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It was Teachers' day in 2014 and I was participating in a speaking contest where we had to each talk a few lines about our favourite teacher. While everybody talked about how "well" their teacher taught them, and how close to them and approachable their teacher was, one participant stood out for me. A good friend of mine up until today, she didn't refer to any of her teachers and based her talk on the premise that "Life was the best teacher". Why? Because while conventional teachers taught lessons first and then gave tests. While life did the opposite; it gave a test first and then left the learner to derive whatever meaning from these "tests". Of course, she didn't win. She wasn't even in the top three, but she did give me food for thought.

Back then, I was curious about very small things, albeit about how potatoes were supposed to make a light bulb glow or about how the lead character of Interstellar survived after falling into a black hole. I could not even attempt to grasp the physics behind that scene back then, while now I can. But I don't want to. Or maybe, I don't need to (because my major is mathematics and I don't need to go out of my way to understanding something so incredibly complex). But does one imply the other? Do I not want to, because I do not need to? Nevertheless, I was forced to learn many things I did not like in the process, although I grew to temporarily like them very much… (because they gave me a higher score in the exam for less effort!) Like inorganic chemistry. The subject does have logic but without memorizing a large number of facts, the logic seems useless. This often leads to inorganic chemistry being the most hated section of chemistry by science students in India.

But then again, one says that it is "expected" of science students in India to learn physics, chemistry and mathematics in great depth and detail to try and get into a good college. Here again, learning is given second priority by the general public, who only think the goal in life is to get into a "good enough" college. And unsurprisingly, the story does not end there. It does not start in high school either, in fact, it begins much before. All schools in India have science as a mandatory subject until the 10th standard. But what is going wrong? Why do an astounding majority of students end up despising STEM? And why do they continue to work in STEM even if they do? Most STEM students end up doing projects they don't even like, just because it is a part of the prescribed "course curriculum" which is a must for receiving an all important STEM degree. Often, students are expected to perform very long computations and calculations which is thought to "better" their understanding of the topic, given that a numerical perspective may be grasped better by the student. Is this a good idea or is it a better idea to scaffold this process altogether?

I shall attempt to integrate STEM fields into school curriculum itself. Not the subjects, but rather the softer aspect- the social, embodied spheres so that students can feel in place when they pursue a college degree in STEM later on. I remember how I wanted to be a doctor like my parents back when I was small- but soon deviated away from it in pursuit of mathematics (for which I am thankful!) But a lot of my childhood friends wanted to become doctors too, but now they only despise science. Is it because of a lack of a like-minded community? Not that 14 year olds are supposed to have their lives figured out… But would it help if they were nudged in the right direction at the correct time so that they figure out what their interests really are?

The field that I want to focus on is high school Mathematics. With the advent of innumerable coaching institutes for high school students in India that start as early as eighth standard to prepare young adults for competitive examinations in STEM courses in India. The main question I want to address is, if advanced mathematics education should be started earlier than the norm. Here are a few observations I have made after speaking to a lot of peers over the years:

- Certain students who are very inspired in the classes of 7^{th} and 8^{th} start reading elementary calculus textbooks on their own
- Those who wait until 11^{th} standard to learn calculus **may or may not** grasp it and apply it well

  I started learning calculus in 10^{th} standard and I would say that it helped me a lot to beat out my competition in my coaching institute.
- The Indian education system is very inflexible, with students not given liberty to do courses suited to their calibre; However, coaching institutes make very advanced mathematics available to students as soon as 8^{th} standard and leave it in their interest to pursue it. Should the formal education system do the same?
- Certain skills in mathematics can go a long way in grasping calculus content on the first go; so, proficiency in these skills can make first-time learning of calculus very fruitful and effective

Therefore, there is a need of an innovative way to introduce advanced Pre-calculus and raw mathematical concepts in school so that students might grasp calculus better, which is the basis of all advanced mathematics.

The theories that can be of some use here are:

Constructionism/constructivism, Embodied learning, Problem/project based learning and social theories, Learning by design

Learning goals in the conventional system:

- To have great familiarity with manipulating equations
- To memorize certain theorems and use them to prove certain equalities/facts in an exam
- To be great at performing computations, and having a lot of short cut formulae at one's disposal
  Integration Mathematical Formula - Math Shortcut Tricks (math-shortcut-tricks.com)

Learning goals in the novel system:

- To pay homage to the **history** of this great subject by learning about great mathematicians, their lives and their ideals; and correspondingly to interact with contemporary minds to broaden one's view of mathematics from a mere subject in school to a "**way of life**" in conjunction with social theories

  While studying for my graduate level Algebra course, I drew inspiration from Srinavasa Ramanujan'slife. This has led me to perform relatively well so far.

- To understand the **genesis** and **motivation** of basic theorems in mathematics rather than how they are used in problems
- To emphasize **logical proof-writing** rather than drilling on computational skill

  Computational skill should also be drilled but in subjects which require computation ex: chemistry, physics
- To try and expand the scope of a certain theorem and understand in what case it works and what case it doesn't; To **generalize** problems from specific examples and to **design** new problems

  Say, the specific problem is to find the number of possible arrangements of 5 distinct books on a shelf. The general problem would then be : To find the number of possible permutations of n **not necessarily distinct** objects

Potential Challenges faces by learners:

- Proof-writing is a hard skill, that must be honed over years, but is instrumental to advanced mathematics
- All students might not be inspired by the history of the subject or by interacting with contemporary mathematicians; their sole objective might be to get a good score in the competitive exam
- Generalising problems is very hard; there is no limit to generalising, and given a problem, once can always produce a more general problem; generalising problems might not resonate with all of the students
- Each student might have a different approach to a problem, and in cases of hard subjects like mathematics, their approach is bound to be wrong. This creates a challenge for students to construct knowledge confidently

Learning Experience

Formalized:

- Precalculus education at a early high school level; Students will be able to seamlessly explain advanced mathematical concepts like functions, graphs, etc. at the high school level
- Students will be able to make logical arguments as to why a certain statement is true; Will learn the difference between **necessary** and **sufficient** implications

  Consider the statement: Every quadratic function is increasing, A student at the high school level is tempted to test the hypothesis on a few known examples and conclude that the statement is true. But he/she must understand that this is verifying only for a certain class of examples, and there might be one that was forgotten, say $y = -x\text{^}2$, which is decreasing.
- Students will be made to interact with younger students who are interested in similar topics
- Students will be able to see the big picture in a certain theorem and explain it in very simple words, that should be understood by any other student taking the same course or maybe by students who are younger and have a passion for the subject
- Computational skill being a very necessary component shall also be honed, but in different courses. There shall be a digression : Mathematics vs Computation, and the computation course shall cover lengthy calculations and playing around with equations. The Computation course will be compulsory for whoever takes the Mathematics course, but not vice-versa.

Informal:

- Motivated students shall develop a sense of love towards the subject and feel indebted to contribute, hence increasing the chances of pursuing a college degree in mathematics later
- The interaction sessions between contemporary mathematicians will be very organic, and not just subject based- The students will get to know mathematicians as **people**, and not mathematicians as they are seen by the rest of the world
- The students see the **big picture** of mathematics and not just see it as a means of simplifying complex- looking equations or multiplying very large numbers

Learning Theories employed:

Social Theories

In the informal interaction sessions, students will get to know every aspect entailing being a mathematician. Students shall observe and try to imitate certain behaviours that they exhibit, and tend to idolize them.

Constructionism:

When students are able to understand the motivation and the genesis of theorems, they can explain it as a big picture view to other students who are in the same course or "newcomer" students who have a passion for the subject. This enhances the knowledge structure in the students' minds and makes further knowledge retrieval easier.

Constructivism:

When students are exposed to abstract mathematical concepts, logical centres in their brain become more organised. Considering the diversity of these concepts and their versatility, students are bound to consciously construct their own knowledge when being introduced to them. When students try to generalise problems, they are trying to solve the given specific problem in more general contexts, and thereby this is also voluntary knowledge construction.

Pedagogies:

Problem-based learning:

Mathematics essentially began because of existence of logical problems. Students are given an open ended problem in this form: " Solve this specific problem as generally as possible" ; the extent to which the students can generalise reflects the amount of thought and hard work they have put in. Through this experience of solving an open ended problem, many critical mathematical skills are inculcated in the students – The spirit of research is also kindled.

Learning by design:

Students also get to **design** their own problems that they find interesting, and this is in fact how mathematics was born. So, they are also paying homage to the history of the field, by engaging in raw problem-designing themselves. After designing a problem, they will need to think of a solution to that problem, thus, making it a problem-based learning scenario.

Legitimate peripheral participation:

During the interactions with the facilitators and the volunteers, students will situate themselves in the social learning space use their metacognition to judge their expertise. Newcomer students will slowly gravitate towards the “expert” realm.

The role of the facilitator (instructor) would be to formalize mathematical concepts like in a regular school, to comment on the problems designed by the students and to explain it to the rest of the class and brainstorm ideas from the class as a whole. The instructor shall play a very important role because he/she is the sole point of contact between the entire sphere of mathematical knowledge and the students.

Volunteers have a very important role in context of social theories, as explained earlier. They shall influence the students academically and non-academically.

Scaffolds:

Instructors may use animations other than embodiment to aid students’ understanding of the concepts. These animations may include graphics, pop ups and live simulations which help the students concretely align the “bricks” of knowledge in their minds, and scaffolds would be like temporary supports which support the knowledge structures until they have been “cemented” and can stand strong on their own. Further knowledge can be constructed upon these knowledge structures similarly, in perhaps some later course.

How does learning happen?

- In the context of learning theories and the curiosity of the students; the sky is the limit and student are free to explore whatever question they want
- Students learn by constructing artifacts while explaining certain concepts to other students. This may also aid in embodied cognition as the student tries to make the other students understand by various bodily hints/ movements
- Learning in a problem/project based scenario has plenty of scope in a field like mathematics where questions form the foundation of the subject. Any question would serve as an ill-framed question which the student can ponder about
- When students design problems, they will need to keep in mind the feasibility of finding a solution to that problem, because that should be their intent. When they solve their own problem, it will be a problem-based learning scenario.
- Unlike a conventional education system, it is the students who construct their own knowledge at their own pace
- Interaction with the facilitators (instructor) which helps to address their queries and doubts, and misconceptions
- Interaction with the volunteers which aids them in the big picture- to give them the taste of the subject as a whole personality – every social aspect of it