

Optimization Assignment - 2

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Problem Statement - A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is $8m^3$. If building of tank costs Rs 70 per sq metres for the base and Rs 45 per square metre for sides. What is the cost of least expensive tank?

Solution

Theoretical approximation

Let l, b and h are the length, width and height of tank
Let R_b be the base cost and R_s be the sides cost.

Given

$$h = 2, V = 8 \quad (1)$$

$$V = lbh \quad (2)$$

where

$$8 = 2l \quad (3)$$

$$lb = 4 \quad (4)$$

So,

$$b = \frac{4}{l}, l = \frac{4}{b} \quad (5)$$

Total least cost is given by,

$$S(l) = 280 + 180 \left(l + \frac{4}{l} \right) \quad (6)$$

$$S'(l) = 180 \left(1 - \frac{4}{l^2} \right) \quad (7)$$

$$S''(l) = 180 \left(\frac{8}{l^3} \right) > 0 \quad (8)$$

Thus, $S(l)$ has a minimum which can be obtained from (7) as

$$180 \left(1 - \frac{4}{l^2} \right) = 0 \quad (9)$$

$$\implies l^2 - 4 = 0 \quad (10)$$

$$\implies l = 2 \quad (11)$$

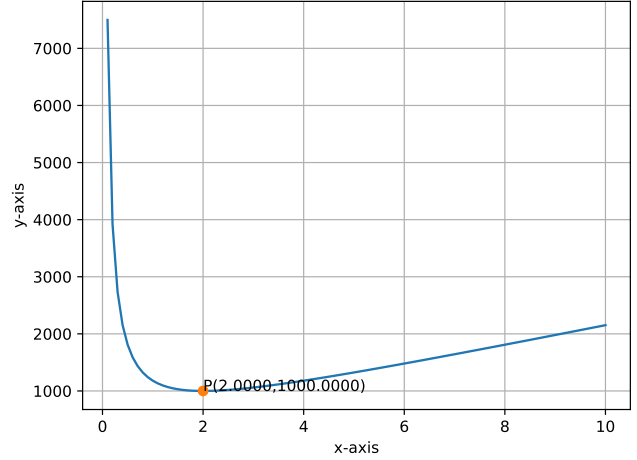


Figure 1: Graph of $f(x)$ in (??)

Substituting l in equation in (6) we will get the total minimum cost and its verified

$$C_{min} = 1000 \quad (12)$$

$$l = 2 \quad (13)$$

Gradient descent

Let l be the length of tank
The Total least cost of tank is expressed as

$$f(l) = 280 + 180 \left(l + \frac{4}{l} \right) \quad (14)$$

Using gradient ascent method we can find its minima

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \quad (15)$$

$$\implies x_{n+1} = x_n + \alpha \left(180 \left(1 - \frac{4}{l^2} \right) \right) \quad (16)$$

Taking $x_0 = 2, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\text{Minima} = 1000 \quad (17)$$

$$\text{Minima Point} = 2 \quad (18)$$