Determinants of Off-Diagonal Triangular Matrices

Consider a matrix of the following consisting of only nonzero elements on the diagonal and off-diagonal:

$$A = \begin{bmatrix} a_{11} & a_{12} & & & \\ a_{21} & a_{22} & a_{23} & & \\ & a_{32} & \ddots & a_{(n-2)(n-1)} & \\ & & & a_{(n-1)(n-1)} & a_{(n-1)n} \\ & & & a_{n(n-1)} & a_{nn} \end{bmatrix}.$$

Problem. What is the determinant of A?

Solution. We will derive a recurrence for the diagonal elements d_i after having row reduced to eliminate the values below the diagonal. Following this row reduction, the determinant will simply be

$$\det A = d_1 d_2 \cdots d_n.$$

Our initial condition shall be that $d_1 = a_11$. Observe then that

$$d_i = a_{ii} - a_{i(i-1)} \cdot \frac{a_{(i-1)i}}{d_{i-1}},$$

which suffices for our recurrence.

If one of the d_i values are 0, then the determinant doesn't exist (?). Actually this isn't true hmmmm I suppose we could row swap up in this case.