SECTION-A

Short Answer Type Questions

Ques1:- Explain Peano's axioms with suitable example.

Ans:- Peano axioms, also known as Peano's postulates, in number theory, five axioms introduced in 1889 by Italian mathematician Giuseppe Peano. Like the axioms for geometry devised by Greek mathematician Euclid, the Peano axioms were meant to provide a rigorous foundation for the natural numbers (0, 1, 2, 3, ...) used in arithmetic, number theory and set theory. In particular, the Peano axioms enable an infinite set to be generated by a finite set of symbols and rules.

The five Peano axioms are:

1. Zero is a natural number.
2. Every natural number has a successor in the natural numbers.

Example:- 0 is a natural number, and its successor is 1,

Similarly, the successor of 1 is 2 and so on.

Successor means '+1' in the natural number.

3. Zero is not the successor of any natural number.
4. If the successor of two natural numbers is the same, then the two original numbers are the same.
5. If a set contains zero and the successor of every number is in the set, then the set contains the natural numbers.

The fifth axiom is known as the principle of induction because it can be used to establish properties for an infinite number of cases without having to give an infinite number of proofs. In particular, given that P is a property and zero has P and that whenever a natural number has P its successor also has P, it follows that all natural numbers have P.

Ques2:- By means of truth table, show that $(p \vee q) \wedge \sim(p \vee q)$ is a contradiction.

Ans:-

TRUTH TABLE

p	q	$p \vee q$	$\sim(p \vee q)$	$(p \vee q) \wedge \sim(p \vee q)$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

Since, in the above truth table, $(p \vee q) \wedge \sim(p \vee q)$ contains all false value. Therefore, It is a contradiction.

Hence proved

Ques3:- Solve the recurrence relation :

$$4a_n - 20a_{n-1} + 17a_{n-2} - 4a_{n-3} = 0$$

Ans:- Find characteristic equation of given recurrence relation.

$$4\alpha^n - 20\alpha^{n-1} + 17\alpha^{n-2} - 4\alpha^{n-3} = 0$$

Here, the highest order is 3.

$\therefore\ 4\alpha^3 - 20\alpha^{3-1} + 17\alpha^{3-2} - 4\alpha^{3-3} = 0$

$4\alpha^3 - 20\alpha^2 + 17\alpha - 4\alpha^0 = 0$

$4\alpha^3 - 20\alpha^2 + 17\alpha - 4 = 0 \quad (\because n^0 = 1)$

Now, the factors of 4 are $\pm 1, \pm 2, \pm 4$

(i) put $\alpha = 1$.

$4(1)^3 - 20(1)^2 + 17(1) - 4 = 0$

$\cancel{4} - 20 + 17 - \cancel{4} = 0$

$-3 \neq 0$

(ii) put $\alpha = 2$

$4(2)^3 - 20(2)^2 + 17(2) - 4 = 0$

$4 \times 8 - 20 \times 4 + 34 - 4 = 0$

$32 - 80 + 34 - 4 = 0$

$32 - 46 - 4 = 0$

$32 - 50 = 0$

$-18 \neq 0$

(iii) put $\alpha = 4$

$4(4)^3 - 20(4)^2 + 17(4) - 4 = 0$

$4 \times 64 - 20 \times 16 + 68 - 4 = 0$

$256 - 320 + 68 - 4 = 0$

$256 - 252 - 4 = 0$

$\cancel{256} - \cancel{256} = 0$

$0 = 0 \quad$ (L.HS = R.H.S)

$\therefore$ $(\alpha-4)$ is a factor of $4\alpha^3-20\alpha^2+17\alpha-4$

Now, we have to find out other two factors

$$
\begin{array}{r|l}
 & 4\alpha^2-4\alpha+1 \\
\hline
\alpha-4 & 4\alpha^3-20\alpha^2+17\alpha-4 \\
 & \cancel{4\alpha^3}-16\alpha^2 \\
 & \ominus \quad \oplus \\
\hline
 & -4\alpha^2+17\alpha-4 \\
 & -\cancel{4\alpha^2}+16\alpha \\
 & \oplus \quad \ominus \\
\hline
 & \cancel{\alpha}-\cancel{4} \\
 & \alpha-4 \\
 & \ominus \quad \oplus \\
\hline
 & \times \\
\hline
\end{array}
$$

$(\cancel{\alpha-4})$

$$
\begin{aligned}
4\alpha^3-20\alpha^2+17\alpha-4 &= (\alpha-4)(4\alpha^2-4\alpha+1) \\
&= (\alpha-4)(4\alpha^2-(2+2)\alpha+1) \\
&= (\alpha-4)(4\alpha^2-2\alpha-(2\alpha-1)) \\
&= (\alpha-4)(2\alpha(2\alpha-1)-1(2\alpha-1)) \\
&= (\alpha-4)(2\alpha-1)(2\alpha-1) \\
&= (\alpha-4)(2\alpha-1)^2
\end{aligned}
$$

For zeroes, $\alpha-4=0$

$$\boxed{\alpha=4}$$

Or $2\alpha-1=0$

$$\boxed{\alpha=\frac{1}{2}}$$

Since, roots are real and distinct.

Therefore, the homogenous solution of a linear difference equation is given by.

$$\boxed{a_n^{(n)} = A_1\alpha_1^n + A_2\alpha_2^n + \dots\dots A_k\alpha_k^n}$$

Now, the solution is.

$$\boxed{a_n = A_1(4)^n + A_2\left(\frac{1}{2}\right)^n + A_3\left(\frac{1}{2}\right)^n}$$

Ques 4:- Write short notes on
(a) Predicates
(b) Quantifiers

Ans:-

PREDICATES

A predicate is an expression of one or more variables defined on some specific domain. A predicate with variables can be made a proposition by either assigning a value to the variable or by quantifying the variable.

- We had a problem before with the truth of "That guy is going to the store."
  - ⇒ The problem was that we couldn't decide if it was true or false, because the

sentence didn't specify ~~two~~ who "that guy" is.
Some are going to the store, and some are not.

- We also have similar things elsewhere in maths.
  - ⇒ We say things like "$x/2$ is an integer".
    That is true for some x but not others.
  - ⇒ It's not a proposition either.
- A predicate is a statement with variables.
  - ⇒ Written with a capital letter and the variables listed as arguments, like $P(x, y, z)$.
- We can express these two things using predicates.
  - ⇒ Let $P(x)$ be true if x is going to the store.
  - ⇒ Let $Q(x)$ be true if $x/2$ is an integer.

QUANTIFIERS

The variable of predicates is quantified by quantifiers. There are two types of quantifier in predicate logic - Universal Quantifier and Existential Quantifier.

Universal Quantifiers

Universal quantifier states that the statements within its scope are true for every value of the specific variable. It is denoted by the symbol $\forall$.

$\forall x\, P(x)$ is read as for every value of x, $P(x)$ is true.

<u>Example :-</u> "Man is mortal" can be transformed into the propositional form $\forall x\, P(x)$ where $P(x)$ is the predicate which denotes x is mortal and the universe of discourse is all men.

<u>Existential Quantifier</u>

Existential quantifier states that the statements within its scope are true for some values of the specific variable. It is denoted by the symbol $\exists$.

$\exists x\, P(x)$ is read as for some values of x, $P(x)$ is true.

<u>Example :-</u> "Some people are dishonest" can be transformed into the propositional form $\exists x\, P(x)$ where $P(x)$ is the predicate which denotes x is dishonest and the universe of discourse is some people.

SECTION - B

Long Answer Type Questions

Ques 1:- State and prove De-Morgan's laws and also shows that distributive laws holds over three sets.

Ans:- De Morgan's law

The complement of the union of two sets is equal to the intersection of their complements and the complement of the intersection of two sets is equal to the union of their complements. These are called De Morgan's laws.

For any two finite sets A and B;

(i) $(A \cup B)' = A' \cap B'$ (which is a De Morgan's law of union).

(ii) $(A \cap B)' = A' \cup B'$ (which is a De-Morgan's law of intersection).

Proof of De Morgan's law I:

$(A \cup B)' = A' \cap B'$

Let $P = (A \cup B)'$ and $Q = A' \cap B'$

Let x be an arbitrary element of P then $x \in P$

$\Rightarrow x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$
$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \in A'$ and $x \in B'$
$\Rightarrow x \in A' \cap B'$
$\Rightarrow x \in Q$
Therefore, $P \subset Q$ .... (i)
Again, let y be an arbitrary element of Q then
$y \in Q \Rightarrow y \in A' \cap B'$
$\Rightarrow y \in A'$ and $y \in B'$
$\Rightarrow y \notin A$ and $y \notin B$
$\Rightarrow y \notin (A \cup B)$
$\Rightarrow y \in (A \cup B)'$
$\Rightarrow y \in P$
Therefore, $Q \subset P$ ...... (ii)
Now combine (i) and (ii)
we get; $P = Q$ i.e. $(A \cup B)' = A' \cap B'$

<u>Proof of De Morgan's law</u> II :

$$(A \cap B)' = A' \cup B'$$

Let $M = (A \cap B)'$ and $N = A' \cup B'$
Let x be an arbitrary element of M then $x \in M$
$\Rightarrow x \in (A \cap B)'$
$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ or $x \notin B$
$\Rightarrow x \in A'$ or $x \in B'$
$\Rightarrow x \in A' \cup B'$
$\Rightarrow x \in N$
Therefore, $M \subset N$ .... (i)
Again, let y be an arbitrary element of N then
$y \in N$
$\Rightarrow y \in A' \cup B'$
$\Rightarrow y \in A'$ or $y \in B'$
$\Rightarrow y \notin A$ or $y \notin B$
$\Rightarrow y \notin (A \cap B)$
$\Rightarrow y \in (A \cap B)'$
$\Rightarrow y \in M$
Therefore, $N \subset M$ ..... (ii)
Now combine (i) and (ii)
we get; $M = N$ i.e. $(A \cap B)' = A' \cup B'$

<u>Distributive laws holds over three sets</u> :

<u>Distributive law</u>

- $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
- $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

Proof : 2

- $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

Let us assume that x be an arbitrary ~~constant~~ element of the set $(A \cup B) \cap C$

$x \in (A \cup B) \cap C$

$x \in A \cup B$ and $x \in C$

$x \in A$ or $x \in B$ and $x \in C$

$x \in A$ and $x \in C$ or $x \in B$ and $x \in C$

$x \in (A \cap C)$ or $x \in (B \cap C)$

$x \in (A \cap C) \cup (B \cap C)$

Hence proved

- $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

Let us assume that x be an arbitrary element of the set $(A \cap B) \cup C$.

$x \in (A \cap B) \cup C$

$x \in A \cap B$ or $x \in C$

$x \in A$ and $x \in B$ or $x \in C$

$[x \in A$ or $x \in C]$ and $[x \in B$ or $x \in C]$

$x \in (A \cup C)$ and $x \in (B \cup C)$

$x \in (A \cup C) \cap (B \cup C)$

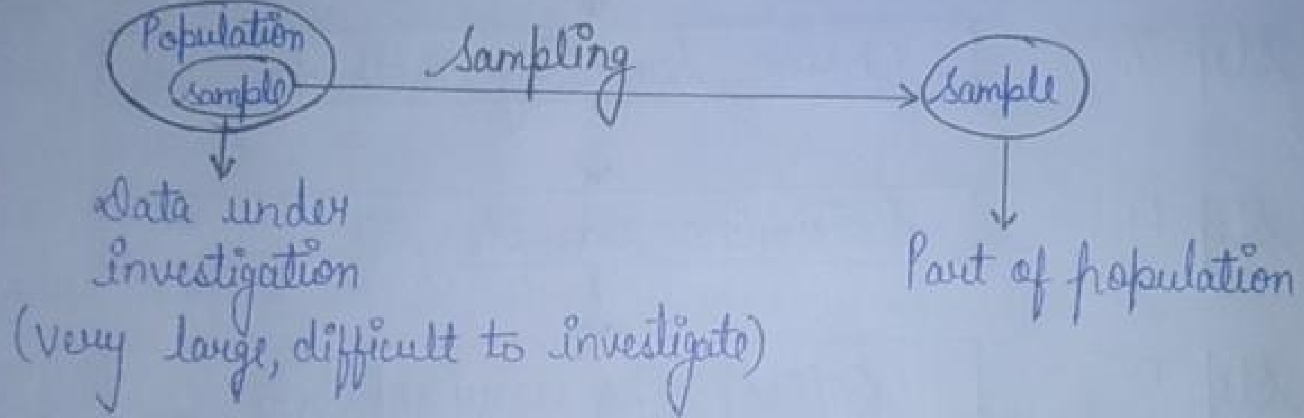
Hence proved.

Ques 2:- What is sampling? What are its types? Explain in detail.

Ans:-

SAMPLING

Sampling is a method that allows us to get information about the population based on the statistics from a subset of the population (sample), without having to investigate every individual.



Why do we need Sampling?

Sampling is done to draw conclusions about populations from samples, and it enables us to determine a population's characteristics by directly observing only a portion (or sample) of the population.

- Selecting a sample requires less time than selecting every item in a population.
- Sample selection is a cost-efficient method.

• Analysis of the sample is less cumbersome and more practical than an analysis of the entire population.

Steps involved in Sampling

Step 1 — Identify and define Target Population

↓

Step 2 — Select Sampling Frame

↓

Step 3 — Choose Sampling methods

↓

Step 4 — Determine Sample size

↓

Step 5 — Collect the required data

Different Types of Sampling Techniques

Sampling Methods

Probability Sampling	Non-Probability Sampling
1. Simple Random	1. Convenience
2. Systematic	2. Quota
3. Stratified	3. Judgement
4. Cluster	4. Snowball

Probability Sampling

In probability sampling, every element of the population has an equal chance of being selected. Probability sampling gives us the best chance to create a sample that is truly representative of the population.

Non-Probability Sampling

In non-probability sampling, all elements do not have an equal chance of being selected. Consequently, there is a significant risk of ending up with a non-representative sample which does not produce generalizable results.

For example, let's say our population consists of 20 individuals. Each individual is numbered from 1 to 20 and is represented by a specific color (red, blue, green or yellow). Each person would have odds of 1 out of 20 of being chosen in probability sampling.

With non probability sampling, these odds are not equal. A person might have a better chance of being chosen than others.

Types of Probability Sampling

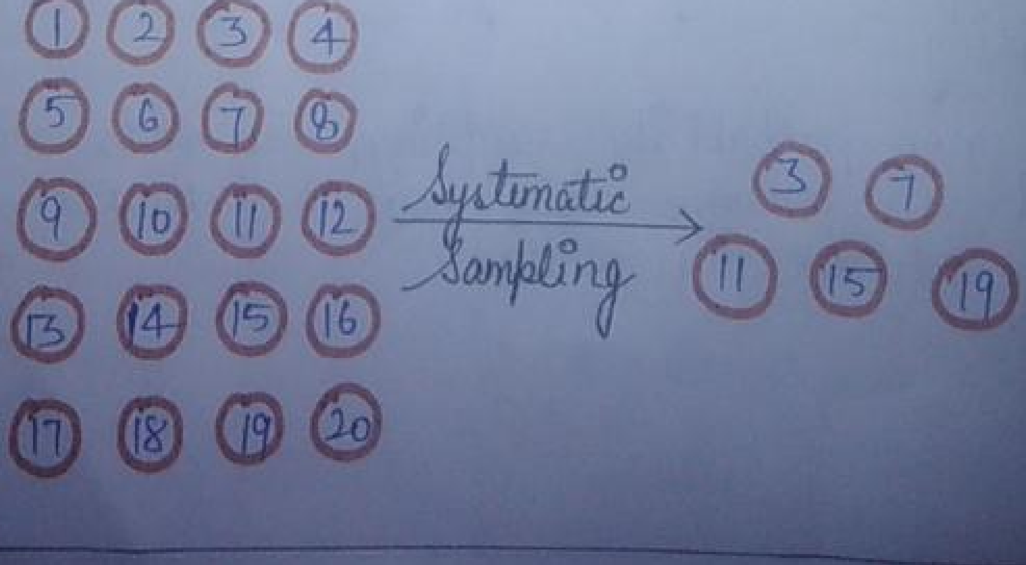
Simple Random Sampling

This is a type of sampling technique you must have come across at some point. Here, every individual is chosen entirely by chance and each member of the population has an equal chance of being selected.

Systematic Sampling

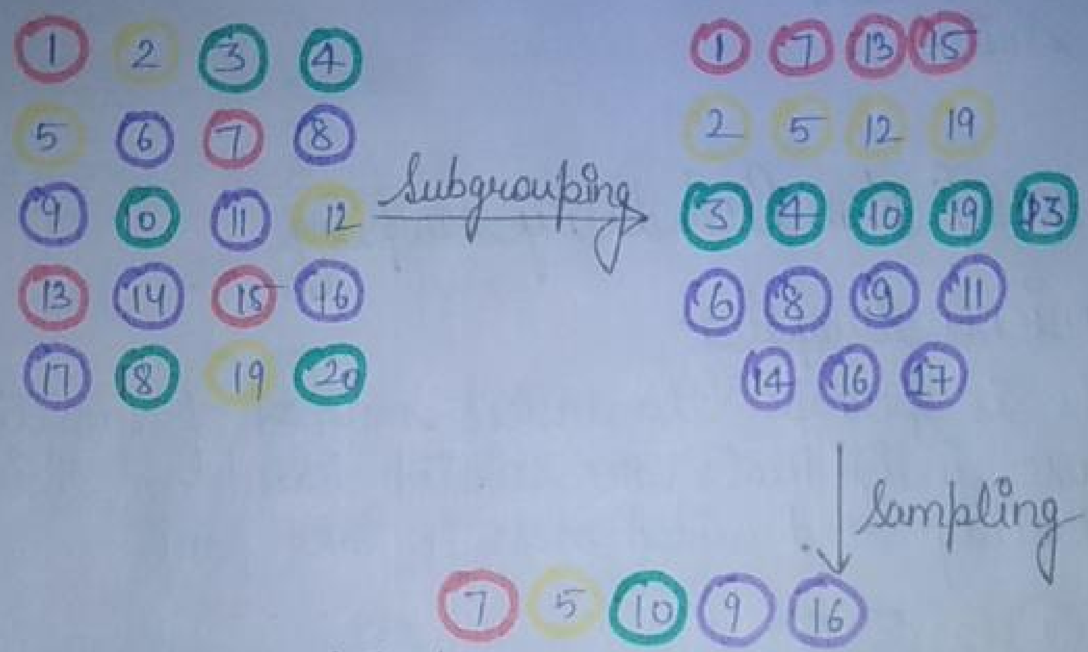
In this type of sampling, the first individual is selected randomly and others are selected using a fixed 'sampling interval'.

Say our population size is x and we have to select a sample size of n. Then, the next individual that we will select would be x/nth intervals away from the first individual. We can select the rest in the same way.



Stratified Sampling

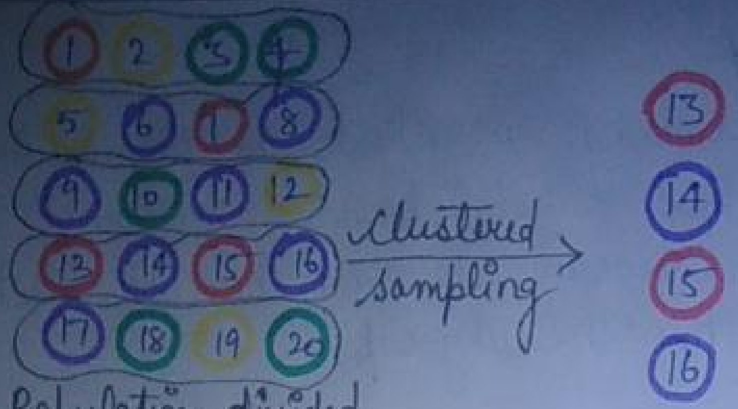
In this type of sampling, we divide the population into subgroups (called strata) based on different trials like gender, category, etc. And then we select the sample from these subgroups.



Here, we first divided our population into subgroups based on different colors of red, yellow, green and blue. Then, from each colour, we selected an individual in the proportion of their numbers in the population.

Cluster Sampling

In a clustered sample, we use the subgroups of the population as the sampling unit rather than individuals. The population is divided into subgroups, known as clusters, and a whole cluster is randomly selected to be included in the study :



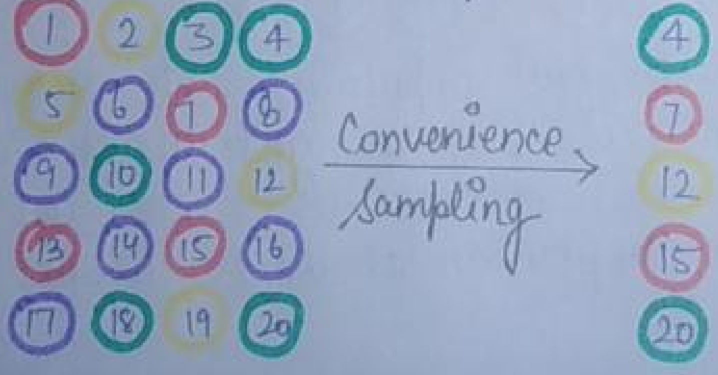
Population divided into clusters

A cluster is selected as our sample

Types of Non-Probability Sampling

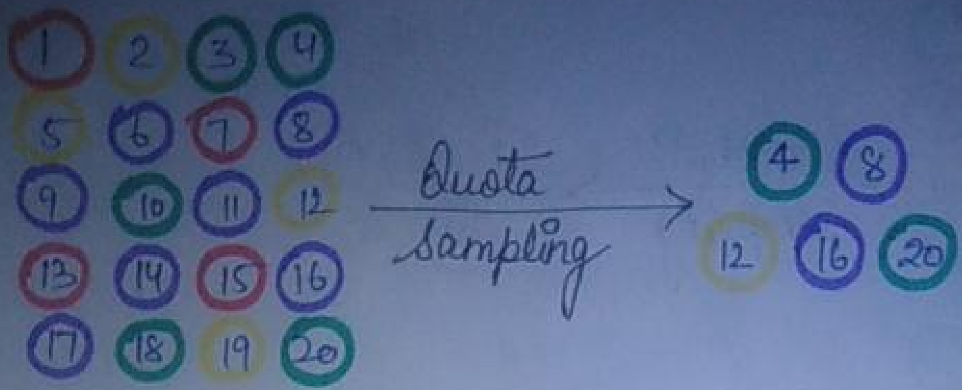
Convenience Sampling

This is perhaps the easiest method of sampling because individuals are selected based on their availability and willingness to take part.



Quota Sampling

In this type of sampling, we choose items based on predetermined characteristics of the population. Consider that we have to select individuals having a number in multiples of four for our sample:

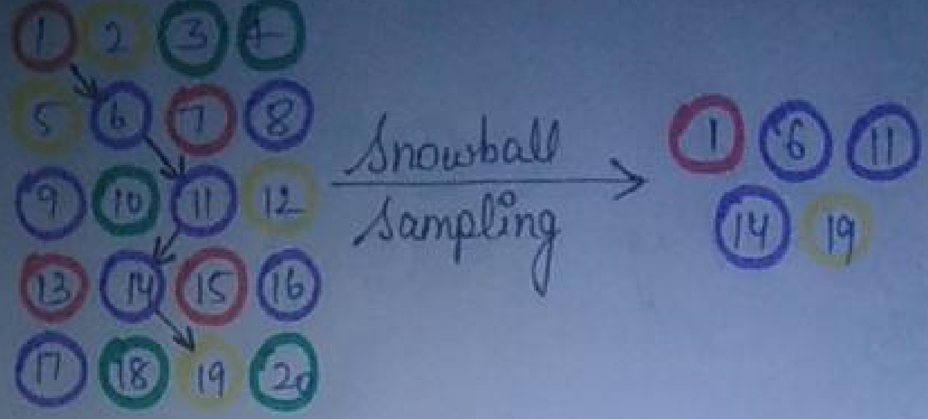


Judgement Sampling

It is also known as selective sampling. It depends on the judgment of the experts when choosing whom to ask to participate.

Snowball Sampling

Existing people are asked to nominate further people known to them so that the sample increases in size like a rolling snowball. this method of sampling is effective when a sampling frame is difficult to identify.



Here, we had randomly chosen person 1 for our sample, and then he/ she recommended person 6, and person 6 recommended person 11, and so on.

Ques 3:- State and proof Baye's Theorem.

Ans:- Baye's theorem

Baye's theorem describes the probability of occurence of an event related to any condition. It is also considered for the case of conditional probability. Bayes theorem is also known as the formula for the probability of "causes".

For example :- if we have to calculate the probability of taking a blue ball from the second bag out of three different bags of balls, where each bag contains three different colour balls viz. red, blue, black. In this case, the probability of occurence of an event is calculated depending on other conditions is known as conditional probability.

Bayes Theorem Statement

Let $E_1, E_2, \dots E_n$ be a set of events associated with a sample space S, where all the events $E_1, E_2, \dots E_n$ have non zero probability of occurrence and they form a partition of S. Let A be any event associated with S, then according to Bayes theorem,

$$P(E_i/A) = \frac{P(E_i)\,P(A|E_i)}{\sum_{k=1}^{n} P(E_k)\,P(A|E_k)}$$

for any $k = 1, 2, 3, \dots n$

Bayes Theorem Proof

According to the conditional probability formula,

$$P(E_i|A) = \frac{P(E_i \cap A)}{P(A)} \quad \text{---------- (1)}$$

Using the multiplication rule of probability,

$$P(E_i \cap A) = P(E_i)\,P(A|E_i) \quad \text{---------- (2)}$$

Using total probability theorem,

$$P(A) = \sum_{k=1}^{n} P(E_k)\,P(A|E_k) \quad \text{---------- (3)}$$

Putting the values from equations (2) and (3) in eq (1)

$$\boxed{P(E_i|A) = \frac{P(E_i)\,P(A|E_i)}{\sum_{k=1}^{n} P(E_k)\,P(A|E_k)}}$$

Besides statistics, the Baye's theorem is also used in various disciplines, with medicine and pharmacology as the most notable examples. In addition, the theorem is commonly employed in different fields of finance.

Formula for Bayes Theorem

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

where,

- $P(A|B)$ = the probability of event A occuring, given event B has occured.
- $P(B|A)$ = the probability of event B occuring, given event A has occured.
- $P(A)$ = the probability of event A.
- $P(B)$ = the probability of event B.