A Unified View of Frequency Estimation and their Attacks on Local Differential Privacy

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Outline

Introduction:
- Differential Privacy
- Local Differential Privacy
- Pure LDP Framework
- Attack Problem

Frequency Estimation Techniques:
- RAPPOR
- K Randomized Response (kRR)
- Optimized Unary Encoding (OUE)
- Optimized Local Hashing (OLH)

Attacks:
- General Attack Formulation
- Attacking kRR
- Attacking OUE
- Attacking OLH

Evaluation:
- Comparison between Estimators
- Gain from Attacks
- Impacts of Parameters on Attacks

Conclusion:

Introductory Concepts

Differential Privacy

Dataset, $\mathcal{D}$	
Features	$x_1, x_2, \dots, x_n$
Labels	$y_1, y_2, \dots, y_m$

Subset of Labels	
S	y_1, y_4, y_{11}

$$\Pr\left[\mathrm{M}\left(\begin{array}{c} \boldsymbol{x} \\ \hline sample_1 \\ sample_2 \\ sample_3 \\ sample_4 \\ sample_5 \end{array} \right) \in \mathrm{S} \right] \leq e^{\epsilon} \Pr\left[\mathrm{M}\left(\begin{array}{c} \boldsymbol{y} \\ \hline sample_1 \\ sample_2 \\ sample_3 \\ sample_4 \\ sample_{11} \end{array} \right) \in \mathrm{S} \right] + \delta$$

Local Differential Privacy

$$\Pr\left[\mathrm{M}\left(\begin{array}{|c|}\hline x \\ \hline sample_1 \\ \hline\end{array}\right) \in \mathrm{y}\right] \leq e^{\epsilon} \Pr\left[\mathrm{M}\left(\begin{array}{|c|}\hline y \\ \hline sample_{11} \\ \hline\end{array}\right) \in \mathrm{y}\right]$$

Protocols for Local Differential Privacy

Encode

Rice
Bean
Soda

Rice	Bean	Eggs	Soda	Oreo	Taco
1	1	0	1	0	0

Perturb

1	1	0	1	0	0

1	0	0	1	1	0

Aggregate

1	0	0	1	1	0
1	0	1	1	0	1
0	0	0	1	1	0

2	0	1	3	2	1

Frequency Estimation Problem

Pure Differential Privacy

- A unified framework --- Can be used to define all frequency protocols.

- Defined based on two fixed probabilities p^* and q^*

$$\Pr[PE(v_1) \in \{y \,|v_1 \in Support(y)\}] = p^*$$
$$\Pr[PE(v_2) \in \{y \,|v_1 \in Support(y)\}] = q^*$$

- Based on this framework, we will describe 4 protocols – RAPPOR, kRR, OUE, OLH.

Attacking Estimators – Problem Formulation

- Assume: $n\ real\ users$
- Inject: $m\ fake\ users$
- To increase frequency of r items: $T = \{t_1, t_2, \dots, t_r\}$

- Goal:

$$\Delta\tilde{f}_t = \tilde{f}_{t,after} - \tilde{f}_{t,before} \qquad \forall t \in T$$

$$\max_{Y} \sum_{\{t \in \mathrm{T}\}} \mathbb{E}\left[\Delta\tilde{f}_t\right]$$

Foundational Work

Google's RAPPOR

Encode

$\mathcal{H} = \{H_1, H_2, \ldots, H_m\}$

0	1	0	0	0	0

$B_0[i] = 1 \quad if\ \exists H \in \mathcal{H}, s.t., H(v) = i$

Perturb

0	1	0	0	0	0

0	1	0	0	1	0

$$\Pr[B_1[i]] = 1 = \begin{cases} p = 1 - \frac{f}{2} & if\ B_0[i] = 1 \\ q = \frac{f}{2} & if\ B_0[i] = 0 \end{cases}$$

Aggregate

- Using Linear Regression
- Using LASSO Regression

Limitations of RAPPOR

- Use of Bloom filter reduced communication cost

However,

- Accuracy decreased significantly
- Computation cost of the aggregation step increased significantly

SOTA Frequency Estimators

K Randomized Response

Encode

A	B	C	D	E	F

A	B	C	D	E	F

Perturb

A	B	C	D	E	F

A	D	C	C	F	E

$$\Pr[PE(v) = i] = \begin{cases} p = \dfrac{e^{\epsilon}}{e^{\epsilon} + d - 1} & if\ i = v \\ q = \dfrac{1}{e^{\epsilon} + d - 1} & otherwise \end{cases}$$

Aggregate

$$\tilde{f}_v = \frac{\frac{1}{n}\sum_{\{i=1\}}^{n} \mathbb{I}_{\{Support(y_i)\}}(v) - q^*}{p^* - q^*}$$

For kRR,

$$Support(y_i) = \{y\}$$

Optimal Unary Encoding

Encode

Item

$\downarrow$

0	0	1	0	0	0

Perturb

0	0	1	0	0	0

$\downarrow$

0	0	1	0	1	0

$$\Pr[PE(v) = 1] = \begin{cases} p = \frac{1}{2} & if\ i = v \\ q = \frac{1}{e^{\epsilon} + 1} & otherwise \end{cases}$$

Aggregate

$$\tilde{f}_v = \frac{\frac{1}{n}\sum_{\{i=1\}}^{n} \mathbb{I}_{\{Support(y_i)\}}(v) - q^*}{p^* - q^*}$$

For OUE,

$$Support(y_i) = \{v \mid v \in [d]\ and\ y_v = 1\}$$

Optimal Local Hashing

Encode

$\mathcal{H} = \{H_1, H_2, \dots, H_m\}$

H_3

0	0	1	0	0	0

Perturb

0	0	1	0	0	0

0	0	0	0	0	0

$$\Pr[y = \langle H, x \rangle] = \begin{cases} p = \dfrac{e^{\epsilon}}{e^{\epsilon} + d - 1} & if\ x = i \\ q = \dfrac{1}{e^{\epsilon} + d - 1} & otherwise \end{cases}$$

Aggregate

$$\tilde{f}_v = \frac{\frac{1}{n} \sum_{\{i=1\}}^{n} \mathbb{I}_{\{Support(y_i)\}}(v) - q^*}{p^* - q^*}$$

For OLH,

$Support(y_i)$
$= \{v \mid v \in [d]\ and\ H(v) = x\}$

Maximum Gain Attacks on Frequency Estimators

Frequency Estimators Under Attack

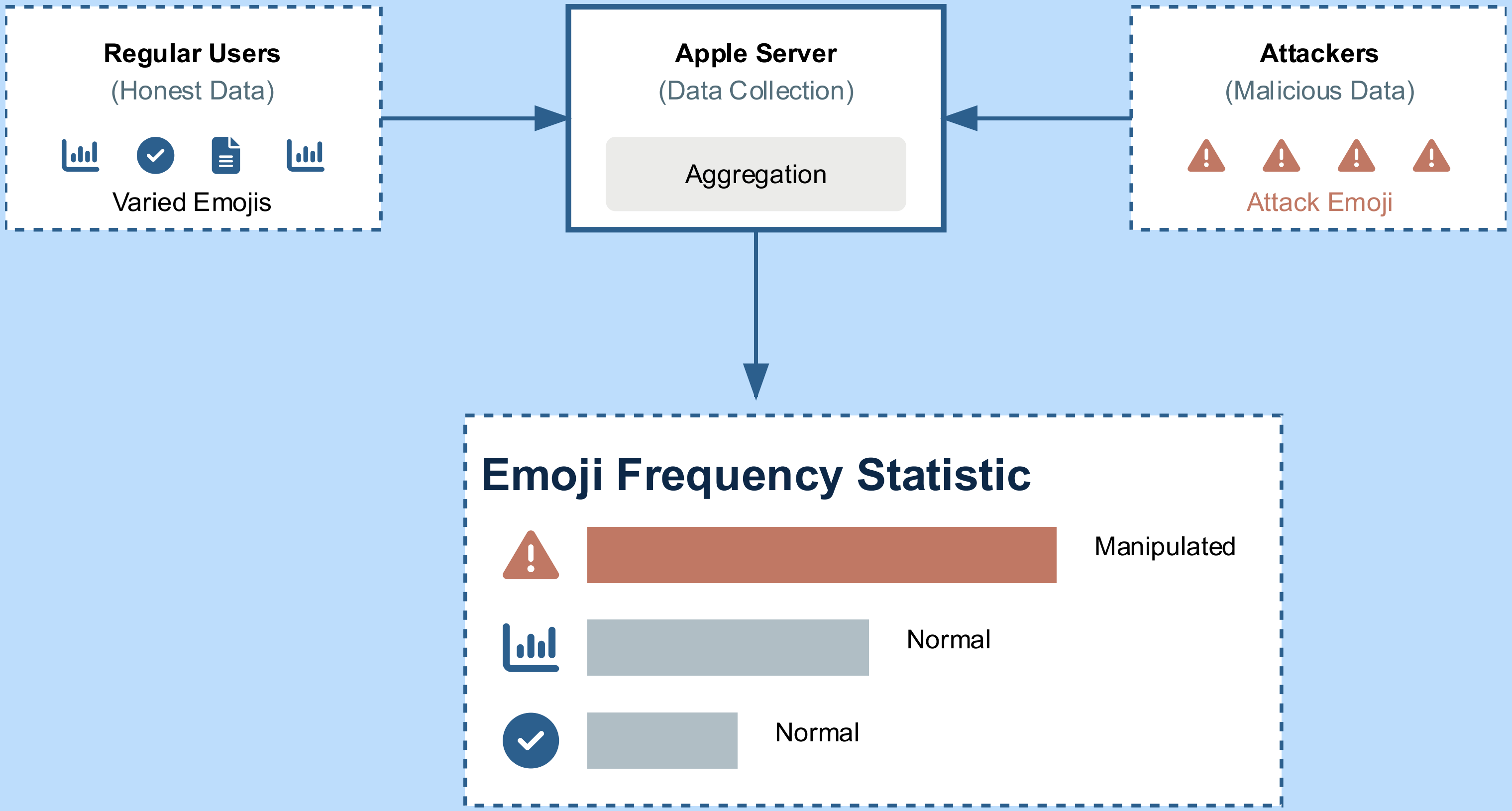


Illustration: Apple introduced LDP to estimate popular emojis in 2016

Frequency Gain from an Attack

General Formula:

$$G = \frac{\sum_{\{i=n+1\}}^{\{n+m\}} \mathbb{E}\left[\mathbb{I}_{Support(y_i)}(t)\right]}{(n+m)(p^* - q^*)} - \frac{m \sum_{\{i=1\}}^{\{n\}} \mathbb{E}\left[\mathbb{I}_{Support(y_i)}(t)\right]}{n(n+m)(p^* - q^*)}$$

(Accounts for Fake Users' Impact)

(Accounts for Genuine Users' Impact Dilution)

MGA tries to maximize this gain

Maximum Gain Attack: Overview

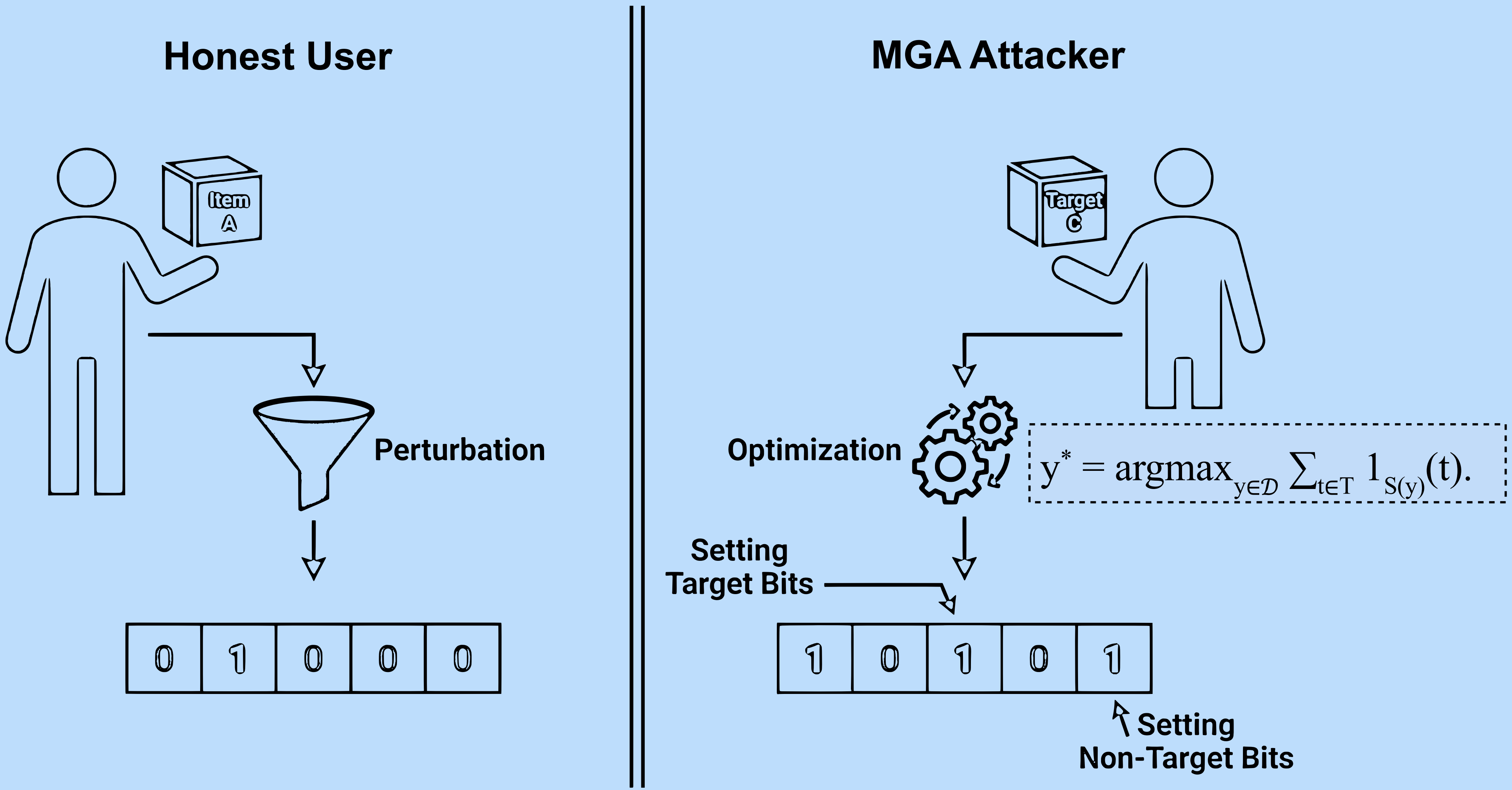


Illustration: MGA attack on the OUE protocol

Attacking kRR

- The support set is:

$$\sum_{\{t \in T\}} \mathbb{I}_{Support(y_i)}(t) = 1$$

- Gain becomes:

$$G = \frac{m}{(n+m)(p^* - q^*)} - c$$

- Plugging in p^* and q^* gives:

$$G = \beta(1 - f_T) + \frac{\beta(d-r)}{e^\epsilon - 1}$$

Attacking OUE

- The support set is:

$$\sum_{\{t \in T\}} \mathbb{I}_{Support(y_i)}(t) = r$$

- Gain becomes:

$$G = \frac{rm}{(n+m)(p^* - q^*)} - c$$

- Plugging in p^* and q^* gives:

$$G = \beta(2r - f_T) + \frac{2\beta r}{e^{\epsilon} - 1}$$

Attacking OLH

- The support set is:

$$\sum_{\{t \in T\}} \mathbb{I}_{Support(y_i)}(t) = r$$

- Gain becomes:

$$G = \frac{rm}{(n+m)(p^* - q^*)} - c$$

- Plugging in p^* and q^* gives:

$$G = \beta(2r - f_T) + \frac{2\beta r}{e^{\epsilon} - 1}$$

Evaluation

Estimators: Utility vs. Security

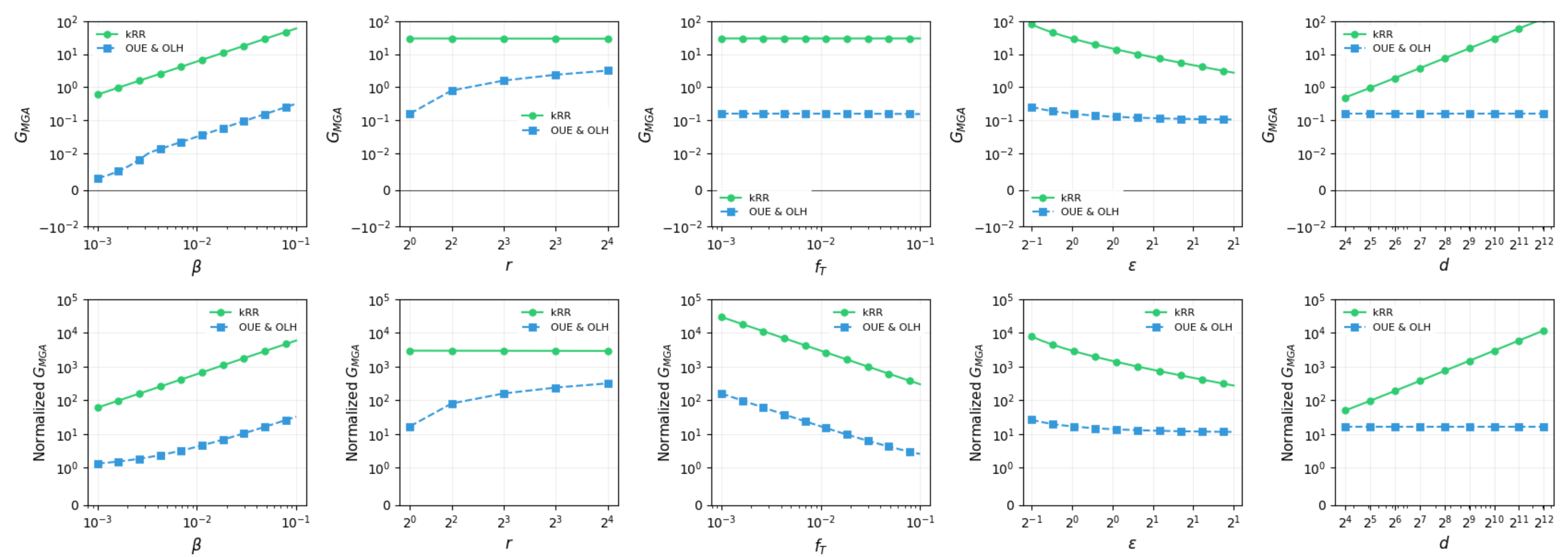
	kRR	OUE	OLH
Communication Cost	$\theta(\log d)$	$\theta(d)$	$\theta(\log n)$
Variance	$n \cdot \frac{d-2+e^{\epsilon}}{(e^{\epsilon}-1)^2}$	$n \cdot \frac{4e^{\epsilon}}{(e^{\epsilon}-1)^2}$	$n \cdot \frac{4e^{\epsilon}}{(e^{\epsilon}-1)^2}$
Gain of MGA	$\beta(1-f_T)+\frac{\beta(d-r)}{e^{\epsilon}-1}$	$\beta(2r-f_T)+\frac{2\beta r}{e^{\epsilon}-1}$	$\beta(2r-f_T)+\frac{2\beta r}{e^{\epsilon}-1}$

n = number of real users
m = number of fake users
$\beta = \frac{m}{n+m}$ = fraction of fake users

d = domain size
T = set of target items
$r = |T|$

$$f_T = \sum_{\{t \in T\}} f_t$$

MGA Comparison



Conclusion

Summary and Open Questions

- Different Frequency Estimation Protocols under LDP
- Attacks on Estimators
  - kRR performs poorly when the domain size increases
  - OUE, OLH perform poorly when the target items increases
- Open Question:
  - Large domain, Large target items
  - How to handle complex data types
  - Utility – Privacy trade-offs

Thank You