SOUND INTENSITY

F. J. FAHY

Institute of Sound and Vibration Research,
The University, Southampton, UK

ELSEVIER APPLIED SCIENCE
LONDON and NEW YORK

Of course, the maximum spatial gradient of pressure phase lies in the direction of the mean intensity vector: δ_{pI}, *as measured*, is therefore a function of both the *form of field* and of the *orientation of the intensity probe axis* within that field.

4.7 EXAMPLES OF IDEALISED SOUND INTENSITY FIELDS

The following examples are presented in order to illustrate the various characteristics of harmonic intensity fields revealed by the aforegoing analyses.

4.7.1 The Point Monopole

Expressions for the pressure and radial particle velocity fields, presented earlier in Chapter 3, are repeated here for the convenience of the reader.

$$p(r, t) = (A/r)\exp[i(\omega t - kr)]$$

$$u_r(r, t) = (A/\omega\rho_0 r)(k - i/r)\exp[i(\omega t - kr)]$$

Application of eqns (4.16) and (4.18) yields the following expressions for the active and reactive intensity components, which are purely radially directed:

$$I_a(r, t) = (A^2/2r^2\rho_0 c)[1 + \cos 2(\omega t - kr)] \tag{4.52}$$

and

$$I_r(r, t) = (A^2/2r^3\rho_0\omega)\sin 2(\omega t - kr) \tag{4.53}$$

The ratio of the magnitudes $|I_a|/|I_r| = I/Q = kr$, which shows that the reactive intensity dominates in the near field, and the active component dominates in the far field. The relationship between I and p^2 is the same as in a plane progressive wave. The curl of $\mathbf{I}$ is zero because the intensity is directed purely radially. The Lagrangian equals $A^2/\omega\rho_0 r^4$, indicating that the divergence of $\mathbf{Q}$ decreases very rapidly with r.

4.7.2 The Compact Dipole

The ideal dipole source illustrated in Fig. 4.9 comprises two point monopoles of equal strength and opposite polarity in close proximity to each other in terms of a wavelength ($kd \ll 1$). The pressure field of

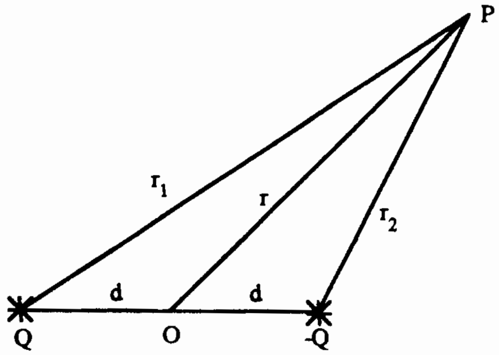

FIG. 4.9. Compact dipole of strength $2i\omega Qd$.

a harmonic dipole is expressed by

$$p(r_1, r_2, t) = [i\omega\rho_0 Q/4\pi][(1/r_1)\exp(-ikr_1) - (1/r_2) \times \exp(-ikr_2)]\exp(i\omega t) \quad (4.54)$$

In terms of the general expression $p = P\exp(i\phi_p)\exp(i\omega t)$

$$P = (\omega\rho_0 Q/4\pi)[1/r_1^2 + 1/r_2^2 - (2/r_1 r_2)\cos k(r_1 - r_2)]^{1/2} \quad (4.55)$$

and

$$\phi_p = \tan^{-1}[(r_2\cos kr_1 - r_1\cos kr_2)/(r_2\sin kr_1 - r_1\sin kr_2)] \quad (4.56)$$

Expressions for radial and tangential components of active and reactive intensity may be derived by the application of eqns (4.16) and (4.18) in the appropriate directions. Alternatively, eqns (4.34c) and (4.34d) may be employed with the radial component of complex intensity being given by

$$C_r = I_r + iQ_r = \tfrac{1}{2}(PU_r^*) \quad (4.57a)$$

and the tangential component as

$$C_\theta = I_\theta + iQ_\theta = \tfrac{1}{2}(PU_\theta^*) \quad (4.57b)$$

The complex pressure amplitude may be written as

$$P = (P_{1r} + P_{2r}) + i(P_{1i} + P_{2i}) \quad (4.58)$$

where

$$P_{1r} = (\omega\rho_0 Q\sin(kr_1))/(4\pi r_1)$$

$$P_{2r} = -(\omega\rho_0 Q\sin(kr_2))/(4\pi r_2)$$

$$P_{1i} = (\omega\rho_0 Q\cos(kr_1))/(4\pi r_1)$$

$$P_{2i} = -(\omega\rho_0 Q\cos(kr_2))/(4\pi r_2)$$

The complex amplitude of the radial velocity component is given by

$$U_r = U_{rr} + iU_{ri}$$

where

$$\begin{aligned}U_{rr} &= (1/\omega\rho_0)\{(P_{1i}/r_1 + kP_{1r})(1/r_1)(r + d\cos\theta) \\ &\quad + (P_{2i}/r_2 + kP_{2r})(1/r_2)(r - d\cos\theta)\} \\ U_{ri} &= (1/\omega\rho_0)\{(-P_{1r}/r_1 + kP_{1i})(1/r_1)(r + d\cos\theta) \\ &\quad + (-P_{2r}/r_2 + kP_{2i})(1/r_2)(r - d\cos\theta)\}\end{aligned} \tag{4.59}$$

The complex amplitude of tangential velocity is given by

$$U_\theta = U_{\theta r} + iU_{\theta i}$$

where

$$\begin{aligned}U_{\theta r} &= (1/r\omega\rho_0)\{(P_{1i}/r_1 + kP_{1r})(-d\sin\theta/r_1) \\ &\quad + (P_{2i}/r_2 + kP_{2r})(d\sin\theta/r_2)\} \\ U_{\theta i} &= (1/r\omega\rho_0)\{(-P_{1r}/r_1 + kP_{1i})(-d\sin\theta/r_1) \\ &\quad + (-P_{2r}/r_2 + kP_{2i})(d\sin\theta/r_2)\}\end{aligned} \tag{4.60}$$

The resulting expressions for I and Q can be decomposed into components corresponding to the two individual monopoles, plus 'interference' terms which introduce tangential components absent in the individual monopole fields. Examples of the distributions of I and Q are presented in Figs 4.10(a,b).

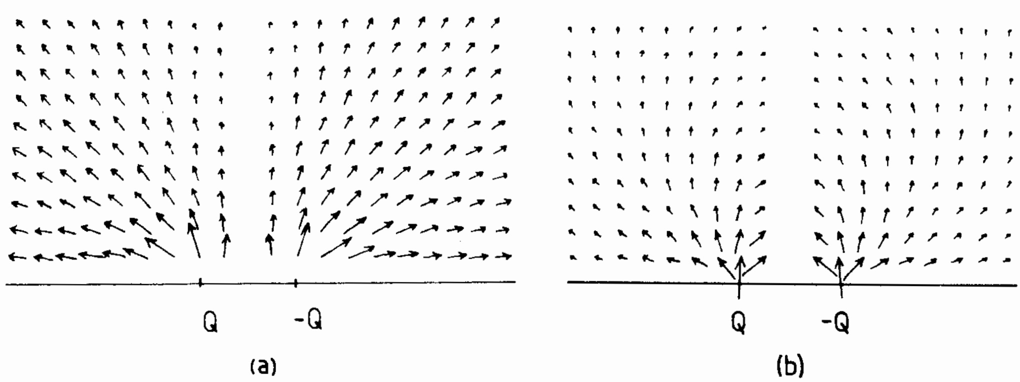

FIG. 4.10. Distributions of (a) mean intensity and (b) reactive intensity in the field of a compact dipole ($kd = 0{\cdot}2$; vector scales $\propto I^{1/4}$, $Q^{1/4}$: (a) scale 16 times (b) scale).

4.7.3 Interfering Monopoles

As mentioned in Chapter 3, it is possible mathematically to synthesise any complex source from an array of point monopoles of suitable amplitude and phase. The form of the intensity field produced by the superposition of such elementary wave fields may be illustrated by the case of two point monopoles of variable relative amplitude and phase. Figure 4.11(a) illustrates the mean intensity field of monopoles of equal strength and phase at a non-dimensional separation distance of $kd = 0{\cdot}2$ (the plotted vector magnitudes are proportional to $I^{1/4}$). The effect of doubling the strength of one of the sources is illustrated in Fig. 4.11(b), and Fig. 4.11(c) shows what happens when the phase of one of the pair is reversed (scales $\frac{1}{16}$ relative to Fig. 4.10(a)). The result clearly demonstrates the fact that the power radiated by an elementary volumetric source is affected by the pressure field imposed upon it by other coherent (phase-related) sources in its proximity. The magnitude of the mutual influence depends upon the separation distance because of the inverse square law. In the last case the weaker

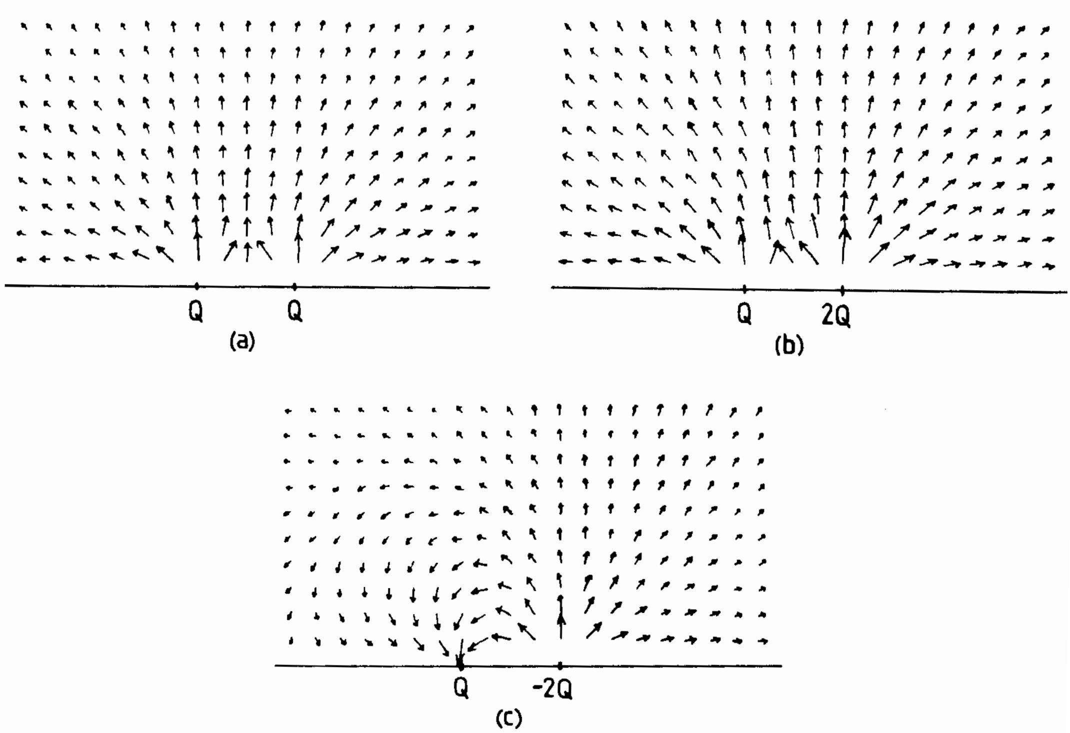

FIG. 4.11. Distribution of mean intensity in the fields of (a) two monopoles, (b) monopoles of different strengths but equal phase and (c) monopoles of different strengths and opposite phase.

monopole constitutes an active *sink*. (The phenomenon can easily be demonstrated with two small loudspeakers and an intensity meter.)

These results demonstrate an inter-dependence between coherent source regions which, in principle, makes it impossible, and even illogical, to identify any one region as *the source* of sound power, because the total power is a consequence of the simultaneous action of all the regions. An implication of considerable import for the practice of noise control is that the suppression of any portion of a total source array does not necessarily reduce the radiated power; indeed, it may increase it. A related consequence is that measurements of intensity in regions close to an extended source may not be well suited to the task of estimating the total radiated power because they are 'contaminated' by components of active energy flow which do not leave the vicinity of the source.

In case the reader is about to cast away this 'unhelpful' volume in frustration or despair, it should be pointed out that traverses of intensity probes over the surfaces of real sources often produce useful information. Among the reasons for this apparent contradiction of the above strictures are the following: (i) interdependence (in technical terminology 'mutual radiation impedance') exists only between coherently fluctuating source regions, i.e. those having time-stable, unique phase relationships; (ii) in cases of broad band sources, a multi-frequency 'smearing effect', illustrated in the following section, operates so as to reduce the degree and extent of near field recirculation of energy which is characteristic of narrow frequency bands.

4.7.4 Intensity in Ducts

As shown in Chapter 3, acoustic fields within uniform ducts may be considered to consist of summations of the characteristic duct modes, the amplitudes and phases of which are determined by the forms and locations of the sources driving the fields. These modes are simply interference patterns which satisfy the particular duct boundary conditions. Equation (3.35) indicates that the axial wavenumber components of modes other than the plane wave (the higher order modes) are, at frequencies higher than their cut-off frequencies, greater than the acoustic wavenumber $k = \omega/c$. At first sight, the implication appears to be that sound can propagate along the duct axis at speeds higher than c: this is, however, not so, because it is not the acoustic disturbances forming the modal interference pattern which are propagating at this speed, but the pattern itself. The group speed