

Eliminate x'^2 from

given $x'^2 = y'^2 \left[\left(\frac{n}{p_y} \right)^2 - 1 \right] - 1$

$$y'^2 = \frac{x'^2 (n^2 - p_x^2) - p_x^2}{p_x^2}$$

$$y'^2 = x'^2 \left[\left(\frac{n}{p_x} \right)^2 - 1 \right] - 1$$

$$y'^2 = \underbrace{\left[\left(\frac{n}{p_x} \right)^2 - 1 \right]}_{\text{call this } \gamma_x} \left\{ \underbrace{\left[\left(\frac{n}{p_y} \right)^2 - 1 \right]}_{\text{call this } \gamma_y} y'^2 - 1 \right\} - 1$$

$$y'^2 = \gamma_x (\gamma_y y'^2 - 1) - 1$$

$$y'^2 = \gamma_x \gamma_y y'^2 - \gamma_x - 1$$

$$y'^2 (1 - \gamma_x \gamma_y) = -\gamma_x - 1$$

$$y'^2 = \frac{1 + \gamma_x}{\gamma_x \gamma_y - 1} = \frac{1 + \left(\frac{n^2}{p_y^2} \right) - 1}{\frac{n^4}{p_x^2 p_y^2} - \frac{n^2}{p_x^2} - \frac{n^2}{p_y^2}}$$

$$\left[\left(\frac{n}{p_x} \right)^2 - 1 \right] \left[\left(\frac{n}{p_y} \right)^2 - 1 \right] = \frac{n^4}{p_x^2 p_y^2} - \frac{n^2}{p_x^2} - \frac{n^2}{p_y^2} + 1$$

$$y'^2 = \frac{\frac{n^2}{p_y^2} \div n^2}{\left(\frac{n^4}{p_x^2 p_y^2} - \frac{n^2}{p_x^2} - \frac{n^2}{p_y^2} \right) \div n^2}$$

$$= \frac{1 \cdot p_x^2}{p_y^2 \left(\frac{n^2}{p_x^2 p_y^2} - \frac{1}{p_x^2} - \frac{1}{p_y^2} \right) p_x^2}$$

$$y'^2 = \frac{p_x^2}{n^2 - p_y^2 - p_x^2}$$

Thus $y' = \frac{p_x}{\sqrt{n^2 - p_x^2 - p_y^2}}$

Similarly for $x' = \frac{p_y}{\sqrt{n^2 - p_x^2 - p_y^2}}$