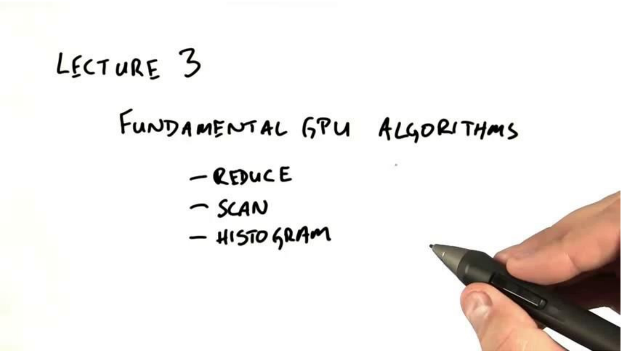
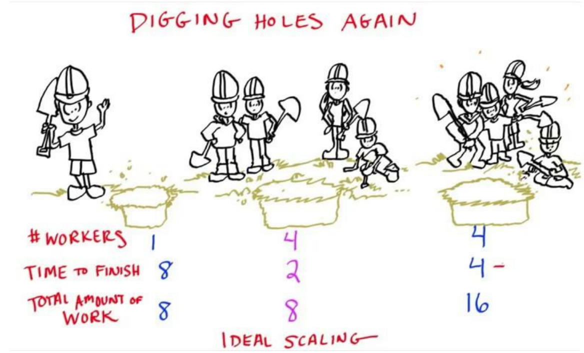
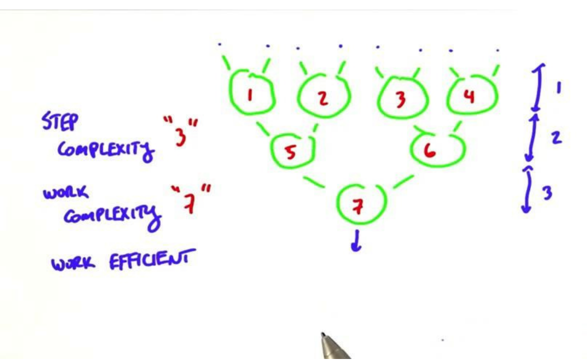
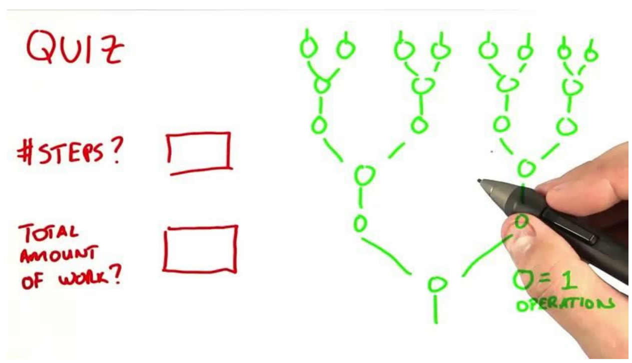
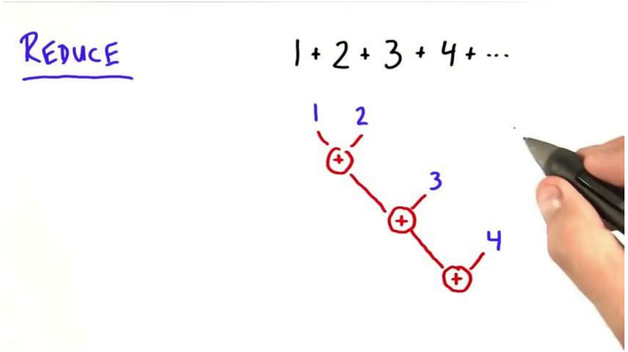
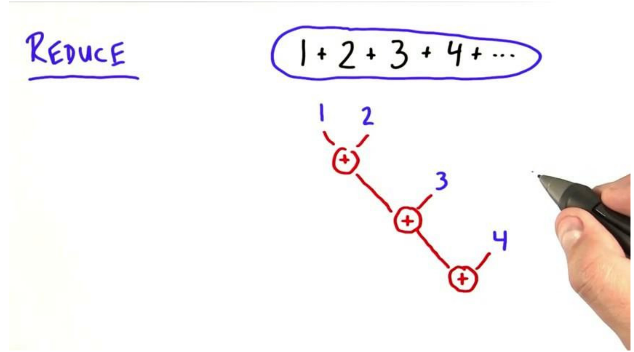

LECTURE 3
FUNDAMENTAL GPU ALGORITHMS
- REDUCE
- SCAN
- HISTOGRAM

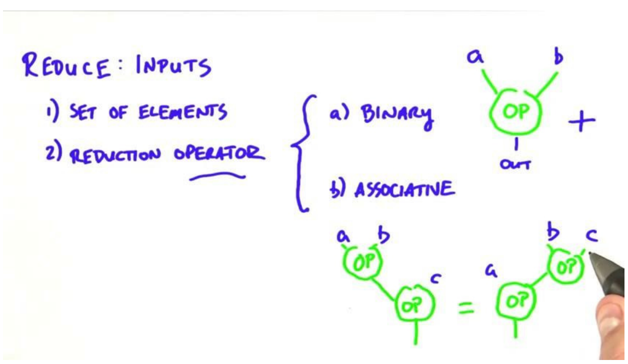

DIGGING HOLES AGAIN
# WORKERS 1 4 4
TIME TO FINISH 8 2 4
TOTAL AMOUNT OF WORK 8 8 16
IDEAL SCALING

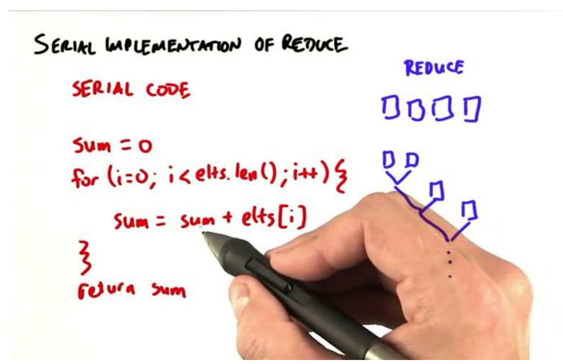

STEP COMPLEXITY "3"
WORK COMPLEXITY "7"
WORK EFFICIENT
1 2 3 4 5 6 7
1
2
3

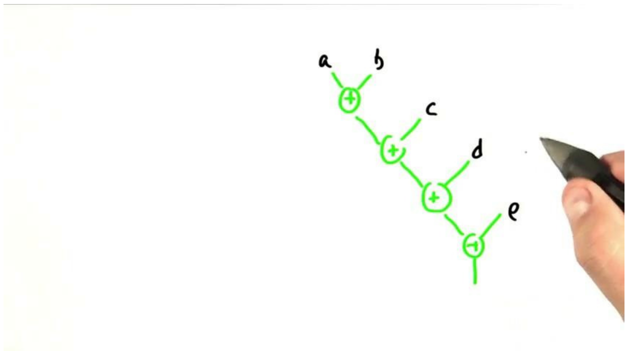

QUIZ
# STEPS?
TOTAL AMOUNT OF WORK?
O = 1 OPERATIONS

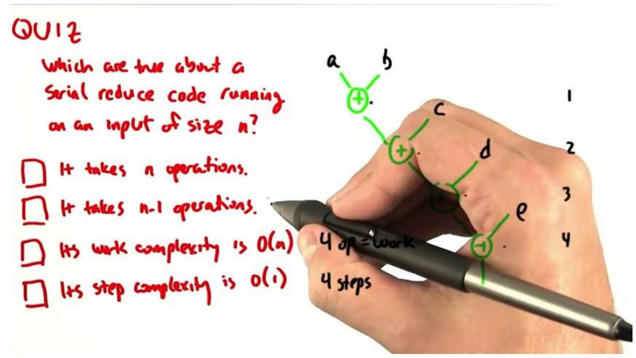

REDUCE
1 + 2 + 3 + 4 + ...
1
2
+
3
+
4
+

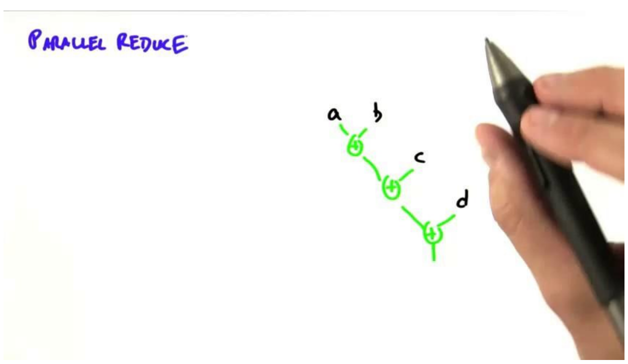

REDUCE
1 + 2 + 3 + 4 + ...
1
2
+
3
+
4
+

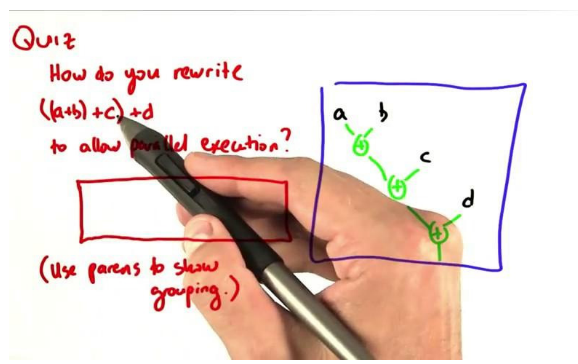

REDUCE: INPUTS
1) SET OF ELEMENTS
2) REDUCTION OPERATOR
a) BINARY
a
b
OP
OUT
+
b) ASSOCIATIVE
a
b
OP
c
OP
=
a
b
c
OP
OP

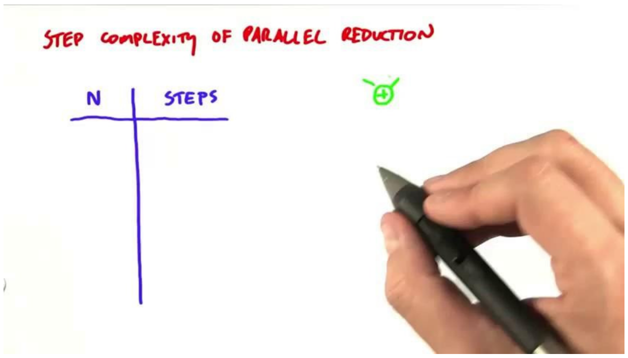

SERIAL IMPLEMENTATION OF REDUCE
SERIAL CODE
sum = 0
for (i=0; i<elts.len(); i++) {
sum = sum + elts[i]
}
return sum
REDUCE

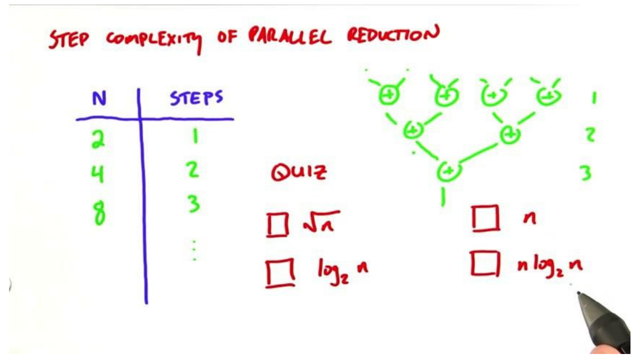

a
b
c
d
e

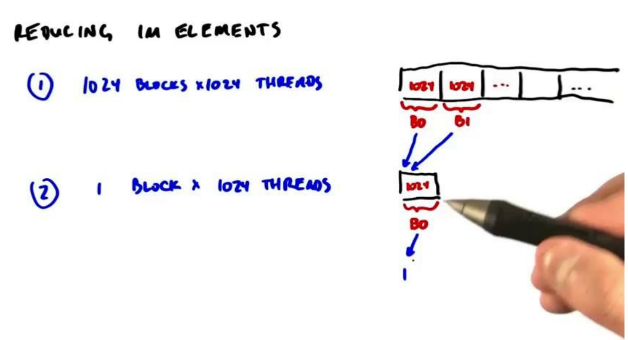

QUIZ
Which are true about a serial reduce code running on an input of size n?
It takes n operations.
It takes n-1 operations.
Its work complexity is O(n)
Its step complexity is O(1)
a
b
c
d
e
1
2
3
4
4 op = work
4 steps

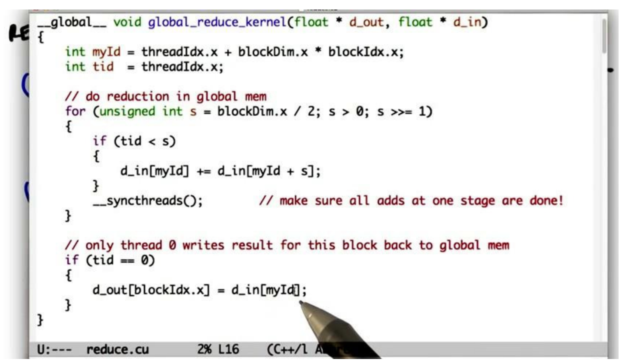

PARALLEL REDUCE
a
b
c
d


QUIZ
How do you rewrite
((a+b)+c)+d
to allow parallel execution?
(Use parens to show grouping.)
a
b
c
d

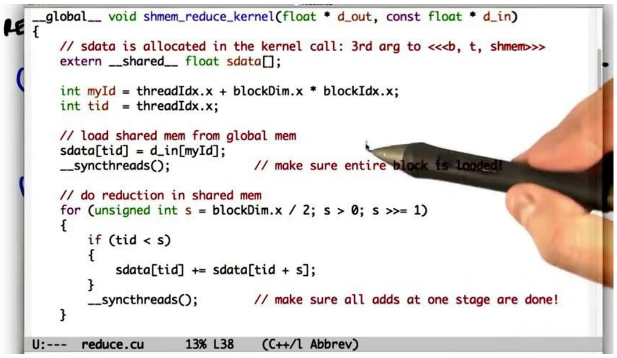


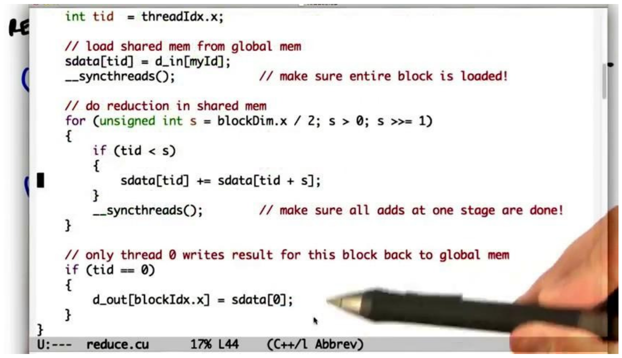



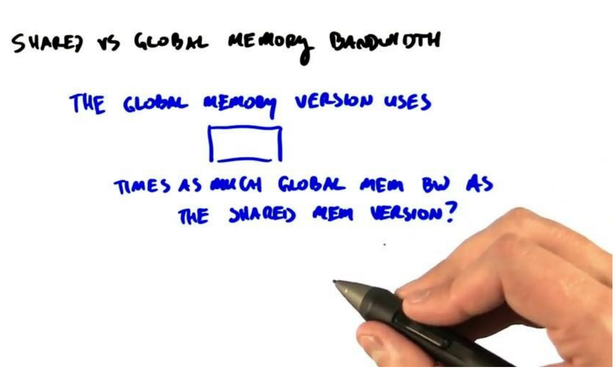
```
__global__ void global_reduce_kernel(float * d_out, float * d_in)
{
    int myId = threadIdx.x + blockDim.x * blockIdx.x;
    int tid  = threadIdx.x;

    // do reduction in global mem
    for (unsigned int s = blockDim.x / 2; s > 0; s >>= 1)
    {
        if (tid < s)
        {
            d_in[myId] += d_in[myId + s];
        }
        __syncthreads();        // make sure all adds at one stage are done!
    }

    // only thread 0 writes result for this block back to global mem
    if (tid == 0)
    {
        d_out[blockIdx.x] = d_in[myId];
    }
}
U:--- reduce.cu      2% L16     (C++/l A
```



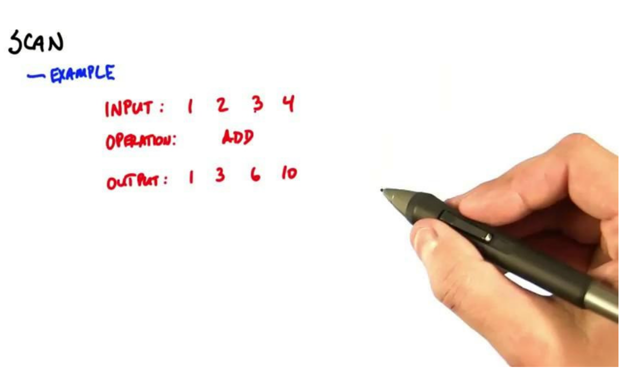
__global__ void shmem_reduce_kernel(float * d_out, const float * d_in)
{
// sdata is allocated in the kernel call: 3rd arg to <<<b, t, shmem>>>
extern __shared__ float sdata[];
int myId = threadIdx.x + blockDim.x * blockIdx.x;
int tid = threadIdx.x;
// load shared mem from global mem
sdata[tid] = d_in[myId];
__syncthreads(); // make sure entire block is loaded!
// do reduction in shared mem
for (unsigned int s = blockDim.x / 2; s > 0; s >>= 1)
{
if (tid < s)
{
sdata[tid] += sdata[tid + s];
}
__syncthreads(); // make sure all adds at one stage are done!
}
U:--- reduce.cu 13% L38 (C++/l Abbrev)

int tid = threadIdx.x;
// load shared mem from global mem
sdata[tid] = d_in[myId];
__syncthreads(); // make sure entire block is loaded!
// do reduction in shared mem
for (unsigned int s = blockDim.x / 2; s > 0; s >>= 1)
{
if (tid < s)
{
sdata[tid] += sdata[tid + s];
}
__syncthreads(); // make sure all adds at one stage are done!
}
// only thread 0 writes result for this block back to global mem
if (tid == 0)
{
d_out[blockIdx.x] = sdata[0];
}
}
U:--- reduce.cu 17% L44 (C++/l Abbrev)

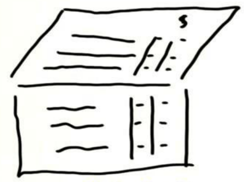
SHARED VS GLOBAL MEMORY BANDWIDTH
THE GLOBAL MEMORY VERSION USES
TIMES AS MUCH GLOBAL MEM BW AS
THE SHARED MEM VERSION?

SCAN
– EXAMPLE
INPUT: 1 2 3 4
OPERATION: ADD
OUTPUT: 1 3 6 10

SCAN

- EXAMPLE
  - INPUT : 1 2 3 4
  - OPERATION: ADD
  - OUTPUT : 1 3 6 10
- ADDRESSES SET OF PROBLEMS OTHERWISE DIFFICULT TO PARALLELIZE
- NOT USEFUL IN SERIAL WORLD BUT VERY USEFUL IN PARALLEL
- TODAY: EXPLAINING WHAT + HOW
  BUT NOT WHY (NEXT LECTURE)




your CHECKBOOK

| TRANSACTION | BALANCE |
|---|---|
| $ 20 | 20 |
| 5 | 25 |
| -11 | 14 |
| -9 | 5 |
| -3 | 2 |
| 15 | 17 |
| INPUT | OUTPUT |

INPUTS TO SCAN

- INPUT ARRAY
- BINARY ASSOCIATIVE OPERATOR } LIKE REDUCE
- IDENTITY ELEMENT [ I op a = a ]

| OP | I | BECAUSE |
|---|---|---|
| + | Ø | Ø + a = a |
| min (on unsigned chars) | ØxFF | min(ØxFF, a) = a |

QUIZ

WHAT IS THE IDENTITY FOR ...

Multiply $\boxed{\phantom{xx}}$

Logical or $\boxed{\phantom{xx}}$

Logical and $\boxed{\phantom{xx}}$

## What Scan Does

Input: array A, operator $\oplus$, identity $I$

$$[a_0 \quad a_1 \quad a_2 \quad a_3 \quad \cdots \quad a_{n-1}] \quad \text{INPUT}$$

$$[I \quad a_0 \quad a_0 \oplus a_1 \quad a_0 \oplus a_1 \oplus a_2 \quad \cdots \quad a_0 \oplus a_1 \oplus a_2 \oplus \cdots a_{n-2}] \quad \text{OUTPUT}$$

## What Scan Does

Input: array A, operator $\oplus$ (PLUS), identity $I$ (Ø)

$$[a_0 \quad a_1 \quad a_2 \quad a_3 \quad \cdots \quad a_{n-1}] \quad \text{INPUT}$$

$$[I \quad a_0 \quad a_0 \oplus a_1 \quad a_0 \oplus a_1 \oplus a_2 \quad \cdots \quad a_0 \oplus a_1 \oplus a_2 \oplus \cdots a_{n-2}] \quad \text{OUTPUT}$$

IN + [3 1 4 1 5 9]

OUT [0 3 4 8 9 14]

## QUIZ

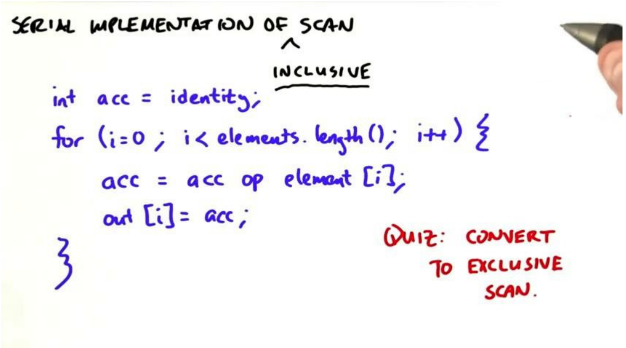
MAX-SCAN ON UNSIGNED INTS [3 1 4 1 5 9]

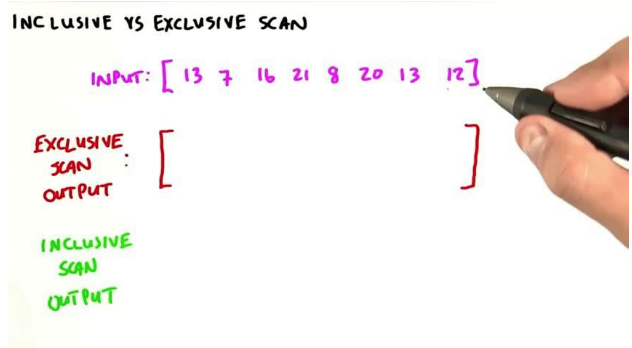
IDENTITY? $\boxed{\phantom{x}}$

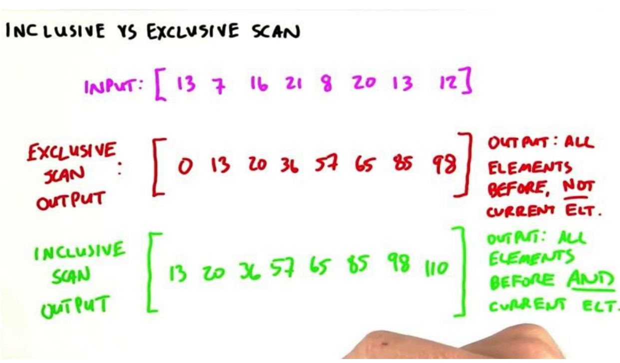
OUTPUT? [ $\boxed{\phantom{x}}$ $\boxed{\phantom{x}}$ $\boxed{\phantom{x}}$ $\boxed{\phantom{x}}$ $\boxed{\phantom{x}}$ $\boxed{\phantom{x}}$ ]

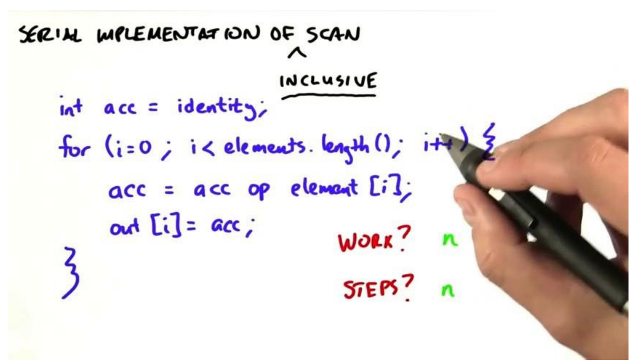
## SERIAL IMPLEMENTATION OF SCAN

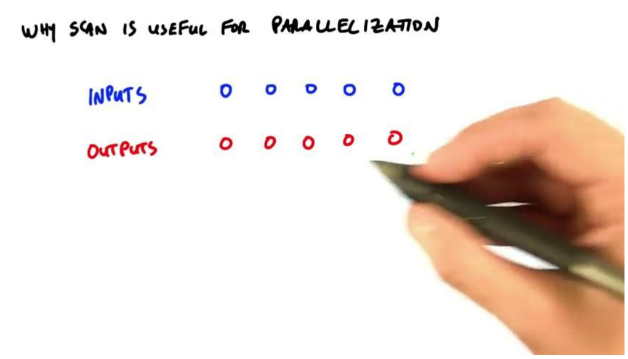
```
int acc = identity;
for (i=0; i< elements.length(); i++) {
    acc = acc op element[i];
    out[i] = acc;
}
```

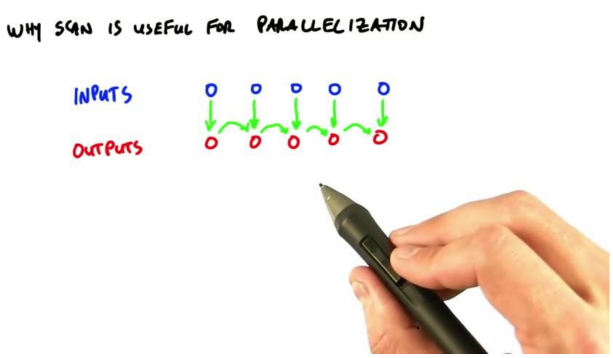

SERIAL IMPLEMENTATION OF SCAN
INCLUSIVE
int acc = identity;
for (i=0 ; i< elements.length(); i++) {
acc = acc op element [i];
out [i] = acc;
}
QUIZ: CONVERT TO EXCLUSIVE SCAN.

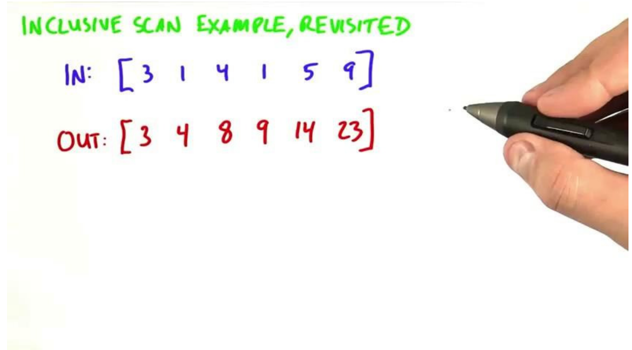

INCLUSIVE VS EXCLUSIVE SCAN
INPUT: [ 13 7 16 21 8 20 13 12 ]
EXCLUSIVE SCAN OUTPUT: [ ]
INCLUSIVE SCAN OUTPUT

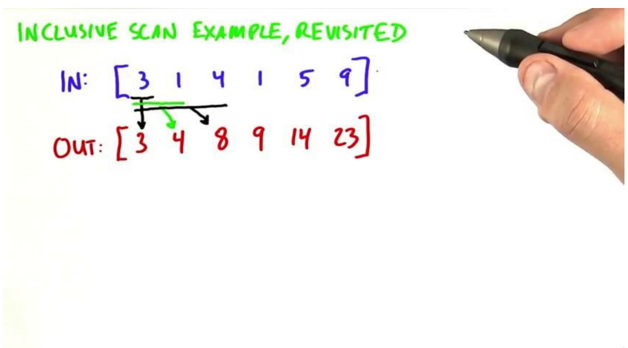

INCLUSIVE VS EXCLUSIVE SCAN
INPUT: [ 13 7 16 21 8 20 13 12 ]
EXCLUSIVE SCAN OUTPUT: [ 0 13 20 36 57 65 85 98 ]
OUTPUT: ALL ELEMENTS BEFORE, NOT CURRENT ELT.
INCLUSIVE SCAN OUTPUT [ 13 20 36 57 65 85 98 110 ]
OUTPUT: ALL ELEMENTS BEFORE AND CURRENT ELT.

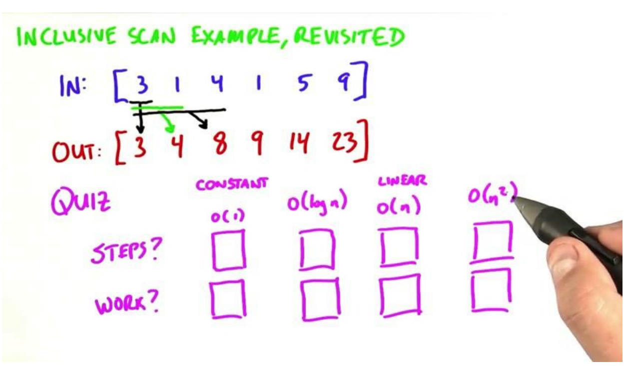

SERIAL IMPLEMENTATION OF SCAN
INCLUSIVE
int acc = identity;
for (i=0 ; i< elements.length(); i++) {
acc = acc op element [i];
out [i] = acc;
}
WORK? n
STEPS? n


WHY SCAN IS USEFUL FOR PARALLELIZATION
INPUTS
OUTPUTS

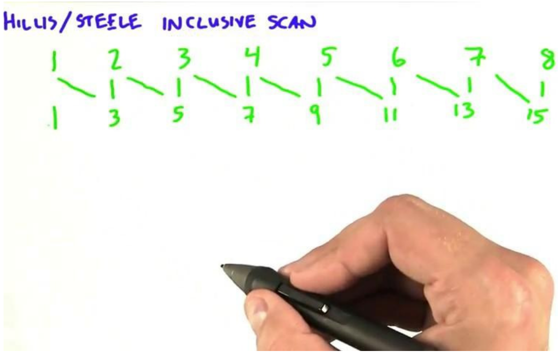

WHY SCAN IS USEFUL FOR PARALLELIZATION
INPUTS
OUTPUTS

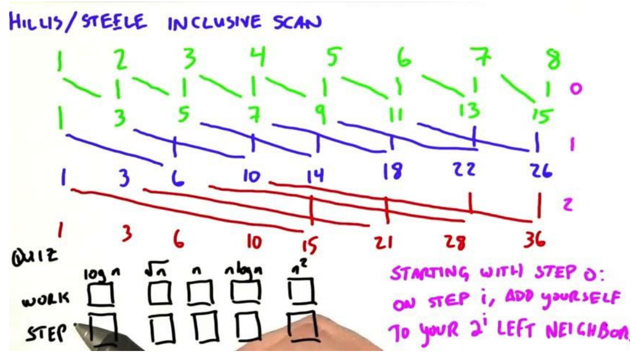

INCLUSIVE SCAN EXAMPLE, REVISITED
IN: [3 1 4 1 5 9]
OUT: [3 4 8 9 14 23]

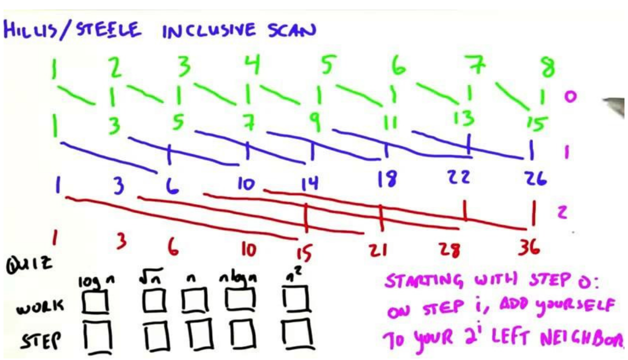

INCLUSIVE SCAN EXAMPLE, REVISITED
IN: [3 1 4 1 5 9]
OUT: [3 4 8 9 14 23]

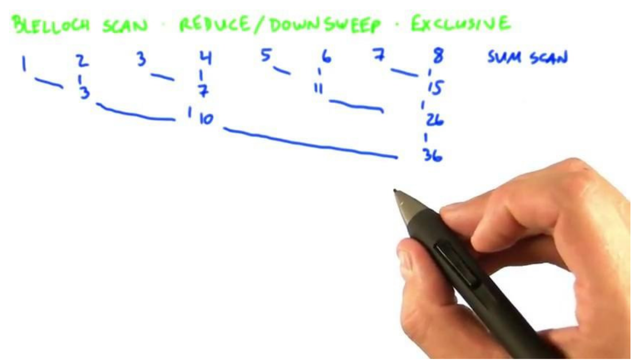

INCLUSIVE SCAN EXAMPLE, REVISITED
IN: [3 1 4 1 5 9]
OUT: [3 4 8 9 14 23]
QUIZ
CONSTANT
O(1)
O(log n)
LINEAR
O(n)
O(n²)
STEPS?
WORK?

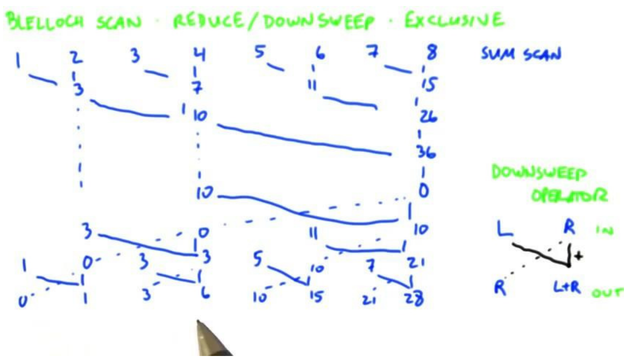

TWO PARALLEL SCAN ALGORITHMS
MORE STEP-EFFICIENT
MORE WORK-EFFICIENT
HILLIS + STEELE
BLELLOCH
X
X

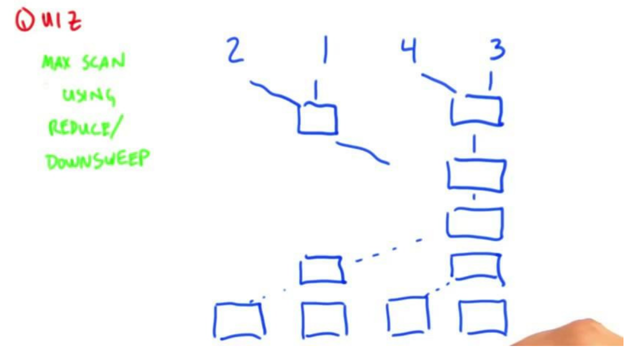

HILLIS/STEELE INCLUSIVE SCAN
1 2 3 4 5 6 7 8
1 3 5 7 9 11 13 15


HILLIS/STEELE INCLUSIVE SCAN
1 2 3 4 5 6 7 8
0
1 3 5 7 9 11 13 15
1
1 3 6 10 14 18 22 26
2
1 3 6 10 15 21 28 36
QUIZ
log n √n n n log n n²
WORK
STEP
STARTING WITH STEP 0:
ON STEP i, ADD YOURSELF
TO YOUR 2^i LEFT NEIGHBOR

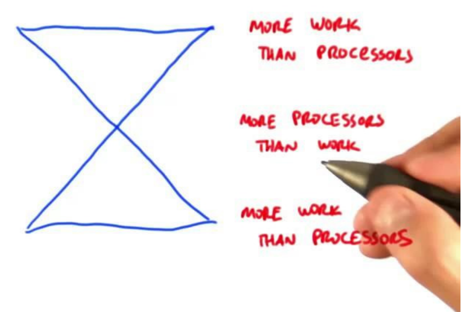

HILLIS/STEELE INCLUSIVE SCAN
1 2 3 4 5 6 7 8
0
1 3 5 7 9 11 13 15
1
1 3 6 10 14 18 22 26
2
1 3 6 10 15 21 28 36
QUIZ
log n √n n n log n n²
WORK
STEP
STARTING WITH STEP 0:
ON STEP i, ADD YOURSELF
TO YOUR 2^i LEFT NEIGHBOR

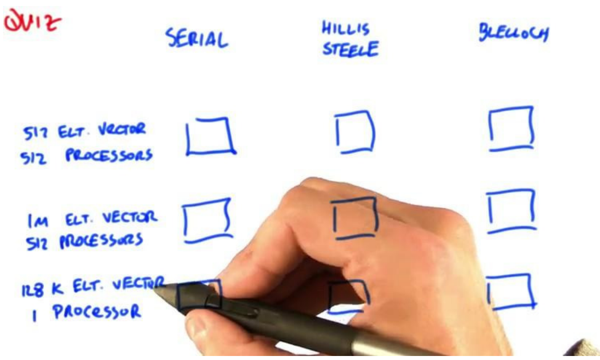

BLELLOCH SCAN · REDUCE/DOWNSWEEP · EXCLUSIVE
1 2 3 4 5 6 7 8
SUM SCAN
3 7 11 15
10 26
36

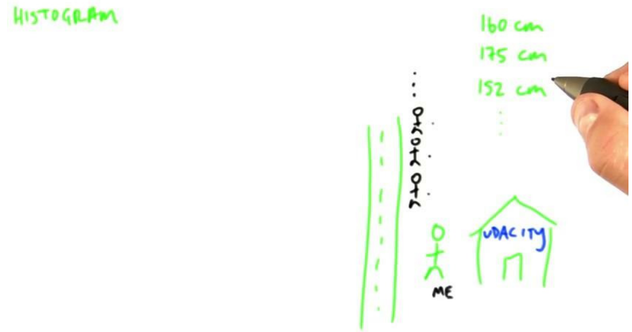

BLELLOCH SCAN · REDUCE/DOWNSWEEP · EXCLUSIVE
SUM SCAN
DOWNSWEEP OPERATOR
L R IN
R L+R OUT

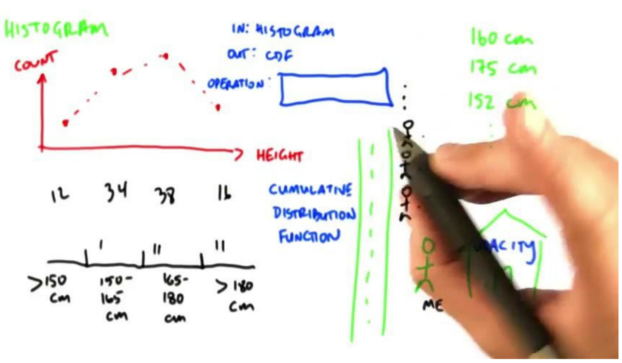

QUIZ
MAX SCAN USING REDUCE/ DOWNSWEEP
2 1 4 3

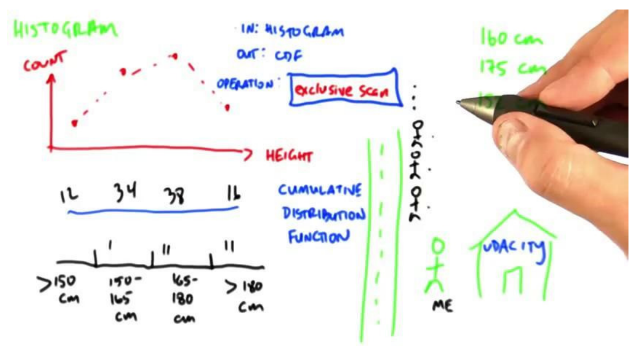

BLELLOCH SCAN · REDUCE/DOWNSWEEP · EXCLUSIVE
SUM SCAN
REDUCE
STEPS:
WORK:
DOWNSWEEP
STEPS:
WORK:

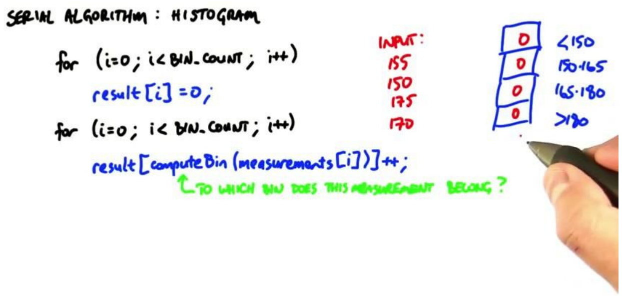

MORE WORK THAN PROCESSORS
MORE PROCESSORS THAN WORK
MORE WORK THAN PROCESSORS

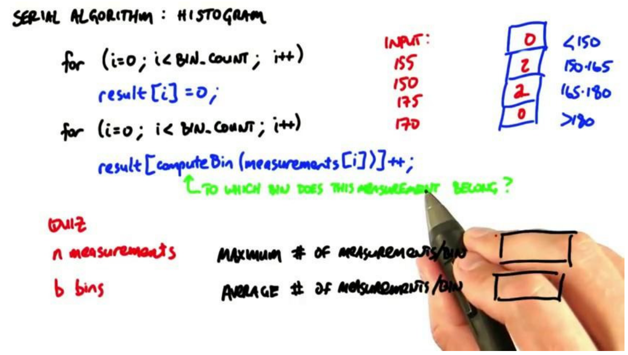

QUIZ
SERIAL
HILLIS STEELE
BLELLOCH
512 ELT. VECTOR
512 PROCESSORS
1M ELT. VECTOR
512 PROCESSORS
128 K ELT. VECTOR
1 PROCESSOR

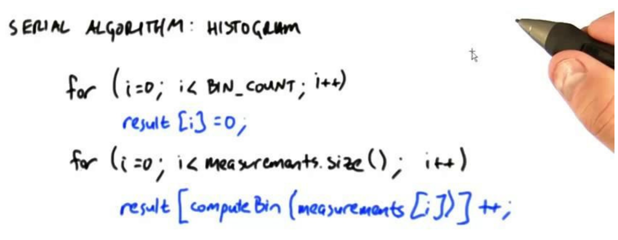

HISTOGRAM
160 cm
175 cm
152 cm
ME
UDACITY

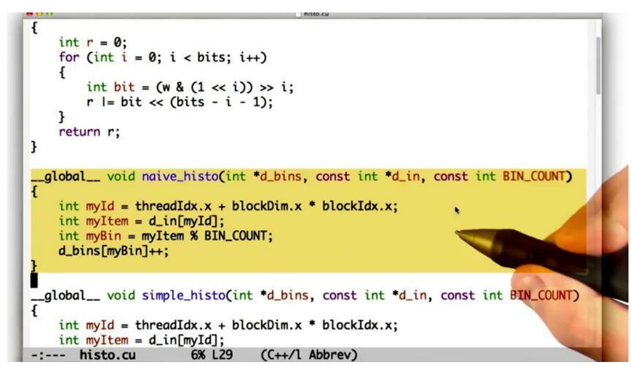

HISTOGRAM
COUNT
HEIGHT
IN: HISTOGRAM
OUT: CDF
OPERATION:
160 cm
175 cm
152 cm
12 34 38 16
CUMULATIVE
DISTRIBUTION
FUNCTION
>150 cm
150-165 cm
165-180 cm
>180 cm
ME

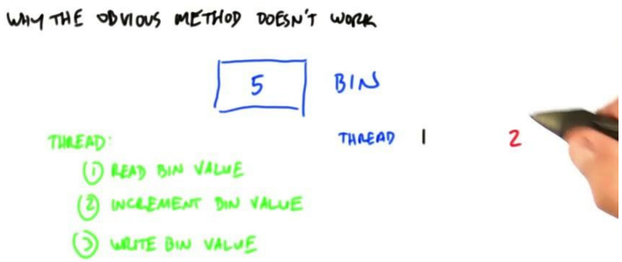

HISTOGRAM
COUNT
HEIGHT
IN: HISTOGRAM
OUT: CDF
OPERATION: exclusive scan
160 cm
175 cm
12 34 38 16
CUMULATIVE
DISTRIBUTION
FUNCTION
>150 cm
150-165 cm
165-180 cm
>180 cm
ME
UDACITY

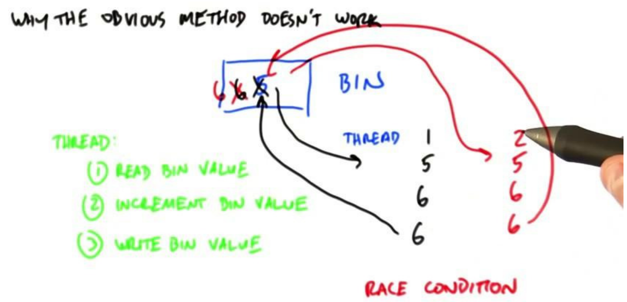

SERIAL ALGORITHM : HISTOGRAM
for (i=0; i< BIN_COUNT; i++)
result[i] = 0;
for (i=0; i< BIN_COUNT; i++)
result[computeBin(measurements[i])]++;
TO WHICH BIN DOES THIS MEASUREMENT BELONG?
INPUT:
155
150
175
170
0
0
0
0
<150
150-165
165-180
>180

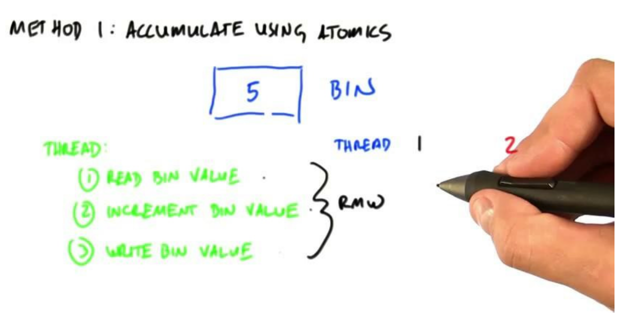

SERIAL ALGORITHM : HISTOGRAM
for (i=0; i< BIN_COUNT; i++)
result[i] = 0;
for (i=0; i< BIN_COUNT; i++)
result[computeBin(measurements[i])]++;
TO WHICH BIN DOES THIS MEASUREMENT BELONG?
INPUT:
155
150
175
170
0
2
2
0
<150
150-165
165-180
>180
QUIZ
n measurements
b bins
MAXIMUM # OF MEASUREMENTS/BIN
AVERAGE # OF MEASUREMENTS/BIN

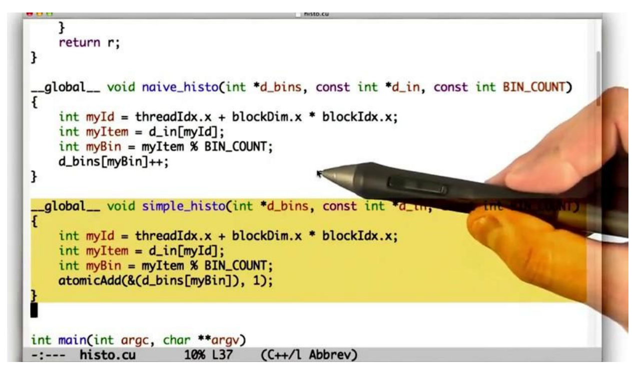

SERIAL ALGORITHM: HISTOGRAM
for (i=0; i< BIN_COUNT; i++)
result[i] = 0;
for (i=0; i< measurements.size(); i++)
result[computeBin(measurements[i])]++;

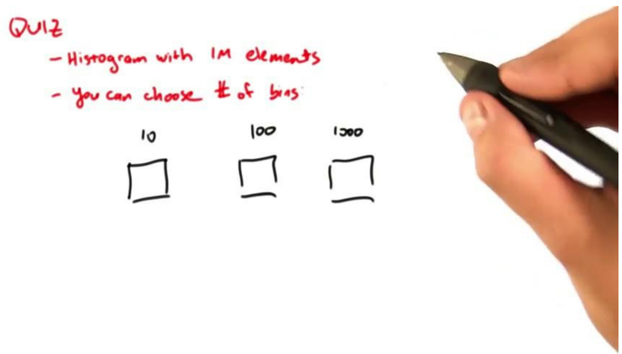

{
int r = 0;
for (int i = 0; i < bits; i++)
{
int bit = (w & (1 << i)) >> i;
r |= bit << (bits - i - 1);
}
return r;
}
__global__ void naive_histo(int *d_bins, const int *d_in, const int BIN_COUNT)
{
int myId = threadIdx.x + blockDim.x * blockIdx.x;
int myItem = d_in[myId];
int myBin = myItem % BIN_COUNT;
d_bins[myBin]++;
}
__global__ void simple_histo(int *d_bins, const int *d_in, const int BIN_COUNT)
{
int myId = threadIdx.x + blockDim.x * blockIdx.x;
int myItem = d_in[myId];
-:--- histo.cu 6% L29 (C++/l Abbrev)

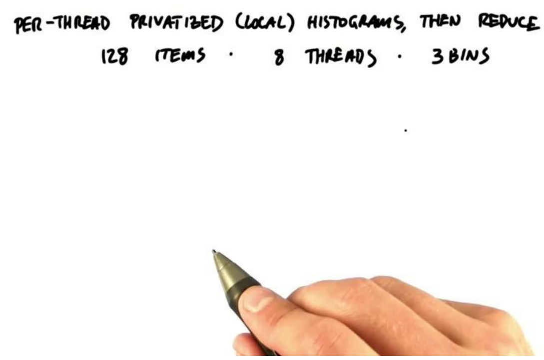

WHY THE OBVIOUS METHOD DOESN'T WORK
5
BIN
THREAD:
1 READ BIN VALUE
2 INCREMENT BIN VALUE
3 WRITE BIN VALUE
THREAD 1
2

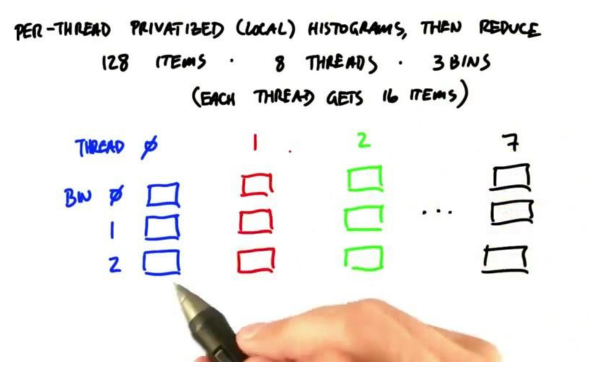

WHY THE OBVIOUS METHOD DOESN'T WORK
BIN
THREAD:
1 READ BIN VALUE
2 INCREMENT BIN VALUE
3 WRITE BIN VALUE
THREAD 1
5
6
6
2
5
6
6
RACE CONDITION

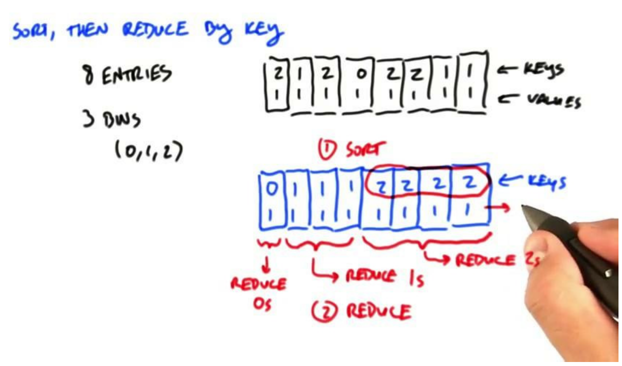

METHOD 1: ACCUMULATE USING ATOMICS
5
BIN
THREAD:
1 READ BIN VALUE
2 INCREMENT BIN VALUE
3 WRITE BIN VALUE
RMW
THREAD 1
2

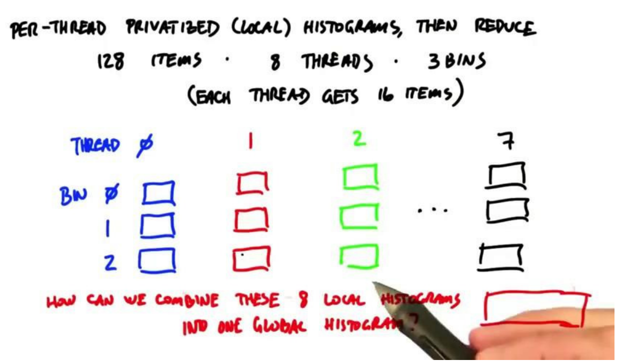

}
return r;
}
__global__ void naive_histo(int *d_bins, const int *d_in, const int BIN_COUNT)
{
int myId = threadIdx.x + blockDim.x * blockIdx.x;
int myItem = d_in[myId];
int myBin = myItem % BIN_COUNT;
d_bins[myBin]++;
}
__global__ void simple_histo(int *d_bins, const int
{
int myId = threadIdx.x + blockDim.x * blockIdx.x;
int myItem = d_in[myId];
int myBin = myItem % BIN_COUNT;
atomicAdd(&(d_bins[myBin]), 1);
}
int main(int argc, char **argv)
-:--- histo.cu 10% L37 (C++/l Abbrev)

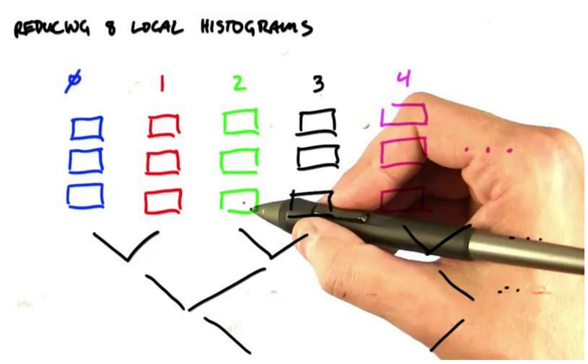

QUIZ
- Histogram with 1M elements
- You can choose # of bins
10
100
1000

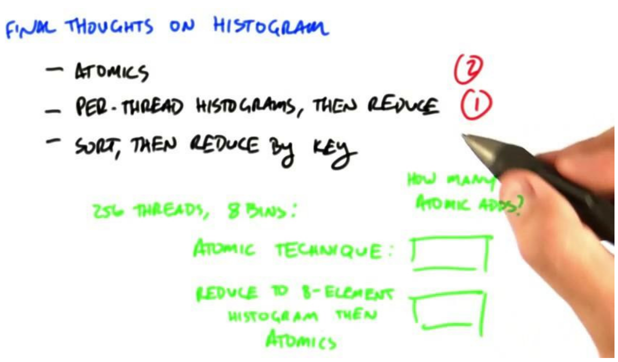

PER-THREAD PRIVATIZED (LOCAL) HISTOGRAMS, THEN REDUCE
128 ITEMS · 8 THREADS · 3 BINS


PER-THREAD PRIVATIZED (LOCAL) HISTOGRAMS, THEN REDUCE
128 ITEMS · 8 THREADS · 3 BINS
(EACH THREAD GETS 16 ITEMS)
THREAD Ø
1
2
7
BIN Ø
1
2
...


SORT, THEN REDUCE BY KEY
8 ENTRIES
3 BINS
(0,1,2)
← KEYS
← VALUES
① SORT
← KEYS
REDUCE 0s
REDUCE 1s
REDUCE 2s
② REDUCE


PER-THREAD PRIVATIZED (LOCAL) HISTOGRAMS, THEN REDUCE
128 ITEMS · 8 THREADS · 3 BINS
(EACH THREAD GETS 16 ITEMS)
THREAD Ø 1 2 ... 7
BIN Ø
1
2
HOW CAN WE COMBINE THESE 8 LOCAL HISTOGRAMS
INTO ONE GLOBAL HISTOGRAM?


REDUCING 8 LOCAL HISTOGRAMS
Ø 1 2 3 4


FINAL THOUGHTS ON HISTOGRAM
– ATOMICS (2)
– PER-THREAD HISTOGRAMS, THEN REDUCE (1)
– SORT, THEN REDUCE BY KEY
HOW MANY ATOMIC ADDS?
256 THREADS, 8 BINS:
ATOMIC TECHNIQUE:
REDUCE TO 8-ELEMENT HISTOGRAM THEN ATOMICS