Optimization Assignment - 2

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Problem Statement **- A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is $8m^3$. If building of tank costs Rs 70 per sq metres for the base and Rs 45 per square metre for sides. What is the cost of least expensive tank?**

Solution

Theoretical approximation

Let l,b and h are the length,width and hight of tank
Let R_b be the base cost and R_s be the sides cost.

Given

$$h = 2, V = 8 \tag{1}$$

$$V = lbh \tag{2}$$

where

$$8 = 2l \tag{3}$$
$$lb = 4 \tag{4}$$

So,

$$b = \frac{4}{l}, l = \frac{4}{b} \tag{5}$$

Total least cost is given by,

$$S(l) = 280 + 180\left(l + \frac{4}{l}\right) \tag{6}$$

$$S'(l) = 180\left(1 - \frac{4}{l^2}\right) \tag{7}$$
$$S''(l) = 180\left(\frac{8}{l^3}\right) > 0 \tag{8}$$

Thus, S(l) has a minimum which can be obtained from (7) as

$$180\left(1 - \frac{4}{l^2}\right) = 0 \tag{9}$$
$$\implies l^2 - 4 = 0 \tag{10}$$
$$\implies l = 2 \tag{11}$$

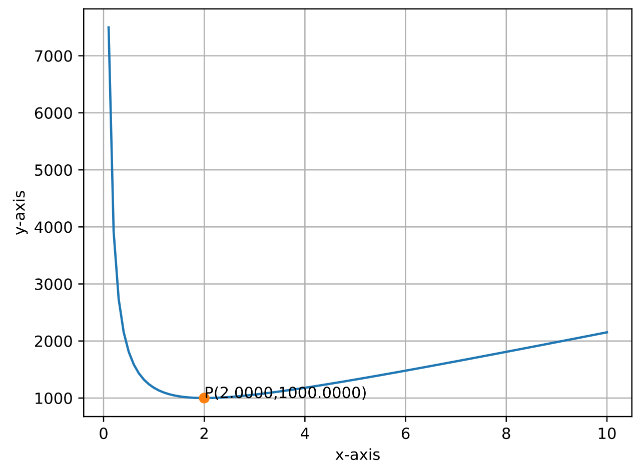

Figure 1: Graph of $S(l)$ vs l

Substituting l in equation in (6) we will get the total minimum cost and its verified

$$\boxed{C_{min} = 1000} \tag{12}$$
$$\boxed{l = 2} \tag{13}$$

Gradient descent

Let l be the length of tank
The Total least cost of tank is expressed as

$$f(l) = 280 + 180\left(l + \frac{4}{l}\right) \tag{14}$$

Using gradient ascent method we can find its minima

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \tag{15}$$
$$\implies x_{n+1} = x_n - \alpha\left(180\left(1 - \frac{4}{l^2}\right)\right) \tag{16}$$

Taking $x_0 = 2, \alpha = 0.001$ and precision $= 0.00000001$, values obtained using python are:

$$\boxed{\text{Minima} = 1000} \tag{17}$$
$$\boxed{\text{Minima Point} = 2} \tag{18}$$