle/HtQkS7YGFXxEgpKx7

Acknowledgments

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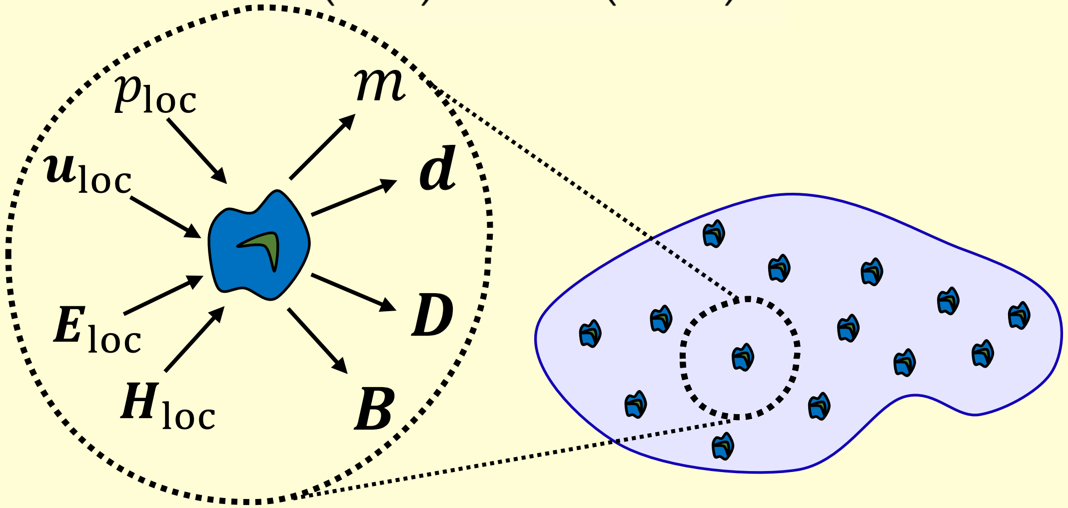
TEXAS
The University of Texas at Austin

Direct notation

Direct notation

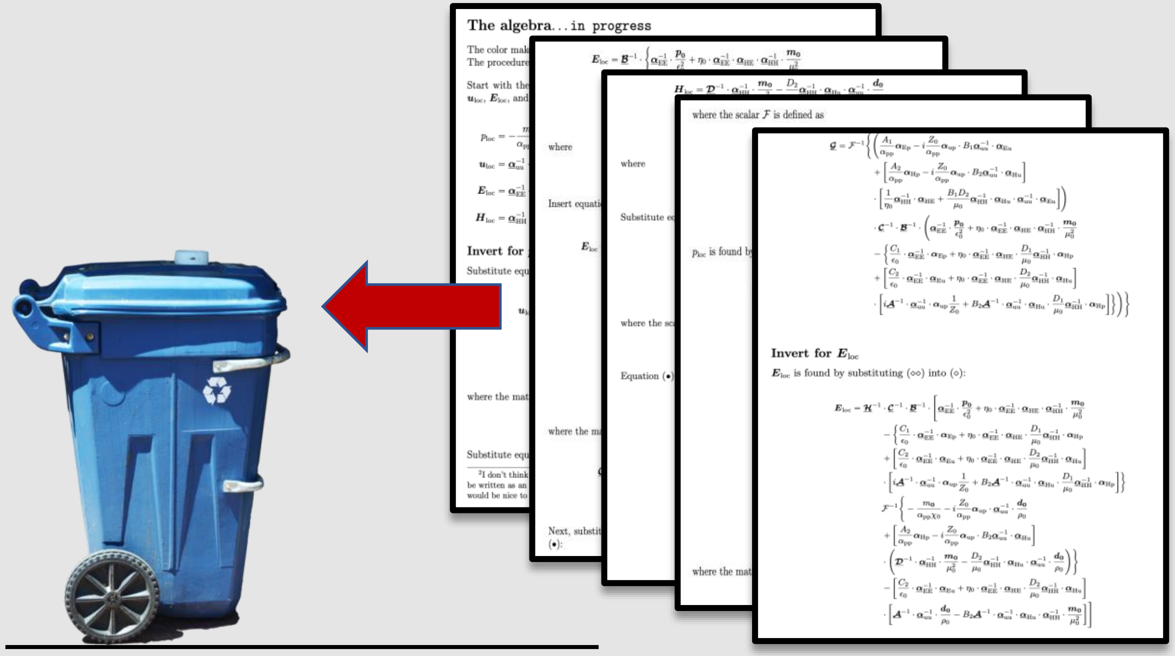
$$\begin{pmatrix} m_0 \\ \boldsymbol{d}_0 \\ \boldsymbol{P}_0 \\ \boldsymbol{M}_0 \end{pmatrix} = \underline{\boldsymbol{\alpha}} \begin{pmatrix} p_{\text{loc}} \\ \boldsymbol{u}_{\text{loc}} \\ \boldsymbol{E}_{\text{loc}} \\ \boldsymbol{H}_{\text{loc}} \end{pmatrix}$$



$$\begin{pmatrix} \varepsilon_{\text{eff}} \\ \mu_{\text{eff}} \\ \boldsymbol{D}_{\text{eff}} \\ \boldsymbol{B}_{\text{eff}} \end{pmatrix} = \begin{pmatrix} -\beta & \boldsymbol{\gamma}^{\mathsf{T}} & \boldsymbol{b}^{\mathsf{T}} & \boldsymbol{d}^{\mathsf{T}} \\ -\boldsymbol{\eta} & \underline{\boldsymbol{\rho}} & \underline{\boldsymbol{v}} & \underline{\boldsymbol{m}} \\ \boldsymbol{c} & \underline{\boldsymbol{w}} & \underline{\boldsymbol{\epsilon}} & \underline{\boldsymbol{\xi}} \\ \boldsymbol{e} & \underline{\boldsymbol{n}} & \underline{\boldsymbol{\zeta}} & \underline{\boldsymbol{\mu}} \end{pmatrix} \begin{pmatrix} p_{\text{eff}} \\ \boldsymbol{u}_{\text{eff}} \\ \boldsymbol{E}_{\text{eff}} \\ \boldsymbol{H}_{\text{eff}} \end{pmatrix}$$

Cons of direct notation

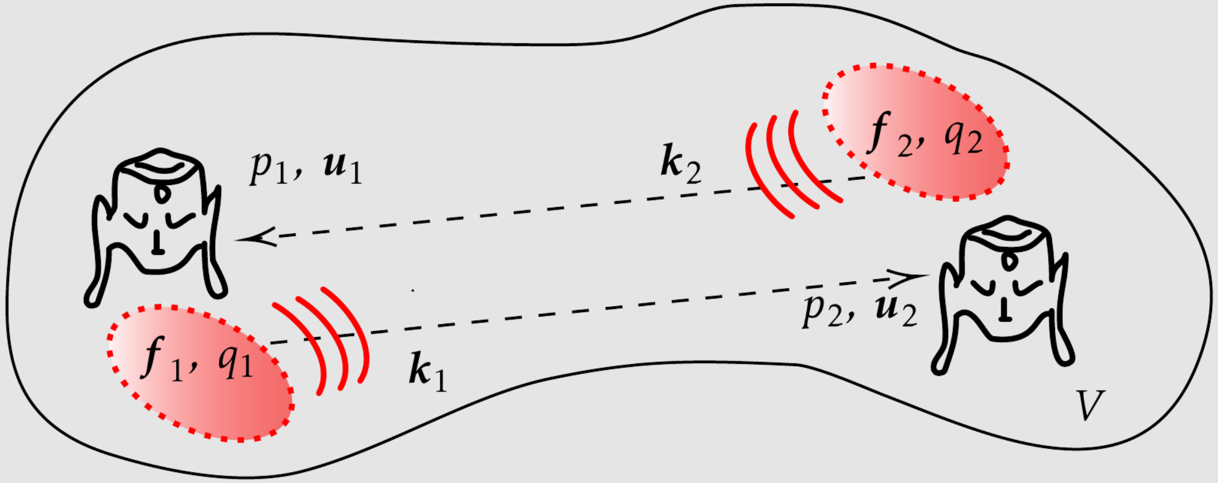
- *Very difficult* to directly invert $\underline{\boldsymbol{\alpha}}$
- Lattice sums exist and converge in 1D
- Experiment is performed in 1D[1]
- Follow analysis Alù[2] and Sieck et al.[3]

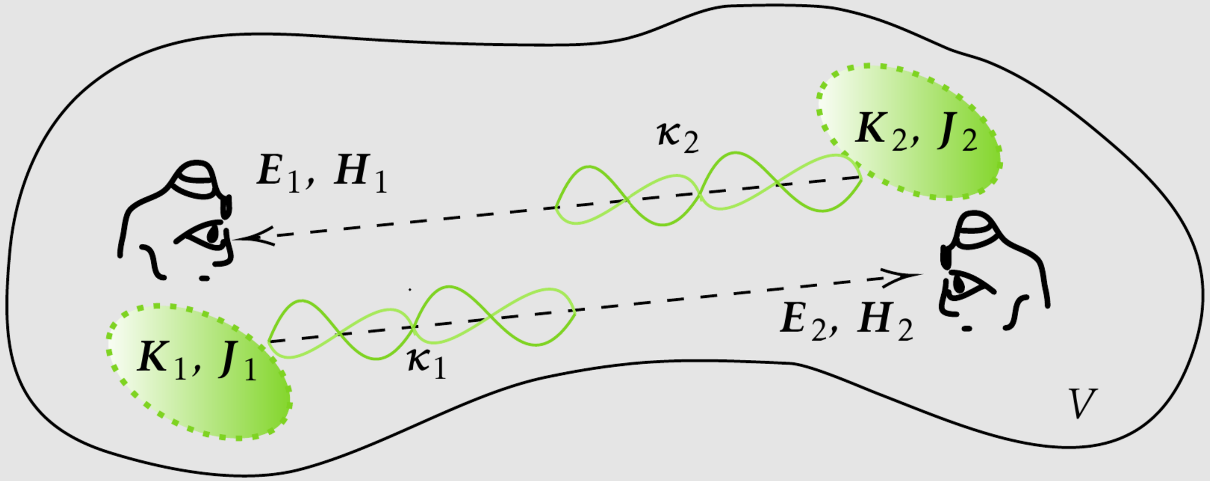


1 M. A. Casali et al. JASA **151**, A94 (2022)
2 A. Alu, PRB **84**, 075153 (2011)
3 Sieck et al., PRB **96**, 104303 (2017)

Is it *physical*?

acoustic & electrodynamic reciprocity





$$\cancel{\int_A (p_1\boldsymbol{u}_2 - p_2\boldsymbol{u}_1 + \boldsymbol{E}_1\times\boldsymbol{H}_2 - \boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}\,dA} = 0 \text{ by Sommerfeld radiation condition}$$

$$= \cancel{\int_V (\boldsymbol{u}_2\cdot\boldsymbol{f}_1 - \boldsymbol{u}_1\cdot\boldsymbol{f}_2 + p_1q_2 - p_2q_1 + \boldsymbol{E}_2\cdot\boldsymbol{J}_1 - \boldsymbol{E}_1\cdot\boldsymbol{J}_2 + \boldsymbol{H}_1\cdot\boldsymbol{K}_2 - \boldsymbol{H}_2\cdot\boldsymbol{K}_1)dV}$$

= 0 by reciprocity

$$-i\omega\int_V \underbrace{(\boldsymbol{u}_1\cdot\boldsymbol{\mu}_2 - \boldsymbol{u}_2\cdot\boldsymbol{\mu}_1 + p_1\varepsilon_2 - p_2\varepsilon_1 + \boldsymbol{H}_1\cdot\boldsymbol{B}_2 - \boldsymbol{H}_2\cdot\boldsymbol{B}_1 - \boldsymbol{E}_1\cdot\boldsymbol{D}_2 + \boldsymbol{E}_2\cdot\boldsymbol{D}_1)}_{=\,0 \text{ also}}dV$$

$$\underline{\rho} = \underline{\rho}^{\mathsf{T}}, \quad \underline{\mu} = \underline{\mu}^{\mathsf{T}}, \quad \underline{\epsilon} = \underline{\epsilon}^{\mathsf{T}}$$

$$\begin{aligned} \gamma &= \eta^{\mathrm{o}} - \eta^{\mathrm{e}} \\ \underline{\xi} &= (\underline{\zeta}^{\mathrm{o}})^{\mathsf{T}} - (\underline{\zeta}^{\mathrm{e}})^{\mathsf{T}} \\ \underline{\boldsymbol{v}} &= (\underline{\boldsymbol{w}}^{\mathrm{o}})^{\mathsf{T}} - (\underline{\boldsymbol{w}}^{\mathrm{e}})^{\mathsf{T}} \\ \underline{\boldsymbol{m}} &= (\underline{\boldsymbol{n}}^{\mathrm{e}})^{\mathsf{T}} - (\underline{\boldsymbol{n}}^{\mathrm{o}})^{\mathsf{T}} \\ \boldsymbol{b} &= \boldsymbol{c}^{\mathrm{o}} - \boldsymbol{c}^{\mathrm{e}} \\ \boldsymbol{d} &= \boldsymbol{e}^{\mathrm{e}} - \boldsymbol{e}^{\mathrm{o}} \end{aligned}$$

Sieck et al., PRB **96**, 104303 (2017)
Kong, Proc. IEEE **60**, 9 (1972)

Is it *physical*?

lossless and passive acoustic & electromagnetic medium

$$\Re\mathfrak{e}\left(\nabla\cdot\boldsymbol{I}+\nabla\cdot\boldsymbol{S}\right)=0$$

$$\boldsymbol{I}=\frac{1}{2}p\boldsymbol{u}^{*} \qquad \boldsymbol{S}=\frac{1}{2}\boldsymbol{E}\times\boldsymbol{H}^{*}$$

$$\boldsymbol{\mu}\cdot\boldsymbol{u}^{*}-\boldsymbol{\mu}^{*}\cdot\boldsymbol{u}+\varepsilon^{*}p-\varepsilon p^{*}+\boldsymbol{B}\cdot\boldsymbol{H}^{*}-\boldsymbol{B}^{*}\cdot\boldsymbol{H}-\boldsymbol{D}^{*}\cdot\boldsymbol{E}+\boldsymbol{D}\cdot\boldsymbol{E}^{*}=0$$

$$\begin{pmatrix}\varepsilon\\ \boldsymbol{\mu}\\ \boldsymbol{D}\\ \boldsymbol{B}\end{pmatrix}=\begin{pmatrix}-\beta & \boldsymbol{\gamma}\cdot & \boldsymbol{b}\cdot & \boldsymbol{d}\cdot\\ -\boldsymbol{\eta} & \underline{\boldsymbol{\rho}} & \underline{\boldsymbol{v}} & \underline{\boldsymbol{m}}\\ \boldsymbol{c} & \underline{\boldsymbol{w}} & \underline{\boldsymbol{\epsilon}} & \underline{\boldsymbol{\xi}}\\ \boldsymbol{e} & \underline{\boldsymbol{n}} & \underline{\boldsymbol{\zeta}} & \underline{\boldsymbol{\mu}}\end{pmatrix}\begin{pmatrix}p\\ \boldsymbol{u}\\ \boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}$$

$$\begin{aligned}\Im\mathfrak{m}\,\beta&=0\\ \underline{\boldsymbol{\rho}}^{\dagger}&=\underline{\boldsymbol{\rho}}\\ \boldsymbol{\eta}&=\boldsymbol{\gamma}^{*}\\ \underline{\boldsymbol{\mu}}&=\underline{\boldsymbol{\mu}}^{\dagger}\\ \underline{\boldsymbol{\epsilon}}&=\underline{\boldsymbol{\epsilon}}^{\dagger}\\ \underline{\boldsymbol{\zeta}}&=\underline{\boldsymbol{\xi}}^{\dagger}\end{aligned}
\qquad
\begin{aligned}\underline{\boldsymbol{w}}^{\dagger}&=\underline{\boldsymbol{v}}\\ \underline{\boldsymbol{n}}^{\dagger}&=\underline{\boldsymbol{m}}\\ \boldsymbol{c}&=-\boldsymbol{b}^{*}\\ \boldsymbol{d}&=-\boldsymbol{e}^{*}\end{aligned}$$

Sieck et al., PRB **96**, 104303 (2017)
Kong, Proc. IEEE **60**, 9 (1972)

Is it *physical*?

Reciprocity

$$\underline{\rho} = \underline{\rho}^{\mathsf{T}}$$
$$\underline{\mu} = \underline{\mu}^{\mathsf{T}}$$
$$\underline{\epsilon} = \underline{\epsilon}^{\mathsf{T}}$$
$$\gamma = \eta^{\mathrm{o}} - \eta^{\mathrm{e}}$$
$$\underline{\xi} = (\underline{\zeta}^{\mathrm{o}})^{\mathsf{T}} - (\underline{\zeta}^{\mathrm{e}})^{\mathsf{T}}$$
$$\underline{v} = (\underline{w}^{\mathrm{o}})^{\mathsf{T}} - (\underline{w}^{\mathrm{e}})^{\mathsf{T}}$$
$$\underline{m} = (\underline{n}^{\mathrm{e}})^{\mathsf{T}} - (\underline{n}^{\mathrm{o}})^{\mathsf{T}}$$
$$\boldsymbol{b} = \boldsymbol{c}^{\mathrm{o}} - \boldsymbol{c}^{\mathrm{e}}$$
$$\boldsymbol{d} = \boldsymbol{e}^{\mathrm{e}} - \boldsymbol{e}^{\mathrm{o}}$$

+

Passivity

$$\begin{aligned}-\mathfrak{Re}(\nabla\cdot\boldsymbol{I}+\nabla\cdot\boldsymbol{S}) = -\frac{i\omega}{4}\big[&-\boldsymbol{u}^*\cdot(\underline{\rho}^\dagger-\underline{\rho})\boldsymbol{u}+2i\,\mathfrak{Im}(\beta)|p|^2-(\eta-\gamma^*)\cdot p\boldsymbol{u}^*+(\eta^*-\gamma)\cdot p^*\boldsymbol{u}\\ &+\boldsymbol{E}^*\cdot(\underline{\epsilon}-\underline{\epsilon}^\dagger)\boldsymbol{E}+\boldsymbol{H}^*\cdot(\underline{\mu}-\underline{\mu}^\dagger)\boldsymbol{H}-\boldsymbol{E}^*\cdot(\underline{\zeta}^\dagger-\underline{\xi})\boldsymbol{H}+\boldsymbol{H}^*\cdot(\underline{\zeta}-\underline{\xi}^\dagger)\boldsymbol{E}\\ &+\boldsymbol{u}^*\cdot(\underline{v}-\underline{w}^\dagger)\boldsymbol{E}+\boldsymbol{u}^*\cdot(\underline{m}-\underline{n}^\dagger)\boldsymbol{H}+\boldsymbol{E}^*\cdot(\underline{w}-\underline{v}^\dagger)\boldsymbol{u}+\boldsymbol{H}^*\cdot(\underline{n}-\underline{m}^\dagger)\boldsymbol{u}\\ &+p(\boldsymbol{b}^*+\boldsymbol{c})\cdot\boldsymbol{E}^*+p(\boldsymbol{d}^*+\boldsymbol{e})\cdot\boldsymbol{H}^*-p^*(\boldsymbol{b}+\boldsymbol{c}^*)\cdot\boldsymbol{E}-p^*(\boldsymbol{d}+\boldsymbol{e}^*)\cdot\boldsymbol{H}\big]\geqslant 0\end{aligned}$$

=

All $\mathfrak{Im}$ parts vanish for lossless case

$$\begin{aligned}-\mathfrak{Re}(\nabla\cdot\boldsymbol{I}+\nabla\cdot\boldsymbol{S}) = \omega\Big[&\frac{1}{2}\boldsymbol{u}^*\cdot\mathfrak{Im}(\underline{\rho})\boldsymbol{u}+\frac{1}{2}\mathfrak{Im}(\beta)|p|^2-\mathfrak{Im}(\boldsymbol{\chi}^{\mathrm{o}}_{pu})\cdot\mathfrak{Re}(p\boldsymbol{u}^*)+\mathfrak{Im}(\boldsymbol{\chi}^{\mathrm{e}}_{pu})\cdot\mathfrak{Im}(p\boldsymbol{u}^*)\\ &+\frac{1}{2}\boldsymbol{E}^*\cdot\mathfrak{Im}(\underline{\epsilon})\boldsymbol{H}+\frac{1}{2}\boldsymbol{H}^*\cdot\mathfrak{Im}(\underline{\mu})\boldsymbol{H}-\mathfrak{Im}(\underline{\chi}^{\mathrm{o}}_{EH}):\mathfrak{Re}(\boldsymbol{H}^*\otimes\boldsymbol{E})-\mathfrak{Im}(\underline{\chi}^{\mathrm{e}}_{EH}):\mathfrak{Im}(\boldsymbol{H}\otimes\boldsymbol{E}^*)\\ &+\mathfrak{Im}(\underline{\chi}^{\mathrm{o}}_{uE}):\mathfrak{Re}(\boldsymbol{E}\otimes\boldsymbol{u}^*)+\mathfrak{Im}(\underline{\chi}^{\mathrm{e}}_{uE}):\mathfrak{Im}(\boldsymbol{E}\otimes\boldsymbol{u}^*)-\mathfrak{Im}(\underline{\chi}^{\mathrm{o}}_{uH}):\mathfrak{Im}(\boldsymbol{H}^*\otimes\boldsymbol{u})+\mathfrak{Im}(\underline{\chi}^{\mathrm{e}}_{uH}):\mathfrak{Re}(\boldsymbol{H}\otimes\boldsymbol{u})\\ &-\mathfrak{Im}(\boldsymbol{\chi}^{\mathrm{o}}_{pE})\cdot\mathfrak{Im}(p\boldsymbol{E}^*)+\mathfrak{Im}(\boldsymbol{\chi}^{\mathrm{e}}_{pE})\cdot\mathfrak{Re}(p\boldsymbol{E}^*)+\mathfrak{Im}(\boldsymbol{\chi}^{\mathrm{o}}_{pH})\cdot\mathfrak{Re}(p\boldsymbol{H}^*)-\mathfrak{Im}(\boldsymbol{\chi}^{\mathrm{e}}_{pH})\cdot\mathfrak{Im}(p\boldsymbol{H}^*)\Big]\geqslant 0\end{aligned}$$

Sieck et al., PRB **96**, 104303 (2017)

EM bianisotropy: invariant under Lorentz transformation

$$\begin{pmatrix} cD' \\ H' \end{pmatrix} = \begin{pmatrix} \underline{I}c\epsilon & 0 \\ 0 & \underline{I}/c\mu \end{pmatrix} \begin{pmatrix} E' \\ cB' \end{pmatrix}$$ ...Isotropic

$$\underline{\mathcal{L}} \begin{pmatrix} cD \\ H \end{pmatrix} = \underline{\mathcal{L}} \begin{pmatrix} \underline{I}c\epsilon & 0 \\ 0 & \underline{I}/c\mu \end{pmatrix} \begin{pmatrix} E \\ cB \end{pmatrix}$$ ...Lorentz transformation

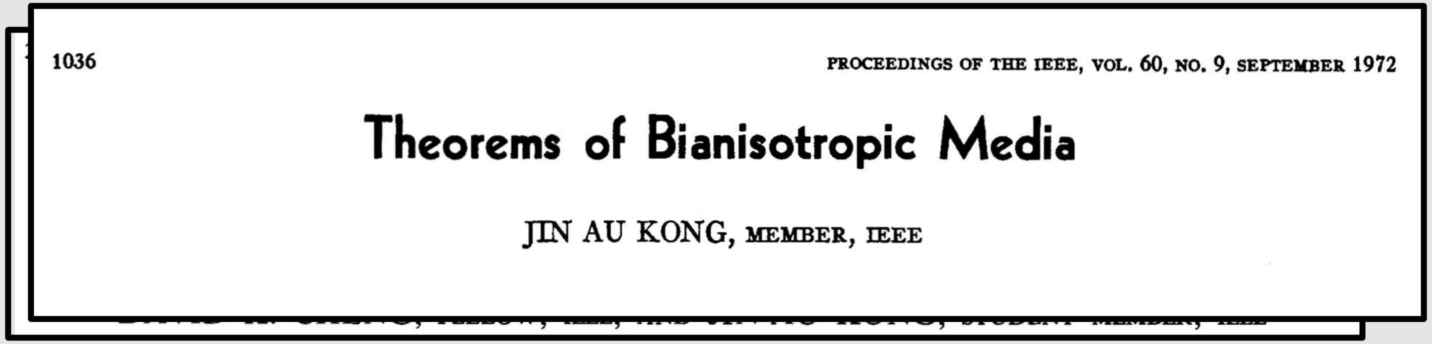
$$\begin{pmatrix} cD \\ H \end{pmatrix} = \begin{pmatrix} \underline{P} & \underline{L} \\ \underline{M} & \underline{Q} \end{pmatrix} \begin{pmatrix} E \\ cB \end{pmatrix}$$ ....Bianisotropic

$$\begin{pmatrix} D \\ B \end{pmatrix} = \begin{pmatrix} \underline{\epsilon} & \underline{\xi} \\ \underline{\zeta} & \underline{\mu} \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix}$$ ....*E-H* form

J. Kong

[A] medium in which the field vectors ***D*** and ***H*** depend on both ***E*** and ***B*** but are parallel to neither be described as **bianisotropic**. A moving medium, even if it is isotropic in its rest frame, then appears bianisotropic to the laboratory observer.

—D. K. Cheng and J. Kong
[Proc. IEEE **56**, 248-251, 1968]



1036 PROCEEDINGS OF THE IEEE, VOL. 60, NO. 9, SEPTEMBER 1972

Theorems of Bianisotropic Media

JIN AU KONG, MEMBER, IEEE

Willis coupling: invariant under coordinate transformations

WAVE MOTION 3 (1981) 1–11
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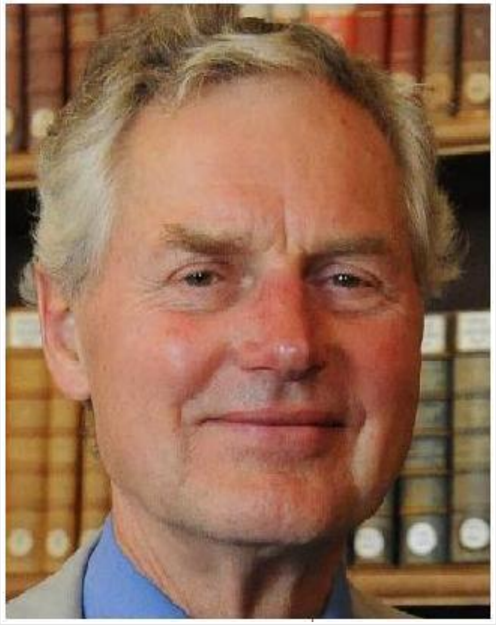
VARIATIONAL PRINCIPLES FOR DYNAMIC PROBLEMS FOR INHOMOGENEOUS ELASTIC MEDIA

J.R. WILLIS

School of Mathematics, University of Bath, Bath BA2 7AY, England



J. R. Willis

New Journal of Physics
The open-access journal for physics

On cloaking for elasticity and physical equations with a transformation invariant form

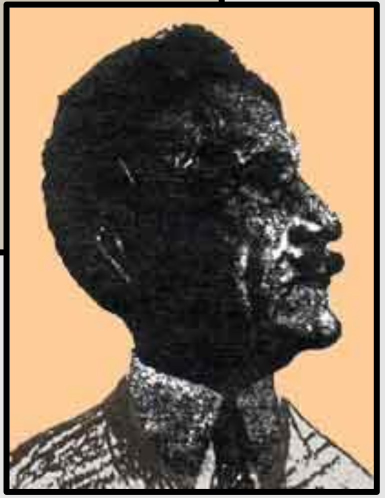
Graeme W Milton[1], Marc Briane[2] and John R Willis[3]

It seems rather amazing that the elastodynamic equations under a curvilinear coordinate transformation map to equations that are a special case of those of Willis.

—Milton et al
[New J. Phys. **8**, 248 (2006)]

The other J. R. Willis

Joseph Roy (J. R.) Willis (1876-1960) was a legend in *Albuquerque*. He was a slim gent with a small mustache. He wore capes. He twirled canes. He smoked Pall Malls (cigarettes) in an ivory holder. He loved, more than anything, to discuss his paintings. Albuquerque old- timers, who were fascinated by his foppish bearing, remember him. He had flair, a flakiness that today would be considered weird. J. R. Willis was just different. He was an artist. He came from the Southeast to chronicle the Great Southwest. Willis learned art on slates. He made fashion drawings for the Atlanta Constitution. He painted backdrops for Universal Studios in Hollywood. In 1917, Willis left for New York to discuss cartoon syndication. He stopped in Arizona to sketch *Hopis*. He decided to stay in the Great Southwest. He moved to Gallup and set himself up as a photographer, and toured the reservations to photograph *Navajos*, Hopis, *Zunis*, *Lagunas* and *Acomas*. He would then use the photos as sources for his paintings.



J. R. Willis

https://www.adobegallery.com/artist/Joseph_Willis_1876_196021236000

An exotic but potentially bianisotropic medium

Hence the complete set of MHD equations reads

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad \text{continuity} \tag{22.1}$$

302

Chapter 22 / Hydromagnetic Equations and Hydromagnetic Waves 303

momentum

$$\rho\left[\frac{\partial \mathbf{u}}{\partial t} + \nabla\left(\frac{|\mathbf{u}|^2}{2}\right) + (\nabla \times \mathbf{u}) \times \mathbf{u}\right] = -\rho\nabla\mathcal{V} - \nabla P + \nabla \cdot \overleftrightarrow{\pi} + \mathbf{f}_{\rm rad} + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}, \tag{22.2}$$

$$\rho T\left(\frac{\partial s}{\partial t} + \mathbf{u} \cdot \nabla s\right) = -\nabla \cdot \mathbf{F}_{\rm cond} + \Gamma - \Lambda + \Psi + \frac{\eta}{4\pi}|\nabla \times \mathbf{B}|^2, \tag{22.3}$$

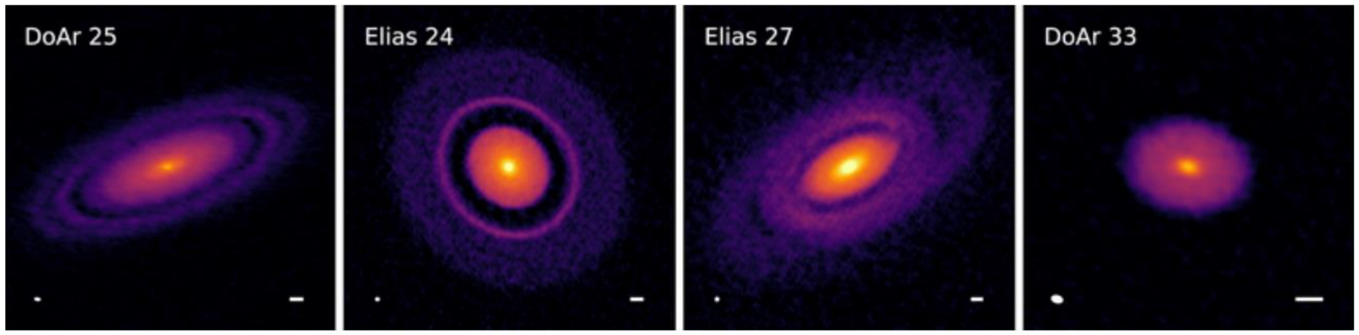
$$\nabla^2 \mathcal{V} = 4\pi G(\rho + \rho_{\rm ext}), \tag{22.4}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{u}) = -\nabla \times (\eta \nabla \times \mathbf{B}), \tag{22.5}$$

Maxwell-Ampere

with

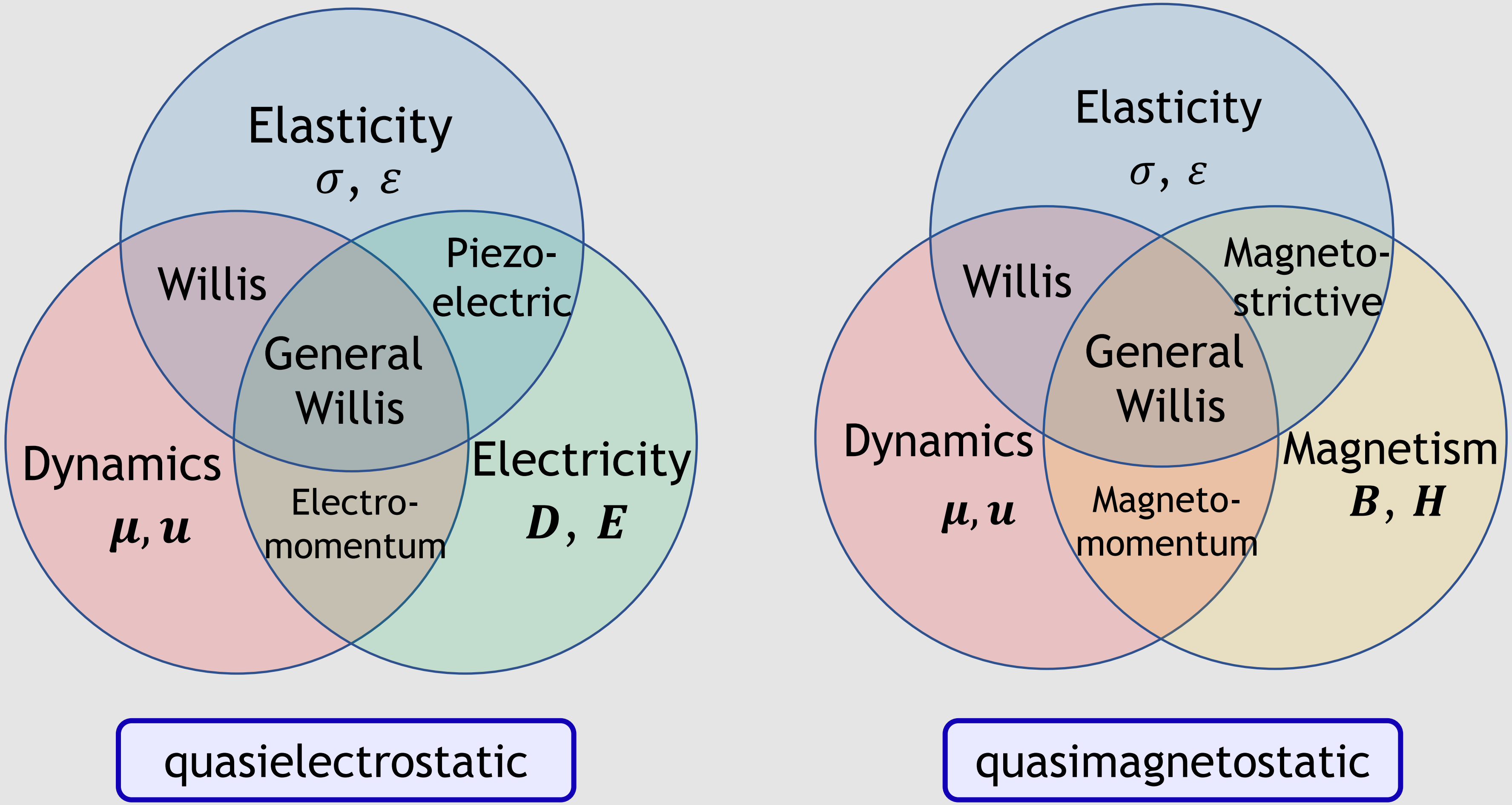
$$\nabla \cdot \mathbf{B} = 0 \tag{22.6}$$



~~nonlinearity~~, ~~viscosity~~, ~~body forces~~

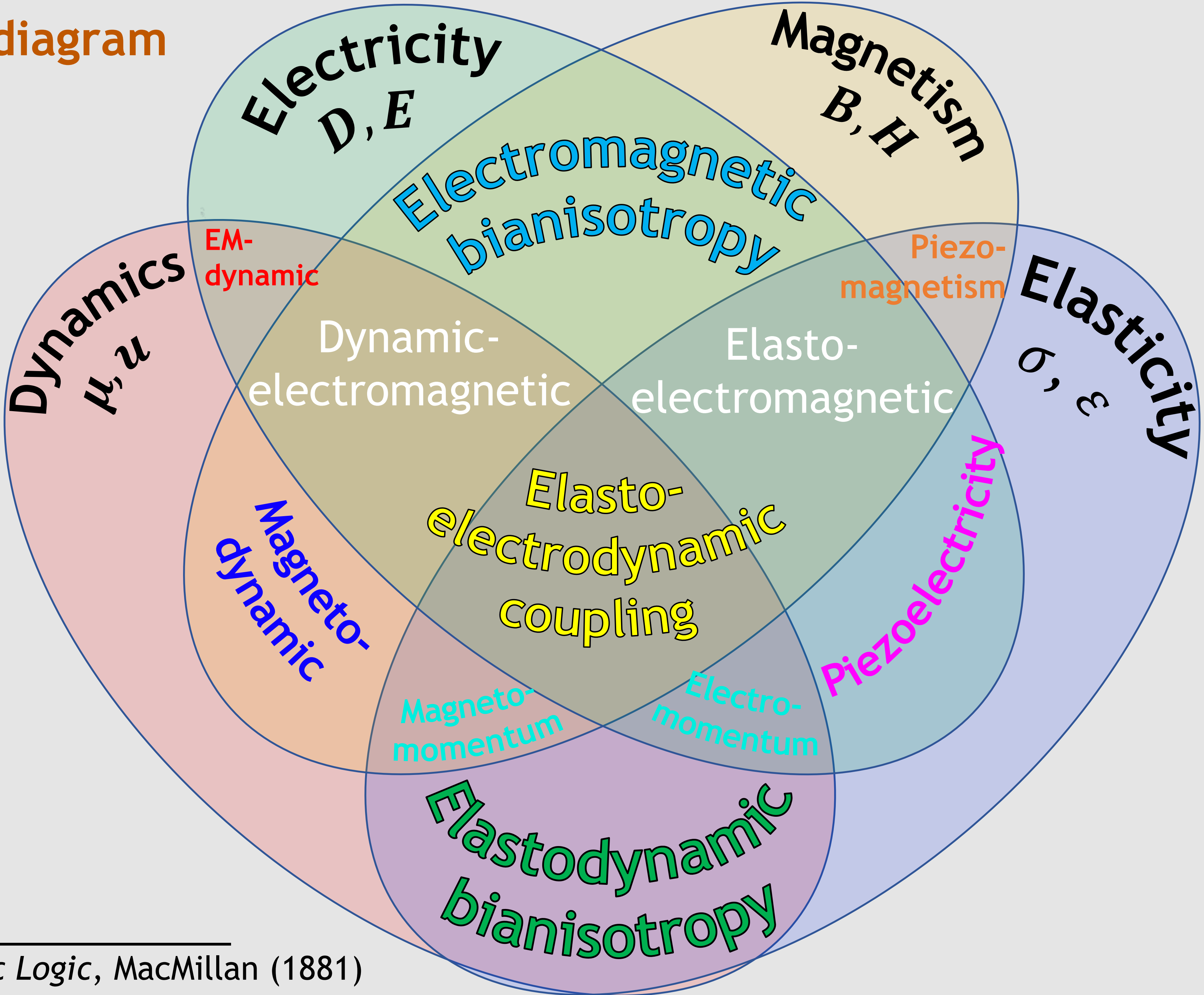
J. D. Jackson, *Classical Electrodynamics*, Wiley, 3rd ed. (1999)
S. M. Andrews, *Annu. Rev. Astron. Astrophys.*, **58**, 483 (2020)
A. S. Mohov, Master's thesis, Department of Physics, UNLV. (2023)

Generalized Willis coupling does not include electromagnetic bianisotropy



Adapted from Fig. 1 of Pernas-Salomón and Shmuel, PRA **14**, 064005, 2020

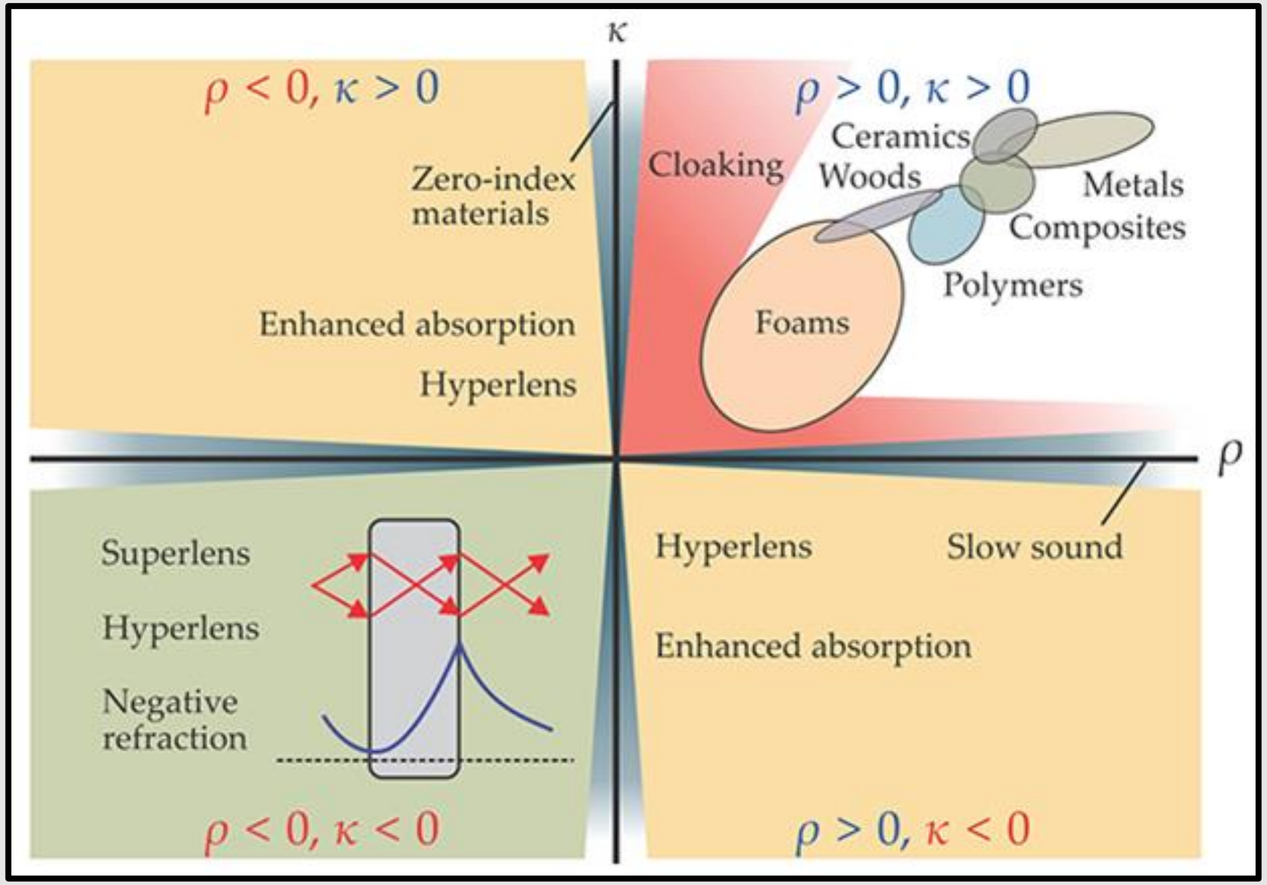
4-way Venn diagram



J. Venn, *Symbolic Logic*, MacMillan (1881)

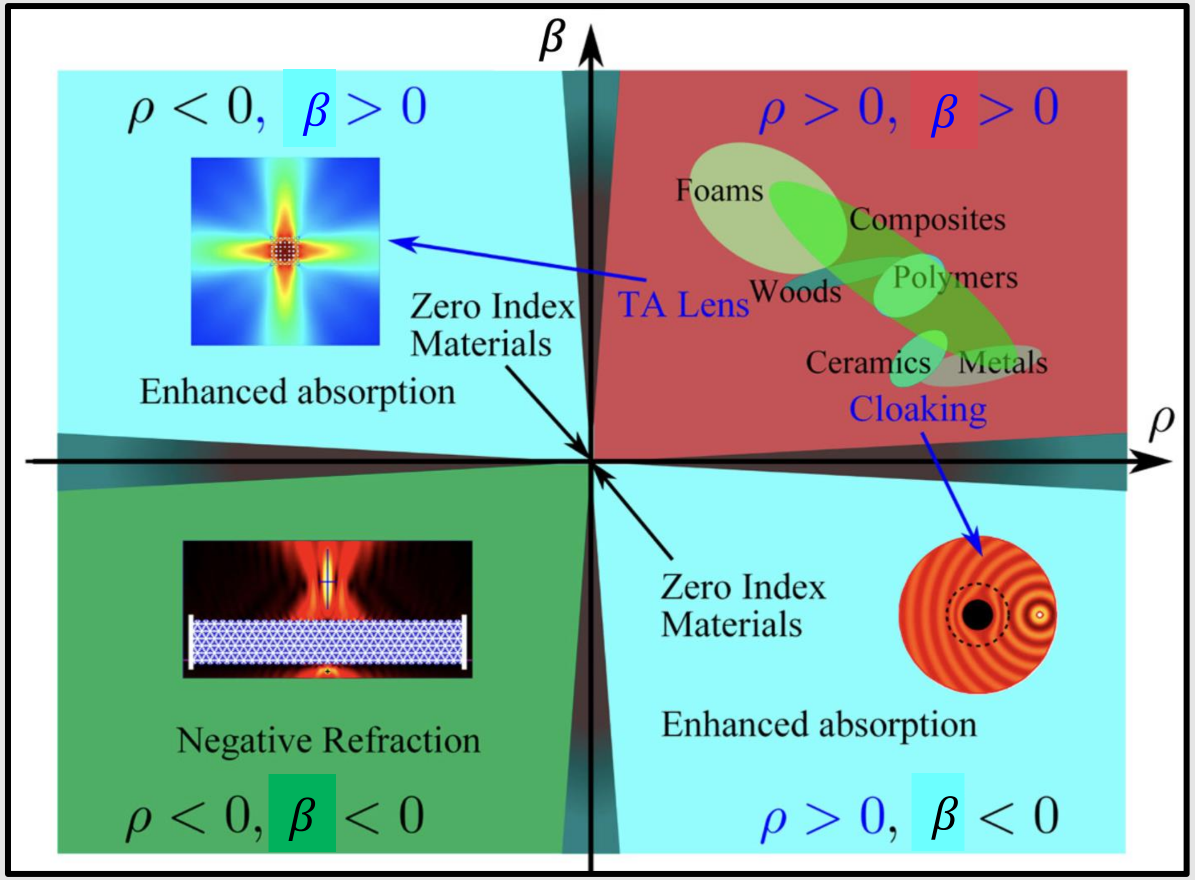
Ashby charts

Stiffness vs. density



M. R. Haberman & M. D. Guild, Physics Today **69**, 2016

Compressibility vs. density



M.R. Haberman & A. N. Norris, Acoustics Today, **12**, 3 (2016)

Green’s functions

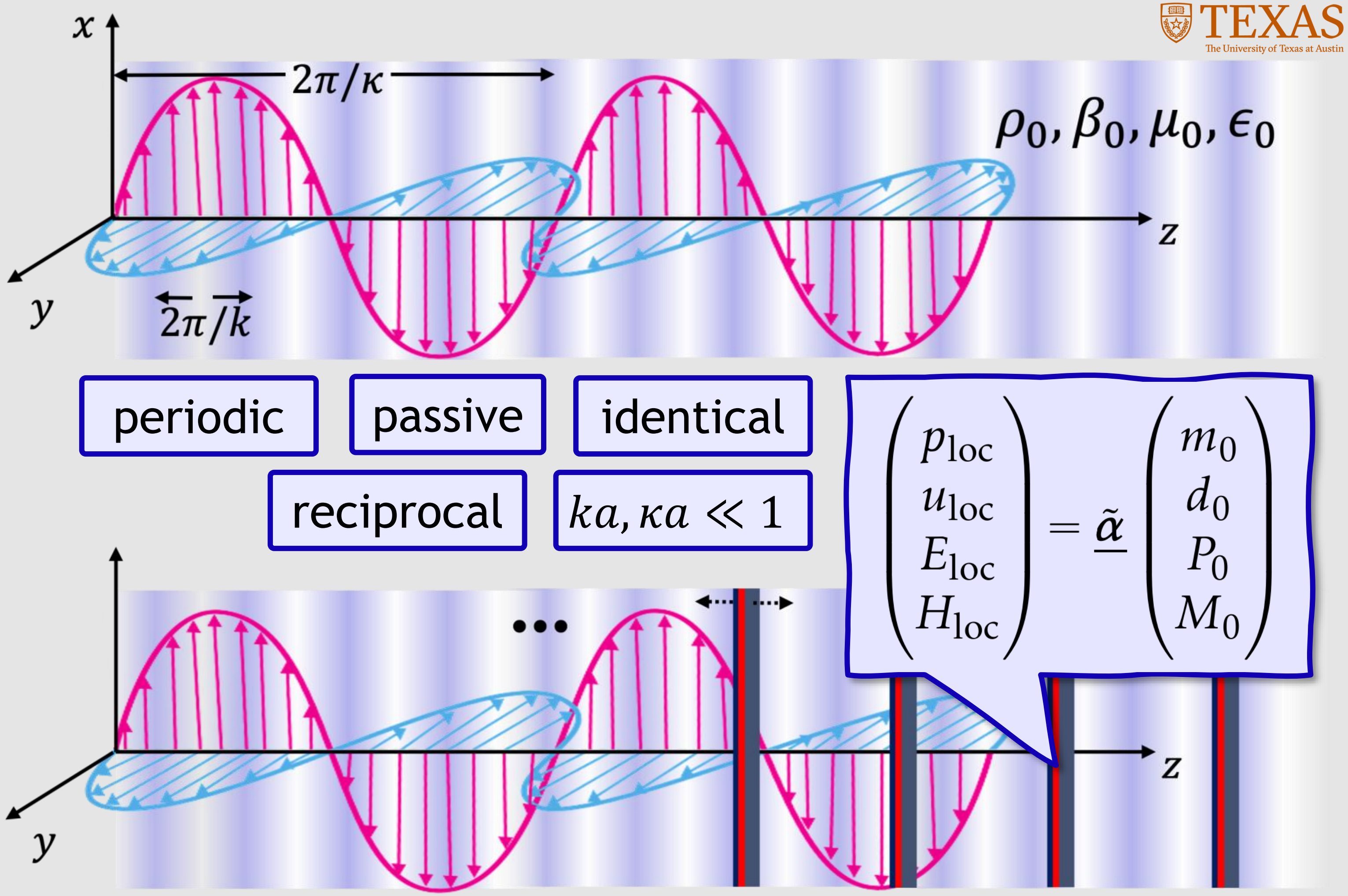
$$p_{\text{loc}} = p_{\text{ext}} + \sum_{n\neq 0} \left(-\beta_0^{-1} G_{\text{ac}}^{0n} m_n + c_0 G_{\text{ac}}^{0n} d_n \right)$$

$$u_{\text{loc}} = u_{\text{ext}} + \sum_{n\neq 0} \left(-c_0 G_{\text{ac}}^{0n} m_n + \rho_0^{-1} G_{\text{ac}}^{0n} d_n \right)$$

$$E_{\text{loc}} = E_{\text{ext}} + \sum_{n\neq 0} \left(\epsilon_0^{-1} G_{\text{em}}^{0n} P_n + C_0 G_{\text{em}}^{0n} M_n \right)$$

$$H_{\text{loc}} = H_{\text{ext}} + \sum_{n\neq 0} \left(C_0 G_{\text{em}}^{0n} P_n + \mu_0^{-1} G_{\text{em}}^{0n} M_n \right)$$

scattered fields



Governing equations in vectorial form

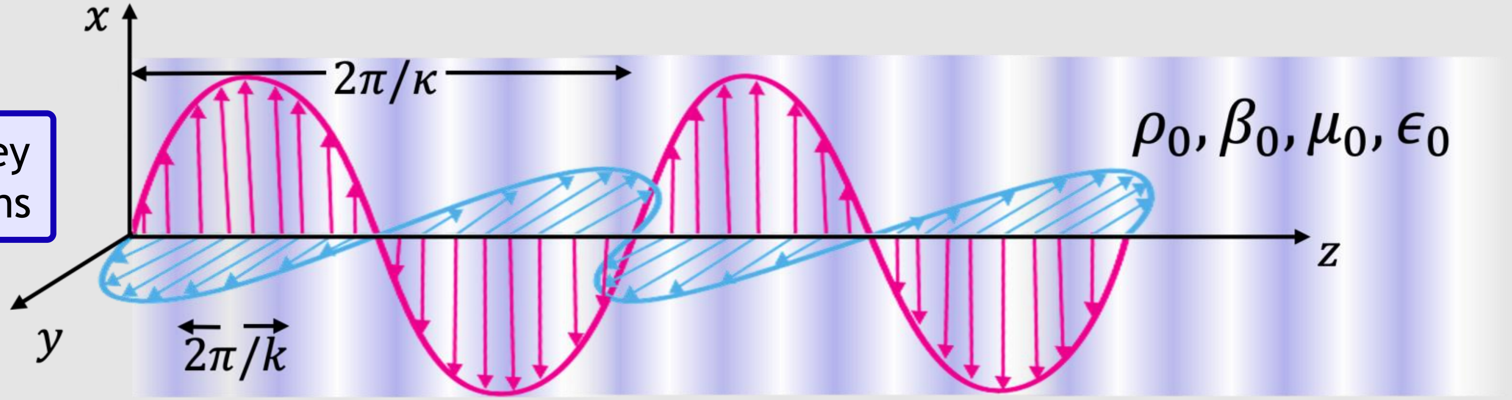
$$\nabla p_{\mathrm{ext}} + \rho_0 \frac{\partial \boldsymbol{u}_{\mathrm{ext}}}{\partial t} = \boldsymbol{f}_{\mathrm{ext}}$$

$$\nabla \cdot \boldsymbol{u}_{\mathrm{ext}} + \beta_0 \frac{\partial p_{\mathrm{ext}}}{\partial t} = q_{\mathrm{ext}}$$

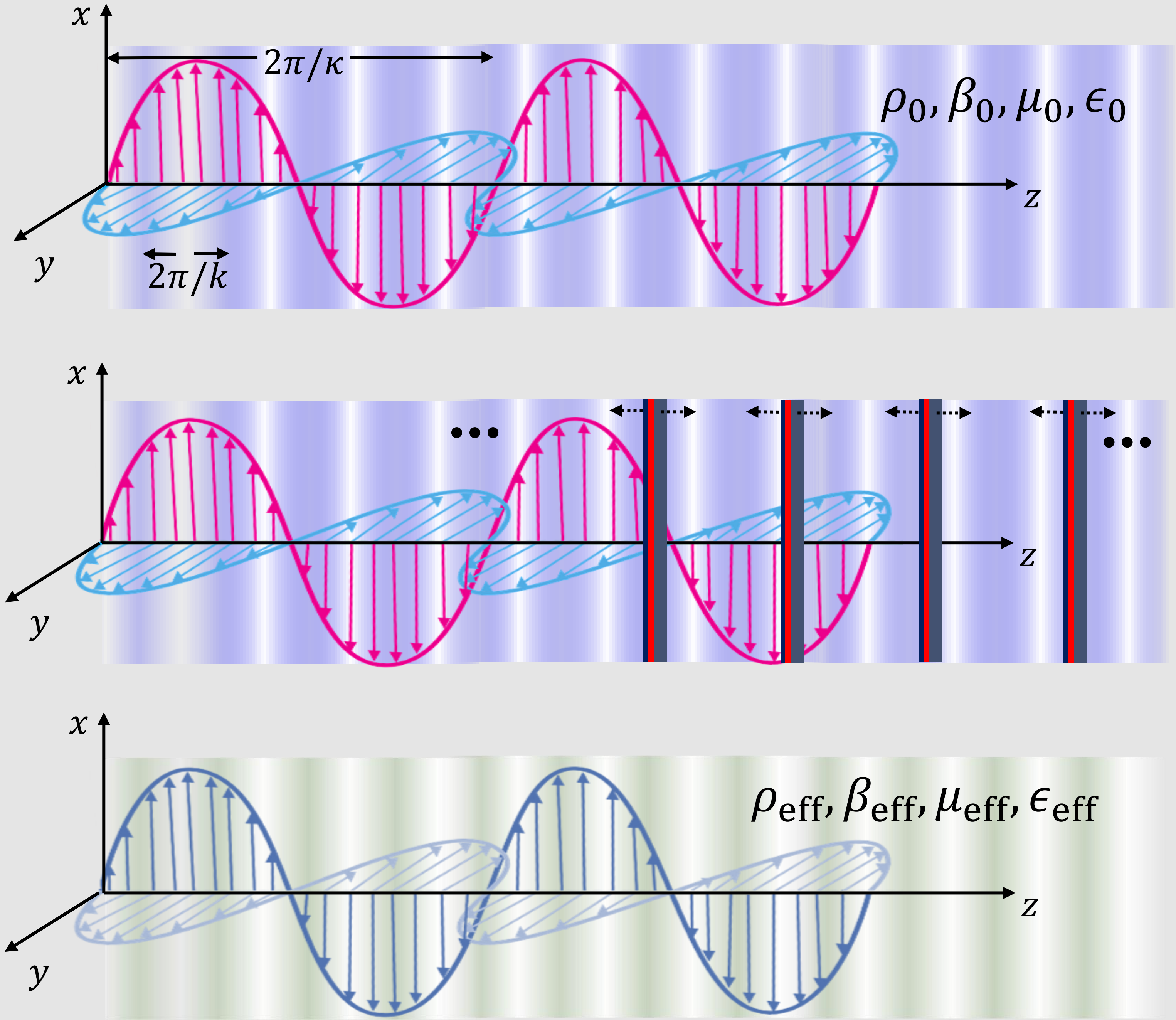
$$\nabla \times \boldsymbol{E}_{\mathrm{ext}} + \mu_0 \frac{\partial \boldsymbol{H}_{\mathrm{ext}}}{\partial t} = -\boldsymbol{K}_{\mathrm{ext}}$$

$$\nabla \times \boldsymbol{H}_{\mathrm{ext}} - \epsilon_0 \frac{\partial \boldsymbol{E}_{\mathrm{ext}}}{\partial t} = \boldsymbol{J}_{\mathrm{ext}}$$

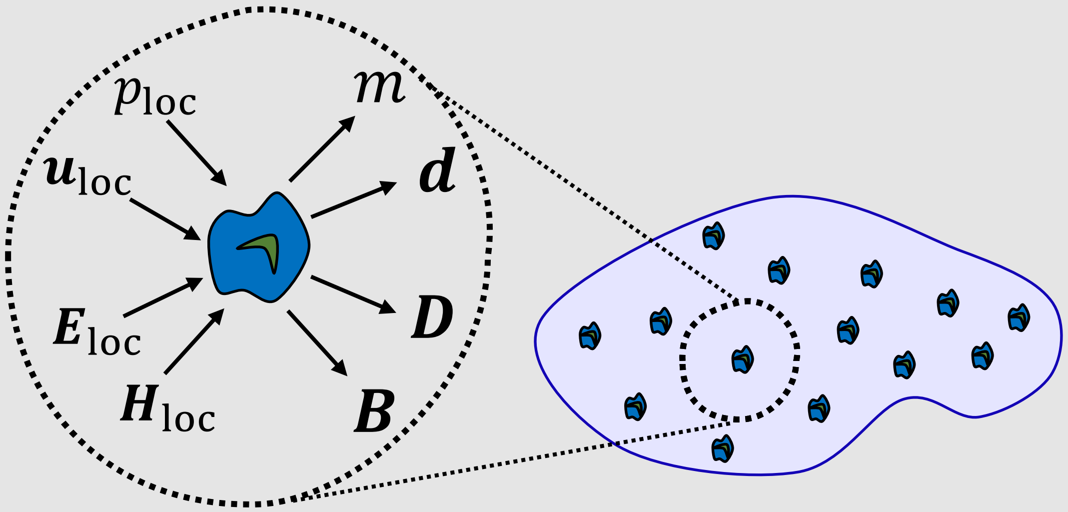
external fields obey governing equations

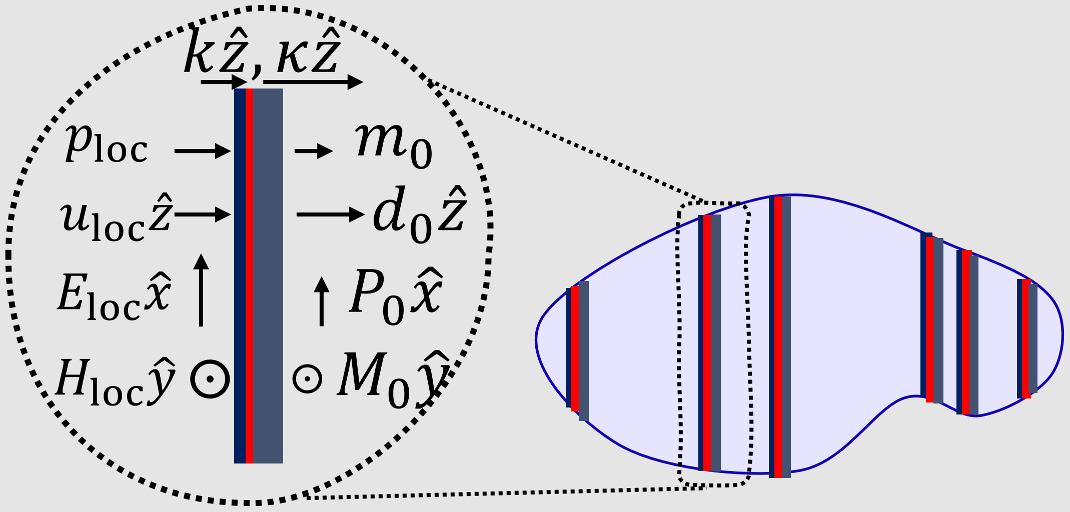


Homogenization procedure schematic



General and 1D/TEM scatterers





How does this relate to other homogenization schemes?

Homogenization of piezoelectric planar Willis materials undergoing antiplane shear

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[b] GROC-UJI, Institut de Noves Tecnologies de la Imatge, Universitat Jaume I, 12071 Castelló de la Plana, Spain

Generalized effective dynamic constitutive relation for heterogeneous media: Beyond the quasi-infinite and periodic limits

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PHYSICAL REVIEW APPLIED **14,** 064005 (2020)

Fundamental Principles for Generalized Willis Metamaterials

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