

## 21-242 HW 2

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Due September 8, 2025

**Question 1.** Consider  $\mathbb{R}^2$  as a vector space over  $\mathbb{R}$ , and let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a linear transformation of this vector space. Prove that if  $\vec{w}$  is the midpoint (viewing  $\mathbb{R}^2$  as the Euclidean plane) of  $\vec{v}$  and  $\vec{u}$ , then  $T(\vec{w})$  is the midpoint of  $T(\vec{v})$  and  $T(\vec{u})$ .

**Question 2.** Considering again  $\mathbb{R}^2$  as a vector space over  $\mathbb{R}$ , suppose  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is a linear transformation satisfying

$$T \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \text{and} \quad T \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$$

1. How do you know such a transformation  $T$  exists?
2. What would  $T \begin{pmatrix} 9 \\ 10 \end{pmatrix}$  be? Prove your answer.