Advances in Stochastic Geometry for Cellular Networks

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Overview of Research Contributions

- Thrust-I: 3GPP-inspired HetNet models
- Thrust-II: Mm-wave integrated access and backhaul network models
- Thrust-III: Machine learning meets stochastic geometry

Publications (1)

Book

[B1] H. S. Dhillon, **C. Saha**, and M. Afshang, *Poisson Cluster Processes: Theory and Application to Wireless Networks*, under preparation. Cambridge University Press, 2021.

Journals

[J10] **C. Saha**, M. Afshang, and H. S. Dhillon, "Meta Distribution of Downlink SIR in a Poisson Cluster Process-based HetNet Model", under revision, *IEEE Wireless Commun. Letters*. Available online: arxiv.org/abs/2007.05997.

[J9] **C. Saha**, H. S. Dhillon, "Load on the Typical Poisson Voronoi Cell with Clustered User Distribution", *IEEE Wireless Commun. Letters*, to appear.

[J8] **C. Saha**, H. S. Dhillon, "Millimeter Wave Integrated Access and Backhaul in 5G: Performance Analysis and Design Insights ", *IEEE Journal on Sel. Areas in Commun.*, vol. 37, no. 12, pp. 2669-2684, Dec. 2019.

[J7] **C. Saha**, H. S. Dhillon, N. Miyoshi, and J. G. Andrews, "Unified Analysis of HetNets using Poisson Cluster Process under Max-Power Association", *IEEE Trans. on Wireless Commun.*, vol. 18, no. 8, pp. 3797-3812, Aug. 2019.

Publications (2)

- [J6] **C. Saha**, M. Afshang, and H. S. Dhillon, "Bandwidth Partitioning and Downlink Analysis in Millimeter Wave Integrated Access and Backhaul for 5G", *IEEE Trans. Wireless Commun.*, vol. 17, no. 12, pp. 8195-8210, Dec. 2018.
- [J5] M. Afshang, **C. Saha**, and H. S. Dhillon, "Equi-Coverage Contours in Cellular Networks", *IEEE Wireless Commun. Letters*, vol. 7, no. 5, pp. 700-703, Oct. 2018.
- [J4] **C. Saha**, M. Afshang, and H. S. Dhillon, "3GPP-inspired HetNet Model using Poisson Cluster Process: Sum-product Functionals and Downlink Coverage", *IEEE Trans. on Commun.*, vol. 66, no. 5, pp. 2219-2234, May 2018.
- [J3] M. Afshang, **C. Saha**, and H. S. Dhillon, "Nearest-Neighbor and Contact Distance Distributions for Matérn Cluster Process", *IEEE Commun. Letters*, vol. 21, no. 12, pp. 2686-2689, Dec. 2017.
- [J2] **C. Saha**, M. Afshang, and H. S. Dhillon, "Enriched K-Tier HetNet Model to Enable the Analysis of User-Centric Small Cell Deployments", *IEEE Trans. on Wireless Commun.*, vol. 16, no. 3, pp. 1593-1608, Mar. 2017.
- [J1] M. Afshang, **C. Saha**, and H. S. Dhillon, "Nearest-Neighbor and Contact Distance Distributions for Thomas Cluster Process", *IEEE Wireless Commun. Letters*, vol. 6, no. 1, pp. 130-133, Feb. 2017.

Publications (3)

Conference Proceedings

[C8] S. Mulleti, **C. Saha**, H. S. Dhillon, Y. Elder, "A Fast-Learning Sparse Antenna Array", in Proc. IEEE RadarConf20, to appear.

[C7] **C. Saha** and H. S. Dhillon, "Interference Characterization in Wireless Networks: A Determinantal Learning Approach", in Proc. IEEE Int. Workshop in Machine Learning for Sig. Processing, Pittsburgh, PA, Oct. 2019.

[C6] **C. Saha** and H. S. Dhillon, "Machine Learning meets Stochastic Geometry: Determinantal Subset Selection for Wireless Networks", in Proc. IEEE Globecom, Waikoloa, HI, Dec. 2019.

[C5] **C. Saha** and H. S. Dhillon, "On Load Balancing in Millimeter Wave HetNets with Integrated Access and Backhaul", in Proc. IEEE Globecom, Waikoloa, HI, Dec. 2019.

[C4] **C. Saha**, M. Afshang and H. S. Dhillon, "Integrated mmWave Access and Backhaul in 5G: Bandwidth Partitioning and Downlink Analysis", in Proc. IEEE ICC, Kansas City, MO, May 2018.

Publications (4)

[C3] **C. Saha**, M. Afshang and H. S. Dhillon, "Poisson Cluster Process: Bridging the Gap Between PPP and 3GPP HetNet Models", in Proc. ITA, San Diego, CA, Feb. 2017.

[C2] **C. Saha** and H. S. Dhillon, "D2D Underlaid Cellular Networks with User Clusters: Load Balancing and Downlink Rate Analysis", in Proc. IEEE WCNC, San Francisco, CA, March 2017.

[C1] **C. Saha** and H. S. Dhillon, "Downlink Coverage Probability of K-Tier HetNets with General Non-Uniform User Distributions", in Proc. IEEE ICC, Kuala Lumpur, Malaysia, May 2016.

Other publications

[C10] K. Bhogi, **C. Saha**, H. S. Dhillon, "Learning on a Grassmann Manifold: CSI Quantization for Massive MIMO Systems", available online: arxiv.org/abs/2005.08413.

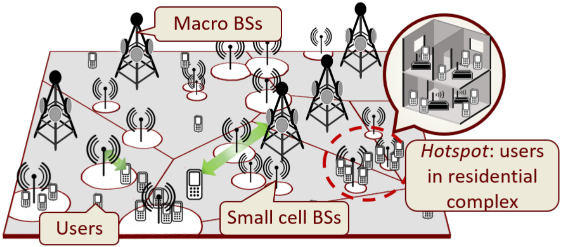
Section I

Introduction

Stochastic Geometry | Random Spatial Models of Cellular Networks | Poisson Point Process Models | Spatial Couplings

Modeling and Analysis of Cellular Networks

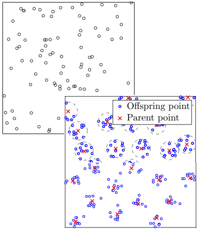
- A heterogeneous cellular network (HetNet) consists of a variety of base stations (BSs) such as macro BSs (MBSs) and small cell BSs (SBSs) and users (e.g. pedestrians, hotspots).
- Performance characterization:
  - Extensive system-level simulations
  - Simulation settings standardized by 3GPP



- Analytical performance characterization: Stochastic geometry
  - Assumes locations of BSs and users as realization of point processes.

Point Processes

A point process $\Phi = \{x_1, x_2, \dots\}$ is a random sequence of points in $\mathbb{R}^2$ (in general $\mathbb{R}^d$).



Example of point processes

Measure formalism. A point process is a random counting measure.

- If $B \subset \mathbb{R}^2$ is a Borel set, $\Phi(B) = \#$ of points of Φ in B is a random variable.
- More formally Φ is a measurable mapping from a probability space $(\Omega, \mathcal{A}, \mathbb{P})$ to the counting measure space.

How to specify the distribution of Φ?

- Moments: $\mathbb{E}\left[\left(\sum_{x\in\Phi\cap B} f(x)\right)^n\right]$
- Probability generating functional (PGFL): $G_\Phi[v] := \mathbb{E}\left[\prod_{x\in\Phi} v(x)\right]$
- Sum product functional (SPFL): $S_\Phi[g, v] := \mathbb{E}\left[\sum_{x\in\Phi} g(x) \prod_{y\in\Phi} v(y)\right]$

First Comprehensive Model (*PPP Model*)

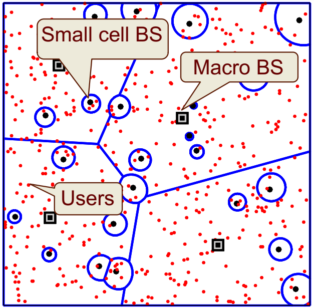
Poisson point process (PPP)

Φ is a PPP with intensity measure Λ if

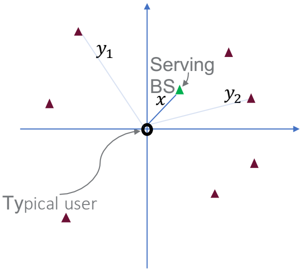
- $\Phi(B) \sim \texttt{Poisson}(\Lambda(B))$
- $\Phi(B_1), \Phi(B_2)$ are independent if B_1 and B_2 are disjoint

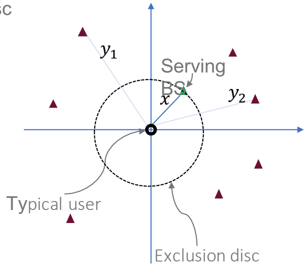
- When $\Lambda(B) = \lambda vol(B)$, Φ is a homogeneous PPP.
- Roughly speaking, PPP places points uniformly at random independently of each other.

- Model different types of BSs as independent Poisson Point Processes (PPPs).
- BS locations of the k-th tier are modeled as a homogeneous PPP Φ_k of density λ_k.
- Model users as a PPP ($\Phi_{\rm u}$) independent of the BSs.
- Assumes independence across everything.
- Very tractable.



Why is a PPP model tractable?





Interfering BSs outside the exclusion disc are still PPP.

$$\mathbb{P}(\mathtt{SINR} > \beta) \text{ where } \mathtt{SINR} = \frac{\overbrace{P h_{x^*} \|x^*\|^{-\alpha}}^{\text{signal power}}}{\underbrace{\mathtt{N}_0}_{\text{noise power}} + \underbrace{\sum_{x \in \Phi \setminus \{x^*\}} P h_x \|x\|^{-\alpha}}_{\text{interference}}}.$$

Assuming Rayleigh fading, *, $\mathrm{P}_{\mathrm{cPPP}} = \frac{\pi}{C(\alpha)} \frac{\sum_{i=1}^{K} \lambda_i P_i^{\frac{2}{\alpha}} \beta_i^{-\frac{2}{\alpha}}}{\sum_{i=1}^{K} \lambda_i P_i^{\frac{2}{\alpha}}}$.

*"**Modeling and analysis of K-tier downlink heterogeneous cellular networks,**" **IEEE JSAC, 2012.**

Current State-of-the-art

The original PPP-based model/approach has been enhanced in many ways [†].

PPP-based HetNet Model

Channel Models	Metrics	Technology and applications
Multi-slopes pathloss	SINR distribution	Cache-enabled BSs
Rayleigh, Nakagami	Area spectral efficiency	Energy harvesting
$\kappa-\mu$ shadowed fading	Rate distribution	Massive MIMO
mm-Wave	Local delay	Localization

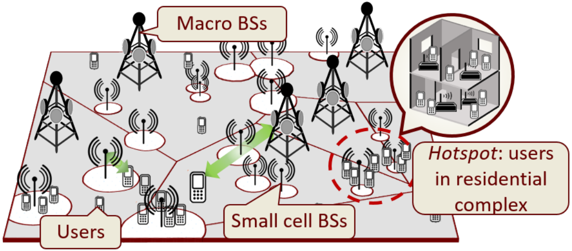
All these extensions are based on the assumption that the BSs and users are independent PPPs, which is not very realistic. See next slide.

[†] J. G. Andrews, A. K. Gupta and H. S. Dhillon, "A Primer on Cellular Network Analysis Using Stochastic Geometry". Available online: arxiv.org/abs/1604.03183.

PPP Model: How "accurate" it is?

Key features missing in this "baseline" PPP model:

- Non-uniformity in the user distribution
  - Modeling all users as an independent PPP is not realistic.
  - Fraction of users form spatial clusters (Hotspots), e.g. users in public places and residential areas.



- Correlation between small cell BS (SBS) and user locations
  - Operators deploy SBSs (e.g., picocells) at the areas of high user density.
- Inter and intra BS-tier dependence:
  - BS locations are not necessarily independent.
  - Site planning for deploying BSs introduces correlation in BS locations.

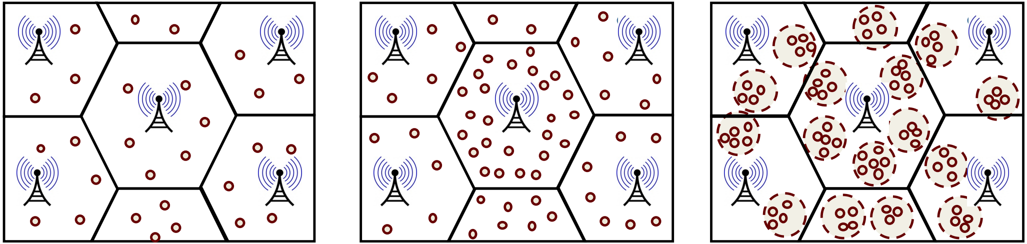
Section II

3GPP Models of HetNets

Existing standards for spatial configurations of users and base stations in network simulation

3GPP Models: User Distributions

3GPP considers different configurations of SBSs and users in HetNet simulation models [‡] .



Users "uniform" across macro cells

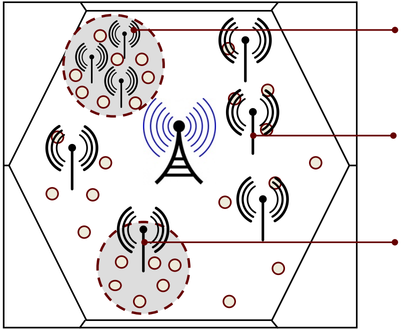
Users "non-uniform" across macro cells

Users forming clusters within a disc

User configurations usually considered in 3GPP HetNet models.

[‡] 3GPP TR 36.932 V13.0.0 , "3rd generation partnership project; technical specification group radio access network; scenarios and requirements for small cell enhancements for E-UTRA and E-UTRAN (release 13)," Tech. Rep., Dec. 2015.

3GPP Model: SBS Distribution



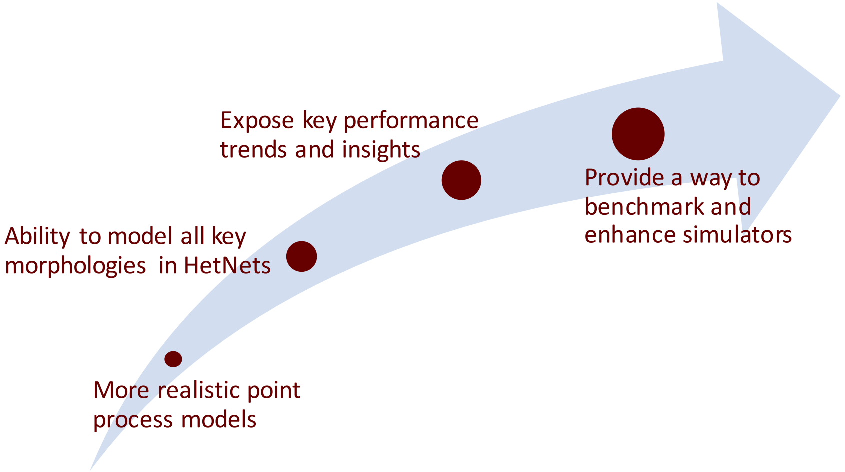
SBSs deployed at higher density in certain areas (e.g., indoor models)

SBSs deployed randomly or under some site planning.

SBSs deployed at the centers of user hotspots.

SBS Configurations in 3GPP HetNet Model.

Thinking beyond Homogeneous PPP



Is there something that is almost as tractable as a PPP but could model these other morphologies accurately?

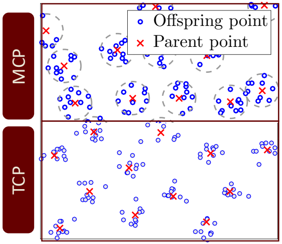
The Answer is Poisson Cluster Process

Poisson Cluster Process (PCP) is more appropriate abstraction for user and BS distributions considered by 3GPP.

Definition (PCP)

A PCP is generated from a PPP $\Phi_{\rm p}$ called the **Parent PPP**, by replacing each point $\mathbf{z}_i$ by a finite offspring point process $\mathcal{B}_i$ where each point is independently and identically distributed around origin.

$$\Phi = \bigcup_{\mathbf{z}_i \in \Phi_{\rm p}} \mathbf{z}_i + \mathcal{B}_i.$$



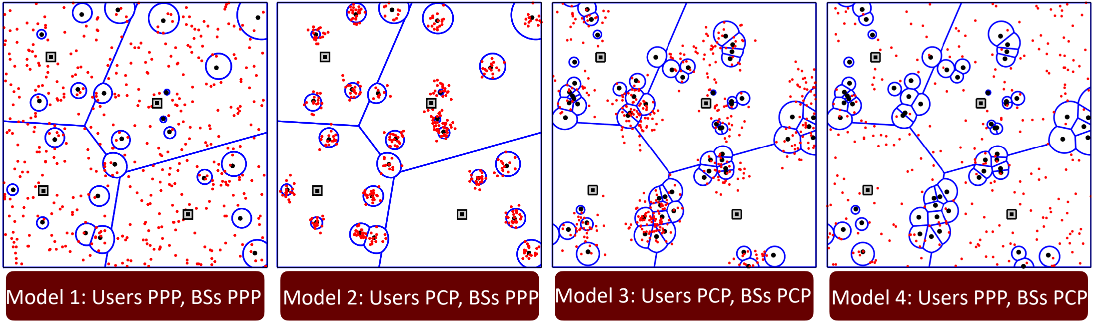
Examples of PCPs

Special Cases

- $\mathcal{B}_i(\mathbb{R}^2) \sim \texttt{Poisson}(\bar{m})$.
- **Matérn Cluster Process**: Each point in $\mathcal{B}_i$ is uniformly distributed inside disc of radius $\texttt{R}$ centered at origin.
- **Thomas Cluster Process**: Each point in $\mathcal{B}_i$ is normally distributed (with variance σ^2) around the origin.

PCP-based Canonical HetNet Models

New HetNet models generated by combining PCPs with PPPs have closer resemblance with 3GPP HetNet models§.



§Model 1: [Dhillon2012]
Model 2: [Saha2017] Saha *et. al.* "Enriched K-tier HetNet Model to Enable the Analysis of User-Centric Small Cell Deployments", *IEEE TWC*, 2017.
Models 3,4: [Saha2019] Saha *et. al.*,"Unified Analysis of HetNets using Poisson Cluster Process under Max-Power Association", *IEEE TWC*, 2019.
Unified: [Saha2018] Saha *et. al.*, ., "3GPP-inspired HetNet Model using Poisson Cluster Process: Sum-product Functionals and Downlink Coverage", *IEEE TCOMM*, 2018.

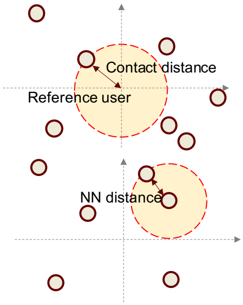
Section III

PCP-based General HetNet Models

PPP and PCP as BS and user distributions | Coverage Analysis

PCP Fundamentals: Distance Distributions

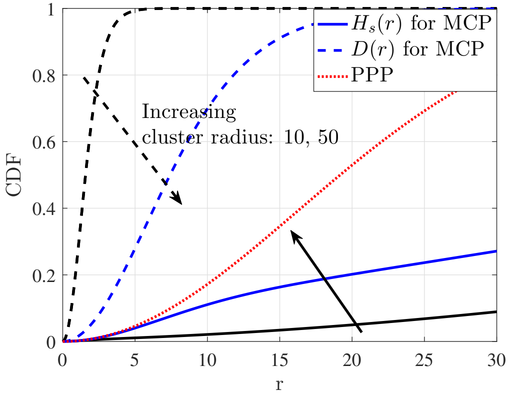
- The first step in the analysis under max-power association is to find the distribution of the distance from the typical user to the BS providing the maximum received power.
- Mathematically, this corresponds to determining nearest-neighbor and/or contact distance distribution of the point process.



- For PPP, these distributions are the same and are $\mathrm{Rayleigh}((2\pi\lambda)^{-1})$.
- For PCP, these distributions were not well-studied. We have characterized these distributions for TCP and MCP¶.

¶ Afshang, Saha, Dhillon, "Nearest-Neighbor and Contact Distance Distributions for Thomas Cluster Process", *IEEE Wireless Commun. Letters*, 2017.
Afshang, Saha, Dhillon, "Nearest-Neighbor and Contact Distance Distributions for Matérn Cluster Process", *IEEE Commun. Letters*, 2017.

Contact and Nearest Distance Distributions of MCP

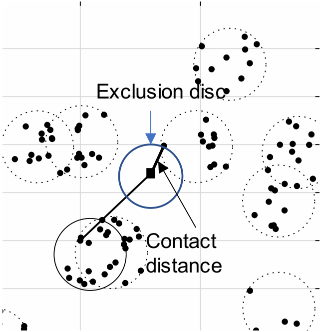


CDF of contact and nearest distance distribution of MCP in $\mathbb{R}^2$. For the MCP, $\lambda_{\mathrm{p}} = 20 \times 10^{-6}, \bar{m} = 30$.

The nearest-neighbor and contact distance distributions of PCP significantly differ from that of PPP.

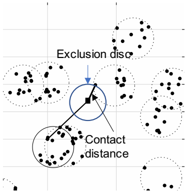
Coverage Analysis when BSs are PCP: A Toy Example

Consider a single tier network where BSs are distributed as PCP. The typical user is located at the origin (no user BS coupling).



We assume the small scale fading on each link is i.i.d. Rayleigh, i.e. $h_x \overset{\text{i.i.d.}}{\sim} \exp(1)$.

Coverage Analysis when BSs are PCP: Challenges



Assuming interference-limited network,

$$\mathtt{P_c} = \mathbb{P}\left(\frac{h_{x^*}\overbrace{\|x^*\|^{-\alpha}}^{\text{contact distance}, R}}{\sum_{x\in\Phi\setminus\{x^*\}} h_x\|x\|^{-\alpha}} > \beta\right)$$

$$= \mathbb{E}\left[\prod_{x\in\Phi\cap b(0,R)} \frac{1}{1+\beta\frac{\|x\|^{\alpha}}{R^{\alpha}}}\right]$$

Challenge: Conditioned on R, the PCP outside the disc $b(0,R)$ is not a PCP.
Solution: Conditioned on the locations of the points of the parent PPP $\Phi_{\rm p}$, a PCP $\Phi(\lambda_{\rm p}, \bar{m}, f)$ is a PPP in $\mathbb{R}^d$ with intensity function $\lambda(x|\Phi_{\rm p}) = \bar{m}\lambda_{\rm p}\sum_{v\in\Phi_{\rm p}} f(x-v)$.

- First, condition on the parent PPP of BS PCP, derive conditional coverage: $\mathtt{P_c}|\Phi_{\rm p}$.
- Finally, decondition $\mathtt{P_c}|\Phi_{\rm p}$ w.r.t. the distribution of $\Phi_{\rm p}$.

It is important that $\mathtt{P_c}|\Phi_{\rm p}$ is a standard functional of $\Phi_{\rm p}$ (such as PGFL, SPFL etc).

New Spatial Models of Cellular Networks: PCP meets PPP

The general K-tier HetNet model:

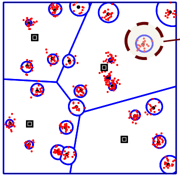
- Two sets of BS PPs:
  - K_1 BS tiers are modeled as independent PPPs. The index set: $\mathcal{K}_1$.
  - K_2 BS tiers are modeled as independent PCPs. The index set: $\mathcal{K}_2$.
- User Distribution:
  - Type1: Users can be independent PPP.
  - The BSs of the q-th tier (Φ_q) are coupled with the user locations.
    - Type2: Users are PCP with BSs at the cluster center ($q \in \mathcal{K}_1$)
    - Type3: Users and BSs are PCP with same ($q \in \mathcal{K}_2$)

Models 1-4 from the previous slide are special cases of this *general* model.

How to handle the Spatial Coupling?

The typical user, under spatial coupling will observe the Palm version of $\Phi^{(q)}$.

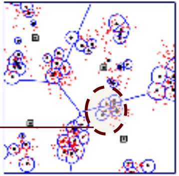
Isolate the spatial coupling by define a virtual 0-th tier Φ_0.



Type 2 users

If users form a PCP around BS locations, the 0th tier is a BS at the cluster center of the typical user.

If both users and BSs are modeled as (coupled) PCPs, the 0th tier is the set of BSs belonging to the same cluster center of the typical user.



Type 3 users

$$\Phi^{(0)} := \begin{cases} \emptyset; & \text{for Type 1 users,} \\ \{z_o\}; & \text{for Type 2 users,} \\ z_o + \mathcal{B}^{(q)}; & \text{for Type 3 users.} \end{cases}$$

Max Average Received Power based Association

Cell Association: if x^* is the location of the BS serving the typical user,

$$x^* = \arg\max_{\{\widetilde{x}_k, k \in \mathcal{K}\}} P_k \|\widetilde{x}_k\|^{-\alpha},$$

where $\widetilde{x}_k = \arg\max_{x \in \Phi^{(k)}} P_k \|x_k\|^{-\alpha} = \arg\min_{\{x \in \Phi^{(k)}\}} \|x\|$ is the location of candidate serving BS in $\Phi^{(k)}$.

Next Steps

- Construct Φ_0 to handle user BS coupling.
- If $\mathcal{S}_k = \mathbf{1}(x^* \in \Phi^{(k)})$ denotes k-th tier association event,

$$\mathtt{P_c} = \mathbb{P}\left(\bigcup_{k \in \mathcal{K}_1 \cup \mathcal{K}_2 \cup \{0\}} \{\mathtt{SIR}(x^*) > \beta_k, \mathcal{S}_k\} \right) = \sum_{k \in \mathcal{K}_1 \cup \mathcal{K}_2 \cup \{0\}} \mathbb{P}\left(\mathtt{SIR}(x^*) > \beta_k, \mathcal{S}_k \right),$$

Coverage Analysis: Max Avg. Power

- Conditioning step: Condition on the locations of all the parent points of $\Phi^{(i)}, \forall\ i \in \mathcal{K}_2$. Derive conditional coverage probability:

$$\mathtt{P_c}^{\mathrm{cond}}\left(\cup_{i\in\mathcal{K}_2'}\Phi_{\mathrm{p}}^{(i)}\right) := \sum_{k\in\mathcal{K}_1\cup\mathcal{K}_2\cup\{0\}} \underbrace{\mathbb{P}\left(\{\mathtt{SIR}(x^*) > \beta_k, \mathcal{S}_k\}\middle|\Phi_{\mathrm{p}}^{(i)}, \forall i \in \mathcal{K}_2'\right)}_{\text{Per-tier coverage}, \mathtt{P_c}_k}.$$

- Deconditioning step: Decondition the conditional coverage w.r.t. the distributions of $\Phi_{\mathrm{p}}^{(i)},\ \forall\ i \in \mathcal{K}_2$. Thus, $\mathtt{P_c} = \mathbb{E}\left[\mathtt{P_c}^{\mathrm{cond}}\left(\cup_{i\in\mathcal{K}_2'}\Phi_{\mathrm{p}}^{(i)}\right)\right]$.

Coverage Analysis: Max Avg. Power

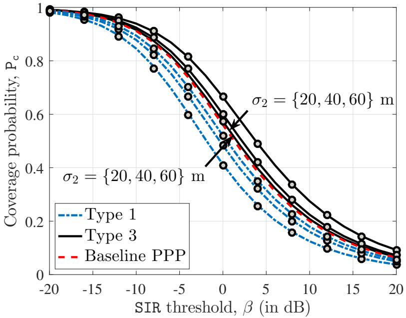
We are able to express coverage as a product of PGFLs and SPFLs of the BS PPPs (for $k \in \mathcal{K}_1$) or parent PPPs of BS PCPs (for $k \in \mathcal{K}_2$).

One representative result for Type 1 and 3 users, when $k \in \mathcal{K}_2 \cup \{0\}$,

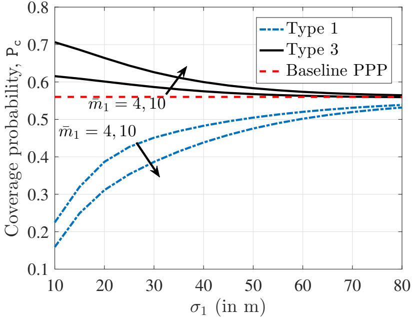
$$\mathrm{P}_{\mathrm{c}k} = \bar{m}_k \int_0^\infty \overbrace{\prod_{j_1 \in \mathcal{K}_1} \mathbb{E}\Big[\exp\Big(-\pi r^2 \lambda_{j_1} \bar{P}^2_{j_1,k} \rho(\beta_k, \alpha)\Big]}^{\text{PGFL of } \Phi^{(j_1)}} \prod_{j_2 \in \mathcal{K}_2' \setminus \{k\}} \overbrace{\mathbb{E}\Big[\prod_{z \in \Phi_{\mathrm{p}}^{(j_2)}} \mathcal{C}_{j_2,k}(r, \|z\|)\Big]}^{\text{PGFL of } \Phi^{(j_2)}}$$
$$\times \underbrace{\mathbb{E}\Big[\Big(\sum_{z \in \Phi_{\mathrm{p}}^{(k)}} f_{\mathrm{d}_k}(r \| \|z\|) \prod_{z \in \Phi_{\mathrm{p}}^{(k)}} \mathcal{C}_{k,k}(r, \|z\|)\Big)\Big]}_{\text{SPFL of } \Phi^{(k)}} \mathrm{d}r.$$

See [Saha2019] for the definitions of $\rho(\cdot)$ and $\mathcal{C}_{ij}(\cdot, \cdot)$.

Results for Type 1 and 3 Users



$\mathtt{P_c}$ vs. $\mathtt{SIR}$ threshold ($\alpha = 4,\ P_2 = 10^3 P_1$, $\lambda_1 = 1$ km^{-2}, $\lambda_{\rm p2} = 25$ km^{-2}, and $\bar{m}_2 = 4$).



$\mathtt{P_c}$ vs. σ_2 ($\alpha = 4,\ P_1 = 10^3 P_2,\ \lambda_1 = 1$ km^{-2}, $\lambda_{\rm p2} = 25$ km^{-2}, and $\beta = 0$ dB).

Add some insights

Convergence to the baseline PPP Model

Proposition 1 (Weak Convergence of PCP to PPP)

If $\Phi^{(k)}$ is a PCP with parameters $(\lambda_{\mathrm{p}_k}, f_{k,\xi}, \bar{m}_k)$ then

$$\Phi_k \to \bar{\Phi}_k \ (weakly) \text{ as } \xi \to \infty,$$

where $\bar{\Phi}_k$ is a PPP of intensity $\bar{m}_k \lambda_{\mathrm{p}_k}$ if $\sup(f_k) < \infty$.

Proposition 2

The limiting PPP $\bar{\Phi}^{(k)}$ and the parent PPP $\Phi_{\mathrm{p}}^{(k)}$ of $\Phi^{(k)}$ $(k \in \mathcal{K}_2)$ are independent, i.e.,

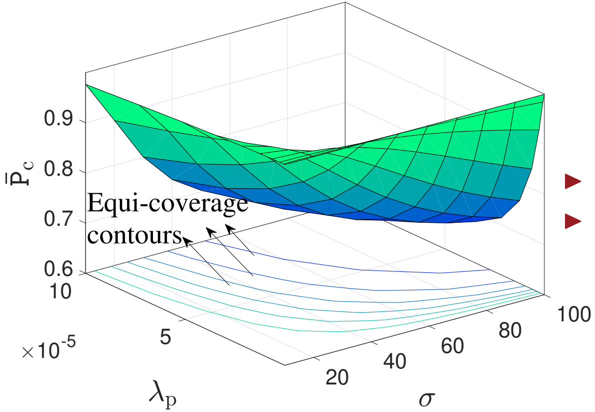
$$\lim_{\xi \to \infty} \mathbb{P}\big(\Phi_k(A_1) = 0, \Phi_{\mathrm{p}_k}(A_2) = 0\big) = \mathbb{P}\big(\bar{\Phi}_k(A_1) = 0)\mathbb{P}(\Phi_{\mathrm{p}_k}(A_2) = 0\big),$$

where $A_1, A_2 \subset \mathbb{R}^2$ are arbitrary closed compact sets.

Equi-Coverage Contours

For the general HetNet model, it is possible to find *equi-coverage contours*) in the parameter space[‖].

For a single tier network with PCP distributed BSs, the contours are:



- TCP: $\lambda_{\rm p}\sigma^2 =$ constant.
- MCP: $\lambda_{\rm p}{\rm R}^2 =$ constant.

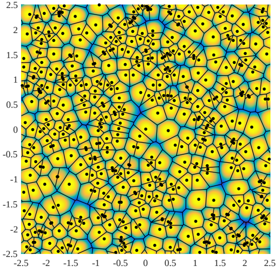
‖ **Afshang, Saha, Dhillon, "Equi-Coverage Contours in Cellular Networks",** ***IEEE Wireless Commun. Letters*, 2018.**

SIR Meta Distribution

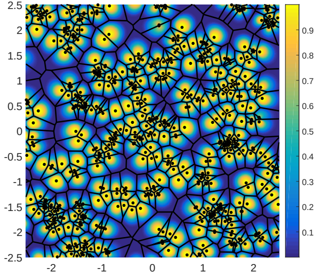
Definition

The meta distribution of SIR is the CCDF of the conditional success probability $P_s(\beta) \triangleq \mathbb{P}(\mathtt{SIR} > \beta|\Phi)$, i.e., $\bar{F}(\beta,\theta) = \mathbb{P}[P_s(\beta) > \theta], \beta \in \mathbb{R}^+, \theta \in (0,1]$.

Because of erogodicity, $\bar{F}(\beta, x)$ denotes the fraction of users that achieve an SIR of β with probability at least x in each network realization:



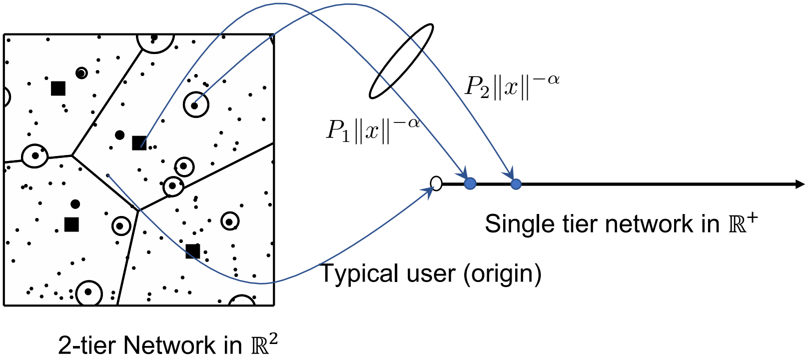
(a) Link reliability for PPP BSs



(b) Link reliability for PCP BSs

$$\bar{F}(\beta, x) = \lim_{r\to\infty} \frac{\sum_{u\in\Phi_{\rm u}\cap rB} \mathbf{1}(\mathbb{P}(\mathtt{SIR}(u) > \beta) > x)}{\mathbb{E}[\Phi_{\rm u}(rB)]}, \ \forall\ B \in \mathfrak{B}(\mathbb{R}^2).$$

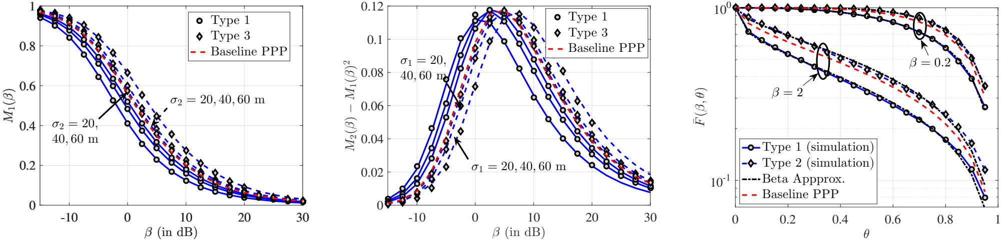
SIR Meta Distribution for General HetNet Model (2)



- Like coverage (which is $M_1(\beta)$), $M_b(\beta)$ is also expressed as the product of PGFLs and SPFLs of BS PPPs (when $i \in \mathcal{K}_1$) and parent PPPs of BS PCPs (when $i \in \mathcal{K}_2$)**.
- From the b-th moments, the CDF of meta distribution can be obtained by moment matching a beta kernel.

** [Saha2020a] Saha, Afshang, Dhillon, "Meta Distribution of Downlink SIR in a Poisson Cluster Process-based HetNet Model", arxiv.org/abs/2007.05997.

SIR Meta Distribution for General HetNet Model (3)



(a) Mean of meta distribution. (b) Variance of meta distribution. (c) Meta distribution ($\sigma_2 = 40$ m)

Meta distribution of SIR for Type 1 and Type 3 users in a two tier network. Details of the network configuration: $K = 2$, $\mathcal{K}_1 = \{1\}$, $\mathcal{K}_2 = \{2\}$, $q = 2$ for Type 3, $\alpha = 4$, $P_2 = 10^2 P_1$, $\lambda_{\rm p_2} = 2.5$ km^{-2}, $\lambda_{\rm p_1} = 1$ km^{-2}, $\bar{m}_2 = 4$, and $\sigma_2 = \sigma_{\rm u}$. Markers indicate the values obtained from Monte Carlo simulations.

IAB

Load Distribution (1)

- Load on a typical BS is the number of users associated with that BS.
- The load on a typical BS is equal to the number of points of the user point process ($\Phi_{\rm u}$) falling in the typical Poisson Voronoi cell ($\mathcal{C}_0$) of Φ (assuming single tier and max-power based association).
- When Φ and $\Phi_{\rm u}$ are PPPs, the PMF of load is known. $\Phi_{\rm u}(\mathcal{C}_0) \sim {\rm Poisson}({\rm vol}(\mathcal{C}_0))$.
- When $\Phi_{\rm u}$ is PCP, we characterized the mean

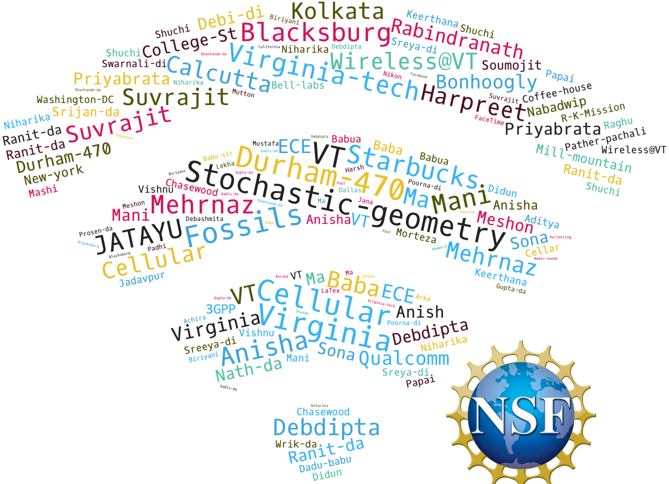
Load Analysis (2)

Load Analysis (3)

Key Take-Aways

Acknowledgments

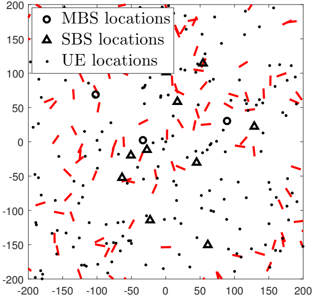
- Committee members.
- National Science Foundation (Grants CNS-1617896 and CNS- 1923807).
- Current and Past lab-mates.
- Friends and Family.

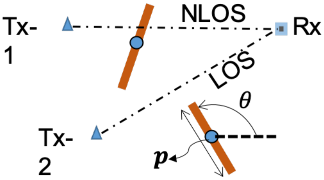


Thank You!

Appendix

- MBSs are modeled as a homogeneous PPP $\Phi_{\rm m}$ with density $\lambda_{\rm m}$.
- SBSs are modeled as a homogeneous PPP $\Phi_{\rm s}$ with density $\lambda_{\rm s}$.
- Blockages are distributed as a Germ-grain model $\Phi_{\rm bl}(\mathbf{p}, L_{\rm bl}, \theta)$.





Pathloss of a link of distance z can be expressed as:

$$L(z) = \begin{cases} z^{\alpha_\ell}, & \text{if } s = \ell, \ i.e. \ \#(\Phi_{\rm bl} \cap \overline{\mathbf{x}, \mathbf{y}}) = 0 \\ z^{\alpha_{\rm n}}, & \text{if } s = {\rm n}, \ i.e. \ \#(\Phi_{\rm bl} \cap \overline{\mathbf{x}, \mathbf{y}}) > 0 \end{cases},$$

Received $\mathtt{SNR}$

Received $\mathtt{SNR}$ at $\mathbf{y}$ from a BS at $\mathbf{x}$:

Access Link

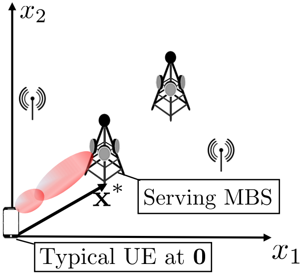
$$\mathtt{SNR}_{\rm a} = \frac{P_i G_i G_{\rm u} \beta h_{\mathbf{x},\mathbf{y}} L_{{\rm a}_i}(\|\mathbf{x}-\mathbf{y}\|)^{-1}}{\mathtt{N}_0 W}, i \in \{{\rm m},{\rm s}\}$$

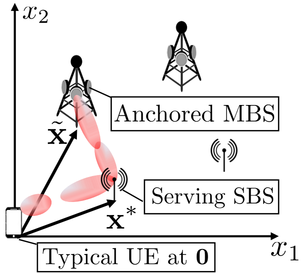
Backhaul Link

$$\mathtt{SNR}_{\rm b} = \frac{P_{\rm m} G_{\rm m} G_{\rm s} \beta h_{\mathbf{x},\mathbf{y}} L_{\rm b}(\|\mathbf{x}-\mathbf{y}\|)^{-1}}{\mathtt{N}_0 W}$$

- G_i = antenna gain for i-th tier BS ($i \in \{{\rm m},{\rm s}\}$)
- $G_{\rm u}$ = antenna gain of UE
- $L_{\rm a}, L_{\rm b}$ = pathloss for access and backhaul links
- $\mathtt{N}_0$ = noise PSD
- W = system BW
- β= reference pathloss at 1 m
- $h_{\mathbf{x},\mathbf{y}}$ = small scale fading

Association Policy





- The typical user is located at origin.
- The user connects to the BS that provides maximum average received power:

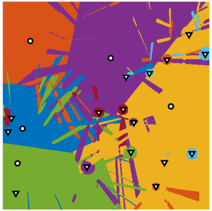
$$\mathbf{x}^* = \arg \max_{\substack{\mathbf{x}\in\Phi_i \\ i\in\{\mathrm{s},\mathrm{m}\}}} P_i T_i \beta G_i G_{\mathrm{u}} L_{\mathrm{a}_i}(\|\mathbf{x}\|)^{-1},$$

where T_i is the bias factor to the i-th tier.

- The SBS connects to the MBS that provides maximum average received power. Hence conditioned on $\mathbf{x}^* \in \Phi_{\mathrm{s}}$, the location of the anchored MBS:

$$\tilde{\mathbf{x}} = \arg \max_{\mathbf{x}\in\Phi_{\mathrm{m}}} P_{\mathrm{m}} \beta G_{\mathrm{m}} G_{\mathrm{s}} L_{\mathrm{b}}(\|\mathbf{x} - \mathbf{x}^*\|)^{-1}.$$

Association Cells



The association cell of a BS located at $\mathbf{x}$ is a closed subset in $\mathbb{R}^2$, where the BS at $\mathbf{x}$ provides the maximum received power.

Access Association Cell

$$\mathcal{C}_{\mathrm{a}_i}(\mathbf{x}) = \{\mathbf{z} \in \mathbb{R}^2 : P_i T_i \beta G_i L_{\mathrm{a}_i}(\|\mathbf{z}-\mathbf{x}\|)^{-1} \geq P_j T_j \beta G_j L_{\mathrm{a}_j}(\|\mathbf{z}-\mathbf{y}\|)^{-1}, \forall\, \mathbf{y} \in \Phi_j, j \in \{\mathrm{m}, \mathrm{s}\}\}$$

Association Probability: $\mathcal{A}_i = \mathbb{P}(\mathbf{x}^* \in \Phi_i), i \in \{\mathrm{m}, \mathrm{s}\}$.

Backhaul Association Cell

$$\mathcal{C}_{\mathrm{b}}(\mathbf{x}) = \{\mathbf{z} \in \mathbb{R}^2 : L_{\mathrm{b}_\mathrm{m}}(\|\mathbf{z}-\mathbf{x}\|)^{-1} \geq L_{\mathrm{b}_\mathrm{m}}(\|\mathbf{z}-\mathbf{y}\|)^{-1}, \forall\, \mathbf{y} \in \Phi_\mathrm{m} | \mathbf{x} \in \Phi_\mathrm{m}\}.$$

Resource Partitioning (1/2)

- W is shared between the access and backhaul links.
- Available resource at the BS is equally divided among the connected users (also known as the BS load) by a round robin scheduling.
- If $\mathbf{x}^* \in \Phi_{\rm m}$), the resource allocated for the typical user by the tagged MBS is inversely proportional to $(\Phi_{\rm u}(\mathcal{C}_{\rm a_m}(\mathbf{x}^*)) + \sum_{\mathbf{x}\in\Phi_{\rm s}\cap\mathcal{C}_{\rm b}(\mathbf{x}^*)} \Phi_{\rm u}(\mathcal{C}_{\rm a_s}(\mathbf{x})))$.
- If $\mathbf{x}^* \in \Phi_{\rm s}$, resource fraction allocated for the backhaul of tagged SBS: resource fraction allocated for the backhaul of tagged SBS:

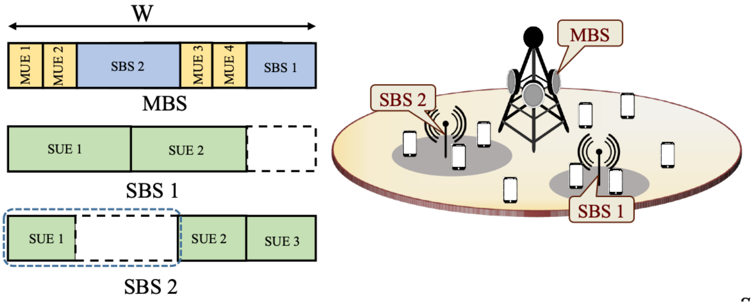
$$\omega = \frac{\Phi_{\rm u}(\mathcal{C}_{\rm a_s}(\mathbf{x}^*))}{\Phi_{\rm u}(\mathcal{C}_{\rm a_m}(\tilde{\mathbf{x}})) + \sum\limits_{\mathbf{x}\in\Phi_{\rm s}\cap\mathcal{C}_{\rm b}(\tilde{\mathbf{x}})} \Phi_{\rm u}(\mathcal{C}_{\rm a_s}(\mathbf{x}))}.$$

  Remaining $(1-\omega)$ fraction of resource is split equally between the users connected to the SBS.

$$\texttt{Rate} = \begin{cases} \frac{W}{\Phi_{\rm u}(\mathcal{C}_{\rm a_m}(\mathbf{x}^*)) + \sum\limits_{\mathbf{x}\in\Phi_{\rm s}\cap\mathcal{C}_{\rm b}(\mathbf{x}^*)} \Phi_{\rm u}(\mathcal{C}_{\rm a_s}(\mathbf{x}))} \log\left(1+\mathtt{SNR}_{\rm a}(\mathbf{0})\right), & \text{if } \mathbf{x}^* \in \Phi_{\rm m}, \\ \frac{W}{\Phi_{\rm u}(\mathcal{C}_{\rm a_s}(\mathbf{x}^*))} \min\left(\omega \log\left(1+\mathtt{SNR}_{\rm b}(\mathbf{x}^*)\right), (1-\omega)\log\left(1+\mathtt{SNR}_{\rm a}(\mathbf{0})\right)\right), & \text{if } \mathbf{x}^* \in \Phi_{\rm s}. \end{cases}$$

- Rate coverage: $\mathtt{P_r}(\rho) = \mathbb{P}(\texttt{Rate} > \rho)$.

Resource Partitioning (2/2)



Resource partition strategies for a toy example: MBS with two SBSs and four macro users (MUEs), SBS 1 has two users (denoted as SUEs) and SBS 2 has three users.

SNR Distribution (1/2)

Assumption

All link states in the network are independently distributed Bernoulli random variables with the LOS probability of a link of distance $r > 0$ being $p(r) = \exp(-r/\mu)$, where μ is the LOS range constant.

Definition (Pathloss Process)

The sequence $\mathcal{L}_{k_i} = L_{k_i}(\|\mathbf{x}\|) : \mathbf{x} \in \Phi_i\}$ ($k \in \{\mathrm{a}, \mathrm{b}\}$, $i \in \{\mathrm{m}, \mathrm{s}\}$) is called the pathloss process.

Lemma 1

$\mathcal{L}_{\mathrm{k}_i}$ is a PPP in $\mathbb{R}^+$ with intensity measure: $\Lambda_{k_i}([0, l)) =$

$$2\pi\lambda_i\left[\mu\left(\mu - e^{-\frac{l^{\frac{1}{\alpha_{k_i,\ell}}}}{\mu}}\left(l^{\frac{1}{\alpha_{k_i,\ell}}} + \mu\right)\right) + \frac{l^{\frac{2}{\alpha_{k_i,\mathrm{n}}}}}{2} - \mu\left(\mu - e^{-\frac{l^{\frac{1}{\alpha_{k_i,\mathrm{n}}}}}{\mu}}\left(l^{\frac{1}{\alpha_{k_i,\mathrm{n}}}} + \mu\right)\right)\right],$$

and density $\lambda_{k_i}(l) = 2\pi\lambda_i\left(\frac{1}{\alpha_{k_i,\ell}} l^{\frac{2}{\alpha_{k_i,\ell}}-1} e^{-\frac{l}{\mu}} + \frac{1}{\alpha_{k_i,\mathrm{n}}} l^{\frac{2}{\alpha_{k_i,\mathrm{n}}}-1}\left(1 - e^{-\frac{l}{\mu}}\right)\right).$

SNR Distribution (2/2)

Lemma 2

The association probability of the typical access link to a BS of Φ_i is expressed as:
$\mathcal{A}_i = \int\limits_0^\infty e^{-\sum\limits_{j\in\{\mathrm{m},\mathrm{s}\}} \Lambda_{\mathrm{a}_j}((0,\Omega_{j,i} l])} \lambda_{\mathrm{a}_i}(l)\mathrm{d}l$, where $\Omega_{j,i} = \frac{P_j T_j \beta G_j}{P_i T_i \beta G_i}$.

Lemma 3

The MBS coverage is expressed as: $\mathbb{P}(\mathtt{SNR}_{\mathrm{a}}(\mathbf{0}) > \tau | \mathbf{x} \in \Phi_{\mathrm{m}}) =$

$$\int_{l>0} \bar{F}_H\left(-\frac{\mathrm{N}_0 W \tau l}{P_{\mathrm{m}} G_{\mathrm{m}} G_{\mathrm{u}} \beta}\right) f_{L_{\mathrm{a}}^*}(l|\mathbf{x}^* \in \Phi_{\mathrm{m}})\, \mathrm{d}l,$$

and, the joint SBS and backhaul coverage is expressed as:
$\mathbb{P}(\mathtt{SNR}_{\mathrm{a}}(\mathbf{0}) > \tau_1, \mathtt{SNR}_{\mathrm{b}}(\mathbf{x}^*) > \tau_2 | \mathbf{x} \in \Phi_{\mathrm{s}}) =$

$$\int_{l_1>0} \bar{F}_H\left(-\frac{\mathrm{N}_0 W \tau_1 l_1}{P_{\mathrm{s}} G_{\mathrm{s}} G_{\mathrm{u}} \beta}\right) f_{L_{\mathrm{a}}^*}(l|\mathbf{x}^* \in \Phi_{\mathrm{s}})\mathrm{d}l_1 \int_{l_2>0} \bar{F}_H\left(-\frac{\mathrm{N}_0 W \tau_2 l_2}{P_{\mathrm{m}} G_{\mathrm{m}} G_{\mathrm{s}} \beta} - \Lambda_{\mathrm{b_m}}(0, l_2]\right) \lambda_{\mathrm{b_m}}(l_2)\mathrm{d}l_2.$$

Assumed that independence between $\mathtt{SNR}_{\mathrm{a}}(\mathbf{0})$ and $\mathtt{SNR}_{\mathrm{b}}(\tilde{\mathbf{x}})$.

Load Distribution (1/2)

- Since the spatial distribution of blockages is stationary, the association cells generated by $\Phi_{\rm m}$ and $\Phi_{\rm s}$ form a stationary partition of $\mathbb{R}^2$.

Proposition 3

, $\mathbb{E}[\mathcal{C}_{\rm b}(\mathbf{0})] = \frac{1}{\lambda_{\rm m}}$ and $\mathbb{E}[\mathcal{C}_{{\rm a}_i}] = \frac{\mathcal{A}_i}{\lambda_i}, (i \in \{{\rm m}, {\rm s}\})$.

- The analytical characterization of $\mathcal{A}_i$ is difficult for the Germ grain blockage model.
- We evaluate $\mathcal{A}_i$ using Monte Carlo simulation of the network.
- We find μ for which $\mathcal{A}_i$ under the independent blocking assumption is close to $\mathcal{A}_i$ for correlated blocking.

Load Distribution (2/2)

Lemma 4

Under Proposition 3, the PMFs of $\Phi_{\rm u}(\mathcal{C}_{\rm a_m}(\mathbf{x}^*))$ and $\Phi_{\rm u}(\mathcal{C}_{\rm a_s}(\mathbf{x}^*))$ are given as: $\mathbb{P}(\Phi_{\rm u}(\mathcal{C}_{{\rm a}_i}(\mathbf{x}^*)) = n) = \mathtt{K}_t\big(n; \frac{\lambda_i}{\mathcal{A}_i}, \lambda_{\rm u}\big), \mathbb{P}(\Phi_{\rm u}(\mathcal{C}_{\rm a_m}(\tilde{\mathbf{x}})) = n) = \mathtt{K}\big(n; \frac{\lambda_i}{\mathcal{A}_{\rm m}}, \lambda_{\rm u}\big),\ i \in \{{\rm m}, {\rm s}\}$, where $\mathtt{K}_t(n; \lambda, \lambda_{\rm u}) =$

$$\frac{3.5^{3.5}}{(n-1)!}\frac{\Gamma(n+3.5)}{\Gamma(3.5)}\left(\frac{\lambda_{\rm u}}{\lambda}\right)^{n-1}\left(3.5+\frac{\lambda_{\rm u}}{\lambda}\right)^{-n-3.5}, n \geq 1,$$

and $\mathbb{P}(\Phi_{\rm s}(\mathcal{C}_{\rm b}(\mathbf{x}^*)) = n) \approx \mathbb{P}(\Phi_{\rm s}(\mathcal{C}_{\rm b}(\tilde{\mathbf{x}}) = n) = \mathtt{K}(n; \lambda_{\rm m}, \lambda_{\rm s}),\ n \geq 0$, where $\mathtt{K}(n; \lambda, \lambda_{\rm u}) =$

$$\frac{3.5^{3.5}}{n!}\frac{\Gamma(n+3.5)}{\Gamma(3.5)}\left(\frac{\lambda_{\rm u}}{\lambda}\right)^{n}\left(3.5+\frac{\lambda_{\rm u}}{\lambda}\right)^{-n-3.5}, n \geq 0.$$

Rate Coverage

Theorem 1

Rate coverage of a typical user is expressed as, $\mathtt{Pr}(\rho) =$

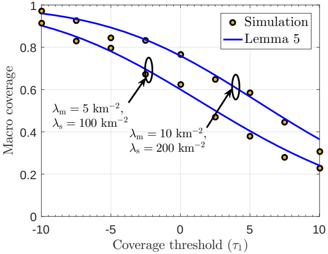
$$\mathcal{A}_{\rm m}\sum_{n=1}^{\infty}\mathbb{P}\left(\mathtt{SNR}_{\rm a}(\mathbf{0}) > 2^{\frac{\rho}{W}\left(n+\frac{\mathcal{A}_{\rm s}\lambda_{\rm u}}{\lambda_{\rm m}}\right)} - 1 \middle| \mathbf{x}^* \in \Phi_{\rm m}\right)\mathtt{K}_t\left(n; \frac{\lambda_{\rm m}}{\mathcal{A}_{\rm m}}, \lambda_{\rm u}\right)$$

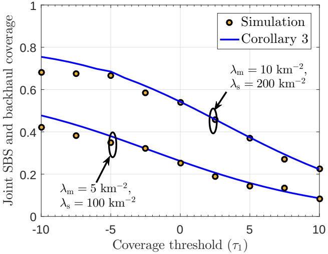
$$+\mathcal{A}_{\rm s}\sum_{n=1}^{\infty}\mathbb{P}\left(\mathtt{SNR}_{\rm a}(\mathbf{0}) > 2^{\frac{\rho n}{W}\left(1+\frac{\lambda_{\rm m} n}{\lambda_{\rm u}}\right)} - 1 \middle| \mathbf{x}^* \in \Phi_{\rm s}\right)\mathbb{P}\left(\mathtt{SNR}_{\rm b}(\mathbf{0}) > 2^{\frac{\rho\left(n+\frac{\lambda_{\rm u}}{\lambda_{\rm m}}\right)}{W}} - 1\right)\mathtt{K}_t\left(n; \frac{\lambda_{\rm s}}{\mathcal{A}_{\rm s}}, \lambda_{\rm u}\right).$$

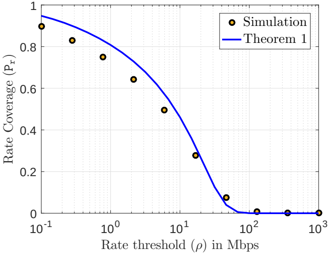
Validation of the Analytical Expressions

Table: Key system parameters and default values

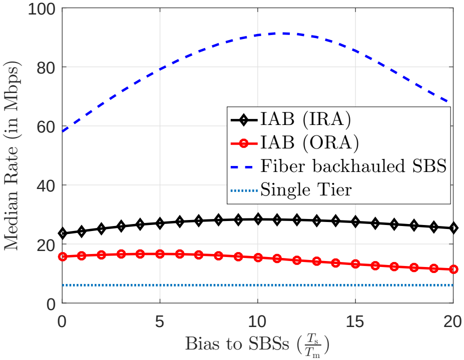
Parameter	Value
$P_{\mathrm{m}},\ P_{\mathrm{s}}$	40, 20 dBm
$\alpha_{k_{i,\ell}}, \alpha_{k_{i,\mathrm{n}}}$ $(\forall\ k \in \{\mathrm{a},\mathrm{b}\},\ i \in \{\mathrm{m},\mathrm{s}\})$	3.0, 4.0
f_c	28 GHz
$G_{\mathrm{m}},\ G_{\mathrm{s}}$	18 dB
G_{u}	0 dB
$\mathtt{N}_0 W$	-174 dBm/Hz+ $10\log_{10} W$ $+10$ dB (noise-figure)
$\{\lambda_{\mathrm{m}}, \lambda_{\mathrm{s}}, \lambda_{\mathrm{u}}\}$	$\{10, 50, 1000\}$ km^{-2}
$T_{\mathrm{m}}, T_{\mathrm{s}}$	1,1
λ_{u}	1000 km^{-2}
$L_{\mathrm{bl}}, \lambda_{\mathrm{bl}}$	5 m, 1500 km^{-2}
W	1 GHz





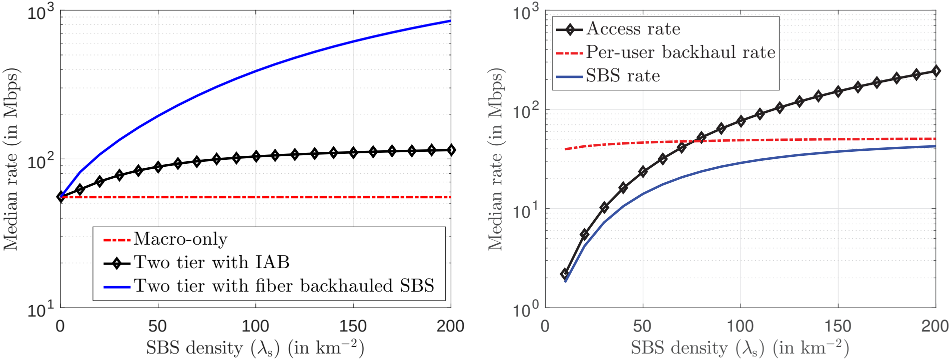


Effect of Bias Factor



- Median rate (ρ_{50}) is maximized for some bias which ensures load balancing between the MBS and SBS tiers.
- For IAB, ρ_{50} does not change as prominently as that of the HetNet with fiber-backhauled SBSs.
- This is because the users offloaded from the BSs in Φ are not completely disappearing from the macro load, but are coming back to the macro load in terms of the increased backhaul load.

Limits of Network Densification



For IAB, the SBS densification does not improve rate beyond some saturation limit.

Conclusion

- Developed an analytical model of mm-wave two-tier HetNet with IAB.
- Derived the downlink rate coverage of this network.
- Introduced reasonable assumptions to handle load distributions under correlated blockage.
- Bias factors do not play a prominent role in boosting data rate in IAB-enabled HetNets.
- The improvement in rate quickly saturates with SBS densification.

Future Works.

- Analysis of multi-hop IAB networks.
- Characterize the delays occurring in the IAB networks.

References

Journal Extension of this paper:

[1] C. Saha and H. S. Dhillon, "Millimeter Wave Integrated Access and Backhaul in 5G: Performance Analysis and Design Insights", *IEEE Journal on Sel. Areas in Commun.*, to appear.

Relevant Prior Arts:

[2] C. Saha, M. Afshang, and H. S. Dhillon, "Bandwidth partitioning and downlink analysis in millimeter wave integrated access and backhaul for 5G," *IEEE Trans. on Wireless Commun.*, vol. 17, no. 12, pp. 8195-8210, Dec. 2018.

[3] S. Singh, F. Baccelli, and J. G. Andrews, "On association cells in random heterogeneous networks," *IEEE Wireless Commun. Letters*, vol. 3, no. 1, pp. 70-73, Feb. 2014.

[4] S. Singh, H. S. Dhillon, and J. G. Andrews, "Offloading in heterogeneous networks: Modeling, analysis, and design insights," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 5, pp. 2484-2497, May. 2013.

[5] S. Aditya, H. S. Dhillon, A. F. Molisch, and H. M. Behairy, "A tractable analysis of the blind spot probability in localization networks under correlated blocking," *IEEE Trans. on Wireless Commun.*, vol. 17, no. 12, pp. 8150-8164, Dec. 2018.

Contact and Nearest Distance Distributions of MCP

Theorem 2

If Φ is a MCP in $\mathbb{R}^d$, the CDF of its contact distance is given by

$$H_s(r) = 1 - \exp\Bigg(-c_d\lambda_{\mathrm{p}}\Bigg((r+\mathtt{R})^d - |r-\mathtt{R}|^d \exp\left(-\frac{\bar{m}}{\mathtt{R}^d}(\min(r,\mathtt{R}))^d\right) - d\int_{|r-\mathtt{R}|}^{r+\mathtt{R}} \exp\left(-\frac{\bar{m}}{c_d\mathtt{R}^d}\Upsilon(r,\mathtt{R},y)\right) y^{d-1}\mathrm{d}y\Bigg)\Bigg), \quad (1)$$

where $\Upsilon(r_1, r_2, t)$ the area of intersection of two discs of radii r_1, r_2 and distance between the centers t. The CDF of its nearest distance distance is given by

$$D(r) = \begin{cases} 1-(1-H_s(r))R^{-d}\Bigg(\exp\left(-\frac{\bar{m}(\min(\mathtt{R},r))^d}{\mathtt{R}^d}\right)|\mathtt{R}-r|^d \\ +d\int_0^{\mathtt{R}}\exp\left(-\frac{\bar{m}}{c_d\mathtt{R}^d}\Upsilon(r,\mathtt{R},x)\right)x^{d-1}\mathrm{d}x\Bigg), & \text{if } r \leq 2\mathtt{R}, \\ 1-(1-H_s(r))e^{-\bar{m}}, & \text{if } r > 2\mathtt{R}. \end{cases} \quad (2)$$

Contact and Nearest Distance Distributions of TCP

Theorem 3

If Φ is a TCP in $\mathbb{R}^d$, the CDF of its contact distance is given by

$$H_s(r) = 1 - \exp\left(-\lambda_{\rm p} 2^d c_d \int_0^\infty \left(1 - \exp\left(-\bar{m}(1 - Q_{\frac{d}{2}}(\sigma^{-1}y, \sigma^{-1}r))\right)\right) y^{d-1} {\rm d}y\right), \quad (3)$$

where $Q_{\frac{d}{2}}$ is the Marcum-Q function. The CDF of the nearest neighbor distance is given by

$$D(r) = 1 - (1 - H_s(r)) \int_0^\infty \exp\left(-\bar{m}\left(1 - Q_{\frac{d}{2}}(\sigma^{-1}y, \sigma^{-1}r)\right)\right) \chi_d(\sigma^{-1}y)\, {\rm d}y, \quad (4)$$

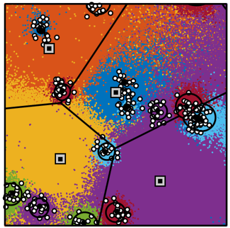
where $\chi_d(\cdot)$ is the PDF of a Chi distribution with d degrees of freedom.

Max Instantaneous Power based Association

SIR of the link between the typical user and a BS at $x \in \Phi^{(k)}$:

$$\mathtt{SIR}(x) = \frac{P_k h_x \|x\|^{-\alpha}}{\underbrace{\mathcal{I}(\Phi^{(k)} \setminus \{x\}) + \sum_{j \in \mathcal{K} \setminus \{k\}} \mathcal{I}(\Phi^{(j)})}_{\mathcal{I}(\Phi^{(i)}) = \sum_{y \in \Phi^{(i)}} P_i h_y \|y\|^{-\alpha} : \textbf{aggregate interference}}}. \quad (5)$$

Cell Association: If x^* denotes the location of the BS serving the typical user at origin,



$$\max_{k \in \Phi^{(k)}} P_k h_k \|x_k\|^{-\alpha} = \arg \max_{x \in \cup_{k \in \{\mathcal{K}_1 \cup \mathcal{K}_2\}} \Phi^{(k)}} \mathtt{SIR}(x).$$

- Since small scale fading is part of the association rule, the cells are *irregular* shaped.
- This strategy is *coverage-optimal*.
- Lends to a nice structure of analysis (next slide).

Downlink Coverage

Definition (Coverage probability)

Assuming β is the $\mathtt{SIR}$-threshold, coverage probability is defined as:

$$\mathtt{P_c} = \mathbb{P}(\mathtt{SIR}(x^*) > \beta | o \in \Phi_{\rm u}) = \mathbb{P}\Bigg(\max_{\substack{x\in\Phi^{(k)},\\ k\in\mathcal{K}_1\cup\mathcal{K}_2}} \mathtt{SIR}(x) > \beta \Bigg| o \in \Phi_{\rm u} \Bigg)$$

$$= \mathbb{E}^{!o}_{\Phi_{\rm u}}\Bigg[\mathbf{1}\Bigg(\bigcup_{k\in\mathcal{K}_1\cup\mathcal{K}_2} \bigcup_{x\in\Phi^{(k)}} \{\mathtt{SIR}(x) > \beta\} \Bigg)\Bigg].$$

Assuming $\beta > 1$, $\mathtt{P_c} = \sum_{k\in\mathcal{K}} \mathtt{P_c}_k$, where $\mathtt{P_c}_k$ = per-tier coverage [Dhillon2012].
Under i.i.d. Rayleigh fading assumption,

$$\mathtt{P_c}_k = \underbrace{\mathbb{E}\Bigg[\sum_{x\in\Phi^{(k)}} \prod_{y\in\Phi^{(k)}\setminus\{x\}} \mu_{k,k}(x,y) \Bigg]}_{\text{Sum-product functional (SPFL)}} \times \prod_{k_1\in\mathcal{K}_1\setminus\{k\}} \overbrace{\mathbb{E}\Bigg[\prod_{y\in\Phi^{(k_1)}} \mu_{k_1,k}(x,y) \Bigg]}^{\text{Probability generating functional (PGFL)}},$$

where $\mu_{i,j}(x,y) = \frac{1}{1+\frac{\beta P_i \|x\|^\alpha}{P_j \|y\|^\alpha}}$.

Sum-product functional

Definition

If Φ is a point process in $\mathbb{R}^d$, the SPFL of Φ is $S_\Phi[g,v] := \mathbb{E}\left[\sum_{x\in\Phi} g(x) \prod_{y\in\Phi} v(x,y)\right]$, where $v : \mathbb{R}^d \times \mathbb{R}^d \to (0,1]$ and $g : \mathbb{R}^d \to \mathbb{R}^+$ are measurable.

Lemma 5

The expressions of reduced SPFL for different PPs are given as: $S^!_\Phi[g,v] =$

$$\int_{\mathbb{R}^d} g(x)\underbrace{G_\Phi[v(x,\cdot)]}_{\text{PGFL of } \Phi}\Lambda(\mathrm{d}x) \qquad \Phi \text{ is PPP with intensity measure } \Lambda,$$

$$\bar{m}\lambda_{\mathrm{p}} \int_{\mathbb{R}^d} g(x)G_\Phi[v(x,\cdot)]G_{\mathcal{B}^!_o}[v(x,\cdot)]\mathrm{d}x \qquad \Phi \text{ is PCP with parameters } (\lambda_{\mathrm{p}}, \bar{m}, f),$$

$$\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} g(x)G_{\Phi^{(0)}|z_o}[v(x,\cdot)]$$

$$\times \left(\bar{m}_q \int_{\mathbb{R}^2} v(x,y)f(y|z_o)\mathrm{d}y + 1\right) f(x|z_o)f_{\mathrm{u}}(z_o)\,\mathrm{d}x\,\mathrm{d}z_o, \qquad \text{Type 3 users, } \Phi^{(0)}.$$

Coverage Probability: One Representative Result

Theorem 4 (Coverage Probability: Type 2 Users)

$\mathrm{P_c}$ of a typical user for $\beta > 1$ is –

$$\mathrm{P_c} = \sum_{k \in \mathcal{K}} \mathrm{P_c}_k,$$

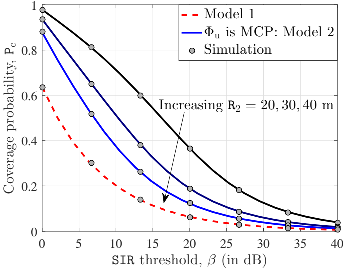
where $\mathrm{P_c}_k$ is the per-tier coverage:

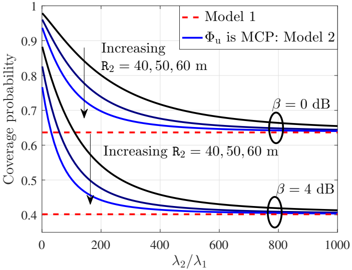
$$k = 0 : \mathrm{P_c}_k = \int_{\mathbb{R}^2} \prod_{k_1 \in \mathcal{K}_1} G_{\Phi^{(k_1)}}[\mu_{k_1,k}(z_o, \cdot)] \prod_{k_2 \in \mathcal{K}_2} G_{\Phi^{(k_2)}}[\mu_{k_2,k}(z_o, \cdot)] f_{\mathrm{u}}(z_0) \mathrm{d}zo,$$

$$k \in \mathcal{K}_1 : \mathrm{P_c}_k = \lambda_k \int_{\mathbb{R}^2} \prod_{k_1 \in \mathcal{K}_1} G_{\Phi^{(k_1)}}[\mu_{k_1,k}(x, \cdot)] \prod_{k_2 \in \mathcal{K}_2} G_{\Phi^{(k_2)}}[\mu_{k_2,k}(x, \cdot)] G_{\Phi^{(0)}}[\mu_{q,k}(x, \cdot)] \, \mathrm{d}x,$$

$$k \in \mathcal{K}_2 : \mathrm{P_c}_k = \bar{m}_k \lambda_{\mathrm{p}_k} \int_{\mathbb{R}^2} \prod_{k_1 \in \mathcal{K}_1} G_{\Phi^{(k_1)}}[\mu_{k_1,k}(x, \cdot)] \prod_{k_2 \in \mathcal{K}_2} G_{\Phi^{(k_2)}}[\mu_{k_2,k}(x, \cdot)]] \times G_{\mathcal{B}_o^{(k)!}}[\mu_{k,k}(x, \cdot)] G_{\Phi^{(0)}}[\mu_{q,k}(x, \cdot)] \mathrm{d}x.$$

Insights





add insights..

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