

21-242 HW 2

Instructor: Wesley Pegden

Due September 8, 2025

Question 1. Consider \mathbb{R}^2 as a vector space over \mathbb{R} , and let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation of this vector space. Prove that if \vec{w} is the midpoint (viewing \mathbb{R}^2 as the Euclidean plane) of \vec{v} and \vec{u} , then $T(\vec{w})$ is the midpoint of $T(\vec{v})$ and $T(\vec{u})$.

Question 2. Considering again \mathbb{R}^2 as a vector space over \mathbb{R} , suppose $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation satisfying

$$T\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \text{and} \quad T\left(\begin{pmatrix} 5 \\ 6 \end{pmatrix}\right) = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$$

1. How do you know such a transformation T exists?
2. What would $T\left(\begin{pmatrix} 9 \\ 10 \end{pmatrix}\right)$ be? Prove your answer.