

# Assignment

## knapsack

Solve the following instance of knapsack problem with 4 objects and knapsack capacity = 10

- \* weights of 4 objects = 3, 4, 2, 3 respectively
- \* profit of 4 objects = 30, 45, 25, 36 respectively
- Do not use memory functions

objects	weights	Profit
1	3	30
2	4	45
3	2	25
4	3	36

$$v[i, j] = \begin{cases} 0 & \text{if } i=0, j=0 \\ v[i-1, j] & \text{if } w_i > j \\ \max\{v[i-1, j], v[i-1, j-w_i] + p_i\} & \text{if } w_i \leq j \end{cases}$$

n \ m	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	30	30	30	30	30	30	30	30
2	0	0	0	30	45	45	45	75	75	75	75
3	0	0	25	25	25	55	70	75	75	100	100
4	0	0	25	36	36	61	70	75	91	106	111



~~Step 1: when  $i=1$ ,  $w_1=8$ ,  $P_1=30$~~

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$$J=1, v[1,1] = v[0,1] = 0$$

$$J=2, v[1,2] = v[0,2] = 0$$

$$J=3, v[1,3] = \max\{v[0,3], v[0,0] + 30\} = 30$$

$$J=4, v[1,4] = \max\{v[0,4], v[0,1] + 30\} = 30$$

$$J=5, v[1,5] = \max\{v[0,5], v[0,2] + 30\} = 30$$

$$J=6, v[1,6] = \max\{v[0,6], v[0,3] + 30\} = 30$$

$$J=7, v[1,7] = \max\{v[0,7], v[0,4] + 30\} = 30$$

$$J=8, v[1,8] = \max\{v[0,8], v[0,5] + 30\} = 30$$

$$J=9, v[1,9] = \max\{v[0,9], v[0,6] + 30\} = 30$$

$$J=10, v[1,10] = \max\{v[0,10], v[0,7] + 30\} = 30$$

Step 2: when  $i=2$ ,  $w_2=4$ ,  $P_2=45$

$$J=1, v[2,1] = v[1,1] = 0$$

$$J=2, v[2,2] = v[1,2] = 0$$

$$J=3, v[2,3] = v[1,3] = 30$$

$$J=4, v[2,4] = \max\{v[1,4], v[1,0] + 45\} = 45$$

$$J=5, v[2,5] = \max\{v[1,5], v[1,1] + 45\} = 45$$

$$J=6, v[2,6] = \max\{v[1,6], v[1,2] + 45\} = 45$$

$$J=7, v[2,7] = \max\{v[1,7], v[1,3] + 45\} = 75$$

$$J=8, v[2,8] = \max\{v[1,8], v[1,4] + 45\} = 75$$

$$J=9, v[2,9] = \max\{v[1,9], v[1,5] + 45\} = 75$$

$$J=10, v[2,10] = \max\{v[1,10], v[1,6] + 45\} = 75$$

Step 3: when  $i=3$ ,  $w_3=2$ ,  $P_3=25$

$$J=1, v[3,1] = v[2,1] = 0$$

$$J=2, v[3,2] = \max\{v[2,1], v[2,0] + 25\} = 25$$

$$J=3, v[3,3] = \max\{v[2,3], v[2,1] + 25\} = 25$$



$$\begin{aligned}
 J=4, \quad v[3,4] &= \max\{v[2,4], v[2,2] + 25\} = 25 \\
 J=5, \quad v[3,5] &= \max\{v[2,5], v[2,3] + 25\} = 55 \\
 J=6, \quad v[3,6] &= \max\{v[2,6], v[2,4] + 25\} = 70 \\
 J=7, \quad v[3,7] &= \max\{v[2,7], v[2,5] + 25\} = 75 \\
 J=8, \quad v[3,8] &= \max\{v[2,8], v[2,6] + 25\} = 75 \\
 J=9, \quad v[3,9] &= \max\{v[2,9], v[2,7] + 25\} = 100 \\
 J=10, \quad v[3,10] &= \max\{v[2,10], v[2,8] + 25\} = 100
 \end{aligned}$$

Step 4<sup>th</sup> when  $i=4, w_4=3, P_4=36$

$$\begin{aligned}
 J=1, \quad v[4,1] &= v[3,1] = 0 \\
 J=2, \quad v[4,2] &= v[3,2] = 25 \\
 J=3, \quad v[4,3] &= \max\{v[3,3], v[3,0] + 36\} = 36 \\
 J=4, \quad v[4,4] &= \max\{v[3,4], v[3,1] + 36\} = 36 \\
 J=5, \quad v[4,5] &= \max\{v[3,5], v[3,2] + 36\} = 61 \\
 J=6, \quad v[4,6] &= \max\{v[3,6], v[3,3] + 36\} = 70 \\
 J=7, \quad v[4,7] &= \max\{v[3,7], v[3,4] + 36\} = 75 \\
 J=8, \quad v[4,8] &= \max\{v[3,8], v[3,5] + 36\} = 91 \\
 J=9, \quad v[4,9] &= \max\{v[3,9], v[3,6] + 36\} = 106 \\
 J=10, \quad v[4,10] &= \max\{v[3,10], v[3,7] + 36\} = 111
 \end{aligned}$$

Optimal Solution

$$\begin{aligned}
 v(n, m) &= v[4, 10] \\
 &= 111
 \end{aligned}$$

4<sup>th</sup> object is selected as maximum profit

$$\text{then } 111 - 36 = 75$$

$$\text{then } v[3, 7] = v[2, 7] = 75$$

Therefore 3<sup>rd</sup> object is not selected

$$v[2, 7] \neq v[1, 7]$$

2<sup>nd</sup> object is selected

$$75 - P_2 = 75 - 45 = 30$$



$$\sqrt{1.3} \neq \sqrt{0.3}$$

$$30 \neq 0$$

1st Object is selected

$$\begin{matrix} x_1 & x_2 & x_3 & x_4 \\ \{ 1 & 1 & 0 & 1 \} \end{matrix}$$