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Assignment-4

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Triangle Law of Vector addition

The triangle law of vector addition says that when two vectors are represented as two sides of a triangle with the same order of magnitude and direction, then the magnitude and direction of the resultant vector is represented by the third side of the triangle taken in reverse order..

$$\mathbf{R} = \mathbf{A} + \mathbf{B} \quad (1)$$

Problem Statement:

Construct a right triangle whose base is 12cm and sum of its hypotenuse and other side is 18cm.

SOLUTION:

Let A,B and C be the vertices of right triangle with with coordinates $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ respectively.

OB-Base. AB-Hypotenuse. OA-Side.

Given:

Since its a right triangle $OA \perp OB$

Since base is 12cm length of $OB = 12$
i.e,

$$b = 12 \quad (2)$$

Sum of length OA and AB = 18cm
i.e,

$$a + c = 18 \quad (3)$$

To Find

The magnitude of a i.e

STEP-1

Let k be the unknown point in the vertex A

Then coordinates of vertices O,A and B are :

$$O \begin{pmatrix} 0 \\ 0 \end{pmatrix} B \begin{pmatrix} 12 \\ 0 \end{pmatrix} A \begin{pmatrix} 0 \\ k \end{pmatrix}$$

We know that $a + c = 18$

So, Let $m=18$

$$a + c = m \quad (4)$$

We know that ,

$$c^2 = a^2 + b^2 \quad (5)$$

STEP-2

(1.) If we find 'a' we can get k in $A(0,k)$

$$O \begin{pmatrix} 0 \\ 0 \end{pmatrix} B \begin{pmatrix} 12 \\ 0 \end{pmatrix} A \begin{pmatrix} 0 \\ k \end{pmatrix} \quad (6)$$

By using equation (5)

$$c^2 = a^2 + b^2$$

$$b^2 = c^2 - a^2 \quad (7)$$

We know that $c^2 - a^2 = (c - a)(c + a)$

Since, $c + a = m$

$$b^2 = c - a(m) \quad (8)$$

$$c - a = \frac{b^2}{m} \quad (9)$$

And ,

$$c + a = m \quad (10)$$

Using equation (9) and (10),

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} c \\ a \end{pmatrix} = \begin{pmatrix} m \\ \frac{b^2}{m} \end{pmatrix} \quad (11)$$

STEP-2

(3) Using equation (11)

Let,

$$\mathbf{P} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (12)$$

$$\mathbf{y} = \begin{pmatrix} c \\ a \end{pmatrix} \quad (13)$$

$$\mathbf{Q} = \begin{pmatrix} x \\ \frac{b^2}{m} \end{pmatrix} \quad (14)$$

We know that,

$$\mathbf{P} * \mathbf{y} = \mathbf{Q} \quad (15)$$

And,

$$P^{-1}P = I \tag{16}$$

multiplying P^{-1} on both sides in equation (15)

$$y = P^{-1}Q \tag{17}$$

Using equation (17) we get ,

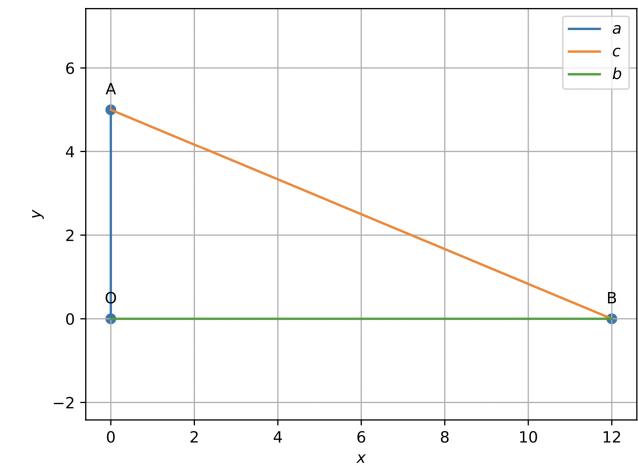
$$c = 13 \tag{18}$$

$$a = 5 \tag{19}$$

The Coordinates of $A \begin{pmatrix} 0 \\ k \end{pmatrix}$ is ,

$$A \begin{pmatrix} 0 \\ 5 \end{pmatrix} \tag{20}$$

Result



implementation

Equation no	Role
5	Pythagorean Theorem
11	Matrix form of Linear equation
16	Results Identity matrix
18	Length of c
19	Length of a
20	substituting k=5

Construction

vertex	coordinates
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
A	$\begin{pmatrix} 0 \\ 5 \end{pmatrix}$
B	$\begin{pmatrix} 12 \\ 0 \end{pmatrix}$

Download the code
Github link: Assignment-4.