

# 3GPP-inspired Stochastic Geometry Models for Heterogeneous Cellular Networks

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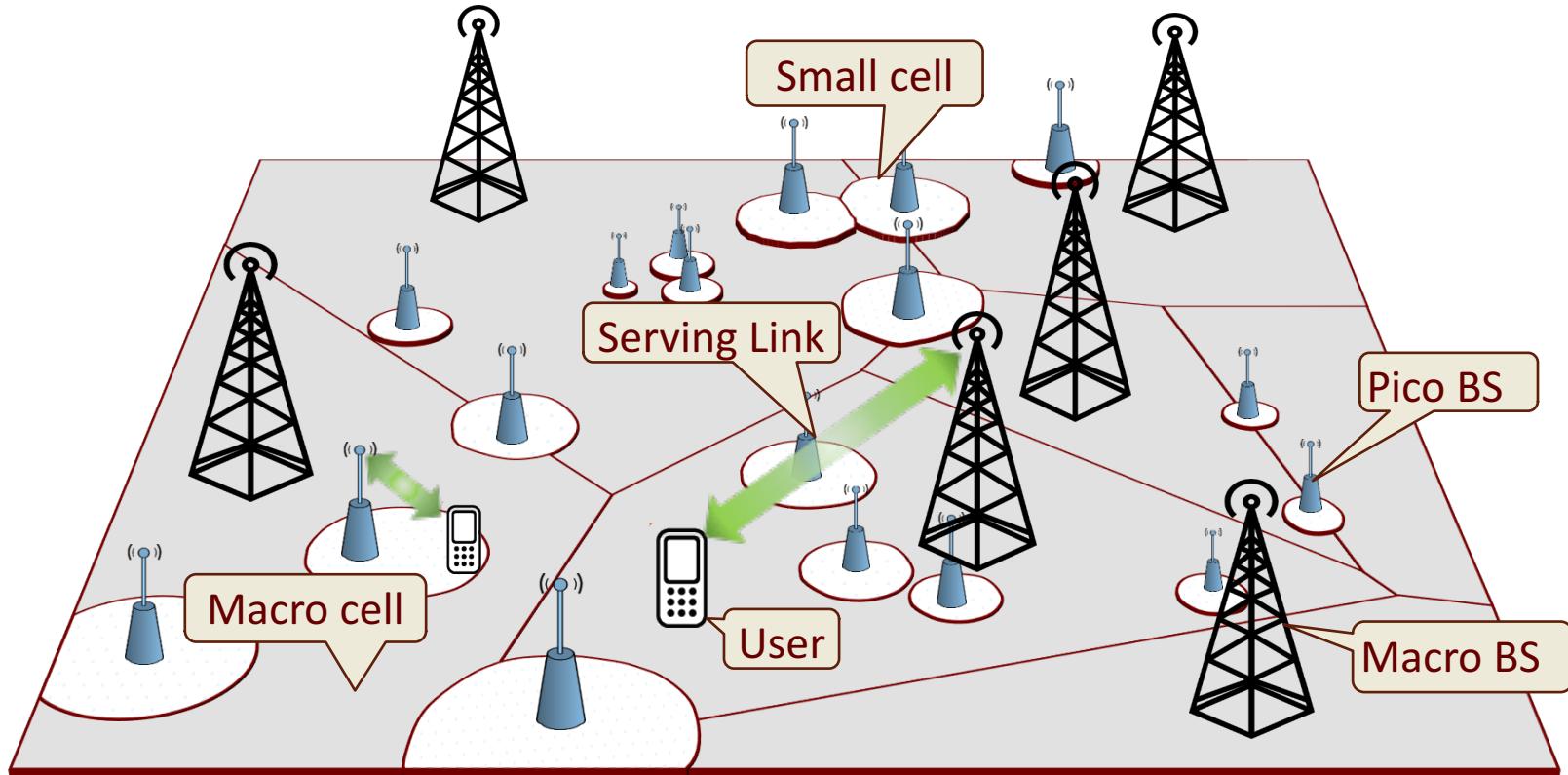
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**Joint work with Chiranjib Saha and Mehrnaz Afshang**

# Heterogeneous Network (HetNet)



HetNet formed by different types of Base Stations (BSs)

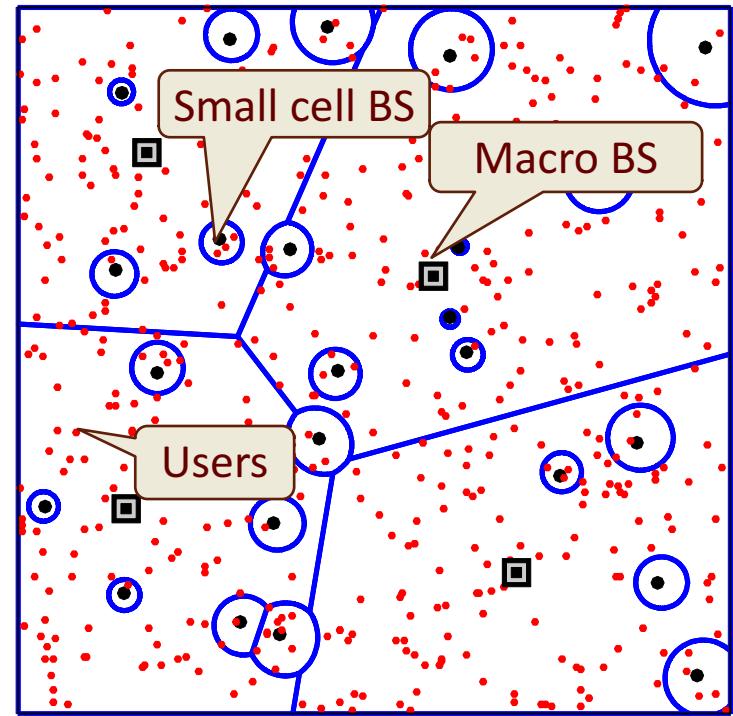
# Our Goal

All models are wrong but some are useful.  
-- George Box

- \* Goal of this talk (in the above context): Develop models for HetNets that are “less wrong” than the current models but are still almost as “useful”.
- \* In other words, develop models that are realistic enough to capture key characteristics of HetNets while being tractable enough to reveal useful insights.

# First Comprehensive Model (*PPP Model*)

- \* Model different types of BSs as independent Poisson Point Processes (PPPs).
- \* Model users as a PPP independent of the BSs.
- \* *Roughly speaking, PPP places points uniformly at random independently of each other*



“Wrong”: Assumes independence across everything.  
“Useful”: Very tractable. One example is given on the next slide.

[1] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, “Modeling and analysis of K-tier downlink heterogeneous cellular networks,” *IEEE Journal on Sel. Areas in Commun.*, 2012.

# Downlink Coverage for this PPP Model

- \* Assuming Rayleigh fading and maximum SIR-based cell selection, the coverage probability for a typical randomly located user is

$$P_{c(\text{PPP})} \left( \frac{\text{BS density}}{\{\lambda_i\}}, \frac{\text{Transmit power}}{\{\beta_i\}}, \frac{\text{Target SIR}}{\{P_i\}} \right) = \frac{\pi}{C(\alpha)} \frac{\sum_{i=1}^K \lambda_i P_i^{2/\alpha} \beta_i^{-2/\alpha}}{\sum_{i=1}^K \lambda_i P_i^{2/\alpha}}$$

- \* If target SIRs for all the tiers are the same, the coverage probability is  $P_{c(\text{PPP})} = \frac{\pi}{C(\alpha) \beta^{2/\alpha}}$

The coverage probability becomes independent of the BS densities  $\{\lambda_i\}$  and transmit powers  $\{P_i\}$  when target target SIRs for all the tiers are the same.

Demonstrates remarkable tractability of this model. Hence “very useful”.

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[1] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, “Modeling and analysis of K-tier downlink heterogeneous cellular networks,” *IEEE Journal on Sel. Areas in Commun.*, 2012.

# Current State-of-the-Art

The original PPP-based model/approach  
has been enhanced in many ways

## Channel Models

Multi-slopes pathloss

Rayleigh, Nakagami

$\kappa-\mu$  shadowed fading

mm-Wave

## Metrics

SINR distribution

Area spectral efficiency

Rate distribution

Local delay

## Technology and applications

Cache-enabled BSs

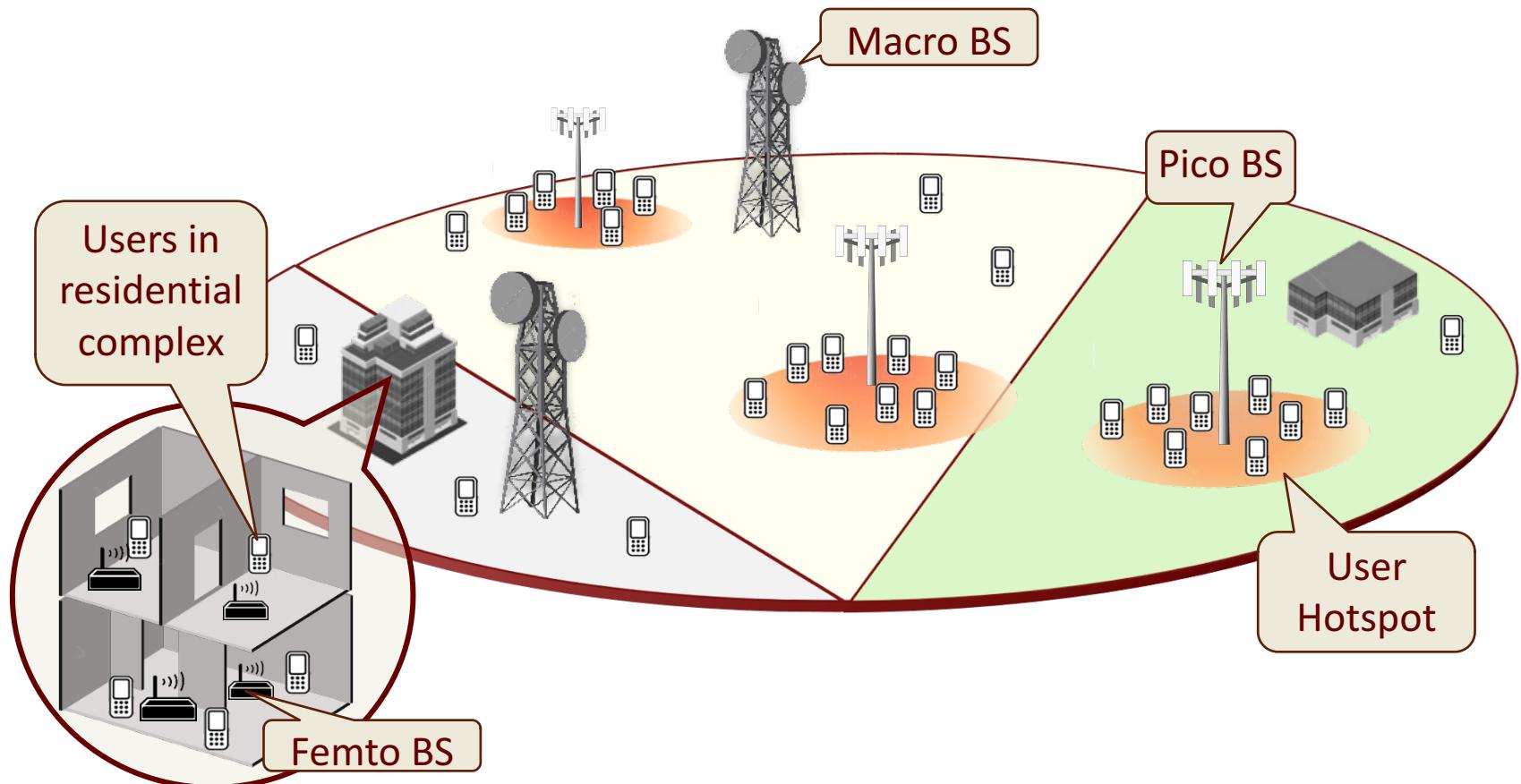
Energy harvesting

Massive MIMO

Localization

All these extensions are based on the assumption that the BSs and users are independent PPPs, which is not very realistic. See next slide.

# PPP Model: How “accurate” is it?



A simple illustration of a HetNet

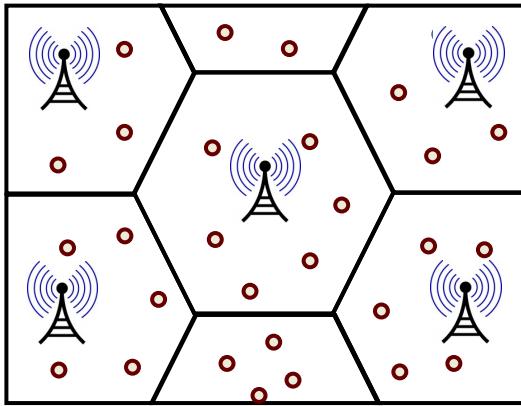
# PPP Model: How “accurate” is it?

- \* Key features missing in this “baseline” PPP model:
- \* **Non-uniformity in the user distribution**
  - \* Modeling all users as an independent PPP is not realistic.
  - \* Fraction of users form spatial clusters (*Hotspots*), e.g. users in public places and residential areas.
- \* **Correlation between small cell BS (SBS) and user locations**
  - \* Operators deploy SBSs (e.g., picocells) at the areas of high user density.
- \* **Inter and intra BS-tier dependence:**
  - \* BS locations are not necessarily independent.
  - \* Site planning for deploying BSs introduces correlation in BS locations.

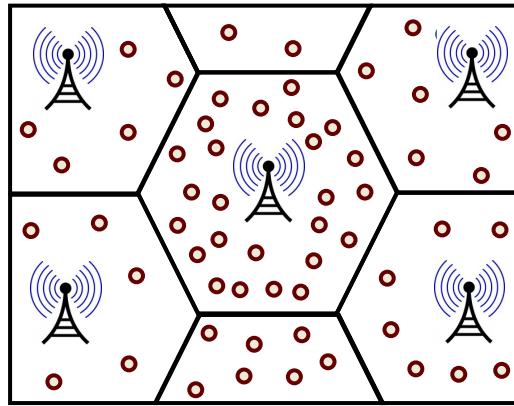
In contrast, 3GPP simulation models are much more accurate!

# 3GPP Models: User Distribution

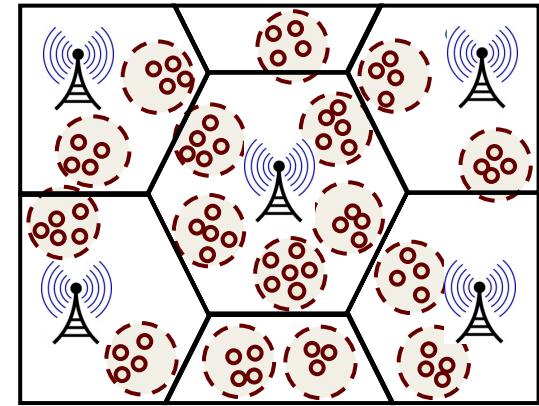
- \* 3GPP considers different configurations of SBSs and users in HetNet simulation models.



Users “uniform” across macro cells



Users “non-uniform” across macro cells

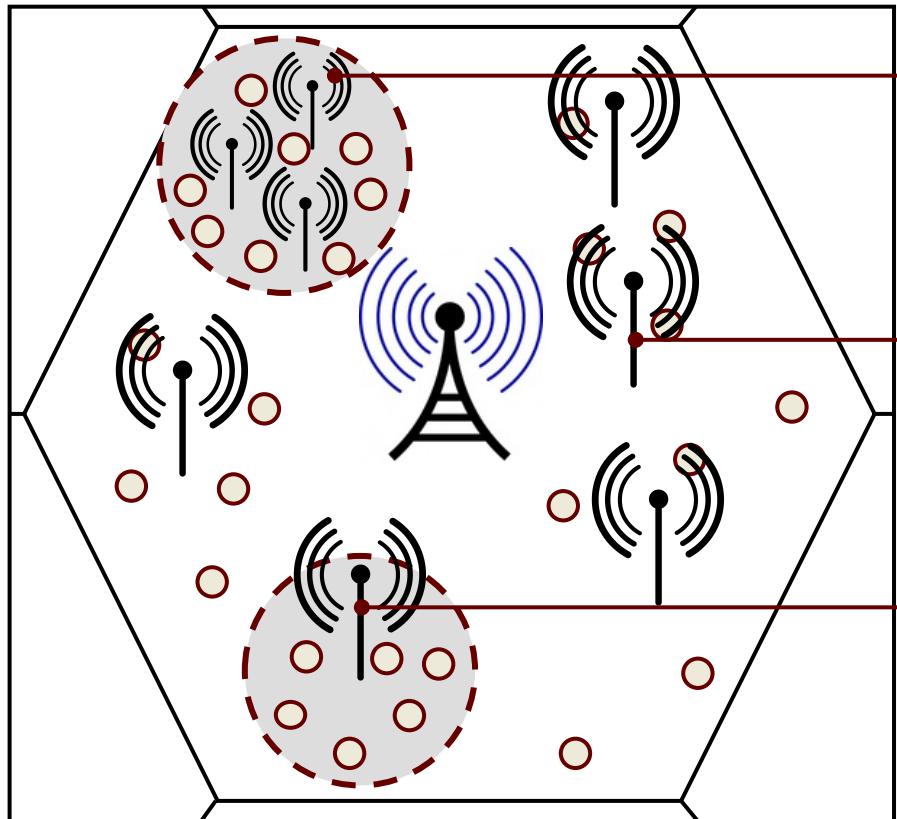


Users forming clusters within a disc

User configurations usually considered in 3GPP HetNet models

[2] 3GPP TR 36.932 V13.0.0 , “3rd generation partnership project; technical specification group radio access network; scenarios and requirements for small cell enhancements for E-UTRA and E-UTRAN (release 13),” Tech. Rep., Dec. 2015

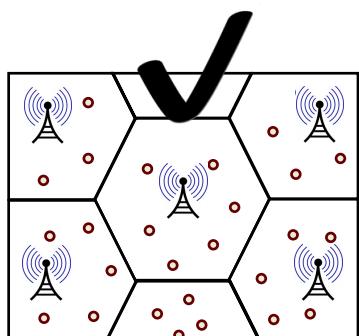
# 3GPP Model: SBS Distribution



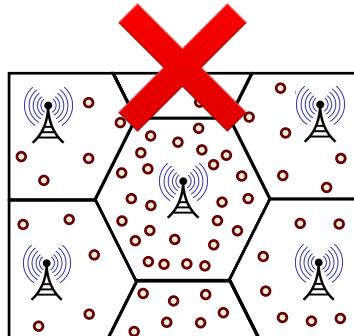
- SBSs deployed at higher density in certain areas (e.g., indoor models)
- SBSs deployed randomly or under some site planning.
- SBSs deployed at the centers of user hotspots.

SBS Configurations in 3GPP HetNet Model

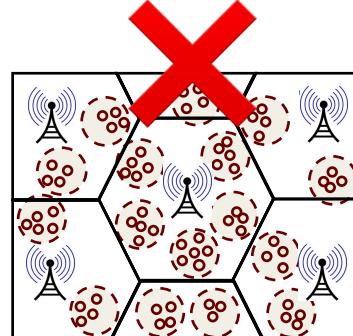
# How does PPP model hold up?



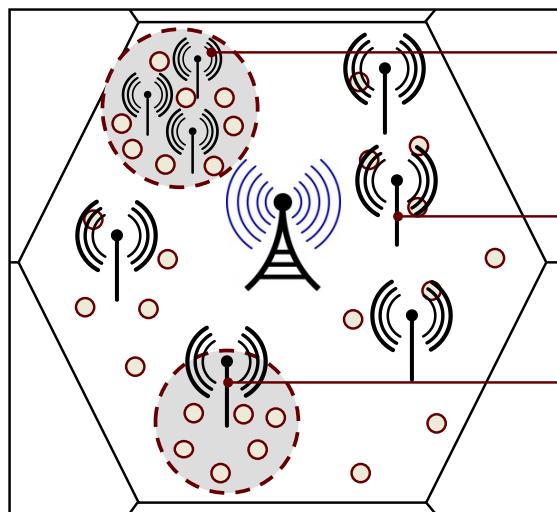
Users “uniform” across  
macro cells



Users “non-uniform”  
across macro cells



Users forming clusters  
within a disc

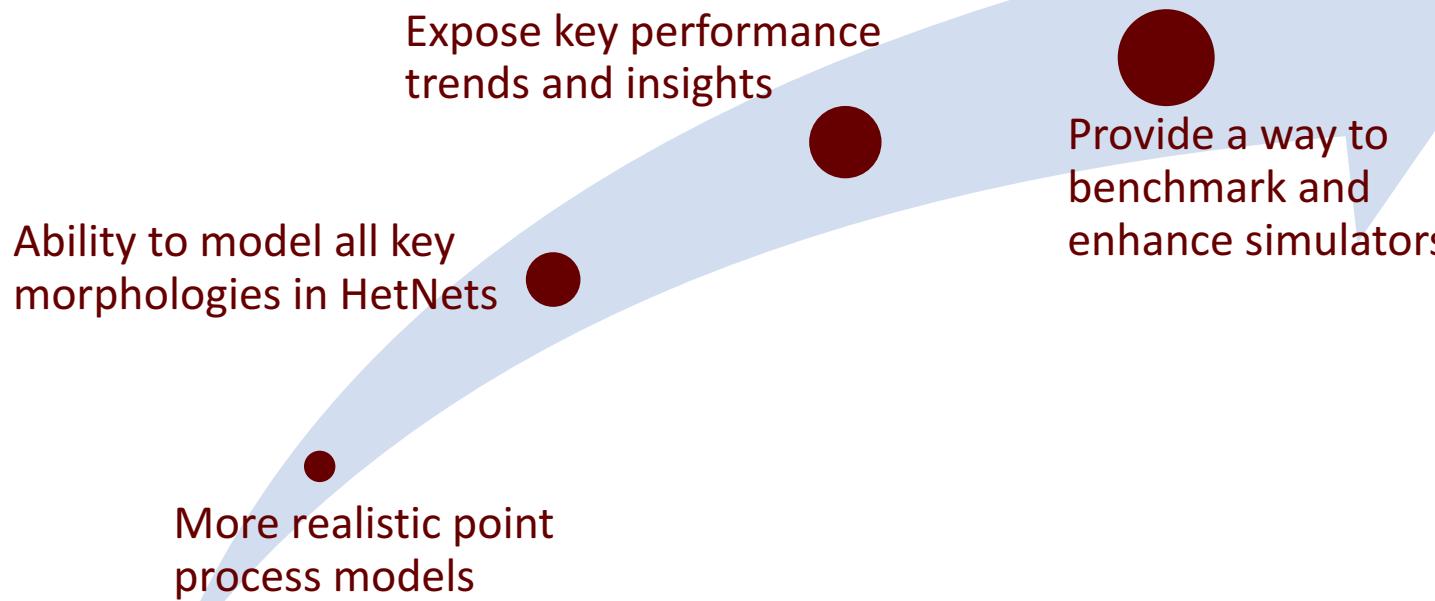


- SBSs deployed at high density at certain areas (indoor models)
- SBSs deployed randomly or under some site planning.
- SBSs at the center of user hotspot.



Therefore, while  
PPP model is very  
useful, it is highly  
restrictive

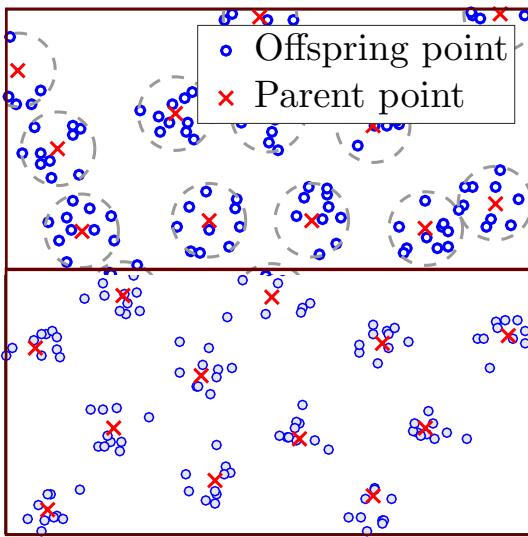
# Thinking Beyond Homogeneous PPP



Q: Is there something that is almost as tractable as a PPP but could model these other morphologies accurately?

# The Answer is Poisson Cluster Process

- \* **Poisson Cluster Process (PCP)** is more appropriate abstraction for user and BS distributions considered by 3GPP.



Examples of PCPs

## Definition: Poisson Cluster Process (PCP)

A PCP is generated from a PPP  $\Phi_p$  called the **Parent PPP**, by replacing each point  $z_i$  by a finite offspring point process  $\mathcal{B}_i$  where each point is independently and identically distributed around origin.

$$\Phi = \bigcup_{z_i \in \Phi_p} z_i + \mathcal{B}_i.$$

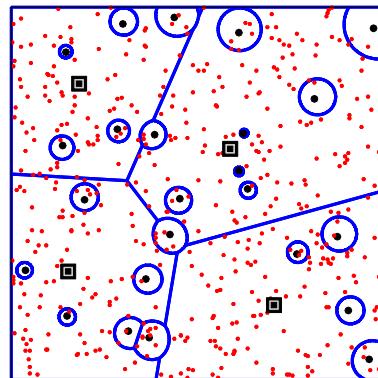
## Special cases

- $\#\mathcal{B}_i \sim \text{Poisson}(\bar{m})$
- **Matern Cluster Process:** Each point in  $\mathcal{B}_i$  is **uniformly** distributed inside disc of radius  $r_d$  centered at origin.
- **Thomas Cluster Process:** Each point in  $\mathcal{B}_i$  is **normally** distributed (with variance  $\sigma^2$ ) around the origin.

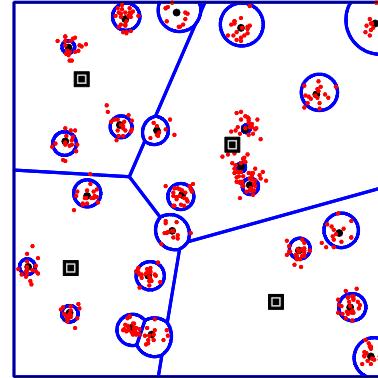
# PCP-based Canonical HetNet Models

- \* New HetNet models generated by combining PCPs with PPPs have closer resemblance with 3GPP HetNet models.

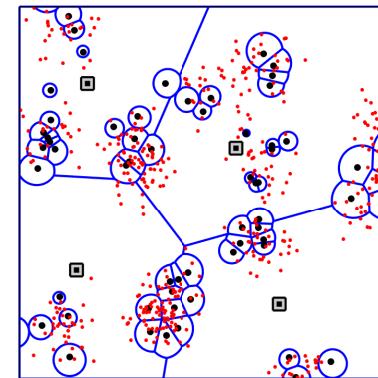
Model 1: Users PPP,  
BSs PPP (Baseline)



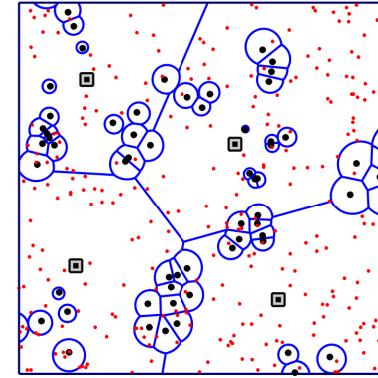
Model 2: Users PCP,  
BSs PPP



Model 3: Users PCP,  
BSs PCP



Model 4: Users PPP,  
BSs PCP



Model 1: H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Modeling and analysis of K-tier downlink heterogeneous cellular networks," *IEEE JSAC*, 2012.

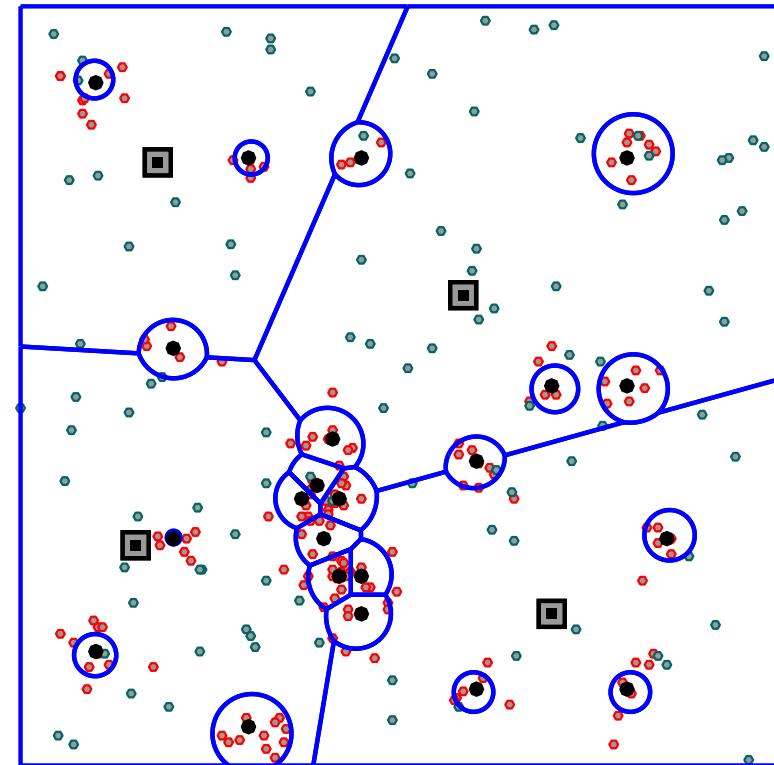
Model 2: C. Saha, M. Afshang, and H. S. Dhillon, "Enriched K-Tier HetNet Model to Enable the Analysis of User-Centric Small Cell Deployments", *IEEE TWireless* 2017.

Model 3: M. Afshang, H. S. Dhillon, "Poisson Cluster Process Based Analysis of HetNets with Correlated User and Base Station Locations", submitted, 2017.

Unified: C. Saha, M. Afshang, H. Dhillon, "3GPP-inspired HetNet Model using Poisson Cluster Process: Sum-product Functionals and Downlink Coverage", submitted, 2017.

# Unifying these models: PCP meets PPP

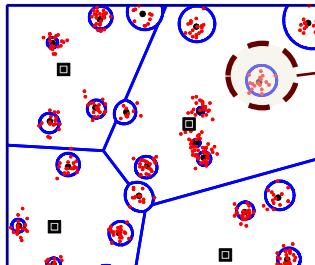
- \* We consider K tier HetNet with two sets of BS PPs:
  - \*  $K_1$  BS tiers are modeled as independent PPPs.
  - \*  $K_2$  BS tiers are modeled as independent PCPs.
- \* User Distribution:
  - \* Users can be independent PPP.
  - \* Users can be a PCP correlated with the BS distribution.



Models 1-4 from the previous slide are special cases of this “unified” model.

# Analysis of the Unified Model

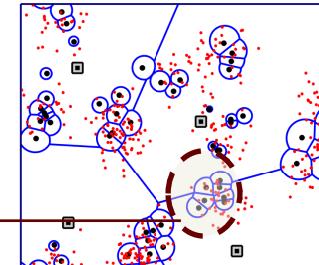
**Handling coupling in the point processes:** Can be done by defining a virtual 0<sup>th</sup> tier  $\Phi_0$  followed by applying Slivynak's theorem.



BSs PPP, Users PCP

If users form a PCP around BS locations, the 0<sup>th</sup> tier is a BS at the cluster center of the typical user.

If both users and BSs are modeled as (coupled) PCPs, the 0<sup>th</sup> tier is the set of BSs belonging to the same cluster center of the typical user.



BSs PCP, Users PCP

## Key next steps

Find the distribution of the distance from the typical user to the serving and interfering BSs

Max Power

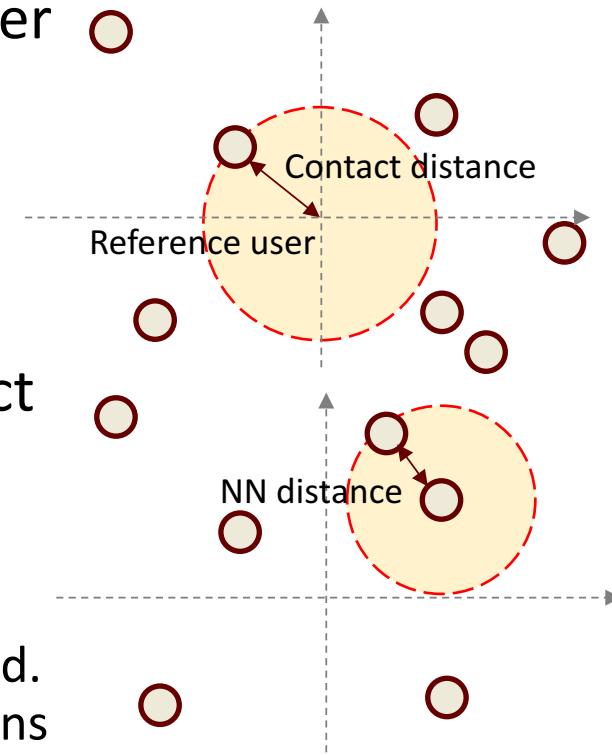
Characterize sum-product functional of a PCP and PPP

Max SIR

We will introduce important distance distributions of interest to the max-power case next and then go into the analysis of max-SIR case.

# Nearest-neighbor and Contact Distance Distribution of PCP

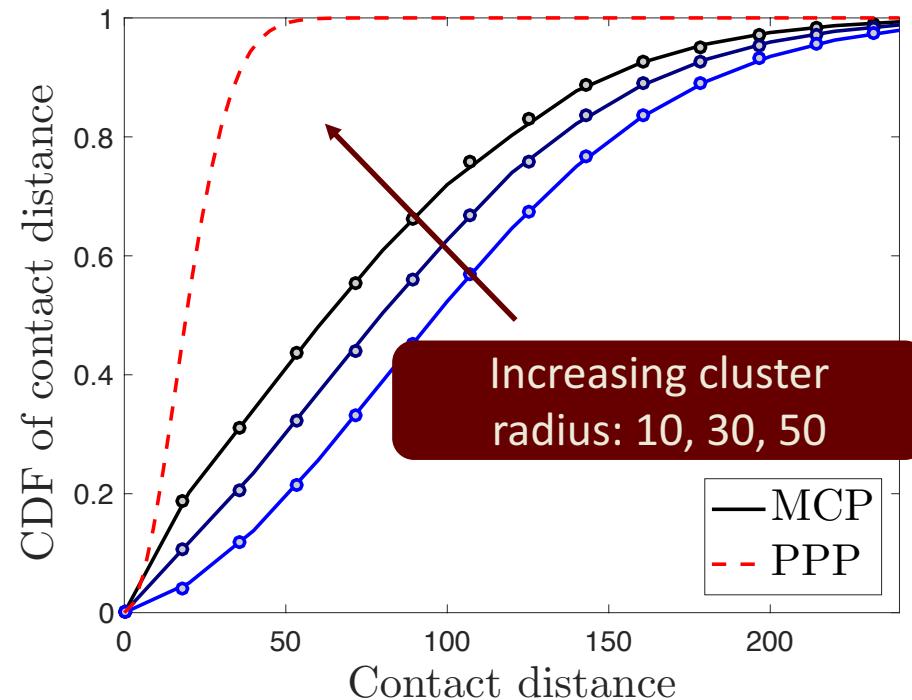
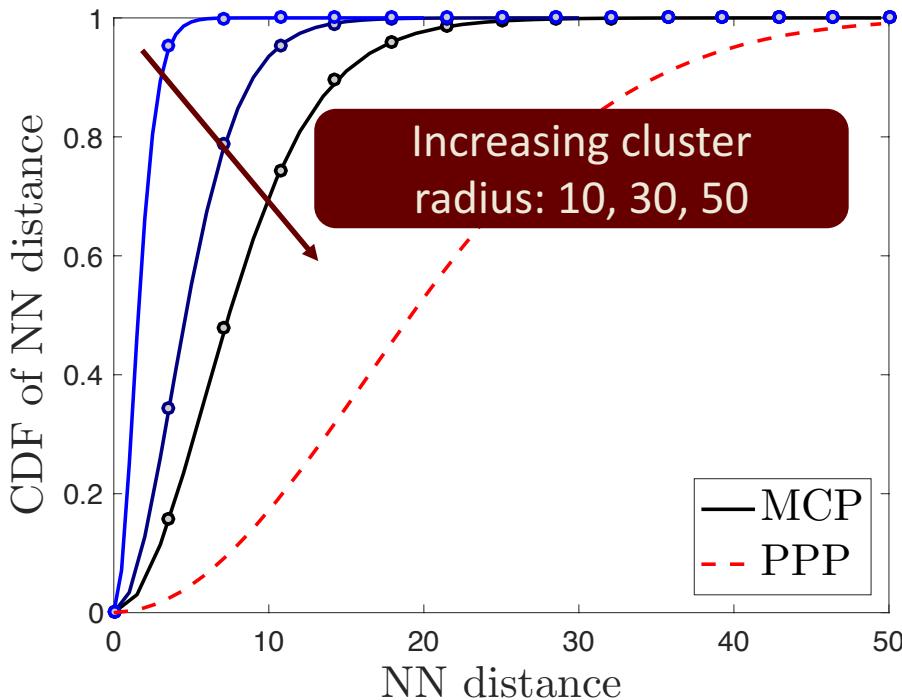
- \* The first step in the analysis under max-power association is to find the distribution of the distance from the typical user to the BS providing the maximum received power.
- \* Mathematically, this corresponds to determining nearest-neighbor and/or contact distance distribution of the point process.
  - \* For PPP, these distributions are same and are  $\text{Rayleigh}((2\pi\lambda)^{-1})$ .
  - \* For PCP, these distributions were not well-studied. We have recently characterized these distributions for TCP and MCP in [3,4].



[3] M. Afshang, C. Saha, H. S. Dhillon, “Nearest-Neighbor and Contact Distance Distributions for Thomas Cluster Process”, *IEEE Wireless Communications Letters*, 2017

[4] M. Afshang, C. Saha, H. S. Dhillon, “Nearest-Neighbor and Contact Distance Distributions for Matérn Cluster Process”, *IEEE Communications Letters*, 2017

# Nearest-neighbor and Contact Distance Distribution of PCP



The nearest-neighbor and contact distance distributions of PCP significantly differ from that of PPP.

For the plot, we considered  $\lambda_p=20$  clusters per Km<sup>2</sup>

# Max-SIR Case: Coverage Probability

\* Coverage probability, or equivalently CCDF of SIR , is:

$$P_c = \mathbb{P}(\text{SIR} \geq \beta) = \mathbb{P}\left(\bigcup_{k \in \mathcal{K}} \bigcup_{\mathbf{x} \in \Phi_k} \frac{P_k h_{\mathbf{x}} \|\mathbf{x}\|^{-\alpha}}{\mathcal{I}(\Phi_k \setminus \{\mathbf{x}\}) + \sum_{j \in \mathcal{K} \setminus \{k\}} \mathcal{I}(\Phi_j)} > \beta\right)$$

where  $\mathcal{I}(\Phi_i) = \sum_{\mathbf{y} \in \Phi_i} P_i h_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha}$  is the interference from all BSs in  $\Phi_i$ .

\* For  $\beta > 1$ , union bound holds with equality and we can express  $P_c$  in terms of the *per-tier coverage probability*:

$$P_c = \sum_{k \in \mathcal{K}} P_{c(k)} = \sum_{k \in \mathcal{K}} \mathbb{E}\left[\sum_{\mathbf{x} \in \Phi_k} \mathbf{1}\left(\frac{P_k h_{\mathbf{x}} \|\mathbf{x}\|^{-\alpha}}{\mathcal{I}(\Phi_k \setminus \{\mathbf{x}\}) + \sum_{j \in \mathcal{K} \setminus \{k\}} \mathcal{I}(\Phi_j)} > \beta\right)\right]$$

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[5] C. Saha, M. Afshang, H. S. Dhillon, “3GPP-inspired HetNet Model using Poisson Cluster Process: Sum-product Functionals and Downlink Coverage”, 2017, available online: arxiv.org/pdf/1705.01699.

# Coverage Probability

- \* By Rayleigh fading assumption for the serving link:

$$\sum_{k \in \mathcal{K}} \mathbb{E} \left[ \sum_{\mathbf{x} \in \Phi_k} \exp \left( -\frac{\beta}{P_k} (\mathcal{I}(\Phi_k \setminus \{\mathbf{x}\})) \right) \exp \left( -\frac{\beta}{P_k} \left( \sum_{j \in \mathcal{K} \setminus \{k\}} \mathcal{I}(\Phi_j) \right) \|\mathbf{x}\|^\alpha \right) \right]$$

- \* Taking expectation with respect to fading for the interfering links:

$$\sum_{k \in \mathcal{K}} \mathbb{E} \left[ \sum_{\mathbf{x} \in \Phi_k} \prod_{\mathbf{y} \in \Phi_k \setminus \{\mathbf{x}\}} \frac{1}{1 + \frac{\beta}{P_k} \left( \frac{\|\mathbf{x}\|}{\|\mathbf{y}\|} \right)^{-\alpha}} \prod_{j \in \mathcal{K} \setminus \{k\}} \exp \left( -\frac{\beta}{P_k} (\mathcal{I}(\Phi_j)) \|\mathbf{x}\|^\alpha \right) \right]$$

$P_{c(k)}$ : Per-tier Coverage

# Per-tier Coverage: Sum-Product Functional

$$\mathbb{E} \left[ \sum_{\mathbf{x} \in \Phi_k} \mathbb{E} \left[ \prod_{j \in \mathcal{K} \setminus \{k\}} \exp \left( -\frac{\beta}{P_k} (\mathcal{I}(\Phi_j)) \|\mathbf{x}\|^\alpha \right) \right] \right] \prod_{\mathbf{y} \in \Phi_k \setminus \{\mathbf{x}\}} \frac{1}{1 + \frac{\beta}{P_k} \left( \frac{\|\mathbf{x}\|}{\|\mathbf{y}\|} \right)^{-\alpha}}$$

$$g(\mathbf{x}) \qquad \qquad \qquad v(\mathbf{x}, \mathbf{y})$$

## Definition

Sum-product functional of a point process  $\Psi$  can be defined as:

$$\mathbb{E} \left[ \sum_{\mathbf{x} \in \Psi} g(\mathbf{x}) \prod_{\mathbf{y} \in \Psi \setminus \{\mathbf{x}\}} v(\mathbf{x}, \mathbf{y}) \right],$$

where  $g(\mathbf{x}) : \mathbb{R}^2 \mapsto [0, 1]$  and  $v(\mathbf{x}, \mathbf{y}) : [\mathbb{R}^2 \times \mathbb{R}^2] \mapsto [0, 1]$  are measurable.

- \* There are more general versions of sum-product functionals\*.
- \* There will be different expressions of the sum-product functional depending on  $\Phi_k$  ( $k \in \mathcal{K}$ ).

\* U. Schilcher, S. Toumpis, M. Haenggi, A. Crismani, G. Brandner and C. Bettstetter, "Interference Functionals in Poisson Networks," in *IEEE Trans. Inf. Theory*, Jan. 2016.

# Sum product functional

\* We require sum-product functional of  $\Phi_k$  when:

Case 1:

$\Phi_k$  is PPP

Case 2:

$\Phi_k$  is PCP

Case 3:

$\Phi_k = \Phi_0$

Campbell-Mecke Theorem can be applied to evaluate sum-product functional.

- \* The third case occurs when the users are modeled as PCP having same parent PPP as that of some PCP distributed BS tier.
- \* Has to be evaluated separately as  $\Phi_0$  is a **finite** PP.

# Sum product functional: PPP

## Sum Product Functional: PPP

The sum product functional of  $\Psi$  when  $\Psi$  is a PPP can be expressed as follows:

$$\mathbb{E} \left[ \sum_{\mathbf{x} \in \Psi} g(\mathbf{x}) \prod_{\mathbf{y} \in \Psi \setminus \{\mathbf{x}\}} v(\mathbf{x}, \mathbf{y}) \right] = \int_{\mathbb{R}^2} g(\mathbf{x}) G(v(\mathbf{x}, \mathbf{y})) \Lambda(d\mathbf{x}),$$

where  $\Lambda(\cdot)$  is the mean measure of  $\Psi$ ,  $G(\cdot)$  denotes the PGFL of  $\Psi$ .

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[1] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, “Modeling and analysis of K-tier downlink heterogeneous cellular networks,” *IEEE Journal on Sel. Areas in Commun.*, 2012.

# Sum product functional: PCP

- \* In PCP, selecting a point  $\mathbf{x}$  implies selecting a tuple  $(\mathbf{x}, \mathbf{z})$ , where  $\mathbf{z}$  is the cluster center of  $\mathbf{x}$ .

## Sum Product Functional: PCP

The sum product functional of  $\Psi$  when  $\Psi$  is a PCP can be expressed as follows:

$$\mathbb{E} \left[ \sum_{\mathbf{x} \in \Psi} g(\mathbf{x}) \prod_{\mathbf{y} \in \Psi \setminus \{\mathbf{x}\}} v(\mathbf{x}, \mathbf{y}) \right] = \iint_{\mathbb{R}^2 \times \mathbb{R}^2} g(\mathbf{x}) G(v(\mathbf{x}, \mathbf{y})) \tilde{G}_c(v(\mathbf{x}, \mathbf{y}) | \mathbf{z}) \Lambda(d\mathbf{x}, d\mathbf{z}),$$

with

$$\Lambda(d\mathbf{x}, d\mathbf{z}) = \lambda_{p_k} \bar{m}_k f_k(\mathbf{x} - \mathbf{z}) d\mathbf{z} d\mathbf{x}$$

where  $G(\cdot)$  is the PGFL of  $\Psi$  and  $\tilde{G}_c(\cdot | \mathbf{z})$  is the PGFL of a cluster of  $\Psi$  centered at  $\mathbf{z}$  under its reduced Palm distribution.

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[5] C. Saha, M. Afshang, H. S. Dhillon, “3GPP-inspired HetNet Model using Poisson Cluster Process: Sum-product Functionals and Downlink Coverage”, 2017, available online: <https://arxiv.org/pdf/1705.01699>

# Sum product functional: A special Case

- \* Conditioning on the number of offspring points is the key to the analysis of sum product functional of **finite** PP  $\Phi_0$ , i.e., BSs in  $\Phi_k$  belonging to the same cluster center of the typical user.

## Sum Product Functional: Finite PP

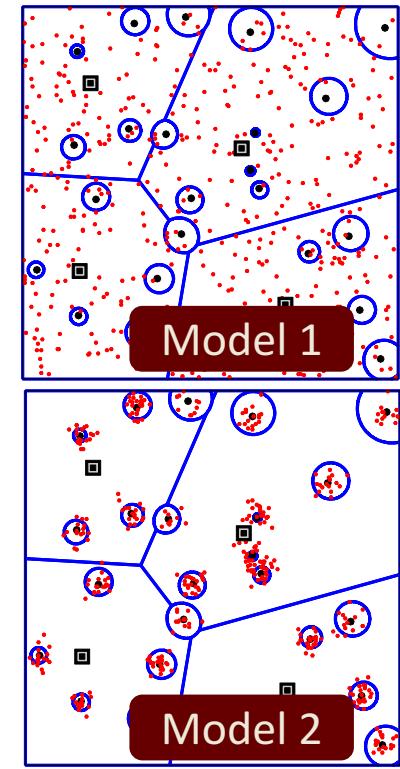
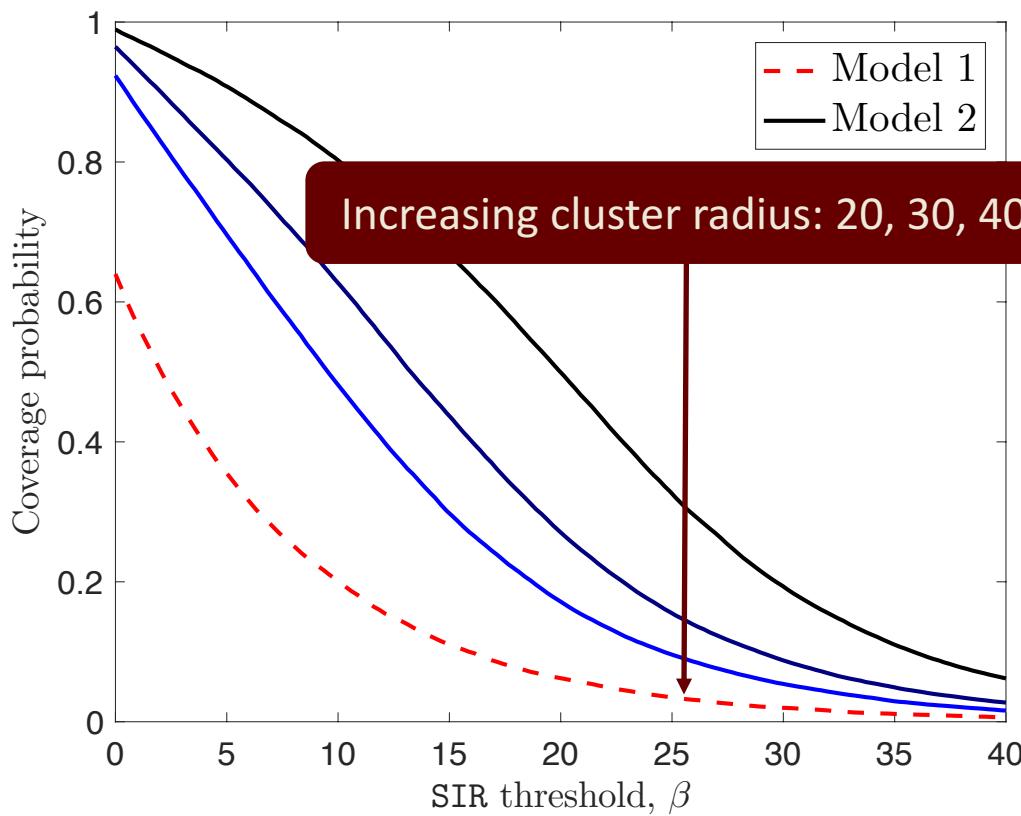
The sum product functional of  $\Psi$  with cluster center located at  $\mathbf{z}$  can be expressed as follows:

$$\begin{aligned} & \mathbb{E} \left[ \sum_{\mathbf{x} \in \Psi} g(\mathbf{x}) \prod_{\mathbf{y} \in \Psi \setminus \{\mathbf{x}\}} v(\mathbf{x}, \mathbf{y}) \right] \\ &= \sum_{n=1}^{\infty} \int_{\mathbb{R}^2} g(\mathbf{x}) \left( \int_{\mathbb{R}^2} (v(\mathbf{x}, \mathbf{y}) f_k(\mathbf{y} - \mathbf{z}) d\mathbf{y} \right)^{n-1} f_k(\mathbf{x} - \mathbf{z}) d\mathbf{x} n^2 \frac{p_k(n)}{\bar{m}_k} \end{aligned}$$

[5] C. Saha, M. Afshang, H. S. Dhillon, “3GPP-inspired HetNet Model using Poisson Cluster Process: Sum-product Functionals and Downlink Coverage”, 2017, available online: <https://arxiv.org/pdf/1705.01699>.

[6] C. Saha, M. Afshang, and H. S. Dhillon, “Poisson Cluster Process: Bridging the Gap Between PPP and 3GPP HetNet Models”, in *Proc. ITA*, San Diego, CA, Feb. 2017.

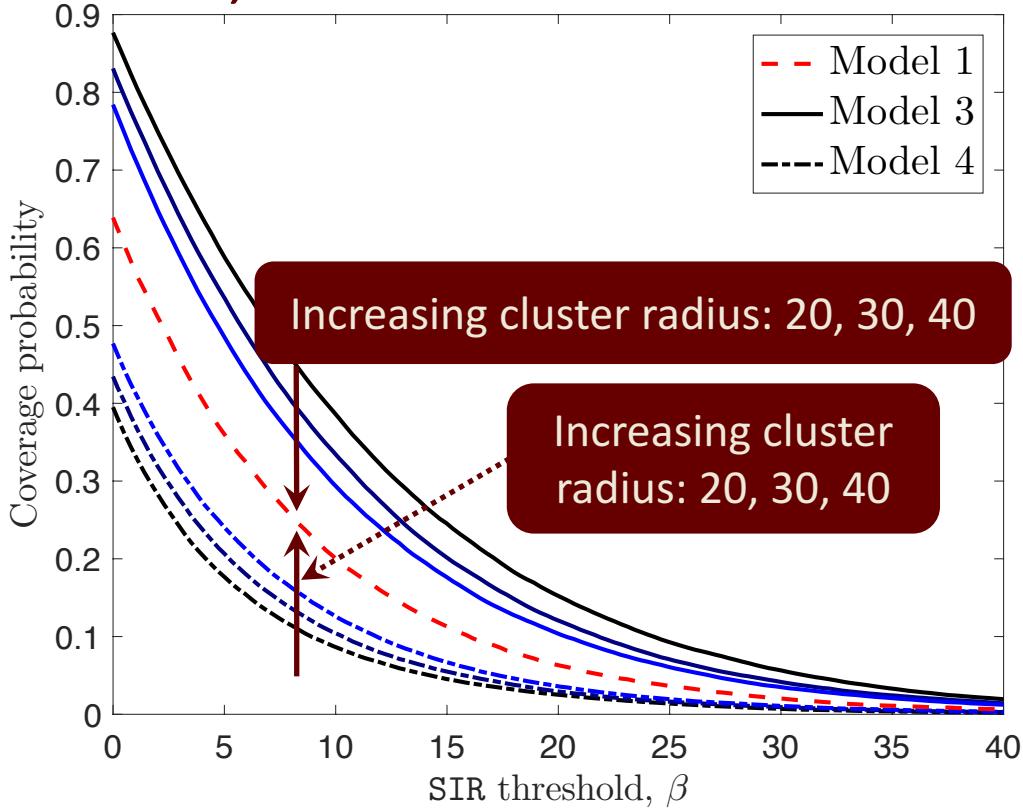
# Coverage Probability: Model 1 vs. Model 2



In Model 2, the coverage probability decreases as cluster radius increases and converges towards that of Model 1.

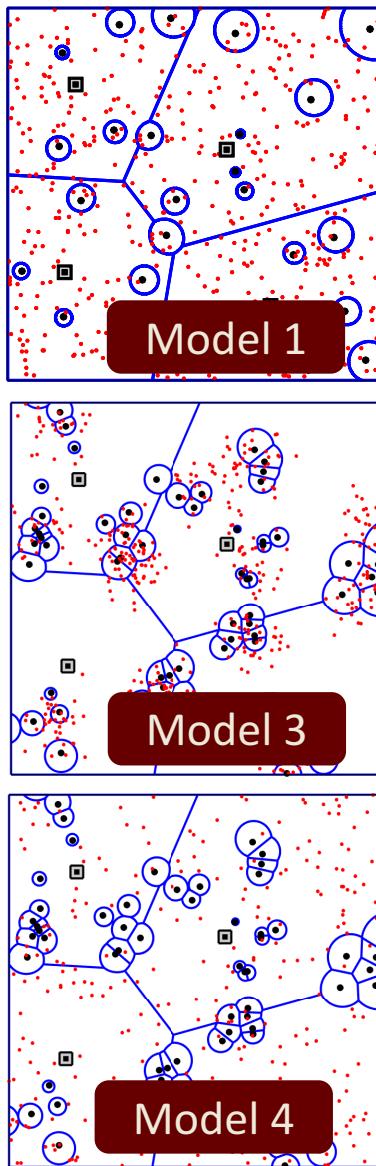
For the plot, we considered  $\lambda_1 = 1 \text{ Km}^{-2}$ ,  $P_1 = 1000 P_2$ , and  $\lambda_2 = 100 \lambda_1$

# Coverage Probability: Models 1, 3 and 4



Increasing cluster radius has a conflicting effect on coverage probability of Model 3 and 4: coverage probability of Model 4 increases while that of Model 3 decreases.

For the plot, we considered  $\lambda_1 = 1 \text{ Km}^{-2}$ ,  $P_1 = 1000 P_2$ ,  $m_2=3$ , and  $\lambda_2 = 100 \lambda_1$



# Limiting Behavior: Unified HetNet Model

- \* As the cluster size tends to infinity:
  - \* The PCP weakly converges to a PPP
  - \* The limiting PPP and the parent PPP become independent point processes.
- \* As a result, coverage probability of unified HetNet model reduces to

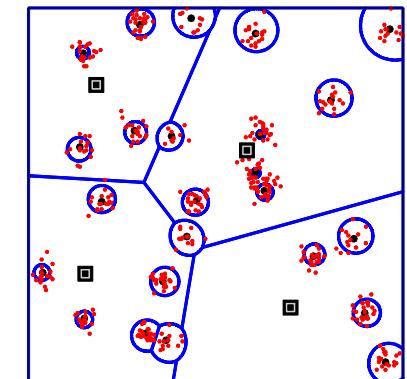
$$P_c = \frac{\pi}{C(\alpha)} \frac{\sum_{k \in \mathcal{K}_1} \frac{\lambda_k P_k^{\frac{2}{\alpha}}}{\beta_k^{\frac{2}{\alpha}}} + \sum_{k \in \mathcal{K}_2} \frac{\bar{m}_k \lambda_{p_k} P_k^{\frac{2}{\alpha}}}{\beta_k^{\frac{2}{\alpha}}}}{\sum_{j \in \mathcal{K}_2} \lambda_j P_j^{\frac{2}{\alpha}} + \sum_{j \in \mathcal{K}_2} \bar{m}_j \lambda_{p_j} P_j^{\frac{2}{\alpha}}},$$

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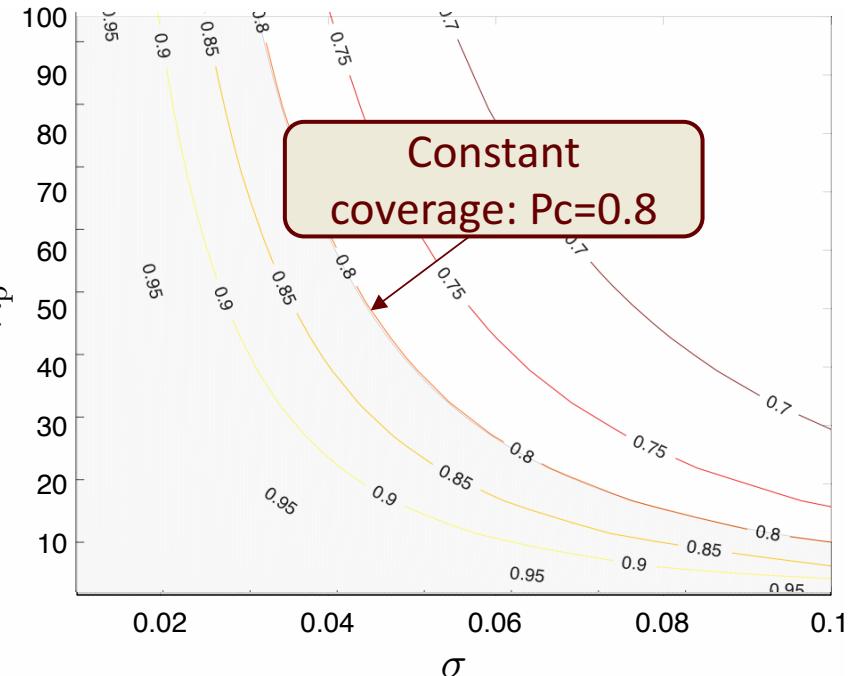
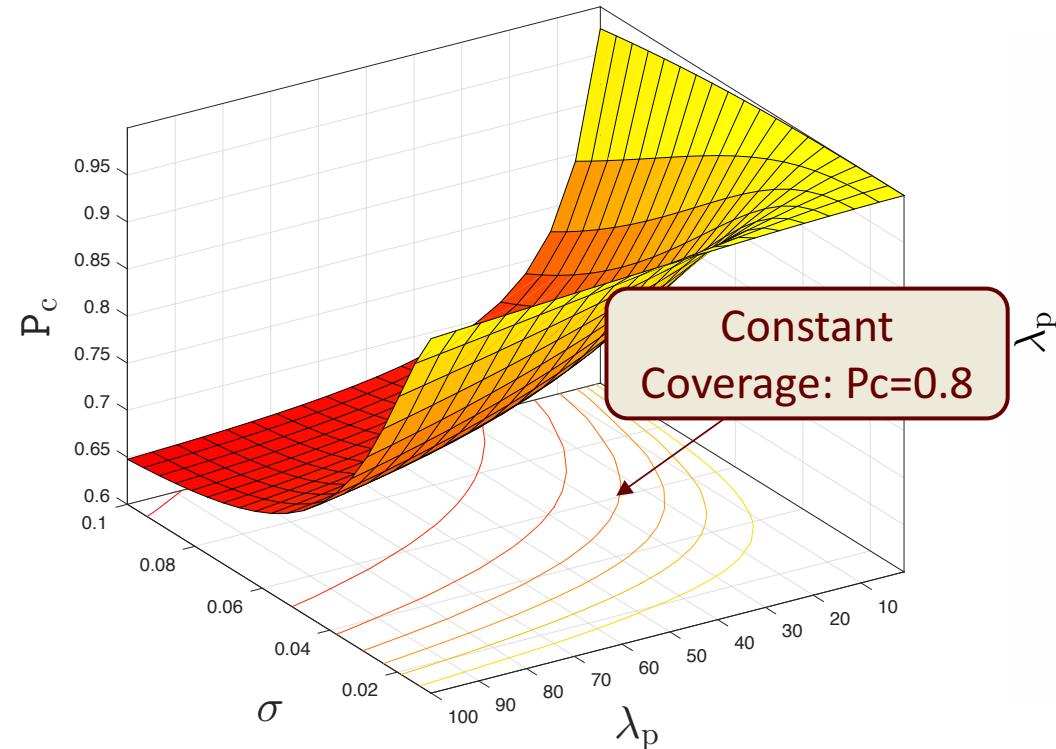
[5] C. Saha, M. Afshang, H. S. Dhillon, “3GPP-inspired HetNet Model using Poisson Cluster Process: Sum-product Functionals and Downlink Coverage”, 2017, available online: <https://arxiv.org/pdf/1705.01699.pdf>.

# Representative Insight: Scale Invariance of Coverage Probability

- \* While the result itself is more general, we introduce this in the context for Model 2 for the sake of simplicity
- \* Consider a two tier HetNet (Model 2) with tier 1 and tier 2 being homogeneous PPPs of densities  $\lambda_1$  and  $\lambda_p$  and users form PCP around BSs of tier 2.
  - \* Users form TCP around tier 2 BSs with cluster variance  $\sigma^2$
  - Scale invariance when  $\lambda_P \sigma^2 = \text{constant}$**
  - \* MCP with cluster radius  $r_d$
  - Scale invariance when  $\lambda_P r_d^2 = \text{constant}$**
- \* This result is valid for Models 3 and 4 as well.



# Scale-invariance of Coverage Probability



Coverage probability remains the same if  $\lambda_p \sigma^2$  and/or  $\lambda_p r_d^2$  are constants.

For the plot, we considered SIR threshold,  $\beta = 0$  dB.

# Summary and Conclusions

- \* Introduced new spatial models that are much closer to the ones used in 3GPP simulations while being almost as tractable as the popular PPP model
- \* Presented some represented analytical results
- \* The performance trends are highly sensitive to the choice of spatial models (not surprising at all)
  - \* PPP model is clearly not sufficient to study all possible configurations of users and small cell BSs.
- \* These new models also provide a bridge between stochastic geometry and 3GPP simulations

# Most Relevant Publications

## Heterogeneous Cellular Networks

- \* H. S. Dhillon, R. K. Ganti, F. Baccelli and J. G. Andrews, “**Modeling and Analysis of K-Tier Downlink Heterogeneous Cellular Networks**”, *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 3, pp. 550-560, Apr. 2012.
- \* C. Saha, M. Afshang, H. S. Dhillon, “**Enriched K-Tier HetNet Model to Enable the Analysis of User-Centric Small Cell Deployments**”, *IEEE Trans. on Wireless Commun.*, vol. 16, no. 3, pp. 1593-1608, Mar. 2017.
- \* C. Saha, M. Afshang, and H. S. Dhillon, “**Poisson Cluster Process: Bridging the Gap Between PPP and 3GPP HetNet Models**”, in *Proc. ITA*, San Diego, CA, Feb. 2017.
- \* C. Saha, M. Afshang, H. S. Dhillon, “**3GPP-inspired HetNet Model using Poisson Cluster Process: Sum-product Functionals and Downlink Coverage**”, submitted. Available online: arxiv.org/pdf/1705.01699.
- \* M. Afshang, H. S. Dhillon, “**Poisson Cluster Process Based Analysis of HetNets with Correlated User and Base Station Locations**”, submitted. Available online: arxiv.org/abs/1612.0728.

## Distance Distributions in PCPs

- \* M. Afshang, C. Saha, and H. S. Dhillon, “**Nearest-Neighbor and Contact Distance Distributions for Thomas Cluster Process**”, *IEEE Wireless Commun. Letters*, 2017.
- \* M. Afshang, C. Saha, and H. S. Dhillon, “**Nearest-Neighbor and Contact Distance Distributions for Matern Cluster Process**”, *IEEE Commun. Letters*, 2017.

## Applications of PCPs to D2D Networks

- \* M. Afshang, H. S. Dhillon, and P. H. J. Chong , “**Modeling and Analysis of Clustered Device-to-Device Networks**” *IEEE Trans. on Wireless Commun.*, 2016.
- \* M. Afshang, H. S. Dhillon, and P. H. J. Chong, “**Fundamentals of Cluster-Centric Content Placement in Cache-Enabled Device-to-Device Networks**” *IEEE Trans. on Commun.*, 2016.

# Thank You for Your Attention

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