

Codeforces Problem 1967A

Problem. Let n be some fixed natural number. We are given frequency counts of integers 1 to n as well as the choice of k more counts.

Let the *score* of some arrangement of numbers denote the number of subarrays of the arrangement that are a permutation of the numbers 1 to n and let counts be some arrangement of the given and chosen numbers as mentioned above. Find the maximum score of counts for which it is optimally chosen.

Solution. While I shall not try to prove that it is the most optimal as that would take a lot of thinking, we have decent intuition to believe that a sliding window technique achieves the highest score. Let $c[i]$ denote the frequency counts given to us and $s[i]$ denote the additional k chosen counts. We then have the following claim.

Claim. Let the optimal maximal minimal shared value obtained by choice of $s[i]$ be

$$\text{value} = \max_{s[i]} \min_i (c[i] + s[i]).$$

We then have that the maximal score is

$$1 + (n - 1)(\text{value}) + \varepsilon,$$

where ε is a residual term that may range from 0 to $2n - 1$ inclusive.