Algorithm for Calculating the Cost of Surveillance Based on the Single Server Queueing Model and a Stepwise Constant Arrival Rate Function

MTR - Sept. 29, 2014

This algorithm is based on the queueing model of the StochMod 2014 presentation by Rieders and Levy [1]. The calculations follow the approach by Brill [2]. Please, note that all rates need to refer to the same time period.

[1.] Rieders, Maria and Michael Levy, *Analysis of Household Participation in a Public Health Intervention Program for Eliminating the Spread of an Epidemic Disease*,Presented at EURO Working Group on Stochastic Modeling,Mannheim/Germany, June 2014.

[2.] Brill, P. H., *Single-Server Queues with Delay-Dependent Arrival Streams*, Probability in the Engineering and Informational Sciences, **2**, 1988, 231-247.

Input:

- M number of discretization points for arrival rate;
- $\{a_j,\ j = 1, \ldots, M\}$ age levels;
- $\{\beta_j,\ j = 0, \ldots, M\}$ redispersion rates for each given age level;
- α rate of infection in attack zone;
- c participation rate;
- p prevalence of infection;
- μ service rate.

Output:

- the expected number of houses treated per time period.

Note: we can also calculate the distribution of the amount of time a house will spend waiting for and receiving treatment.

Calculations:

1. Initialization
   Set $\alpha' \leftarrow \alpha \times (1-c) \times p$ (residual infestation rate).
   For $j = 0, \ldots, M$, set $\lambda_j \leftarrow \alpha' + \beta_j$ (combined arrival rates).
   Set $a_0 \leftarrow 0$.
   Set $b_M \leftarrow 1$; $B_M \leftarrow (e^{-\mu a_M})/\mu$ (Equation (3.14) in [2]).
   Set $G_M \leftarrow \frac{e^{-(\mu-\lambda_M)a_M}}{(\mu-\lambda_M)}$.

2. For $j = M-1, \ldots, 0$, calculate

   a. (Equation (3.17) in [2])

$$\begin{aligned} b_j \leftarrow & -\mu \left(\frac{\lambda_{j+1}-\lambda_j}{\lambda_{j+1}-\lambda_j+\mu} \right) e^{(\lambda_{j+1}-\lambda_j+\mu)a_{j+1}} B_{j+1} \\ & + \mu \sum_{k=j+2}^{M} \left[\frac{\lambda_k-\lambda_{j+1}}{\lambda_k-\lambda_{j+1}+\mu} - \frac{\lambda_k-\lambda_j}{\lambda_k-\lambda_j+\mu} \right] e^{(\lambda_k-\lambda_j+\mu)a_{j+1}} B_k \\ & + e^{(\lambda_{j+1}-\lambda_j)a_{j+1}} b_{j+1}; \end{aligned}$$

   b. (Equation (3.13) in [2])

$$B_j \leftarrow \mu \sum_{k=j+1}^{M} \left(\frac{e^{(\lambda_k-\lambda_j)a_{j+1}} - e^{(\lambda_k-\lambda_j)a_j}}{\lambda_k-\lambda_j+\mu} \right) B_k + \left(\frac{e^{-\mu a_j} - e^{-\mu a_{j+1}}}{\mu} \right) b_j;$$

   c. (Equation (3.15) in [2])

$$G_j \leftarrow \mu \sum_{k=j+1}^{M} \frac{(\lambda_k-\lambda_j)(e^{\lambda_k a_{j+1}} - e^{\lambda_k a_j})}{(\lambda_k-\lambda_j+\mu)\lambda_k} B_k + \left(\frac{e^{-(\mu-\lambda_j)a_j} - e^{-(\mu-\lambda_j)a_{j+1}}}{\mu-\lambda_j} \right) b_j.$$

3. Calculate the normalization constant

$$c_M \leftarrow \left(\sum_{j=0}^{M} \frac{\mu B_j}{\lambda_j} + G_j \right)^{-1}.$$

4. Calculate expected number of treated houses per time period

$$\sum_{j=0}^{M} \lambda_j \times G_j \times c_M.$$