Introduction to Deontic Logic: A Beginner's Tutorial

Deontic Logic is a branch of modal logic that deals with normative concepts such as obligations, permissions, and prohibitions. It is widely used in fields like ethics, law, and artificial intelligence to formalize reasoning about rules and responsibilities.

Key Concepts in Deontic Logic

1. **Obligation (O):** Represents that an action is obligatory.
   Example: *O(A)* means "Action A is obligatory."
2. **Permission (P):** Represents that an action is permitted.
   Example: *P(A)* means "Action A is permitted."
3. **Prohibition (F):** Represents that an action is forbidden.
   Example: *F(A)* means "Action A is forbidden."
4. **Negation (¬):** Used to represent the opposite of a statement.
   Example: ¬*O(A)* means "Action A is not obligatory."

Basic Syntax and Semantics

Deontic Logic uses the standard propositional logic syntax with additional modal operators:

Symbol	Meaning	Example
O(A)	A is obligatory	*O(pay_taxes)*
P(A)	A is permitted	*P(vote)*
F(A)	A is forbidden	*F(steal)*
¬	Negation	¬*O(lie)* (It is not obligatory to lie)

Deontic Axioms and Rules

1. **Obligation and Prohibition Relationship:**
   $O(A) \leftrightarrow \neg P(\neg A)$
   If action *A* is obligatory, then not doing *A* is not permitted.
2. **Permission and Obligation Relationship:**
   $P(A) \rightarrow \neg O(\neg A)$
   If action *A* is permitted, then it is not obligatory to refrain from *A*.
3. **Prohibition Definition:**
   $F(A) \leftrightarrow O(\neg A)$
   An action is forbidden if and only if refraining from it is obligatory.

Example Use Case: Traffic Rules

Scenario:

- **Rule 1:** Stopping at a red light is obligatory. (*O(stop_at_red)*)

- **Rule 2:** Running a red light is forbidden. (*F(run_red)*)
- **Rule 3:** Turning right on red is permitted unless otherwise posted. (*P(turn_right_on_red)*)

Formalization:

1. *O(stop_at_red) → F(run_red)*
   If stopping is obligatory, running the light is forbidden.
2. *P(turn_right_on_red) → ¬O(stop_completely)*
   If turning right is permitted, stopping completely may not be obligatory.

Querying Deontic Logic:

- Is it allowed to run a red light?
  Answer: ¬P(run_red) (Not permitted, based on the rules.)

Practical Applications

- **Ethics and Legal Reasoning:** Model complex legal systems to derive whether actions are permissible.
- **Artificial Intelligence:** Design autonomous agents that follow ethical guidelines.
- **Policy Compliance Systems:** Automate rule-checking and compliance verification in organizations.

By using Deontic Logic, one can reason about complex normative scenarios systematically, making it a powerful tool in formal logic and applied fields.