Assignment 4

Level set method formulation

Assume the equation of the surface ϕ (x) and C-level set of this surface given by

$$\phi (x) = C$$

We can write the zero-level set of ϕ as

$$\phi (x) = 0$$

Movement of the zero-level set with the time can be represent by obtaining the derivative of above equation.

$$\frac{\partial \phi(x(t),t)}{\partial t} = 0$$

by applying chain rule,

$$\frac{\partial \phi}{\partial x(t)} \frac{\partial x(t)}{\partial t} + \frac{\partial \phi}{\partial t} = 0$$

We can write this equation as

$$\nabla\phi.X_t + \phi t=0 \text{ --------------- } 1$$

X_t means the speed of the vector and it is given by

$$X_t = g. \frac{\nabla\phi}{\|\nabla\phi\|}$$

By substituting to equation 1

$$g\|\nabla\phi\| + \phi t=0$$

$$\phi_t = - g\|\nabla\phi\|$$

by solving this equation, we can show that

$\phi' = \phi + \Delta t * g * \|\nabla\phi\|$ (we can see that, this equation doing the gradient descent on ϕ with respect to t(time))

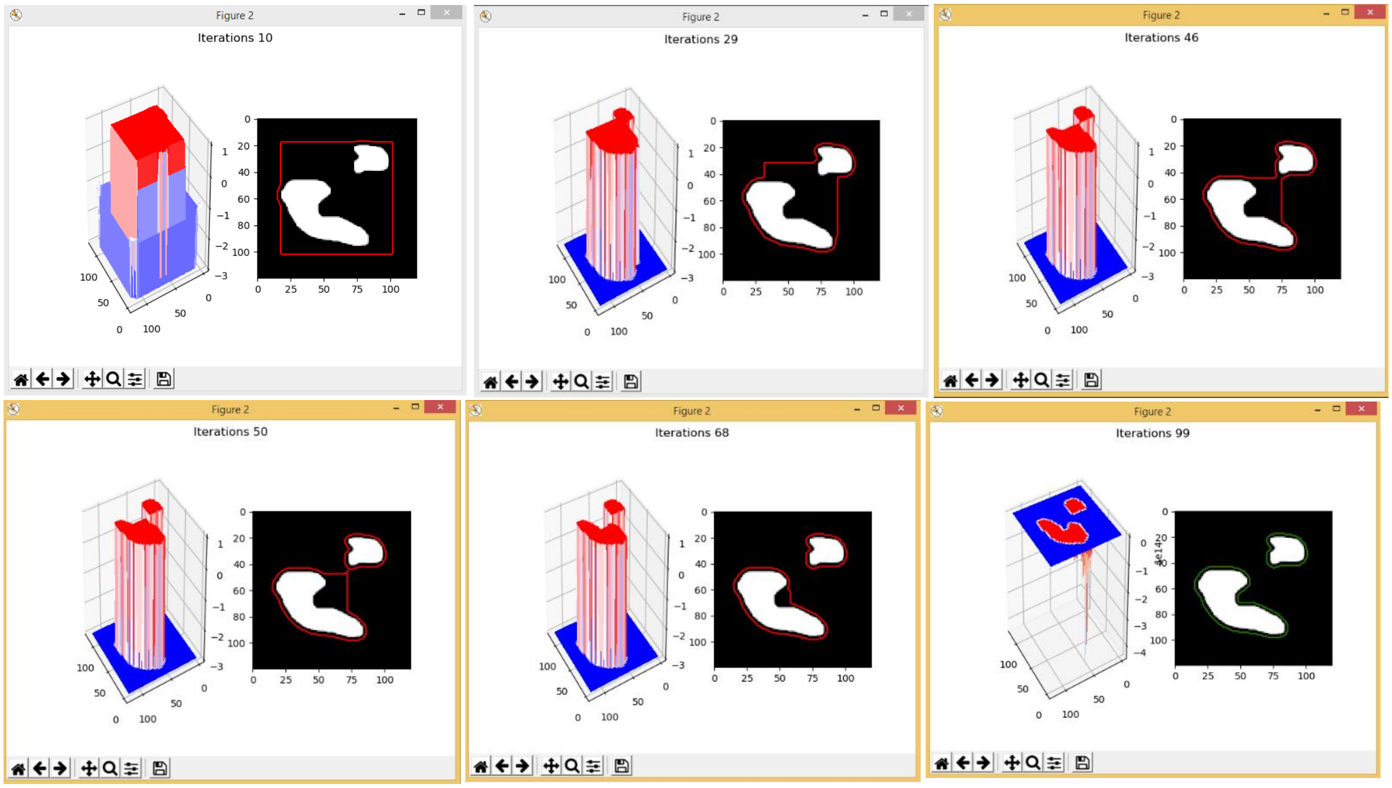
Question 1.

In the given code g means vector field where at every point it tells us to the direction and the magnitude of movement of surface ϕ.

For that we want to derive g from the image. As g is velocity field, we want g to be high at all the regions that are not the border of the object and we want to segment and slowly propagate near the border and want to stop at the border of the image.
For that g derive from edge detector.

ϕ_t gives how the speed of surface evoluation of ϕ.

```
phi_t = - g * grad_phi_norm
```



Question 2.

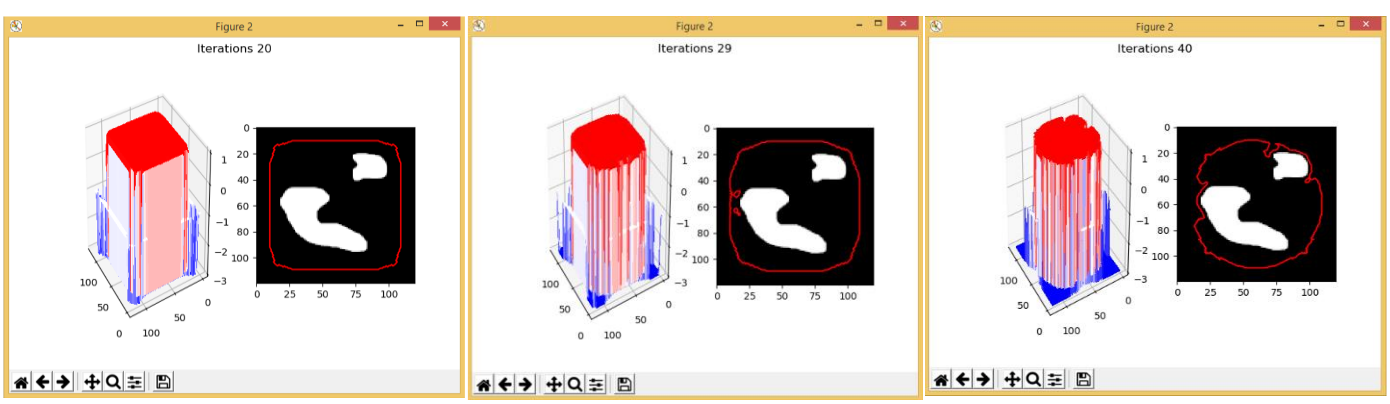
We want to build function for divergence.

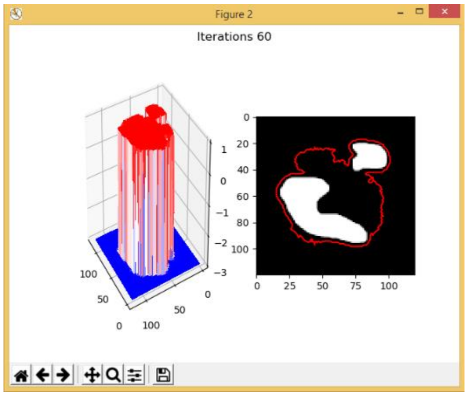
```
def div(x):
  dy = np.gradient(x[0], axis = 0)
  dx = np.gradient(x[1], axis = 1)
  return abs(dx + dy)
```

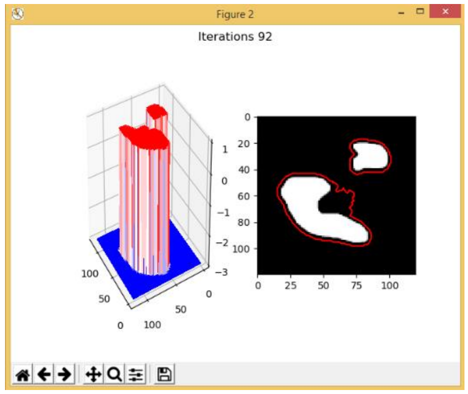
I. $\phi_t = g\kappa|\nabla\phi|$

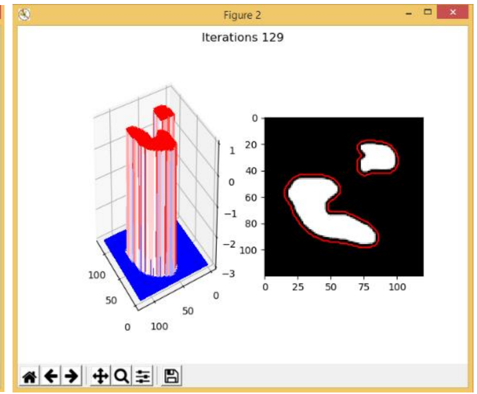
coding part:

```
phi_t  =  - g * div(np.nan_to_num(grad_phi / grad_phi_norm)) * grad_phi_norm
```





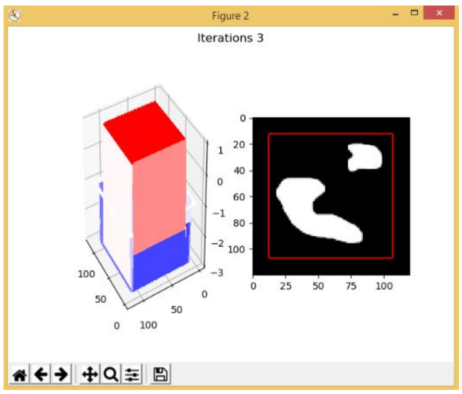


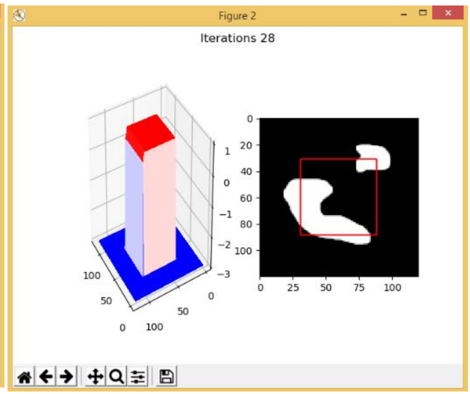


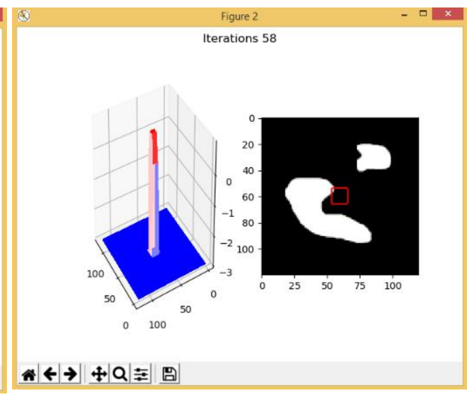
II. $\phi_t = |\nabla\phi|$

Code part:

```
phi_t  =  - grad_phi_norm
```



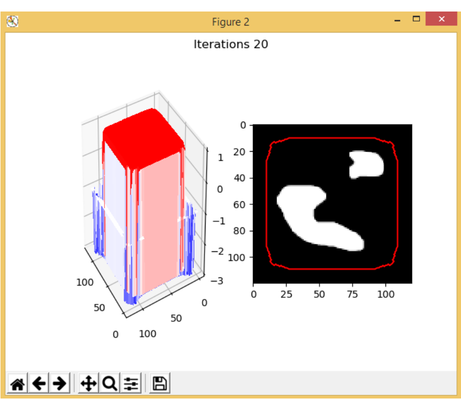


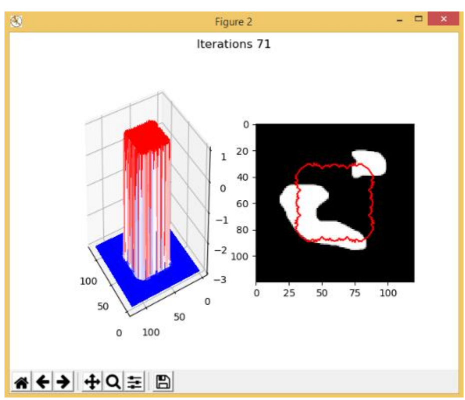


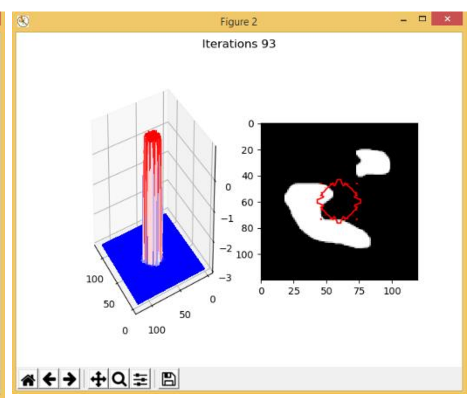
III. $\phi_t = \mathrm{div}(\frac{\nabla\phi}{|\nabla\phi|}) * |\nabla\phi|$

Code part:

```
phi_t = - div(np.nan_to_num(grad_phi / grad_phi_norm)) * grad_phi_norm
```



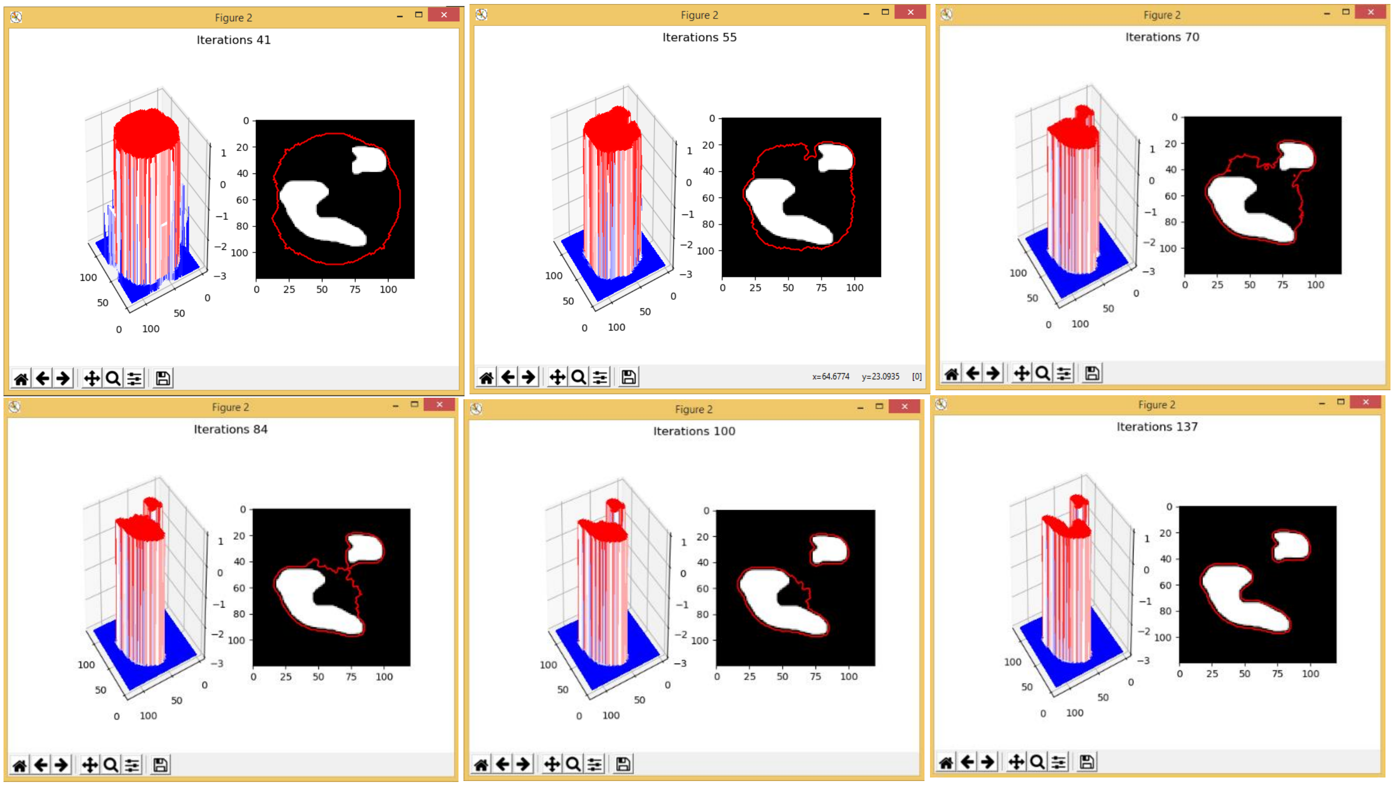




IV. $\phi_t = \mathrm{div}(g * \frac{\nabla\phi}{|\nabla\phi|}) * |\nabla\phi|$

Code part:

```
phi_t = - div(g * (np.nan_to_num(grad_phi / grad_phi_norm))) * grad_phi_norm
```



Question 3.

Image	No of iteration needed			
	$\phi_t = g\kappa\lvert\nabla\phi\rvert$	$\phi_t = \lvert\nabla\phi\rvert$	$\phi_t = \mathrm{div}(\frac{\nabla\phi}{\lvert\nabla\phi\rvert}) * \lvert\nabla\phi\rvert$	$\phi_t = \mathrm{div}(g * \frac{\nabla\phi}{\lvert\nabla\phi\rvert}) * \lvert\nabla\phi\rvert$
	129	Fails to segment (without stopping function cannot segment)	Fails to segment (without stopping function cannot segment)	139

Question 4.

	$\phi_t = g\kappa\lvert\nabla\phi\rvert$	$\phi_t = \lvert\nabla\phi\rvert$	$\phi_t = \mathrm{div}(\frac{\nabla\phi}{\lvert\nabla\phi\rvert}) * \lvert\nabla\phi\rvert$	$\phi_t = \mathrm{div}(g * \frac{\nabla\phi}{\lvert\nabla\phi\rvert}) * \lvert\nabla\phi\rvert$
Image examples where each of the velocity terms fails to segment