High-level multi-stage optimization for Chagas disease Exploration/Exploitation dilemma

The "Inglorious Bug Hunters"

1 Introduction

T. infestans detection and control are very complex. In this paper, we apply optimization techniques to solve the high-level planning problem. The model attempts to abstract away the targeting of specific houses, and rather consider general strategies that we could use.

2 Model overview

As we discussed over the hike, we could abstractly model the planning problem in the following way:

- there is a weekly budget b for what we could perform.
- time t runs $1, 2$, etc weeks. we could do this for days just as well.
- for simplicity, we divide all the houses into two classes: "high yield" and everything else.
  - High-yield houses are those produced by reliable reports, or next to cases with instation. These are the houses that have a relatively good chance of being infested.
  - H^t =number of "high-yield" houses that we know of at time t. If inspected, such houses "yield" infestations with probability y_h.
  - We start with a basic supply of "high-yield" houses, H^1. Spontaneous reports are effectively like a strategy that produces a lot of high-yield houses.
  - there is also a virtually inexhaustible supply of other houses of which we know nothing, and which yield with probability $y_l \ll y_h$.
  - $y_l = 0.02$, $y_h = 0.2$
- there are multiple "strategies". Abstractly all strategies have the following parameters:

- s_i^t = number of "units of strategy i" performed in week t. There is a minumum ($0 \le s_i$) and a maximum ($s_i \le m_i$).
- u_i =number of high-yield houses inspected for T. infestans per unit of strategy i. A strategy that involves going to one HY house would have $u_i = 1$. Many strategies are not as maximizing, and have a mix of LY and HY.
- v_i =number of low-yield houses inspected ... per unit of strategy i. If strategy i involves visiting one house, then $v_i + u_v = 1$.
- c_i = the unit "cost" of strategy i. In general, there could be various kinds of costs, including time, money, favors and so on. For now, let's assume there is only one resource.
- the gain from a strategy has, abstractly two dimensions:
- $q_i(s_i)$ =expected number of "high-yield" houses that we identify for future visits. In the simplest case, there is a linear relationship: $= q_i s_i$.

- types of strategies

  1. advertise on the radio
  2. do active grid-based surveillance
  3. visit "high-yield" houses
  4. visit 2 hops from a source of an infestation

Some representative values:

#	Strategy	Costc_i	HY Visits u_i	LY Visits v_i	New HY (q_i)	m_i	
1	Radio ad	10	0	0	20	1	
2	Grid survey	1	0.1	0.9	0.1	∞	
3	Visit known high yield	1	1	0	2	∞	
4	Visit 2 hops from infestation	1	0.5	0.5	1	∞	
5	Coerce houses missed in campaign	2	0.9	0	0	∞	
6	Follow up on passive reporting	15	10	5	20	1	

- "Collect passive reporting" assumes that we receive information about 15 houses and must visit all of them. 10 of the houses are HY, and 5 are low yield.
- Generally, "exploration" strategies are those that involve a low number of visits to HY houses.
- Notice that some strategies might be dominated, b/c they have higher cost and lower yields than other strategies. Therefore, they would never be chosen in an optimal solution.

3 Optimization Formulation

The simplest model is the two stage (two weeks) model. This model is not practically very useful, but it would help us to formulate the many-stage model:

$$\max \sum_{t\in\{1,2\}} \sum_i (y_h u_i + y_l v_i) s_i^t$$

subject to all of the following:

$$\begin{aligned}
\sum_i c_i s_i^1 &\leq b \text{ the budget constraint for week 1}\\
\sum_i c_i s_i^2 &\leq b \text{ the budget constraint for week 2}\\
H^0 - \sum_i u_i s_i^1 &= H^1, \quad \text{our starting supply of HYs}\\
H^{t-1} + \sum_i q_i s_i^{t-1} - \sum_i u_i s_i^{t-1} &= H^t, \quad t = 2 \text{ we can only visit HYs if we have previously found them}\\
0 &\leq H^t \leq \infty, \quad t = 1, 2\\
0 &\leq s_i^t \leq m_i \quad \forall i, t
\end{aligned}$$

- This problem seems computationally easy to solve, and could by done with a very long look-ahead. It should be still doable even if we have non-linear functions like $q_i(s_i^1)$ instead of $q_i s_i^1$.
- Notice that exploration increases the stock of high-yield houses, which is then drained by exploration.
- The version with the long look-ahead is an immediate generalization of those equations.
- The model could be dry-runned:
  - abstractly
  - using ground-truth data, and integrated with the risk-map simulator

We can express this problem as a linear integer program [1] and solve using the the R optimization package.

4 Results

For a simple experiment, we solve the 4-week model with a budget of 5, 25, 120. The starting number of HY sites is 3.

Budget=5. The final objective value is 3.64, which refers to the expected number of infested sites found over 5 weeks.of searching. The strategy map is as follows. In the map, entry m_{it} for row of strategy i indicates the number of times that strategy is used on week t.

#		Costc_i		Week 1	Week 2	Week 3	Week 4	
1	Radio ad	10						
2	Grid survey	1						
3	Visit known high yield	1		1	5	5	5	
4	Visit 2 hops from infestation	1		4				
5	Coerce houses missed in campaign	2						
6	Follow up on passive reporting	15						
	Remaining bag of HY sites	n/a		0	1	6	11	

Budget=25. Objective value: 15.82

#		Costc_i		Week 1	Week 2	Week 3	Week 4	
1	Radio ad	10		1				
2	Grid survey	1						
3	Visit known high yield	1			25	25	25	
4	Visit 2 hops from infestation	1		4				
5	Coerce houses missed in campaign	2						
6	Follow up on passive reporting	15						
	Remaining bag of HY sites	n/a		0	0	25	50	

Budget=120. Objective value: 56.24

#		Costc_i		Week 1	Week 2	Week 3	Week 4	
1	Radio ad	10		2	2			
2	Grid survey	1			10			
3	Visit known high yield	1				100	100	
4	Visit 2 hops from infestation	1		6	90	5	5	
5	Coerce houses missed in campaign	2						
6	Follow up on passive reporting	15						
	Remaining bag of HY sites	n/a		0	0	18.5	131.0	

5 Discussion

- Initially (week 1 or 2), the solver indicates using strategies that generate a lot of high-yield sites. In subsequent weeks those sites are exploited.
- If the budget is large, one can use the highly-efficient strategy of a radio ad.
- A problem with the existing formation is that the exploitation strategy seems to dominate other strategies in later rounds. This is because of the assumption that it generates high returns as well as lots of other sites that are high-yield
- In reality, the returns from exploitation decline because the hotspot/focus is exhausted. We need to find a way of modeling this exhaustion. This is

a spatial phenomenon. We might divide the HY sites into "freshly-found" vs. "near other HY sites". The latter generate only few new HY sites.

- Pitfalls:
  - most importantly, solving this would not give necessarily the best houses to visit, only the right strategy mixture. This targeting problem might be done by eye-balling, or with a separate model.

6 Extensions

- The strategy mixture would change with time. Initially, the strategies would focus on "exploration"/"lead generation", and in later stages on exploitation.
- We could introduce a Bayesian update steps to the constants, such as yield.
- Even though the optimization has multiple times, after completing one week, we recompute this with the new data. For example, we could solve this for $t = 1, 2, 3, \dots, T$, then do one week of work, and compute it for $t = 2, 3, 4, \dots T$. When we recompute, we update the variables and parameters based on actual work performed and detection results. For example, if we inspected a HY house and detected nothing, then we could update the risk map, and update the H based on the new data.
- Hotspots tend to be exhausted. To model that, we create a separate bag of high-yield house for each hotspot.
- We could consider L^t =number of "low-yield" houses, and M^t, medium-yield that we know at time t. More broadly, we don't even need to discretize the yield of the houses. rather we could say that every strategy produces a distribution of yields, d_i (this is a function, or at least a vector for the expected number of houses of each quality), per unit effort, which is added to the stock we have access to: $D^{t+1} = D^t + s_i d_i$.
- We could introduce a penalty for delaying visits to HY house. This would prevent "hoarding" of targets without following them soon, which would piss off people who report them (reduce cooperation?). Need to think how to best write equations for this.

7 Extension to account for space

We need to introduce space to answer two questions:

- the selection of the specific houses among high yield and low yield
- the difference in potential for new "high yield" uncovered

The general idea:

References

[1] Laurence A Wolsey. *Integer programming*, volume 42. Wiley New York, 1998.