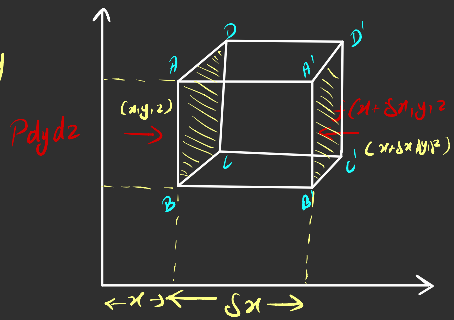
Q1, Conservation of momentum :-

So, X, Y, Z are external force along the x, y, z axis

$\rho \rightarrow$ density of the fluid

$P \rightarrow$ pressure on parallel pip



So, force due to pressure on (ABCD) :- $f(x,y,z)$

$$\Rightarrow \boxed{P\,dydz = f(x,y,z)}$$

and, force due to pressure on (A'B'C'D') :- $f(x+\delta x, y, z)$

So, resultant force due to bothe the sites :-

$$\Rightarrow f(x,y,z) - f(x+\delta x, y, z)$$

$$\Rightarrow -\delta x \frac{df}{dx}$$

now,

$$\frac{df}{dx} = \frac{dp}{dx}\delta y \delta z \quad — (1)$$

on using (1),

$$\Rightarrow \boxed{-\delta x \delta y \delta z \frac{dp}{dx}}$$

now, the external force along x-axis :- $\rho\, dxdydz\, X$

$$\frac{Du}{Dt} = \text{acceleration along } x\text{-axis}$$

now, by newton's 2^{nd} lay

mass x acceleration = total force ($\rightarrow$ x-axis)

$$\Rightarrow \quad \rho\,\delta x\,\delta y\,\delta z \times \frac{Du}{Dt} = X\rho\,\delta x\,\delta y\,\delta z - \delta x\,\delta y\,\delta z\,\frac{dp}{dx}$$

on solving these equation we get.

$$\Rightarrow \quad \frac{Du}{Dt} = X - \frac{1}{\rho}\frac{dp}{dx}$$

now, by using $\Rightarrow \dfrac{D}{Dt} = \dfrac{\delta}{\delta t} + \dfrac{u\,\delta}{\delta x}$

$$\Rightarrow \quad \rho\,\cancel{\delta x\,\delta y\,\delta z} \times u\left(\frac{\delta}{\delta t} + \frac{u\,\delta}{\delta x}\right) = \cancel{\delta x\,\delta y\,\delta z}\left(X\rho - \frac{dp}{dx}\right)$$

$$\Rightarrow \quad \cancel{\rho}\,u\frac{\delta}{\delta t} + \cancel{\rho}\,u^2\frac{\delta}{\delta x} = X\cancel{\rho} - \frac{1}{\rho}\times\frac{dp}{dx}$$

$$\Rightarrow \quad \boxed{X = \frac{1}{\rho}\frac{dp}{dx} + \frac{\delta u}{\delta t} + \frac{u \cdot \delta u}{\delta x}}$$

now, in respective to x, y, z we have, $x \rightarrow u$, $y \rightarrow v$, $z \rightarrow w$

$\rightarrow$ along x-axis $\Rightarrow \dfrac{Du}{Dt} = X - \dfrac{1}{\rho}\dfrac{dp}{dx}$

$$X = \frac{1}{\rho}\frac{dp}{dx} + \frac{\delta u}{\delta t} + \frac{u \cdot \delta u}{\delta x}$$

→ along y - axis ⇒ $\frac{Dv}{Dt} = Y - \frac{1}{\rho}\frac{dp}{dy}$

$$Y = \frac{1}{\rho}\frac{dp}{dy} + \frac{\delta v}{\delta t} + \frac{v \cdot \delta v}{\delta x}$$

→ along z - axis ⇒ $\frac{Dw}{Dt} = Z - \frac{1}{\rho}\frac{dp}{dx}$

$$Z = \frac{1}{\rho}\frac{dp}{dz} + \frac{\delta w}{\delta t} + \frac{w \cdot \delta w}{\delta z}$$