

Poisson Cluster Process: Bridging the Gap Between PPP and 3GPP HetNet Models

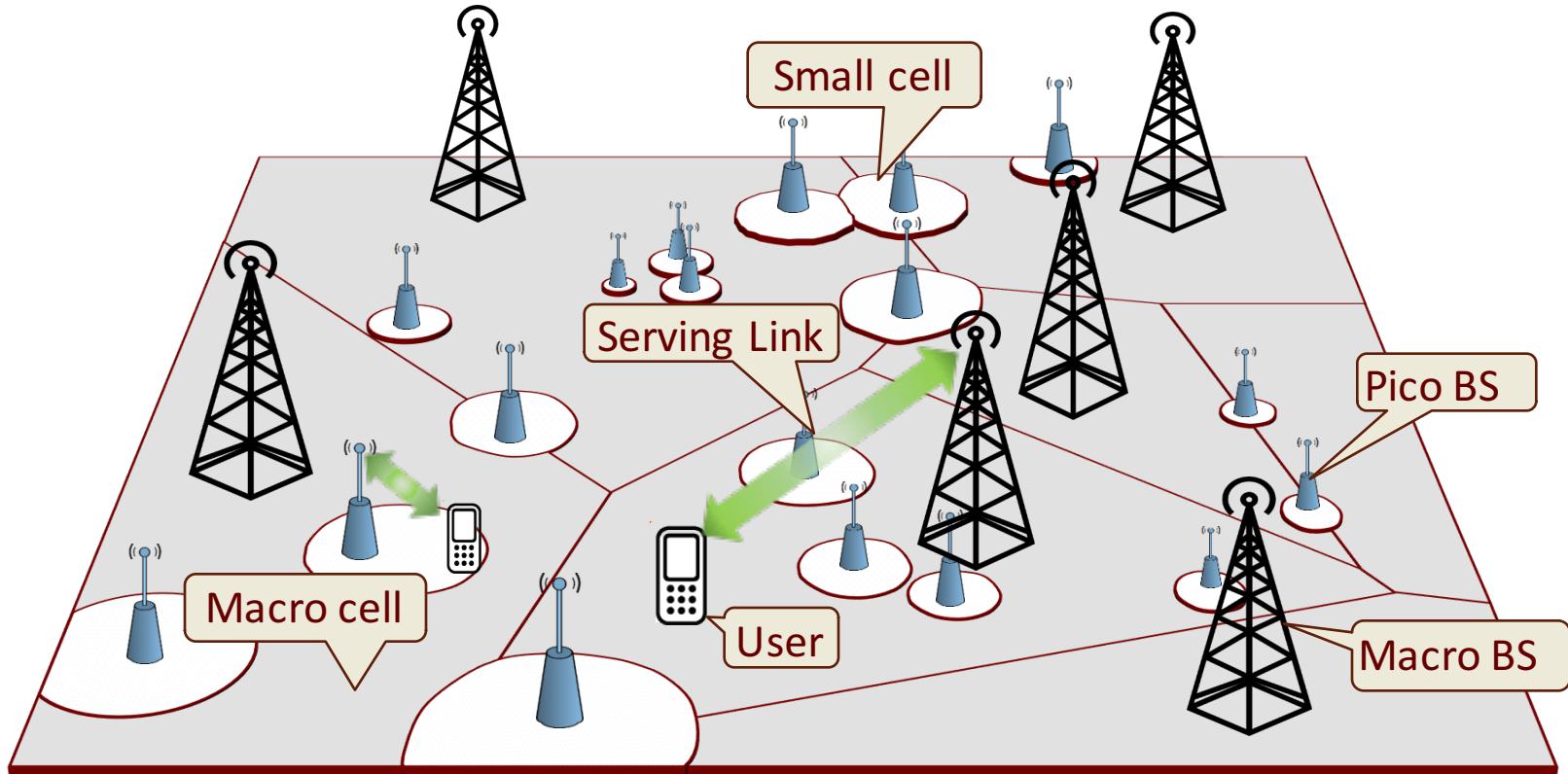
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Joint work with Chiranjib Saha and Mehrnaz Afshang

Wireless @ Virginia Tech

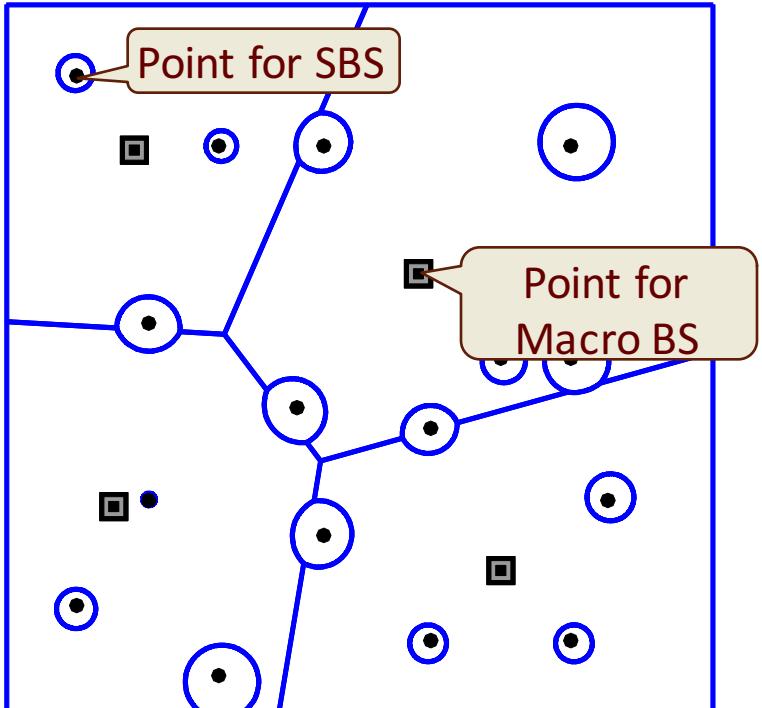
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HetNet: Different Components



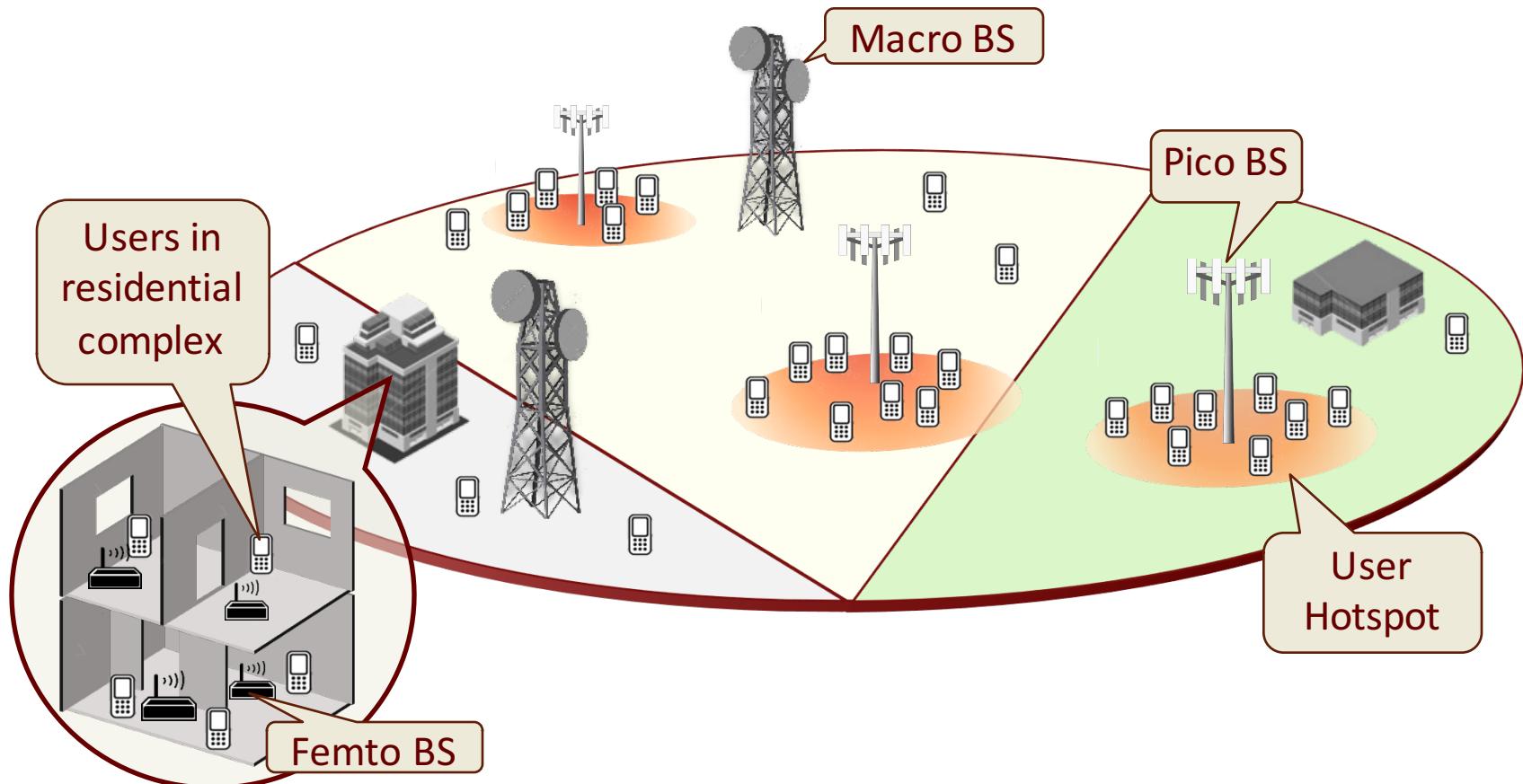
HetNet with different BSs and coverage regions

PPP Model of HetNets



- * BSs of different tiers (macro and small cells) and users are modeled as homogeneous Poisson point process (PPP) over \mathbb{R}^2 .
- * The statistics of SINR at the origin is computed.
- * Due to stationarity, this result is used to characterize coverage probability of a typical user of the network.

PPP Model: How far from actual HetNet?



A Simple Illustration of HetNet

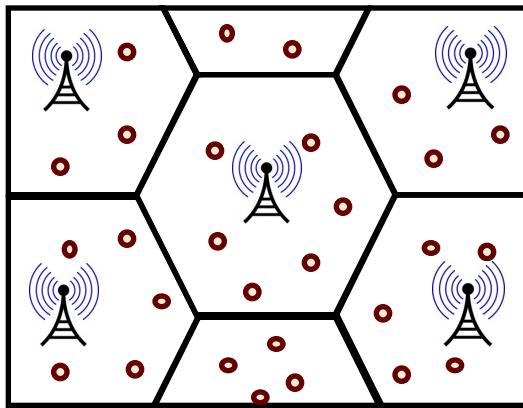
PPP Model: How far from actual HetNet?

- * Key components missing in the baseline PP model:
 - * Non-uniformity in user distribution:
 - * Modeling all users as an independent PPP is not realistic.
 - * Fraction of users form spatial clusters (*Hotspots*), e.g. users in public places and residential areas.
 - * Correlation between small cell BS (SBS) and user location:
 - * Operators deploy pico BSs at higher density at user hotspot.
 - * Users install their own femto BSs.
 - * Inter and Intra BS-tier Dependence: *
 - * BS locations are not necessarily independent.
 - * Site planning for deploying BSs introduces correlation in BS locations.

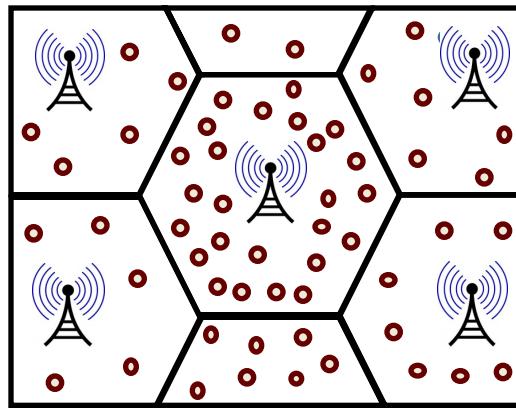
* N. Deng, W. Zhou, and M. Haenggi, "Heterogeneous cellular network models with dependence," IEEE JSAC, 2015.

3GPP Models: User Distribution

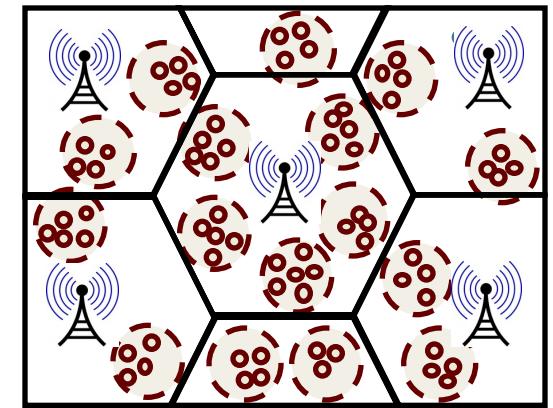
- * 3GPP considers different configurations of SBSs and users in HetNet simulation model.



Users “uniform” across
macro cells



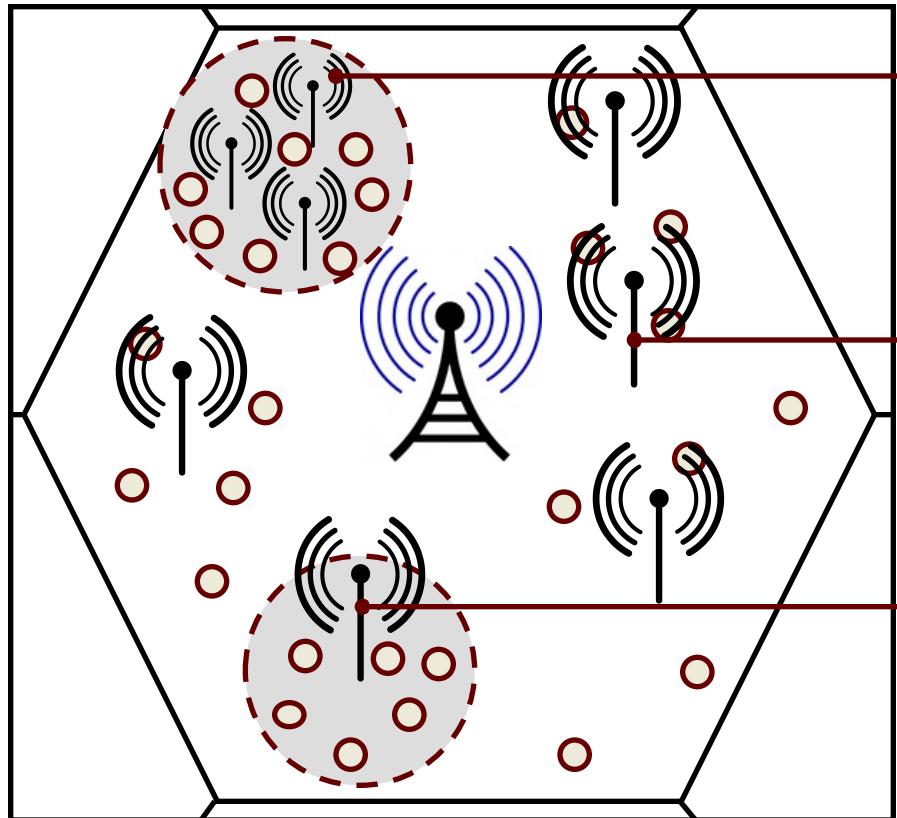
Users “non-uniform”
across macro cells



Users forming clusters
within a disc

User Configurations in 3GPP HetNet Model

3GPP Model: SBS Distribution

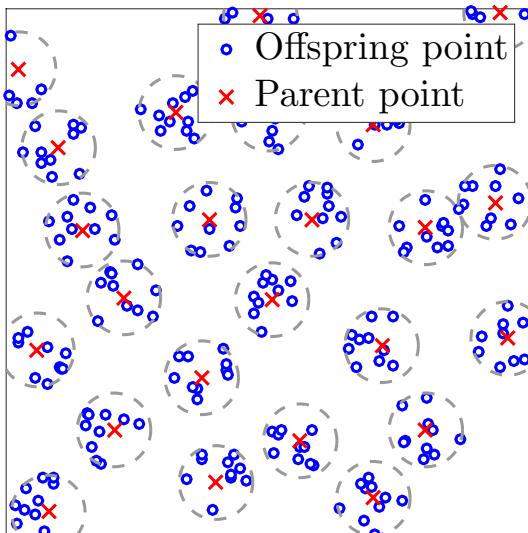


- SBSs deployed at higher density at certain areas (indoor models)
- SBSs deployed randomly or under some site planning.
- SBSs at the center of user hotspot.

SBS Configurations in 3GPP HetNet Model

How can PCP enhance HetNet Model?

- * **Poisson Cluster Process (PCP)** is more appropriate abstraction for user and BS distributions considered by 3GPP.



Matern Cluster Process:
An example of PCP

Definition: Poisson Cluster Process (PCP)

A PCP is generated from a PPP Φ_p called the **Parent PPP**, by replacing each point z_i by a finite offspring point process \mathcal{B}_i where each point is independently and identically distributed around origin.

$$\Phi = \bigcup_{z_i \in \Phi_p} z_i + \mathcal{B}_i.$$

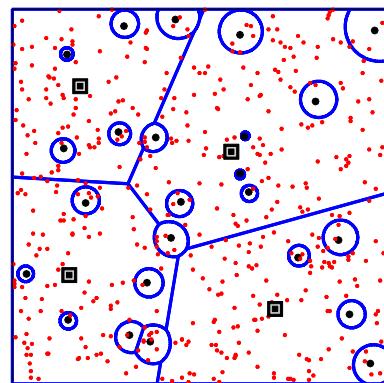
Definition: Matern Cluster Process

- $\#\mathcal{B}_i \sim \text{Poisson}(\bar{m})$
- Each point in \mathcal{B}_i is uniformly distributed inside disc of radius r_d centered at origin.

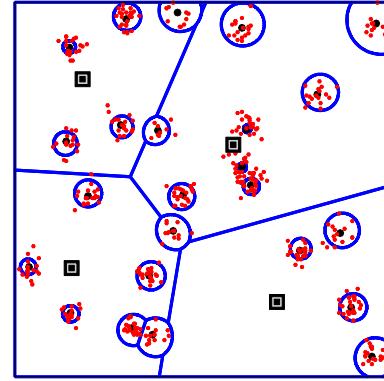
PCP Based HetNet Models

- * New HetNet models with combination of PPP and PCP have closer resemblance to 3GPP HetNet models.

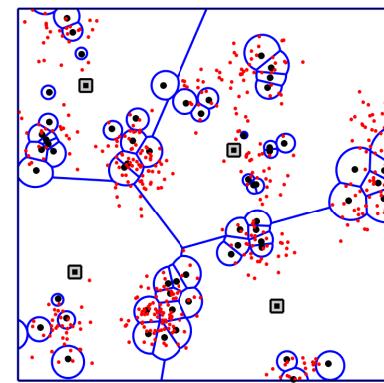
Model 1: Users PPP,
BSs PPP (Baseline)



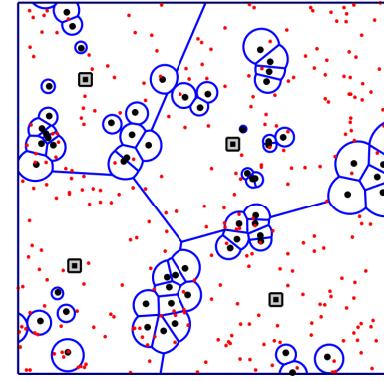
Model 2: Users PCP,
BSs PPP



Model 3: Users PCP,
BSs PCP

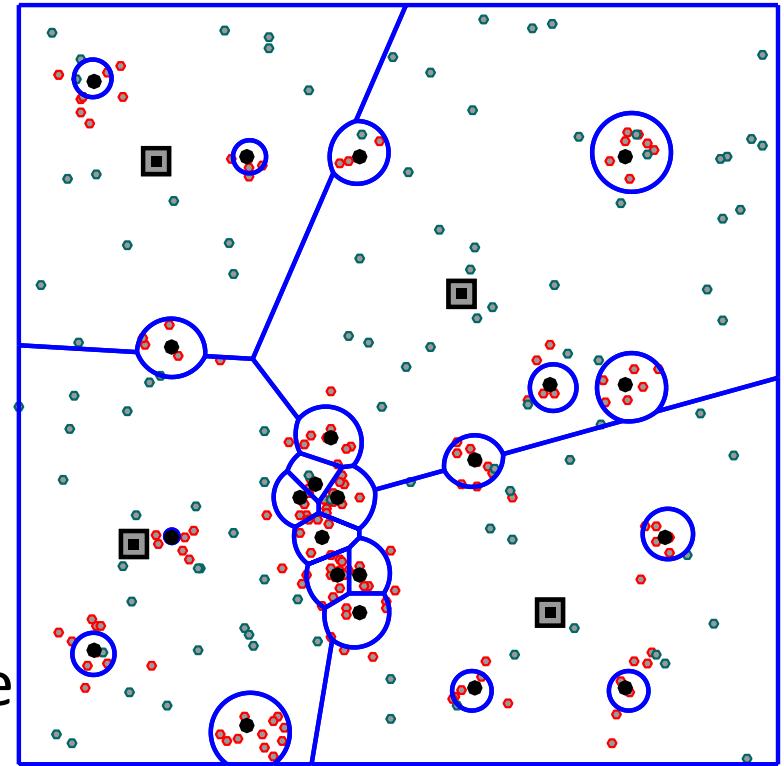


Model 4: Users PPP,
BSs PCP



Unified HetNet Model: PCP meets PPP

- * We consider K tier HetNet with two sets of BS PPs:
 - * K_1 BS tiers are modeled as independent PPPs.
 - * K_2 BS tiers are modeled as independent PCPs.
- * User Distribution:
 - * Users can be independent PPP.
 - * Users can be a PCP correlated with the BS distribution.
- * For this setup, we derive coverage probability.



Models 1-4 appear as special cases of this unified framework.

User Association: Max SIR and Max Power

- * Two types of cell association are considered.
 - * **Max Power:** Connect to the BS providing maximum average received power.
 - * Contact and Nearest distance distributions of PCP and PPP are the key components of analysis*.
 - * **Max SIR:** Connect to the BS providing maximum instantaneous SIR (assuming interference limited network).
 - * The analysis is well-known for the Baseline model **.
 - * We generalize this approach to enable coverage analysis for the proposed Unified Model.

* M. Afshang, C. Saha, and H. S. Dhillon, “Nearest-Neighbor and Contact Distance Distributions for Thomas Cluster Process”, IEEE Wireless Commun. Letters, 2016.

** H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, “Modeling and analysis of K-tier downlink heterogeneous cellular networks,” IEEE Journal on Sel. Areas in Commun., 2012.

Construction of 0th tier

- * If the users are PCP, the user PP is correlated with one BS PPP Φ_k .
- * Correlation can be handled by constructing the 0th tier (Φ_0).
 - * If users are clustered around Φ_k which is a PPP, Φ_0 is only the BS of Φ_k at cluster center (z) of the typical user.
 - * If users same parent PPP of Φ_k which is a PCP, Φ_0 is the set of BSs in Φ_k belonging to the same cluster center (z) of the typical user.

Coverage Probability

* Coverage probability, or equivalently CCDF of SIR , is:

$$P_c = \mathbb{P}(\text{SIR} \geq \beta) = \bigcup_{k \in \mathcal{K}} \mathbb{E} \left[\mathbf{1} \left(\sum_{\mathbf{x} \in \Phi_k} \frac{P_k h_{\mathbf{x}} \|\mathbf{x}\|^{-\alpha}}{\mathcal{I}(\Phi_k \setminus \{\mathbf{x}\}) + \sum_{j \in \mathcal{K} \setminus \{k\}} \mathcal{I}(\Phi_j)} > \beta \right) \right]$$

where $\mathcal{I}(\Phi_i) = \sum_{y \in \Phi_i} P_i h_y \|\mathbf{y}\|^{-\alpha}$ is the interference from all BSs in Φ_i .

* For $\beta > 1$, P_c can be expressed as sum of probability that serving BS belongs to $\Phi_k (k \in \mathcal{K})$.

$$P_c = \sum_{k \in \mathcal{K}} P_{c(k)} = \sum_{k \in \mathcal{K}} \mathbb{E} \left[\mathbf{1} \left(\sum_{\mathbf{x} \in \Phi_k} \frac{P_k h_{\mathbf{x}} \|\mathbf{x}\|^{-\alpha}}{\mathcal{I}(\Phi_k \setminus \{\mathbf{x}\}) + \sum_{j \in \mathcal{K} \setminus \{k\}} \mathcal{I}(\Phi_j)} > \beta \right) \right]$$

Coverage Probability

- * By Rayleigh fading assumption

$$\sum_{k \in \mathcal{K}} \mathbb{E} \left[\sum_{\mathbf{x} \in \Phi_k} \exp \left(-\frac{\beta}{P_k} (\mathcal{I}(\Phi_k \setminus \{\mathbf{x}\})) \right) \exp \left(-\frac{\beta}{P_k} \left(\sum_{j \in \mathcal{K} \setminus \{k\}} \mathcal{I}(\Phi_j) \right) \|\mathbf{x}\|^\alpha \right) \right]$$

- * Taking expectation with respect to fading

$$\sum_{k \in \mathcal{K}} \mathbb{E} \left[\sum_{\mathbf{x} \in \Phi_k} \prod_{\mathbf{y} \in \Phi_k \setminus \{\mathbf{x}\}} \frac{1}{1 + \frac{\beta}{P_k} \left(\frac{\|\mathbf{x}\|}{\|\mathbf{y}\|} \right)^{-\alpha}} \right] \mathbb{E} \left[\prod_{j \in \mathcal{K} \setminus \{k\}} \exp \left(-\frac{\beta}{P_k} (\mathcal{I}(\Phi_j)) \|\mathbf{x}\|^\alpha \right) \right]$$

$P_{c(k)}$: Per-tier Coverage

Coverage Probability

$$P_{c(k)} = \mathbb{E} \left[\sum_{\mathbf{x} \in \Phi_k} \prod_{\mathbf{y} \in \Phi_k \setminus \{\mathbf{x}\}} \frac{1}{1 + \frac{\beta}{P_k} \left(\frac{\|\mathbf{x}\|}{\|\mathbf{y}\|} \right)^{-\alpha}} \right] \prod_{j \in \mathcal{K} \setminus \{k\}} \mathbb{E} \left[\exp \left(-\frac{\beta}{P_k} \mathcal{I}(\Phi_j) \|\mathbf{x}\|^{\alpha} \right) \right]$$

This term belongs to a general family of functional over point processes called **sum-product functional**.

Can be expressed in terms of **probability generating functional (PGFL)** of Φ_j .

Sum-Product Functional

$$\mathbb{E} \left[\sum_{\mathbf{x} \in \Phi_k} \prod_{\mathbf{y} \in \Phi_k \setminus \{\mathbf{x}\}} \frac{1}{1 + \frac{\beta}{P_k} \left(\frac{\|\mathbf{x}\|}{\|\mathbf{y}\|} \right)^{-\alpha}} \right] \equiv \mathbb{E} \left[\sum_{\mathbf{x} \in \Psi} g(\mathbf{x}) \prod_{\mathbf{y} \in \Psi \setminus \{\mathbf{x}\}} v(\mathbf{x}, \mathbf{y}) \right],$$

Definition

We define Sum-product functional of a point process Ψ can be defined as:

$$\mathbb{E} \left[\sum_{\mathbf{x} \in \Psi} g(\mathbf{x}) \prod_{\mathbf{y} \in \Psi \setminus \{\mathbf{x}\}} v(\mathbf{x}, \mathbf{y}) \right],$$

where $g(\mathbf{x}) : \mathbb{R}^2 \mapsto [0, 1]$ and $v(\mathbf{x}, \mathbf{y}) : [\mathbb{R}^2 \times \mathbb{R}^2] \mapsto [0, 1]$ are measurable.

- * There are more general versions of Sum-product functional *.
- * There will be different expressions of the sum-product functional depending on Φ_k ($k \in \mathcal{K}$).

* U. Schilcher, S. Toumpis, M. Haenggi, A. Crismani, G. Brandner and C. Bettstetter, "Interference Functionals in Poisson Networks," in *IEEE Trans. Inf. Theory*, Jan. 2016.

Sum product functional

* We require sum-product functional of Φ_k when:

Case 1:

Φ_k is PPP

Case 2:

Φ_k is PCP

Case 3:

$\Phi_k = \Phi_0$

Campbell-Mecke Theorem can be applied to evaluate sum-product functional.

- * The third case occurs when the users are modeled as PCP having same parent PPP as that of some PCP distributed BS tier.
- * Has to be evaluated separately as Φ_0 is a **finite** PP.

Sum product functional: PPP

Lemma 1:

The sum product functional of Ψ when Ψ is a PPP can be expressed as follows:

$$\mathbb{E} \left[\sum_{\mathbf{x} \in \Psi} g(\mathbf{x}) \prod_{\mathbf{y} \in \Psi \setminus \{\mathbf{x}\}} v(\mathbf{x}, \mathbf{y}) \right] = \int_{\mathbb{R}^2} g(\mathbf{x}) G(v(\mathbf{x}, \mathbf{y})) \Lambda(d\mathbf{x}),$$

where $\Lambda(\cdot)$ is the mean measure of Ψ , $G(\cdot)$ denotes the PGFL of Ψ .

H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, “Modeling and analysis of K-tier downlink heterogeneous cellular networks,” IEEE JSAC, 2012.

Sum product functional: PCP

- * In PCP, selecting a point from \mathbf{x} implies selecting a tuple (\mathbf{x}, \mathbf{z}) , where \mathbf{z} is the cluster center of \mathbf{x} .

Lemma 2:

The sum product functional of Ψ when Ψ is a PCP can be expressed as follows:

$$\mathbb{E} \left[\sum_{\mathbf{x} \in \Psi} g(\mathbf{x}) \prod_{\mathbf{y} \in \Psi \setminus \{\mathbf{x}\}} v(\mathbf{x}, \mathbf{y}) \right] = \iint_{\mathbb{R}^2 \times \mathbb{R}^2} g(\mathbf{x}) \tilde{G}(v(\mathbf{x}, \mathbf{y}) | \mathbf{z}) \Lambda(d\mathbf{x}, d\mathbf{z})$$

with

$$\Lambda(d\mathbf{x}, d\mathbf{z}) = \lambda_{p_k} \bar{m}_k(\mathbf{x} - \mathbf{z}) d\mathbf{z} d\mathbf{x}$$

$\tilde{G}(\cdot)$ denotes PGFL of Ψ with respect to its reduced Palm distribution.

Sum product functional: A special Case

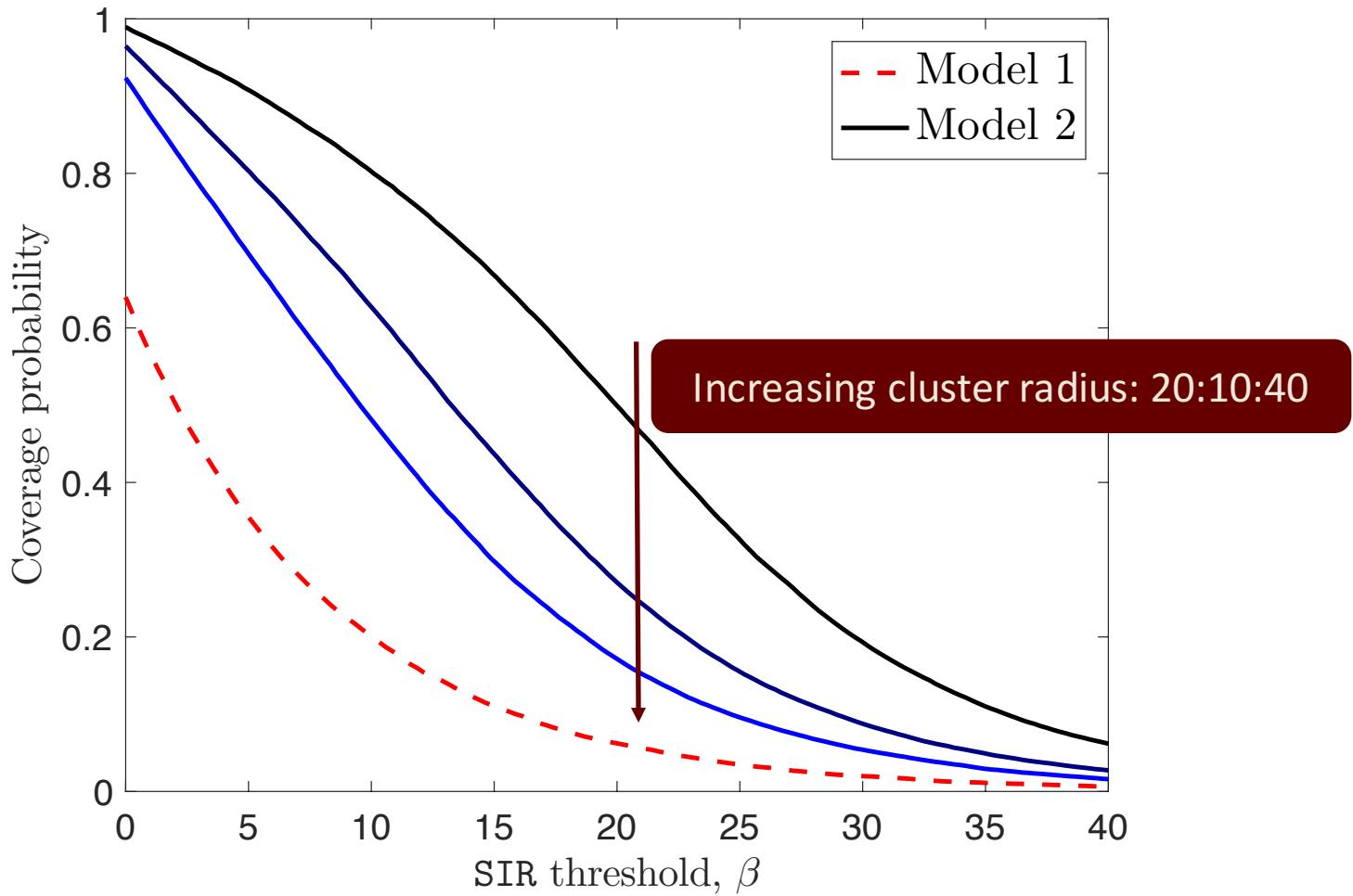
- * Conditioning on the number of offspring points is the key to the analysis of sum product functional of **finite** PP Φ_0 , i.e., BSs in Φ_k belonging to the same cluster center of the typical user.

Lemma 3:

The sum product functional of Ψ when Ψ is a cluster of a randomly chosen point of Φ_k ($k \in \mathcal{K}_2$) can be expressed as follows:

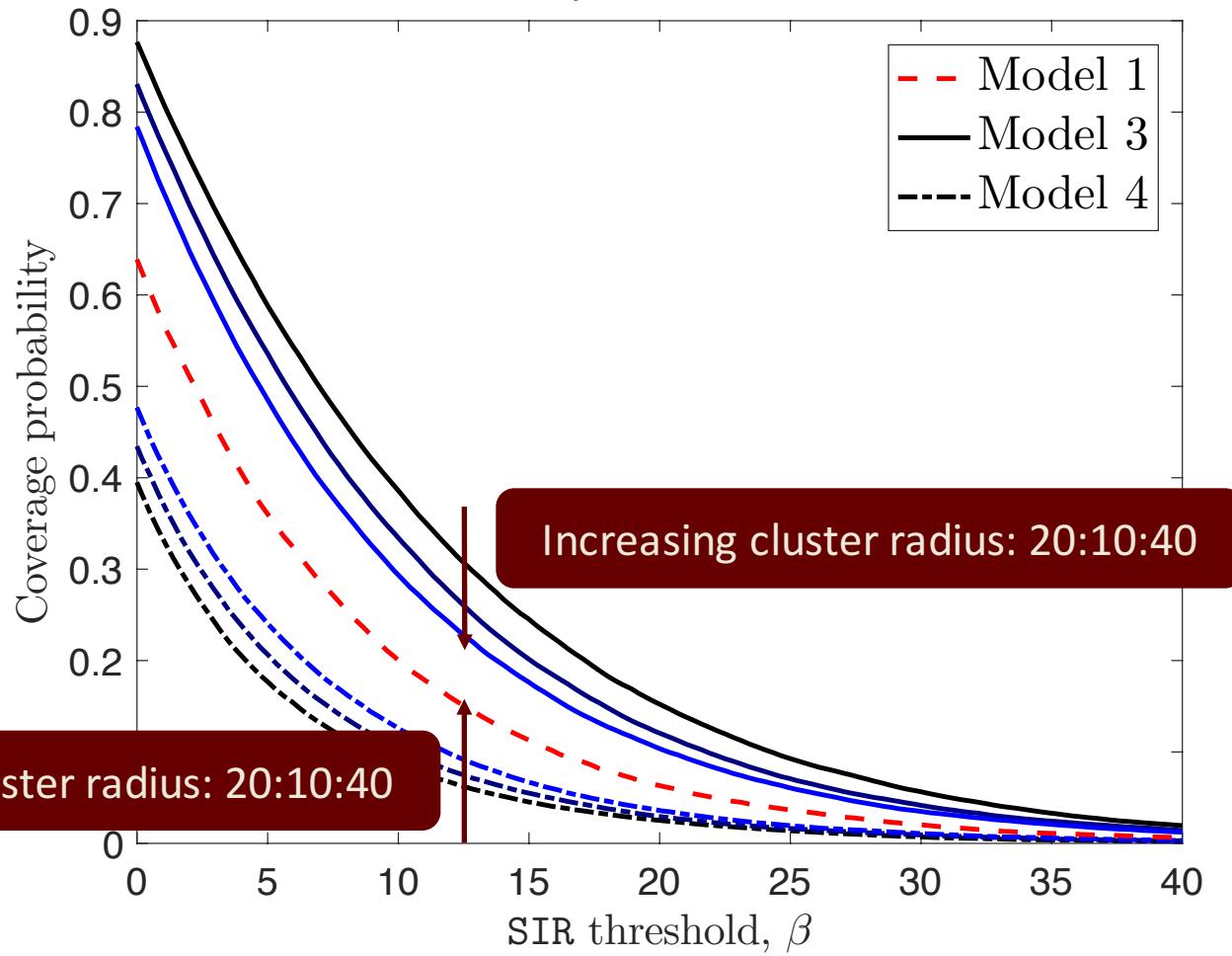
$$\begin{aligned} & \mathbb{E} \left[\sum_{\mathbf{x} \in \Psi} g(\mathbf{x}) \prod_{\mathbf{y} \in \Psi \setminus \{\mathbf{x}\}} v(\mathbf{x}, \mathbf{y}) \right] \\ &= \sum_{n=1}^{\infty} \int_{\mathbb{R}^2} g(\mathbf{x}) \left(\int_{\mathbb{R}^2} (v(\mathbf{x}, \mathbf{y}) f_k(\mathbf{y} - \mathbf{z}) d\mathbf{y} \right)^{n-1} f_k(\mathbf{x} - \mathbf{z}) d\mathbf{x} n^2 \frac{p_{k(n)}}{\bar{m}_k} \end{aligned}$$

Results: Model 1 Vs. Model 2



In Model 2, the coverage probability decreases as cluster radius increases and shifts towards that of Model 1.

Results: Models 1,3 and 4



Increasing cluster radius: 20:10:40

Increasing cluster radius: 20:10:40

Increasing cluster radius has a conflicting effect on coverage probability of Model 3 and 4: coverage probability of Model 4 increases whereas that of Model 3 decreases.

Conclusive Remarks

- * We propose a unified framework for modeling K -tier HetNet.
- * Derived coverage probability for this model under max-SIR connectivity.
 - * Sum-product functionals for PCP and finite point processes were derived as key components of analysis.
- * Special cases of the Model closely resemble BS and user configurations in 3GPP simulations of HetNet.
- * Future Directions:
 - * Rate analysis
 - * Incorporation of shadowing and general channel models
 - * Extension to uplink
 - * Developing unified framework for Max-power association
 - * Modeling spatial separation of BSs *

* M. Afshang and H. S. Dhillon, “Spatial modeling of device-to-device networks: Poisson cluster process meets Poisson hole process,” in Proc., IEEE Asilomar, Nov. 2015,

Relevant Publications

- * M. Afshang, H. S. Dhillon, “**Poisson Cluster Process Based Analysis of HetNets with Correlated User and Base Station Locations**”, submitted, Available online: arxiv.org/abs/1612.07285
- * C. Saha, M. Afshang, H. S. Dhillon, “**Enriched K-Tier HetNet Model to Enable the Analysis of User-Centric Small Cell Deployments**”, *IEEE Trans. on Wireless Commun.*, 2017.
- * M. Afshang, C. Saha, and H. S. Dhillon, “**Nearest-Neighbor and Contact Distance Distributions for Thomas Cluster Process**”, *IEEE Wireless Commun. Letters*, 2016.
- * M. Afshang, H. S. Dhillon, and P. H. J. Chong , “**Modeling and Analysis of Clustered Device-to-Device Networks**” *IEEE Trans. on Wireless Commun.*, 2016.
- * M. Afshang, H. S. Dhillon, and P. H. J. Chong, “**Fundamentals of Cluster-Centric Content Placement in Cache-Enabled Device-to-Device Networks**” *IEEE Trans. on Commun.*, 2016.

Thank You for Your Attention

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Coverage Probability

$$= \sum_{k \in \mathcal{K}} \mathbb{E} \left[\mathbf{1} \left(\sum_{\mathbf{x} \in \Phi_k} \frac{P_k h_{\mathbf{x}} \|\mathbf{x}\|^{-\alpha}}{\mathcal{I}(\Phi_k \setminus \{\mathbf{x}\}) + \sum_{j \in \mathcal{K} \setminus \{k\}} \mathcal{I}(\Phi_j)} > \beta \right) \right]$$

By $\beta > 1$ assumption

$$= \sum_{k \in \mathcal{K}} \mathbb{E} \left[\sum_{\mathbf{x} \in \Phi_k} \mathbb{P} \left(h_{\mathbf{x}} > \frac{\beta}{P_k} (\mathcal{I}(\Phi_k \setminus \{\mathbf{x}\}) + \sum_{j \in \mathcal{K} \setminus \{k\}} \mathcal{I}(\Phi_j)) \|\mathbf{x}\|^{\alpha} \right) \right]$$

$$= \sum_{k \in \mathcal{K}} \mathbb{E} \left[\sum_{\mathbf{x} \in \Phi_k} \exp \left(- \frac{\beta}{P_k} (\mathcal{I}(\Phi_k \setminus \{\mathbf{x}\})) \right) \right]$$

$$\times \exp \left(- \frac{\beta}{P_k} \left(\sum_{j \in \mathcal{K} \setminus \{k\}} \mathcal{I}(\Phi_j) \right) \|\mathbf{x}\|^{\alpha} \right)$$

By Rayleigh
assumption

$$= \sum_{k \in \mathcal{K}} \mathbb{E} \left[\sum_{\mathbf{x} \in \Phi_k} \prod_{\mathbf{y} \in \Phi_k \setminus \{\mathbf{x}\}} \frac{1}{1 + \frac{\beta}{P_k} \left(\frac{\|\mathbf{x}\|}{\|\mathbf{y}\|} \right)^{-\alpha}} \right] \mathbb{E} \left[\prod_{j \in \mathcal{K} \setminus \{k\}} \exp \left(- \frac{\beta}{P_k} (\mathcal{I}(\Phi_j)) \|\mathbf{x}\|^{\alpha} \right) \right]$$

Sum product functional is the key to the derivation of coverage

Sum product functional: A special Case

- * In order to model the correlation between user and BS, the sum product functional of the cluster within which user is located should be handled separately.

Lemma 3:

The sum product functional of Ψ when Ψ is a cluster of a randomly chosen point of Φ_k ($k \in \mathcal{K}_2$) can be expressed as follows:

$$\begin{aligned} & \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \exp \left(-\bar{m}_k \left(\int_{\mathbb{R}^2} \left(1 - v(\mathbf{x}, \mathbf{y}) \right) f_k(\mathbf{y} - \mathbf{z}) d\mathbf{y} \right) \right) \\ & \times \left(\bar{m}_k \int_{\mathbb{R}^2} v(\mathbf{x}, \mathbf{y}) f_k(\mathbf{y} - \mathbf{z}) d\mathbf{y} + 1 \right) f_k(\mathbf{x} - \mathbf{z}) f_k(\mathbf{z}) d\mathbf{x} d\mathbf{z}. \end{aligned}$$