Jewell Approach

November 12, 2014

The overall goal is to use active surveillance to target high-risk households, which is thought to be more efficient than random targeting. However, it is unclear whether this would be more efficient than treating houses around an infected house. Below I have summarized the approach along with questions that remain.

1. Data needed
   - Covariate data
     - location
   - Observed data
     - detection of bugs at time points $(N(t)_i)$
     - spray time of houses $(R(t)_i)$
   - Unobserved data
     - infection times of notified houses $(I(t)_i)$
     - infected houses that are not yet detected

   *we also have known uninfected: In their model, S is unobserved but in ours it is partially observed

   Houses are either

   - Susceptible (S): not infected (partially observed)
   - Infected (I): infected (unobserved)
   - Notified (N): known infected (observed)
   - Recovered (R): known treated (observed)

   Transmission rate between infected i and susceptible j and observed (notified) i and susceptible j:

$$\beta_{ij} = \begin{cases} \text{hop rate } \lambda * (1 - \phi) & \text{if } \rho_{ij} < C \\ \text{jump rate } \lambda * \phi & \text{if } \rho_{ij} > C \end{cases}$$

   where λ is the number of dispersal events per house, and ϕ is the proportion of jumps. For now, we assume $C = 30$ meters. We can use estimates from the synethic likelihood approach: $\phi = 0.188$ and $\lambda = 0.0102$ (invasions per occupied house per week).

where ρ_{ij} is the distance between houses i and j in meters.

They include time-dependent infectivity function $h(\cdot)$ describing how the infectivity increases as the length of infection increases. Let $K_i(t)$ be the number of bugs observed in the ith house at time t. I think it makes sense to think of $h(\frac{K_i(t)}{max(K_i(t)})$, with the outcoming ranging from 0 to 1 (fractions of max infectivity of 1).

There are several models used to estimate population growth rates:

- Ricker model:

$$N_{t+1} = N_t * exp(r(1 - \frac{N_t}{k}))$$

  where r is the intrinsic growth rate and k is the carrying capacity.

- Hassell model:

$$N_{t+1} = \frac{a * N_t}{(1 + b * N_t)^c}$$

- Beverton-Holt model:

$$N_{t+1} = \frac{a * N_t}{1 + b * N_t}$$

  This is the same as the Hassell model with $c = 1$.

Thus, total infection pressure becomes

$$\tau_j = \beta_0 + \sum_{I_i < I_j < R_i} \beta_{ij} \cdot h(I_j - I_i)$$

2. Likelihood

Rate of notification conditional on an infection is $F_D(d)$ =???
f_D is the distribution of $D_i = N_i - I_i$.

In homogenous mixing Markov model, $f_d \sim exp(\lambda)$. $F_D(d) = \int_d^\infty f_D(y)dy$ (as far as my understanding – this should be the cdf but seems to be 1- cdf). Jewell uses a variation of the exponential distribution. In our case, this is some function of the number of bugs and time of notification.

$$f(D_i) = f(\text{\# bugs observed at } N_i)$$

$$\begin{aligned} f(\mathbf{I}, \theta | \mathbf{N}, \mathbf{R}) &= \prod_{j=1, j \neq k}^{[\mathbf{I}]} \big(\tau_j(I_j^-)\big) exp\Big(- \int_{I_k}^{T_{obs}} \Big(\sum_{j=1 j \neq k}^{[\mathbf{S(T_{obs})}]} \tau_j(t)\Big) dt \Big) \\ &\times \prod_{j=1}^{m} f_D(N_j - I_j) \times \prod_{j=m+1}^{[\mathbf{I}]} (1 - F_D(T_j - I_j)) \times prior \end{aligned}$$

where

$$T_j = \begin{cases} T_{obs} & \text{if j is currently presumed susceptible but is unknowingly infected} \\ C_j & \text{if j is sprayed before infestation status is known} \end{cases}$$

$k =$ initial infection (the likelihood is conditional on index case). We don't know the index case, so how do we handle? Either epicenter regression or use 2010 data as index cases. I haven't seen a situation with multiple first cases, so not sure if second option would work.

What about prior? The Jewell paper uses Gamma prior for θ. Synthetic likelihood results for informative prior?

We are interested in R_i. Define R_i as the expected number of further premises a premises i would infect were it the index infection in a hypothetical infection where all other houses started as susceptibles, conditional on all parameters.

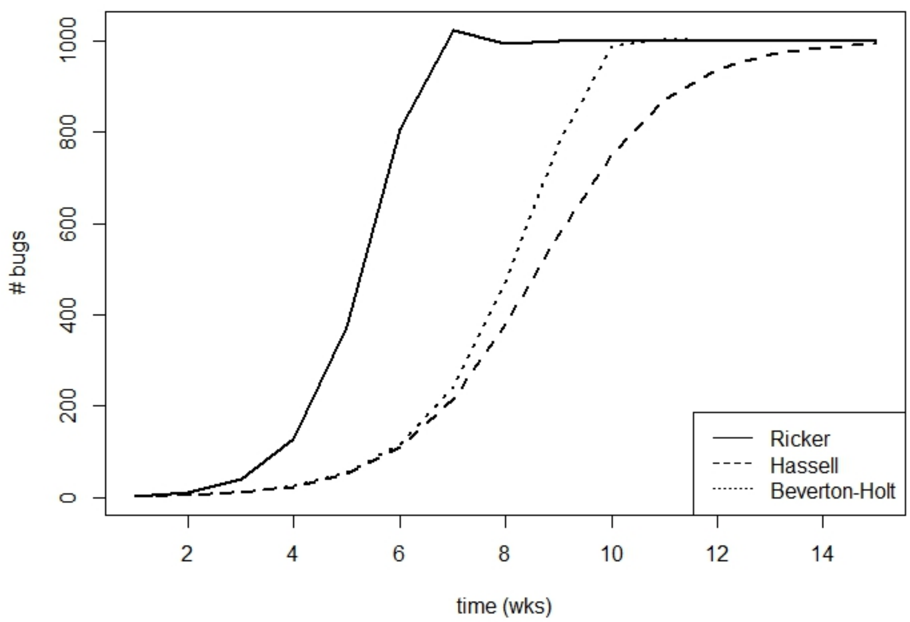

Figure 1: Models when $k = 1$ and $r = 1.23$ (estimated from Rabinovich, 1972)